DISCUSSION PAPER #305

OLIGOPOLY AND COMPETITION IN LARGE MARKETS

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October 1977

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Roberts's research was supported by the National Science Foundation under grant SOC 76-20953.

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ABSTRACT

In this paper we study questions of oligopoly and competition in a general equilibrium framework. In particular, we consider the Nash equilibria of a model of non-cooperative exchange in the context of a measure space of economic agents which incorporates both atoms, representing large traders or organized syndicates of traders, and a non-atomic continuum of infinitesimal individual traders. In this framework we consider some qualitative properties of these Nash equilibria including their optimality, the impact of the size of a syndicate on the consumption its members receive, and the incentives to form and to depart from syndicates.

INTRODUCTION

In this paper we study questions of oligopoly and competition in a general equilibrium framework. In particular, we consider the Nash equilibria of a model of non-cooperative exchange in the context of a measure space of economic agents which incorporates both atoms, representing large traders or organized syndicates of traders, and a non-atomic continuum of infinitesimal individual traders.

Benyamin Shitovitz (14) (15) introduced this type of measure theoretic model to study situations in which some but not all agents may have market power. Traditional general equilibrium treatments of such situations [see, e.g., Chapter 6 of Arrow and Hahn (1)] have been deficient in that they have simply assumed *a priori* that certain agents behave as price-takers while others act non-competitively, with no formal explanation being given as to why a particular agent should behave one way or the other. Shitovitz's approach represents an important contribution in pointing to an explicit formulation leading to such differences in behavior. In this paper we seek to explore the use of this type of model in studying issues of oligopoly in a general equilibrium framework. A specific focus of our work is in illuminating how either perfectly or imperfectly competitive behavior may emerge endogenously in this model, depending on the characteristics of the agent and his place in the economy.

Shitovitz's analysis concentrated on the core of the economy, that is, the set of allocations which no group of agents can improve upon by using only its own resources to achieve a distribution of commodities which each of its members prefers to the allocation in question. This solution concept has, of course, been widely applied in economics, and the equivalence between the core and competitive equilibria in the absence of large traders or syndicates is well-known. [See Hildenbrand (6) for a presentation of these results]. Most of the succeeding work with Shitovitz's mixed measure-theoretic model has also been concerned with the core. [See, for example, Drèze and Gabszewicz (3), Jaskold-Gabszewicz (7), Aumann (2), Postlewaite and Rosenthal (11), and Drèze, Gabszewicz and Postlewaite (4)].

While Shitovitz's model would seem especially appropriate for studying oligopoly, he concentrated not so much on market power phenomena per se as on the possibility that all the core outcomes would still be competitive allocations despite the presence of atoms. Some of the results he obtained in studying this issue appear so counter-intuitive as to seem to call into question the use of this model with atoms and a non-atomic ocean in studying oligopoly. For example, Theorem B in (14) indicates that if there are two large traders or syndicates with the same endowment densities and preferences over consumption bundle densities, then the core and competitive allocations coincide, no matter what the relative sizes of the two traders. Thus, the presence of an arbitrarily small (but not infinitesimal) rival can be viewed as completely cancelling out the market power of a large trader who might represent all but a tiny fraction of the market's potential supply of a commodity.

Of course, if one finds such a result unsatisfying, one need not question the model of atoms and a continuum. Rather, one might object to the use of the core as the solution concept. Our work here indicates that this is probably the right response. In place of the core, we take as our solution concept the Nash equilibrum of a non-cooperative model of exchange. In this framework, we are able to obtain general equilibrium results concerning the qualitative properties of oligopoly which are both intuitively appealing and in accord with the insights offered by partial equilibrium analyses.

The model of exchange we use is derived from one introduced by Shubik (16) in which a good is exchanged for money. Each seller offers a quantity of the good for sale and receives a proportion of the total amount of money bid for the good by buyers equal to the proportion of the total amount of the good that he supplied. Similarly, a buyer bids an amount of money for the good and receives the fraction of the total supply offered that his bid is of the total amount bid. This model has been extended by Shapley and Shubik (13) to encompass many commodities, with one of the goods being used as commodity money.

With a finite number of agents, the Nash equilibria of the game defined by this rule of exchange are not typically Pareto-optimal. One source of this inefficiency corresponds to imperfectly competitive behavior and is a central concern in this paper. However, the fixed initial supply of the commodity money can also lead to an inefficiency which may prevail even in large economies [see Jaynes, Okuno and Schmeidler (8) for an analysis of this question]. However, a variant of the model introduced by Pazner and Schmeidler (9) eliminates this source of inefficiency by replacing the commodity money by units of account, and it is with this model that we will be concerned here.

Within this model, Postlewaite and Schmeidler (12) have shown that as the number of traders becomes large and each individual becomes small relative to the market, the inefficiency arising from imperfectly competitive behavior disappears asymptotically, while Dubey and Shapley (5) have shown that the Nash and competitive equilibria coincide if there a are an infinite number of traders, each of whom is negligible relative to the market. These results may be viewed as general equilibrium, pure exchange versions of the classical results on the competitiveness of the Cournot equilibrium in large economies. Our analysis focuses, on the other hand, not on the competitiveness of the outcomes of imperfectly competitive behavior, but rather on the nature of oligopoly where a small numbers of large traders face one another and a large mass of unorganized agents.

In the following section we specify the model we will be considering. We next present a simple example which illustrates the model. This example satisfies the assumptions of Shitovitz's Theorem B, but the Nash allocations (as opposed to the core) are not Pareto optimal, let alone competitive. In the succeeding section, we establish this property in general. We further show that under the assumption of no negative income effects, the larger of two syndicates made up of the same type of traders will restrict its sales and purchases more than does the smaller group. Finally we address two aspects of the incentives to form cartels. First, we show that any individual agent can gain by breaking from a cartel once it is formed. Second, we examine whether it is always advantageous for a group to collude or whether, instead, disadvantageous syndicates can exist.

Our aim in this paper is not to achieve the greatest possible generality. Rather, we seek to show how the framework introduced by Shitovitz, when combined with an appropriate model of non-cooperative behavior, can lead to useful insights concerning oligopoly. For this reason, we will work with the simplest possible model, even though some of our arguments are applicable to more general situations.

THE MODEL

We consider an exchange economy with 2 commodities in which the set of traders is indexed by the unit interval $T = [0,1]$ endowed with Lebesgue measure μ. Intuitively, we interpret $\mu(S)$ to be the proportion of the total set of traders belonging to the set S. (Without further mention, we take all subsets of T and functions on T to be measurable with respect to the relevent measure. If atoms are present, this measure is not the Lebesgue measure itself, but is derived from it in a standard way).

Each agent $t \in T$ is endowed with a strictly positive amount $w_i(t)$ of each good, i = 1,2, and has preferences represented by a smooth, monotonic utility function $U^t(.,.)$.

To simplify further, we assume that there are but two types of agents, who are indexed respectively by the disjoint sets T^o and T^1, $T^o \cup T^1 = T$. The members of T^o will be treated as unorganized traders throughout the discussion. Since our main interest in this paper is the effect of oligopolies or syndicates in markets, we will consider various different organizations of the traders in T^1. At the one extreme, T^1 may also consist of unorganized traders who act individually. Alternatively, T^1 might consist of a collection $(T^{11}, \ldots, T^{1n})$ of syndicates, non-insignificant groups of traders who coordinate their actions within their respective groups. This would represent a situation of oligopoly. As a third possibility there might be an organized portion of T^1 as well as a collection of syndicates. The key point is that the members of a syndicate act in concert in deciding on the common trades they will all make while those in the unorganized portion of the market act individually.

Trade takes place through a market in which the two goods are exchanged against one another. Each trader sends some amount of commodities to market. These supplies are aggregated, and a price ratio is obtained as the ratio of these aggregate supplies. Each trader sending good 1 to the market then receives the amount of good 2 his supply will purchase at the effective price, and similarly for suppliers of good 2. For an unorganized trader, his supply decisions have a completely negligible effect on the market aggregates and thus on the price ratio. Consequently, he faces a budget constraint that coincides with that of the usual Walrasian price-taker. However, organized groups of traders acting in concert exert a non-negligible impact on prices: in general, an increase in the supply of a good by a syndicate will lower the relative price of the good. Therefore, the budget set facing an organized trader is characterized by a strictly concave curve rather than by a straight budget line. Agents recognize this phenomenon, and, consequently, when organized as a syndicate they adopt behavior which is not perfectly competitive although, when unorganized, the same agents with the same characteristics and motivation would be price-takers.

More specifically, let $s : T \to R^2_+$ be an integrable function associating with each trader a strategy, that is $s_i(t)$ is the amount of the i-th good that trader t sends to the market. We will call such a function a supply profile and denote it $(s_1(\cdot), s_2(\cdot))$. If t and t′ both belong to $S^j \subset T^1$, then $s_i(t) = s_i(t') = s^j_i$ by assumption. If an unorganized trader t supplies (σ_1, σ_2) while the supply profile of the economy is $(s_1(\cdot), s_2(\cdot))$, this unorganized trader will receive

$$\sigma_2 \int_T s_1(\tau)d\mu \Big/ \int_T s_2(\tau)d\mu = b^t_1(\sigma, s(.))$$

of the first good, and

$$\sigma_1 \int_T s_2(\tau)d\mu \Big/ \int_T s_1(\tau)d\mu = b^t_2(\sigma, s(.))$$

of the second good. A trader t who belongs to an organized group S $(\mu(S)=\mu_S > 0)$, each member of which is supplying (σ_1,σ_2), receives

$$b^t_1(\sigma, s(.)) = \sigma_2 [\int_{T\sim S} s_1(\tau)d\mu_S + \sigma_1\mu_S]/[\int_{T\sim S} s_2(\tau)d\mu + \sigma_2\mu_S]$$

and

$$b_2^t(\sigma,s(.)) = \sigma_1[\int_{T\sim S} s_2(\tau)d\mu + \sigma_2\mu_S]/[\int_{T\sim S} s_1(\tau)d\mu + \sigma_1\mu_S]$$

of the two goods. A Nash equilibrium supply profile, given the organization of T^1, is a pair $(s_1(.)), s_2(.))$ such that each unorganized trader t is maximizing his utility with respect to his choice $(s_1(t),s_2(t))$, and each syndicate S^i is choosing a common trade $(s_1(t),s_2(t))=(s_1^j,s_2^j)$ for each member of the syndicate such that the common utility function of all members of S^j is maximized by this choice. More formally, let the sets Σ^t be defined by $\Sigma^t = \{s \in R_+^2 | s \leqq w(t)\}$. Define the pay-off function h^t for any trader by

$$h^t(\sigma,s)=U^t(w(t) + b^t(\sigma,s)-s).$$

Then, given the organization of the economy into syndicates and unorganized traders, a Nash equilibrium is a supply profile $(s_1(.),s_2(.))$ such that for each unorganized t,

$$h^t(s(t),s) \geq h^t(\sigma,s)$$

for all $\sigma \in \Sigma^t$ and such that for each syndicate $S \subset T^1$ there does not exist σ' such that $\sigma' \in \Sigma^t$, $t \in S$, and

$$h^t(\sigma',s) > h^t(s(t),s)$$

for the traders in S. (Recall Σ^t and h^t are constant on $S \subset T^1$).

It should be noted that if $s_i(t) \equiv 0$ for all t, then this game is not well-defined. By introducing an arbitrary market allocation scheme to deal with such situations, one can avoid this problem. However, this procedure usually results in the creation of trivial Nash equilibria in which no trade occurs $(s_1(t)=s_2(t)=0$ for all $t)$. In our discussion we will consider only non-trivial Nash equilibria (except when the initial endowment is pareto optimal).

AN EXAMPLE

At this point, an example may be useful. Suppose $\mu(T^o)=1/2$ and that T^1 is organized into two syndicates, S^1 and S^2, where $\mu(S^1)=\mu(S^2)=1/4$. The agents in T^1 have straight indifference curves with slope $dx_2/dx_1=-2$, while those in T^o have indifference curves whose slope is -3/2 for $x_2 > 3x_1$ and 0 for $x_2 < 3x_1$. The endowments densities are (2,2) for all agents.

The unique competitive equilibrium in this economy gives consumption densities of (2/3,4) to each agent in T^o and of (10/3,0) to each in T^1. However, this allocation does not correspond to a Nash equilibrium. Rather the allocation giving (1,3) to the agents in T^o and (3,1) to all in T^1 is a Nash equilibrium allocation here. To verify this, consider first the options open to the first syndicate. If it considers supplying an amount σ_2 in per capita terms of good 2, while S^2 continues to supply 1 unit per capita of good 2 and T^o supplies 1 unit per capita of the first good, then the members of S^1 will receive in return

$$\sigma_2 \frac{(1/2)1}{(1/4)1 + 1/4\,\sigma_2} = \frac{2\sigma_2}{1+\sigma_2}$$

of good one per capita. If on the other hand S^1 considers supplying σ_1 per capita of good one, each member will receive in return

$$\sigma_1 \frac{(1/4)1}{(1/2)1 + 1/4\,\sigma_1} = \frac{\sigma_1}{2+\sigma_1}$$

per capita of good two. Given the fixed supplies from T^o and S^2, these exchange possibilities give rise to consumption densities of

$$(x_1, x_2) = \left(2 + \frac{2\sigma_2}{1+\sigma_2} - \sigma_1,\ 2 + \frac{\sigma_1}{2+\sigma_1} - \sigma_2\right)$$

The corresponding attainable set is diagramed in Figure 1. It is readily verified that the intercepts are as indicated and that the frontier is given by the differentiable, strictly concave function $x_2 = (3x_1 - 10)/(x_1 - 4)$. Thus, the marginal cost of good one in terms of good two is $-2/(x_1-4)^2$. At the specified point, (3,1), this is equal to -2, the MRS of any agent in T^1. Thus, as claimed, the consumption density (3,1) is optimal for S^1. Of course, the same is true for S^2.

Now consider any trader $t \in T^o$. Such a trader faces a Walrasian budget constraint passing through his initial endowment and having slope $dx_2/dx_1 = -p = -\int s_1 / \int s_2$. Given supply profile here, this ratio is $(1/2)/(1/2)=1$. With the specified preferences for the agents in T^o, the optimal consumption is, as claimed, (1,3). Thus, the specified allocation is a Nash equilibrium and, indeed, it is the unique symmetric one with equal prices for the two goods.

This Nash equilibrium is not competitive: indeed, it is not even Pareto optimal, since it involves too low a level of trade. In contrast, if the core were our solution concept, then all solution allocations would, by Shitovitz's Theorem B, be competitive. Thus, the non-cooperative model leads to sharply differing results from the cooperative model of exchange. Moreover, the nature of the resulting outcomes is consistent with the intuition suggested by partial equilibrium analysis. Those agents with market power take into account the effect of an increase in supply on their part on the terms of trade. Thus their trades are not the Walrasian trades they would make if they had no impact.

OLIGOPOLISTIC EQUILIBRIA AND THE INCENTIVES TO ORGANIZE

It is clear that if some agents are organized the equilibria of this model cannot in general be Pareto optimal, since marginal prices equal the average price for unorganized agents while they differ for organized traders. Thus, the marginal rates of substitution will differ between agents at equilibrium. In this section we obtain further qualitative properties of the Nash equilibria.

At a competitive equilibrium, all agents with given tastes and endowments receive the same consumption (or, if preferences are not strictly convex, at least bundles that are indifferent). This property does not hold at the Nash equilibrium, as we now show.

<u>PROPOSITION</u>: Let S^1 and S^2 be two syndicates in T^1 of positive but differing sizes and let s(.) be a Nash equilibrium supply profile resulting in a strictly positive per capita consumption vector for the members of one of these syndicates. Then $x^1 \neq x^2$ where x^i is the equilibrium consumption vector for $t \in S^2$.

<u>Proof</u>: Let s be a Nash equilibrium and suppose $x^1 = x^2$ so $s^1 = s^2$. If for any agent we have $s_1(t)s_2(t) \neq 0$, we may change his strategy so that $s_i'(t)=0$ for that commodity for which $x_i(t) > w_i$ at the equilibrium and $s_j'(t)=w_j-x_j$ for the other commodity. This yields the same opportunity sets and final consumption vectors for all agents as the original strategies, so we will assume $s_1(t)s_2(t)=0$ for all t. Then since $x^1 = x^2$ we must have $s^1 = s^2$. We will further assume without loss of generality that $s_1^1 = s_1^2 = 0$, that is the organized traders are suppliers of the second good and receivers of the first.

Now, if the syndicate S^2 supplies σ_2^2 per capita to the market while the other agents supply the amounts from the equilibrium profile, then it would obtain the amount

$$b_1^2 = \sigma_2^2 \frac{\int s_1(t)}{\mu^2\sigma_2^2 + \mu^1 s_2^1}$$

of good one per capita in return. Therefore, the marginal price ratio between good one and good two for S^2 is

$$\frac{d\sigma_2^2}{db_1^2} = \frac{\mu^2\sigma_2^2 + \mu^1 s_2^1}{\int_{\sim(S^1 \cup S^2)} s_1(t)} \cdot \frac{\mu^2\sigma_2^2 + \mu^1 s_2^1}{\mu^1 \sigma_2^1}$$

At the Nash equilibrium strategy, $\sigma_2^2 = s_2^2$, so

$$\frac{d\sigma_2^2}{db_1^2} = p \frac{\mu^2 s_2^2 + \mu^1 s_2^1}{\mu^1 s_2^1},$$

where p is the average exchange ratio between goods one and two.

Since the Nash equilibrium results in maximization of per capita utility, we have (assuming $x^2 > 0$) $MRS^2 = p(\mu^2 s_2^2 + \mu^1 s_2^1)/\mu^1 s_2^1$. By the same reasoning, $MRS^1 = p(\mu^1 s_2^1 + \mu^2 s_2^2)/\mu^2 s_2^2$.

By assumption, the per capita consumption allocations are the same to the two syndicates, and since the utility functions are the same, we must have $MRS^1 = MRS^2$ at equilibrium. But this cannot happen, given $s^1 = s^2$ and $\mu^1 \neq \mu^2$. Thus, assuming an interior solution, the agents in the two unequal-sized syndicates must be treated differently. ||

It is perhaps worth noting that this proposition is valid in much more general situations than those specified here.

It seems intuitively appealing that the large of two syndicates should restrict its per capita supply more than the smaller one, since its supply has a larger marginal impact on price. In the case that neither good shows negative income effects for the members of T^1, this is in fact the case. At a Nash equilibrium, the actual or average price ratio is the same for all agents, so both syndicates' trades lie along the line with this slope through the common endowment. Given the assumed normality, the MRS is strictly decreasing along this budget line as x_1 increases. Thus, MRS^i/s_2^i is also decreasing. But $\mu^1/\mu^2 = \left(\frac{MRS^1}{s_2^1}\right)/\left(\frac{MRS^2}{s_2^2}\right)$ at equilibrium.

Thus, the larger organized group will make a smaller trade per capita and achieve a lower per capita equilibrium level of utility.

There is an interesting sidelight to the above calculations. We saw that $db_1^2/d\sigma_2^2$ (which can be thought of as marginal revenue) is equal to

$$\frac{1}{p}\left[\frac{\mu_1 s_2^1}{\mu_1 s^1 + \mu_2 s_2^2}\right] = \frac{1}{p}\left[1 - \frac{\mu_2 s_2^2}{\mu_1 s_2^1 + \mu_2 s_2^2}\right]$$

$$= \frac{1}{p}\ [1 - \text{the firm's share of the market}].$$

Thus we see that marginal revnue of a syndicate can be written in terms of the price and its "degree of market control." The expression is reminiscent of the expression derived by partial equilibrium techniques for marginal revenue for a firm, $MR = p(1 - \frac{1}{\eta})$ where η is the elasticity of demand. Here $1/\eta$ is referred to as the degree of monopoly.

The fact that an agent's equilibrium consumption depends on whether or not he belongs to an organized group and, if he does, on the size of the group to which he bleongs, raises the issue of stability of syndicates. We consider two aspects of this question: first, we look at the incentives facing a trader to join or break from a syndicate individually, and second we consider whether a group of agents will automatically gain from forming a syndicate and withholding supply. As might be expected, there is reason for an individual to act as a free rider and break from a syndicate as long as he expects the cartel to stay otherwise intact. However, we also show that the formation of a syndicate may prove disadvantageous to all of its members in that they all receive a less preferred consumption bundle when organized than they would if they all acted competitively. We then present a sufficient condition for this apparent anomaly not to arise.

A syndicate typically may be expected to withhold supply, trading less than it would under competitive behavior at the prevailing (average) exchange ratio, since it faces a marginal price for the good it is purchasing that is higher than the average price at the equilibrium. This is illustrated in Figure 2, where w is the endowment and x is the consumption chosen by the syndicate in per capita terms.

Now, an individually-infinitesimal, unorganized trader outside the syndicate faces the achievable set given by the average price line AA′, as opposed to that faced by the syndicate, which is given by BB′. The former set clearly contains points preferred to x. Moreover, if the syndicate will continue in existence, the defection of a single agent will not alter the average price ratio, so these preferred points would be attainable to him. Thus, any member of any syndicate has an incentive to leave the syndicate.

This result may,however, over-state the extent of cartel instability in finite economies, since in actuality no trader is truly negligible. Thus, he cannot automatically assume that the syndicate will not dissolve, and as well, his adopting competitive behavior will affect the price ratio. Both of these effects tend to lessen the expected gains to departing from the syndicate, and to impart some greater stability to syndicates. For a similar analysis in a somewhat different framework, see Postlewaite and Roberts (10).

However, even if syndicates are unstable in this sense, one might expect that there would still be an incentive to organize syndicates. The instability noted above can be explained in terms of the individual agent acting as a free-rider, gaining the benefits of the syndicate's restriction of supply without contributing to the burden of maintaining this restriction. However, given a group of agents with identical tastes and endowments, one might expect that they would collectively gain from collusion, even if, once the group is formed, an individual might gain from departing from it.

In fact, the situation is much more complicated than might seem. We illustrate this by considering two economies, each of which may be organized in differing ways.

The first economy is that given in the earlier example, in which all agents have an endowment (2,2), half the agents (those in T^o) have preferences with MRS = 3/2 for $x_2 > 3x_1$ and 0 for $3x_1 > x_2$ and the other half (T^1) have straight line indifference curves with slope -2. As noted earlier, the competitive equilibrium in this economy, which is also the Nash equilibrium when all agents are unorganized, yields consumption vectors of (2/3, 4) for traders in T^o and (10/3, 0) for those in T^1.

If all the traders in T^1 organize into two equal-sized syndicates, the outcome is (1,3) for T^o and (3,1) for each member of each syndicate. In this case, the T^1 agents gain from being organized, since they prefer (3,1) to (10/3,0).

The second example is identical to the first, except that for agents in T^o the MRS is 6/5 for $x_2>3x_1$, rather than 3/2. In this economy, the competitive outcome gives (1/3,4) to the agents in T^o and (11/3,0) to those in T^1, while the outcome when the T^1 agents are organized in two equal-sized syndicates is as in the first example, (3,1) for $t \in T^1$. In this case, the organized agents are all worse off as a result of being organized, since they prefer (11/3,0) to (1,3).

Another possible organization of these economies shows further complexities. Suppose that in each economy half the T^1 agents are organized as a syndicate while the other half and the agents in T^o are unorganized. In the first economy this results in a Nash equilibrium yielding (10/9, 10/3) to the agents in T^o, (10/3,0) to the unorganized agents in T^1 and (22/9,4/3) to the organized agents. In the second economy these vectors are approximately (.62,3.67), (3.67,0) and (3.11,.67) respectively. Using the utility function $u^1(x_1, x_2)$ $u^1(x_1, x_2)=2x_1 + x_2$ and taking the T^1 agents to consist of two equal-sized groups, either of which can be organized or unorganized, we can represent the outcomes to the T^1 agents in tabular form as below.

ECONOMY 1

	U	O
U	6.67 / 6.67	6.67 / 6.22
O	6.22 / 6.67	7 / 7

ECONOMY 2

	U	O
U	7.33 / 7.33	7.33 / 6.89
O	6.89 / 7.33	7 / 7

Thus, in the first economy, if the agents represented by the rows are organized (O) while those represented by the columns are unorganized (U), the outcome gives utility 6.22 to agents in the first group and 6.67 for those in the second.

In the first economy, the best outcome from the point of view of a T^1 agent is when both groups are organized. If he (and his group) is unorganized, it is a matter of indifference to him whether or not the other group organizes or not. The worst outcome is for him to be in an organized group while the other group's members are acting competitively. Thus, starting from a competitive organization, one would not expect either group to form unilaterally. If one starts from a position of full cartelization, this too is stable. And one might expect that an organized syndicate facing a competitive fringe would dissolve if it were able to predict the outcome that would thereby result.

In the second economy the ranking of the outcomes differs. Here the T^1 agents are best off if they are completely unorganized. Any agent in one group is, however, indifferent as to whether the other group organizes or not, given that his group is unorganized. Either of these situations is preferred by him to the situation in which both syndicates are organized, while the worst outcome again results from being a member of an organized group while the other group behaves competitively. In this economy, then, the cartelized outcome is unstable, while the competitive outcome is stable. Any movement towards organization hurts those attempting it.

This matter of the formation of a syndicate hurting the agents involved in the syndicate is reminiscient of the phenomenon of disadvantageous syndicates noted in connection with the core by Aumann (2) and investigated by several other authors since. Generally the formation of a syndicate will enlarge the set of core allocations since the coalitions involving only some of the members of the syndicate are no longer available for blocking. One then says that a syndicate is disadvantageous if the new allocations entering the core are worse for the members of the syndicate than the allocations originally there.

A major focus of research on the issue of disadvantageous syndicates has been to explain their origin and existence in economic terms. In the context of our non-cooperative model, there seems to be an obvious explanation. In particular, an agent (or syndicate) in deciding on his best action takes the bids forthcoming from the rest of the economy as fixed, i.e., he assumes demand for the good he supplies is of unitary elasticity. This is, in general, clearly not a correct expectation and in fact, a syndicate's impact is such that this misperception can become important.

Suppose that, in fact, the offer curve from the unorganized portion of the economy actually did have the assumed shape, at least in a region of the Nash equilibrium. This is illustrated in Figure 3, which may be interpreted in terms one side of the market consisting either of a single syndicate and an organized group of equal size or of two equal-size syndicates. Here w is the endowment point, x is the Nash equilibrium and the line connecting them gives the average price ratio. The curve II′ is the indifference curve of a member of the syndicate through the equilibrium, which necessarily is steeper than the price line since the marginal price exceeds the average for the syndicate. The horizontal line OO′ is the offer curve, in per capita terms, of the unorganized part of the economy.

The intersection of the competitive offer curve of the syndicate with OO′ would be given the competitive equilibrium. Since this offer curve must meet II′ below and to the right of x, must pass through w, and must lie above the plane through x whose slope is the MRS there, it is clear that it must intersect OO′ below II′ . Thus, there will be a competitive allocation which is worse for the syndicate than the Nash allocation.

In fact, as long as the offer curve passes through x and never doubles back so as to be above II′ again below and to the right of x, there will be such a competitive allocation. In this regard, it is useful to compare the two examples given earlier: this condition is met in the first example when both groups are organized, and the organized outcome is better than the competitive one; it is not met in the second, and the syndicates are disadvantageous. Of course, this condition on offer curves near the equilibrium is unsatisfactory, since it involves the equilibrium. We leave it as an interesting but open question as to whether conditions on preferences and endowments can be devised to insure that organizing is beneficial to the traders involved.

CONCLUSIONS

We have examined a very simple general equilibrium model of exchange in which some agents, by acting collectively, have monopoly power, while others, acting individually, are competitive price-takers. The results and examples we present indicate to us that this model is a useful one for examination of oligopoly. In particular, the results on the qualititative nature of the equilibria - that they are non-optimal and that, in the normal case, large groups restrict supply more than small ones - would seem to be useful results. The issue of cartel stability which we have discussed is also significant and worthy of further work. In particular, our results indicate that cartelization is not always profitable, and this raises the difficult but fascinating issue of building a model in which one can explain endogenously which cartels will emerge.

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Figure 1

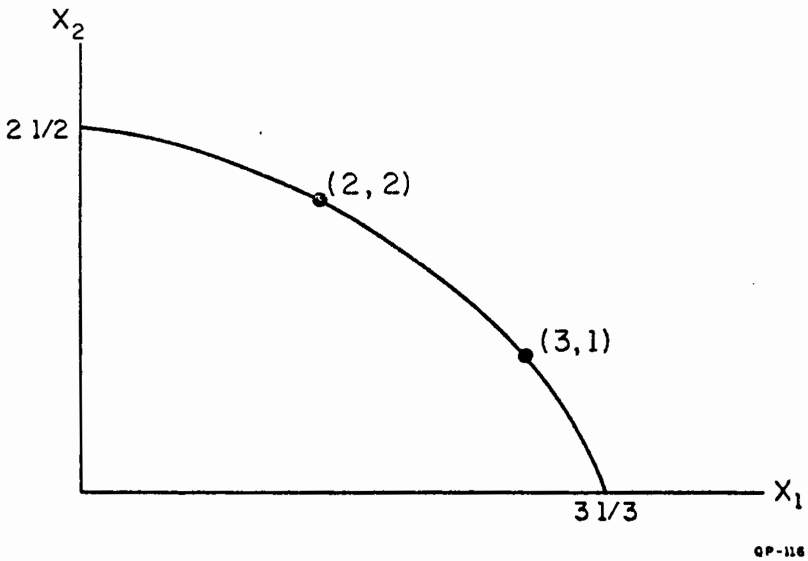

Figure 2

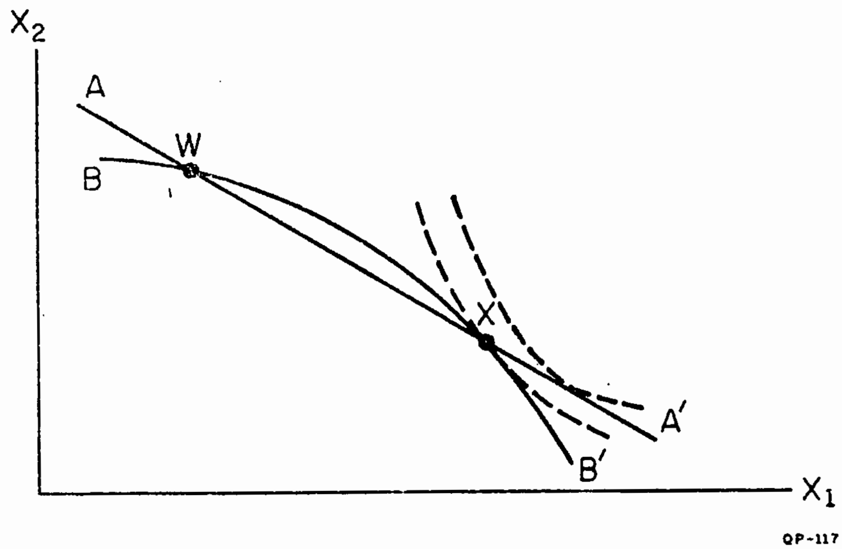

Figure 3