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# ON THE THEORY OF LAYOFFS

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## 1. Introduction

Layoff unemployment, particularly its "temporary" aspect, is a common phenomena that seems to defy rigorous explanations by most existing theories. In the neoclassical market framework, unemployment can occur in response to a fall in the value of labor productivity only if wage rates fail to respond in the manner hypothesized by the theory. Although forms of frictional and speculative unemployment can be explained by recent elaborations of the approach that account for information imperfections, claims that these models provide an insight into an employer's tendency to respond to a cyclic downturn by issuing proverbial pink slips is somewhat dubious. Even the recent attempt to extend the general equilibrium theory of exchange and production to the case of non-market clearing prices does not provide a convincing explanation.

Against this background of unsatisfactory success, the recent interest in the "implicit contract" theory of the employment relationship is understandable. The claim by some of its proponents that both "sticky" wage rates and layoffs can be explained by employment and wage payment strategies designed to cope with unanticipated but inevitable fluctuations in output demand can hardly fail to excite. The fact that these strategies can survive competition only if there is a long term attachment between employer and employee only adds to the credibility of the approach among those with a detailed understanding of the operation and institutions of the labor market.

In spite of this enthusiasm and the considerable literature that has been generated by it, the following question has not been clearly answered. What are the necessary and sufficient conditions for the existence of layoff unemployment? Akerlof and Miyazoki [1977] argue that worker risk aversion, a condition used to explain wage rates that do not instantaneously clear the labor market in the original papers by Baily [1974], Azariadis [1975] and Gordon [1974], implies full employment.Both Feldstein [1976] and Baily [1977] argue that the transfer of risk from worker to employer is not an essential element of the approach. Feldstein considers the case in which workers self insure income and Baily uses a utility function that is linear in wage income. Although the focus of Feldstein's analysis is on the effect of unemployment insurance schemes on layoff rules, he also suggests that the distinction between men and hours in productions is an important consideration. Baily argues that the value of the worker's opportunity to search for alternative employment implies that workers are laid off when the value of their productivity falls too low.

In this paper I attempt to sort out and relate these apparently conflicting claims. This purpose is pursued in the context of a model that includes both Baily's and Feldstein's as specific cases. The preferences of the typical worker are represented by a utility function defined on consumption and leisure. In general, the output of the typical firm is a function of both man-hours used in production and the number of employed workers. Search is an activity that uses worker time as its only input. The state of final output demand,as characterized by the price of output, is regarded as a random variable with a known probability distribution.

Workers are attached to their jobs because of the costs of finding alternative employment. Because of the attachment, firms compete for workers *ex ante* by offering contracts that specify the wage to be paid, the hours worked by those who will be employed and the proportion to be employed in every demand state. In equilibrium, the contracts offered by each employer must be efficient in the sense that the expected utility of the typical worker is maximum given the employer's expected profit rate. Whether expected profits or the expected utility of the contracts offered are equalized across firms by competition is not important for our purpose.

The results of this inquiry can be stated as follows. Given no worker search, every efficient contract is characterized by full employment if (1) workers do not prefer risk in consumption and leisure and (2) the marginal product attributable to an increase in employment, holding the total man-hours used in production constant, is non-negative. That both conditions are generally necessary as well is demonstrated by constructing examples for which layoffs do occur when the price of final output is sufficiently low. In the examples, either each worker prefers to 'bunch' his leisure and/or 'sharing the work' yields less total output when in either case the number of hours per employed worker required to maintain full employment is sufficiently small. These results generalize to the case in which workers are allowed to search for alternative employment opportunities if and generally only if the search technology used is convex and independent of the worker's employment status. All results are independent of the possibilities that the workers have to insure their incomes.

Although understanding the fact that efficient contracts are characterized by full employment under theoretical standard conditions is important, the fact that layoffs can occur when only one of the conditions fails is equally significant. The analysis also illustrates that layoffs to the extent that they are explained by the theory of efficient contracts reflect a rational response to fluctuations in the value of labor productivity given tastes and technology. Finally, the possibility that an efficient contract may insure labor income is of no consequence.

## 2. The Model

Consider a labor market composed of a given number of employing firms and a fixed number of workers. Workers are assumed to be homogeneous in production. Each firm is characterized by a production function $f(hx,x)$, where $x$ denotes the number of workers employed in a specified time period and $h$ represents the fraction of the period worked by each, and a probability distribution function defined on the possible set of output prices $P$. Let $p \in P$ denote the price of the firm's output realized during the period. Workers have identical preferences that can be represented by the utility function $u(c,\ell)$ defined on the set of possible consumption-leisure combinations.

At the beginning of the period each worker must select to join a particular firm. Within the period, all workers are attached to the firm initially chosen. Both the contract offered by each firm and the probability distribution on its set of possible prices are known to the worker at the beginning of the period, but the realized price that the firm will receive for its output during the period is not known with certainty. Firms compete at the beginning of the period by offering attractive contracts.

A contract is a rule that determines for each possible $p \in P$ the probability of being employed during the period $q(p)$, the fraction of the period spent working by each employed worker $h(p)$, and the compensation paid during the period. The wage paid depends in general on the worker's employment status as well as the realized price of output. Let $w_0(p)$ and $w_1(p)$ denote the compensation if unemployed and employed respectively. Note that we are allowing for the possibility of 'private' unemployment compensation in the model even though Feldstein is not specific on this point and Baily does not allow it. In defense, our specification is both more general and more consistent with the spirit of the contract idea. In sum, a <u>contract</u> is a function $(h,q,w_0,w_1) : P \to R^4_+$ such that $h(p) \leq 1$ and $q(p) \leq 1$.

Preferences of the typical worker over the set of feasible contracts can be induced as follows. The worker realizes the consumption-leisure combination $(c_0(p),1)$ if unemployed and $(c_1(p), 1-h(p))$ if employed where $(c_o,c_1) : P \to R^2_+$ is the consumption plan that maximizes expected utility given the contract and the worker's budget constraint. Suppose that the worker is able to fully insure his labor income, as Feldstein assumes. In this case, the worker's optimal plan satisfies

$$v = E \{ \max_{(c_0,c_1) \geq 0} [q\, u(c_1, 1-h) + (1-q)\, u(c_0,1)] \} \tag{1}$$

subject to

$$y = E \{qw_1 + (1-q)w_0\} \geq E_p \{ q\, c_1 + (1-q)\, c_0 \} \tag{2}$$

where $E\{\cdot\}$ is the mathematical expectation of $\{\cdot\}$ with respect to the probability distribution on P. The budget constraint (2) simply requires that expected income, $y$, be sufficient to cover the worker's expected consumption. 1/

Of course, $v$ as defined in (1) is the <u>indirect utility</u> or <u>utility value</u> of a contract. A firm is assumed to rank contracts according to their expected profit. Since $x = qn$ where $n$ is the size of the firm's work force, expected profit is

$$\pi = E\{pf(hqn, qn) - w_1 qn - w_0(1-q)n\}. \tag{3}$$

A <u>contract is said to be efficient for a firm</u> and a given labor force if there exists no other preferred by both the employer and the labor force. In other words, the value of an efficient contract is at least as large as the value of any other that yields the same expected profit. Given competition, the contract offered by every firm in equilibrium is efficient. Since the workers have identical preferences, the contracts offered by all firms that are able to attract a labor force must have the same value to the workers. Hence, each employer maximizes his expected profit by offering a contract that is efficient given the common value of all the contracts offered by the others. The labor force size desired by each firm also maximizes the employer's expected profit given the common value. Finally, the equilibrium value of the contracts is such that the sum of the demands for workers is equal to the total number who wish to be attached to some firm. [2]

Of course, the hypothesis that actual contracts are efficient does not require a competitive equilibrium of the type just described. It is always in the interest of both the firm's workers and their employer to agree to an efficient contract. Consequently, the results presented in this paper concerning the properties of any efficient contract are of more general interest.

For every efficient contract that yields expected profit $\pi$, there exists a map $(c_o, c_1, \ell, q) : P \to R_+^4$ that solves

$$E\{\max_{(c_0,c_1,\ell,q)\geq 0}[\,qu(c_1,\ell) + (1-q)u(c_0,1)]\} \tag{4}$$

subject to

$$E\{pf((1-\ell),q) - qc_1 - (1-q)c_0\} \geq \pi \tag{5.a}$$

and

$$(\ell(p),q(p)) \leq (1,1) \text{ for all } p \in P \tag{5.b}$$

by virtue of (1) - (3), given the normalization $n = 1$. This map is the consumption-leisure allocation associated with an efficient contract. Call any such map an <u>efficient allocation</u>.

When workers are able to fully and costlessly insure labor income, then the efficient wage rule is determined only up to the expected wage payment. Specifically, given an efficient contract, any other contract with the same hours and employment rule but a different wage rule is efficient if and only if it offers an expected wage payment $E\{w_1 q + (1-q)w_0\}$ equal to that implicit in the original. However, when workers are not able to insure income but payments to unemployed workers are allowed, then an efficient contract pays a wage in every contingency equal to the worker's desired consumption in that contingency because employers are indifferent to wage fluctuations. Formally, in this case the contract $(h(p),q(p),w_0(p),w_1(p))$ is efficient given $\pi$ if and only if it is equal to $(1-\ell(p),q(p),c_0(p),c_1(p))$ where $(c_0(p),c_1(p),\ell(p),q(p))$ is an efficient allocation given $\pi$.

These observations illustrate the relationship between the Baily-Feldstein model and the models in the 'implicit' contract literature.[3/] The set of efficient allocations have the property that efficient hours and employment rules are independent of any assumptions one cares to make about the availability of income insurance because a risk neutral employer is always willing to provide it. However, if workers are able to self insure or have access to perfect insurance markets, then the wage rule need not equal the optimal consumption plan and in general is indeterminant.

In the remainder of the paper $u(c,\ell)$ is a strictly increasing function with derivatives denoted as $u_1(c,\ell)$ and $u_2(c,\ell)$ such that

$$\lim_{\ell \to 0} u_2(c,\ell) = \infty. \tag{6}$$

The production function $f(hq,q)$ is strictly increasing in its first argument and possesses derivatives denoted as $f_1(hq,q)$ and $f_2(hq,q)$. Finally, all possible prices are positive, $P \subset R_+$. The boundary condition (6) rules out the possibility that worker demand no leisure.

3. Full Employment: Sufficient Conditions

In this section we show that every efficient contract is characterized by full employment when the following assumptions hold:

A1: $u(c,\ell)$ is (strictly) concave.

A2: $f_2(hq,q)$ is (strictly) non-negative.

Given A1, the workers are averse to risk in consumption and leisure in the sense that none prefer a probability distribution on the set of feasible consumption leisure pairs to its mathematical expectation with certainty. Of course, the assumption implies that the marginal rate of substitution between consumption and leisure diminishes (or at least does not increase) everywhere. Given A2, an increase in employment obtained by decreasing hours worked per employed worker so as to maintain total man-hours used does not decrease output. Hence, the standard case in which man-hours is the only labor input is included. 4/

The assumptions appear to be weak probably because they are standard in competitive market formulations. But, since they imply no unemployment in the standard competitive model, we should not be surprised that they have the same force here. Indeed, full employment obtains in both the standard and the contract market models because A1 implies that no worker prefers one consumption-leisure combination during the fraction of his work life that he spends employed and another combination during the remaining fraction spent unemployed to the average of the two combinations all the time and because A2 implies that there is no real cost involved in catering to this preference. However, it is plausible that neither of these conditions holds.

A formal demonstration of the assertion that layoffs do not occur given A1 and A2 exploits the necessary conditions for an efficient allocation. When $u(\cdot)$ and $f(\cdot)$ are differentiable and (6) holds, the map $(c_o(p), c_1(p), \ell(p), q(p))$ given $\pi$ is an efficient allocation only if the following hold for all $p \in P$ :

$$(1-q)[u_1(c_o,1) - \lambda] \leq 0, \text{ with equality if } c_o)p) > 0. \tag{7.a}$$

$$q[u_1(c_1,\ell) - \lambda] \leq 0, \text{ with equality if } c_1(p) > 0. \tag{7.b}$$

$$u_2(c_1,\ell) - \lambda p f_1(q(1-\ell),q) \geq 0, \text{ with equality if } \ell(p) < 1. \tag{7.c}$$

$$\begin{aligned} \eta &= [u(c_1,\ell) - u(c_o,1)] \\ &\quad + \lambda\,[p(1-\ell)f_1(\cdot) + pf_2(\cdot) - (c_1 - c_o)] \geq 0, \end{aligned} \tag{7.d}$$

with equality if $q(p) < 1$. $\lambda$ is the non-negative multiplier associated with the expected profit constraint (5.a). As such, it is independent of the realized price of final output.

That the marginal utility of consumption is equalized across all possible prices for which consumption is positive, conditions (7.a) and (7.b), is an implication of the fact that workers are able to fully insure income either directly or indirectly via the employer's wage payments policy. Condition (7.c) is the classical requirement that the marginal rate at which the worker is willing to substitute leisure for income, $u_2(\cdot)/\lambda$, equals the value of the marginal product of work time, $pf_1(\cdot)$, if hours worked are positive. However, unlike the traditional formulation, a "change" in the hourly "wage," $pf_1(\cdot)$, has no income effect because $\lambda$ is independent of its realized value.

If the marginal product of work time at zero hours is bounded, $f_1(0,q) < \infty$, then the number of hours worked by the employed equals zero for all sufficiently low output prices given that consumption is positive by virtue of (7.a) - (7.b). Although one violates the langugage by speaking of employees who work zero hours as employed, neither the neo-classical nor the Keynesian literature regard this case as unemployment because the value of the marginal product, the standard demand price, is less than the marginal rate at which the worker is willing to substitute leisure for consumption, the supply price. We see no reason to deviate from this tradition.

Definition: An efficient contract is said to provide for layoffs if and only if a possible price exists such that the fraction of the labor force that works a positive number of hours is strictly less than unity.

The multiplier $\eta(p)$ is that associated with the upper bound on the fraction that can be employed, $q(p) \leq 1$. It may be interpreted as the indirect utility of a marginal increase in the fraction of the employed labor force when the output price is p. Its two terms, as expressed in (7.d), are respectively the difference between the expost utility of being employed and of being unemployed and the indirect or imputed utility of the change in expost profit attributable to an increase in the fraction of the labor force employed. Hence, if layoffs were to occur for some p, then either workers prefer to be laid off expost or profits can be increased expost by reducing employment. More precisely, the gain (loss) in utility attributable to becoming employed balances the utility value of the profit loss (gain) resulting in the case of the marginally employed member of the work force. It is this condition that is novel to the contract theory of the employment relationship. However, when A1 and A2 obtain, gains from employment always outweigh losses.

Theorem 1: No efficient contract provides for layoffs if both A1 and A2 obtain and at least one holds strictly.

Proof. We need only show that $\eta(p) > 0 \ \forall \ p \in P$. Since conditions (7.b) and (7.c) respectively imply $c_1[u_1(c_1,\ell) - \lambda] = 0$ and $(1-\ell)[\lambda p f_1(\cdot) - u_2(c_1,\ell)] = 0$, (7.d) may be rewritten as

$$\eta = [u(c_1,\ell) - u(c_o,1) - (c_1 - c_o)u_1(c_1,\ell) - (\ell - 1)u_2(c_1,\ell)] \tag{8}$$

$$+ \lambda p f_2(q(1-\ell),q) + c_o[\lambda - u_1(c_1,\ell)].$$

Every tangent plane bounds a concave function from above. Hence, the first term in (8) is non-negative by virtue of A1 and positive when $\ell < 1$ and A1 hold strictly. Since $u_1(\cdot) > 0$ and (7.b) imply $\lambda > 0$, the second term is non-negative from A2 and positive when A2 is strict. Finally, (7.b) and $c_o \geq 0$ implies that the last term is non-negative. 5/

Q.E.D.

Note that (7.a) is not used in the proof. Since $c_o$ can be chosen arbitrarily, the theorem is valid even if no wage is paid to laid off workers and consumption in every contingency is equal to labor income. Indeed,

$c_0$ could equal the benefit paid by a public unemployment insurance scheme provided that the benefit is taxed at the same marginal rate as wage income and that the tax scheme used to finance the benefit is fully experience rated.

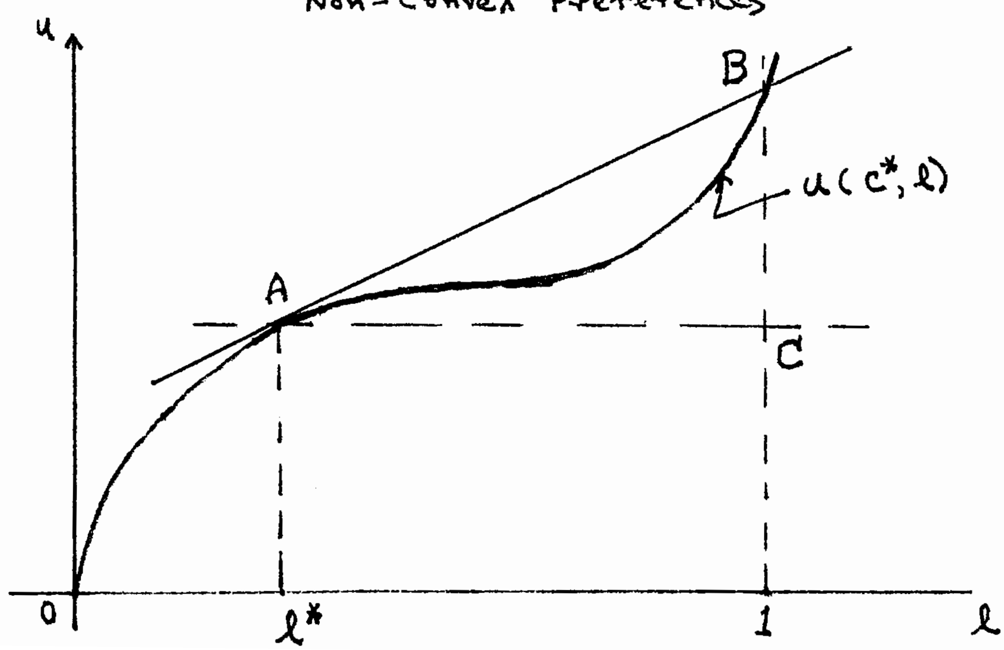
## 4. Layoff Unemployment: Examples

In this section we show that layoff unemployment can occur if either hypothesis of the Theorem is relaxed by constructing plausible examples. In the first example, the utility function $u(c,\ell)$ is not convex everywhere. In the second, the second partial of $f(hq,q)$ is not non-negative everywhere. The examples illustrate that layoff unemployment is explained by tastes and/or technology in the model.

Example No. 1. Suppose that the typical worker prefers to take leisure in "bunches." Specifically, a combination of periodic vacations and work periods at longer hours per week is preferred to continuous work at a small number of hours per week. This case arises when leisure increases in value at the margin in a neighborhood of full leisure as illustrated in Figure 1.

FIG. 1

Non-Convex Preferences



The curve labelled $u = u(c^*,\ell)$ represents total utility given a consumption rate held constant at $c^*$. By construction, the line through the points A and B is tanget to the curve at $\ell = \ell^*$. Consequently, the worker prefers the gamble, $(c^*,\ell^*)$ with probability $q$ and $(c^*,1)$ with probability $1-q$, to the average combination $(c^*,q\ell^* + (1-q)1)$ with certainty. Formally, $qu(c^*,\ell^*) + (1-q)u(c^*,1)$, which is the vertical distance to AB at $\ell = q\ell^* + (1-q)1$, exceeds $u(c^*,q\ell^* + (1-q)1)$.

That preferences of the type illustrated can "explain" layoff unemployment is easily established. For the purpose of constructing an example, assume that $\lim_{c\to 0} u_1(c,\ell) = \lim_{\ell\to 0} u_2(c,\ell) = \infty$, that $u(c,\ell)$ is strictly concave in $c$ but is non-concave in $\ell$ as in Figure 1, and that $u(c,\ell)$ is strongly separable in $c$ and $\ell$ $(u_{12}(\cdot) = u_{21}(\cdot) \equiv 0)$. Furthermore, assume that $f(\cdot)$ depends only on man-hours employed and is strictly concave, with an infinite derivative at zero hours, i.e., $f_2(\cdot) \equiv 0$, and $f_{11}(\cdot) < 0$, and $\lim_{h\to 0} f_1(\cdot) = \infty$. Of course, the assumptions regarding preferences together with (7.a) and (7.b) imply that $c_0 = c_1 = c^*$ is optimal for all $p \in P$ where $c^*$ is the unique solution to

$$u_1(c^*,1) = \lambda . \tag{9.a}$$

In addition, the unique value of $\ell^* < 1$ illustrated in Figure 1 is the solution to

$$u(c^*,\ell^*) - u(c^*,1) - (\ell^* - 1)u_2(c^*,\ell^*) = 0 \tag{9.b}$$

since the line segment BC is equal to both $u(c^*,1) - u(c^*,\ell^*)$ and

$(1 - \ell^*)u_2(c^*,\ell^*)$ by construction.

The optimal leisure-employment rule $(\ell,q) = (\ell(p),q(p))$ is unique and solves

$$u_2(c^*,\ell(p)) - \lambda p f_1(q(p)(1-\ell(p))) = 0 \tag{10.a}$$

and

$$\eta(p) = u(c^*,\ell(p)) - u(c^*,1) - (\ell(p) - 1)u_2(c^*,\ell(p)) \geq 0, \tag{10.b}$$

with strict equality if $q(p) < 1$, by virtue of (7.c), (7.d), $c_0 = c_1 = c^*$ and $f_2(\cdot) \equiv 0$.

For all $p$ such that $q(p) = 1$, (10.a) determines $\ell(p)$. But if $q(p) < 1$, then $\ell(p) = \ell^* < 1$, by virtue of (9.b) and the fact that (10.b) holds as an equality, and $q(p)$ is determined by (10.a). Since $q(p)$ is increasing in $p$ when $\ell(p) = \ell^*$ given $f_{11}(\cdot) < 0$, the following characterization of the efficient quantity rules is valid. A critical positive output price

$$p^* = u_2(c^*,\ell^*)/\lambda f_1(1 - \ell^*) \tag{11}$$

exists such that

$$q(p) = 1 \text{ and } \ell(p) < 1 \quad \forall \quad p \geq p^* \tag{12.a}$$

and

$$q(p) < 1 \text{ and } \ell(p) = \ell^* < 1 \ \forall \quad 0 < p < p^* . \tag{12.b}$$

Hence, layoff unemployment occurs when the price of output is sufficiently low because workers prefer to combine random vacations with periods of work at longer hours to continuous employment when hours worked per week required to maintain full employment are small.

Example No. 2. Formally, the marginal product of men, given man-hours, is negative if and only if the elasticity of output with respect to hours per man exceeds the output elasticity of men. Feldstein [1976,p.943] argues that this case is empirically relevant and tells the following story by way of explanation. Capital-labor substitution is limited in the short run because the stock of machines is fixed and the average weekly hours of utilization cannot be chosen independently of the length of the work week. Indeed, given only one work shift, the traditional neo-classical production function with arguments man-hours, and machine-hours is of the form $G(hx,hK)$ where $x$ and $K$ represent the numbers of men and machines employed respectively. If $G(\cdot)$ is homogeneous of degree one and concave, then $G(\cdot) = hnG(q,k) = hng(q)$ where $k = K/n$, $n$ is the size of the firm's available work force, and $g(q)$ is concave. Hence, by letting $n = 1$, we have $f(hq,q) \equiv hg(q)$, which implies $f_2(\cdot) < 0$. More generally, if worker "fatigue" is important--effective hours per day increase with actual hours worked, but at a decreasing rate--then $f_2(\cdot)$ is negative when $h$ is small but may be positive when $h$ is large.

Let us suppose that $f(hq,q)$ is concave, that $\lim_{h \to 0} f_1(hq,q) = \infty$, that $f_2(hq,q) < 0$ for all small values of $h$, and that $f_2(hq,q) > 0$ for large values given any $q > 0$. Further assume that $u(c,\ell)$ is concave and homogeneous of degree one. (The latter restriction implies risk neutrality in the Arrow-Pratt sense.) Our assumptions imply that the optimal allocation is uniquely determined by the necessary conditions. Furthermore, since

$$u(c,\ell) = cu_1(c,\ell) + \ell u_2(c,\ell)$$

globally by virtue of Euler's Theorem, the necessary conditions are

$$(1-q)[u_1(c_o,1) - \lambda] = 0 \tag{13.a}$$

$$q[u_1(c_1,\ell) - \lambda] = 0 \tag{13.b}$$

$$u_2(c_1,\ell) - \lambda p f_1(q(1-\ell),q) = 0 \tag{13.c}$$

$$\eta = \lambda p f_2(q(1-\ell),q) \geq 0, \tag{13.d}$$

with strict equality holding if $q(p) < 1$, by virtue of (7).

Suppose $q(p) = 1$ for some $p \in P$. In this case (13.b), (13.c), and the assumptions imply

$$\frac{\partial \ell(p)}{\partial p} = \frac{u_{11}(\cdot)\lambda f_1(\cdot)}{\Delta} < 0,$$

where

$$\Delta = \begin{vmatrix} u_{11}(\cdot) & u_{12}(\cdot) \\ u_{21}(\cdot) & u_{22}(\cdot) + p\lambda f_{11}(\cdot) \end{vmatrix} = p\lambda f_{11}(\cdot)u_{11}(\cdot) > 0. \text{ 6/}$$

Since $f_2(\ ) > 0$ when $h = 1-\ell$ is large by assumption, it follows that $q(p) = 1$ when $p$ is large. However, (13.c) also implies that $\ell \to 1$ as $p \to 0$ given $q = 1$. Hence, the assumption that $f_2(\cdot) < 0$ when $h = 1-\ell$ is small implies that $\ell(p) < 1$ and $q(p) < 1$ for all $p < p^*$ where $p^*$ is given by (11) and $(c^*,\ell^*)$ is the solution to

$$u_1(c^*,\ell^*) = \lambda \tag{14.a}$$

and

$$f_2(1-\ell^*,1) = 0 \tag{14.b}$$

In this case, layoff unemployment is due to the fact that an increase in employment obtained by work sharing reduces total output when the hours per worker required to maintain full employment are sufficiently small.

The only explicit example of layoff unemployment presented by Feldstein [1976] is one in which hours worked by each worker employed is fixed. Fixing hours worked or more generally bounding them away from zero, which is all that one needs for the purpose of the demonstration, is an ad hoc assumption contrary to the spirit of the efficiency hypothesis. But if either tastes or technology are such that contracts provide for layoffs, then efficient hours worked per hour are bounded away from zero for all positive output prices .

5. Worker Search

Consider a generalized version of the model in which workers, whether employed or not, have the ability and incentive to seek alternative employment. For this purpose, think of the time allocated to search activity as an input in a process designed to generate alternative employment opportunities. Then, because the opportunity cost of time may exceed its return in the search process, both Feldstein's case -- no search -- and Baily's case -- search only when unemployed -- are possibilities.

Time is divided into a sequence of discrete intervals of unit length denoted as $t = 1,2,\ldots$ . As of the beginning of period $t$, the typical worker is attached to a particular employer and $V_t$ denotes the maximal expected utility value of the worker's entire future consumption-leisure stream given this attachment. The utility flow obtained is $u(c,\ell)$ where $(c,\ell) = (c_1,\ell_1)$ if employed and $(c,\ell) = (c_o,\ell_o)$ if laid off during the period. In general, $\ell_1 = 1 - h - s_1$ where $h$ is the fraction of the period spent working and $s_1$ is the fraction of the period allocated to search given employment, and $\ell_o = 1 - s_o$ where $s_o$ denotes the fraction of the period spent searching given unemployment. Future utility flows are discounted at a fixed positive rate. The discount factor is $\beta$. Finally, the probability of employment during a period is denoted by $q$. In each period, the entire vector $(c_o,c_1,h,s_o,s_1,q)$ is chosen conditional on the firm's realized demand state represented by the random output price $p$.

The probability that the worker finds one or more alternative offers during any time period is a function of the time allocated to search activity. Let $g(s)$ denote this probability. At the beginning of period $t$ neither the number nor the value of any offer obtained during the period are known with certainty. Let U denote the random expected utility of the best offer obtained by the worker during a period. Since $V_{t+1}$ denotes the expected utility given that the worker continues his attachment with his current employer beyond period t, the worker accepts an alternative offer and quits his current job if and only if the realized value of U exceeds $V_{t+1}$.

The model outlined above is similar to the simple wage search model in which the typical worker is unaware of the precise location and identity of particular alternative employers. As in the standard model, we assume that this information is collected sequentially by a process of randomly sampling from the set of such opportunities. The formulation differs from the standard one by allowing for the possibility that more than one alternative employer is located per period, by making the expected number located a choice variable and by permitting search while employed. 7/

Even though competititon is imperfect when the workers do not know their alternative employment opportunities with certainty, the employer has an incentive to offer an efficient contract. Were he not to do so his workers are more frequently attracted to other employers. Because replacement is generally costly to the employer due to specific training and recruiting costs, the employer minimizes the capital loss attributable to quits only by offering the workers contracts that maximize their utility for a given expected future profit stream.

In this dynamic generalization of the model, an <u>efficient contract</u> maximizes the worker's expected utility of continuing his current attachment subject to an expected future profit flow constraint. Associated with every efficient contract is an allocation $(c_o(p),c_1(p),\ell_o(p),\ell_1(p),q(p))$ such that

$\ell_1(p) = 1 - h(p) - s_1(p)$ and $\ell_o(p) = 1 - s_o(p)$ where $(h(p),q(p))$ are the hours and employment rules embodied in the contract and $(c_o(p),c_1(p),s_o(p),s_1(p))$ is the worker's optimal consumption-search plan given the contract and his budget constraint. This allocation is efficient in the sense that the worker's discounted expected future stream of utility is maximum in each period given the employer's expected future profit stream. By virtue of Bellman's principle of dynamic optimality, the joint map $z(p) = (c_o(p),c_1(p),h(p),s_o(p),s_1(p),q(p))$ is a solution to the maximization problem implicit in the following recursive definition of the indirect utility value of attachment. 8/

$$V_t = \underset{t}{E} \underset{z\geq 0}{\text{Max}}_t \{q[u(c_1,\ell_1) + \beta[g(s_1) \underset{t+1}{E} \text{Max}[U,V_{t+1}] + (1 - g(s_1))V_{t+1}]]$$

$$+ (1 - q)[u(c_o,\ell_o) + \beta[g(s_o) \underset{t+1}{E} \text{Max}[U,V_{t+1}] + (1 - g(s_o))V_{t+1}]]\} \tag{15}$$

subject to

$$\underset{t}{E} \{pf(hq,q) - qc_1 - (1 - q)c_o\} \geq \pi_t \tag{16.a}$$

$$\ell_o = 1 - s_o \geq 0,\ \ell_1 = 1 - s_1 - h \geq 0,\ \text{and}\quad q \leq 1. \tag{16.b}$$

The structure of (15) reflects the following facts. First, the utility of the worker's future consumption-leisure stream at the beginning of period $t$ is equal to the utility flow realized during that period $u(c,\ell)$ plus the present value of the end of the period expected utility of the future consumption-leisure stream as of date $t+1$. Second, the worker quits if and only if he obtains at least one alternative offer and the expected utility value of the best offer, $U$, exceeds the utility he can expect by remaining with his current employer. Since $g(s)$ is the probability of obtaining at least one alternative offer, the worker's expected end of period utility is

$$g(s(t)) \underset{t+1}{E} \text{ Max } [U, V_{t+1}] + (1 - g(s(t))V_{t+1}$$

where $\underset{t+1}{E} \text{ Max } [U, V_{t+1}]$ is the mathematical expectation taken with respect to the probability distribution of U given his information at date $t+1$. Finally, the values of $c, \ell$ and s are chosen contingent on whether or not the worker will be laid off and on the output price that will obtain during the period.

The probability of locating an alternative employer per period, $g(s)$, is an increasing differentiable function and $g(0) = 0$ by assumption. To extend the Theorem to the case in which workers may search, we need only add

Assumption A3 : $g(s)$ is(strictly) concave.

In other words, the "marginal productivity" of search time does not increase.

Theorem 2: No efficient contract provides for layoffs if A1, A2 and A3 obtain and at least one holds strictly.

Proof: The argument is an extension of that used in the no search case. Conditions (7.a), (7.b), and (7.c) are still necessary for an efficient allocation, except that $(c_0, \ell_0)$ replaces $(c_0, 1)$. In addition, the time allocated to search given unemploymemt and employment respectively must satisfy

$$- u_2(c_1, \ell_1) + \beta g'(s_1) \underset{t+1}{E} \text{ Max } [U - V_{t+1}, 0] \leq 0, \tag{17.a}$$

with equity if $s_0 > 0$, and

$$-u_2(c_1,\ell_1) + \beta g'(s_1) \underset{t+1}{E} \text{Max } [U - V_{t+1}, 0] \leq 0 , \tag{17.b}$$

with equality if $s_1 > 0$ , where $g'(s)$ is the derivative of $g(s)$. These new conditions require the equality of the marginal cost of search time, the marginal utility of leisure, and marginal gain in expected future utility attributable to search if the time allocated to the search activity is positive. Note that if the worker's current job yields a utility value, $V_{t+1}$ , which is sufficiently high relative to the value of alternatives available, no search may be optimal in either employment state. Furthermore, if the marginal utility of leisure when employed exceeds the marginal utility when laid off, $u_2(c_1,\ell_1) > u_2(c_0,\ell_0)$, which is a plausible possibility, then the worker may search when laid off but not search when employed.

Finally, the employment rule is such that

$$\begin{aligned} \eta &= u(c_1,\ell_1) - u(c_0,\ell_0) \\ &+ \lambda [ pf_1(\cdot) + pf_2(\cdot) - (c_1 - c_0) ] \\ &+ \beta [ g(s_1) - g(s_0) ] \underset{t+1}{E} \text{ Max } [U - V_{t+1}, 0] \geq 0, \end{aligned} \tag{18}$$

with equality if $q(p) < 1$. The last term is the difference between the <u>total</u> expected future gain in utility attributable to search given that the worker is employed rather than laid off. Obviously, it is negative if the worker searches only if laid off. When no search is optimal (18) reduces to (7.d) given $g(0) = 0$.

Since $c_1[u_1(c_1,\ell_1) - \lambda] = 0$ and $h[\lambda pf_1(\cdot) - u_2(c_1,\ell_1)] = 0$ from (7.b) and (7.c) respectively, and since

$s_1[\beta g'(s_1) \underset{t+1}{E} \max [U - V_{t+1}, 0] - u_2(c_1, \ell_1)] = 0$ from (17.b), (18) may be rewritten as

$$\eta = u(c_1,\ell_1) - u(c_0,\ell_0) - (c_1 - c_0)u_1(c_1,\ell_1) + (h + s_1)u_2(c_1,\ell_1)$$

$$+ p\lambda f_2(hq,q) + c_0[\lambda - u_1(c_1,\ell_1)]$$

$$+ \beta [ g(s_1) - g(s_0) - s_1 g'(s_1)] \underset{t+1}{E} \max [ U - V_{t+1}, 0]$$

Finally, $\ell_1 = 1 - s_1 - h$ and $\ell_0 = 1 - s_0$ imply

$$\eta = [u(c_1,\ell_1) - u(c_0,\ell_0) - (c_1 - c_0)u_1(c_1,\ell_1) - (\ell_1 - \ell_0)u_2(c_1,\ell_1)] \quad (19)$$

$$+ p\lambda f_2(hq,q) + \beta[g(s_1) - g(s_0) - (s_1 - s_0)g'(s_1)] \underset{t+1}{E} \max [U - V_{t+1}, 0]$$

$$+ c_0[\lambda - u \;(c_1,\ell_1)] + s_0[u_2(c_1,\ell_1) - \beta g'(s_1) \underset{t+1}{E} \max[U - V_{t+1}, 0] ] .$$

One of the first two terms is positive if either A1 or A2 holds strictly. The first three terms are non-negative given A1 , A2 , and A3 respectively. Conditions (7.b) and (17.b) imply respectively that the last two terms are non-negative. Q.E.D.

Baily [ 1977 ] suggests that layoffs can occur if the typical worker searches while laid off but does not search while working. Since Baily's specific model satisfies consumption A1 and A2, we have established that the claim is false when the search technology as characterized here is convex. Indeed, since the last term in (19) is strictly positive given $s_1 = 0$ and $s_o > 0$ in general neither A1 nor A2 need hold strictly .

However, in our model the search technology available to the worker is the same whether laid off or not. Another interpretation of Baily's claim is that search while working is less productive than search while laid off. Formally, $\hat{g}(s) < g(s)$ for all $s > 0$ where $\hat{g}(s)$ denotes the probability of obtaining an offer while employed and $g(s)$ is the probability when laid off. 9/ For the purpose of showing that contracts generally will provide for layoffs when the search technology depends on employment status, we consider the extreme case in which no offer can be obtained while employed; i.e., $\hat{g}(s) \equiv 0$ for all $s \geq 0$. Further, assume that $u(c,\ell)$ is concave and homogeneous of degree one and that $f(hq,q)$ depends only on man-hours and is concave. The latter assumptions imply that the first two terms of (19) are zero. Finally, because both the average and marginal productivity of search while employed are zero, one can show that

$$\eta = s_o u_2(c_1,\ell_1) - \beta g(s_o) \underset{t+1}{E} \text{Max}\, [U - V_{t+1}, 0] \tag{20}$$

given $c_1 > 0$.

That efficient contracts do provide for layoffs in this case can now be established by contradiction. Suppose that $q(p) = 1$ for all $p \in P$. Then $(c_1,\ell_1)$ is independent of $E\,\text{Max}\,[U - V_{t+1},0]$ from (7.a) and (7.b). By virtue of (17.a), $s_o > 0$ for all sufficiently large values of $E\,\text{Max}\,[U - V_{t+1},0]$ but, of course, $s_o \leq 1$. Hence, these results and (20) imply that $\eta(p) < 0$ for all sufficiently large value of $E\,\text{Max}\,[U - V_{t+1},0]$ if $q(p) = 1$ which is a contradiction.

## 6. Summary

That layoff unemployment cannot be explained by the Baily-Feldstein theory under conditions regarded as "standard" is certainly an important conclusion of the paper. Standard conditions include convex worker preferences, a production technology with man-hours as the only labor input and a worker search technology that is convex and independent of a worker's employment status. However, one can reasonably argue that none of these conditions hold globally. Furthermore, quite robust examples of layoff unemployment can be constructed by relaxing any one of the three sufficient conditions.

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FOOTNOTES

1/ Throughout the paper we abstract from the existence of a public unemployment insurance scheme. The effect of such a scheme on incentives to layoff workers is adequately treated by Feldstein [1976]. Furthermore, his analysis implies that the results presented in this paper hold when there is unemployment insurance if the tax used to finance it is fully experience rated and the benefit is taxed at the same marginal rate as earned income.

2/ This description of an "equilibrim" in a competitive market for contracts follows Baily [1977]. Alternative conceptualizations in Feldstein [1976] and in Akerlof and Miyazaki [1977] presume free entry and exit of employing firms.

3/ Specifically, that stimulated by Azariadis [1975], Baily [1974] and Gordon [1974].

4/ Both Baily [1977] and Feldstein [1976] maintain A1. In addition, both assume that $u(\cdot)$ is strongly separable in its two arguments. Instead of explicitly allowing for self insurance of income as Feldstein does, Baily assumes that $u(\cdot)$ is linear in c which has the same affect. In Baily's version of the model $f_2(\cdot) \equiv 0$ since man hours is the only labor input. Although Feldstein argues that $f_2 < 0$ is a reasonable possibility, he does not consider its implications. Finally, Baily allows for worker search but Felstein does not. This section and the next deals with the latter case.

5/ The theorem is a generalization of that established by Akerlof and Miyazaki [1977].

6/ Remember that $u_{11}u_{22} - n_{12}n_{21} = 0$ if $u(\cdot)$ is homogeneous of degree one.

7/ Papers that deal with models which embody one or none of these generalizations include those by Wilde [ 1977], Burdett [ 1977] and Mortensen [ 1977].

8/ $E_t\{\cdot\}$ denotes the expectation conditional on information available at the beginning of the period while $E_{t+}\{\cdot\}$ is the conditional expectations given informations available at the end of the period. Both the price of output realized during the period and the number of workers in the firm's labor force at the end are uncertain at the beginning of the period. Since $V_{t+1}$ depends on both in general, it is random at the beginning of the period. In addition, knowledge of both may condition the probability distribution on the set of possible values of U.

9/ In other words, locating alternatives is more "difficult" while employed. A possible rationale for this case might be based on the fact that it is almost impossible to apply for a job except during "normal" working hours.