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A MATHEMATICAL-PROGRAMMING MODEL FOR OPTIMAL SELECTION OF TOURISM PLANS AND GATEWAYS FOR THE AIRLINE INDUSTRY

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ABSTRACT

A major concern of organizations considering the introduction of several new plans is the level of participation by consumers with different "requirements" in each of the different plans. The selection of a package of plans to be offered by the organization is contingent upon the consumer's expected choice which, in turn, depends on the totality of packages offered. Thus, the decision process is circular. In this paper we model this decision process as a mathematical program. Exploiting the special structure of the constraints, a relatively simple integer programming algorithm is presented to obtain an optimal package of new plans to be offered. The model is applied to the problem of selecting gateways in the Rockies for ski tourists by a major airline.*

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I. INTRODUCTION

New product or service introductions are essential to the continued growth and survival of organizations. Pessemier and Root [25] indicate that new products/services account for a sizeable percentage of annual sales. Formal procedures and models are increasingly being used to improve the success rate of new products/services within organizations. One of the key areas where modelling is used is that of positioning new products/services within the market. Assmus [4], Urban [30] and Burger [6], have investigated this problem for a single product. When a firm is considering several new products or services, the interaction between the proposed new products/services and the existing ones makes this problem even more complex. The work done in this area as reported in Pessemier and Root is for one new product/service only.

Programming efforts related to these models have focussed on determining ideal points, or attribute weights, or both. Srinivasan and Shocker [28,29] used as inputs t-component vectors of the attribute values of each brand, and a series of forced-choice paired comparisons of brands, where the consumer is required to choose the preferred brand from two presented, for all possible pairs of brands. The same authors [28] use this same procedure (LINMAP) to determine new product ideas. If the attributes included in the model are 'actionable' from the company's point of view, then choice of a new product concept can be predicted by discovering its relationship to other brands, and to the ideal levels of each attribute. The potential new product concepts can therefore be represented abstractly by means of the attributes employed by the consumer in evaluation.

Pekelman and Sen [24] use a model similar to the LINMAP model in estimating attribute weights for product classes, except that they ask the consumer to estimate his ideal point for each attribute rather than having it determined by the model. Their model then either minimizes distance violations from the ideal point for the preference pairs (as in LINMAP) or it can also minimize the number of preference pairs which are violated by the estimating equation. In a recent study Shugan and Balachandran [27] propose a mathematical programming model for determining optimal product line. They consider situations where consumers give their preference for a firm's products. If the first product is not available, consumers are assumed to buy their second preference, the firm cannot satisfy their second preference consumers go to a competitor. Thus in their model the analysis facilitates the simultaneous introduction of at most two products or services.

In the next section we restrict, this generalization to the Airline industry's "gateway" determination problem. Later we provide a model, algorithm and illustration. Finally we show other related applications.

II. GATEWAY SELECTION PROBLEM

Consider the selection process by an airline manager who initiates a set of tourist packages to be available to customers (tourists/vacationers) from among a multitude of alternate plans that are contemplated. All of them hinge on the fact of determining a few "gateways" for which the "main" ticket is issued. From among the gateways to be selected each tourist/customer decides that gateway to which they make their "main" travel. When a set of alternate gateways are available, they lead to different package plans that include lodging at a resort, ski-lifts etc., with a price tag. The tourist has the option of choosing the package plan that is most "attractive" among the ones offered. Such package plans are released for different ski resorts by different airlines. (United [1], TWA, AA, and also by professional associations, agencies [2]). The airline can offer a number of plans with varying plan parameters, such as different types of locations, types of slopes, availability of cross-country, ski ambience, type and duration of lodging facilities, the availability of ski training schools and nurseries, natural or man-made slopes, and above all the price tag of the package including airline ticket to that gateway and the number of lift tickets associated. Thus the airline has the option of offering a large number of package plans with different feasible combinations of the above parameters, all hinging on a few FAA approved, jet landable commercial gateways. The major decisions that are most relevant for the airline are

i) Which of the new gateways are to be serviced; and/or

ii) Which of the existing gateways might be discontinued, if necessary.

Administratively, financially and due to FAA imposed regulations, it is not feasible for the airline to operate to all the gateways. Thus the airline, in consideration of customers behavior patterns and preferences, and other marketing considerations including the gateways that competitors fly into, has to decide the set of gateways to be serviced. Hence the problem that the airline faces when considering the simultaneous introduction of several new gateways is complicated by the expected consumers reaction to its decisions of the gateways operated. One way of analyzing this problem using the existing models, is to view each combination of possible gateways as ONE hypothetical new plan. By appropriate modifications the previous models can be used to determine the relative desirability of introducing these new hypothetical plans, and one such hypothetical plan could be chosen. However, if the number of gateways considered is large this procedure leads to, too many combinations. For example if 30 gateways are to be evaluated the number of hypothetical plans to be considered would be greater than 1 billion. Thus this approach is computationally infeasible in any real context. From among such feasible combinations the airline is interested in selecting that combination which maximizes the total traffic volume. Other criteria of selection and multiple criteria [21] such as total revenue or total profit, could easily be incorporated in the model. Though our model formulation, analysis and algorithm development is general enough in many new product/service introduction contexts, for illustrative purposes and algorithm explanation we specifically consider the ski tourists gateways in the Rocky area (Colorado). (See Figure 1, showing the gateways)

Let N denote the total number of existing and proposed alternate gateways. The decision regarding the gateways to operate is dependent on the preferences of tourists. Since the number of gateways and the tourist population is large and unknown, it is operationally impractical to obtain the preferences on a one to one basis. For purposes of operationalisation, it is customary in the marketing literature to use a preference model. The model that we use is similar to the linear compensatory model often referred to as the "Fishbein" type model. To obtain such a preference model, we follow the sequence of steps given below:

(i) Tourist Segmentation:

A tourist's choice for selecting a gateway depends on his characteristics which can be identified by a set of factors, such as: age, income, marital status, skiing proficiency, geographic location of their residence, etc. These characteristics are chosen so that tourists having the same characteristics have similar preferences in selection of gateways. The problem of determining relevant customer characteristics and market segmentations have been investigates in the marketing literature [13, 25]. Such segmentations of tourists based on the chosen characteristics leads to identification of consumer classes. A consumer class is defined to be a subset of consumers such that any member in that subset has the same interval of consumer attributes and the same preference function. Let M denote the total number of consumer classes, and let w_j denote the number of consumers in class j, j = 1, 2,...M. Associated with each consumer class j, j = 1,...M and <u>any gateway i</u>, i = 1, 2,...N, let p_j (i) be the preference function of consumers in class j for gateway i. We define p_j (i) as a function and not as a constant p_{ij} for purposes of generality. It is conceivable that an airline may have data for p_{ij} if the airline obtains consumer class preferences over all possible gateways. However, if the number of gateways is large, for example 50, then even though the consumer classes are identified, it is infeasible to get consumer class preferences for each of the different gateways. Thus, it is customary to identify "fundamental" attributes k, k = 1, 2,...K of gateways and obtain a preference function for each consumer class over the attributes such as accessibility to different resort areas, connecting modes of travel, cost, city offerings, etc.

(ii) Design: Identification and Relative Valuation of Gateway Attributes.

Product attribute identification and their relative importance are crucial for new product planning. The attributes are identified through the use of focus groups using consumers, technical experts, open-ended responses on questionnaires, brainstorming and other qualitative techniques [11,13,14, 17]. The number of attributes identified through such methods is often large and for operational reasons of data acquisition these attributes need to be aggregated [5,13,17,21]. Basic attributes can be reduced into a set of <u>fundamental attributes</u> through "factor analysis" (correlation) [11,13, 17], to possess discriminatory ability (discriminant analysis) [12], and/or by a stress or dissimilarity measure leading to fundamental dimensions (non-metric scaling) [11,12,13,14].

The relative importance of these fundamental attributes is accomplished through a market survey, and/or expert opinion over the gateways wherein the data pertaining to "b_{ijk}" where, b_{ijk} represents consumer class j's belief as to the extent to which attribute k is offered by gateway i are obtained for all i,j,k combinations needed.

(iii) Estimation of Preference Functions

An additional input in obtaining the preference function is to obtain the rank preference of a selected subset of gateways by consumer classes. Let, p_{ij} represent rank preference of gateway i by consumer class j.

Through the use of logit analysis [22,23] the relative weight I_{jk} corresponding to the importance of attribute k to consumer class j, can be obtained to get the preference functions for each consumer class over all the gateways. This is of the form:

$$P_j\ (i) = \sum_k I_{jk}\, b_{ijk} \qquad \text{for } i = 1 \ldots N;\ j = 1 \ldots M$$

Logit analysis has to be used since rank preferences are the only data that can be obtained from consumers. If preference data on a continuous interval scale can be obtained, other techniques such as monotonic regression [19,29], conjoint analysis [14] or tradeoff analysis [20] could be used.

(iv) Prediction of Tourist Choice

From (i) above we obtain the rating of each relevant attribute related to a particular gateway, so that we can rate each of the gateways on the basis of 'how much' of that attribute the gateway possesses. From (iii) we obtain the importance factors (weights) of the relevant attributes for each tourist stratum. Multiplying these gateway ratings times the 'importance' of the particular attributes, and summing over attributes, results in a total score, or the level of tourist's 'affect' for the particular gateway. Prediction of the tourist's behavior depends on using the gateway with the highest preferences <u>from amongst the ones offered</u> provided no competitor gateway has a higher score.

III. THE GATEWAY DETERMINATION MODEL

The airline is interested in maximizing the total number of tourists participation in the gateways, subject to certain sets of constraints given below:

(i) Minimal level of sales or participation before the gateway is feasible for the airline.

(ii) Certain policy constraints restricting the total number of gateways that will be offered. These constraints may be due to administrative convenience and FAA regulation regarding airline industry.

(iii) All consumers in a consumer class will select that gateway that yields the highest preference value from amongst the gateways offered, including those offered by competitors.

A model for the above selection process is formulated next. Define

$Y_0 = 1$ corresponds to competitors gateways with the highest preference that are available.

For $i = 1, 2, \ldots N$, define

$$Y_i = \begin{cases} 0 & \text{if gateway } i \text{ is not offered by the airline} \\ 1 & \text{if gateway } i \text{ is offered by the airline} \end{cases}$$

For $i = 0, 1, \ldots N$, and $j = 1, 2, \ldots M$, let

$$X_{ij} = \begin{cases} 0 & \text{if gateway } i \text{ is not chosen by individuals in tourist class } j \\ 1 & \text{otherwise} \end{cases}$$

We have to ensure that individuals in each tourist class choose that gateway from the ones offered, that enables them to reach their highest preference. This can be ensured by two sets of conditions being satisfied; namely, no individual chooses a gateway that is not offered, and that each individual in the tourist class chooses that gateway which maximizes his preference.

$$X_{ij} - Y_i \leq 0 \text{ for } j = 1, 2, \ldots, M,\ i = 1, 2, \ldots, N, \text{ and}$$

$$\sum_{i=1}^{N} P_{ij}X_{ij} - P_{ij}Y_i \geq 0 \text{ for } j = 1, 2, \ldots, M,\ i = 0, 1, 2, \ldots, N.$$

Further, we also want to ensure that individuals in each tourist class choose not more than one gateway, and that the airline does not offer more than a maximum number, K, of gateways due to regulations. These conditions can be represented by

$$\sum_{i=1}^{N} X_{ij} \leq 1 \qquad \text{for } j = 1, 2, \ldots, M, \text{ and}$$

$$\sum_{i=1}^{N} Y_i \leq K$$

Finally, we add the constraint that the airline would consider gateway i only if a certain minimum number of individuals M_i, participate in that gateway. This can be represented by

$$\sum_{j=1} W_j X_{ij} \geq M_i Y_i \qquad \text{for } i = 1, 2, \ldots, M.$$

Thus, the choice and selection process can be represented by the following integer programming problem.

(1) Maximize $\sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} X_{ij}$

subject to

(2) $X_{ij} - Y_i \leq 0$ for $j = 1, 2, \ldots, M$, $i = 0, 1, 2, \ldots, N$,

(3) $\sum_{i=1}^{N} p_{ij} X_{ij} - p_{ij} Y_i \geq 0$ for $j = 1, 2, \ldots, M$, $i = 0, 1, 2, \ldots, N$,

(4) $\sum_{i=0}^{N} X_{ij} = 1$ for $j = 1, 2, \ldots, M$.

(5) $$\sum_{i=1}^{N} Y_i \qquad \leq K$$

(6) $$\sum_{j=1}^{M} w_j X_{ij} \qquad \geq M_i Y_i \quad \text{for } i = 1, 2, \ldots, M.$$

(7) $0 \leq X_{ij} \leq 1$, $0 \leq Y_i \leq 1$, X_{ij}, Y_i interger variables

where $c_{0j} = 0$ and $c_{ij} = w_j$ for each i,j, $i \neq 0$ since the airline's objective is to maximize participation.

For a reasonable problem with about 30-50 gateways being considered and 100 tourist classes the formulation would give between 3000-5000 variable and an even larger number of constraints. These problems are clearly too large to solve by conventional integer programming techniques. We exploit the special structure of the problem and devise an implicit enumeration scheme for obtaining the solution. We first observe the following:

Proposition I. If individuals in each customer class have strictly ordered preferences amongst the gateways, that is, no p_{ij}'s are equal for a fixed j, then restricting Y_i to be integer would imply that in any feasible solution to (1)-(7), X_{ij} would also be integer.

Proof: Follows immediately from constraint set (3) which states that individuals in each customer class choose the gateway that gives them the highest preference.

Since most individuals would just choose one gateway to participate in, it is not unreasonable to assume that their preferences over the gateways are a strict order. Henceforth, we assume this. With this assumption and the above proposition, the number of integer variables is considerably reduced. In the example considered earlier when 30-50 gateways were being considered for 100 tourist classes, the number of integer variables that we have to consider decreases from 3000-5000 to 30-50. In general, we are left with N integer variables, where N is the number of gateways being considered by the institution. With this reduction there are a finite number $\sum_{k=0}^{K}\binom{N}{k}$ of solutions in the Y variables that satisfy (5), and exhaustive enumeration provides a finite procedure for determining the optimal combination of gateways to offer. However, $\sum_{k=0}^{K}\binom{N}{k}$ can turn out to be a large number. When N = 30, and K = 5, we have about 175,000 possible combinations which would take a long time to evaluate. We therefore provide an implicit enumeration technique with an additive algorithm to obtain an optimal solution.

IV. THE ENUMERATIVE ALGORITHM

As a consequence of Proposition I, it is sufficient to obtain the integer solution to the Y_i variables, in order to solve the problems (1)-(7). Thus, we provide an enumerative algorithm to obtain the optimal solution of Y_i only for i = 1,...,N. Optimal values of X_{ij}'s are incorporated implicitly within the algorithm. The enumerative algorithm is based on an elementary tree search similar to those given in [9 , 10]. (The reader is assumed to be familiar with Graph theory [15], terminology).

<u>Definition 1</u>. A <u>solution</u> is defined to be any feasible assignment of values of 0 or 1 to all Y_i's, feasible for (2)-(7).

Adding variable Y_0 to constraint (5) we have

$$(8) \qquad \sum_{i=0}^{N} Y_i \le K + 1$$

The tree search traces a path of nodes, until either a new solution is obtained or a node is reached which yields information that all solutions in which that particular node is included may be ruled out of consideration. Thereupon the process backtracks to the unique node that immediately precedes the one ruled out, and embarks on a different path, unless none are left and it becomes necessary to backtrack further. Once the process is pushed back to the starting node, and information is obtained that forbids tracing out any more branches of the tree, the procedure terminates.

To elaborate the branching process more precisely we will use the following standard notation and conventions [9 , 10]. The term i will be used to denote $Y_i = 1$, and the complementary term $\bar{i}$ will be used to denote $Y_i = 0$. If a variable Y_i is fixed at a value then it is underlined in the solution sequence.

Definition 2. A solution sequence is defined to be a sequence of the integer variables in which (i) no term appears more than once, (ii) the corresponding 0-1 assignment to some or all of the Y variables is well defined (i.e., not both i and $\bar{i}$ can appear in the sequence and for each i, $0 \le i \le N$), and (iii) the number of terms i is less than or equal to K + 1.

Let S_n denote this solution sequence at node n. Associated with a solution sequence S_n, let R_n denote the set of gateways at level 1, in S_n, Z_n denote the set of gateways at level 0, in S_n, and F_n denote the set of gateways not in S_n--viz. the gateways which are free to take values 0 or 1 for completion of the solution sequence. (Note: $R_n \cup Z_n = S_n$; $R_n \cap Z_n = \phi$;

$F_n \cup S_n = \{0, 1, \ldots, M\}$; and $F_n \cap S_n = \phi$) .

At any node r in the tree search, the best feasible solution obtained until that node is stored, together with its objective function value, L_n and called the <u>incumbent solution</u>. A <u>terminal solution</u> sequence is defined to be a solution sequence for which there exists no 0-1 assignment of the free variables F_n that will produce a feasible solution better than the incumbent.

For a given solution sequence S^n corresponding to node n, we partition the set of tourist classes into the following mutually exclusive and exhaustive subsets.

$$(9) \qquad A_i^n = \{j | p_{ij} \geq p_{kj} \text{ for all } k \in F^n \cup R^n\} \text{ for each } i \in R^n,$$

and

$$(10) \qquad B^n = \{j | j \notin A_i^n \quad \text{for any } i \in R^n\}.$$

Thus the set A_i^n consists of tourist classes who would choose the i^{th} plan (which is a level 1 in the solution sequence), regardless of whether any additional plans are added to the solution sequence. We also let

$$(11) \qquad w_{i,N+1}^n = \sum_{j \in A_i^n} w_j$$

and

$$(12) \qquad c_{i,N+1}^n = \sum_{j \in A_i^n} c_{ij} \quad .$$

Further, we denote the current lower bound at node n by L^n.

An algorithm for determining an optimal solution to the problems (1)-(7) is provided next. In this algorithm the special structure of the problem is exploited to eliminate certain branches from consideration. Initially all the gateways except the one with the 0^{th} index are free. The 0^{th} gateway, which corresponds to not offering any gateway is fixed at level 1, and is the initial incumbent solution.

Step 0. Let the node number $n = 0$; $S^n = \{\underline{0}\}$, $R^n = \{0\}$, $F^n = \{1, 2, \ldots, M\}$, $Z^o = \Phi$ and $L^n = 0$. Determine A^o_o and B^o and $W^o_{o,N+1}$ and $C^o_{o,N+1}$.

Step 1. At node n, form a tableau with rows corresponding to every index in $R^n \cup F^n$ and the columns corresponding to tourist classes in B^n. If $|R^n| = K+1$, form the tableau with rows corresponding to indices in R^n only. In each cell (k,j) of the tableau, corresponding to gateway k and tourist class j, store c_{kj}, w_j, and p_{kj}. Add to this tableau another column and store $c^n_{i,N+1}$, $w^n_{i,N+1}$, and where these values are given by (11) and (12)

Step 2. For each column $j \in B^n$ determine the unique index i, such that $p_{ij} > p_{kj}$ for $k \neq i$. Set the corresponding $X^n_{ij} = 1$. For each row i in the tableau let

$$c^n_i = \sum_{\{j \in B^n \text{ and } X_{ij} = 1\}} c_{ij} + c^n_{i,N+1}$$

$$w^n_i = \sum_{\{j \in B^n \text{ and } X_{ij} = 1\}} w_{ij} + w^n_{i,N+1}$$

Step 3. Upper Bound Check

The upper bound of the objective function for all nodes along this branch equals $\sum_{i=1}^{M} c^n_i$. If this value is less than L^n go to Step 7; else go to Step 4.

Step 4. Feasibility Check for Current Solution.

Let $U^n = \{i | X_{ij} = 1 \text{ or } i \in R_n\}$. If $w^n_i \geq M_i$ for each $i \in U^n$, and if $|U^n| \leq K + 1$, then the current solution is feasible. If the current solution is feasible go to Step 7, else go to Step 5.

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Step 5. Branching

Determine $k \in F^n$, such that $C_k^n = \underset{i \in F^n}{\text{maximum}}\ C_i^n$. Let $n = n+1$, augment the old solution sequence by adding index k to its right. Then $R^{n+1} = R^n \cup \{k\}$, $Z^{n+1} = Z^n$, and $F^{n+1} = F^n - \{k\}$. Determine A_i^{n+1} for each $i \in R^{n+1}$, and B^{n+1}. Let $L^{n+1} = L^n$, and go to Step 2.

$$A_i^{n+1} = \{A_i^n\} \cup \{j | x_{ij}^n = 1\},\ c_{i,N+1}^n = c_i^n,\ w_{i,N+1}^n = w_i^n \quad \text{for each } i$$

Step 6. Incumbent Solution

If $\sum_{i=1}^{M} c_i^n > L^n$, let $L^{n+1} = \sum_{i=1}^{M} c_i^n$, and store the solution sequence and values of the X_{ij}'s, otherwise let $L^n = \sum_{i=1}^{M} c_i^n$. Go to Step 7.

Step 7. Backtracking

Determine the rightmost term in the solution sequence that is not underlined. If none exists, terminate, else place the complement of that term in the solution sequence, underline it, and remove all terms to the right of it. Replace n by n+1 and update S^{n+1}, F^{n+1}, Z^{n+1}, R^{n+1}, B^{n+1}, and let $L^{n+1} = L^n$. Also determine A_i^{n+1} for $i \in R^{n+1}$. Go to Step 2.

The above algorithm terminates in a finite number of steps, since no node is ever repeated. The proof of the finiteness of the algorithm is similar to the proof given in [10].

Remarks:

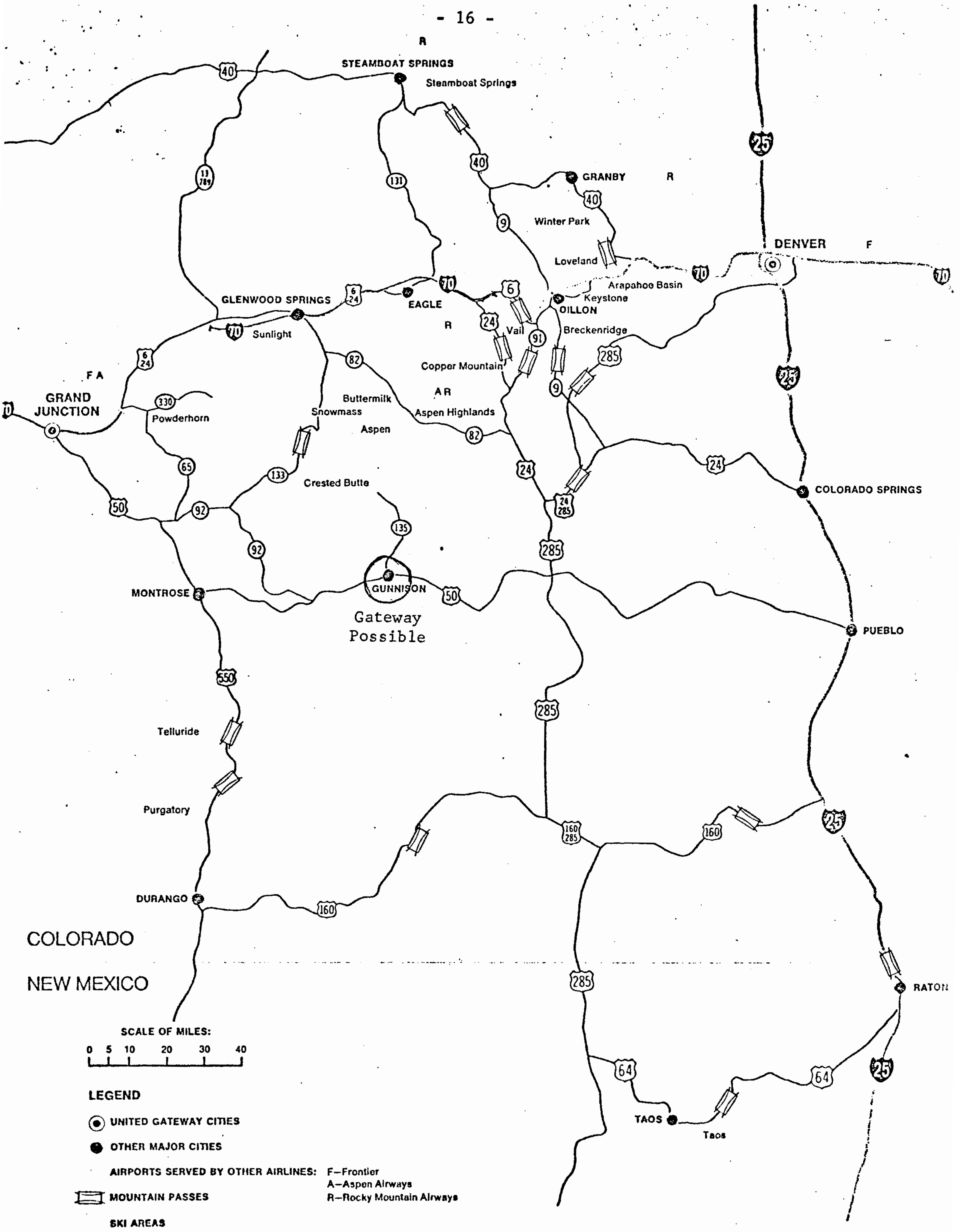
1. Observe that the tableau contracts in size as we go down a branch. This is because the variables corresponding to plans fixed at level 0 can

be excluded from the tableau. Further, the columns corresponding to individuals who choose from the fixed plans need not be considered when branching down from a node.

2. A variety of combinatorial constraints such as not offering a plan in conjunction with another, or offering at least one from certain classes of plans, can be very easily taken care of in the enumeration tree. For example, if the airline does not want to offer both the gateways i_0 and i_1, then as soon as i_0 or i_1 enters the solution sequence, $\underline{\bar{i}}_1$ or $\underline{\bar{i}}_0$ can be respectively added to the solution sequence.

3. If the airline wishes to offer exactly K gateways then the test in Step 4 can be modified to check whether the number of indices in J is exactly K + 1 or not before allowing it to take the place of the incumbent.

4. In any solution of Y_i's, Step 2 and A_i^n provide solution values of the X_{ij} variables.


R
STEAMBOAT SPRINGS
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Winter Park
Loveland
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Arapahoe Basin
Keystone
DILLON
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24
Vail
91
Breckenridge
285
9
GLENWOOD SPRINGS
6 24
EAGLE
R
Sunlight
F A
GRAND JUNCTION
330
Powderhorn
65
82
Buttermilk
Snowmass
Aspen
Aspen Highlands
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133
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24 285
COLORADO SPRINGS
50
92
MONTROSE
GUNNISON
Gateway Possible
285
PUEBLO
550
Telluride
Purgatory
160 285
160
25
DURANGO
COLORADO
NEW MEXICO
RATON
64
TAOS
Taos
SCALE OF MILES:
0 5 10 20 30 40
LEGEND
UNITED GATEWAY CITIES
OTHER MAJOR CITIES
AIRPORTS SERVED BY OTHER AIRLINES: F—Frontier
A—Aspen Airways
R—Rocky Mountain Airways
MOUNTAIN PASSES
SKI AREAS

V. Illustrative Example

We consider the airlines problem of determining the gateways to operate for tourists going to the Ski slopes in the Rockies in Colorado. A map of these regions is shown in Figure 1. At the present time the airline flies into two gateways in this region, Denver and Grand Junction. A third gateway for commercial jets in this region is Gunnison and competing airlines fly into this gateway. The problem we consider in this example is that of determining which 2 gateways the airline should fly into so as to maximize total participation, taking into account tourist preferences, and an administrative/financial constraint that a gateway would be operated only if a certain (say 10) percent of this airline's customers would fly into the gateway.

For purposes of this example we consider six tourist classes and the preferences of these tourist classes for the three gateways, and for flying with a competitor are given in Table I. In Table I a high preference number indicates greater desirability.

Table I

Tourist Preference Data

Tourist Class	Tourist Category	Preferences: Denver Plan I	Grand Junction Plan 2	Gunnison Plan 3	Competitor Plan 0	Percentage of Tourist
Single, Short Stay	1	2.5	1.5	3.0	2.8	15
Single, Long Stay	2	3.2	2.5	1.5	3.0	10
Couple, Short Stay	3	3.2	3.0	2.0	2.8	16
Couple, Long Stay	4	3.4	2.0	1.5	3.0	34
Family, Short Stay	5	2.0	3.0	2.5	2.7	5
Family, Long Stay	6	2.5	3.3	2.1	2.4	20

Solution by our Algorithm

Objective Fynction: Maximize Participation

i.e. $c_{ij} = w_j$ for $i = 1,2,3$; $c_{ij} = 0$ for $i = 0$.

Since competitors plans are always available - we treat competitors plan as one always available, i.e. like Y_o.

Step 0. $n = 0,\ S^o = \{\underline{0}\}, R^o = \{0\},\ F^o = \{1,2,3\}$

$L^o = 0,\ A^o_o = \Phi,\ B^o = \{1,2,3,4,5,6\};\ w^o_{o,N+1} = c^o_{o,N+1} = 0,\ Z^o = \Phi$

Step 1. Tableau(4)

Plans \ Tourist Class	1	2	3	4	5	6	7
1	15 / 2.5	10 / 3.2	34 / 3.4	5 / 2.0	5 / 2.0	20 / 2.5	0
2	15 / 1.5	10 / 2.5	16 / 3.0	34 / 2.0	5 / 3.0	20 / 3.3	0
3	15 / 3.0	10 / 1.5	16 / 2.0	34 / 1.5	5 / 2.5	20 / 2.1	0
0	0 / 2.8	0 / 3.0	0 / 2.8	0 / 3.0	0 / 2.7	0 / 2.4	0

Step 2. Looking at tableau we have

$X^o_{31} = 1,\ X^o_{12} = 1,\ X^o_{13} = 1,\ x^o_{10} = 1,\ x^o_{25} = 1,\ x^o_{26} = 1$

$w^o_1 = c^o_1 = 60;\ w^o_2 = c^o_2 = 25;\ w^o_3 = c^o_3 = 15;\ w^o_0 = c^o_0 = 0\ .$

Step 3. Upper Bound for all nodes along this path $= \sum_{i=1}^{3} c^o_i = 100$

Step 4. $U^o = \{0,1,2,3\},\quad w^o_1 = 60 \geq 10;\ w^o_2 = 25 \geq 10;\ w^o_3 = 15 \geq 0$

$w^o_0 = 0$ - feasible (no minimal participation.)

Since $|| U^o || = 4$, the solution is not feasible.

Step 5. $c^o_1 = \underset{i=\{1,2,3\}}{\text{Max}} \{60,25,15\},\ n = 1$

$S^1 = \{\underline{0},1\},\quad R^1 = \{0,1\},\quad F^{n+1} = \{2,3\}$

$A^1_1 = \{2,3,4\},\quad A^1_0 = \Phi,\quad B^1_1 = \{1,5,6\},\ L^1 = 0\ .$

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(4) Since $c_{kj} = w_j$ only two numbers appear in each cell w_j and p_{kj}

2nd Iteration. Step 1.

Plans \ Tourist Class	1		5		6		7
1	15	2.5	5	2.0	20	2.5	60
2	15	1.5	5	3.0	20	3.3	0
3	15	3.0	5	2.5	20	2.1	0
0	0	2.8	0	2.7	0	2.4	0

Step 2. $w_1^1 = c_1^1 = 60$ Looking at tableau, $X_{31}^1 = X_{25}^1 = X_{26}^1 = 1$

$w_2^1 = c_2^1 = 25$; $w_3^1 = c_3^1 = 15$; $w_0^1 = c_0^1 = 0$.

Step 3. Upper Bound = 100 - hence go to Step 4.

Step 4. $U^1 = \{0,1,2,3\}$ and $w_1^1 = 60 \geq 10$; $w_2^1 = 25 \geq 10$; $w_3^1 = 15 \geq 0$; $w_0^1 = 0 \geq 0$

$||U^1|| = 4$, solution not feasible.

Step 5. $c_2^1 = \underset{i=2,3}{\text{Max}} \{25,15\}$, $n = 2$, $S^n = \{\underline{0},1,2\}$

$R^2 = \{0,1,2\}$, $F^2 = \{3\}$, $A_1^2 = \{2,3,4\}$, $A_2^2 = \{5,6\}$, $B^2 = \{1\}$, $L^2 = 0$, $Z^2 = \Phi$.

3rd Iteration. Since $||R^2|| = 3$, form tableau with rows in R^n only.

Plans \ Tourists	1		7
1	15	2.5	60
2	15	1.5	25
0	0	2.8	0

$X_{01}^3 = 1$; $c_1^3 = 60$; $c_2^3 = 25$; $c_o^3 = 0$.

Step 4. Solution is feasible.

Step 6. Since 85 > 0, $L^3 = 85$ and we have $X_{01} = 1$, $X_{12} = 1$, $X_{13} = 1$, $X_{14} = 1$, $X_{25} = 1$, $X_{26} = 1$ as current solution, together with $Y_o = 1$, $Y_1 = 1$, $Y_2 = 1$, i.e. $S^3 = \{\underline{0},1,2\}$, and now we go to Step 7 (Backtrack).

Step 7. $S^4 = \{\underline{0},1,\bar{2}\}$; $F^4 = \{3\}$; $R^4 = \{0,1\}$; $B^4 = \{1,5,6\}$; $L^4 = 85$; $A_1^4 = \{2,3,4\}$; $Z^4 = \{2\}$

4th Iteration.

Plans \ Tourist	1	5	6	7
1	15 / 2.5	5 / 2.0	20 / 2.5	60
3	15 / 3.0	5 / 2.5	20 / 2.1	0
0	0 / 2.8	0 / 2.7	0 / 2.4	0

Step 2. Looking at tableau

$X_{31}^4 = X_{05}^4 = X_{16}^4 = 1;$

$w_1^4 = c_1^4 = 80$; $w_3^4 = c_3^4 = 15$; $w_0^4 = 5$; $c_0^4 = 0$

Step 3. Upper Bound along this node = 95

Step 4. $U^4 = \{1,3,0\}$ and $w_1^4 = 80 \geq 10$; $w_3^4 = 15 \geq 10$; $w_0^4 = 0 \geq 0$.

Hence solution is feasible and we go to Step 6.

Step 6. Since 95 > 100, $L^4 = 95$

$S^4 = \{\underline{0}1,\bar{2},3\}$

$X_{12} = X_{13} = X_{14} = X_{16} = X_{31} = X_{05} = 1$ is current solution, and we go to Step 7.

Step 7. $S^5 = \{\underline{0},1,\bar{2},\bar{3}\}$, $F^5 = \Phi$, $R^5 = \{0,1\}$, $Z^5 = \{2\}$, $B^5 = \{1\}$, $L^5 = 95$, $A_1^5 = \{2,3,4,6\}$, $A_0^5 = \{5\}$.

<u>5th Iteration</u> Tableau is

Plan \ Tourist	1	7
1	15 2.5	80
0	0 2.8	0

<u>Step 2.</u> Looking at tableau

$$X_{01} = 1, \text{ Hence } w_1^5 = 80 = c_1^5$$

$$w_0^5 = 20, \; c_0^5 = 0$$

<u>Step 3.</u> Upper Bound along this is 80, which is less than 95,

hence go to step ..

<u>Step 7.</u> $S^6 = \{ \underline{0}, \underline{\bar{1}} \}$, $L^6 = 95$, $R^6 = \{0\}$, $F^6 = \{2,3\}$, $Z^n = \{1\}$, $B^6 = \{1,2,3,4,5,6\}$

<u>6th Iteration</u>. form Tableau

Plans \ Tourist Class	1	2	3	4	5	6	7
2	15 1.5	10 2.5	16 3.0	34 2.0	5 3.0	20 3.3	0
3	15 3.0	10 1.5	16 2.0	34 1.5	5 2.5	20 2.1	0
0	0 2.8	0 2.8	0 2.8	0 3.0	5 2.7	0 2.4	0

Looking at tableau:

Step 2. $X_{31} = X_{02} = X_{23} = X_{04} = X_{25} = X_{26} = 1$

$c_2^6 = 41 \qquad w_2^6 = 41$

$c_3^6 = 15 \qquad w_3^6 = 15$

$c_0^6 = 0 \qquad w_0^6 = 44$

Step 3. Since $\sum_{i=1}^{3} c_i^6 = 56$, go to Step 7.

Step 7. Backtrack not possible, hence current incumbent is optimal.

Optimal Solution. Objective function value = 95

Plans 1, 3 are to be offered, (i.e. Gateways at Denver and Gunnison)

also $X_{12} = X_{13} = X_{14} = X_{16} = X_{15} = X_{05} = 1$

Remarks: In this problem - if plans were evaluated singly

Plan 1 is most preferred; Plan 2 is 2nd most preferred

Plan 3 is 3rd most preferred; but in terms of combination of plans, 1 and 3 are better than Plans 1 and 2.

VI. CONCLUSION

It is easy to see that the above model can be applied to a much broader class of problems. The major result is the reduction of the size of the 0-1 variables from a very large number that included the X_{ij}'s to only Y_i's. Some of the common applications include the following:

(i) General routes and flight schedule decisions for airlines on a national basis

(ii) New Product planning and product line structuring which includes the existing products of the firm and those of the competitors, e.g., automobiles, consumer appliances, T.V., etc.

(iii) Modes of Transportation and Transportation routes for a city (may include time schedules) considering existing modes and routes in conjunction with proposed routes and modes.

(iv) Location of branches (facilities) for banks, supermarkets, fast food chains, etc.

It is to be noted, that the "cannibalism" effect is explicitly addressed in this model, in the above decision analysis. Thus our model is more suited to the above situations since such cannibalism effect is always present.

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