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A SIMPLE MODEL OF EQUILIBRIUM PRICE DISPERSION

by

Jennifer F. Reinganum

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Jennifer F. Reinganum*

Northwestern University, Evanston, IL 60201

1. Introduction

Since the publication of Stigler's seminal article "The Economics of Information" (15), many authors have examined optimal search strategies for agents facing stochastic prices, wages or demand conditions (6,7,10,11,12). While the models developed were insightful analyses of the problems of individual agents, most have suffered from the fact that the price dispersion upon which they are predicated disappears when the strategies are implemented in a simple market setting (see 10 for examples). It will be shown below that the reasons for these failures lie in part with the strategies themselves and in part with the market settings in which they are typically placed. Specifically, we are interested in demonstrating that price dispersion may exist even within the framework of a very simple model. Furthermore, the persistence and degree of price dispersion in this particular model will be shown to depend crucially upon two concepts that are conspicuously absent from most search models: the existence of differing marginal costs among firms and the nonzero elasticity of consumers' demand curves.

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As Butters (1) correctly noted, some deviation from the deadly "simplest model"--wherein identical consumers with unitary demand search sequentially, at a fixed cost, over identical monopolistically competitive firms--is required in order to obtain a nondegenerate price distribution. That is, imperfect information <u>alone</u> is insufficient to support price dispersion. Several recent papers have presented models which include such deviations: e.g., Wilde (16), using nonsequential search; and Salop ((14), where the supply side consists of a monopolist rather than competing sellers--with interesting results.

This paper, in extremely simple terms and with considerable generality, focuses on and demonstrates the existence of price dispersion for a model which is remarkable similar to the "simplest" one. Specifically, the model which follows differs from the "simplest model" in but two respects:

1) the peculiar assumption of unitary demand is abandoned; buyers are assumed to purchase goods according to their (elastic) demand curves.

2) firms are not identical; each has constant marginal costs and each value of marginal costs occurs with some frequency among the firms in the industry.

These assumptions, along with the remaining elements of the "simplest model" (and some technical assumptions), will be shown to be sufficient to support a nondegenerate distribution of product prices in equilibrium.

2. <u>Buyers' Behavior</u>

Assume that there is a continuum of identical buyers, each possessed of a strictly quasi-concave, twice continuously differentiable utility function over commodities. Further assume that buyers' demand functions for all commodities are defined , downward sloping in their own prices and continuously differentiable. The utility function can then be rewritten in terms of prices and wealth as $U(W,p_0,p)$ where W denotes wealth, p_0 is the vector of fixed prices for all other goods and p is the price paid by the buyer for the "product," to be defined below. We will make the further simplifying assumption that the indirect utility function $U(W,p_0,p)$ can be written as $V(p_0,p) + W$, where V is strictly decreasing in its second argument. The assumption of additive separability in wealth does not appear to be strictly necessary but it allows easy transition from the more familiar formulae of Rothschild (10,12) and DeGroot (3).(Compare Eqn.s 1 and 1' below).

Buyers are assumed to have perfect information in the markets for all commodities except one, hereafter referred to as the "product" market. The product market is characterized by a distribution of prices. Each consumer, in attempting to maximize the expected utility of his limited wealth, engages in search behavior in the product market. Insofar as this activity increases expected utility, buyers attempt to ascertain a minimum price for the product.

Rothschild (11) has discussed the case in which buyers do not know the distribution of prices from which they are sampling. If buyers are allowed sampling with recall (that is, they are allowed to choose any one among the prices they have been quoted), then the optimal search procedure is not significantly different from the one obtained when buyers know the distribution of product prices. Therefore it will be assumed that buyers sample with recall from a known price distribution. Denote this distribution by $F(p)$ where $F(.)$ is continuously differentiable almost everywhere and has positive density on the closed interval $[\underline{p},\overline{p}]$ for some $\underline{p}$ and $\overline{p}$ ($> \underline{p}$) in R_+.

Since nonsequential search is optimal only if there are economies of scale in sampling, it will be assumed that each buyer follows a sequential search strategy, soliciting price quotations so long as the expected increase in utility from doing so is positive. More precisely, if the fixed sampling cost per observation is k, and if the lowest price encountered on the first n searches is S, then the expected gain in utility from searching once more is

$$h(S) = \int_{\underline{p}}^{S} [V(p_o,p) + W - (n+1)k]dF(p)$$

$$+ \int_{S}^{\overline{p}} [V(p_o,S) + W - (n+1)k]dF(p)$$

$$- [V(p_o,S) + W - nk]$$

which reduces to

$$h(S) = \int_{\underline{p}}^{S} [V(p_o,p) - V(p_o,S)]dF(p) - k. \qquad (1)$$

Assuming that W and $V(p_o,\underline{p})$ are finite, it can be shown that the optimal stopping rule for the sequential search procedure exists and is myopic (see Appendix). That is, there exists a unique reservation price p_r such that $h(p_r) = 0$ and the optimal strategy is to buy if the sampled price p is less than or equal to p_r, and to continue searching if p is greater than p_r. Since all buyers are identical, they have a common reservation price p_r and common demand curves for all goods. The quantity demanded (wealth and all other prices held constant) by each buyer when quoted price p will be

$$d(p) = \begin{cases} 0 & \text{for } p > p_r \\ q(p) & \text{for } p \leq p_r \end{cases}$$

where q(p) is continuously differentiable (except perhaps at p_r).

The usual assumption that

$$d(p) = \begin{cases} 0 & \text{for } p > p_r \\ 1 & \text{for } p \leq p_r \end{cases}$$

along with a utility function of the form $U(W,p_o,p) = W - g(p_o) - 1p$ generates the familiar "searching for the lowest price" rule: search until a price m is encountered such that

$$\int_{\underline{p}}^{m} [m-p]dF(p) - k \leq 0 \ . \qquad (1')$$

This assumption of unitary demand, so devastating to price dispersion models, is a strange one indeed. Even the stock example of an automobile is not satisfactory--while it is true that one buys a single car (not .9 or 2.3 cars), one can and does add and remove various options. These add-ons constitute buying "more" car. Therefore, we have eliminated this assumption, thus allowing for the interplay of various substitution effects among all commodities as well as the income effects of obtaining a lower product price. This difference will prove critical to the market's ability to support a distribution of prices for the homogeneous product.

3. Sellers' Behavior

Assume that there is a continuum of firms which offers a homogeneous product for sale. Let the continuum of firms be the set J, a finite closed interval of the real line. Each firm j in J has a constant marginal cost c_j at which it can instantaneously produce the product, where $c_j \in [\underline{c},\overline{c}]$ for some $\underline{c}$ and $\overline{c}$ ($> \underline{c}$) in R_+; and for each $c_i \in [\underline{c},\overline{c}]$, there exist some firms which have marginal costs c_i. Further assume that there exists a σ-algebra $\mathcal{M}$ of subsets of J and define $E_c \in \mathcal{M}$ as follows:

$$E_c = \{ j \text{ in } J \text{ such that } c_j \leq c \} \ .$$

Then the Lebesgue measure of J, $\mu(J)$, is finite and equal to the length of the interval. Similarly, $\mu(E_c)$ is finite and $\mu(E_c) \leq \mu(J)$ for all c in $[\underline{c},\overline{c}]$. Define the function $G(c) = \mu(E_c)/\mu(J)$. Note that $G(c) = 0$ for all $c \leq \underline{c}$ and $G(c) = 1$ for all $c \geq \overline{c}$; $G(.)$ is monotone nondecreasing and right continuous. Hence G is a cumulative distribution function for c. In addition, suppose that G is continuously differentiable almost everywhere, with density denoted by $dG(c)$.

Sellers are assumed to be perfectly informed of buyers' reservation prices and demand curves and they exploit this knowledge in their price-setting behavior. Each seller j in J is assumed to choose his product price p_j so as to maximize his expected profits π_j.

$$E[\pi_j] = \begin{cases} (p_j - c_j)q(p_j)E[N_j] & \text{for } p_j \leq p_r \\ 0 & \text{for } p_j > p_r \end{cases} \tag{2}$$

where N_j is defined to be the number of buyers who sample firm j. While both the "number" of buyers and the "number" of sellers in the market are infinite, the proper interpretation is that we are actually considering very large finite numbers of both: say, n sellers and m buyers. Then forming the ratio $\lambda = m/n$ and letting m and n approach infinity, holding λ constant, gives a measure of the average number of buyers per firm.

4. Equilibrium Conditions

For the purposes of this paper, it will be assumed that a market equilibrium exists and it will be shown that the optimality conditions for both buyers and sellers are consistent with the existence of price dispersion in equilibrium.

The notion of equilibrium to be used here is that there should be no incentive for any seller to alter his quoted price, and that both consumers and sellers are maximizing their respective payoffs.

Recalling that firms know the reservation price p_r and the demand function $d(p)$, and that they have acted so as to maximize their expected profits, from Eqn. (2) we see that in equilibrium, p_j will be at or below the reservation price p_r for all sellers in the market. Since $p_j \leq p_r$, all buyers who search j on the first search will buy from j. Because buyers have no prior knowledge of who quotes which price, it is reasonable to require that sellers be treated symmetrically in the sense that, on average, each seller is approached by the same number of buyers on the first search. Therefore, $E[N_j] = \lambda$ for all j in J, and

$$E[\pi_j] = (p_j - c_j)q(p_j)\lambda .$$

Maximizing with respect to p_j yields the familiar condition that

$$p_j^*(1 + 1/e(p_j^*)) = c_j ,$$

where $e(p_j^*)$ is the elasticity of demand at the maximizing price p_j^*. Since we wish to demonstrate that price dispersion may exist even within the framework of an extremely simple model (and is not due entirely to some eccentricities in consumer demand), we assume that the demand curve has constant elasticity $e < -1$. Then $p_j^* = \left(\frac{e}{1+e}\right) c_j$. Define $\underline{p} = \left(\frac{e}{1+e}\right) \underline{c}$.

If $p_j^* \leq p_r$, then a cumulative distribution function $F(p)$ is induced upon prices by the distribution $G(c)$ over costs:

$$F(p) = G\left(\left(\frac{e+1}{e}\right)p\right).$$

And if $p_j^* > p_r$, then the seller's maximal expected profits are obtained by setting the price $p_j = p_r$. Define $\overline{p} = \min \left\{ p_r, \left(\frac{e}{1+e}\right)\overline{c} \right\}$.

Thus the cumulative distribution function $F(p)$ over the interval $[\underline{p}, \overline{p}]$ is the one induced by $G(c)$ except at the upper endpoint p_r. But this discontinuity is inconsequential--$F(p)$ is continuously differentiable almost everywhere.

We have established that sellers will be willing to offer an "interval" of prices $[\underline{p},\overline{p}] = [\left(\frac{e}{1+e}\right)\underline{c}, \min\left\{p_r, \left(\frac{e}{1+e}\right)\overline{c}\right\}]$. Since $p_r \geq \overline{p}$, buyers will be willing to buy from the first seller sampled. All that remains to be shown is that $\overline{p} > \underline{p}$--i.e., that the "interval" $[\underline{p},\overline{p}]$ is a proper interval rather than a single point.

Proposition: $\overline{p} > \underline{p}$.

Proof: Case 1. $\overline{p} = \left(\frac{e}{1+e}\right)\overline{c}$. Since $\overline{c} > \underline{c}$, and $\frac{e}{1+e}$ is a positive constant,

$$\overline{p} = \left(\frac{e}{1+e}\right)\overline{c} > \left(\frac{e}{1+e}\right)\underline{c} = \underline{p}.$$

Case 2. $\overline{p} = p_r$. Recall that

$$h(S) = \int_{\underline{p}}^{S} [V(p_o,p) - V(p_o,S)]dF(p) - k.$$

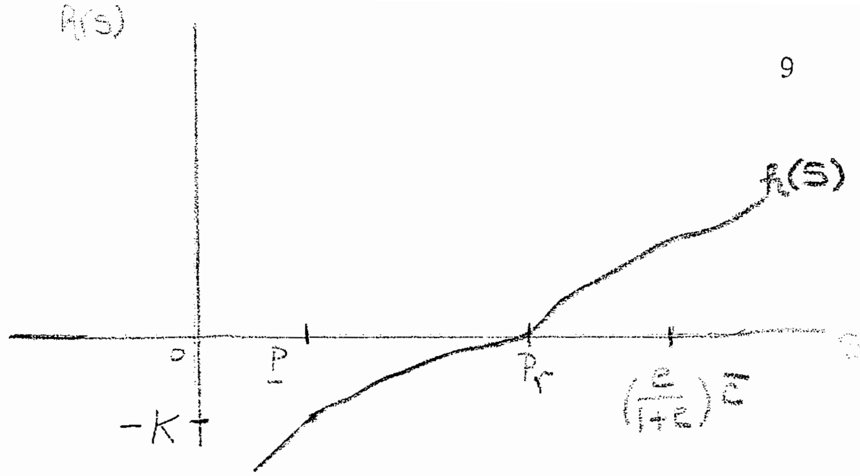
Let us examine h(S) as S changes. Since h(S) is differentiable, we can apply Liebniz' rule:

$$h'(S) = -V_2(p_o,S)F(S) > 0.$$

The inequality follows because V is strictly decreasing in its second argument. Then $h(\underline{p}) = -k$, $h(p_r) = 0$ and h(S) is strictly increasing. By the continuity of h(.) at $\underline{p}$, there exists a subinterval of $[\underline{p},\overline{p}]$ throughout which h has the same sign as $h(\underline{p})$. Since $h(p_r) = 0$ and $h(\underline{p}) < 0$, $p_r \neq \underline{p}$. Furthermore, if p_r were strictly less than $\underline{p}$, then $h(p_r)$ would be negative by the fact that h(.) is increasing. Hence it must be that $p_r > \underline{p}$. ///

As usual, as search costs increase, the reservation price p_r increases; similarly, as search costs k approach zero, so will p_r approach $\underline{p}$. As a simple illustration, assume that h(S) is as shown below:

Fig. 1



Thus it has been shown that there exists a true interval of prices $[\underline{p},\bar{p}]$ distributed according to F(p) which is consistent with a single search on the part of buyers. Furthermore, firms with costs distributed according to G(c) over an interval $[\underline{c},\bar{c}]$ will offer the product for sale at prices distributed according to F(p) over the interval $[\underline{p},\bar{p}]$. So neither buyers nor sellers have any incentive to modify their behavior, and the market equilibrium is characterized by a distribution of prices F(p) rather than a single market price.

5. Conclusion

Assuming the optimal behavior of buyers and sellers and given a distribution of marginal costs, we have demonstrated that there exists an (induced) equilibrium distribution of prices which is nondegenerate. This result is not as trivial as it may seem--it is not equivalent to the statement that "for any finite positive level of sampling costs k and for an infinte number of prices, it will never pay to search until the minimum price is encountered"--this much is obvious. The set of sellers who offer the product at the minimum price has measure zero. One key to the existence of price dispersion in this model is that buyers do not purchase the same amount of the product for all prices at or below the reservation price.

For if they did, then the expected profit maximizing price would be the reservation price p_r for <u>all</u> firms, since in that case

$$E[\pi_j] = (p_j - c_j)Q\lambda$$

where Q is the fixed quantity demanded per buyer. Then expected profits are clearly maximized when p_j is set as high as the market will bear--p_r. In other words, in this model not only is the product price <u>not driven down</u> to the minimum price (this due to the existence of search costs) but the price is also prevented from <u>jumping up</u> to the reservation price p_r by the substitution effects of other goods--i.e., by the more acceptable proposition that buyers consult demand functions rather than purchase the same quantity regardless of its price.

Of course, eliminating the assumption of different marginal costs will also reverse the result. Both a cost dispersion and elastic demand are essential.

6. Discussion

One might think that an anlogous model would involve consumers with different reservation prices and identical firms. This will not yield price dispersion, however, since the firms all face the same decision problem and must therefore draw the same conclusions regarding their optimal price.

Many simplifying assumptions have been made above. The first, that buyers are identical and have simple demand and utility functions, is not objectionable. Although it is patently unrealistic, one can only expect that differences among buyers would reinforce the result rather than reverse it. The same can be said of the assumptions of sellers with constant marginal costs and perfect information.

The source of the sampling cost k has been left unspecified. It may be interpreted as the cost of making a trip to the store, the cost of a telephone call or the cost of crossing the street to ask a neightbor. In any case, it is assumed that buyers are unable to trade information at a cost less than k per observation. Thus the model does not preclude communication among buyers.

While labor market-job search models typically assume that one either accepts a job offer or rejects it--i.e., 0-1 demand--if one considers the number of hours supplied as an elastic function of the wage rate, then the above-developed model is clearly applicable.

A plausible story for the existence of different costs is not difficult to imagine for the short run--anything from different wage contracts to varying age of equipment to different locations with respect to the factory may be invoked. In the long run, this assumption may be more difficult to justify. However, a cost distribution is certainly necessary to this model and similar devices will be required in other attempts at market analyses of price dispersion (1,8,14,16). Underlying any distribution of prices, wages, etc., is the assumption that agents differ--one cannot assume complete symmetry among agents and expect to derive an asymmetrical result like the existence of equilibrium price dispersion.

APPENDIX

An optimal stopping rule will exist for the unbounded sequential sampling procedure if

i) $$E\left|\sup_{p_i \in \{p_1, p_2, \ldots\}} V(p_o, p_i) + W - ik\right| = M < \infty \qquad \text{and}$$

ii) $$\lim_{n\to\infty} \max_{p_i \in \{p_1, p_2, \ldots p_n\}} V(p_o, p_i) + W - nk = -\infty \quad \text{with probability } 1 .$$

However, it is conceivable that $h(S) = 0$ for some S after n observations--that is, it doesn't pay to observe one more price--yet there are gains to be made by sampling several more times. If we want the optimal stopping rule to be myopic, then we must require that there be no gains to sampling an additional (finite) number of times. That is,

iii) if $$\int_{\underline{p}}^{S} \{V(p_o, p) - V(p_o, S)\}\, dF(p) - k \leq 0$$

for some S after n trials (where S is the minimum price quoted in the first n trials), then for all finite integers

z, $$\int_{\underline{p}}^{S} \{V(p_o, S') - V(p_o, S)\}\, d\Phi(S') - zk \leq 0$$ where z is the size of the additional sample and S' is minimum price quoted in the last z trials.

Claim: Assuming W and $V(p_o, p)$ are finite, then there exists an optimal stopping rule for the search strategy defined above. Furthermore, the o.s.r. is myopic.

<u>Proof</u>: (i) $$E\left|\sup_{p_i \in \{p_1,\, p_2,\, \ldots\}} V(p_o, p_i) + W - ik\right| \le V(p_o, \underline{p}) + W \quad .$$

Since W and $V(p_o, \underline{p})$ are assumed to be finite, $V(p_o, \underline{p}) + W = M' < \infty$ and (i) follows, by transitivity of the real numbers.

(ii) $$\lim_{n\to\infty} \max_{p_i \in \{p_1,\, p_2,\, \ldots\, n\}} V(p_o, p_i) + W - nk \le \lim_{n\to\infty} V(p_o, \underline{p}) + W - nk = -\infty$$

(iii) The cumulative distribution function of

$$S' = \text{minimum } \{p_{n+1}, p_{n+2}, \ldots p_{n+z}\} \quad \text{is}$$

$$\Phi(S') = 1 - [1 - F(S')]^z$$

where $F(\cdot)$ is the cumulative distribution function of the prices. Therefore,

$$d\ (S') = z[1 - F(S')]^{z-1}\, dF(S') \quad .$$

Given that

$$\int_{\underline{p}}^{S} \{V(p_o, p) - V(p_o, S)\}\, dF(p) - k \le 0 \quad ,$$

we wish to show that for all z,

$$\int_{\underline{p}}^{S} \{V(p_o, S') - V(p_o, S)\}\, z\, [1 - F(S')]^{z-1}\, dF(S') - zk \le 0 \quad .$$

But $[1 - F(S')]^{z-1} \le 1$, so

$$\int_{\underline{p}}^{S} \{V(p_o, S') - V(p_o, S)\} z [1 - F(S')]^{z-1} dF(S') - zk$$

$$\leq \int_{\underline{p}}^{S} \{V(p_o, S') - V(p_o S)\} z dF(S') - zk$$

$$= z\left[\int_{\underline{p}}^{S} \{V(p_o, S') - V(p_o S)\} dF(S') - k\right] \leq 0$$

The last line follows since the term in square brackets is given to be nonpositive. Hence, there is no finite number of searches in excess of n which will yield an additional increase in expected utility; hence, the optimal stopping rule exists and is myopic.

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