

Discussion Paper Number 320

SPECIFIC CAPITAL, BARGAINING, AND  
LABOR TURNOVER

by

Dale T. Mortensen

March, 1978

Dale T. Mortensen received the B.A. from Willamette University and the Ph.D. in economics from Carnegie-Mellon University. His current research focuses on information and matching problems in the labor market.

While preparing an earlier version of the paper, the author received financial support from the U.S. Department of Labor. This version was partially supported by the National Science Foundation Grant Number SOC-77-08896. Constructive comments from F. Brechling, R.J. Gordon, K. Burdett and H. Grossman are acknowledge. Responsibility for all remaining errors rests with the author.

## 1. Introduction

In a world of heterogeneous workers and jobs, a problem of matching the two in some best way exists. A centralized competitive market sorts workers among jobs in a manner that maximizes aggregate output, appropriately defined, when information about technology and the abilities of individual workers is perfect. When the locations of particular jobs and workers are not known, the instantaneous attainment of a solution that is optimal in this sense is not economic. Because it is not in the interest of either employer or worker to wait until the best alternative is located, imperfect matches are formed. The fact that better alternatives generally exist for both the employer and the worker involved in a job-worker match motivates search by both. A separation at some future date occurs when either worker or employer finds a preferred alternative match. Turnover of this type is a characteristic of the dynamic process by which job-worker matches are improved in a decentralized labor market. The purpose of this paper is to present a framework for analyzing the interrelationship between the choices of search strategies by the two parties involved in an existing match and the nature of the wage bargaining problem.

Given imperfect mobility of the kind just described, future streams of quasi-rents accrue to both parties involved in an existing match. The expected present values of these streams are specific human capital measures of the degree to which each party is attached to the match. This observation is the basis for the principal hypothesis of the theory of labor turnover. The probability that either party will terminate the match at some future date is a decreasing function of his own share of the specific

capital value of the match.<sup>1</sup> The general validity of this proposition is challenged on both theoretical and empirical grounds in this paper.

The theoretical analysis presented in the paper is based on the following ideas. Given an employment agreement, a definition of the relationship between employer and worker and a specification of the wage, the worker's and employer's problem of choosing search strategies is a two person game. The solution to each game determines the value of the specified employment agreement to the worker and the employer. In this way the sets of feasible and likely outcomes of the bargaining over employment agreements are generated. This approach accounts for the simultaneity between wage determination and search behavior, on the one hand, and generates a far richer theory of the employment relationship along the lines suggested by Simon [1951], on the other.<sup>2</sup>

In the simple model developed in Section 2, the quit probability and the dismissal probability are related to the search strategies chosen by worker and employer respectively. In each case the search strategy has two components, a criterion for acceptance of alternative matching opportunities and a measure of search intensity that determines the frequency with which alternatives are located. Search is viewed as a process of sampling randomly from the distribution of alternative opportunities. Searching more intensely is assumed to be more costly at the margin.<sup>3</sup> The principal results presented in the section follow.

If the employment agreement is simply a specification of the wage rate, then the non-cooperative Nash solution to the game of search strategy choice is such that the quit and the dismissal probabilities are respectively decreasing functions of the worker's and the employer's shares of the specific capital value of the match as others have suggested. However, this solution

does not maximize the sum of worker and employer expected wealth because neither party, when terminating the match, takes account of the capital loss that his action imposes on the other. The joint wealth maximizing search strategies are such that both turnover probabilities are smaller than those implied by the noncooperative solution to the game, both decline with the total capital value of the match and neither depends on its division between worker and employer.

In Section 3, the following question is asked. Does a mechanism exist that will induce both worker and employer to pursue joint wealth maximizing search behavior? Two simple ones, neither of which requires direct monitoring of search activities, are studied. The first is the practice sometimes observed of matching the alternative offers obtained by one's partner in the match. The second is an ex ante agreement by each party to compensate the other as a precondition for separation. Severance pay requirements and the existence of non-vested pension plans can be interpreted as such schemes. Although the first mechanism does have the property that the non-cooperative acceptance criterion is the joint wealth maximizing one, each party still has an incentive to search too intensively. However, the non-cooperative solution to the game in which contingent compensation is part of the ex ante employment agreement is joint wealth maximizing.

Will the employment agreement adopted include compensation in the event of a future separation or some other mechanism that will induce cooperative search behavior? In Section 4 we argue that such an agreement

is a likely outcome of a bilateral bargaining problem but need not obtain if the employer has monopsony power. However, in the latter case the firm's workers have an incentive to organize as a means of achieving an employment agreement that motivates joint wealth maximizing turnover rates.<sup>4</sup> This observation suggests the hypothesis that unionized firms have lower quit and dismissal rates, other things equal, as recently suggested by Freeman and Medoff [1977].

The fact that the joint wealth maximizing hypothesis implies that neither the quit probability nor the dismissal probability depends on the division of specific capital between worker and employer is clearly its differentiating empirical implication. The consistency of this implication with the existing empirical evidence is reviewed in Section 5. At this point in time the evidence in its entirety does not contradict the hypothesis and, indeed, some of it offers strong support.

## 2. A Simple Turnover Model

In this section we formulate a model of joint search for alternatives by the worker and employer involved in an existing match. The model is designed to emphasize the interrelationship between the decisions of the two parties regarding the criterion used for acceptance of alternatives and the resources allocated to finding alternatives. Formally, this joint decision problem is a sequentially repeated two person non-sum zero game. Initially, we consider only one period in the sequence. Two standard solution concepts, the Nash

non-cooperative and the cooperative joint wealth maximizing solutions, are shown to have distinguishable empirical implications.

Consider a current match involving two agents that benefit from some form of exchange. Suppose that each can value the future net benefit flows associated with the match as well as those associated with any alternative prospective match. For each agent assume that alternative opportunities randomly arrive at an average rate per unit time that can be controlled at a cost by the agent. Think of each opportunity that does arrive as a random draw from the set of all possible alternatives and assume that each opportunity must either be accepted or rejected on arrival. The match in question survives a short future time interval if and only if no acceptable alternative is received by either agent during the interval. Each agent must decide on the definition of an acceptable alternative and must set the expected rate at which alternatives arrive.

The framework sketched above embodies the idea that neither party knows the location of an alternative that might be preferred to his existing match and the notion that the process of locating a preferred alternative is costly and time consuming. That the decision problems of two matched parties is a game follows from the observation that the welfare of either party is generally affected by the other's decision to accept an alternative. For the purpose of making the latter point concrete in our case, we assume that each party values any match according to the expected capitalized net income stream associated with it.

Let  $y_1$  and  $y_2$  represent the worker's and the employer's respective capitalized future income flows were their match to continue into the future.

Assume that both apply the same discount rate to future income flows so that

$$y = y_1 + y_2 \tag{1}$$

can be meaningfully interpreted as the total or joint capital value of the match. In this section, we suppose that both  $y$  and its division between worker and employer are given. Let  $x_1$  denote the capital value of a randomly located alternative in the worker's case and interpret  $x_2$  analogously in the employer's case. Each is a random variable ex ante characterized by the c.d.f.  $F_i(x) = \Pr\{x_i \leq x\}$ . Let  $\bar{x}_i$  denote the upper bound on the support of  $F_i(x)$ , the best alternative available to agent  $i$ . Finally, null alternatives exist for each party to an existing match; let  $\underline{x}_1$  and  $\underline{x}_2$  represent the capital values of these. By definition, the null alternative is costlessly available at any point in time. In the worker's case the null alternative can be search while unemployed. Holding the job vacant while seeking an acceptable worker for it is the counterpart in the employer's case.

Assume that the arrivals of alternative opportunities to the two parties can be described by two independent Poisson processes, one for each agent. Let  $\lambda_1$  and  $\lambda_2$  denote the means of these two processes; i.e.,  $\lambda_i$  is the expected arrival rate or the average frequency of arrival for agent  $i$ . Hence, in any small time interval of length  $h$ , the probability that the worker (employer) will receive some alternative matching opportunity is approximately  $\lambda_1 h (\lambda_2 h)$ . Since the arrival of two or more arrivals for one agent or an arrival for both is negligible when  $h$  is small; i.e.,  $\lambda_i \lambda_j h^2 / h \rightarrow 0$  as  $h \rightarrow 0$ , the probability that neither agent will receive an

alternative matching opportunity during the interval is  $1 - \lambda_1 h - \lambda_2 h$ .<sup>5</sup>

Let  $\eta_i$  denote the criterion for acceptance of an alternative matching opportunity in the case of agent  $i$ . Specifically, if agent  $i$  should receive an alternative during the future interval of length  $h$  such that  $x_i > \eta_i$ , then the existing match terminates by his decision to accept it. Since either agent may receive an alternative during the interval, the match terminates during the interval if either receives an acceptable alternative. Given a viable match, one for which

$$(y_1, y_2) > (x_1, x_2), \quad (2)$$

the expected end of interval wealth of agent  $i$  as of the beginning of any interval of length  $h$  is

$$\begin{aligned} & \lambda_i h [\Pr\{x_i > \eta_i\} E\{x_i | x_i > \eta_i\} + \Pr\{x_i \leq \eta_i\} y_i] \\ & + \lambda_j h [\Pr\{x_j > \eta_j\} x_i + \Pr\{x_j \leq \eta_j\} y_i] + (1 - \lambda_1 h - \lambda_2 h) y_i \\ & = y_i + \int_{\eta_i}^{\bar{x}_i} (x - y_i) dF_i(x) + q_j h (x_i - y_i), \quad j \neq i \end{aligned}$$

where  $q_i h$ ,  $i = 1, 2$ , is the probability that agent  $i$  will terminate the match during the interval. Specifically,

$$q_i = \lambda_i [1 - F_i(\eta_i)], \quad i = 1, 2 \quad (3)$$

is the instantaneous quit rate when  $i = 1$  and the instantaneous dismissal rate when  $i = 2$ . To summarize, we note that the end of interval expected wealth of agent  $i$  is the sum of three terms: the end of interval capital value of the match to agent  $i$ , the expected capital gain attributable to



the possibility that he will receive an acceptable alternative during the interval and the expected loss in capitalized future rents attributable to the possibility that the other party will terminate the match. Of course, if the match is not viable in the sense that one of its parties prefers his null alternative, then the match terminates with certainty at the end of the interval in any event.

As of the beginning of the interval, the capital value of the match to each party is simply the sum of the net income obtained during the interval plus the present value of his expected wealth at the end of the interval.

Formally, these are

$$v_1 = [w - c_1(\lambda_1)]h + \beta(h) \left[ y_1 + \int_{\eta_1}^{\bar{x}_1} (x - y_1) dF_1(x) + q_2 h(x_1 - y_1) \right] \quad (4.a)$$

and

$$v_2 = [p - w - c_2(\lambda_2)]h + \beta(h) \left[ y_2 + \int_{\eta_2}^{\bar{x}_2} (x - y_2) dF_2(x) + q_1 (\underline{x}_2 - y_2) \right] \quad (4.b)$$

where  $p$  denotes the value of the product flow attributable to the match,  $w$  is the wage paid the worker,  $c_i(\lambda_i)$  is the cost of search to agent  $i$  and  $\beta(h) = 1/(1 + rh)$  is the common discount factor. Throughout the remainder of the paper the following are maintained:

Assumption 1:  $F_i(x)$ ,  $i = 1$  and  $2$ , is differentiable and has a non-degenerate, compact and convex support.

Assumption 2:  $c_i(\lambda)$ ,  $i = 1$  and  $2$ , is an increasing strictly convex function such that  $c_i(0) = c_i'(0) = 0$ .

Assumption 3: The best alternative available to both agents is viable; i.e.  $(\bar{x}_1, \bar{x}_2) > (\underline{x}_1, \underline{x}_2)$ .

The crucial and interesting assumptions are non-degeneracy of the alternative distributions and convexity of search costs. Viability of the best alternatives is innocuous. All the others are not necessary but serve to simplify the analysis.

The search strategy for either party to the match during the time interval in question is a choice of acceptance criterion and an expected arrival frequency, the pair  $(\lambda_i, \eta_i)$ . Because one party's strategy affects the other's expected wealth by determining the probability of a capital loss attributable to termination, the joint search strategy choice problem is formally a game. The Nash non-cooperative solution is a pair of search strategies, one for each party designated as  $(\lambda_i^0, \eta_i^0)$ ,  $i = 1$  and  $2$ , and a vector of associated payoffs  $(v_1^0, v_2^0)$  that satisfy

$$v_1^0 = \max_{(\lambda_1, \eta_1) \geq 0} \{ [w - c_1(\lambda_1)] h + \beta(h) [y_1 + \lambda h \int_{\eta_1}^{\bar{x}_1} (x - y_1) dF_1(x) + q_2^0 h (\underline{x}_1 - y_1)] \} \quad (5.a)$$

and

$$v_2^0 = \max_{(\lambda_2, \eta_2) \geq 0} \{ [p - w - c_2(\lambda_2)] h + \beta(h) [y_2 + \lambda h \int_{\eta_2}^{\bar{x}_2} (x - y_2) dF_2(x) + q_1^0 h (\underline{x}_2 - \underline{y}_2)] \} \quad (5.b)$$

where  $q_i^0 = \lambda_i^0 [1 - F_i(\eta_i^0)]$ . In other words, the strategy choice of each maximizes his own expected wealth taking the other's choice as given. Because assumptions 1 and 2 imply that  $v_i, i = 1$  and  $2$ , is bounded from above on

the joint strategy space, is concave in his own strategy and is continuous in the other agent's strategy, a Nash non-cooperative equilibrium always exists. Indeed, the equilibrium strategies are unique for both and satisfy the following first-order conditions:

$$c'_i(\lambda_i^0) = \int_{\eta_i^0}^{\bar{x}_i} (x - y_i) dF_i(x) \quad (6.a)$$

$$\eta_i^0 = y_i \quad (6.b)$$

$i = 1$  and  $2$ . The reservation value of the match to agent  $i$ ,  $\eta_i^0$ , is that part of the capital value of continuing the match that accrues to agent  $i$ . The optimal average arrival frequency,  $\lambda_i^0$ , is such that the cost and return attributable to a larger frequency are equal at the margin. The following result is an immediate consequence of (3) (6) and the assumptions:

Proposition 1: Given the Nash non-cooperative equilibrium, the quit (dismissal) rate depends only on  $\lambda_i^0$  and decreases with the worker's (employer's) share of the capital value of the match.<sup>6</sup>

There are at least three reasons for questioning the appropriateness of the Nash non-cooperative solution to the two person non-sum zero game. First, by cooperating the two players can generally improve the payoffs to both. Second, the unspecified costs of coordinating cooperation are likely to be minimal in the two person case. Finally, the implicit assumption that the strategy choice of one player will not affect the choice of the other is not reasonable. All three reasons would seem to have force in our particular case.

If complete cooperation could be achieved, then the appropriate criterion for their choice would be the maximization of the total value of the

match at the beginning of the interval,  $v = v_1 + v_2$  . By simply adding the respective sides of the two equations of (4) , it follows that the joint wealth maximizing strategy pair  $(\lambda_i^*, \eta_i^*)$  ,  $i = 1$  and  $2$  , and the associated maximal total value of the match,  $v^*$  , satisfy

$$v^* = \max_{(\lambda_i, \eta_i)_{i=1,2}} \{ [p - c_1(\lambda_1) - c_2(\lambda_2)]h + \beta(h) [y + \lambda_1 h \int_{\eta_1}^{\underline{x}_1} (x + \underline{x}_2 - y) dF_1(x) + \lambda_2 h \int_{\eta_2}^{\underline{x}_2} (x + \underline{x}_1 - y) dF_2(x)] \} \quad (7)$$

where  $y$  is the total capital value of the match at the end of the future time interval as defined in (1) . Since  $v$  is a separable function of the two agents' strategies and assumption 1 and 2 imply that it is also concave in both, a joint wealth maximizing solution to the game of search satisfies

$$c_i'(\alpha_i^*) = \int_{\eta_i^*}^{\bar{x}_i} (x + \underline{x}_j - y) dF_i(x) \quad (8.a)$$

$$\eta_i^* = y - \underline{x}_j \quad (8.b)$$

$j \neq i$  ,  $i = 1$  and  $2$  .

A quick comparison of (6) and (8) reveals the fact that the Nash solution does not maximize joint wealth if the capital value of the match exceeds the sum of the values of both agents' null alternatives. By pursuing a Nash non-cooperative strategy, agent  $i$  ignores the fact that agent  $j$  would lose the capitalized future flow of expected rents,  $y_j - \underline{x}_j$  , associated with the match were agent  $i$  to terminate. The joint wealth maximizing strategy requires that an alternative to agent  $i$  be sufficiently attractive to com-

penalizes agent  $j$  for this loss. In addition, the expected gain in total rather than private wealth attributable to a marginal increase in the arrival frequency is equated to the marginal cost of search in the joint wealth maximizing case.

The principal empirical difference between the hypothesis that joint wealth maximizing strategies are pursued and the hypothesis of Nash non-cooperative search behavior is revealed by comparing the following implications of (8) with Proposition 1.

Proposition 2: Given the joint wealth maximizing strategies, both the quit rate and the dismissal rate decrease with the total capital value of the match and are independent of its division.

Because each agent undervalues the match given non-cooperative behavior, turnover rates are "too high".

Proposition 3: Both the quit rate and the dismissal rate associated with joint wealth maximizing search are smaller than those associated with non-cooperative search strategies if  $y = y_1 + y_2 < \bar{x}_1 + \bar{x}_2$ .

Proof. A comparison of (6) and (8) reveals that Proposition 3 holds only if  $\eta_i^* = y - \underline{x}_j = y_i + y_j - \underline{x}_j \cong \eta_i^0 = y_i^0$ ,  $i = 1$  and  $2$  and if the inequality is strictly for one agent. But this condition is implied by (1), the definition of a viable match, and holds strictly if  $y = y_1 + y_2 < \bar{x}_1 + \bar{x}_2$ , the total capital value of the match is less than the sum of the agents' best alternatives. Q.E.D.

### 3. Mechanisms for Cooperation

What arrangement could be made that would motivate cooperative search

behavior? The issue is one of appropriate incentives. Because of the problems of monitoring either the extent of search activity or the actual acceptance criterion used by either party, we reject any simple agreement to pursue joint wealth maximizing strategies as unenforceable. Instead we consider mechanisms for cooperation that do not require direct monitoring. Two are studied. The first and most obvious is an ex ante agreement by each party to make a counter offer when the other receives an attractive alternative matching opportunity. The second is an agreement to compensate the other as a precondition for separation.

The issue of incentives is investigated with the aid of the following behavior assumption. Relative to the mechanism each party to a match chooses his best Nash non-cooperative strategy. The logic of the method is as follows. Each mechanism can be thought to define a new game of search strategy choice. The a priori argument for the empirical relevance of the joint wealth maximizing strategies is strengthened if a mechanism exists such that the Nash equilibrium relative to the mechanism is the joint wealth maximizing strategy pair. In the sequel we show that requiring compensation has this property but that counter offering does not.

Consider first the case in which each party is permitted to make a counter offer. Then, each is willing to give up that part of his share of the capitalized specific rents,  $y_i - \underline{x}_i$ , required to prevent the other from terminating the match. Specifically, given the division of the total capital value of the match in the event that no alternatives arrive during the next interval of length  $h$ , the counter offer made by agent  $j$  to agent  $i$  in response to an alternative of value  $x_i$  is  $x_i$  if  $y_i + y_j - \underline{x}_j \geq x_i > y_i$ .

Obviously, no counter offer needs to be made if  $x_i \cong y_i$  and none will be made if  $x_i > y - \underline{x}_j$ . As a consequence, agent  $i$  accepts the alternative and terminates the match if and only if  $x_i > y - \underline{x}_j = \eta_i^*$ . Hence, the reservation values of the match are joint wealth maximizing.

However, the optimal counter offer mechanism just described implies that the expected ex ante end of the interval wealth for agent  $i$  is

$$\begin{aligned} & \lambda_i h E\{\max(x_i, y_i)\} + \lambda_j h E\{\max(y - x_j, \underline{x}_i)\} + (1 - \lambda_1 h - \lambda_2 h) y_i \\ &= y_i + \lambda_i h \int_{y_i}^{\bar{x}_i} (x - y_i) dF_i(x) + \lambda_j h \int_{y_j}^{\bar{x}_j} \max[y_j - x, \underline{x}_i - y_i] dF_j(x), \end{aligned}$$

$j \neq i$  and  $i = 1$  and  $2$ . Agent  $i$  makes a gain whenever he obtains an alternative greater than his end of interval value of the match ( $x_i > y_i$ ) and takes a loss whenever agent  $j$  finds an alternative of greater value than his end of interval capital value of the match ( $x_j > y_j$ ). Although the expected capital loss attributable to the possibility that the other party will find an attractive offer is smaller than in the original formulation, an increase in the other's arrival frequency still increases that loss at the margin. Since neither takes this effect into account, both have an incentive to search "too much."

Formally, the non-cooperative choices of the two arrival frequencies  $(\hat{\lambda}_1, \hat{\lambda}_2)$  and their associated payoff vector  $(\hat{v}_1, \hat{v}_2)$  satisfy,

$$\begin{aligned} \hat{v}_1 = & \max_{\lambda_1 \cong 0} \{ [w - c_1(\lambda_1)]h + \beta(h) [y_1 \\ & + \lambda_1 h \int_{y_1}^{\bar{x}_1} (x - y_1) dF_1(x) + \hat{\lambda}_2 h \int_{y_2}^{\bar{x}_2} \max[y_2 - x, \underline{x}_1 - y_1] dF_2(x) ] \} \quad (9.a) \end{aligned}$$

and

$$\begin{aligned} \hat{v}_2 &= \max_{\lambda_2 \geq 0} \{ [p - w - c_2(\lambda_2)]h + \beta(h) [y_2 \\ &+ \hat{\lambda}_1 h \int_{y_1}^{\bar{x}_1} \max[y_1 - x, \underline{x}_2 - y_2] dF_1(x) + \lambda_2 h \int_{\bar{x}_2}^{\bar{x}_1} (x - y_2) d\bar{F}(x) \} \end{aligned} \quad (9.b)$$

Hence (9), (8) and (6) imply

$$\hat{\lambda}_i = \lambda_i^0 > \lambda_i^* \quad , \quad i = 1 \text{ and } 2 \quad , \quad \text{and, consequently,}$$

$$\hat{q}_i = \lambda_i^0 [1 - F(\eta_i^*)] > q_i^* = \lambda_i^* [1 - F(\eta_i^*)] \quad i = 1 \text{ and } 2 \quad .$$

Proposition 4: Given the counter offer mechanism, both turnover rates exceed those associated with the joint wealth maximizing strategy pair if  $y = y_1 + y_2 < \bar{x}_1 + \bar{x}_2$  .<sup>7</sup>

In the case of a compensation scheme, each agent is required to compensate the other for any capital loss arising as a consequence of an actual separation. In other words, if agent  $i$  accepts an alternative of value  $x_i$  , its value net of the required compensation is  $[x_i - (y_j - \underline{x}_j)]$  . Since agent  $i$  will also forego his share,  $y_i$  , were he to separate, he will terminate if and only if  $x_i > \eta_i^* = y - \underline{x}_j$  . Again the reservation values are joint wealth maximizing. Moreover, in this case the agent  $j$  obtains  $y_j$  whether the match continues or not due to the compensation received in the event of a separation. Specifically, given compensation, the expected end of interval wealth for either agent is given by

$$\begin{aligned} &\lambda_i h [\Pr\{x_i > \eta_i^*\} E\{x_i - (y_j - \underline{x}_j) \mid x_i > \eta_i^*\} + \Pr\{x_i \leq \eta_i^*\} y_i] \\ &+ \lambda_j h [\Pr\{x_j > \eta_j^*\} [\underline{x}_i + (y_i - \underline{x}_i)] + \Pr\{x_j \leq \eta_j^*\} y_i] + (1 - \lambda_1 h - \lambda_2 h) y_i \\ &= y_i + \lambda_i h \int_{\eta_i^*}^{\bar{x}_1} (x + \underline{x}_j - y) dF_i(x) \quad . \end{aligned}$$



It equals the agent's share of the end of interval capital value of the match plus the expected joint capital gain attributable to the agent's search. Consequently, each agent's Nash choices of his own arrival frequency is  $\lambda_i^*$ , the joint wealth maximizing value.

The Nash choices of the arrival frequencies given compensation and the division of the end of interval capital value of the match  $(y_1, y_2)$  determine the division of the beginning of interval total expected wealth  $v^*$  between the employer and worker. Formally, the Nash equilibrium payoffs given compensation are

$$v_1^* = \max_{\lambda_1 \geq 0} \{ [w - c_1(\lambda_1)] + \beta(h) [y_1 + \lambda_1 h \int_{\eta_1^*}^{\bar{x}_1} [x + \underline{x}_2 - y] dF_1(x)] \} \quad (11.a)$$

and

$$v_2^* = \max_{\lambda_2 \geq 0} \{ [p - w - c_2(\lambda_2)]h + \beta(h) [y_2 + \lambda_2 h \int_{\eta_2^*}^{\bar{x}_2} [x + \underline{x}_1 - y] dF_2(x)] \} \quad (11.b)$$

Clearly, the optimal arrival frequencies are  $(\lambda_1^*, \lambda_2^*)$  by virtue of (8) and  $v_1^* + v_2^* = v^*$  by virtue of (7). In sum, we have established

Proposition 5: Given the compensation mechanism, the associated Nash non-cooperative equilibrium is joint wealth maximizing.

The practice of granting severance pay to a dismissed worker is precisely the type of contingent terminal payment suggested. Other restrictions on the employer's freedom to initiate a termination are consistent with the general framework. Arrangements that require direct compensation of the employer before quitting are not common, however. In part, the lack of such compensation reflects the fact that contracts that give the employer a property right to human capital are not legally enforceable. However, paid vacations and nonvested pension plans, payments made contingent on specified periods of previous employment, serve to raise the cost of quitting to the worker. All of these practices are equivalent to paying some portion of the worker's income share into a contingent escrow account. That sum, then, at least partially compensates the employer if the worker does quit.

#### 4. The Employment Agreement

An employment agreement is an ex ante specification of all relevant aspects of the relationship between the worker and employer including the wage to be paid. Since the specification may be contingent on future uncertain events the concept of an employment agreement, which I borrowed from Simon [1951], can be regarded as a generalization of the notion of an "implicit contract" introduced by Azariadis [1975] and Baily [1974]. In the context of the simple model developed in this paper, the agreement specifies the future wage stream to be paid at each future date and possibly a decision to adopt the compensation mechanism, described in the previous section, as a means of motivating joint wealth maximizing search by both parties. If side payments were allowed, the employment agreement would certainly include compensation in the event of a future separation or some alternative arrangement with equivalent effect. Under what conditions will the employment agreement include compensation in the more realistic no side payment case?

The analysis presented in this section suggests that compensation or an equivalent arrangement can be expected when the wage determination problem is one of bilateral bargaining. However, because the typical firm employs many workers and has power relative to his employees given imperfect mobility, the bilateral bargaining framework may not be the appropriate one except in the case of a unionized firm. In the case of a monopsonist, we show that the employment agreement need not induce joint wealth maximizing turnover rates because the workers will agree to compensation only if they receive a share of the rents. But, if the monopsony solution is not joint wealth maximizing, then the workers have an incentive to organize as a means of increasing total capital value of each job-worker match. In sum, the analysis formalizes one aspect of the argument that an incentive and an effect of unionization is reduced turnover as recently expressed by Freeman and Medoff [1977].

The terms of the employment agreement can be viewed as set ex ante at the date

of the inception of the match in question. To characterize the values to both employer and worker of all possible employment agreements, one must recognize that the choice of search strategies, formulated above as a single period game conditional on the agreement, is repeated in every subsequent period of the remaining tenure of the match. One can account for this fact by noting that the search games are sequentially interrelated. The capital value of the match to either agent at the end of any time interval  $(t, t + h)$  is his expected wealth given that the match continues beyond date  $t + h$ . In other words,  $v_i(t) = y_i(t)$ ,  $i = 1$  and  $2$ , for all  $t$  in which the match will continue to at least date  $t + h$ , in the notation of the previous sections. This observation implies that the end of interval expected capital values of the match are conditional on the search strategies that both worker and employer will pursue in the future.

Specifically, if at the inception of the match contingent compensation in the event of a future separation is not agreed to, then the associated Nash search strategies in each interval  $(t, t+h)$  satisfy

$$v_1^o(t) = y_1^o(t) = \underset{(\lambda_1, \eta_1)}{\text{Max}} \{ [w(t) - c_1(\lambda_1)]h + \beta(h)[y^o(t+h) + \lambda_1 h \int_{\eta_1}^{\bar{x}_1} [x - y_1^o(t+h)] dF_1(x) + q_2^o h [\underline{x}_1 - y_1^o(t+h)] \}$$

and

$$v_2^o(t) = y_2^o(t) = \underset{(\lambda_2, \eta_2)}{\text{Max}} \{ [p - w(t) - c_2(\lambda_2)]h + \beta(h)[y^o(t+h) + \lambda_2 h \int_{\eta_2}^{\bar{x}_2} [x - y_2^o(t+h)] dF_2(v) + q_1^o h [\underline{x}_2 - y_2^o(t+h)] \}$$

for all  $t$  by virtue of (5) where  $w(t)$  is the agreed on wage for the

interval and  $(y_1^0(t+h), y_2^0(t+h))$  are the end of interval capital values of the match conditioned on the future wage stream given no agreement to compensate in the future. These conditions, of course, are implied by Bellman's principle of dynamic optimality. They together with appropriate right hand end point conditions define the entire sequence of Nash equilibria given no compensation.

For the sake of simplicity, assume that both agents have infinite horizons and that all relevant parameters including the wage are stationary. Since all the games in the sequence are identical in this case, so are the equilibria; i.e.,  $y_i^0(t) = y_i^0(t+h)$  for all  $t$  and  $i = 1$  and  $2$ . Given this fact and  $\beta(h) = 1/(1+rh)$ , one can show that the equilibrium capital values and search strategies in the continuous time stationary infinite horizon case satisfy

$$ry_1^0 = \text{Max}_{(\lambda_1, \eta_1)} \{w - c_1(\lambda_1) + \lambda_1 \int_{\eta_1}^{\bar{x}_1} (x - y_1^0) dF_1(x) + q_2^0(\underline{x}_1 - y_1^0)\} \quad (12.a)$$

and

$$ry_2^0 = \text{Max}_{(\lambda_2, \eta_2)} \{p - w - c_2(\lambda_2) + \lambda_2 \int_{\eta_2}^{\bar{x}_2} (x - y_2^0) dF_2(x) + q_1^0(\underline{x}_2 - y_2^0)\} \quad (12.b)$$

by rearranging the equations above and then letting  $h \rightarrow 0$ . In each case, the right hand side is the permanent expected income of each party to the match given no compensation.

If compensation is agreed to both now and in the future, then  $v_i^*(t) = y_i(t)$  for all  $t$ . Hence, an analogous argument and the equations of (11) imply the following capital values of the match in the infinite horizon stationary case

$$ry_1^* = \text{Max}_{(\lambda_1, \eta_1)} \{w - c_1(\lambda_1) + \lambda_1 \int_{\eta_1}^{\bar{x}_1} (x + \underline{x}_2 - y^*) dF_1(x)\} \quad (13.a)$$

$$ry_2^* = \text{Max}_{(\lambda_1, \eta_2)} \{p - w - c_2(\lambda) + \lambda_2 \int_{\eta_2}^{\bar{x}_2} (x + \underline{x}_1 - y^*) dF_2(x)\} \quad (13.b)$$

where, of course,  $y^* = y_1^* + y_2^*$  is the maximal total capital value of the match.

Note that equations (12) and (13) implicitly define the capital values of the match to both worker and employer as functions of the value of its product flow,  $p$ , and the average,  $w$ , for the case of no agreement to compensate and for the case of compensation respectively. By adding the equations of (13), one can easily verify that the joints maximal capital value of the match,  $y^* = y_1^* + y_2^*$ , is a strictly increasing function of  $p$  and is independent of  $w$ . In other words, the wage serves only to divide the total between the two parties given a contingent compensation agreement. Similarly, the equations of (12) imply that the total capital value of the match, given no compensation,  $y_1^0 + y_2^0$ , increases with the value of the product flow. However, in this case the total also depends on the wage because an increase in  $w$ , by increasing the worker's share and decreasing the employer's, results in a decrease in the instantaneous quit rate and an increase in the instantaneous dismissal rate by virtue of Proposition 1. In general, the impact of these two effects on the total capital value are not offsetting. Finally, Propositions 3 and 5 imply that  $y^* \geq y_1^0 + y_2^0$  in general with strict equality holding when the maximal total capital value of the match is less than the sum of the workers' and employers' best alternatives ( $y^* < \bar{x}_1 + \bar{x}_2$ ).

Consider the following concepts:

$$W^0 = \{w \in R + \mid (y_1^0, y_2^0) \cong (\underline{x}_1, \underline{x}_2)\} \quad (14)$$

$$W^* = \{w \in R + \mid (y_1, y_2) \cong (\max(\underline{x}_1, y_1^0), \max(\underline{x}_2, y_2^0))\} \quad (15)$$

Since the match is viable only if its capital value to each party exceeds the value of his null alternative,  $W^0$  is the set of wage rates that are feasible given no agreement to compensate. Both employer and worker will agree to the match and to compensate if and only if the wage paid is an element of  $W^*$ .

Obviously,  $y^* \cong \underline{x}_1 + \underline{x}_2$  is necessary for the existence of a match. The following result implies that it is also sufficient. Furthermore, the fact that the set of wage rates for which compensation in the event of a future separation is preferred by both is a proper subset of the set of feasible wage rates implies that each party requires a positive share of the rents, as measured by  $y^* - \underline{x}_1 - \underline{x}_2$ , in return for an agreement to compensate.

Proposition 6: If  $y^* > \underline{x}_1 + \underline{x}_2$ , then  $W^0$  is not empty and  $W^*$  is a non-empty proper subset of  $W^0$ .

The proof, which is given in the Appendix, establishes that the set of possible wage rates is a closed sub interval of  $[0, p]$  and that compensation will not be agreed to by one of the parties if the wage is near either end of this sub interval. Hence, if the compensation is part of the employment agreement, the rents of the match,  $y^* - \underline{x}_1 - \underline{x}_2$ , are necessarily shared. The proposition also suggests that each party can use his power to agree or not to contingent compensation in the event of a future separation as a threat in the wage negotiations. Figure 1 is constructed in part for the purpose of

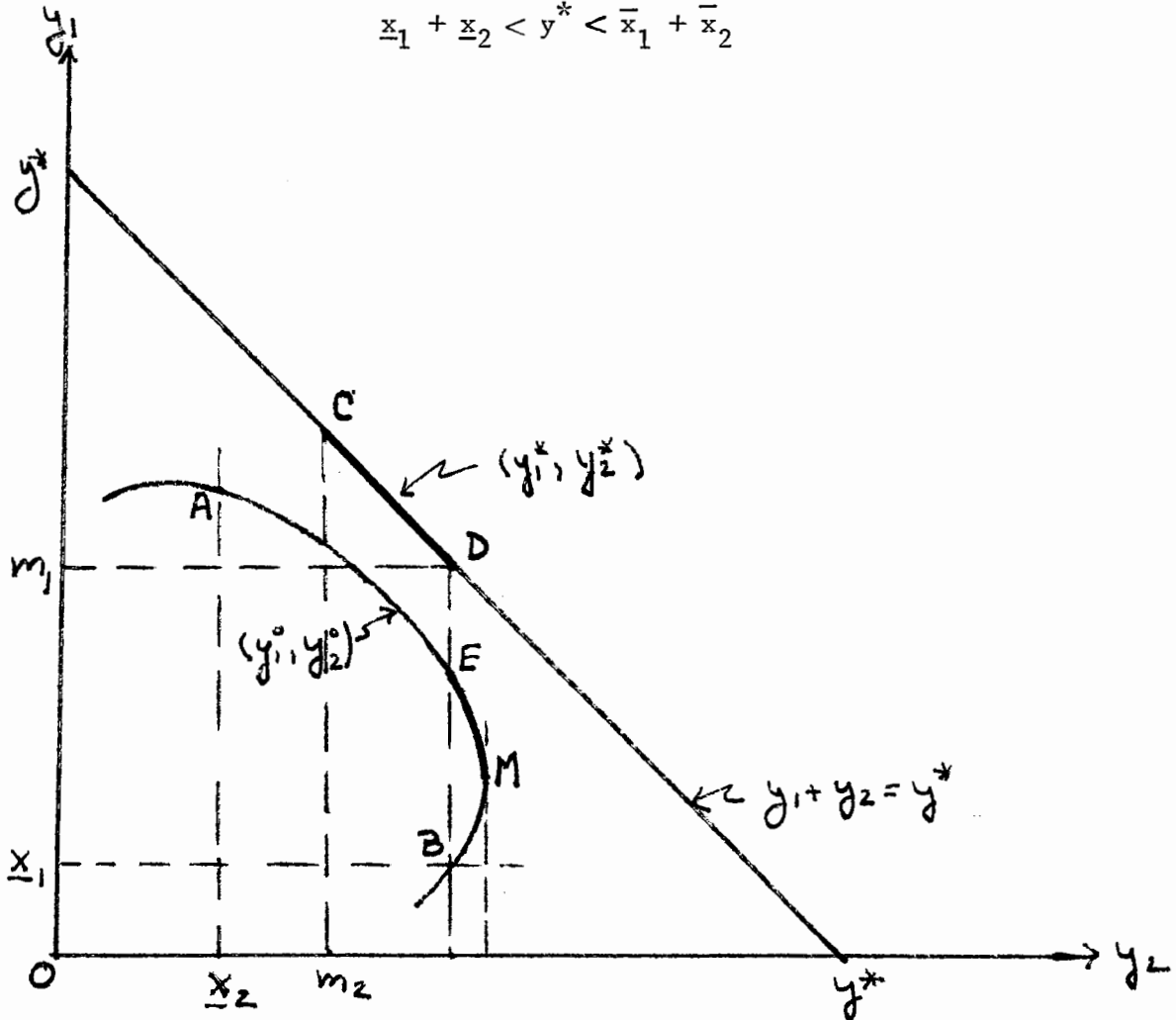
illustrating the latter point.

The curve AB in the Figure is the locus of all solutions of (12) associated with the set of feasible wage rates  $W^0$ . In other words, the parameter of the curve is  $w$  and the curve is generated by varying  $w$  from one end of the feasible wage interval to the other. It represents the set of feasible capital value combinations given no agreement to compensate. That  $y_1^0 + y_2^0$  is always maximum for some wage in the interior of the feasible wage interval can be derived from (12). That either  $y_1^0$  or  $y_2^0$  is maximum on the interior, as illustrated, is a possibility although neither is a necessary property.

The set of all combinations such that  $y_1 + y_2 = y^*$  lie on a line above and to the right of AB because  $y^* > y_1^0 + y_2^0$  from Propositions 3 and 5. Those combinations associated with wage rates for which the agreement to compensate is preferred by both, the set  $W^*$ , are represented by the line segment CD. At the wage rate for which  $m_1 = y_1^0 = y_1^*$ , the worker is indifferent between compensation and no compensation. Similarly, the wage rate at which  $m_2 = y_2^0 = y_2^*$  defines the point C. That the smallest capital value which either agent can expect given compensation,  $m_i$ , exceeds the value of his null alternative  $\underline{x}_i$  is, of course, implied by Proposition 6. Because it can be shown that the points C and D both converge to  $(\bar{x}_1, \bar{x}_2)$ , the best alternative combination, on the maximal capital value of the match,  $y^*$ , converges to their sum  $\bar{x}_1 + \bar{x}_2$ , the combinations on the segment CD do not generally vector dominate all combinations on the curve AB, as illustrated. In other words, an agreement to compensate does not generally Pareto dominate no compensation because sharing

Figure 1: Feasible Capital Values

$$\underline{x}_1 + \underline{x}_2 < y^* < \bar{x}_1 + \bar{x}_2$$



the rents is necessary to get such an agreement.

Suppose worker and employer have symmetric power so that reaching an agreement is a bilateral bargaining problem. Presumably their agreement will be Pareto optimal which implies that they agree either to compensate and select a wage that yields a point on the line segment CD or not to compensate and select a wage that yields a point on the curve segment EM. However, a fully



informed worker observing Figure 1 would threaten not to join the match unless the agreement included compensation since he could do no worse than  $y_1 = m_1$ , which dominates all possibilities for him on the curve of segment EM. Because  $m_2$ , the worst capital value that the employer would receive from such an agreement, dominates the value of holding the job vacant,  $x_2$ , the worker's threat is effective. Because  $(m_1, m_2) > (x_1, x_2)$  and any undominated parts of AB are at the extreme ends, this argument for agreement to compensate as the outcome of a symmetric bilateral bargain, though heuristic, is general.

However, if the problem is not symmetric and specifically if the employer has the power to set the wage, the solution is different. In this case, the employer obtains his maximum capital value at the monopsony combination represented by the point M in Figure 1. He does so even though the workers refuse to cooperate by accepting a mutual ex ante agreement to compensate. It is of interest to note that M, which is the solution that Pencavel [1972] studies, may be such that rents are shared to some extent; i.e.,  $(y_1, y_2) > (x_1, x_2)$  at M.

Although the firm with many employees has monopsony power given the immobility responsible for specific human capital, the workers have an obvious incentive to organize. But, notice that if they do and thereby achieve through bargaining a mutual agreement to compensate, the effect is more than a simple redistribution of rents in the workers' favor. In fact, the total value of each job-worker match increases as a consequence of the reduction in turnover. Hence, the analysis suggests two hypotheses. First, unionization is likely when the maximal total capital value of each match is high relative to that associated with the monopsony solution. Second, unionized firms are more likely to adopt mechanisms that encourage joint wealth maximizing turnover behavior than non-unionized firms of similar employee size and composition.

. The Empirical Evidence

Whether or not wealth maximizing search strategies are pursued by the parties to a match is ultimately an empirical question. In this section, clues to an answer are sought in the existing empirical literature on labor turnover. The studies reviewed for this purpose include recent work by Viscusi [1976], Bartel and Borjas [1976], Medoff [1976] and Freeman [1978] as well as those discussed by Parsons [1977] in his recent review article. Those of particular interest are cross section studies using data on either individual workers or industries.

An important empirical implication common to the specific human capital approach, whether search is joint wealth maximizing or not, is that all turnover rates decline with the specific capital value of the match.

---

In the joint wealth maximizing version of the model, this implication is obtained as follows. First, both the quit probability and dismissal probability for any match decline with the maximal capital value of the match by virtue of Proposition 5. Second, the equations of (13), when added, imply that the maximal capital value of any match increases with the value of the match specific flow,  $p$ , holding the distributions of alternatives available to both employer and worker constant. If the wage paid to the worker and the profit obtained by the employer both increase with the value of the joint product flow as Parsons [1972], Pencavel [1972] and others have argued, then the implication holds for the non-joint wealth maximizing case as well by virtue of Proposition 1 and the equations of (12).

The empirical problem of testing this implication of the theory arises as a consequence of the fact that specific human capital is not directly observable. Standard worker characteristic variables, such as education, age, etc., account for differences in general rather than specific human capital. Although the value of the product of a specific match may increase with the general human capital of the worker, so do the capital values of all the worker's alternatives. The predicted effect on the quit probability is ambiguous although the dismissal probability should decline with general ability.

The variable almost universally used in turnover studies interpretable as an indicator of specific human capital differences is the duration of the match or job tenure. The argument justifying this interpretation is that the extent of job specific training increases with the tenure of the match through a process of learning on the job. Although there are reasonable alternative interpretations of the phenomena, all existing evidence supports the prediction that turnover rates decline with tenure.

Certainly the major empirical implication that distinguishes the joint wealth maximizing hypothesis from the alternative is that both the probability of a worker initiated separation and the probability of termination by the employer are independent of at least marginal changes in the wage rate. Evidence from most of the earlier work on quit behavior, studies that include Stoikov and Raimon [1968], Burton and Parker [1969] and Pencavel [1972], seem to contradict this implication. The most important contradictory evidence is Parsons' [1972] result that quit rates are negatively associated and layoff rates are positively associated with wages across industries. This last study

offers the strongest support for the proposition that neither party takes account of the loss that his search behavior imposes on the other. However, more recent evidence when interpreted in the context of the theory developed here suggest another explanation.

Essentially, the basis of the alternative explanation is that the wage proxies for unobserved determinants of the capital value of the match. The details of the argument are best expressed in terms of the following simple econometric model. Let  $z$  represent a vector of relevant variables sufficient to explain all systematic differences between the maximal capital values of all possible matches. Assume that  $y = \alpha_0 + \alpha z + \epsilon_1$  where  $\epsilon_1$  is a random error with zero mean uncorrelated with  $z$ . Without loss of generality, the elements of  $z$  can be defined so that the vector of coefficients,  $\alpha$ , is positive. In other words, an increase in the  $k^{\text{th}}$  element, the tenure of the match for example, increases the maximal capital value. If the total capital value of each match is "shared" by worker and employer, then the wage also increases with  $z$  across job-worker matches; i.e.,  $w = \beta_0 + \beta z + \epsilon_2$ ,  $\beta > 0$  where  $\epsilon_2$ ,  $E\epsilon_2 = 0$ , represents non-systematic random variation not correlated with  $z$ .

In any given study not all the components of  $z$  are observed. Partition the vector  $z$  into its observed component,  $z_1$ , and unobserved component,  $z_2$ , respectively. Then

$$y = \hat{\alpha}_0 + a_1 z_1 + u_1, \quad Eu_1 = 0 \quad (16)$$

and

$$w = \hat{\beta}_0 + \beta_1 z_1 + u_2, \quad Eu_2 = 0, \quad (17)$$

where  $\hat{\alpha}_0 = \alpha_0 + \alpha_2 Ez_2$ , and  $\hat{\beta}_0 = \beta_0 + \beta_2 Ez_2$  are new constants and where  $u_1 = \alpha_2(z_2 - Ez_2) + \epsilon_1$  and  $u_2 = \beta_2(z_2 - Ez_2) + \epsilon_2$  are new error terms due in part to incomplete observation. Because  $\alpha_2 > 0$  and  $\beta_2 > 0$ ,  $u_1$  and  $u_2$  are positively correlated if  $\epsilon_1$  and  $\epsilon_2$  are independent. The relationship between the two error terms can always be represented as

$$u_1 = \delta u_2 + \epsilon, \quad \delta > 0, \quad E\epsilon = 0, \quad (18)$$

where by construction  $E(u_2 \cdot \epsilon) = 0$  so that  $\delta^2 = E(u_1 u_2) / E(u_2^2)$ .

Given joint wealth maximization, the two turnover rates decline with  $y$ ; i.e.,

$$q_i = \gamma_{0i} + \gamma_{1i} y; \quad \gamma_{1i} < 0 \quad (19)$$

for  $i = 1$  and  $2$ . By using equations (16), (17), and (18) to eliminate  $y$ ,  $u_1$  and  $u_2$ , one obtains the regression model

$$q_i = \hat{\gamma}_{0i} + \gamma_{1i} \delta w + \gamma_{1i} (\alpha - \gamma \beta_1) z_1 + \gamma_{1i} \epsilon \quad (20)$$

$i = 1$  and  $2$  where  $\hat{\gamma}_{0i} = \gamma_{0i} + \gamma_{1i} [\hat{\alpha}_0 - \delta \hat{\beta}_0]$ .

Since the errors in equations (16) and (17) are positively correlated ( $\delta > 0$ ) and since the quit rate declines with total capital ( $\gamma_{11} < 0$ ), the predicted sign of the wage coefficient in any particular study of quit behavior is negative as **often** observed even though the wage has no causal effect on the quit rate under the joint wealth maximizing hypothesis. Furthermore, the absolute values of the estimated coefficients of  $z_1$  are biased downward as estimates of the true

effects of observed capital value determinants on the quit rate because positive differences in  $z_1$  contribute positively to observed wage differences across job-worker matches. Similar conclusions can be drawn on the assumption that the elements of  $z$  are imperfectly measured. In this case, variations in the wage will enter the regressions as a proxy for these errors in variables.

The most important implication of the econometric model concerns comparisons of estimates obtained from different studies. Formally, as more elements of the sufficient capital value determinant vector  $z$  are included in the observed vector  $z_1$  the correlation between  $u_1$  and  $u_2$  generally declines and, in the limit when  $z = z_1$ , is zero if  $\epsilon_1$  and  $\epsilon_2$  are uncorrelated. Hence, studies in which more and better capital value proxies are used should obtain estimated wage effects and estimated capital value proxy coefficients that are respectively smaller and larger in absolute value.

In the earlier studies of quits referred to above, cross industry data were used. When tenure is included at all, it is crudely measured by rough indicators of tenure distribution differences. More recent studies, such as those of Viscusi [1976] and Bartel and Borjas [1976], use data on individual workers. The data sets include exact job tenure information. Both authors also include measures of fringe benefits and, in the results for older men based on the National Longitudinal Survey, a variable representing coverage by a pension plan. In the light of our theory these latter variables, particularly pension coverage, can be regarded as additional measures of the capital value of the match. In all results reported by both authors the wage effect is weakly negative while tenure, the extent of fringe benefits and coverage by a pension plan all have significant negative coefficients in a quit probability model.

Indeed, the estimated wage effect is neither statistically significant nor robust with respect to sign in the results that Viscusi reports in which quit intentions are used as the dependent variable. The fact that union members are less likely to quit than non-union members, other things equal, reported by Freeman [1978], Medoff [1976] and Viscusi [1976] can be interpreted as additional support for the hypothesis both because the bilateral bargaining formulation is more attractive in the context of an organized industry and because the worker's incentives to organize are greater the larger is the maximal capital value of the match.

Of course, the argument used to obtain equation (16) applies equally well to the case of employer initiated separations ( $i = 2$ ). Here we seem to have a clear contradiction since  $\gamma_{21} < 0$  and  $\delta > 0$  imply that the estimated wage effect is negative but Parsons [1972], in a study of cross industry data on layoff rates, and Bartel and Borjas [1976], in a cross worker study of layoff probabilities, both report a significant positive wage effect in layoff equations.

Our first point in rebuttal is that the theory developed in this paper is not an explanation of layoffs as usually defined. This point is important because theoretical work by Feldstein [1976] and Bailey [1977] and confirming empirical studies by Lilien [1977] and Medoff [1976] respectively suggest and substantiate the hypothesis that the majority of all terminations initiated by the employer "without prejudice" to the worker, as layoffs are defined by the Bureau of Labor Statistics, are temporary. The theory has little to do with the process of job-worker matching. Indeed, Medoff's results imply that layoff rates across industries have a significant positive association with the extent of unionization and that wage rate differences

have no independent explanatory power, which suggest that wage rates have simply acted as proxies for unionization in earlier studies.

Our second point of rebuttal is also an appeal to Medoff's [1976] empirical work. In other regressions reported in the same paper in which the discharge rate, as defined by the Bureau of Labor Statistics, is the dependent variable the wage rate has a significant negative coefficient and the coefficient on the extent of unionization is also negative, though not particularly significant. These results, particularly since differences in tenure are not taken into account, can be interpreted as strong evidence in support of the argument used to derive equation (16) for the case of  $i = 2$ .

Obviously, the arguments and evidence reported in this section do not confirm the joint wealth maximizing hypothesis. However, they do establish that our current empirical knowledge about turnover behavior does not contradict the hypothesis as first impression might suggest. A definitive test is clearly going to require a much more sophisticated methodology. Its development is a topic for future research.



Appendix: Proof of Proposition 6

Define  $w_1^0$  and  $w_2^0$  as the wage rates at which the worker and employer are respectively indifferent between the match and his null alternative, i.e.,

$$y_i^0(p, w_i^0) = \underline{x}_i, \quad i = 1 \text{ and } 2. \quad (21)$$

Because the worker has the option of searching for a job while unemployed and the value of that option can be no less than the value of working at a wage of zero,  $\underline{x}_1 \geq y_1^0(p, 0)$ . The existence of the employer's option to hold the job vacant analogously implies that  $\underline{x}_2 \geq y_2^0(p, p)$ . Since  $q_i^0 = \lambda_i^0 [1 - F_i(\eta_i^0)]$ , the equations of (12) and the Envelope Theorem imply

$$\left. \frac{\partial y_1^0}{\partial w} \right|_{w_1^0} = \frac{1}{r + q_1^0 + q_2^0} > 0$$

and

$$\left. \frac{\partial y_2^0}{\partial w} \right|_{w_2^0} = \frac{-1}{r + q_1^0 + q_2^0} < 0$$

These facts collectively imply that  $w_1^0$  is unique and greater than 0,  $w_2^0$  is unique and less than  $p$ , and  $W^0$  is the closed interval  $[w_1^0, w_2^0]$  if  $w_2^0 \geq w_1^0$  and is empty otherwise by virtue of (14). Let

$$f_i(z) = \max_{(\lambda, \eta)} \left\{ \lambda \int_{\eta}^{\bar{x}_i} (x - z) dF_i(x) - c_i(\lambda) \right\}. \quad \text{Because } f_i(z) \text{ is}$$

decreasing and  $\bar{x}_i > \underline{x}_i$  implies that  $f_i(\underline{x}_i) > 0$ , the equations of (12) and definition (21) imply

$$r\underline{x}_1 = w_1^0 + f_1(\underline{x}_1) > w_1^0 + f_1(y^* - \underline{x}_2) \quad (22.a)$$

and

$$r\underline{x}_2 = p - w_2^0 + f_2(\underline{x}_2) > p - w_2^0 + f_2(y^* - \underline{x}_1) \quad (22.b)$$

if  $y^* > \underline{x}_1 + \underline{x}_2$  and  $(\bar{x}_1, \bar{x}_2) > (\underline{x}_1, \underline{x}_2)$  as assumed. Consequently,

$$\begin{aligned} w_2^0 - w_1^0 &> p + f_1(y^* - \underline{x}_2) + f_2(y^* - \underline{x}_1) - r(\underline{x}_1 + \underline{x}_2) \\ &= r(y^* - \underline{x}_1 - \underline{x}_2) > 0 \end{aligned}$$

by virtue of (13) .

Because  $y^* = y_1^* + y_2^*$  is independent of  $w$  and

$$\frac{\partial y_1^*}{\partial w} = \frac{1}{r} = - \frac{\partial y_2^*}{\partial w}$$

from (13) and because  $y_1^* + y_2^* \cong y_1^0 + y_2^0$  from Proposition 5,  $W^0$  and  $W^*$  have elements in common by virtue of (15). However, because (13) and (22) imply  $r\underline{x}_1 > ry_1^*(p, w_1^0)$  and  $r\underline{x}_2 > ry_2^*(p, w_2^0)$ , a neighborhood on either end of  $[w_1^0, w_2^0]$  is not in  $W^*$ . Q.E.D.

REFERENCES

- Azariadis, Costas, "Implicit Contracts and Underemployment Equilibria," Journal of Political Economy, vol. 83, no. 6, 1975.
- Baily, Martin N., "Wages and Unemployment Under Uncertain Demand," Review of Economic Studies, vol. 41, no. 125, 1974.
- Baily, Martin N., "On the Theory of Layoffs and Unemployment," Econometrica, vol. 45, no. 5, July, 1977.
- Bartel, Ann. P., and George J. Borjas, "Middle-Age Job Mobility: Its Determinants and Consequences" National Bureau of Economic Research, Sept. 1976.
- Becker, Gary S. and Elizabeth M. Landes, "Economics of Marital Instability," NBER Working Paper no. 154, Oct. 1976.
- Becker, Gary S. Human Capital, National Bureau of Economic Research, 1964.
- Bellman, Richard E. Dynamic Programming, Princeton University Press, 1957.
- Burdett, Kenneth, "Employee Search and Quits" forthcoming in American Economic Review, 1977.
- Burton, John, and John Parker, "Interindustry Variations in Voluntary Labor Mobility," Industrial and Labor Relations Review, vol. 22, no. 2, 1969.
- Feldstein, Martin S., "Temporary Layoffs in the Theory of Unemployment," Journal of Political Economy, vol. 84, Oct. 1976.
- Feller, William. An Introduction to Probability Theory and Its Applications, Vol. I, Wiley and Sons, 1968.
- Freeman, Richard B. "The Exit-Voice Tradeoff in the Labor Market: Unionism, Job Tenure, Quits and Separations," Harvard Institute of Economic Research, 1978.
- Freeman, Richard B., and James L. Medoff, "What Do Unions Do? A Progress Report," Harvard Discussion Paper no. 586, Nov. 1977.
- Hall, Robert E. "Turnover in the Labor Force," Brookings Papers of Economic Activity. 3, 1972.
- Lilien, David M., "The Cyclical Pattern of Temporary Layoffs in U.S. Manufacturing," presented at the NBER Conference on Unemployment, Stanford, California, Jan., 1978.

- Medoff, James L., "Layoffs and Alternatives under Trade Unions in U.S. Manufacturing," Harvard Discussion Paper no. 525, Dec., 1976. Forthcoming in the American Economic Review.
- Oi, Walter. "Labor as a Quasi-Fixed Factor," Journal of Political Economy, 70, Dec., 1962.
- Parsons, Donald O. "Specific Human Capital: An Application to Quit and Layoff Rates." Journal of Political Economy, Dec. 1972.
- Parsons, Donald O. "Models of Labor Market Turnover, A Theoretical and Empirical Survey," Research in Labor Economics, Vol. 1, 1977.
- Pencavel, John H. "Wages, Specific Training and Labor Turnover in U.S. Manufacturing Industries." International Economic Review, Feb. 1972.
- Simon, Herbert A., "A Formal Theory of the Employment Relationship," Econometrica, vol. 19, no. 3, July, 1951.
- Stoikov, Vladimir and Robert R. Raimon, "Determinants of the Differences in the Quit Rate Among Industries," American Economic Review, Dec., 1968.
- Viscusi, W. Kip. Employment Hazards: An Investigation of Market Performance, a Harvard Dissertation, 1976.
- Viscusi, W. Kip, "Unions, Labor Market Structure, and the Welfare Implications of the Quality of Work," mimeo, Northwestern University, Nov. 1977.

Footnotes

<sup>1</sup>The hypothesis is implicit in Oi [1962] and is discussed by Becker [1964] but it's stated most explicitly by Parsons [1972]. See Parsons [1977] for a recent review of the literature on the specific human capital approach to labor turnover.

<sup>2</sup>Both the approach and the model developed are applicable to other bilateral matching phenomena. The recent analysis of divorce by Becker and Landes [1976] is very similar in spirit.

<sup>3</sup>The search model used is a generalization of that recently developed by Burdett [1977].

<sup>4</sup>A more general expression of this idea is contained in Viscusi [1977].

<sup>5</sup>A formal justification of this argument is given by Feller [1968, pp. 447-448].

<sup>6</sup>The proof only requires elementary differentiation and the observation that  $c_i''(\lambda) < 0$  by assumption.

<sup>7</sup>Becker and Landes [1976] claim that the counter offer mechanism does induce joint wealth maximizing behavior in their paper on divorce. The assertion is incorrect in our model because search intensities are endogeneous and because the counter offer mechanism does not eliminate the externality with respect to this choice.