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REGULATORY PRICING POLICIES
and
INPUT CHOICES UNDER UNCERTAINTY

by

David P. Baron and Robert A. Taggart, Jr.

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ABSTRACT

This paper presents a model of a firm subject to uncertain demand and *ex ante* rate-of-return regulation. It is shown that if the regulator acts in a "sophisticated" manner, not allowing the firm's capital stock to influence the regulated price, the firm produces efficiently. If the regulator acts "naively," however, and sets price in response to the firm's capital stock, overcapitalization results. Although both types of regulation are effective in holding price below the monopoly level, naive regulation results in a larger capital stock, a higher price and smaller certainty-equivalent output than does sophisticated regulation.

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Analysis of the regulated firm's input choices has been dominated by the deterministic Averch-Johnson (1962, hereafter AJ) model in which the firm's profits are constrained by an allowed rate of return on capital. AJ showed that when the allowed rate of return is set between the cost of capital and the unconstrained monopoly rate of return, the firm will choose a capital-labor ratio larger than that which minimizes cost at the level of output produced.

Extension of the AJ model to a world of uncertainty has not been straightforward, however, because results have proved sensitive to assumptions about regulatory behavior. Under several sets of conditions, the AJ results have been shown not to hold. It has been recognized, for example, that it is more realistic to view the regulator as setting an output price for the firm which he perceives to be consistent with a fair rate of return, in contrast to AJ's assumption that the regulator sets the allowed return and the firm may then choose any price consistent with that return.[1] But since _ex post_ and _ex ante_ returns may diverge under uncertainty, it makes a difference whether the price is set before or after the uncertainty is resolved. Leland (1974) has shown, contrary to AJ, that when price is set to allow a fair _ex ante_ return, the firm can be induced to choose an efficient input combination. Peles and Stein (1976) and Perrakis (1976a), on the other hand, considered models in which price is adjusted _ex post_, and have shown, likewise contrary to AJ, that the optimal capital stock under regulation need not exceed that of an unregulated monopolist.

The present paper deals with a model in which demand is uncertain and shows that the AJ undercapitalization result obtains when the regulator sets price *ex ante* but allows that price to be influenced by the firm's capital stock choice. If the regulator is "sophisticated" and sets the price independent of the firm's actual capital stock choice, then as in Leland's analysis the firm will have an incentive to produce efficiently. But if the regulator acts "naively" and allows the firm's input choices to influence his pricing decision, the firm will be able to anticipate and exploit the regulator's behavior and will find it advantageous to produce inefficiently. The AJ model can be interpreted as a price-influence model of this type.

In addition to analyzing technical efficiency of input choices under sophisticated and naive regulation, the present paper characterizes the optimal solutions to the firm's and the regulator's decision problems. With sophisticated regulation, the firm's optimal capital stock is shown to be a decreasing function of the regulated price, and the optimal regulated price is an increasing function of the allowed rate of return. The optimal price is second-best since it equals certainty-equivalent average cost plus an increment reflecting the difference between the allowed rate of return and the cost of capital. With naive regulation, the firm's optimal capital stock is also a decreasing function and the regulated price an increasing function of the allowed rate of return. The optimal price and capital stock are greater with naive than with sophisticated regulation, but compared with the unregulated monopoly solution naive regulation results in a lower price, larger capital stock and greater certainty-equivalent output. Fair rate of return regulation even of the naive type thus yields some benefits to consumers.

The setting of the model, including the firm's production possibilities, the regulatory environment and the securities market are described in Section 1. Sections 2 and 3 consider the firm's investment decision and the regulated price under sophisticated and naive regulation, respectively and conclusions are offered in the final section.

1. The Firm and the Regulatory Setting

The model considered in this paper focuses on *ex ante* regulation for a single period within a multi-period horizon, where the length of a period is the time needed by the firm to change its capital stock. This might correspond, for example, to the time required for delivery of new capital equipment. Since there are no other adjustment costs besides this waiting time, the firm accumulates the capital stock, K_n, for any period n which it deems optimal for that period, regardless of its previously existing capital stock or its expectations of optimal capital stocks in future periods. Consequently, both the regulator and the firm can treat their decisions during a given period as independent of decisions in other periods. As is indicated below, the regulator and the firm can revise their decisions each period in light of new information. Regulation within a period is thus of the *ex ante* type, while *ex post* revisions are made between periods.

The demand, $q_n(p_n, \theta_n)$, occurring during any period n is negatively related to the regulated price, p_n, and also depends on the state of nature, θ_n. It is assumed that θ_n is not revealed until after the regulated price has been set and the firm has fixed its capital stock for the duration of the period. The capital stock is thus chosen *ex ante*, but since the firm is obligated to satisfy whatever demand occurs, it must use labor, L_n, as an <u>ex post</u> factor of production.[2]

The firm's <u>ex post</u> production function is expressed as $f(K_n,L_n)$, and for a given capital stock the required labor input satisfies

$$q_n = q_n(p_n,\theta_n) = f(K_n,L_n). \tag{1}$$

The standard assumptions are made about the marginal productivities of the factors:[3]

$$\frac{\partial q_n}{\partial K_n} > 0,\ \frac{\partial q_n}{\partial L_n} > 0,\ \frac{\partial^2 q_n}{\partial K_n^2} < 0,\ \frac{\partial^2 q_n}{\partial L_n^2} < 0,\ \frac{\partial^2 q_n}{\partial K_n \partial L_n} > 0.$$

For a given K_n, the amount of labor required to satisfy demand may be written as

$$L_n = L(q_n,K_n),$$

where the properties of this function are derivable from those of the production function.[4]

The relationship between the regulatory process and the firm's production decisions is then as follows: The regulator sets a price, p_n, and the firm provides a capital stock, K_n, both of which take effect at the beginning of period n.[5] The regulated price is set so that the firm is expected to earn a fair rate of return <u>ex ante</u>.[6] The state of nature, θ_n, is then revealed, and the firm employs the variable factor input, in conjunction with its predetermined capital stock, to meet demand. It is further assumed that no adjustment of the regulated price will take effect until the beginning of the next period - that is, not until sufficient time has elapsed for the firm to alter its capital stock. The firm's <u>ex post</u> rate of return in any period can differ from the <u>ex ante</u> fair return, therefore, because no <u>ex post</u> price adjustment is made within the period.[7]

Once θ_n is known, both the regulator and the firm have new information and can revise their expectations of θ_{n+1}. Diverse expectations are aggregated by a securities market in the present model, and it will be shown that the regulator can use market information to set a price, p_{n+1}, for the next period. The new price is not influenced by the firm's existing capital stock, however, and does not take effect until after the firm has had time to alter its capital stock. Thus the analysis can focus on regulation and production for a single period, because the periods are linked only by information about future states of the world contained in present and past states.[8] Accordingly, all time subscripts will hereafter be dropped.

Finally, the securities market in which investors evaluate the firm's prospects is assumed to be complete within a period. As of the beginning of each period a price exists for a dollar of return to be delivered in any of the uncertain states θ which may occur in that period.[9] In equilibrium, marginal rates of substitution between certain and uncertain returns will be equated for all investors, and the present value of a dollar of return if θ occurs will be denoted by $\rho(\theta)/r$, where r is the gross ($r > 1$) risk-free interest rate.

For a given regulated price, p, and capital stock, K, the firm will earn an ex post profit, $\pi(p,K,\theta)$, given by

$$\pi(p,K,\theta) = pq(p,\theta) - wL(q,K), \tag{2}$$

where w is the wage rate. The market value, V, of the firm is the present value of these ex post profits, or[10]

$$V = \int\rho(\theta)\pi(p,K,\theta)d\theta/r. \tag{3}$$

Since $\int\rho(\theta)d\theta = 1$, the $\rho(\theta)$ can be interpreted as market "probabilities," and V may in turn be interpreted as the (present) market "expected value" of the firm's ex post profits.[11]

2. Sophisticated Regulation

A. The Firm's Investment Decision

If the regulator sets a price, p, and the firm perceives that its own investment decision will have no influence on p, then the firm will best serve its shareholders' interests by choosing that capital stock, K, given p, which maximizes the net value of the firm, V-K. That is, the firm's problem may be written as

$$\max_{K}\left(V-K = \int \frac{\rho(\theta)}{r}(pq(p,\theta)-wL(q,K))d\theta - K\right).$$

The optimal K is thus that which satisfies[12]

$$-\int \rho(\theta)\, \frac{\partial L}{\partial K}\, d\theta = \frac{r}{w}\,, \tag{4}$$

which says that the ex ante marginal rate of substitution between labor and capital is equal to the factor price ratio. This establishes the following counterpart of Leland's (1974) result:

Proposition 1: For any demand function, the firm's optimal input combination under sophisticated regulation is ex ante efficient.

B. The Regulated Price and the Allowed Rate of Return

The previous section has established that for any given regulated price the firm will choose the efficient capital stock consistent with that price. The only restrictions on this result are that the firm must perceive the regulated price to be unresponsive to its own decisions and that the price must be neither so high nor so low as to drive the firm out of business. The problem that remains is to choose a price within this range.

The difference between the market value of the firm and the replacement value of its capital stock is a measure of the firm's capitalized monopoly rents. Expressing these rents as a percentage (s-1) of the firm's capital stock, or V - K = (s-1)K, the regulator can restrict the size of the rents by specifying a maximum allowable value of s.[13] The regulator must then choose that price, p*, such that the efficient capital stock consistent with p* is also consistent with V* = sK*.

From (3) a price p* such that V* = sK* implies

$$\frac{\int\rho(\theta)\pi(p^*, K^*, \theta)d\theta}{K^*} = rs. \tag{5}$$

Since the left-hand side of (5) can be interpreted as the firm's market "expected" rate of return on invested capital and the right-hand side as an allowed rate of return (expressed as a percentage of the cost of capital), we have

Proposition 2: For a regulated price that yields V = sK, the market "expected" rate of return equals the allowed rate of return.[14] Since K* must be technically efficient _ex ante_, (4) may be used to eliminate r from (5), yielding

$$\frac{\int\rho(\theta)(p^*q(p^*,\theta) - wL(q, K^*))d\theta}{sK} = -\int\rho(\theta)\,w\,\frac{\partial L}{\partial K}\,d\theta. \tag{6}$$

The optimal price p*, corresponding to a given value of s, must then satisfy

$$p^* = \frac{\int\rho(\theta)wL(q,K^*)d\theta}{\int\rho(\theta)q(p^*,\theta)d\theta}\left(1 - s\,\frac{K\int\rho(\theta)\frac{\partial L}{\partial K}\,d\theta}{\int\rho(\theta)L(q,K^*)d\theta}\right) = w\,\frac{E(L)}{E(q)}\,(1-s\eta), \tag{7}$$

where E(L) is the market "expected" labor requirement, E(q) is "expected" output and η is the elasticity of the labor requirement with respect to the capital stock.[15]

The effect on p* and K* of lowering s can be evaluated by totally differentiating V*(K*(p*),p*) = sK* to obtain

$$\frac{\partial V}{\partial K^*}\frac{dK^*}{dp^*}\,dp^* + \frac{\partial V}{\partial p^*}dp^* = s\frac{dK^*}{dp^*}\,dp^* + K^*ds.$$

For a change in s,

$$\frac{dp^*}{ds} = K^* \Big/ \left(\frac{dK^*}{dp^*}\left(\frac{\partial V}{\partial K} - s\right) + \frac{\partial V}{\partial p^*}\right). \tag{8}$$

The relationship F between K* and p* is implicitly defined by (4) as

$$F(K^*,p^*) \equiv -\int \rho(\theta)w\frac{\partial L}{\partial K}d\theta - r = 0,$$

so

$$\frac{dK^*}{dp^*} = \frac{-\partial F/\partial p^*}{\partial F/\partial K^*}.$$

The denominator, $\partial F/\partial K^*$, is negative by the assumed properties of the production function and the numerator, $\partial F/\partial p^*$, is negative, since expected demand declines with p*. This yields

Proposition 3: A decrease in the regulated price increases the firm's optimal capital stock.[16]

Since the firm maximizes V-K, $\partial V/\partial K = 1$ at K* and the term $\frac{dK^*}{dp^*}(\frac{\partial V}{\partial K} - s)$ in (8) is positive for $s > 1$. To determine the sign of dp^*/ds, it remains to evaluate $\partial V/\partial p^*$ at $K = K^*$.

An unregulated monopolist raises price until $\partial V/\partial p = 0$, so $dp^*/ds > 0$ for $s = s_M$. For values of s marginally below s_M, therefore, $p^* < p_M$ and the firm's maximum attainable value of V-K must be below the monopolist's V-K. Formally, the firm's problem is $\max_{K,p} L = V-K-\mu(p-p^*)$, where μ is a Lagrangian multiplier, and the first order optimality conditions are

$$\frac{\partial L}{\partial K} = \frac{\partial V}{\partial K} - 1 = 0 \tag{9a}$$

$$\frac{\partial L}{\partial p} = \frac{\partial V}{\partial p} - \mu = 0 \tag{9b}$$

$$\frac{\partial L}{\partial \mu} = p - p^* = 0 \tag{9c}$$

The multiplier μ represents the change in the maximum attainable value of V-K given an increase in p*. This must be positive for the value of p* under consideration, because an increase in the regulated price would allow the firm to move along its expansion path (according to 9a) back toward the monopoly solution. From (9b), then, $dp^*/ds > 0$ for values of s marginally below s_M. Furthermore, $d(V-K)/dp = \partial V/\partial p + (\frac{\partial V}{\partial K} - 1)\frac{dK^*}{dp^*} = \partial V/\partial p$ along the expansion path, so as s is lowered further, p goes down and the maximum attainable value of V-K is lowered further. Thus the argument above can be repeated for all s such that $1 \leq s < s_M$. This establishes

Proposition 4: For $1 \leq s \leq s_M$, a decrease in s reduces the optimal regulated price, increases the firm's optimal capital stock, and increases the market "expected" output.

The special case in which s = 1 exhibits two additional properties.[17] The first of these is a corollary of Proposition 4.

Corollary: At s = 1 the regulated price p* that eliminates all monopoly rents ($V^* = K^*$) is the minimum price satisfying V = K and hence, yields the maximum market "expected" output consistent with V = K. To see this, note that $V(p,K) - K = 0$ implicitly defines all (p,K) pairs such that V = K. By implicit differentiation

$$\frac{dp}{dK} = -\left(\frac{\partial V}{\partial K} - 1\right) / \frac{\partial V}{\partial p},$$

which equals zero at $K = K^*$, and

$$\frac{d^2p}{dK^2} = \frac{\partial\left(\frac{dp}{dK}\right)}{\partial p} \cdot \frac{dp}{dK} + \frac{\partial\left(\frac{dp}{dK}\right)}{\partial K} = -\frac{\partial^2 V}{\partial K^2} \Big/ \frac{\partial V}{\partial p},$$

which is positive at $K = K^*$. The price p* that yields $V^* = K^*$ at s = 1 thus results in the greatest market "expected" output. If all monopoly rents are eliminated, however, the firm will be indifferent between going into business in the first place and investing its funds elsewhere in the securities market. To provide the firm with an incentive to raise capital and begin production, the regulator may find it necessary to set s > 1.[18]

The second property involves the relationship between the regulated price and the firm's costs. From (2) and (6), for any p* such that $V = sK^*$,

$$rsK^* = \int\rho(\theta)p^*q(p^*,\theta)d\theta - \int\rho(\theta)wL(q,K^*)d\theta,$$

and
$$p^* = \frac{rsK^* + w\int\rho(\theta)L(q,K^*)d\theta}{\int\rho(\theta)q(p^*,\theta)d\theta} \ . \tag{10}$$

From this follows

Proposition 5: For s = 1, the optimal regulated price equals the average cost of an "expected" unit of output.

The optimal regulated price is not, in general, equal to marginal cost, but setting s = 1 is a second-best procedure which produces the maximum "expected" output consistent with the firm covering its "expected" total cost. If s > 1, then from (11) the regulated price includes a mark-up above average cost which reflects the firm's allowed monopoly rents.

3. Price Influence and Naive Regulation

A. The Firm's Investment Decision

If the regulator acts in a sophisticated manner, he sets an output price so that the best the firm can do is produce efficiently and earn the allowed rate of return. Furthermore, he does not alter this price even if the firm, for some reason, does not choose the optimal capital stock, K*, corresponding to p*. An alternative assumption is that the regulator is concerned only with the allowed rate of return, rs, and that he adjusts price in response to changes in the firm's capital stock so that the market "expected" return is always exactly equal to the allowed return. If the firm were to announce plans for a non-optimal capital stock in response to the regulator's initial price, for example, the regulator might immediately change the price so as to permit the firm a fair return on its intended capital stock. Under this form of passive, or naive, regulation the firm's capital stock decision will then be predicated upon influencing the regulated price.

The firm will wish to increase its capital stock in this case as long as

$$\frac{\partial(V-K)}{\partial K} = \frac{\partial V}{\partial K} - 1 + \frac{\partial V}{\partial p}\ \frac{dp}{dK} \tag{11}$$

is positive, where dp/dK reflects the price-influence effect. The regulated price will vary with K along the constraint V = sK, so differentiating this constraint yields

$$\frac{\partial V}{\partial K} + \frac{\partial V}{\partial p}\frac{dp}{dK} = s,$$

and substituting into (11) yields

$$\frac{\partial(V-K)}{\partial K} = s - 1. \tag{12}$$

The following proposition results:

Proposition 6: For s > 1, the firm will increase its capital stock to the point at which a further increment in K can no longer yield an increase of s in the value of the firm.

The efficiency of the optimal capital stock under naive regulation can be analyzed by comparing this model with the Averch-Johnson model.

B. Naive Regulation and the Averch-Johnson Model

In the AJ model, the firm sets its own price and may choose any price-capital stock combination that satisfies the allowed rate of return constraint. In the context of the model developed here, the AJ firm maximizes V-K with respect to p and K, subject to the constraint $V \leq sK$. Letting λ_N be a Lagrangian multiplier and assuming that the constraint is binding at the optimal (p_N, K_N), the necessary optimality conditions are

$$\frac{\partial V}{\partial K} - 1 - \lambda_N\left(\frac{\partial V}{\partial K} - s\right) = 0, \tag{13}$$

$$\frac{\partial V}{\partial p} - \lambda_N\frac{\partial V}{\partial p} = 0, \tag{14}$$

$$V - sK_N = 0. \tag{15}$$

Since $0 < \lambda_N < 1$ when $s > 1$,[19] (13) may be rewritten as

$$\frac{\partial V}{\partial K} = 1 + \lambda_N(1-s)/(1-\lambda_N).$$

Then, from (2) and (3)

$$-\int\rho(\theta)\,\frac{\partial L}{\partial K}\,d\theta \;=\; r/w + \lambda r(1-s)/(1-\lambda)w. \tag{16}$$

As indicated in Section IIA, the left side of (16) is the <u>ex ante</u> marginal rate of substitution (MRS) between labor and capital. Since $0 < \lambda < 1$, this MRS is less than the factor price ratio for $s > 1$ and overcapitalization results.

The optimal solution in this AJ formulation of the model can be shown to be the optimal solution under naive regulation. The firm facing naive regulation is unconstrained in its choice of K, but the regulator chooses the price p in order to provide the allowed return at the capital stock chosen by the firm. Since the firm knows the regulator's decision rule, it can predict the price that will be set for any capital stock it chooses. But this implies that the firm may manipulate K so as to achieve any price consistent with $V = sK$, and thus the decision problem of the firm under naive regulation is identical to that of the firm in the AJ model. Since the feasible set is the same in both cases, the solutions must be the same. This establishes

Proposition 7: The optimal capital stock and the regulated price under naive regulation, in which price is set in response to K to yield $V = sK$, are the same as the optimal solution to the Averch-Johnson model. Thus overcapitalization results for the firm's market "expected" output when $s > 1$.

The price-influence model is also analogous to the "capture" model of regulation.[20] Even if the regulator does not act for purely self-serving purposes, he can still be effectively captured and manipulated to the firm's advantage if the firm can exploit its knowledge of the regulator's decision rule to influence output price.

C. Changes is the Allowed Rate of Return

The effect of a decrease in the allowed rate of return can be found by totally differentiating V = sK with respect to s. Noting that p is a function of K under naive regulation, differentiation yields

$$\frac{\partial V}{\partial K}\frac{dK}{ds} + \frac{\partial V}{\partial p}\frac{dp}{dK}\frac{dK}{ds} = s\frac{dK}{ds} + K. \tag{17}$$

From (14), $\partial V/\partial p = 0$ at (p_N, K_N),[21] and using (13) to evaluate $\partial V/\partial K$ yields

$$\frac{dK}{ds} = \frac{K(1-\lambda_N)}{1-s} .$$

Since $0 < \lambda_N < 1$, $dK/ds < 0$ for $s > 1$. To evaluate the change in price, totally differentiate (14) with respect to K, p and s to obtain

$$\frac{\partial^2 V}{\partial p^2}\frac{dp}{ds} + \frac{\partial^2 V}{\partial p \partial K}\frac{dK}{ds} = 0.$$

V is assumed to be strictly concave in p and $\partial^2 V/\partial p \partial K$ is negative from the assumed properties of the production function. Thus price is an increasing function of s, establishing

Proposition 8: Under naive regulation, the firm's optimal capital stock increases and the price decreases as the allowed rate of return rs is lowered toward r. The market "expected" output of the firm under naive regulation with $s < s_M$ is thus greater than that for an unconstrained monopolist.[22]

Proposition 8 indicates that, even with naive regulation, restricting the rate of return, a firm can earn yields benefits to consumers.

D. Comparison of Solutions for the Sophisticated Regulation, Naive Regulation and Unregulated Monopoly Cases.

Comparison of the three solutions is facilitated by considering the geometric representation in Figure 1, where the hill depicts feasible values of V-K as a function of (p,K). The unregulated monopolist would maximize

(V-K) by choosing the point M corresponding to the price p_M and the capital stock K_M. The regulated firm must satisfy the constraint V = sK, which is represented by the plane of slope (s-1) in (V-K,K) - space, so the feasible set is the intersection of the (V-K) hill with the V = sK plane, as indicated by the heavy solid and dotted line. The firm facing naive regulation can influence price and will be able to achieve point N, the point of maximum V-K in the feasible set. The corresponding capital stock is K_N, and the firm's choice of K_N causes the regulator to set price at p_N. The sophisticated regulator, however, will choose price p*, the lowest price consistent with both efficient production and V = sK. Given p*, the best the firm can do is to achieve point S by choosing the capital stock K*.

Propositions 4 and 8 indicate that $K_N > K_M$, $K^* > K_M$, $p_N < p_M$ and $p^* < p_M$ for $1 < s < s_M$. To compare the solution (p_N,K_N) resulting from naive regulation with the solution (p^*,K^*) resulting from sophisticated regulation, suppose that a firm facing naive regulation were to start at point S. The optimality condition for K_N for the firm under naive regulation is given by (13) as $\partial V/\partial K = 1 + \lambda_N(1-s)/(1-\lambda_N)$ where $0 < \lambda_N < 1$. But since $\partial V/\partial K = 1$ at S, $\partial V/\partial K$ is too large, and by concavity of V, the firm will wish to increase K in order to approach the optimal solution. As the firm increases K, the regulator will respond by changing p, along the constraint V = sK. Differentiating this constraint with respect to K yields

$$\frac{\partial V}{\partial K} + \frac{\partial V}{\partial p}\frac{dp}{dK} = s,$$

or

$$\frac{dp}{dK} = \frac{s - \frac{\partial V}{\partial K}}{\frac{\partial V}{\partial p}} .$$

Since $\partial V/\partial K = 1$ and $\partial V/\partial p > 0$ at S, the regulator will increase price as the firm increases its capital stock. These results are summarized as Proposition 9: Starting from the sophisticated regulation solution, the firm under naive regulation increases its capital stock, forcing the regulator to increase the regulated price. The optimal solution with naive regulation entails a higher price and a greater capital stock than with sophisticated regulation.

As the allowed rate of return constraint is tightened below s_M, firms facing both naive and sophisticated regulation increase their capital stocks, but as indicated in the contour diagram in Figure 2, they move down different sides of the V-K hill. The concentric contours in Figure 2 represent the intersections of V-K with $V = sK$, with the outer contours corresponding to lower values of s. The firm facing naive regulation moves down the line MN, the locus of maximum capital stocks consistent with each value of s.[23] The firm under sophisticated regulation moves down the line MS, the locus of maximum efficient capital stocks consistent with each value of s.[24] As s approaches unity, there is no reason to expect the firm under naive regulation to move closer to the efficient locus, and at $s = 1$, this firm would be indifferent among any of the (p,K) combinations for which $V = K$. The firm under sophisticated regulation, by contrast, would be indifferent between going into business or not at $s = 1$, but if it did go into business, it would prefer K* in Figure 2 to all other values of K.

4. Conclusions

While regulatory commissions may include the rate of return of a firm among their objectives, their primary instrument of control is the price of the firm's output. For a specified price, uncertainty may cause the firm's *ex post* return to differ from the allowed return. While the regulator will change price if this divergence continues over an extended time, however, an *ex post* return constraint is typically not imposed each period. Instead, regulators may be viewed as setting price to allow a fair *ex ante* rate of return within each period and changing this price in response to developments between periods.

If the firm is investor-owned, the sophisticated regulator may use its securities market valuation to determine the minimum *ex ante* price that is consistent with technical efficiency and at the same time yields an "expected" return equal to the allowed rate of return. That price is equal to the average labor and capital costs plus an increment reflecting any difference between the allowed rate of return and the cost of capital. The firm will then produce efficiently, using a larger capital stock than would an unregulated monopolist, provided it perceives the regulated price to be invariant to its own decisions. If the regulator acts naively, however, and is concerned only with pricing to yield the allowed return, overcapitalization will result, as in the AJ model. The firm under naive regulation will employ a larger capital stock than a firm under sophisticated regulation with the same allowed return, but the naive regulated price will be higher and thus market "expected" output will be lower. Nevertheless, naive regulation results in a lower price, larger capital stock and greater "expected" output than those entailed by the unregulated monopoly solution, and to this extent even regulation of the naive type yields some benefits to consumers.

Figure 1

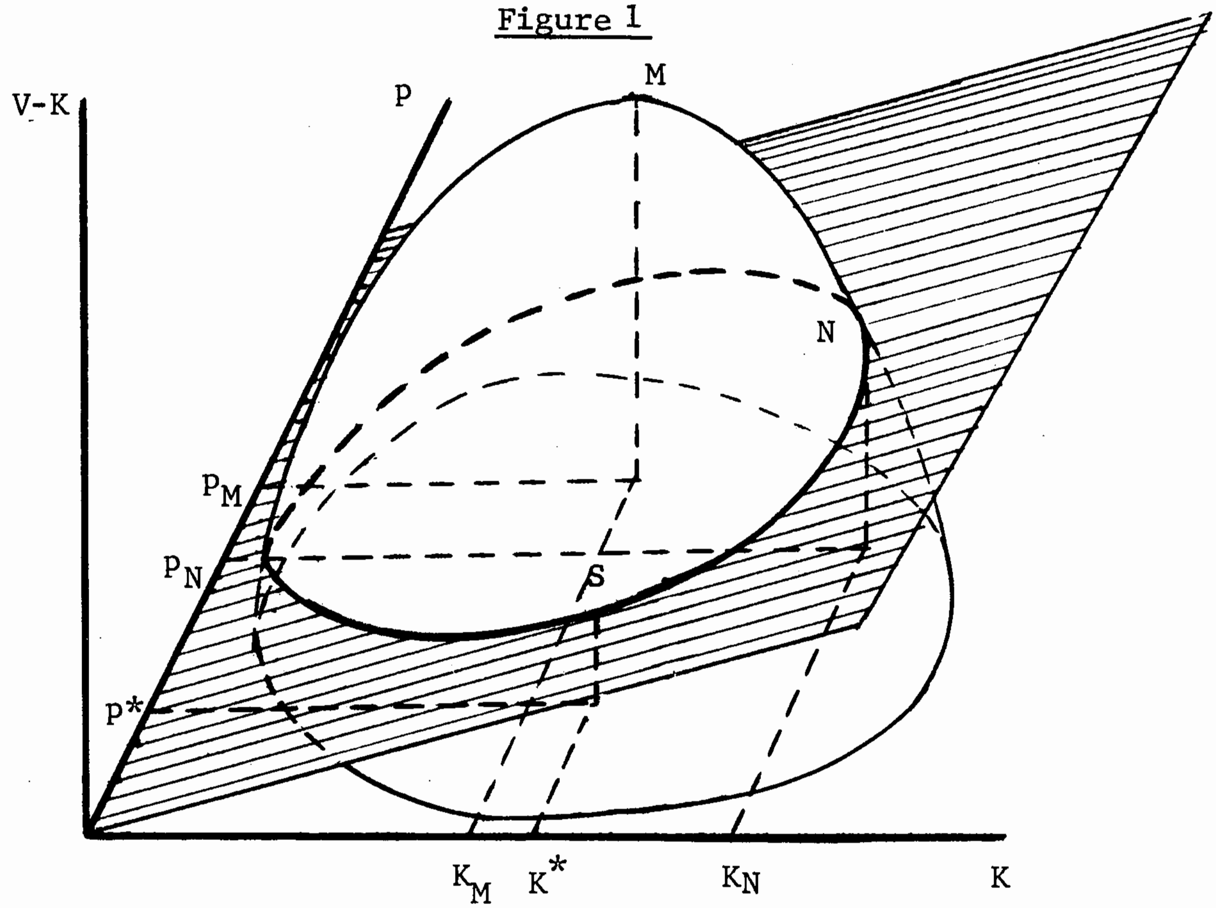
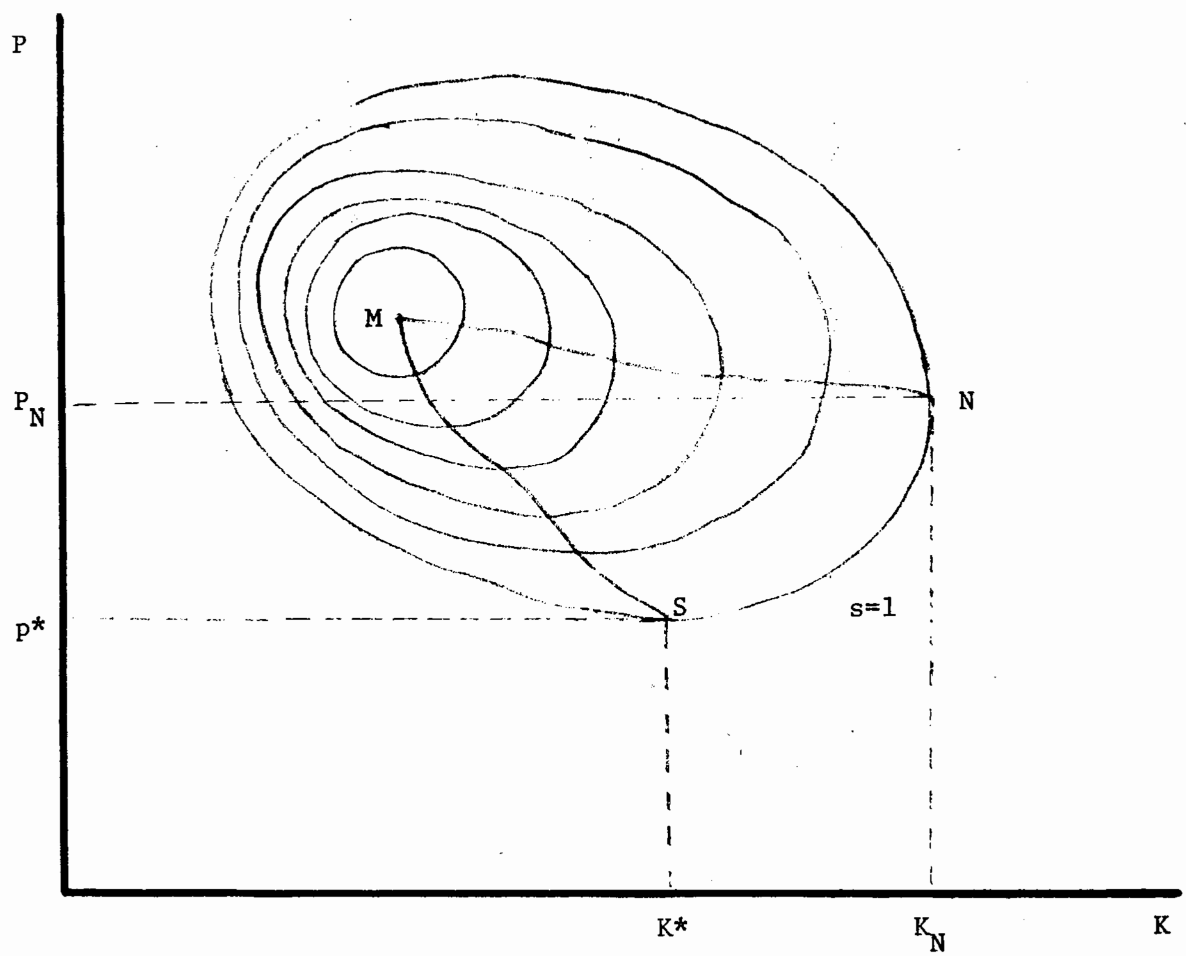


Figure 2



FOOTNOTES

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1. See, for example, Baumol and Klevorick (1970) and Joskow (1974).

2. For an electric utility, fuel would be the ex post input.

3. Although it is assumed that the production function is twice differentiable, less well-behaved functions are also compatible with the analysis. There may be an absolute capacity limit $\bar{q}(K_n)$ for any given capital stock, for example, in which case the firm must be restricted to choose K_n such that $\bar{q}(K_n) \geq q(p_n, \theta_{n\ max})$, the maximum demand that might occur in period n.

4. Specifically, it can be shown that $\partial L_n/\partial q_n > 0$, $\partial L_n/\partial K_n < 0$, $\partial^2 L_n/\partial q^2 > 0$, $\partial^2 L_n/\partial K^2 > 0$ and $\partial^2 L_n/\partial q_n \partial K_n < 0$.

5. As will be seen below, the regulated price and the firm's capital stock are set independently under sophisticated regulation, but are simultaneously determined under naive regulation.

6. Since the ex ante return depends on the uncertain future demand, the regulatory process envisioned here corresponds to the use of a "future test-year." In a recent paper M. G. Subrahmanyam (1977) analyzes a model in which the regulated price for period n + 1 depends on the capital stock and demand which actually prevailed in period n. This corresponds to the use of a "historical test-year."

7. Our reading of the literature suggests that this description is more realistic than one in which price is adjusted so as to satisfy a fair return constraint ex post, as in Peles and Stein (1976) and Perrakis (1976b). Joskow (1974) asserts, for example, that ex post returns have exceeded allowed returns over long periods in the past. Furthermore, as Myers (1972) points out, ex post price adjustment is not even desirable if the regulator's objective is to approximate the competitive solution.

8. The weakness of this description is that for the regulator to be able to announce the price p_{n+1} and for the firm to determine K_{n+1}, they both must be able to predict the market value of the firm at the beginning of period n+1. The state θ_n is thus assumed to be a complete description of the world, including market opportunities.

9. The securities market is not complete across periods, however, so an investor cannot make trades at the beginning of period n conditional on period n + 1 states.

10. The use of a securities market valuation frees the results from dependence on the specific characteristics of investors or of the managers of firms, in contrast to Perrakis (1976b). The complete market assumption used to obtain this V is not as restrictive as it may appear. If the required labor function in (2) is such that $q(p,\theta)$ may be factored out in some way, then (3) can be used to solve for a "market certainty equivalent" of the uncertain output, $\int \rho(\theta)q(p,\theta)d\theta/r$, in terms of market observable information. In that event, investors will agree on the value of the firm even if the state prices, $\rho(\theta)/r$ differ among investors. The advantage of the complete market assumption is that it allows the analysis to proceed with a minimum of restrictions on the production function.

11. See Drèze (1974) for this interpretation.

12. An optimal K exists because the properties of the production function imply that V-K is strictly concave in K. To facilitate comparisons with the firm's decisions in the AJ and unregulated monopoly cases, it will also be assumed that V-K is strictly concave in p. This requires that $2\partial q/\partial p - w(\partial^2 L/\partial q^2)(\partial q/\partial p)^2 - w(\partial L/\partial q)(\partial^2 q/\partial p^2)$ be negative.

13. The restriction is binding only if $s < s_M$ where (s_M-1) is the level of rents, relative to K_M, that would be achieved by an unregulated monopolist. Also, $s \geq 1$ if the firm is to go into business.

14. In the Peles and Stein (1976) and Perrakis (1976) models, by contrast, the allowed return is greater than the expected return, because the regulatory constraint is imposed *ex post* so that the firm can never earn more than the allowed return, while in some states of nature it may earn less.

15. If there is more than one such price, the regulator will presumably choose the lowest one. This will bring forth the maximum market "expected" output consistent with both efficient production and V = sK.

16. A decrease in p increases expected demand which in turn increases the marginal value product (MVP) of capital ($\partial^2 L/\partial K\partial q < 0$). Thus the level of K at which MVP = r increases.

17. The first of these additional properties is consistent with the results reported by Leland (1974), who stipulates that the regulated price be set so that V = K.

18. When $s > 1$, $dp/dK = -(\frac{\partial V}{\partial K} - s)/\frac{\partial V}{\partial p} > 0$. Thus, p* is not the minimum price consistent with V = sK if $s > 1$.

19. To demonstrate that $0 < \lambda_N < 1$, it will first be shown that $\lambda_N \neq 1$ when $(p_N, K_N) > 0$ and $s > 1$. If $\lambda_N = 1$, then (13) implies $1-s = 0$, which contradicts $s > 1$. To show that $\lambda_N \neq 0$, note that $\lambda_N = 0$ yields the monopoly solution in (13) and (14). By assumption, the monopoly solution is not attainable, so $\lambda_N \neq 0$ and thus $\frac{\partial V}{\partial p} = 0$. The bordered Hessian for the system in (13)-(15) is

$$\begin{vmatrix} (1-\lambda_N)\dfrac{\partial^2 V}{\partial K^2} & (1-\lambda_N)\dfrac{\partial^2 V}{\partial p \partial K} & s - \dfrac{\partial V}{\partial K} \\ (1-\lambda_N)\dfrac{\partial^2 V}{\partial p \partial K} & (1-\lambda_N)\dfrac{\partial^2 V}{\partial p^2} & -\dfrac{\partial V}{\partial p} \\ s - \dfrac{\partial V}{\partial K} & -\dfrac{\partial V}{\partial p} & 0 \end{vmatrix}$$

which must be positive. Since $\frac{\partial V}{\partial p} = 0$, expanding about the last column yields $-(s - \frac{\partial V}{\partial K})^2 (1-\lambda_N) \frac{\partial^2 V}{\partial p^2} \geq 0$. Concavity of V implies that $\lambda_N < 1$.

This proof follows the one employed by Baumol and Klevorick (1970) for the AJ model.

20. See G. Stigler (1971) for a discussion of the "capture" model.

21. Even though $\partial V/\partial p = 0$, (p_N, K_N) does not coincide with the unregulated monopoly solution. The latter solution is inadmissible because of the allowed return constraint.

22. That K increases as s decreases is consistent with the results of Takayama (1969), Baumol and Klevorick (1970) and Stein and Borts (1972). The "anti AJ Theorem" of Peles and Stein (1976) does not hold in this case even with multiplicative uncertainty. The reason is that the regulatory constraint is imposed <u>ex ante</u> in the model presented here and <u>ex post</u> in the model of Peles and Stein. That market "expected" output exceeds that of an unregulated monopolist for $s < s_M$ corresponds to Baumol and Klevorick's (alleged) Proposition 4. As Baumol and Klevorick point out, this result hinges on the assumption that $\partial^2 q/\partial K \partial L > 0$.

23. The (p,K) pairs consistent with any value of s are defined by $V = sK$. Differentiating implicitly, we have $dK/dp = -\partial V/\partial p \; / \; (\partial V/\partial K - s) = 0$ (from 14) at any equilibrium point for the firm under naive regulation. Furthermore, $d^2K/dp^2 = -\partial^2 V/\partial p^2 \; / \; (\partial V/\partial K - s) < 0$ at any equilibrium point, so K_N is the maximum K consistent with $V = sK$.

24. If more than one price is consistent with both $V = sK$ and efficient production, the regulator chooses the lowest such price. But then from Proposition (3), K* is the maximum efficient capital stock consistent with $V = sK$.

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