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Input-Output Methods in Forecasting[1]

by

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I. Introduction

Forecasts are more easily produced than they can be reconciled. The number of forecasting techniques available is only exceeded by the number of possible levels and time spans for which forecasts are sought. Forecasts range from very broad macro forecasts of variables such as GNP, inflation rate and gross investment, to product line sales, and material requirements forecasts within a corporation. Choosing the right forecasting technique involves trade-offs between costs (e.g., system development and computational costs), lead time and accuracy. Simple rules of thumb are sometimes offered suggesting, for instance, the use of (1) low-cost time series methods for tracking many disaggregated corporate series; (2) more sophisticated time series models for broader-based series calling for increased accuracy; and (3) full-fledged econometric models for a corporation (or a division thereof), an industry or the national economy. Table 1 summarizes the links between the various forecasting requirements and available models.

Referring to this table the following observations can be made. In general time-series models have been found useful to track a large number of highly disaggregated series--e.g. inventories--in situations where the causal links are stable enough to allow us to safely extrapolate from past observations on that series. When more aggregate series are involved, the impact of a wider range of contributing factors must be allowed for. Econometric techniques are well suited for this purpose. But in this case forecasting requires the knowledge--or the prediction--of exogenous explanatory variables. This additional information requirement makes the resulting forecast conditional; on the other hand it allows more opportunities for sensitivity analysis. As is well known, econometric model specification is a difficult art. Existing macroeconomic theories and corporate planning models are helpful in this task. However, linking macro and micro models is no simple matter. For instance, saying that real GNP will grow at a rate of 5.5% over the next twelve months hides many varied fortunes and misfortunes for different industries. Cor-

Table 1
Hierarchy of Forecasts

Level	Typical Uses	Link to Ensure Forecast Consistency	Usual Techniques	Examples
Aggregate Economic Forecasts Example: Macro-econometric Models	1. National and/or International Economic Policy Formulation 2. Results become Exogenous Inputs for Industry and Corporate Models	Integration of National Macro-Models (e.g. Project LINK) and Input-Output Systems (e.g. INFORUM, LINK [41])	General Simultaneous Equation Econometric Models	[19]
Industry Forecasts	1. Formulation & Testing of Alternate Economic Policies; e.g. (De)regulation of Certain Industries 2. Results Also Used as Exogenous Inputs to Corporate Models	National Input-Output Systems	Input-Output Simultaneous Equation Model Integrating All Interrelations Between Industries Industry-Specific Econometric Models	[4] [48]
Corporate Forecasts	1. Strategic Policy Projection of Income Statements & Balance Sheets for Corporate Planning 2. Results Serve as Inputs to Lower Divisional & Departmental Models	'Market Share' Models Corporate Input-Output Systems	Corporation Specific Econometric Models and Time-Series Techniques	[44]
Product Line and Inventory Item Forecasts			Time Series Techniques	

porations are vitally interested in knowing about these divergent fates as it bears on their growth and/or diversification plans. When it comes to taking account of the full array of interconnections between all industries and the potential impacts they have on each other, no comprehensive econometric model can be specified without some broad theoretical framework to help the econometrician. Input-output tables provide such a framework. At the corporate level input-output analysis (IO) allows corporations to get a detailed, coherent and comprehensive view of their position among other industries and final markets and even, if properly interpreted, within their industry. By quantifying the network of technical interdependence linking all industries within a given geographical area (metropolis, region or more commonly nation), IO analysis enables the corporation to understand the repercussions of changing conditions among its vendors and its clients--and their respective vendors and clients etc. By specifying and measuring the degree of interrelatedness between industries, and firms within these industries, IO analysis is capable of translating broad macro forecasts into the precise industry specific forecasts they imply.

In this paper, we explain the use of the IO methodology in forecasting. Since its inception a number of articles have reported on the usefulness of IO techniques for sales forecasting [1] [43] and financial decision making [45]. Similar uses are, of course, suggested and sketched in Leontief's original exposition of the method [31] [32] [33]. Following the introduction of the static input-output model by Leontief some forty years ago, theoretical and applied research on IO has grown steadily exploring (1) the formal properties of Leontief matrices [22] [39]; (2) their relationships with linear programming and general optimization problems [18] [30]; (3) their application to management problems [44]; and (4) their extensions to regional [29] [37] [38] [50] or metropolitan economic forecasting [26], integrated IO international trade flows forecasting [41], and dynamic systems [34]. Paralleling this growth in research, public and private research and consulting organizations have estimated actual input-output tables and integrated them with forecasting models of varying degrees of complexity. Federal agencies such as the Bureau of Economic Analysis, U.S. Department of Commerce [12], the Bureau of Labor Statistics [15], the Federal Preparedness Agency, and the U.S. Department of Transportation, all have developed IO tables and related series at various levels of industry disaggregation. Universities and research institutes have also developed and/or used their own IO systems; examples include the University of Maryland INFORUM model [4] [5], and the Battelle Memorial Laboratory PREVIEWS model [8]. Finally, IO models and forecasts are currently marketed by various corporations and consulting firms such as Arthur D. Little [35], Data Resources Inc. [20], Wharton [42], Chase Econometrics [28], the General Electric Corporation [36] and the Econoscope Group Inc. [21]. Such widespread use and growing interest in IO techniques for corporate forecasting and planning underscores the need for an up-to-date exposition and numerical illustration of (1) the links between the input-output model and both macro and micro economic models and (2) traditional and new ways of using input-output methods for managerial planning and forecasting. These are discussed in section III and IV respectively. To make this paper self-contained a brief tutorial on the fundamentals of IO analysis is given in Section II. We remark on the limitations and likely future improvements in IO methods in the conclusion.

II. Fundamentals of Input-Output Analysis

This section gives a self-contained presentation of (1) the derivation of an input-output table from national and business accounting data and (2) the interpretation of such tables from a production function or market share standpoint. Also, some of the key IO identities, which hold exactly only if the assumptions discussed throughout this section are met, are summarized for convenient reference. Departure from these key assumptions and the resulting adjustments required are also briefly described to give the reader some feel for the adequacy of the model and the reliability of existing tables.

II.1. The Accounting Framework

Consider the summary income statement of some firm as given in Table 2. A simple modification for inventory changes yields the firm production statement as given in Table 3. With these accounting data for all firms in an economy, it is

Table 2
Income Statement of Firm X

Expenditures	Receipts
I. Purchases from other firms (Intermediate purchases)	1. Sales to industrial users (Other firms and industries)
II. Value Added	1.1. Company #1
1. Employee compensation (wages and salaries)	1.2. Company #2
2. Indirect business taxes	1.3. Company #3
3. Property-type income	2. Sales to final users
3.1. Proprietor's income	2.1. Consumers
3.2. Rental income	2.2. Business on capital account (Investment)
3.3. Corporate profits and inventory valuation adjustment	2.3. Government
3.4. Net interest	· Federal
3.5. Depreciation (Capital consumption allowances)	· State and local
	2.4. Rest-of-the-world
Total Current Expenses	Total Current Receipts

Table 3
Production Statement of Firm X

Allocations	Receipts
Total Current Expenses	Inventory Change
Allocation of Total Value of Production	Total Value of Production

theoretically straightforward to construct an IO matrix. Specifically, if each company produces a homogeneous type of output we can group companies into (say) n 'industries' and record for each such industry the dollar value of sales to all others over a given accounting period. These sales profiles form the n rows of the IO transactions matrix $[x_{ij}]$. A moment's thought will also convince us that the purchases from other firms' (item I) in the income statement need not be separately recorded as it is obtained by labelling the columns of the table with the same industry breakdown as for the rows. The resulting table is thus a square matrix with the same industries (products) over its rows and columns. To balance the system we can also record to the right of the IO table the final sales (item 2) classified as in table 2. This yields the (n X z) matrix, $F = [f_{ik}]$ showing the industrial composition of final sales. Similarly, we can record the various incomes originating in this firm (value added) by adding them to the bottom of each industry column thus obtaining a (s x n) matrix, $V = [v_{hj}]$ of 'primary' inputs to production. Table 4 summarizes the overall accounting scheme for a very aggregated 4-industry version of the actual U.S. 1967 IO table [12].

The IO Transactions Table is in the northwest corner of Table 4. As can be seen from this table, IO data expand the general national income and product accounts by focusing on intermediate transactions (sales/purchases) between industries which

Table 4

Transactions Table, Millions of Dollars 1967

	INTERMEDIATE DEMANDS					FINAL DEMAND				TOTAL FINAL DEMAND	TOTAL GROSS OUTPUT (SALES)
SALES / PURCHASES	Construction	Durables	Nondurables	Services	Total Intermediate Sales	Consumption	Investment	Net Exports	Government		
Construction	30	1793	2290	13583	17696	0	59258	0	26325	85534	103280
Durables	34652	113586	44626	31747	224610	115896	97578	6994	27142	247611	472222
Nondurables	10992	55682	141500	33440	241615	131551	2970	-248	13304	147578	389193
Services	12017	61953	58300	103960	236230	143724	0	760	80737	225220	461450
Total Intermediate Purchases	57690	233014	246717	182731	720153	391171	159808	7506	147509		
Wages	27002	141677	84385	165078	418142						
Property-type Income	14562	76407	45509	82097	225505		GNP				
Indirect Business Taxes	4026	21124	12582	24613	62613						
Total Value Added	45590	239208	142475	278718						705992	
Total Gross Output (Purchases)	103280	47222	389193	461450							1425145

are normally excluded from GNP computations to avoid "double-counting".

II.2. Implementing an IO System

Practical implementation of the above framework requires certain assumptions and compromises to process existing data. Firstly, the question of the industry grouping raises the same problems as the setting up of a Standard Industrial Classification system (SIC). Very few firms are so specialized as to produce a single "product". The SIC classification is used (with some regroupings) to process the quinquennial U.S. Census of Manufactures data which serve as the basic data source for the U.S. tables.(1) The most disaggregated table is at the four-digit SIC level, comprising 484 industries. Aggregated tables for 365 and 81 industries are also made available by the U.S. Department of Commerce. Other organizations have used different aggregation levels to meet their special needs--ranging from about 200 to 35 industries. As explained later, the grouping scheme bears upon the forecast accuracy of an IO system and raises some important theoretical issues [9] [10]. To deal with multi-product firms, the data are collected at the "establishment" [47] level. The 'primary product' of that establishment is recorded in the corresponding industry row and column; any other output--'secondary product'--is considered to be part of the output of some other appropriate industry and 'allocated' accordingly. Secondly, imports for which comparable domestic substitutes are available are similarly allocated, while non-substitutable imports are treated as primary inputs and included in a separate row at the bottom of the table. Thirdly, a few "dummy" industries have to be set up to record such diverse items as 'business travel and entertainment'--a fitting reminder of the pervasive 'expense account' phenomenon!--and small office supplies. To the extent that these allocations can never be made

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(1) Benchmark IO tables exist for 1947 [13], 1958 [24], 1963 [14] and 1967 [12]. The 1972 table is about to be published. Summary annual updates have also been obtained up to 1976 for the Federal Preparedness Agency.

unambiguously, the resulting data contains some unavoidable noise, analogous to the measurement error in any econometric work.(1) A final word of caution about the prices used in the U.S. IO tables should be given. Producers' prices as opposed to purchasers' prices are used and the trade, insurance and transportation margins, which make up the difference between these two, are recorded separately as transactions with three other industries--'trade', 'insurance' and 'transportation'.

II.3. IO Assumptions and Relationships

II.3.1. IO Projections from Exogenous Demand. The transactions data thus recorded are not yet usable for forecasting purposes without some basic assumptions to 'explain' these observed transactions amounts. Traditionally a fixed coefficient production function assumption is made to explain the columns of the n X n transactions matrix $[x_{ij}]$. Specifically it is assumed that, for example, the amount of steel (i) per automobile (j) is fixed in the short run so that any change in automobile output will require a proportionate change in steel input. Each industry production process is thus described by n input coefficients, a_{ij}, $i = 1,2,\ldots,n_j$

$$(1) \quad a_{ij} = \frac{x_{ij}}{x_j} \quad \text{where } x_j = \sum_j x_{ij} + \sum_k f_{ik}$$

and f_{ik} is the final sale of i to the kth final demand category

In theory these a_{ij} coefficients could be computed in physical units e.g. so many pounds of steel or square feet of sheet metal per automobile of a given kind. The huge product breakdown which such a method entails has precluded its implementation so far. Practically, the coefficients are derived in dollar terms (dollar worth of steel per dollar worth of auto output). The resulting n column vectors with n entries (including possibly some 0's) are the production function of the n indus-

(1) The extent of the model specification error--if any--on the other hand depends, among other things, on the adequacy of the fixed coefficient production function assumption as discussed later.

tries. Together they form the <u>input coefficient matrix,</u> A, as shown in Table 5 below. This table represents an aggregation into four 'industries' of the actual U.S. Input-Output tables for 1967 (81 industry level).

Table 5
Direct Coefficients Matrix, 1967

	Construction	Durables	Nondurables	Services
Construction	0.0002	0.0037	0.0058	0.0294
Durables	0.3355	0.2405	0.1146	0.0687
Nondurables	0.1064	0.1179	0.3635	0.0724
Services	0.1163	0.1311	0.1497	0.2252

Although in the sequel only value IO coefficients will be used, we note that the relation between physical, $\tilde{a}_{ij}$, and the value coefficient a_{ij} is

(1') $$a_{ij} = \frac{P_i}{P_j} \tilde{a}_{ij} \quad \text{or} \quad A = \langle P \rangle \tilde{A} \langle P \rangle^{-1}$$

where $\langle P \rangle$ is a diagonal matrix with P_i = price of ith product.

Letting e be a z vector of one's, and taking the final matrix, F, as exogenous for the moment, we obtain the basic 'open' IO model:

(2) Total Output = Intermediate Sales + Final Sales (for each industry)

(2') $$X = AX + Fe$$

(2") $$(I - A)X = Fe$$

(where I is the $n \times n$ identity matrix). Under certain conditions on A ([25][39][46]), which are obviously met by actual tables, the system is 'workable' in the sense that (I-A) is nonsingular so that we can solve for X.

(3) $$X = (I - A)^{-1}(Fe)$$

Note that the coefficients of the inverse ($L = (I-A)^{-1}$) are nonnegative and no less than the corresponding entries in A. Each entry of L is readily interpreted as the total direct and indirect--i.e., including all second-round, third-round etc.--derived input demand for input i per dollar of final sales of product j. These coefficients can also be shown to be a comprehensive measure of overall interrelatedness between industries (i) and (j) [11]. Table 6 illustrates the <u>Leontief inverse matrix,</u> L, for our previous example.

Table 6
Direct and Indirect Requirements, L

	Construction	Durables	Nondurables	Services
Construction	1.0127	0.0157	0.0220	0.0419
Durables	0.5196	1.3949	0.2964	0.1713
Nondurables	0.2995	0.2947	1.6728	0.1940
Services	0.2980	0.2955	0.3769	1.3636

The difference between the indirect and direct requirements for each industry can be quite large. For instance, the primary aluminum industry has no direct sales to the mobile home construction industry, yet it "is among the leaders in total sales generated by mobile home production " [53]. Not all components of final demand need to be assumed exogenous; some can be made endogenous by computing input coefficients for this new "industry" and assuming their short-run stability for forecasting purposes. For instance the household sector can be endogenized by treating personal consumption expenditures in this fashion. Correspondingly, the wages and other household income flows (dividends, etc.) are treated as intermediate inputs; in effect, the household 'industry' is removed from the margin and added as a row and column in the IO matrix. Investment in producers' durables is another obvious case which leads to the computation of a capital coefficient matrix--so much worth of steel per dollar worth of output of a given kind of producer's equipment. Completing this endogenization process for all final demands and corresponding primary inputs elements leads to the 'closed' IO system

$$(4) \qquad X - AX = 0$$

where X and 0 are (n+z) $\times$ 1; A is (n+z) $\times$ (n+z). Clearly solutions to a fully closed system are uninteresting for forecasting purposes since there are no exogenous factors. However, the theoretical properties of such a system are useful in a variety of applications [22].

Returning to the 'open' IO system, note that equations (2') and (3) correspond respectively to the 'structural form' and the 'reduced form', of a simultaneous equation econometric model. A possible model specification error would result from (1) erroneously assuming fixed input coefficients or (2) adopting a sectoring plan--industry grouping--which does not exactly reflect the types of material inputs considered by firms. As explained later, discriminating between these two causes for explaining and correcting IO forecast errors is made difficult by the fact that historically observed coefficient changes can arise from any one of three factors: (1) <u>product mix</u> changes in a given industry; (2) <u>relative input price</u> changes; and (3) <u>actual technological</u> changes in the industry--for instance, as a result of (2) (see [16][17]).Yet the basic interpretation of an open IO system remains: dollar sales for each industry (endogenous variables) are 'explained' (hence, can be forecast) as a linear combination of the prespecified (exogenous) set of industry-specific final demands; the coefficients in this combination being the corresponding industry row vector in the Leontief inverse matrix. This forecast equation contrasts with the standard regression estimation techniques for stochastic simultaneous equation models. Here the reduced form coefficients (entries of the Leontief inverse ℓ_{ij}) are computed from an estimated base year transactions matrix and the forecast is derived as explained. Forecast errors result from (1) variations in the coefficients and (2) incorrect final demand assumptions. This latter source of error is, of course, present in any econometric forecasting equation; even with known coefficients, the forecast is only as accurate as our estimates of the exogenous inputs. Formulas for the model-specific forecast errors are explicitly computed for the empirical example given in Section 4.

II.3.2. <u>IO projections from primary inputs</u>. A less well known view of IO relationships can be developed by considering output coefficients as introduced in [6] [23]. In this case we define the output coefficient b_{ij} as the share of industry i sales to j in industry i total sales:

$$(5) \qquad b_{ij} = \frac{x_{ij}}{x_i}$$

We note the relationship between b_{ij}'s and a_{ij}'s: $b_{ij} = a_{ij} \cdot \frac{x_j}{x_i}$. Thus, if the a's are assumed stable over time the b's cannot be and conversely. The balance condition for the whole system requires that each industry's total sales just exhaust its total outlays on intermediate and primary inputs. For instance

$$x_j = \sum_{i=1}^{n} b_{ij} x_i + \sum_{h=1}^{s} v_{hj} \tag{6}$$

where v_{hj} is the dollar value of the hth type primary input (e.g., the wage bill) in industry j. Equivalently, we can write

$$B'X + V'e = X \tag{6'}$$

where B' is the transpose of the (n X n) <u>output coefficient matrix</u>; and V' is the transpose of the (s X n) primary inputs matrix. Under the same workability conditions as before (eq. 3) we can solve for total sales of each industry

$$(I - B')X = V'e \tag{6''}$$

and

$$X = (I - B')^{-1}(V'e) \tag{7}$$

The output coefficient matrix and the M = (I - B') inverse are illustrated for our example in Tables 7 and 8.

<u>Table 7 - Output Coefficient Matrix, B</u>

	Con-struction	Durables	Non-durables	Ser-vices
Construction	0.0002	0.0173	0.0221	0.1315
Durables	0.0733	0.2405	0.0945	0.0672
Nondurables	0.0282	0.1430	0.3635	0.0859
Services	0.0260	0.1342	0.1263	0.2252

<u>Table 8 - M = $(I - B')^{-1}$</u>

	Con struction	Durables	Non durables	Ser-vice
Construction	1.0127	0.1136	0.0795	0.06
Durables	0.0719	1.3949	0.3576	0.3
Nondurables	0.0831	0.2443	1.6728	0.31
Services	0.1874	0.1674	0.2300	1.3

Interpretation of the M matrix is quite simple. It enables us to explain (predict) total sales of each industry (endogenous variables) by a linear combination of the total dollar value of primary inputs available (or allocated by, for instance, skill-specific job considerations), in every industry. The (reduced form) coefficients in this combination, yielding our industry sales 'forecast', are the entries of each row of the M matrix. Each such entry measures the dollar value of industry i output attainable with primary inputs given by the value added matrix V. Also, we can usefully contrast the two views (input vs. output) of industry sales forecast as given by equations (3) and (7). A final demand-driven model leads to the usual Leontief inverse L, while a primary input-driven model leads to the M matrix. If final demand and primary inputs are both taken to be exogenous, the two sales forecast X_F from equation 3 and X_V from equation 7 will be equal only if industry sales have changed by a scale factor, say λ. In this case $X^{(t)} = \lambda X^{(o)}$ so that the proportions $\frac{X_i^t}{X_j^t} = \frac{X_i^o}{X_j^o}$ for all pairs of industries (i,j)--or equivalently $(Fe)^{(t)} = (Fe)^{(o)}$. In this case, clearly $b_{ij}^{(t)} = \frac{a_{ij}X_j^{(t)}}{X_i^{(t)}} = b_{ij}^{(o)}$. Whereas if $X^{(t)} = TX^{(o)}$ where T is a general linear transformation, then the ratios $b_{ij}^{(o)} = \frac{x_{ij}^{(o)}}{X_i^{(o)}}$ change to $b_{ij}^{(t)} = \frac{x_{ij}^{(t)}}{X_i^{(t)}} \neq \frac{x_{ij}^{(o)}}{X_i^{(o)}}$. Of course, for a given year if all IO relations are measured exactly the system will balance yielding a relationship explaining income flows (value added) by final demand. Presumably, some adjustment process needs to be specified through a macro model to provide the link with F that yields a consistent sales forecast. In any case, the following condition obtained by combining equations (3) and (7), must be satisfied, ex-post:

$$(8) \qquad (V'e) = (I-B')(I-A)^{-1}(Fe) \quad \text{or} \quad (8') \qquad V'e = Q\ (Fe)$$

The matrix, Q, for the sample data is shown in Table 9 below.

For forecasting from a base period into the future, if we adopt the 'input' view of the model and take the a_{ij}'s as constant the translation of an X_F forecast into a $(V'e)$ forecast can be done by using a simple diagonal matrix $\langle\frac{(V'e)_i}{X_i}\rangle$

to premultiply $(I-A)^{-1}(Fe)$, since assuming constant $[a_{ij}]$ is equivalent to assuming constant $\frac{(V'e)_i}{X_i}$. This 'specialized Leontief inverse' $\langle\frac{(V'e)_i}{X_i}\rangle$. L is used in [52] and [19]. On the other hand, if we take the 'output' view, we assume the fixity of the b_{ij}'s, then the a_{ij}'s--and, hence, the ℓ_{ij} coefficients of the inverse--have to change so that (8′) holds with unchanged b_{ij}'s. With either assumption--fixed a's or fixed b's--equation (8′) acts as a constraint that holds ex post and thus can be used to compute the updated b's or a's coefficients.

Table 9 - Matrix Q, 1967

	Construction	Durables	Nondurables	Services
Construction	0.9581	-0.1026	-0.0567	-0.0116
Durables	0.2942	0.9772	-0.0651	-0.0814
Nondurables	0.0814	0.0180	0.9884	-0.0659
Services	0.0370	0.1078	0.1254	1.0227

III. The Interface Between IO Methods and Macro and Micro Forecasts

Having thus described the fundamental IO assumptions and algebra, we are now in a position to specify how the IO model serves as a link between macro forecasts and micro (corporate) forecasting. Actually, this can be done in many ways. Here we do not intend to cover all the conceivable ways of integrating IO in a hierarchy of forecasts. Rather we intend to discuss some of the key submodels needed to 'close' the system. Relevant assumptions bearing on forecast accuracy are also specified. In this manner, the reader will be able to judge for himself how the many existing macroeconometric models can be related to corporate forecasting systems via IO.

III.1. The Macro-IO Interface

At the most aggregate level, a very simple way of translating (say) a GNP forecast into an industry sales forecast is to use the basic open IO relation given in equation (3)

(3) $X = (I - A)^{-1}(F\ e)$

We only need to allocate GNP into its industry components to obtain the final demand vector (Fe). A simple assumption is to consider that the allocation of GNP by industry for some base year will remain unchanged for the forecast year. The percent breakdown coefficients thus obtained are one instance of what is commonly called 'bridge coefficients'. Clearly the assumption underlying (3) is that final demand (Fe) is entirely exogenous to the system. Such demand-dominated models, popularized in textbook expositions of elementary Keynesian economics, are, of course, one-sided.

At the other end of the spectrum, one can consider pure supply-dominated models in the spirit of 'Ricardian' economics. There a basic variable to be explained is the income distribution--globally and by industry. To illustrate, suppose for a moment that we are given the total amount of all primary inputs available--say, for instance, labor measured in man-hours -- then the demand for primary inputs by industry is simply given as a function of gross industry output as in equation (6").

$$(6'') \qquad (V'e) = (I-B')X$$

And, of course, if X is itself explained by some other exogenous variable--as, for instance, final demand--we can write as in equation (8')

$$(8') \qquad (V'e) = Q\ (Fe)$$

One step beyond this would make the primary inputs themselves endogenous as implied by (7). This is done, for instance, for capital in the dynamic Leontief IO system where account is taken of the fact that one component of final demand, gross investment, determines the capital stock over time. Another alternative is to make the drastic Malthusian assumption of an endogenous labor force via feedbacks effects of 'starvation wages' on population.

These extensions, however, point the way to a much broader integration of the IO 'marginals' (V and F)--and components thereof--of the IO table to close the macro system. An obvious method has already been mentioned in the context of the closed IO system. Consider each final demand component as an industry and compute consumption, investment, government and exports 'input coefficients' for each such 'industry'. For instance, the household industry 'production function' consists of the percent breakdown of aggregate consumption by product (industry). This, of course, assumes stable product shares in a typical consumer budget--which may be less realistic than the assumption of fixed IO coefficients for manufacturing industries. Paralleling this endogenization of the right-hand side (F) marginal of IO, a similar process is applied to the bottom marginal, V. If, for instance, households' income consists of, say, wages only, then the wage row in V when divided through by the total output of the column industry describes the labor input coefficient per unit of output for each industry. The resulting system is the closed Leontief system

$$(4) \qquad AX = X \quad \text{or} \quad (I-A)X = 0$$

which has only the trivial (0) or indeterminate solutions--one of the x's, say x_i, must be set in order to determine the others. The extreme solution represented by the closed IO system suggests more general and useful feedback mechanisms to integrate IO within a macro system--thus opening the way to a consistent translation of macro forecasts into industry and corporate forecasts. In all cases 'bridge' models are required. Two broad classes of models can be used.

III.1.1. <u>Constant 'Bridge' Coefficients Tables.</u> In constructing a bridge for V it can be assumed, for instance, that in each industry the labor/output ratio is constant for any level of output--but possibly different across industries. Similarly for other primary inputs the same fixed coefficient assumption can be made. Alternatively other 'bridges' can be defined for V and F; e.g., the wage bill in industry j/total factor payment in j; or consumption of product i/total consumption etc. In all such cases the choice of a particular type of bridge coefficient is guided by (1) what data are assumed known (exogenous) for instance given by an outside source (model, expert judgment, etc.); and (2) if alternative data (disaggregated or aggregated) are equally available, which bridge coefficients can be more safely assumed to be stable. An example of this is given by GNP vs. components of GNP (consumption, investment, government, rest-of-the-world) forecasts. Clearly the more disaggregated the exogenous data are, the greater the amount of information is required to compute the chosen 'bridge' coefficients and then prepare the forecast. Also, as noted earlier assuming certain coefficients to be stable rules out the constancy of other coefficients (e.g. a's vs. b's) so that consistency among assumptions must always be checked. Table 10 below summarizes the variety of industry level forecasts obtainable from alternative assumptions about (1) which variables are taken to be exogenous and (2) which bridge coefficients are used. By and large many of the published and commercially developed industry sales forecasts correspond to the first two rows (I·A and I·B) of this table. An obvious way to extend the bridge coefficients approach short of a full integration within a macro econometric [40] model is to track their time path and adjust them accordingly ([4] [51]). We illustrate the use of these methods with empirical results in the next section.

III.1.2. Fully integrated variable bridge coefficients models. In this class we find many large-scale macroeconometric models which attempt to disaggregate their forecasts at the industry level. Well-known examples include the Brookings model [19], and C. Almon's INFORUM model [3][4][5]. The logic of this type of integration is best illustrated in Figure 1 below, wherein some typical exogenous variables are specified and the most common linkages used to (partly) close the model are shown. As in any simultaneous equation econometric model such linkages can be of the exact non-stochastic type (identities) or they can be assumed behavioral relations--e.g. consumption functions or investment func-

Queries	Structural Assumption	Exogenous Input	Computations: 'Bridge' Coefficients	Computations: Forecasting Equation	Residual Model Error Covariance
(I) Industry Sales Forecast (Final Demand Approach) From GNP	1. Stability of input coefficients (a_{ij}) over time 2. Stability of proportions of final demand in GNP (only necessary for (A) and (B)).	A. Aggregate GNP level B. GNP by major final demand categories (e.g., Consumption, Investment) C. GNP by industry D. GNP by industry & final demand category	$B_F = \frac{1}{e'Fe} \cdot Fe$ $B_F = \frac{1}{e'Fe} \cdot F$	$\hat{X}_F = LB_F e'Fe$ $\hat{X}_F = LB_F (e'F)'$ $\hat{X}_F = L\ Fe$ $\hat{X}_F = L\ \hat{Fe}$	$\Sigma_{\hat{X}_F} = LE[\epsilon_{\widehat{Fe}} \cdot \epsilon'_{\widehat{Fe}}]L'$
(II) Industry Sales Forecast (Primary Inputs Availability Approach) from NI	1. Stability of output coefficients (b_{ij}) over time 2. Stability of proportions of factor payments in NI (only necessary for (A) and (B)	A. Aggregate National Income NI (Value Added) level B. NI by major type of factor payments (e.g. wages, profit, rent, interest, indirect business taxes) C. NI by industry D. NI by industry & type of factor payments	$B_v = \frac{1}{e've} v'e$ $B_v = \frac{1}{e've} \cdot V'$	$\hat{X}_v = MB_v e've$ $\hat{X}_v = MB_v Ve$ $\hat{X}_V = M\ v'e$ $\hat{X}_v = Me'\hat{v}$	$\Sigma_{\hat{X}_F} = ME[\epsilon_{\widehat{e'v}} \cdot \epsilon'_{\widehat{e'v}}]M'$
(III) Components of Value Added--e.g., Corporate Profits--from Final Demand	1. Stability of input coefficients (a_{ij}) over time 2. Stability of proportions of factor payments in NI and possibly: 3. Stability of proportions of final demand in GNP (depending upon level of disaggregation.)	A. GNP at any level of disaggregation (see IA,B,C, D, above)	Bridge for $\widehat{Fe}$ constructed as in I above $\tilde{B}_V = V\langle e'v\rangle^{-1}$	$v'e = Q\ \widehat{Fe}$ $\hat{V} = \tilde{B}_V \langle \widehat{v'e} \rangle$	$\Sigma_{\widehat{e'V}} = QE[\epsilon_{\widehat{Fe}}\ \epsilon'_{\widehat{Fe}}]Q'$
(IV) Components of Final Demand from Available Primary Inputs	1. Stability of input coefficients (a_{ij}) over time 2. Stability of proportions of final demand in GNP and possibly: 3. Stability of proportions of factor payments in NI (depending upon the level of disaggregation of NI forecast)	A. National Income at any level of disaggregation (see IIA,B,C, D, above)	Bridge for $\widehat{v'e}$ constructed as in II above $\tilde{B}_F = \langle Fe\rangle^{-1} \cdot F$	$\widehat{Fe} = Q^{-1}(\widehat{v'e})$ $\hat{F} = \langle \widehat{Fe} \rangle \tilde{B}_F$	$\Sigma_{\widehat{Fe}} = Q^{-1}E[\epsilon_{\widehat{e'V}}\ \epsilon'_{\widehat{e'V}}]Q'^{-1}$

Table 10

Legend:

- ($\wedge$ refers to a forecast value)
- $\langle\ \rangle$ Diagonal matrix
- E Expectation operator

tions, etc. These stochastic behavioral relations are estimated using standard econometric procedures: single equation or, preferably, simultaneous equation procedures.

Figure 1 summarizes the basic relations required to go beyond the fixed, or more generally time-trended, bridge coefficient model. We note the following points:

(i) only the general scheme is shown here and a variety of functions can be postulated depending upon the level of disaggregation of the macro model. For instance, consumption functions can be estimated globally or at the level of specific personal consumption expenditures (PCE) industry categories, as in the INFORUM model. For consumption, typical explanatory variables are, say, income and relative prices for which elasticity estimates are computed. Investment functions can be of the desired capital-output ratio type--with discrete adjustments over time to reach this level. Government expenditures can be assumed all exogenous--a component of the fiscal policy package--or partly endogenous. For instance at the state and local level education expenses may be related to the population by age group distribution (INFORUM) [4]. Imports can be made endogenous by estimating global or sectoral import functions (Brookings) [19]. On the primary inputs side labor demand by industry can be obtained at various levels of sophistication--e.g. different constant or time-trended labor/output, labor/value added or even total value added/output coefficients for each industry (Brookings).

(ii) The role of IO relations is clear. It ensures consistency of the forecast by translating, for instance, the final demand by industry into the gross product-originating (value-added) by industry--which then leads to industry sales forecast, demand for primary inputs (e.g. employment) forecasts, etc. IO relations allow us to achieve industry-by-industry, PCE category-by-industry and a host of other such consistent mappings between specific components of V and F. This point is high-

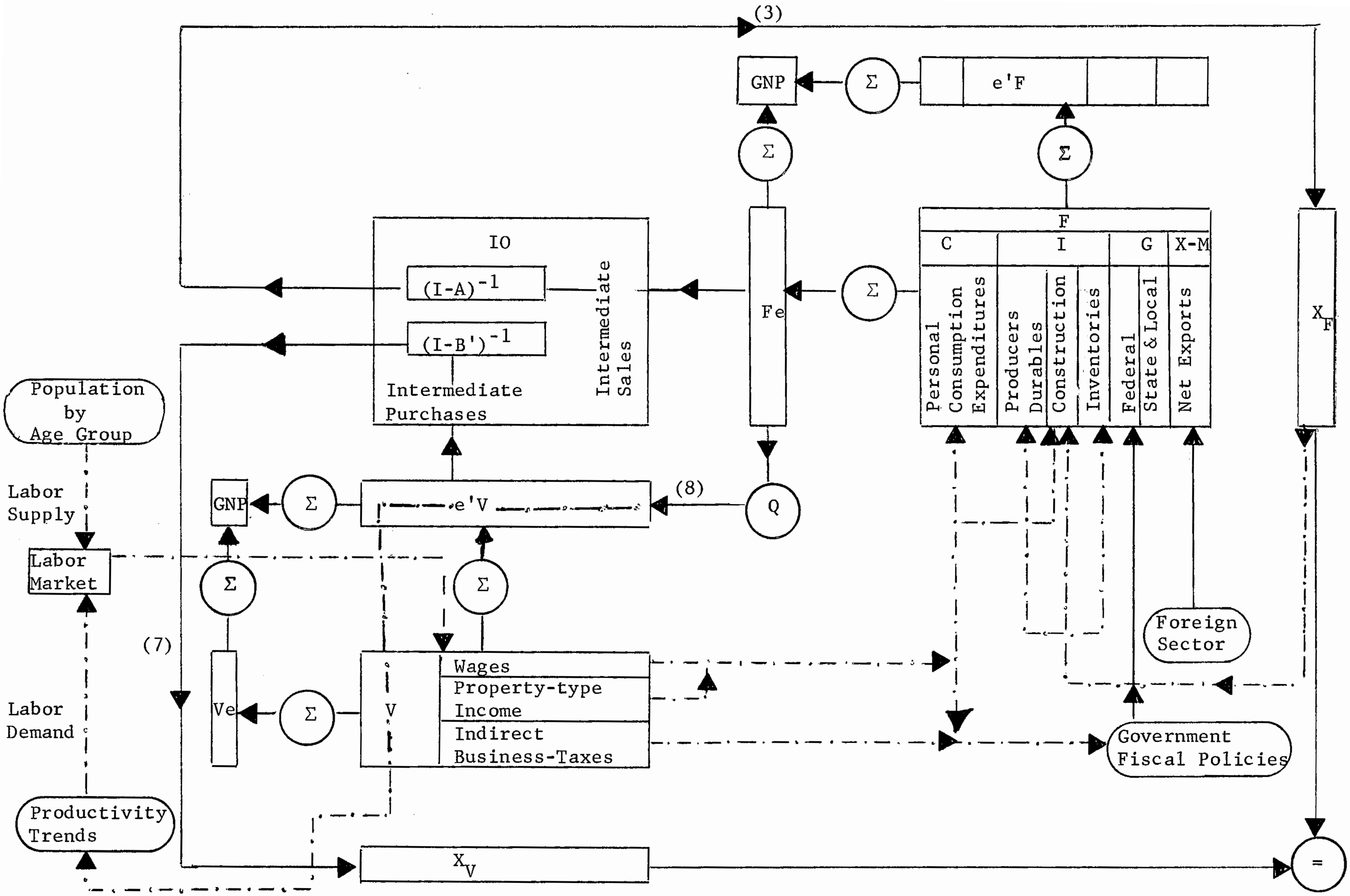

Figure 1. SUMMARY OF IO - MACRO INTERFACE

Legend: Identity relation

Behavioral relation (stochastic)

Exogenous variable

Operation

Numbers in parenthesis refer to equations in text lighted by the fact that equation 8--$V'e = (I-B')(I-A)^{-1}(F\ e)$--acts as a constraint on changes in the b or a coefficients to guarantee such consistency. A direct consequence of that consistency condition is the well-known scalar equality $e'Fe = e'Ve = e'(I-B')(I-A)^{-1}(Fe) = e'QFe$ namely the sum of value added by all industry - Gross Product originating - equals the sum of final demand by industry = GNP.

(iii) The 10-macro interface in Figure 1 ignores some important links involving goods and inputs prices, and the financial sector. This simplification is for brevity only and should not be construed as implying that IO is incapable of accommodating such extensions; models such as Brookings integrate these other factors.

(iv) The user of IO forecasting models can use any number of existing macro models as exogenous inputs or design his own model. In any case he can operate at various levels of industry, final demand categories and value-added component disaggregation depending upon the purpose of his forecasts.

Once these forecasts are obtained, they can be used more or less directly as inputs to corporate forecasts and planning rather than using the common approximation of specifying simple single equation models relating say corporate sales to GNP, unemployment, relative prices, etc. This widely used method entails an unnecessary model specification error resulting in systematic forecast errors.

III.2. The Corporate/Micro IO Interface

A basic difficulty often encountered by the line manager is to translate corporate "economic assumptions" into product forecasts. So far the IO model coupled with a macro model--or at least driven by some broad macro forecast, e.g. GNP growth rate translated into final demand by some bridge coefficients--only provides total dollar sales forecast for the industries that make up the sectoring plan of the IO tables. To date, existing tables are still too aggregated to be directly usable by a corporation, or, even less so, by a division of a corporation. As IO usage has grown among large corporations, the problem has been faced and some good approximate solutions have been devised. Let us consider, for instance, the case of a division of a corporation[(1)] whose sales are in refractories, minerals, foundry equipment and industrial and residential glass as described in [43] [44]. In refractories alone 24 varieties of separate products can be identified ranging from fireclay brick (#1) to bonding mortars (#20). Markets for these products comprise 19 different industries ranging from open-hearth melting (#2) to chemicals (#13). Even if we use the finest IO table, 484 industries at the SIC 4-digit level, we are still left with a 3-digit gap to reach the 24-product breakdown--at the 7-digit level. This means that the direct input coefficient of the Stone and Clay Products industry (IO #36) per dollar of sales of, say, Primary Iron and Steel Manufacturing (IO #37), $a_{36;37} = .00355$, is a weighted average of many different 'mini' coefficients--one for each product market combination in this pair of IO industries (36;37). Further, this product-mix/market (or output--mix) may not accurately reflect the weighted average coefficient for this company as market share differs among vendors to the primary steel industry.[(2)] Blind application of this coefficient, or the corresponding inverse matrix entry ℓ_{ij}, would hardly help the line manager translate a broad economic guideline--e.g., "assume a 10% increase in steel imports next year and a 5% increase in new construction"--into product-specific forecasts. As more disaggregated up-to-date IO tables become available, the problem will be attenuated. For now, a good approximate solution is to build a detailed products-by-consuming-industry sales matrix ($[x_{\iota j}]$) for the company (an "Almon's skirt" [4]); where ι ranges over all the firm's products classified within an

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[(1)] Combustion Engineering, Cermatec

[(2)] As mentioned in Section 2 above, variations in relative prices between these many product-market pairs will also affect the value coefficient even without any change in physical coefficients.

IO industry row, and j corresponds to the IO purchasing industry. This information is easily accessible from product sales data. Coefficients $\alpha_{ij} = \frac{x_{ij}}{X_j}$, of product use per dollar (or ton) of output of each market (e.g. Primary Iron and Steel) can be derived from interviews with technical experts, trade association data or any other source. Grouping these micro-input coefficients into a matrix allows the company to readily translate client-industry sales forecasts $\hat{X}_j$ into product-specific direct sales forecast as shown in Figure 2 below. IO-based energy studies offer a timely example of this type of product/market breakdown. Energy-producing sectors are refined into various types of fuel consuming industrial and final markets. ([27]). Clearly this type of product-market breakdown can also be related to the other basic IO matrices.

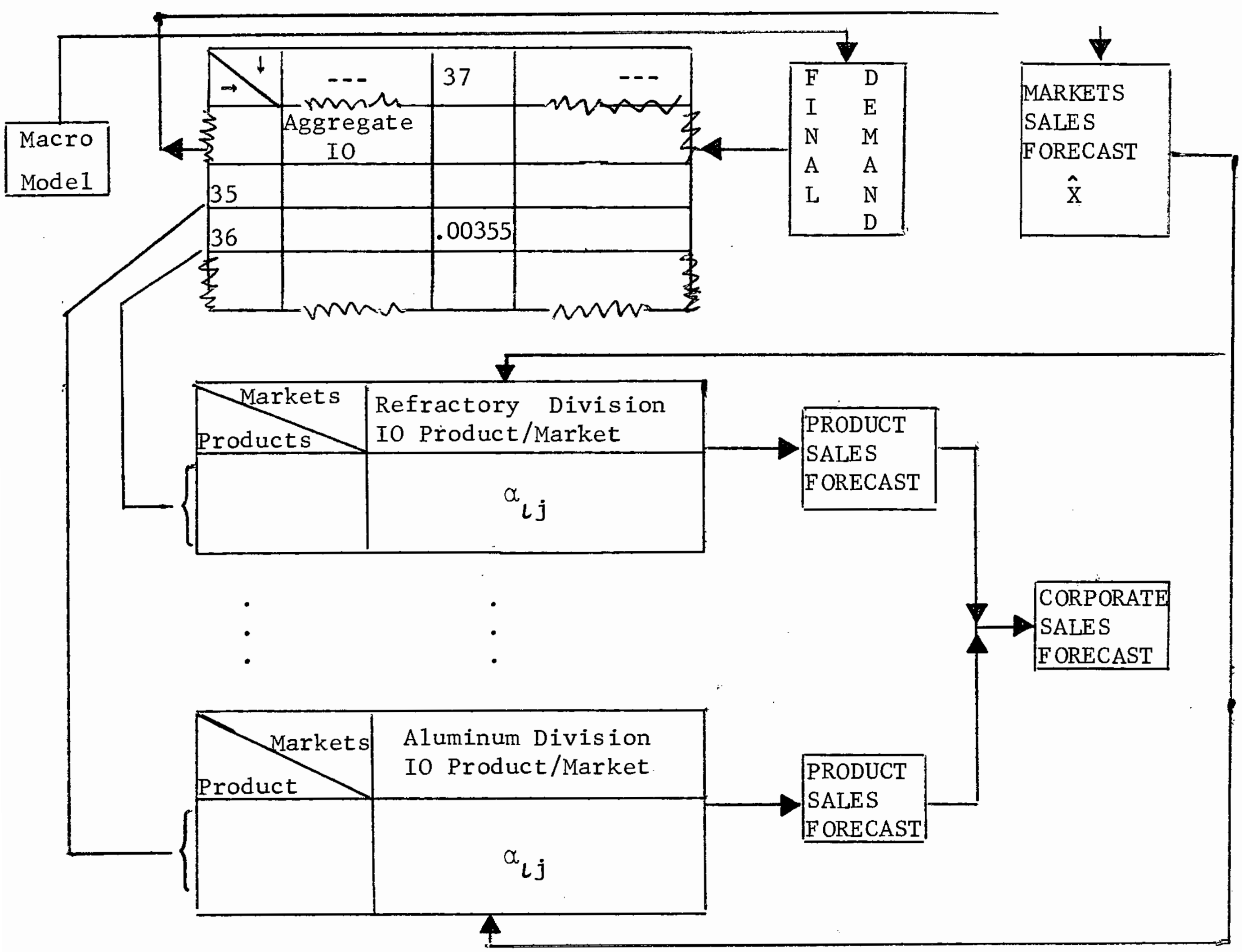

Figure 2. The Corporate/Micro IO Interface

Using the classic Leontief Inverse L, for instance, the impact of, say, a predicted sharp drop in auto sales (IO #59) on the company sales of fireclay brick will be given by:

$$\sum_j \alpha_{\iota j} \cdot \ell_{j59} \cdot \Delta(Fe)_{59}$$

(where ι denotes the fireclay brick product category and j ranges over all IO industry markets for fireclay brick).

The approximation involved in this type of calculation stems from ignoring the upstream effect (and feedbacks) of an increase in output of each refractory product on the refractory division's vendors. The smaller the proportion of this firm's product in the entire IO industry to which it belongs, the better the approximation.(1) For most large diversified firms the method gives a good approximation as reported in [43] [44].

Two final points should be noted. First the accuracy of the coefficients in the Product/Market technology matrix must be checked periodically as results are particularly sensitive to changes therein. Time trend studies are often used in practice, e.g. logistic curve fittings as commonly used in technological forecasting [4]; technical information from industry experts is also useful for those adjustments. Second, another type of IO corporate interface deserves mentioning, as it can easily complement the previous model for large technologically integrated multi-division corporations with many inter-division transactions. A corporate IO table can be set up with divisions, or portions thereof, corresponding to the industries and sales to and purchases from other companies corresponding to final demand and primary inputs respectively [45]. All IO techniques described so far readily apply to this corporate IO table.

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(1) To include such effects would in effect disaggregate the whole IO matrix to the division product detail level which is clearly impractical and too expensive in most cases.

IV. Empirical Results

IV.1. Data Sources, Computations and Results

In this section we describe the forecasts obtained with the IO methodology described in the previous section. The 1967 U.S. matrix was used, at the 81, 42 and 4-industry level. Data on the marginals F and V were obtained from (1) various BEA publications [12], and (2) U.S. Department of Commerce National Income and Product Accounts Time Series. To ensure consistency between SIC-based and IO-based industry data, the original IO matrix was aggregated to a 42-industry breakdown, and the corresponding A, L, M and Q matrices were computed. All data were deflated to 1967 dollars through GNP price deflators. To keep the calculations manageable, we purposely did not attempt to drive our IO forecasts via some larger macro model as described in III.2 above; thus we focused exclusively on the simple bridge coefficient approach of III.1 above, with the bridges $B_F = \frac{1}{e'Fe} \cdot Fe$ and $B_v = \frac{1}{e'Ve} V'$ calculated for the base year 1967. An ex-post static simulation was also carried-out using the actual marginals V and F thus enabling us to gauge the extent of the model-specific error when compared to the results obtained with bridge coefficients.

Table 11 summarizes the results for the period 1967 through 1971 for six leading industries, measured in terms of their sales in total GNP.

Table 11: Percent Deviation of Four Gross Output Estimates and Theil's Optimal Linear Correction for Six Selected Industries

		X_F	$X_F C$	$X_F B$	$X_F BC$	X_V	$X_V C$	$X_V B$	$X_V BC$
1967	AFF	0.6	0.5	0.6	0.5	-0.4	-0.3	-0.5	-0.3
	CC	0.0	-0.1	-0.0	-0.1	0.3	-0.2	-0.3	-0.2
	F&K	0.1	0.1	0.1	0.1	0.2	0.3	0.2	0.3
	PM	0.0	-0.1	0.0	-0.1	-3.4	-3.3	-3.4	-3.3
	MXE	-0.0	-0.1	-0.0	-0.1	-0.5	0.3	0.5	-0.3
	MVE	-0.0	-0.1	-0.0	-0.1	-1.0	-0.8	-1.0	-0.8
1971	AFF	3.0	3.8	8.9	11.4	12.8	8.2	7.8	10.4
	CC	-2.0	-1.2	-5.9	-3.8	6.6	2.4	-6.3	-3.9
	F&K	0.4	1.2	8.7	11.1	17.9	13.2	8.7	11.4
	PM	7.6	8.4	15.8	18.4	37.5	31.9	11.8	14.6
	MXE	3.1	3.9	17.1	19.7	20.1	12.3	16.5	19.4
	MVE	4.1	4.9	-6.8	-4.8	7.6	3.3	-7.7	-5.4

Legend

AFF - Agriculture Forestry & Fisheries
CC - Contract Construction
F&K - Food & Kindred Products
PM - Primary Metals
MXE - Machinery Except Electrical
MVE - Motor Vehicles & Equipment

X_F: X forecast using actual F and equation (3)
$X_F B$: X forecast using bridge-derived F and equation (3)
X_V: X forecast using actual V and equation (7)
$X_V B$: X forecast using bridge-derived V and equation (7)
$X_F C$, $X_F BC$, $X_V C$, $X_V BC$: corresponding corrected forecast using Theil's optimal linear correction [49]

Table 12 summarizes the aggregate forecast error in terms of the percent Root Mean Square Error (% RMSE) for the same years, over all industries.(1)

Table 12
Aggregate Forecast Errors

% RMS Error	67	68	69	70	71
X_F	0.2	2.9	2.4	3.2	3.8
$X_F C$	0.2	2.8	2.3	3.1	4.0
$X_F B$	0.2	5.0	5.0	8.1	12.3
$X_F BC$	0.2	4.9	4.7	8.3	12.6
X_V	0.9	5.7	7.8	13.4	14.9
$X_V C$	0.9	4.4	5.7	10.2	11.8
$X_B B$	0.9	5.0	4.7	8.0	12.2
$X_V BC$	0.9	5.0	4.7	7.8	12.0

(1) Theoretically X_V should equal X_F for the base year, 1967. The difference must be attributed to residual statistical errors in estimating V.

Consistent with other empirical studies, the results show a high degree of accuracy. Comparison of the forecast obtained with the actual F (or V) values for a year and the forecasts derived from estimates of F (and V) using bridge coefficients enable us to assess the extent of the IO specific forecast error. Updating of the bridge and IO coefficients for the years 1968 through 1971 would further reduce the error. It is also worth noting that a simple test of the degree of stability of the a vs. the b coefficients over time is the relative errors in X_F vs. X_V forecasts. The clear superiority of X_F vs. X_V--when using actual F and V--would imply the greater stability of the a coefficients. Yet as we move to bridge-derived F and V this superiority vanishes. Finally this type of comparison is limited by the fact that the a's are in dollar terms whereas the b's are straight proportions as the price terms cancel out in

$$b_{ij} = \frac{p_i \tilde{x}_{ij}}{p_i \tilde{X}_i} .$$

IV.2. <u>Forecast Errors, Sources and Corrections</u>

IV.2.1. <u>Sources of error</u>. It is important to distinguish between three sources of errors in IO forecasts: (1) errors in the IO coefficients; (2) errors arising from the bridge procedure; (3) errors in forecasts of the exogenous variables. Only (1) is attributable to the IO methodology per se. As explained earlier, specification errors--e.g., variable rather than fixed coefficient industry production functions--and/or measurement errors--e.g., a suboptimal sectoring plan or changes in prices, product mix or technology--all lead to inaccurate coefficients. Much research has been done to measure the relative importance of each source of coefficient error in actual tables and correct it. Without attempting to cover all these approaches it should be noted that (1) optimal sectoring plans--i.e., forecast-error-minimizing industry groupings--have been studied and tested [9] [10]; and (2) a number of coefficient adjustment methods have been proposed; e.g., time trend studies, industry expert surveys, and the bi-proportional ('RAS') method. This last method attempts to adjust the base year IO coefficients by a least-squares criterion so that the resulting IO tables for later years are consistent with the recorded marginals F and V for these years [2]. Existing integrated IO systems often use a combination of these approaches to adjust the coefficients.

As regards errors from the use of bridge coefficients, it should be noted that, as shown in Table 10, there are many types of bridge coefficients. They can be computed from total final demand (GNP) or for each component (consumption, investment, etc.) when individual forecasts of these components are known. Further, whichever coefficients are used, they can be adjusted in two ways. Firstly by tracking their time trends and secondly by linking them with a full-fledged macro model. In the latter case, the bridge coefficients become functionally dependent upon the level of activity. For instance, consumption by industry can depend on the level of after tax income ([4]). Such procedures invariably result in appreciable error reduction.

Finally, as regards forecast errors in exogenous variables, the same prescription for reducing them applies as in any simultaneous equation model. Better outside models are needed to limit this source of error. Here again, this may involve more or less sophisticated macro models, e.g., econometric models for simultaneous time series analysis.

IV 2.2 Residual Model Error Covariance. Suppose appropriate adjustments have been made to the bridge coefficients and the IO coefficients and a model has been built to provide forecasts for the exogenous variables. The computed residuals, ϵ (deviations from predicted values), for the forecast variables can be linearly translated into errors in the endogenous variables. For example if we fit a forecasting model to each industry final demand and compute the residual variance-covariance matrix $E[\epsilon_{\widehat{Fe}} \cdot \epsilon'_{\widehat{Fe}}]$, the variance-covariance matrix of $V'e(\Sigma_{V'e})$ and of $X(\Sigma_X)$ are given by:

(9) $$\Sigma_{V'e} = QE[\epsilon_{\widehat{Fe}} \cdot \epsilon'_{\widehat{Fe}}]Q' \quad \text{and} \quad \Sigma_X = LE[\epsilon_{\widehat{Fe}} \cdot \epsilon'_{\widehat{Fe}}]L'$$

where E[ ] represents the expectation operator. For the 4-industry example, table 13 summarizes variance-covariance matrices of X and $V'e$ given the residual variance-covariance matrix obtained after a linear time trend model has been fitted to Fe, for quarterly data from 1946 to 1976.

Table 13 - Error Covariance Matrices for 1967 Data

$E[\epsilon_{Fe} \cdot \epsilon'_{Fe}]$				Σ_X				$\Sigma_{V'e}$			
88	-17	-10	-63	85	27	11	-41	90	-4	-12	-94
-17	154	87	168	27	463	402	528	-4	114	62	140
-10	87	77	124	11	402	412	525	-12	62	63	121
-63	168	124	275	-41	528	525	769	-94	140	121	357

IV.3. Conclusion. The previous discussion has shown the potential of IO-based forecasts. Current limitations stem mostly from the extensive data requirements of IO. The cost of obtaining frequent updates on interindustry transactions is aggravated when further disaggregation is sought. Yet such disaggregation is the key to large-scale routine use of IO by corporations. The corporate/IO interface sketches a partial answer to this problem. New corporate information systems and reporting requirements are likely to provide easier and more frequent access to IO usable data. This may also be the key for compiling enough price information on disaggregated product lines to allow the calculation of physical IO tables. On a more theoretical level, the implications of the distribution of the model-specific errors for certain key issues in industrial organization and corporate finance [7] hold out the promise of a better understanding of widely divergent trends in different industries.

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Abbreviations

AER	American Economic Review
EMA	Econometrica
IER	International Economic Review
JMR	The Journal of Marketing Research
RESTAT	The Review of Economics and Statistics
RES	The Review of Economic Studies
SEJ	The Southern Economic Journal