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The Protective Effect of a Tariff  
Under Uncertainty

by

Elhanan Helpman and Assaf Razin

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Elhanan Helpman  
University of Rochester  
Tel-Aviv University

and

Assaf Razin  
Northwestern University  
Tel-Aviv University

It is well known that in the standard deterministic two-sector economy the imposition of a tariff induces a resource flow from the export industry to the import competing industry if the external terms of trade do not change. This is the small country case. It is also known that in the large country case; i.e., in the case in which a country's import (export) volume influences its external terms of trade, an imposition of a tariff may induce a resource flow out of the import competing industry and into the export industry. This is known as the Metzler Paradox (see Metzler (1949)). In the small country case the imposition of the tariff reduces necessarily the internal terms of trade, because the external terms of trade do not change. Since domestic competitive resource allocation is governed by the internal terms of trade, the deterioration in the internal terms of trade that follows the tariff leads to an expansion of the import competing industry and to a contraction of the export industry. Hence, the tariff is protective in this case. In the large country case the imposition of a tariff may increase the external terms of trade at a rate which exceeds the rate of tariff, in which case the internal terms of trade will improve, thereby reversing the direction of resource

flow. If this happens, the tariff is said to protect the export industry and not the import-competing industry.

It is the purpose of this paper to show that in the presence of uncertainty a tariff need not provide protection to the import competing industry even in the small country case. The situation in which this may occur is one in which there is international trade in commodities but

no international trade in securities. If there is international trade in securities, a tariff does provide conventional protection.

Our analysis relies on the model developed in Helpman and Razin (1978a,b). In this model there is a stock market in which shares of firms are traded. The allocation of factors of production is governed by equity prices and it depends on commodity prices only to the extent that they influence equity prices. In the absence of international trade in securities, domestic equity prices are internally determined, since domestic risks are then fully borne by domestic residents. Now, the imposition of a tariff in a small country worsens necessarily the internal commodity terms of trade in every state of nature. However, its impact on relative equity prices, which determines the interindustry resource flow, depends on whether the tariff will shift the demand for equities towards the import competing sector or away from it. If tariff proceeds are not redistributed back to consumers, then the shift in the demand for equities can go either way, and we show an example in which demand shifts towards the equities of the exportable industry, in which case the tariff does not protect the import competing industry. We also show that when tariff proceeds are redistributed back to consumers, a 'small' tariff protects the import competing industry if both goods are

normal in consumption. (The difference between the two cases, with and without tariff proceeds redistribution, is explained at the end of the example.) This contrasts with the deterministic case in which the redistribution policy is not important for the protective effect of a tariff in the small country case (it is though important for the large country case).

## I. The Model

Our small economy consists of firms and consumers who operate in an uncertain environment generated by random production technology or random world prices. These random elements produce an incentive to develop financial capital markets, whose existence--in the form of stock markets--we assume. Domestic financial capital markets may or may not be integrated into the world's capital markets. If domestic capital markets are not integrated into the world's capital markets (i.e., there exists no international trade in securities), they enable risk-sharing only among domestic residents. However, if domestic capital markets are integrated into the world's capital markets (i.e., there exists international trade in securities), they permit international risk-sharing. Since we deal with international trade, we assume that there is international trade in commodities.

Input decisions have to be made before the resolution of uncertainty. As a result, firms face random profits and cannot undertake profit maximization. Instead, we assume--following Diamond (1967)--that firms choose their input levels so as to maximize their net value on the stock market; this procedure is equivalent to profit maximization whenever the relevant random elements become degenerate (i.e., their value becomes known with certainty). After the resolution of uncertainty returns are realized and the firms distribute them to their final stockholders.

Individuals play a double role in this economy. In the first stage -- before the resolution of uncertainty -- individuals choose a portfolio, by means of trading in the stock market. An equity in a firm entitles the stockholder to a share in the firm's random return. This share equals the inverse of the number of the firm's outstanding equities. This is the stage in which individuals play the role of investors.

In the second stage -- after the resolution of uncertainty -- individuals use the proceeds from portfolios to purchase commodities. This is the stage in which they play the role of consumers.

Clearly, the two roles are interrelated. The ultimate goal of a portfolio chosen in the first stage is to provide consumption in the second stage. Hence, portfolio choice depends on preferences over consumption goods--but it also depends on probability beliefs, price expectations, and attitudes towards risk.

### Firms

Consider a two sector economy which produces two commodities,  $X_1$  and  $X_2$ , by means of labor and capital. Each sector is composed of identical firms, and the output of each firm depends on its employment of capital and labor and on the state of the world that realizes. In particular, in every state of the world  $\alpha$ ,  $\alpha = 1, 2, \dots, S$ , the output of firm  $j$  is:

$$Q_j(\alpha) = \theta_j(\alpha) f_j(L_j, K_j), \quad \text{for } \alpha = 1, 2, \dots, S \quad (1)$$

where

$\theta_j$  = a positive valued random variable

$f_j(\cdot)$  = a standard neoclassical linear homogeneous production function

$L_j$  = labor input in firm  $j$

$K_j$  = capital input in firm  $j$

$Q_j$  = output of firm  $j$ , which is also random

Since all firms in a given sector are identical and  $f_j(\cdot)$  is linear homogeneous, (1) also describes the output of the sector to which firm  $j$  belongs if  $L_j$  and  $K_j$  are interpreted as total factor inputs in this sector. We use this aggregation procedure, and from now on use sectors as the production units. The index  $j$  is used to denote sectors;  $j = 1, 2$ .

Assuming the existence of a stock market, it is explained in Helpman and Razin (1978a,b) that by selling shares in the stock market a firm in sector  $j$  can be viewed as selling real equities of type  $j$ , where one real equity of type  $j$  provides the bundle  $[\theta_j(1), \theta_j(2), \dots, \theta_j(S)]$  of commodity  $X_j$ . The output of real equities of type  $j$  by industry  $j$  is  $Z_j = f_j(L_j, K_j)$  and we can draw a transformation curve between real equities -- TT in Figure 1. The curve TT has all the usual characteristics of a Heckscher-Ohlin type transformation curve.

It was shown in Helpman and Razin (1978a,b) that, given the relative price of type-2 real equities  $q^0$  ( $q^0$  is the price of type-2 real equities divided by the price of type-1 real equities), net value



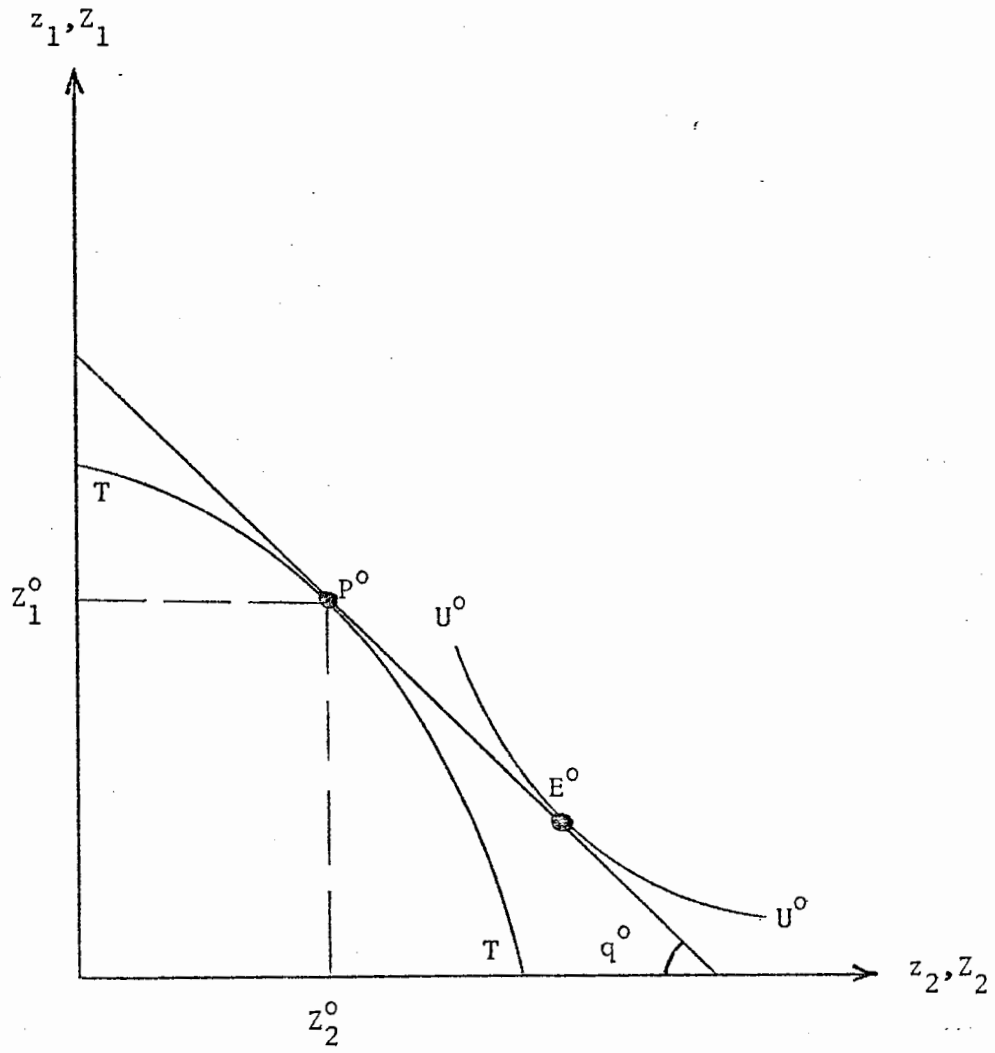


Figure 1

maximizing firms will choose in an equilibrium a point on the transformation curve TT at which the MRT between  $Z_2$  and  $Z_1$  is equal to  $q^0$ . Hence, given  $q^0$ , production of real equities will take place at point  $P^0$  in Figure 1. Corresponding to point  $P^0$  there is an equilibrium wage rate and rental rate on capital, and an equilibrium allocation of the fixed supplies of labor and capital between the sectors. Given  $P^0$ , the output of commodities is not uniquely determined; it depends on the state of nature. If state  $\alpha$  realizes, the output of commodity  $i$  will be  $\theta_i(\alpha)Z_i^0$ ,  $i = 1,2$ .

By varying  $q$  along TT, we trace out the general equilibrium supply functions:

$$Z_j = Z_j(q), \quad j = 1,2 \quad (2)$$

Clearly, for  $q$  which does not result in complete specialization,  $Z_2(\cdot)$  increases in  $q$  (i.e.,  $Z_2'(q) > 0$ ) and  $Z_1(\cdot)$  decreases in  $q$  (i.e.,  $Z_1'(q) < 0$ ).

In addition:

$$Z_1'(q) + qZ_2'(q) \equiv 0 \quad (3)$$

### Consumers

Let  $v(p,I)$  be the representative consumer's indirect utility function, where  $p$  is the price of  $X_2$  in terms of  $X_1$  and  $I$  is income in terms of  $X_1$ . All consumers are assumed to be identical. Then, it is shown in Helpman and Razin (1978b) (equation (7)) that the consumer's portfolio choice is in equilibrium:

$$\max_{z_1, z_2 \geq 0} \text{Ev}[p(\alpha), \theta_1(\alpha)z_1 + \theta_2(\alpha)z_2], \quad \text{s.t. } z_1 + q^0 z_2 \leq Z_1(q^0) + qZ_2(q^0) \quad (4)$$

where  $z_i$  is purchase of type- $i$  real equities and  $E$  is the expectations operator based on subjective probability beliefs. Commodity prices, which may be state dependent,  $p(\alpha)$ , are assumed to be given to our small country.

Assuming risk aversion, we can draw a set of convex to the original assets-indifference curves, where an asset-indifference curve is defined as all combinations of  $(z_1, z_2)$  for which the expected utility is constant. Then the solution to (4) can be represented by the tangency of an assets-indifference curve to an assets-budget line, like point  $E^0$  in Figure 1.  $U^0 U^0$  represents there the highest affordable expected utility level. Observe that points  $P^0$  and  $E^0$  in Figure 1 represent an equilibrium in which there is international trade in equities and in which the rest of the world produces a perfect substitute for domestic type-2 real equities, which are imported. If there is no international trade in equities, the equilibrium domestic relative price  $q$  will be such as to make  $z_i = Z_i(q)$ . Such an equilibrium is represented in Figure 2 by point  $P$  at which an assets-indifference curve is tangent to the transformation curve. Notice, however, that we are still assuming international trade in commodities after the realization of a state of nature at the going world prices  $p(\alpha)$ .

At this point the reader should note that the assets-indifference curves depend on the distribution of relative commodity prices. A shift in the price distribution pivots the entire assets-indifference map.

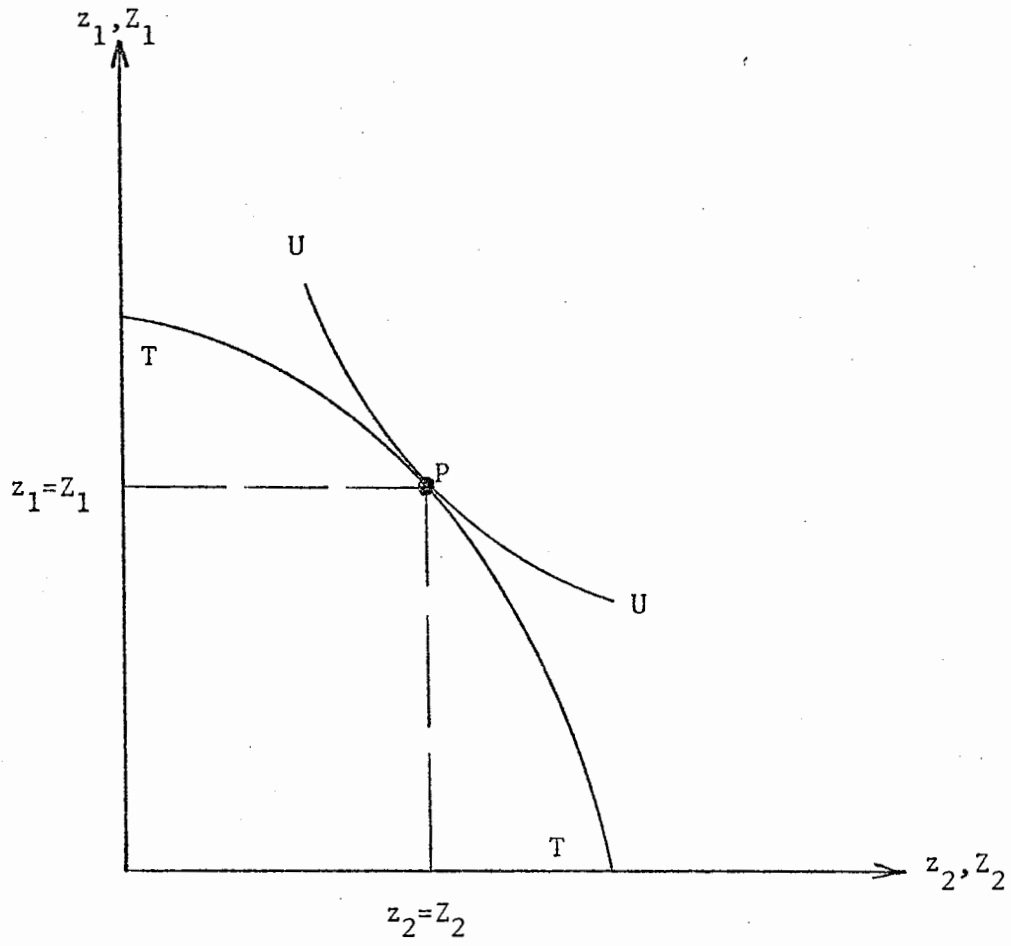


Figure 2

## II. Protection Under Uncertainty

Now, consider an ad valorem tariff on the second commodity, assuming that the second commodity is imported in every state of the world. The effect of the tariff on the allocation of resources between the two sectors differs according to whether international trade in securities takes place. We begin with the case of no international trade in securities, so that domestic residents bear all domestic risks.

### (a) No International Trade in Securities

The tariff-inclusive assets-indifference curves (which, along with the production possibilities curve, help determine the economy's production) are given by:

$$Ev[(1 + t)p(\alpha); \theta_1(\alpha)z_1 + (1 + t)p(\alpha)\theta_2(\alpha)z_2 + T(\alpha, t)] = \text{constant} \quad (5)$$

where,

$t$  = the tariff rate (assumed to be state independent)

$T(\alpha, t)$  = state  $\alpha$  transfer payments

If tariff proceeds are redistributed back to consumers,  $T(\alpha, t)$  equals tariff proceeds in state  $\alpha$ , and it is equal to zero if tariff proceeds are not redistributed.

The tariff-inclusive marginal rate of substitution between real equity 2 and real equity 1, assuming that the individual perceives that the transfers he receives are not affected by changes in his portfolio, is given by:

$$\text{MRS}(z_1, z_2; t) \equiv \frac{E(1+t)p(\alpha)\theta_2(\alpha)v_I[(1+t)p(\alpha); \theta_1(\alpha)z_1 + (1+t)p(\alpha)z_2 + T(\alpha, t)]}{E\theta_1(\alpha)v_I[(1+t)p(\alpha); \theta_1(\alpha)z_1 + (1+t)p(\alpha)\theta_2(\alpha)z_2 + T(\alpha, t)]} \quad (6)$$

Let us start with a discussion of the case in which tariff proceeds are not distributed back to consumers; i.e., the government is using the revenue from tariffs in order to either purchase commodities which do not influence consumer behavior or <sup>1</sup> pays them out to foreigners. Remember that in the small country deterministic model a tariff protects the import competing industry regardless of whether tariff proceeds are redistributed. In the present case:

$$T(\alpha, t) = 0 \text{ for all } t \text{ and } \alpha = 1, 2, \dots, S \quad (7)$$

From (6) and (7) it is readily verified that a change in the tariff rate twists the assets-indifference curves at every point  $(z_1, z_2)$ , and changes the marginal rate of substitution between real equities 2 and 1. This results from the fact that the tariff changes the mean as well as higher moments (such as the variance) of the distribution of the relative internal price of good 2.

In Figure 3, point  $E_S^0$  denotes the pre-tariff stock market equilibrium in which the pre-tariff assets-indifference curve  $U_0U_0$  is tangent to the production possibilities curve TT. If the post-tariff assets-indifference curve, which passes through the initial point  $E_S^0$  is steeper than  $U_0U_0$ , like  $U_1U_1$ , the new equilibrium must be at a point on TT to the right of  $E_S^0$ ; that is, resources are moving away from sector 1 and into the import competing sector, sector 2, which is the standard case. If the post-tariff assets-indifference

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<sup>1</sup>The first case occurs if, for example, the government uses tariff proceeds to provide public goods, and the utility function is additively separable in private and public good

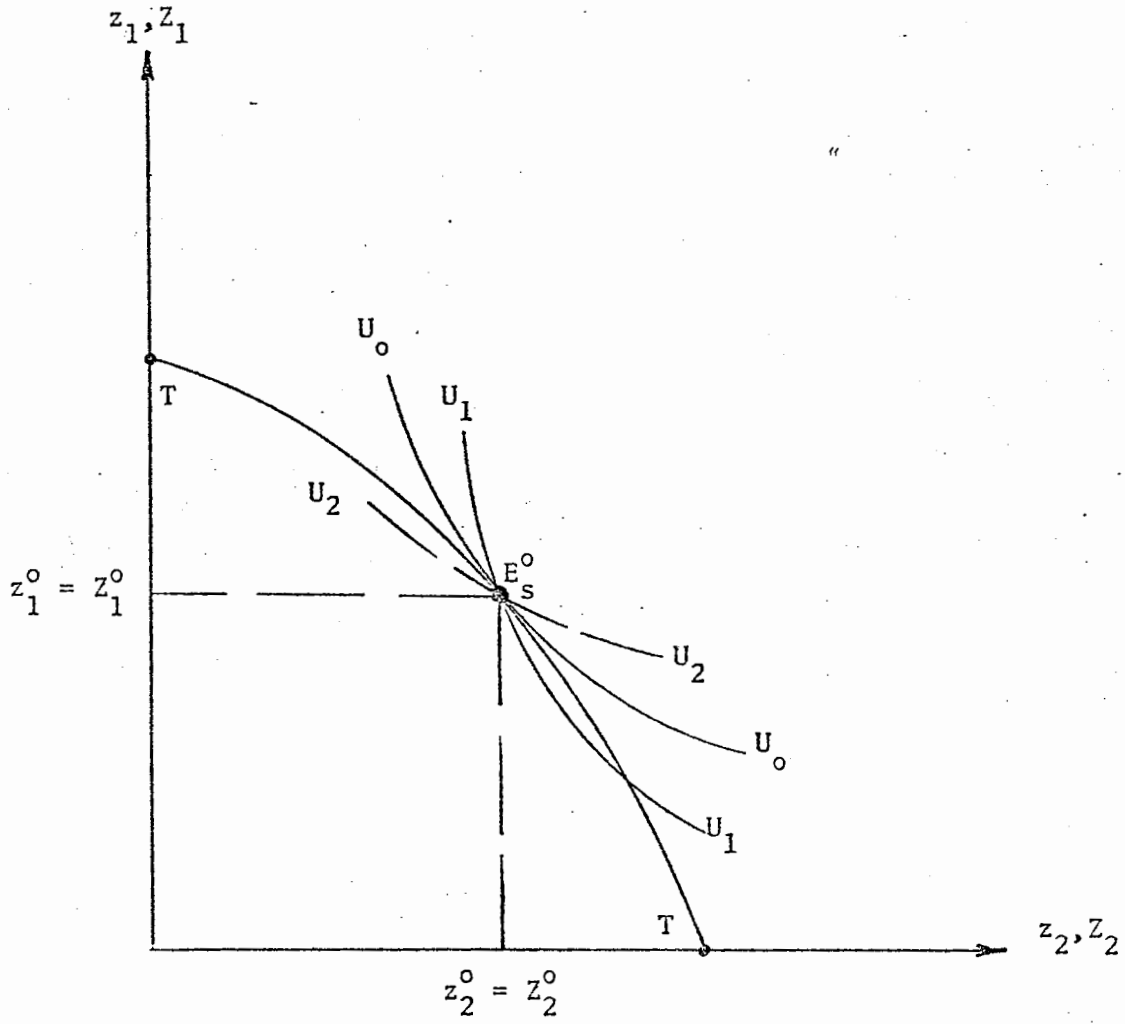


Figure 3

curve, which passes through  $E_S^0$  is flatter than  $U_0^0$ , like  $U_2U_2$ , the new equilibrium must be at a point on TT to the left of  $E_S^0$ , that is, resources are moving away from the importable goods sector and into the exportable goods sector. In the second case a tariff does not protect the import competing sector, contrary to the deterministic case. The following is an example in which this paradoxical result occurs.

Example

Let the utility function be:

$$u = \log (c_1 + \log c_2)$$

This yields the indirect utility function:

$$v = \log \{I - 1 - \log [(1+t)p]\}$$

where I stands for the consumer's disposable income. This implies (using (6) and (7)):

$$\text{MRS}(z_1, z_2, t) \equiv \frac{E[\theta_1(\alpha)z_1 + (1+t)p(\alpha)\theta_2(\alpha)z_2 - 1 - \log(1+t)p(\alpha)]^{-1}\theta_2(\alpha)(1+t)p(\alpha)}{E[\theta_1(\alpha)z_1 + (1+t)p(\alpha)\theta_2(\alpha)z_2 - 1 - \log(1+t)p(\alpha)]^{-1}\theta_1(\alpha)}$$

Assume now  $\theta_2(\alpha) = 1$  for all  $\alpha$ ,  $p(\alpha) = p$  for all  $\alpha$ , and that at the initial equilibrium

$$t^0 = 0$$

$$z_1^0 = 1$$

$$z_2^0 = (1 + \log p)/p$$



appropriate choice

The stockholders' choice of these real equity holdings can be assured by an / of production technologies and factor endowments. Then, the derivative of MRS with respect to  $t$ , evaluated at the initial equilibrium is:

$$\frac{\partial \text{MRS}(z_1^0, z_2^0; t^0)}{\partial t} = E \frac{1}{\theta_1(\alpha)} - (p \log p) [E(\frac{1}{\theta_1(\alpha)})^2 - (E \frac{1}{\theta_1(\alpha)})^2] = E[\frac{1}{\theta_1(\alpha)}] - (p \log p) \text{Var}[\frac{1}{\theta_1(\alpha)}]$$

where Var stands for variance.

Hence, for sufficiently large  $p$  or  $\text{Var}[1/\theta_1(\alpha)]$ , we get:

$$\frac{\partial \text{MRS}(z_1^0, z_2^0; t^0)}{\partial t} < 0$$

This implies that, for a 'small' tariff,  $U_2 U_2$  in Figure 3 is the post-tariff assets-indifference curve. Therefore, the imposition of the tariff leads to a contraction of the imports competing industry and an expansion of the export industry.

In the absence of uncertainty, the variance of  $1/\theta_1(\alpha)$  is zero and the paradoxical result does not arise. In the presence of uncertainty the paradoxical result can arise because of the negative effect that an increase in  $t$  has on the demand for type-2 real equities, holding their returns constant. This can be seen as follows. Write the indirect utility function as

$$v = \log\{\theta_1(\alpha)z_1 + (1+t)pz_2 - 1 - B(t)\}$$

where  $B(t) = \log[(1+t)p]$

It can be shown that an increase in B reduces the demand for type-2 real equities. Now, an increase in the tariff rate has two effects. It increases B, resulting in a decline in the demand for type-2 real equities, and increases the return on type-2 real equities, resulting in an increase in its demand. The first effect, which is a negative income-type effect, dominates in this case when p is large enough.

Consider now the case in which tariff proceeds are redistributed back to consumers. In this case state  $\alpha$  transfers (i.e., the tariff rate times the value of imports) are implicitly given by:

$$T(\alpha, t) = tp(\alpha) \{c_2[(1+t)p(\alpha); \theta_1(\alpha)z_1 + (1+t)\theta_2(\alpha)p(\alpha)z_2 + T(\alpha, t)] - \theta_2(\alpha)Z_2[(1+t)q]\} \quad (8)$$

where

$$c_2[\cdot] = \text{the second commodity demand function}$$

$$\theta_2(\alpha)Z_2[\cdot] = \text{local output of good 2 in state } \alpha$$

Notice that from (8), we get

$$T(\alpha, 0) = 0, \quad \frac{\partial T(\alpha, 0)}{\partial t} = p(\alpha) \{c_2[p(\alpha); \theta_1(\alpha)z_1 + \theta_2(\alpha)p(\alpha)z_2] - \theta_2(\alpha)Z_2'(q)\} \quad (9)$$

That is, a zero tariff rate obviously must give rise to a zero amount of tariff proceeds, and the rate of change in the tariff proceeds for a small tariff is equal to imports evaluated at world prices.

Now, we show that if both goods are normal, the paradoxical result cannot appear in the case of a small tariff. In order to see this, differentiate (6) with respect to t and evaluate it at t=0, using (9), to obtain:

$$\begin{aligned} \frac{\partial \text{MRS}(z_1, z_2; 0)}{\partial t} &= \frac{1}{\text{Ev}_I(\alpha)\theta_1(\alpha)} \{ \text{Ev}_I(\alpha)p(\alpha)\theta_2(\alpha) + \text{Ev}_{I_p}(\alpha)[p(\alpha)]^2\theta_2(\alpha) \\ &+ \text{Ev}_{II}(\alpha)[p(\alpha)]^2c_2(\alpha)\theta_2(\alpha) \\ &- \text{MRS}(z_1, z_2; 0) [\text{Ev}_{I_p}(\alpha)p(\alpha)\theta_1(\alpha) + \text{Ev}_{II}(\alpha)p(\alpha)c_2(\alpha)\theta_1(\alpha)] \} \end{aligned}$$

where  $v_{I_p}$  is the derivative of  $v_I$  with respect to its first argument.

Now,  $v_{I_p} = v_{pI} = \partial(-v_I c_2)/\partial I = -v_{II}c_2 - v_I c_{2I} = \partial c_2/\partial I$  and  $p c_{2I} = 1 - c_{1I}$ . Substituting these relationships in the above expression, we get:

$$\begin{aligned} \frac{\partial \text{MRS}(z_1, z_2; 0)}{\partial t} &= \frac{1}{\text{Ev}_I(\alpha)\theta_1(\alpha)} \{ \text{Ev}_I(\alpha)p(\alpha)\theta_2(\alpha)c_{II}(\alpha) \\ &+ \text{MRS}(z_1, z_2; 0)\text{Ev}_I(\alpha)\theta_1(\alpha)p(\alpha)c_{2I}(\alpha) \} \end{aligned}$$

If both goods are normal, the marginal propensities to spend on them are positive and the above expression is positive. This means that for normal goods a small tariff will twist the assets-indifference curves in Figure 3 so as to make them steeper, like from  $U_0 U_0$  to  $U_1 U_1$ , and thus provide protection to the importable goods sector.

Finally, observe that an equity subsidy -- i.e., a subsidy given to an industry at the financing stage -- will, unambiguously induce the expansion of that industry. In Figure 4 we reconstruct the initial equilibrium shown in Figure 3, the real equity-price ratio being  $q$ . A subsidy to sales of real equity 2 decreases to  $q'$  the relative price of real equity 2

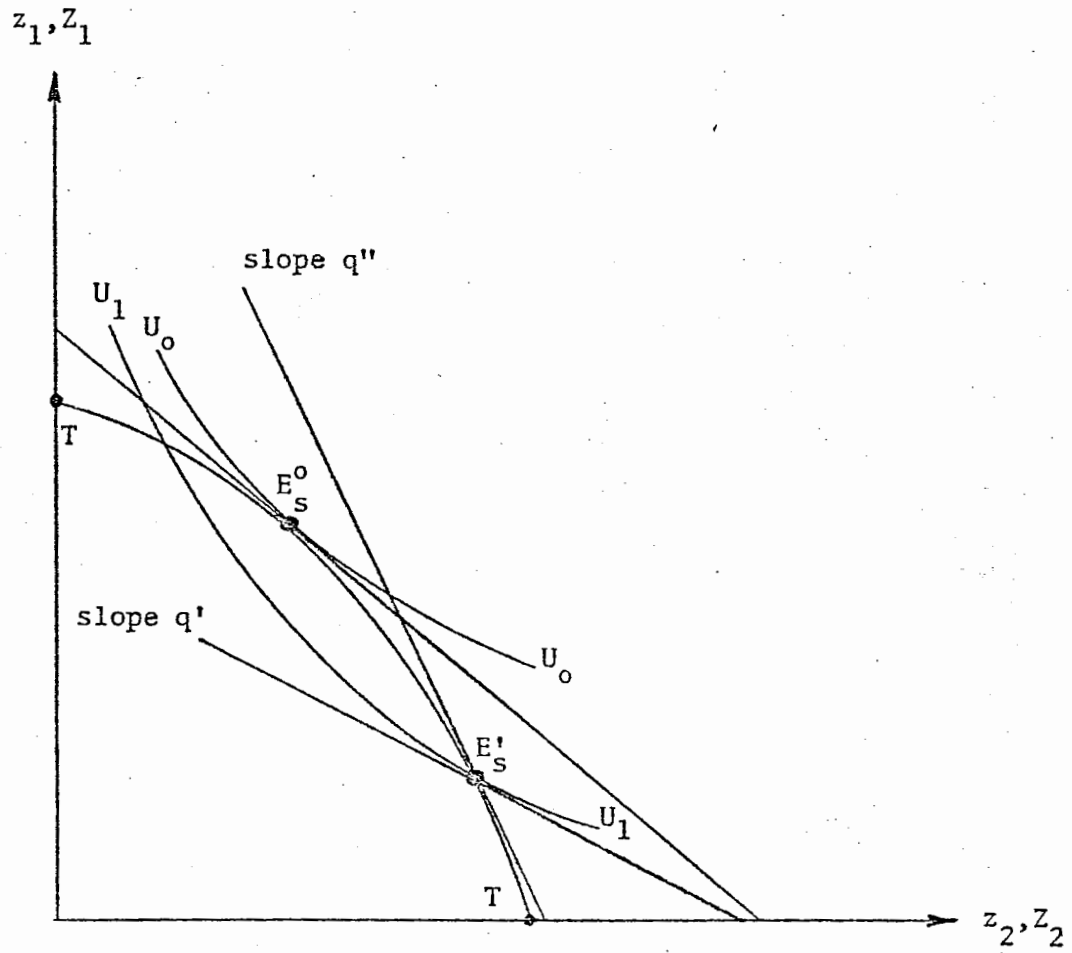


Figure 4

to investors, and drives a wedge between that relative price and the marginal rate of transformation  $q''$ , leading to a new equilibrium  $E'_5$ . Thus, resources will move away from sector 1 and into sector 2.

(b) International Trade in Securities

Now consider the case in which the economy trades with the outside world in both commodities and securities. By the small-country assumption, without a tariff, commodity prices and security prices are given to the home country. A tariff raises the local price of the importable goods; but how does a tariff affect the importable goods industry's stock market value?

It is shown in Helpman and Razin (1978b) that a tariff at a rate of  $100t$  percent, which increases the price of the second commodity by  $100t$  percent in every state of the world, increases by  $100t$  percent the return on each unit of domestic type-2 real equities. This will result in a  $100t$  percent increase in the price of local type-2 real equities in order to eliminate profitable arbitrage. The local type-2 real equity provides a return of  $(1+t)\theta_2(\alpha)p(\alpha)$  in every state  $\alpha$  while the foreign type-2 real equity provides a return of  $\theta_2(\alpha)p(\alpha)$  in state  $\alpha$ . Hence, one unit of local type-2 real equity is now equivalent to  $(1+t)$  units of foreign type-2 real equities.

This means that the price of local type-2 real equities has increased from  $q$  to  $(1+t)q$ . Thus, following a tariff, resources move necessarily away from the exportable-goods industry and into the importable-goods industry, as in the deterministic case.

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