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LABOR SUPPLY UNDER UNCERTAINTY

by

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A. Introduction

The student of labor supply behavior currently has three reasonably well defined tools of analysis at his disposal. The most familiar framework is the theory of time allocation. This adaptation of the neoclassical theory of household demand serves as the principal instrument used in the interpretation of what has become an extensive literature on labor force participation by and the labor supply of household members.[1/] More recent in its origin is the theory of job search. Models of this type focus on the behavior of an individual worker faced with imperfect information about wage rates and job locations.[2/] As a tool of interpretation and explanation, search theory has found its uses in the study of unemployment and turnover behavior.[3/] The availability of panel data on individual work histories has created a need for a dynamic model that allows for uncertain durations of both employment and unemployment periods. Viewing the labor market experience of an individual as a Markov process has proved to be a useful means of describing the "typical" labor market history of different identifiable types of workers.[4/]

As these introductory remarks suggest, the three methods of analysis are currently applied to somewhat different problems. This fact reflects the relative strengths and weaknesses of each approach. The neoclassical model of income - leisure choice and its modern extensions are well suited to the purpose of analyzing the effects of member wage rates, of non-labor income or wealth, and of other household characteristics on the labor supply decisions of its members. However, the approach is limited by the presumption that an individual has complete control over his or her labor market experience. That a worker has only partial control in an uncertain world underlies the job search approach but the time allocation problem in its full complexity is typically ignored. Because of uncertainties concerning the magnitude, timing and frequency of job offers and the duration of jobs, one is not surprised that labor market histories are best described as realizations of a stochastic process. However, existing applications of this framework do not explicitly recognize the fact that an individual worker has some control over this process.

In this paper we present an intertemporal decision model of the labor supply behavior of an individual or household that integrates all three of these approaches. The basic ingredients of the formulation follow. Preferences and employment histories are represented by a given intertemporal utility function defined on time sequences of income - leisure combinations. A strategy is a rule that determines the worker's labor supply and search behavior in every conceivable circumstance that may arise. The worker's future participation history is a realization of a stochastic process partially controlled by the worker's strategy. The strategy selected is assumed to be optimal in the sense that it maximizes the mathematical expectation of lifetime utility takes with respect the probability distribution on possible participation histories induced by the strategy. The techniques of dynamic programming are applied to establish the existence of an optimal strategy and to characterize an optimal strategy.

To give the model structure, we assume that the individual worker's or working household's history in the labor market is generated by a Markov process. Variously defined conditions of employed and unemployed participation are the states of the process. The worker partially controls the time sequence of his experiences in the labor market by taking decisions that influence the transition probabilities.

Specifically, the probability of finding a job per period when unemployed or of finding a better job where employed depend on the fraction of his time endowment that he allocates to search in each period and on the acceptance criterion that he applies to the job offers received. The marriage of search and time allocation theory obtained by including investment in search as an alternative to work and leisure among the time consuming activities produces the following offspring. First, the cost of search is explicitly derived as the utility value of time in the alternative activities. Consequently, the optimal search strategy depends on the worker's preferences, income, and participation state. Second, the imputed costs of income and leisure in each participation state depend on the return to search. Consequently, both the decision to participate and to supply time to the market are influenced by the ease with which a job can be found, the expected duration of a job, and the entire set of possible wage rates that can be earned in the market.

The analysis is presented as follows. In section B, a Markov process model of labor market experience is developed along lines that exist in the literature. The dependence of the transition probabilities on the worker's search strategy is introduced in Section C. The intertemporal decision problem is formulated in Section D. In the case of an employed worker, the first order conditions for work and search require that the marginal rate of substitution of income for leisure be equal to the wage and that the marginal utility cost of search time, the marginal utility of the leisure forgone, be equal to the expected marginal expected future utility gain attributable to a unit of search time. That aversion to risk is required to obtain a solution involving the allocation of time to both work and search is the most interesting result obtained in this section.

In section E, the worker's optimal search strategy is characterized by comparing the criteria for acceptance and the times allocated to search across participation states. The decisions regarding the extent of labor market participation and the amount of time allocated to work are studied in section F. In the extended theory, non-labor income is an important determinant of both search and labor supply decisions. However, the rigorous explanation of "discouraged worker" effects is the model's most important contribution. In both section E and section F, a number of empirical propositions are established; they are all testable, in principle.

In the final substantive section, section G, a two-person household version of the model is considered. The focus of the section is on the effects of a change in one member's employment status on the acceptance and participation decision of the other. The "added worker" effect is only one among a rich array of possible dynamic interactions among the labor market experience of household members. The principle lesson to be learned is that the participation state of any one member depends on the employment histories of all the others in the household.

B. Employment Experience as a Markov Chain

The idea of describing the employment history of a representative individual worker as a stochastic process is clearly expressed in Holt [21]. In applications, the Markov model has been adopted because it provides a method of decomposing unemployment rates into two components, the average frequency and duration of unemployment spells. Specifically, the latter parameters are estimated for each of several different identifiable worker groups. Then the estimates are used to distinguish between differences in duration and frequency as reasons for observed differences in unemployment rates. In this section we introduce the basic model used in this literature.

Let $i \epsilon I = \{1,2,3\}$ represent one of three participation states that the worker may occupy at any date where 1 denotes being unemployed with no job attachment, 2 is the state of being employed and 3 represents unemployment with the possibility of recall to a former job. We assume stochastic movement over time from one state to another. Specifically, the worker's state location is generated by a time-homogeneous Markov process with a standard transition (probability) function.[5/] That is, the probability that the worker is in state i at time $t+\Delta t$ is

$$P_i(t+\Delta t) = \sum_{j=1}^{3} p_{ij}(\Delta t)\, P_j(t), \quad t \geqq 0 \tag{1}$$

where $(P_1(t), P_2(t), P_3(t))$ is the probability frequency distribution on the states at the beginning of time interval $[t, t + \Delta t)$ and $p_{ij}(\Delta t)$ is the transition probability function. The value of the transition function is the probability of making a transition from state j to i during the interval $[t, t + \Delta t)$. Finally, the initial state is known; i.e. $P_i(0) = 1$ for one and only one value of i.

Of course, $p_{ij}(\Delta t) \geqq 0$ and

$$\sum_{i=1}^{3} p_{ij}(\Delta t) = 1 \qquad \forall \quad j \in I \tag{2}$$

since $p_{ij}(\Delta t)$ is a probability and no transition implies remaining in the same state. The process is said to be time-homogeneous where the transition function is independent of t. Let us assume that the transition function can be written as

$$p_{ij}(\Delta t) = a_{ij}\Delta t + 0(\Delta t) \ , \ i \neq j \tag{3}$$

where a_{ij} is a constant and $0(\Delta t)/\Delta t \to 0$ as $\Delta t \to 0$. (2) and (3) imply that the transition function is standard, namely,

$$\lim_{\Delta t \to 0} p_{ij}(\Delta t) = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases} .$$

In other words, the transition function is continuous at $\Delta t = 0$ and, hence, the probability of making any transition in a sufficiently short interval is negligible. When standard, the process remains in each state visited for a strictly positive time interval. Moreover, given (3), more than one state transition in a small time interval of length Δt is of order $0(\Delta t)$; i.e. can be neglected. The usefulness of (3) follows from the fact that it implies a continuous time representation of the probability laws of the process. Specifically, the probability distributions over states at all dates can be represented as the solution to the following linear differential equation system:

$$\lim_{\Delta t \to 0} \left[\frac{P_i(t+\Delta t) - P_i(t)}{\Delta t}\right] = \frac{dP_i}{dt} = -a_{ii}P_i + \sum_{j \neq i} a_{ij}P_j, \ i \in I \tag{4a}$$

where from (2) and (3)

$$a_{jj} = \sum_{i \neq j} a_{ij} \quad , \quad j \varepsilon I. \tag{4b}$$

To give the model content for our purpose we need to interpret (3) and name the parameters $[a_{ij}]$.

Suppose that the job offers received by the worker when unemployed arrive randomly in sequence according to a Poisson process with constant mean α. Then, the number that arrives, n, in an interval of length Δt has probability of occurance

$$\Pr\{n\} = \frac{e^{-\alpha\Delta t}(\alpha\Delta t)^n}{n!} \quad , \quad n = 0, 1, \ldots .$$

where $\alpha\Delta t$ is the expected number that will arrive. If all offers are acceptable, the probability of becoming employed is the probability that one or more will arrive; i.e.,

$$p_{21}(\Delta t) = \sum_{n=1}^{\infty} \Pr\{n\} = 1 - e^{-\alpha\Delta t} .$$

Consequently, the waiting time (duration of unemployment) is a negative exponential random variable with mean equal to the inverse of the arrival frequency, $1/\alpha$. Moreover, because $e^{-\alpha\Delta t} = 1-\alpha\Delta t + 0(\Delta t)$,

$$p_{21}(\Delta t) = \alpha\Delta t + 0(\Delta t) .$$

The parameter a_{21} is equal to α, the expected frequency with which offers arrive.

There are several stories that justify the Poisson offer arrival assumption. For example, suppose that each of N hiring firms randomly contact each worker with the same independent probability $\pi\Delta t$ per period of length Δt. In this case the number of arrivals is a binomial random variable characterized be the parameters $(\pi\Delta t, N)$. But since $\pi\Delta t$ is small when Δt is sufficiently small, the Poisson distribution with mean $\alpha\Delta t = \pi N\Delta t$ is a good approximation provided that the number of firms is large.[6/]

In general, (3) implies that the random time spent waiting in state j for a transition to state i is distributed approximately negative exponential with mean $1/a_{ij}$ when Δt is small. Although the Poisson arrival story used in the case of offers is less convincing in the other cases, this general fact allows an interpretation of (3) and of the parameter a_{ij}.

Let β denote the average recall rate, γ the layoff rate and δ the separation rate. These are respectively the average proportions of the stocks of laid off workers recalled, of employed workers put on temporary layoff and of employed and laid off workers permanently separated from their current or former employer. Suppose in each case that individual workers are selected at random with equal probability so that waiting times are negative exponentials given large stocks. Then, the a_{ij} have the following interpretations, $a_{12} = \delta$, $a_{13} = \delta$, $a_{21} = \alpha$, $a_{23} = \alpha + \beta$, $a_{31} = 0$, $a_{32} = \gamma$. Of course, α is the expected number of offers that arrive per period, the arrived rate. The assumption $a_{23} = \alpha + \beta$ reflects the fact that a laid off worker can become employed either by receiving an outside offer from another employer or by being recalled. Obviously, we are assuming that the chance of a separation is the same whether employed or unemployed but subject to recall; i.e. $a_{12} = a_{13}$. Finally, $a_{31} = 0$ reflects the fact that our unemployed worker can move to the layoff state only by first becoming employed. Since at least two state transitions are required, the probability of such a movement in a short time interval is negligible.

Given these specifications, (4) implies

$$\frac{dP_1}{dt} = -\alpha P_1 + \delta(P_2 + P_3)$$

$$\frac{dP_2}{dt} = \alpha P_1 - (\delta + \gamma) P_2 + (\alpha + \beta) P_3 .$$

Since $P_1 + P_2 + P_3 = 1$ for all t, the third differential equation is redundant. Moreover, the system reduces to

$$\frac{dP_1}{dt} = \delta - (\alpha + \delta)P_1 \tag{5a}$$

$$\frac{dP_2}{dt} = \alpha + \beta - \beta P_1 - (\alpha + \beta + \gamma + \delta)P_2 \tag{5b}$$

$$P_1 + P_2 + P_3 = 1 . \tag{5c}$$

The solution to this system converges to the following steady state values independant of initial conditions:

$$P_1^* = \frac{\delta}{\alpha+\delta} \tag{6a}$$

$$P_2^* = \frac{\alpha(\alpha + \beta + \delta)}{(\alpha + \delta)(\alpha + \beta + \gamma + \delta)} \tag{6b}$$

$$P_3^* = \frac{\alpha\gamma}{(\alpha + \beta)(\alpha + \beta + \gamma + \delta)} . \tag{6c}$$

The steady state probability distribution (P_1^*, P_2^*, P_3^*) has two useful interpretations. For an individual worker, it represents the long run expected fractions of his working life spent in the three participation states. Given a large sample of identical workers, P_i^* is also the expected proportion of the sample to be found in participation state $i \in I$ provided that all have been participating for some time. In both cases the time required to eliminate the effects of initial conditions depends on the speeds of convergence, $\alpha + \delta$ and $(\alpha + \beta + \gamma + \delta)$.

Note that the unemployment rate for workers with no employer or job attachment, P_1^*, is independent of both the temporary layoff note and the recall note. However, the unemployment rate as usually measured is

$$P_1^* + P_2^* = \frac{\delta}{\alpha + \delta} + \left(1 - \frac{\delta}{\alpha + \delta}\right)\left(\frac{\gamma}{\alpha + \beta + \gamma + \delta}\right) .$$

Obviously, it decreases given an increase in either the offer arrived rate, α, or the recall rate, β, and it increases when either the separation rate δ or the temporary layoff rate γ is increased. Differences in any one of these four parameters can account for differences in unemployment rates among identifiable groups of workers. For example, Marston's [26] estimates suggest that the unemployment rate for black males is higher than for their white counterparts primarily because they experience unemployment more frequently; either γ or δ is larger.

Although the assumption that the transition parameters are stationary is unrealistic, it is a useful abstraction if one's purpose is a comparison of the type of employment histories of different identifiable worker groups. However, one would like to know to what extent differences in decisions, made by the workers themselves, might account for observed differences in histories. As a first step toward a framework for an analysis of this question, the model is extended to account for the ways in which a worker can control the transition probabilities in the next section.

C. Job Search and Participation Histories

For simplicity we assume that jobs differ from the worker's viewpoint if and only if the wage rates paid are different; e.g., layoff, separation and recall rates are identical across jobs given the worker type. Moreover, the wage rate earned in any given job is fixed over time. Because the worker is qualified to perform many different jobs in general, he faces a set of different wage rate opportunities. The worker does not know the location of an individual job in this set. Their locations are discovered in the random manner previously described.

Presumably, the worker is more likely to obtain an offer the more intensely he searches for it. As a means of formalizing this notion, we assume that the arrival frequency is αs where s is the fraction of any time period that the worker allocates to search activity and the parameter α is reinterpreted as the expected number of offers received per unit of search time.[7/] Although a linear "search technology" is special, little insight gained by introducing some notion of "diminishing returns" to search time.

Let F(w) be a c.d.f. describing the distribution of wage rates over the worker's potential employers. Then, if each employer contacts the worker with equal probability, F(w) is the probability that any job offered pays a wage of w or less. By definition a wage offer is acceptable if and only if it exceeds the worker's reservation wage, denoted as w^*. Hence, the probability that an offer arriving at random is acceptable equals $[1 - F(w^*)]$. Throughout the remainder of the paper we assume that offers must be either accepted or rejected as they arrive; i.e., search is without recall.

Collectively, our assumptions imply that one acceptable offer arrives in a time interval of length Δt with probability approximately equal to $\alpha s[1 - F(w^*)]\Delta t$ and that the probability of more than one acceptable offer arrives is negligible; i.e., of order $0(\Delta t)$ in the notation of the previous section. One need only replace α with $\alpha s[1 - F(w^*)]$ in the model of the previous section to discover the way in which the worker's choice of reservation wage (w^*) and search intensity (s) affects the process that determines his participation history. However, this procedure implicitly presumes that the worker, when laid off, will choose the same values of (s,w^*) as when unemployed with no employer attachment. Since one suspects that the worker's incentives to search are different in the two states, the assumption is not likely to be consistent with actual behavior.

In general, one expects that the intensity of search chosen when employed at wage w and when unemployed but subject to the possibility of recall to a job paying wage w to depend on both whether or not the worker is employed and on the value of the wage rate. In the sequel, we adopt the notations $(s_i(w), w_i^*(w))$ $i = 2,3$ as a means of suggesting this dependence. When unemployed with no job attachment the worker's search intensity and reservation wage are denoted as s_1 and w_1^* respectively.

If the worker's choice of search intensity and reservation wage given a job attachment depends on the wage paid, then the movements from one participation state to another are no longer described by the process in Section B because the wage changes from time to time as the worker changes job attachments. However, one can construct a new Markov process that again describes a workers employment history by extending the state space to include the set of wage rates that are possible.

Specifically, let the state space be $I \times W$ where W is the set of all possible wage opportunities. Suppose that there are a finite number of job types for which the worker is qualified and let w_k be the wage paid in job type k; i.e., $W = \{w_1, .., w_k, ..., w_m\}$. Both the number of types, m, and the wage paid in each job type w_k, $k = 1,..., m$, are constant over time and finite. With no loss of generality, we can assign the index k so that $k > \hat{k}$ implies $w_k \leqq w_{\hat{k}}$. Finally, it is convenient to include zero in W. Hence,

$$0 = w_m \leqq w_{m-1} \leqq \ldots \leqq w_2 \leqq w_1 < \infty . \tag{7}$$

Think of w_1 as the "first best" wage available, of w_2 as the "second best", etc.

A participation state for the new process is a pair $(i,w) \in I \times W$. A state has the following interpretation. When i = 2, the worker is employed in a job paying wage w. When i = 3, the worker is laid off but may be recalled to a job paying w. When i = 1, the worker is unemployed without job attachment. Hence, when i = 1, the differences among the values of w have no meaning. A formal way of handling this fact is introduced later.

Let $P_j(w,t)$ denote the probability that the worker is in state (j,w) at date t. The transition function is $p_{ij}(\tilde{w}, w, \Delta t)$. Its value is the probability of a transition from state (j,w) to state $(i, \tilde{w})$ during any interval of length Δt. Consequently

$$P_i(\tilde{w},t) = \sum_{j=1}^{3} \sum_{\tilde{w} \in W} p_{ij}(\tilde{w},w,\Delta t) \; P_j(w,t), (i,\tilde{w}) \in I \times W. \tag{8}$$

Obviously, $P_i(\tilde{w},t)$ is the joint distribution over the state space $I \times W$ at time t.

Let $[P_1(t), P_2(t), P_3(t)]$ denote the probability distribution over I as before. Since it is a marginal distribution,

$$P_i(t) = \sum_{w \varepsilon W} P_i(w,t) \qquad i = 1,2,3 \tag{9a}$$

by definition. Furthermore,

$$P_1(t) + P_2(t) + P_3(t) = 1 \; ; \tag{9b}$$

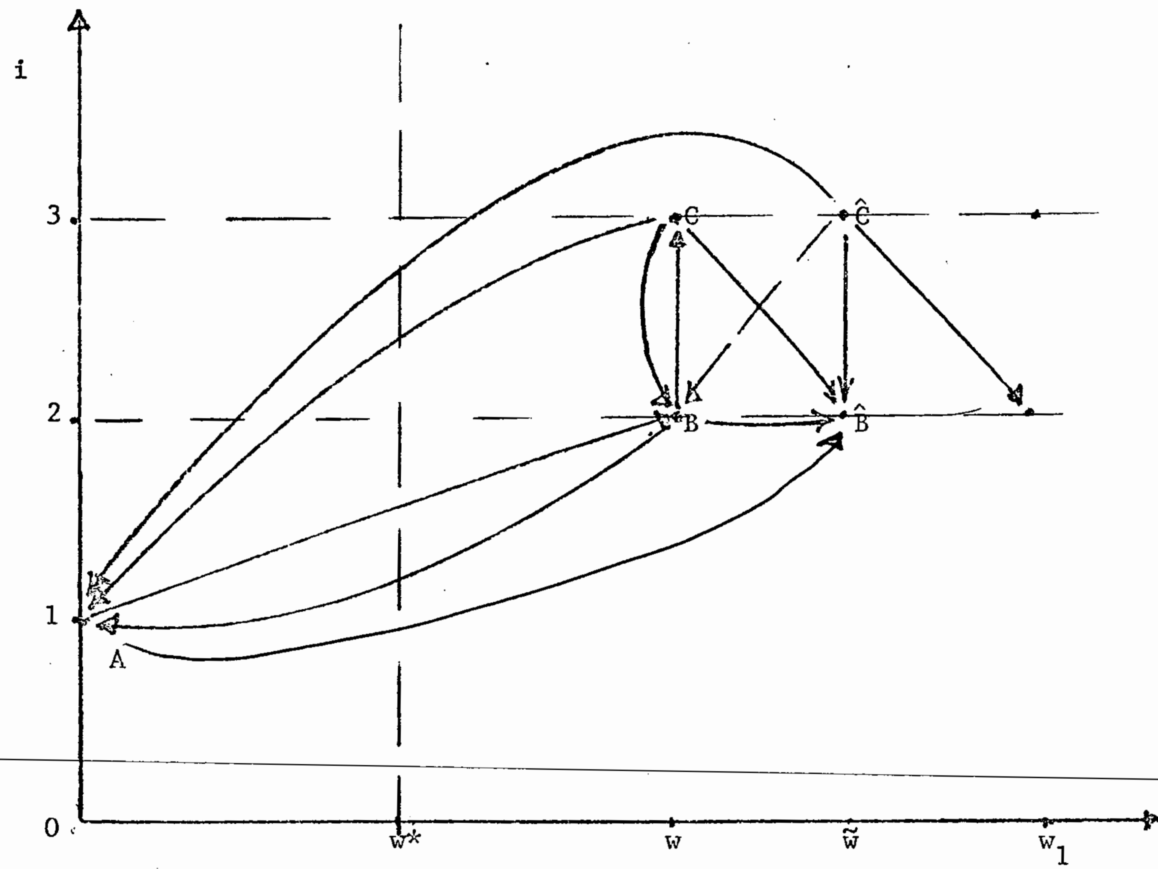
the worker is either unemployed without job attachment, employed or laid off at each date with probabilities P_1, P_2, and P_3 respectively.

The fact that w does not distinguish among states when i= 1 can be handled formally by assuming that no transitions are ever made to or from any unemployment state paying a positive wage; i.e., $p_{ij}(\tilde{w},w) = 0$ for all $\tilde{w} > 0$ and $p_{ij}(\tilde{w},w) = 0$ for all $w > 0$. We further assume that $P_1(w,0) = 0$ for all $w > 0$. This formalism together with (8) and (9) imply that $P_1(t) = P_1(0,t)$ for all t ; i.e., there is only one unemployment state when the worker has no job attachment. By more subtle arguments, the state space can be reduced still further.

Figure 1 is useful for our purpose. The set of possible wage rates offers, W, is represented by various discrete points including w = 0 along the horizontal axis. The set I is the three designated points on the vertical axis. The state space $I \times W$ is the collection of coordinates. The point (1,0) is the only possible unemployment state with no job attachment given our assumptions. It is represented by A in the figure. The points labelled B and $\hat{B}$ are typical employment states, and C and $\hat{C}$ are unemployment states with the possibility of recall to a job paying the wage specified.

FIGURE 1:

STATES AND TRANSITIONS



The arrows to and from the representative states suggest the transitions that are possible in any sufficiently small interval of time. The figure is drawn to take account of the fact that none of the employment or recall states, (i,w) $i = 2$ or 3, will be visited except possibly for an initial instant if the wage is less than or equal to the worker's acceptance wage when unemployed, $w \leqq w_1^*$. Indeed, if the worker was initially unemployed, the only transition possible is to employment (A to B) and that transition will be made if and only if the wage paid is acceptable. Moreover, if unemployed initially, then the worker could be in a state $(3,w)$, $w \leqq w_1^*$, for some $t>0$ only if he were employed at an unacceptable wage at some prior date. Since this implication contradicts revealed preference, we exclude it. The same arguments apply if the worker was initially located in any of the states (i,w) satisfying $i = 2$ or 3 and $w > w_1^*$. Of course, since the initial location is arbitrary the worker may have started out in one of the undesirable states. But, if so, he would immediately quit to become unemployed if $i = 2$ and refuse any recall if $i = 3$. The latter possibility is equivalent to (1,0). In the apt language of Markov theory, these states, (i,w) $i = 2$ or 3 and $w \leqq w^*$, are called instantaneous and transitory. They can be excluded from further consideration because $P_i(w,t) = 0$ on this set for all $t > 0$.

Now, consider the transitions that are possible among the states of interest. For this purpose we need notation that represents the probability that a random wage offer, $\tilde{w}$, is exactly w. Let f(w) serve this purpose. Of course,

$$\Pr\{\tilde{w} \leqq w\} = F(w) = \sum_{\tilde{w} \leqq w} f(\tilde{w}) \tag{10}$$

by definition of the c.d.f. Because the probability that an offer arrives in a small interval Δt is $\alpha s_1 \Delta t$ and because an offer that does arrive pays w with probability f(w), a transition from A to B occurs with probability $\alpha s_1 f(w)\Delta t$ if $w > w_1^*$ and with probability zero otherwise. No other type of transition is possible from A.

From any possible employment state, a point like B in the figure, there are three general possibilities. The worker can leave B either because he was laid off (B to C), because he separates (B to A) or because he received an alternative job offer paying a higher wage (B to $\hat{B}$). The probability in the first case is $\gamma\Delta t$, where γ represents the layoff rate and in the second case is $\delta\Delta t$, where δ is the separation rate. The probability of the transition to an acceptable $\hat{B}$ is $\alpha s_2(w) f(\tilde{w})\Delta t$, since $\alpha s_2(w)\Delta t$ is the probability of receiving an offer and $f(\tilde{w})$ is the probability that it has the value $\tilde{w}$.

Because jobs differ only with respect to the wage rates paid, an employed worker accepts an alternative job offer if and only if a higher wage is paid; i.e., $w_2^*(w) = w$. Consequently, the probability of a transition from $(2,w)$ to $(2,\tilde{w})$ is zero if $w \geqq \tilde{w}$.

When the worker is laid off, at a state like C, the transitions possible are to unemployment without attachment (C to A), to his previous employment state (C to B) and to an alternative acceptable employment state (C to $\hat{B}$). The probability of a transition from C to A is $\delta\Delta t$, where again δ is the separation rate. Since the workers previous employment state is acceptable by revealed preference, the probability of a transition from C to B is simply $\beta\Delta t$, where β is the recall rate. Finally, a transition from C to $\hat{B}$ occurs with probability $\alpha s_3(w)f(\tilde{w})$ provided that $\tilde{w} > w_3^*(w)$. Otherwise, it is zero.

One restriction implicit in the structure that we have designed needs to be pointed out. A given Markov process cannot occupy two or more of the specified states at the same time. Consequently, we have ruled out the possibility that a worker can be both employed in one job and subject to recall to another. Employed but on recall can be included but only by extending and complicating the state space still further. The reader should now be able to see how it could be done.[8/]

The description of the Markov process that arises when the worker has search intensity and reservation wage choice in each participation state is now complete. The reader may find it of interest to "follow the arrows" in Figure 1 as a means of acquiring a feel for the realized employment histories that are possible. Our next task is to consider the problem of calculating the asymptotic probability distribution over the state space.

Since we have already specified the transition probabilities, we will simply write down the special form of the system (8) that represents our particular model without detailed discussion or algebraic manipulation. First, remember that (1,0) is the only possible unemployment state involving no job attachment. Therefore, $P_1(t) = P_1(0,t)$ for all t from (9a). Since transitions from A to only points like $\hat{B}$ or B are possible and from all points like B and C to A occur with the same probability $\delta\Delta t + 0(\Delta t)$,

$$P_1(t + \Delta t) = (1 - s_1[1 - F(w_1^*)])\ P_1(t) + \delta\Delta t[P_2(t) + P_3(t)] + 0(\Delta t)$$

is immediate given (9a) and (10). Of course, by arranging appropriately, by applying (9b), by dividing both sides by Δt and then by taking the limits as $\Delta t \to 0$, we obtain the following analogue of (5a):

$$\frac{dP_1}{dt} = \delta - (\delta + \alpha s_1[1 - F(w_1^*)])P_1 . \tag{11}$$

The asymptotic probability of being unemployed with no job attachment can immediately be derived as the value of P_1 satisfying $dP_1/dt = 0$.

It is convenient to represent the set of acceptable wage rates given unemployment with no job attachment as

$$W_1^* = \{w \in W | w > w_1^*\} = \{w_1, w_2, \ldots, w_n\}$$

where w_n is the smallest acceptable wage rate among those available. Analogously, let

$$W_3^*(\tilde{w}) = \{w \in W | w > w_3^*(\tilde{w})\}$$

and

$$W_2^*(\tilde{w}) = \{w \in W | w > \tilde{w}\}$$

denote the set of acceptable wage rates given participation state $(j,\tilde{w})$, j = 2 or 3. Then for each possible employment state, characterized by some $w \in W_1^*$, we have

$$P_2(w, t + \Delta t) = \left[(1 - \delta\Delta t - \gamma\Delta t - \alpha\Delta t\, s_2(w) \sum_{\tilde{w} \in W_2^*(w)} f(\tilde{w})\right] P_2(w,t)$$

$$+ \alpha\Delta t s_1 f(w) P_1(t) + \beta\Delta t P_3(w,t)$$

$$+ \alpha\Delta t f(w) \sum_{w \in W_3^*(\tilde{w})} s_3(\tilde{w}) P_3(\tilde{w},t)$$

$$+ \alpha\Delta t f(w) \sum_{w \in W_2^*(\tilde{w})} s_2(\tilde{w}) P_2(\tilde{w},t) + 0(\Delta t)$$

where $\sum_{w \in W_i^*(\tilde{w})}$ denote the sum over all $\tilde{w}$ such that w is an element of $W_i^*(\tilde{w})$. The last two non-negligible terms on the right are respectively the probability of arriving at (2,w) from some layoff state other than that associated with the job in question and the probability of arriving from some other job paying a lower wage. By the familiar procedure, one obtains

$$\frac{dP_2(w)}{dt} = -(\delta + \gamma + \alpha s_2(w)[1 - F(w)])P_2(w)$$

$$+ \alpha s_1 f(w) P_1(t) + \beta P_3(w) \tag{12}$$

$$+ \alpha f(w) \sum_{w \in W_3^*(\tilde{w})} s_3(\tilde{w}) P_3(\tilde{w}) + \sum_{w \in W_2^*(\tilde{w})} s_2(\tilde{w}) P_2(\tilde{w}) \quad .$$

The following equation is obtained in an analogous manner

$$\frac{dP_3(w)}{dt} = -(\delta + \beta + \alpha s_3(w)[1 - F(w_3^*(w))])\ P_3(w) + \gamma P_2(w) \tag{13}$$

for each possible layoff state (3,w), $w \in W_1^*$.

Since W_1^* contains n elements, the equations (11), (12) and (13) define a system of 2n + 1 ordinary linear differential equations in the probabilities P_1, $P_2(w_k)$, and $P_3(w_k)$, k = 1, ...,n. The solution is the probability distribution over the possible participation states at each date. Although a unique solution exists for every specification of the initial distribution, the explicit representation requires knowledge of the form of the relationship between the acceptance wage when laid off and the wage paid in the job to which recall is possible, the function $w_3^*(w)$.

The asymptotic distribution can be derived by first setting all three derivatives to zero and then solving the resulting linear equation system for the desired values of the probabilities. The equation system has a recursive structure that can be exploited for the purpose of finding the solution. First, $\frac{dP_1}{dt} = 0$ and (11) immediately yield

$$P_1^* = \delta/(\delta + \alpha s_1[1 - F(w_1^*)]) \cdot \tag{14}$$

Because $\{\tilde{w}|w_n \in W_3^*(\tilde{w})\}$ contains w_n only and $\{\tilde{w}|w_n \in W_2^*(w_n)\}$ is empty, (12) and (13) in the case $w = w_n$ yield solutions for $P_2^*(w_n)$ and $P_3^*(w_n)$ in terms of P_1^*. Given these, $P_2^*(w_{n-1})$ and $P_3^*(w_{n-1})$, can be found using the same procedure in the case $w = w_{n-1}$, etc. The functional forms of the solutions become quite complex as one proceeds and are not particularly revealing.

From the view point of unemployment analysis the following representation of the total probability of being unemployed but subject to recall to some job is of interest:

$$P_3^* = \frac{[1 - P_1^*]\,\gamma}{\delta + \gamma + \beta + \alpha \sum_{w \in W_1^*} s_3(w)[1 - F(w_3^*(w))]P_3^*(w)/P_3^*} \cdot \tag{15}$$

where $u(x_t, \ell_t)$ is the instantaneous utility flow associated with a realized real income-leisure combination (x_t, ℓ_t) during the short interval $[t, t + \Delta t)$ and ρ is the subjective rate at which future utility flows are discounted. If the worker is employed during the interval at a real wage w, then the possible choices of real income x_t, the fraction of the interval devoted to leisure ℓ_t, and the fraction of the interval allocated to search s_t are restricted by the worker's budget and time constraints.

Formally, let $\Gamma(w,y)$ denote the possible set of choices where y is the worker's non-labor income. It is the subset of R^3_+ such that

$$x + w\ell + ws \leqq y + w \tag{17a}$$

$$\ell + s \leqq 1. \tag{17b}$$

The residual fraction of the interval not spent in leisure or search, $h = 1 - \ell - s$, is interpreted as the time spent working. When the worker is not employed, his choices are simply restricted to the set $\Gamma(0,y)$; i.e., $x \leqq y$ and $\ell + s \leqq 1$.

The specification (16) and (17) is the most elementary intertemporal generalization of the simple income-leisure choice model introduced by Robbins [32]. Indeed, if jobs can be located instantaneously, then no time will be allocated to search and the choice of hours worked depends on the wage in the highest paying job available and on non-labor income in ways that are familiar to all students of labor economics. But, if the locations of the various jobs is not known with certainty and if time is required to locate any job, then the decision problem requires that a choice be made in the present, regarding the time allocated to search, that has uncertain effects on the worker's future employment history. Specifically, this choice determines the probability of making a transition in the next short time interval from the worker's current state to every possible employment state. For the purpose of formulating this decision problem, we assume that the optimal strategy maximizes the worker's expected utility at each date, the mathematical expectation of the future discounted instantaneous utility stream.

The appropriate technique for deriving and characterizing the optimal strategy is dynamic programming. For the purpose of applying this technique, we introduce the value function $V_i(w,t)$, defined on the participation state space and on time in general. The value function is the maximal expected utility associated with being in the state $(i,w) \in I \times W$ during the short interval $[t, t + \Delta t)$, that realized when an optimal strategy is pursued subsequent to date t. If we let d(t) denote the decision to be taken during the interval $[t, t + \Delta t)$, then it solves

$$V_i(w,t) = \frac{1}{1 + \rho\Delta t} \max_d \left[u(.)\Delta t + \sum_{j \in I} \sum_{\tilde{w} \in W} p_{ji}(\tilde{w},w,\Delta t)\, V_j(\tilde{w},t + \Delta t) \right] . \qquad (18)$$

by virtue of Bellman's principle of dynamic optimality where $p_{ji}(\tilde{w},w,\Delta t)$ is the probability of the transition from state (i,w) to $(j,\tilde{w})$ during the interval $[t,t + \Delta t)$. The first term on the right of (18) is the present value of the utility flow realized during the interval $[t,t + \Delta t)$. The second term is the present value of the expected maximal end of interval utility at date t given that an optimal strategy is pursued subsequent to $t + \Delta t$, whatever the worker's participation state as of that date. As already noted, both the utility flow realized during the interval and the transition probabilities depend on the worker's current decisions. Condition (18) is simply the statement that the optimal current decision maximizes the expected utility associated with the worker's current state given that an optimal strategy is pursued in the future.

equation (18) implies

$$(1 + \rho\Delta t)V_1(t) = \tag{19.a}$$

$$\max_{(x,\ell,s)\in\Gamma(0,y)} \Big[u(x,\ell)\Delta t + \alpha\Delta t s \sum_{\tilde{w}\in W} f(w) \max[V_2(\tilde{w},t+\Delta t),V_1(t+\Delta t)]$$

$$+ (1-\alpha\Delta t s)V_1(t+\Delta t) + 0(\Delta t)\Big]$$

In particular, the probability of becoming employed at a wage $\tilde{w}$ is

$$p_{21}(\tilde{w},0,\Delta t) = \begin{cases} \alpha s_1(t)\Delta t f(\tilde{w}) + 0(\Delta t) & \text{if } V_2(\tilde{w},t+\Delta t) > V_1(t+\Delta t) \\ 0 \text{ otherwise} & \end{cases}$$

and the probability of remaining unemployed is

$$p_{11}(0,0,\Delta t) = 1 - \sum_{\tilde{w}\in W} p_{21}(\tilde{w},0,\Delta t).$$

The optimal decision given any other participation state at date t is defined in an analogous manner. Suppose the worker is employed at a wage w at date t. Then, with probability $\alpha s_2(t)\Delta t f(\tilde{w}) + 0(\Delta t)$ he can choose to be either employed at wage $\tilde{w}$ or w at the end of the interval. However, with probability $\delta\Delta t + 0(\Delta t)$ he will be separated during the interval and, hence, unemployed with no job attachment at the end. Moreover, the worker is laid off with probability $\gamma\Delta t + 0(\Delta t)$. In all cases, the worker can choose to be unemployed without job attachment, as a non-participant for example, at the end of the interval. Consequently,

$$(1+\rho\Delta t)V_2(w,t) = \max_{(x,\ell,s)\in\Gamma(w,y)}\Big[u(x,\ell)\Delta t \qquad (19.b)$$

$$+ \alpha\Delta ts \sum_{\tilde{w}\in W} f(\tilde{w})\max[V_2(\tilde{w},t+\Delta t),V_2(w,t+\Delta t),V_1(t+\Delta t)]$$

$$+ \gamma\Delta t\max[V_3(w,t+\Delta t),V_1(t+\Delta t)] + \delta\Delta tV_1(t+\Delta t)$$

$$+ (1-\alpha\Delta ts-\gamma\Delta t-\delta\Delta t)\max[V_2(w,t+\Delta t),V_1(t+\Delta t)] + 0(\Delta t)\Big]$$

Finally, if the worker were unemployed at date t but subject to recall to a job paying w, (18) implies

$$(1+\rho\Delta t)V_3(w,t) = \max_{(x,\ell,s)\in\Gamma(0,y)}\Big[u(x,\ell)\Delta t \qquad (19.c)$$

$$+ \alpha\Delta ts \sum_{\tilde{w}\in W} f(\tilde{w})\max[V_2(\tilde{w},t+\Delta t),V_3(w,t+\Delta t),V_1(t+\Delta t)]$$

$$+ \beta\Delta t\max[V_2(w,t+\Delta t),V_1(t+\Delta t)] + \delta\Delta tV_1(t+\Delta t)$$

$$+ (1-\alpha\Delta t-\beta\Delta t-\delta\Delta t)\max[V_3(w,t+\Delta t),V_1(t+\Delta t)] + 0(\Delta t)\Big]$$

where $\beta\Delta t + 0(\Delta t)$ is the probability of recall and $\delta\Delta t + 0(\Delta t)$ is the probability of separating during the interval. Equation (19.c) reflects the assumption that the worker does not have the right to refuse recall or to remain on recall while employed in another job except by ending his job attachment.

In the infinite horizon case, the value function is stationary; i.e. a unique solution $V_i(w)$ to (18) exists such that $V_i(w) = V_i(w,t)$ for all t and that solution is the value function for the infinite horizon problem.[10/] This fact also implies that any optimal strategy is stationary which is precisely the condition needed to characterize the worker's employment history as a Markov process. Although real working lives are finite, one can also show that the infinite horizon value function and, consequently, optimal strategy approximates those associated with the finite horizon problem when the horizon date is sufficiently far in the future.[11/] How far in the future the horizon date must be to obtain a good approximation depends on the speed with which the Markov process converges to its asymptotic distribution. Because the speed of convergence is quite rapid given reasonable values of the relevant parameters (α, β, γ and δ), the approximation would seem to be excellent except for a worker who is very near his or her retirement age. In the sequel we restrict the analysis to the infinite horizon case for this reason. Even so, virtually all of the qualitative results obtained can be generalized to the finite horizon case. The arguments needed to obtain these are, however, more complex.

Given $V_i(w,t) = V_i(w)$ for all t and each participation state, the equations of (19) immediately imply that $V_2(w) \geqq V_1$ and $V_3(w) \geqq V_1$ because unemployment with no job attachment can be chosen by the worker whatever the current participation state. By cancelling the common term $V_i(w)$ on the left and right side of each equation, by dividing both sides of the result by Δt and then by taking the limits of both sides as $\Delta t \to 0$, the equations of (19) can be rewritten in the following form:

$$\rho V_1 = \max_{(x,\ell,s)\,\epsilon\,\Gamma(0,y)} \left[u(x,\ell) + \alpha s \sum_{\tilde{w}\,\epsilon\,W_1^*} (V_2(\tilde{w}) - V_1)f(\tilde{w})\right] \tag{20a}$$

$$\begin{aligned}\rho V_2(w) = \max_{(x,\ell,s)\,\epsilon\,\Gamma(w,y)} &\Big[u(x,\ell) + \alpha s \sum_{\tilde{w}\,\epsilon\,W_2^*(w)} (V_2(\tilde{w}) - V_2(w))f(\tilde{w}) \\ &+ \gamma(V_3(w) - V_2(w)) + \delta(V_1 - V_2(w))\Big].\end{aligned} \tag{20b}$$

$$\begin{aligned}\rho V_3(w) = \max_{(x,\ell,s)\,\epsilon\,\Gamma(0,y)} &\Big[u(x,\ell) + \alpha s \sum_{\tilde{w}\,\epsilon\,W_3^*(w)} (V_2(\tilde{w}) - V_3(w))f(\tilde{w}) \\ &+ \beta(V_2(w) - V_3(w)) + \delta(V_1 - V_3(w))\Big]\end{aligned} \tag{20c}$$

where

$$W_1^* = \{\tilde{w}\ \epsilon\ W \mid V_2(\tilde{w}) > V_1\} \tag{21a}$$

$$W_2^*(w) = \{\tilde{w}\ \epsilon\ W \mid V_2(\tilde{w}) > V_2(w)\} \tag{21b}$$

$$W_3^*(w) = \{\tilde{w}\ \epsilon\ W \mid V_2(\tilde{w}) > V_3(w)\}\ . \tag{21c}$$

Obviously, $W_i^*(w)$ is the set of acceptable wage offers in state (i,w). Consequently, the second term on the right side of each equation of (20) is the total expected gain in utility attributable to search in the current period. It is the product of the expected rate at which acceptable new offers arrive per unit time period, $\alpha s_i \Pr\{\tilde{w}\ \epsilon\ W_i^*(w)\}$, and the conditional mathematical expectation of the utility gain associated with an acceptable new offer,

$$E\{V_2(\tilde{w}) - V_i(w) \mid \tilde{w}\ \epsilon W_i^*(w)\} = \sum_{\tilde{w}\,\epsilon\,W_i^*(w)} [V_2(\tilde{w}) - V_i(w)]f(\tilde{w})/\Pr\{\tilde{w}\ \epsilon\ W_i^*(w)\}.$$

Consequently, in every participation state, the optimal strategy maximizes the current instantaneous utility flow plus the total utility gain attributable to current search time.

The optimal choice of (x,ℓ,s) in each state is unique and continuous in w for each $i \in I$ given (20) if the utility function $u(x,\ell)$ is strictly concave in income and leisure.[12/] Indeed, if the optimal choice is such that time is allocated to all three activites - work, search and leisure - given employment, then in some neighborhood of the optimal (x,ℓ) the utility function u(.) is concave by virtue of the second order necessary conditions. The concavity of the utility function is a cardinal property of preferences and as such has no meaning in the standard theory of household demand under conditions of certainty. However, under conditions of uncertainty, the property can be interpreted as risk aversion.[13/] Specifically, if the worker ranks uncertain income-leisure combinations according to expected utility then he prefers a certain given combination $(\hat{x},\hat{\ell})$ to every joint probability distribution with $(\hat{x},\hat{\ell})$ as its mean if and only if $u(x,\ell)$ is strictly concave. This fact motivates the following definition.

<u>Definition 1</u>: A worker does not prefer (is averse to) risk in income and leisure if and only if $u(x,\ell)$ is (strictly) concave.

In the sequel we maintain that

<u>Assumption</u>: The worker is risk averse in income and leisure; i.e., $u_{11} < 0$, $u_{22} < 0$ and $u_{11}u_{22} - u_{12}u_{21} > 0$ if u(.) is twice differentiable where $u_{ij}(.)$ is the representative second partial derivative of $u(x,\ell)$.

E. The Search Strategy

A search strategy is a pair $(s_i(w),\ W_i^*(w))$ for each participation state $(i,w) \in I \times W$ where $s_i(w)$ is the fraction of the worker's time allocated to search and $W_i^*(w)$ is the set of acceptable offers given state (i,w). As noted in Section C, a search strategy determines the probability of a transition from (i,w) to every employment state $(2,\tilde{w})$. For any small time interval Δt, the transition probability is approximately equal to $\alpha s_i(w)\Delta t f(\tilde{w})$ if $w \in W_i^*(w)$ and is zero otherwise. In Section D an intertemporal decision problem was formulated that generated a search strategy as its solution. The purpose of this section is to characterize the search strategy that is optimal in the sense of that problem.

Our first task is to demonstrate that the worker's optimal acceptance strategy has the so called reservations property as assumed in Section C.

<u>Definition 2</u>: The optimal search strategy has the <u>reservation property</u> if and only if a unique reservation wage rate $w_i^*(w)$ exists such that $W_i^*(w) = \{\tilde{w} \in W | \tilde{w} > w_i^*(w)\}$ for every $(i,w) \in I \times W$.

Given the reservation property, the reservation wage is the lower bound of the set of acceptable wage rates in each participation state. Obviously, (21) implies that the reservation wage is the largest solution to

$$V_2(w_i^*(w)) = V_i(w) \tag{22}$$

where, of course, $w_1^*(w) = w_1^*$ for all w. To establish the reservation property one need only show that $V_2(w)$, expected utility of being employed at wage w, is non-decreasing in w. Below we prove a stronger result, one that also allows us to rank the reservation wage rates across participation states.

Let $g_i(w)$, $i = 1,2,3$, denote the maximal value of the sum of current utility $u(x,\ell)$ and the total return to search in participation state

$(i,w) \varepsilon I \times W$. It is the maximal value of the first two terms of (20a), (20b) and (20c) in each of the three cases. By virtue of the envelope theorem and (17), a function $g_i(w)$ and non-negative Kuhn-Tucker multipliers ($\lambda_i(w)$, $\eta_i(w)$, $\nu_i(w)$) exist such that

$$g_1 = \max_{(x,\ell,s)} [u(x,\ell) + \alpha s \sum_{\tilde{w} \varepsilon W_1^*} (V_2(\tilde{w}) - V_1) f(\tilde{w}) + \lambda_1 (y - x) + \eta_1 (1 - \ell - s) + \nu_1 s] \tag{23a}$$

$$\lambda_1 (y - x) = \eta_1 (1 - \ell - s) = \nu_1 s = 0 , \tag{23b}$$

$$g_2(w) = \max_{(x,\ell,s)} [u(x,\ell) + \alpha s \sum_{\tilde{w} \varepsilon W_2^*(w)} (V_2(\tilde{w}) - V_2(w)) f(\tilde{w}) + \lambda_2 (y + w(1-\ell-s) - x) + \eta_2 (1-\ell-s) + \nu_2 s] \tag{24a}$$

$$\lambda_2 (y + w(1 - \ell - s) - x) = \eta_2 (1 - \ell - s) = \nu_2 s = 0, \tag{24b}$$

and

$$g_3(w) = \max_{(x,\ell,s)} [u(x,\ell) + \alpha s \sum_{\tilde{w} \varepsilon W_3^*(w)} (V_2(\tilde{w}) - V_3(w)) f(\tilde{w}) + \lambda_3 (y - x) + \eta_3 (1 - \ell - s) + \nu_3 s] \tag{25a}$$

$$\lambda_3 (y - x) = \eta_3 (1 - \ell - s) = \nu_3 s = 0. \tag{25b}$$

The multiplier $\lambda_i(w)$ is the imputed utility value of "full income." $\eta_i(w)$ and $\nu_i(w)$ are the Kuhn-Tucker multipliers associated with the non-negativity constraints on work time and search time respectively.

Let $(x_i(w),\ \ell_i(w),\ s_i(w))$ denote the optimal choice of income, leisure time and search time given the participation state. In the sequel, we assume that the optimal choice is unique for each state and continuous in w for each i. A sufficient condition is that $u(x,\ell)$ be strictly concave. The assumption also implies that $g_i(w)$ and $V_i(w)$ are both differentiable almost everywhere with respect to w for each i.[14/] Let $g_i'(w)$ and $V_i'(w)$ denote the derivatives.

Of course $g_1'(w) = V_1'(w) = 0$ since g_1 and V_1 are constants. Equations (24a) and (25a) respectively imply

$$g_2'(w) = \lambda_2[1 - \ell_2 - s_2] - V_2'(w)\alpha s_2 \sum_{\tilde{w}\in W_2^*(w)} f(\tilde{w})$$

and

$$g_3'(w) = -V_3'(w)\alpha s_3 \sum_{\tilde{w}\in W_3^*(w)} f(\tilde{w}) \ .$$

By virtue of (20b) and (24a),

$$\rho V_2'(w) = g_2'(w) + \gamma[V_3'(w) - V_2'(w)] - \delta V_2'(w) .$$

Similarly, (20c) and (25a) imply

$$\rho V_3'(w) = g_3'(w) + \beta[V_2'(w) - V_3'(w)] - \delta V_3'(w) .$$

These four equations form a linear system in the four unknowns $g_2'(w)$, $g_3'(w)$, $V_2'(w)$ and $V_3'(w)$. The solution implies that

$$V_2'(w) \gtreqless V_3'(w) \gtreqless 0 \qquad \text{as} \qquad \lambda_2(w)[1 - \ell_2(w) - s_2(w)] \gtreqless 0 \ . \qquad (26)$$

Since $\lambda_2 = u_1(x,\ell)$ from (25a) where $u_1(.)$ is the partial of u(.) with respect to x, λ_2 is positive given non-satiation in income. Therefore, (26) has the following interpretation. The expected utility of being employed increases with the wage paid more rapidly than does the expected utility of being unemployed and subject to recall to a job paying the same wage if and only if the worker allocates time to work given that wage. Otherwise,

$V_2'(w) = V_3'(w) = 0$. Since $V_2(w) \geqq V_1$ and $V_3(w) \geqq V_1$ from (19), (26) also implies

$$V_2(w) = V_3(w) = V_1 \quad \forall \quad w \leqq w_1^* \tag{27a}$$

and

$$V_2(w) > V_3(w) > V_1 \quad \forall \quad w > w_1^* . \tag{27b}$$

Finally, because the equations of (20) imply that $V_2(w) = V_3(w) = V_1$ if the worker does not allocate time to work when employed at a wage w, we know that the supply of labor

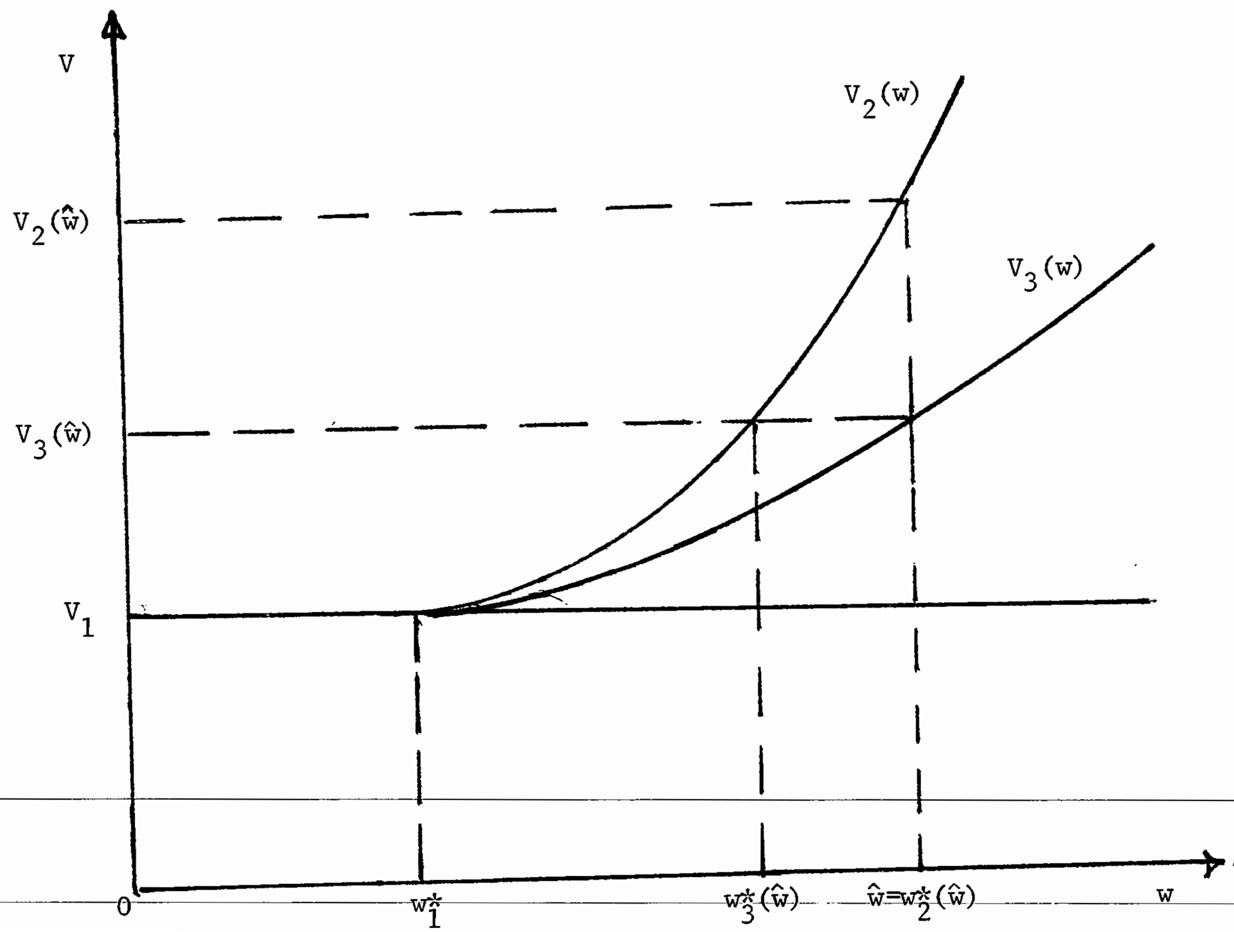
$$h_2(w) = 1 - \ell_2(w) - s_2(w) > 0 \quad \Leftrightarrow \quad w > w_1^* . \tag{28}$$

Hence, $V_2'(w) > V_3'(w) > 0$ if and only if w is acceptable by virtue of (26).

The implications of (22), (26), (27) and (28) for the relationships among w_1^*, w_2^* and w_3^* given any acceptable wage $\hat{w} > w_1^*$ are illustrated in Figure 2.

FIGURE 2:

RESERVATION WAGE RATES



Proposition 1: The optimal search strategy has the reservation property. Moreover, the reservation wage rates are such that $w_1^* = w_3^*(w) = w_2^*(w)$ for all $w \leqq w_1^*$ and $w_1^* < w_3^*(w) < w_2^*(w) = w$ otherwise. Finally, $w_3^*(w)$ increases with w.

Proof. See Figure 2.

Proposition 1 establishes the conjectures used in Section C, the analysis of a worker's employment history as a Markov process. Namely, the reservation wage of an unemployed worker increases with the wage paid on the job to which he may be recalled but that wage is always acceptable. Moreover, a job is acceptable given any state only if it exceeds the reservation wage given unemployment without job attachment. Finally, given employment, the set of acceptable jobs are those that pay a higher wage.

Next, we characterize the optimal allocation of time to search. The optimal solution to the time allocation problem $(x_i(w), \ell_i(w), s_i(w))$ satisfies the first order conditions that follow. Let

$$z_i = \begin{cases} w \text{ if } i = 2 \\ 0 \text{ otherwise} \end{cases}$$

by definition. Then for each state $(i,w) \in I \times W$

$$u_1(x_i,\ell_i) - \lambda_i = 0 \tag{29a}$$

$$u_2(x_i,\ell_i) - z_i\lambda_i - \eta_i = 0 \tag{29b}$$

$$\alpha \sum_{\tilde{w} > w_i^*(w)} [V_2(\tilde{w}) - V_i(w)]f(\tilde{w}) - z_i\lambda_i - \eta_i + \nu_i = 0 \tag{29c}$$

by virtue of (21), (23), (24), (25) and the reservation property. In addition,

$$\lambda_i[y + z_i(1 - \ell_i - s_i) - x_i] = 0 \tag{30a}$$

$$\eta_i[1 - \ell_i - s_i] = 0 \tag{30b}$$

$$\nu_i s_i = 0 \ . \tag{30c}$$

The partial derivatives $u_1(.)$ and $u_2(.)$ are respectively the marginal instantaneous utilities of income and leisure. The multipliers λ_i, η_i, ν_i are imputed marginal utilities associated with the budget constraint, the non-negativity constraint on work time and the non-negativity constraints on search time. The possibilities $x_i = 0$ or $\ell_i = 0$ are ruled out by assumption.

In every state, (29b) and (29c) reduce to

$$u_2(x_i, \ell_i) = \alpha \sum_{\tilde{w} > w_i^*(w)} [V_2(\tilde{w}) - V_i(w)] f(\tilde{w}) + \nu_i \ . \tag{31}$$

The left side is the marginal cost of search, the marginal utility of the leisure foregone. The first term on the right side is the expected gain in utility attributable to the marginal unit of search time. Since $\nu_i = 0$ if $s_i > 0$ from (30c), the investment in search is equal to the expected present value of the future stream of returns attributable to search. Moreover, no time is allocated to search if the investment required exceeds the return for all $0 < s_i < 1$.

The worker is said to be a <u>participant</u> if and only if he searches when unemployed without job attachment; i.e., $s_i > 0$. Consider a participant. Since $z_i = 0$, $(x_1, \eta_1) = (u_1(.), u_2(.)) > 0$ given non-satiation from (29a) and (29b).

Consequently, (30a) and (30b) imply $(x_1, \ell_1) = (y, 1 - s_1)$ and, therefore, (31) is

$$u_2(y, 1-s_1) = \alpha \sum_{\tilde{w}>w_1^*} [V_2(\tilde{w}) - V_1] f(\tilde{w}) \Leftrightarrow s_1 > 0 . \tag{32}$$

When unemployed but subject to recall, the same argument can be used to obtain

$$u_2(y, 1-s_3) = \alpha \sum_{\tilde{w}>w_3^*(w)} [V_2(\tilde{w}) - V_3(w)] f(\tilde{w}) \Leftrightarrow s_3(w) > 0 . \tag{33}$$

Equations (32) and (33) imply that a participant allocates less time to search when unemployed with job attachment then when unemployed without attachment.

Proposition 2: For a participant

$$s_1 > s_3(w) \quad \forall \quad w > w_1^* \quad . \tag{34}$$

and $s_3(w)$ decreases with w when $s_3(w) > 0$.[15/]

Proof. Since $w_3^*(w_1^*) = w_1^*$ and $V_3(w_1^*) = V_1$ from Proposition 1 and (27a), equations (32) and (33) imply that $s_3(w_1^*) = s_1$. Hence, (34) is implied by the second assertion . Because risk aversion implies diminishing marginal utility of leisure, the second statement is implied by (33) and the fact that $V_3'(w) > 0$ when $w > w_1^*$. Q.E.D.

When the worker is employed, (31) is

$$u_2(x_2, \ell_2) = \alpha \sum_{\tilde{w} > w} [V_2(\tilde{w}) - V_2(w)] f(\tilde{w}) \Leftrightarrow s_2(w) > 0 \quad . \tag{35}$$

Of course, $x_2 = y + w(1-\ell_2-s_2) > y$ and $\ell_2 < 1 - s_2$. The effect of the wage on the demand for leisure depends on the effect of income on the marginal utility of leisure because an employed worker has an opportunity to trade income for leisure.

In production theory, two inputs are said to be complements if an increase in one increases the marginal product of the other. This concept and the practice in labor economics of regarding real income (market goods) and leisure (home time) as inputs in a home production process motivate the following definition.

Definition 3: Income and leisure are complements in household production if and only if $u_{12}(x,\ell) = u_{21}(x,\ell) > 0$.[16/]

Given (35) and this definition, we have

Proposition 3: If income and leisure are complements in household production, then

$$s_3(w) \geqq s_2(w) \quad \forall \quad w > w_1^* \tag{36}$$

with strict equality holding when $s_3(w) > 0$.

Proof. The assertion is trivial when $s_2(w) = 0$. However, because (32) and (35) imply $s_2(w_1^*) = s_1$ given $V_1 = V_2(w_1^*)$, $s_2(w) > 0$ for some acceptable wage rates if the worker is a participant. Since $V_2(w) > V_3(w)$ and $w > w_3^*(w)$, $u_2(y, 1 - s_3) > u_2(x_2, \ell_2)$ when s_2 and s_3 are positive from (33) and (35). Hence, $u(y,1 - s_3) > u_2(x_2,\ell_2) > u_2(y,\ell_2) > u_2(y, 1 - s_2)$ given $u_{21}(\cdot) > 0$ and $u_{22}(\cdot) < 0$ because $x_2 > y$ and $\ell_2 < 1 - s_2$. Q.E.D.

By way of summation , we have shown that the possibility of being recalled to a former job is of value to the worker when unemployed. Specifically, the expected utility of being unemployed with job attachment as well as as the expected utility of being employed increases with the wage paid. As a consequence, the return to search in either state if less than when unemployed with no job attachment. This fact explains why a worker searches less and demands a higher wage in an alternative job when currently attached to a job than he does when unemployed with no job attachment.

The marginal return to search when employed is also less than the marginal return when unemployed but subject to recall to a job paying the same wage. Moreover, the marginal cost of search is larger as well when employed if income and leisure are complements in the sense of Definition 3. Hence, the conclusion of Proposition 3 holds even when income and leisure are substitutes in some cases.

Although later we establish that the demand for leisure increases with the wage given the hypothesis to Proposition 3, one cannot show that the time allocated to search declines everywhere with the wage earned even in this case. Because of the income effect of a wage increase, the time allocated to both income and leisure may increase with the wage in some range. Since $h = 1-s-\ell$, the labor supply curve is necessarily "backward bending" in any such range.

F. Participation and the Supply of Labor Time

In the received theory of labor supply, the decision to supply time to work depends only on the highest wage available in the market and the worker's non-labor income. That the supply of time may decrease with the wage rate in some range is the most interesting theoretical implications of this model. This possibility arises only when leisure is a normal good; i.e., its demand increases with non-labor income. In this case, the income effect of a wage increase can offset the substitution effect as every student of labor economics knows. The evidence suggests that a "backward bending" labor supply curve is an empirically relevant case as well as a theoretical possibility.[17/] Moreover, that the supply of labor and indeed participation rates decline with non-labor income in virtually all empirical studies confirm the needed hypothesis.

More recent elaborations of the model include other prices as determinants of labor supply. Possibly the most important of these is the multi-person extension in which the wage rates of all members of a household enter as determinants of the time supplied by each.[18/] Still, as an application of the neo-classical theory of household demand, the emphasis of the theory ~~and the empirical work based on it is on substitution and income effects~~ of prices and wage rates.

However, students of labor force participation have long recognized the importance of measures of labor market conditions in their efforts to explain participation rates. That participation rates decline, <u>ceteris paribus</u>, with measures of unemployment is attributed to the so called "discouraged worker" hypothesis. The idea is that participation is less likely when jobs are hard to find. The discouraged worker effect is not explained by the received neo-classical theory of supply simply because this theory abstracts from the market imperfections that are responsible for discouragement. Specifically, the highest paying job can be found instantaneously at no cost. The model developed in this paper is an extension that takes this imperfection into accounts. As such, it is a natural vehicle on which to perform an analysis of the effects of labor market conditions on both the decision to participate and the decision to allocate time to work.

This purpose is achieved with a greater clarity in exposition if we restrict our analysis to a simpler version of the general model outlined above - one in which temporary layoff states are excluded. Formally, the simplification is obtained by setting the temporary layoff rate, γ, equal to zero. In this version of the model, then, all turnover is permanent. Although the results reported in this section are valid without the restriction, the arguments needed in the proofs are more complex and confusing.

In general, the state of unemployment without job attachment is equivalent to employment at an unacceptable wage by virtue of (27) and (28). Consequently, in the simplified version of the model we have no need for the subscript i. Instead, we simply let $V(w) = V_1$ when $w \leqq w^*$ and $V(w) = V_2(w)$ when $w > w^*$, where w^* is the reservation wage given unemployment. Then (20), (23) and ~~(24) imply~~

$$\rho V(w) = g(w) + \delta[V(w^*) - V(w)] \;\forall\; w \geqq 0$$

where

$$g(w) = \max_{(x,\ell,s)\,\epsilon\,\Gamma(w,y)} [u(x,\ell) + \alpha s \sum_{\tilde{w} > w} [V(\tilde{w}) - V(w)]f(\tilde{w})] .$$

given $\gamma = 0$. Note that these two equations can be rewritten as

$$V(w,.) = \begin{cases} \dfrac{g(w,.)}{\rho+\delta} + \dfrac{\delta}{\rho+\delta} V(w^*,.) & \text{if } w > w^* \\ \dfrac{g(w^*,.)}{\rho} & \text{if } w \leqq w^* , \end{cases} \tag{37a}$$

$$g(w,.) = \max_{(x,\ell,s)\,\epsilon\,\Gamma(w,y)} [u(x,\ell) + s\,r(w,.)] \tag{37b}$$

and

$$r(w,.) = \frac{\alpha}{\rho + \delta} \sum_{\tilde{w} > w} [g(\tilde{w},.) - g(w,.)]f(\tilde{w},.) \tag{37c}$$

by definition.

The equations of (37) all have insightful interpretations. First, g(.) is the indirect utility flow attributable to the optimal solution to the worker's current time allocation problem. It is the intertemporal generalization of the indirect utility function of neo-classical demand theory. Second, the function r(.) is the expected marginal gain in the discounted future utility flow attributable to current search given that time is allocated optimally in the future. The discount rate applied is the sum $\rho + \delta$ because the return to any current investment made in an attempt to find a higher paying job in the present "depreciates" at the expected rate at which the worker will be separated from that job, δ, in the future. Finally, the expected utility of being unemployed, $V(w^*)$, equals the expected present value of the indirect expected utility flow associated with searching for an acceptable job. These are all functions of the wage earned if employed, of non-labor income, and of parameters that characterize the conditions in the labor market faced by the individual worker.

The difficulty of locating a job and the expected duration of any job are reflected in the value of $\theta = \alpha/(\rho + \delta)$. Specifically, θ increases with the offer arrival rate α and decreases with the separation rate δ for a given worker. Hence, an increase in θ, holding the distribution of wage offers constant, reflects an improvement in the worker's labor market.

Let k be a parameter characterizing an improvement in the probability distribution on possible wage offers. As a means of formalizing the notion of an improvement, we assume that the probability of receiving a higher wage offer increases with k; i.e.,

$$\frac{\partial F(w,k)}{\partial k} < 0$$

for all values of w such that $F(w,k) < 1$. Given this definition of k, one can show that

$$\sum_{\tilde{w} > w} \Phi(\tilde{w}) \frac{\partial f(\tilde{w},k)}{\partial k} > 0 \text{ if } F(w,k) < 1$$

where $\Phi(\tilde{w})$ is any increasing function. Hence, an increase in k is an improvement in the wage offer distribution is the sense that an increase induces an increase in $EV(\tilde{w})$, the expected utility of being employed at a randomly sampled wage.

The functional dependence of g(.) and r(.) on the parameters (w,y,θ,k) is implicitly defined by the following rewritten versions of (37b) and (37c):

$$g(w,y,\theta,k) = \max_{(x,\ell,s) \in \Gamma(w,y)} [u(x,\ell) + sr(w,y,\theta,k)] \quad (38a)$$

$$r(w,y,\theta,k) = \theta \sum_{\tilde{w} > w} [g(\tilde{w},y,\theta,k) - g(w,y,\theta,k)] f(\tilde{w},k) . \quad (38b)$$

It is of interest to note that the separation rate, δ, influences behavior only through its affect on θ. Given the assumption that the worker is risk averse in income and leisure in the sense of Definition 1, the optimal solution to the problem defined on the right side of (38a) is determined by the following first order conditions:

$$u_1(x,\ell) - \lambda = 0 \quad (39a)$$

$$u_2(x,\ell) - w\lambda - \eta = 0 \quad (39b)$$

$$r(w,y,\theta,k) - w\lambda - \eta + \nu = 0. \quad (39c)$$

Of course, $(\lambda,\eta,\nu) \geqq 0$,

$$\lambda(y + w(1 - \ell - s) - x) = 0 \quad (40a)$$

$$\eta(1 - \ell - s) = 0 \quad (40b)$$

$$\nu s = 0 \quad (40c)$$

and $w \leqq w^*$ when the worker is unemployed.

We begin the analysis with the familiar case in which the worker is employed but not searching; i.e., $\eta = 0$ and $\nu > 0$. Since $\lambda > 0$, the demand for income and leisure satisfy

$$u_1(x,\ell) - \lambda = 0 \tag{41a}$$

$$u_2(x,\ell) - w\lambda = 0 \tag{41b}$$

$$y + w(1 - \ell) - x = 0. \tag{41c}$$

In words, the marginal rate at which the worker is willing to substitute income for leisure, $u_2(x,\ell)/u_1(x,\ell)$, is equal to the wage. Given this condition, the optimal income-leisure combination is determined by the budget constraint.

When no search is optimal, the demands for income and for leisure (x°, ℓ°) and the imputed value of full income λ° depend only on the wage and on non-labor income. The following Slutsky equations are derived by applying Cramer's rule to the differential form of (41):

$$\frac{\partial x^\circ}{\partial w} = w\lambda^\circ/J_\circ + (1-\ell^\circ)\frac{\partial x^\circ}{\partial y} \tag{42a}$$

$$\frac{\partial x^\circ}{\partial y} = (wu_{12} - u_{22})/J_\circ \tag{42b}$$

$$\frac{\partial \ell^\circ}{\partial w} = -\lambda^\circ/J_\circ + (1 - \ell^\circ)\frac{\partial \ell^\circ}{\partial y} \tag{43a}$$

$$\frac{\partial \ell^\circ}{\partial y} = (u_{21} - wu_{11})/J_\circ \tag{43b}$$

$$\frac{\partial \lambda^\circ}{\partial w} = -\lambda^\circ\frac{\partial \ell^\circ}{\partial y} + (1 - \ell^\circ)\frac{\partial \lambda^\circ}{\partial y} \tag{44a}$$

$$\frac{\partial \lambda^\circ}{\partial y} = (u_{12}u_{21} - u_{11}u_{22})/J_\circ \quad . \tag{44b}$$

The assumption of risk aversion in income and leisure imply that the Jacobian of the system (41) is

$$J_o = -[u_{22} + w^2 u_{11} - w(u_{12} + u_{21})] > 0$$

and that $u_{11} u_{22} - u_{12}u_{21} > 0$.

In the theory of demand, a good is said to be normal if its demand increases with the size of the budget. In our model "expenditures" on income, leisure and search are limited by "full income", y + w, by virtue of (17a). Hence, we have the following

Definition 4: Income, leisure or search time is a normal good if and only if its demand increases with non-labor income.

An increases in the wage, of course, has other than "full income" effects on the demands because the wage is the "market price" of both leisure time and search time as well as the value of the worker's time endowment.

Proposition 4: When an employed worker does not search, then (a) an increase in the wage induces an increase in the demand for income if income is normal and (b) income and leisure are both normal if they are complements in household production.[19/]

Proof: Assertion (a) is implied by the equations of (42) and Definition 4. Since risk aversion implies $u_{11} < 0$ and $u_{22} < 0$ from Definition 1 and income and leisure are complements if and only if $u_{12} = u_{21} > 0$ from Definition 3, (b) follows from (42b) and (43b).

The possibility of a "backward bending" labor supply curve is reflected in (43a). When leisure is normal, the income and substitution effects are of opposite sign except at the corner $\ell^o = 0$. Furthermore, if leisure is normal, the imputed marginal utility of "full income" $\lambda^o(w,y)$ is a decreasing function of the wage given risk aversion by virtue of (44a). This fact plays a role in showing that the return to search decreases with non-labor income in the sequel.

In the received labor supply theory, just outlined above, an individual participates in the labor market if and only if the highest paying job available exceeds the marginal rate at which he is willing to substitute income for leisure given $(x,\ell) = (y,1)$. In other words, the wage must exceed,

$$w_o = u_2(y,1)/u_1(y,1). \tag{45}$$

One can easily show that w_o increases with non-labor income if and only if leisure is normal. As a consequence, the hypothesis that leisure is a normal good is the accepted explanation for the negative empirical associate between participation rates and non-labor incomes.

When time is required to find a job and the location of the job paying each possible wage is uncertain, the existence of a wage offer in excess of w_o is not sufficient. The theory of participation under these conditions is based on the following definition.

Definition 5: An individual participates in the labor market if and only if he searches for a job while unemployed.

In otherwords, $s^o > 0$ while unemployed must be preferred to specialization in leisure. The theory based on this definition provides a somewhat different explanations for the observed relationship between participation rates and non-labor income.

The state of unemployment is formally equivalent to the state of employment at a wage less than or equal to the reservation wage, w^*, as already noted. Because $\ell^o + s^o = 1$ when $w = w^*$, the equations of (39) and (40) imply that the demand for search time by a participant satisfies

$$u_2(y,\ 1 - s^o) = r(w^*,y,\theta,k) \quad . \tag{46}$$

Moreover, the worker is not a participant, $\nu^o > 0$, if the marginal cost of search, $u_2(y.1)$, exceed the marginal return evaluated at the worker's reservation wage, $r(w^*, y, \theta, k)$.

The fraction of each period of unemployment spent searching is obviously a measure of the degree to which a worker participates. The dependence of this measure on non-labor income and on labor market conditions as reflected in the values of θ and k is derived from the dependence of the marginal return to search on these parameters. For the purpose of deriving these effects, we represent (38a) in the following form implied by the envelope theorem:

$$g(w,y,\theta,k) = \max_{(x,\ell,s)} [u(x,\ell) + sr(w,y,\theta,k) + \lambda(y + w(1 - \ell - s) - x) + \eta(1 - \ell - s) + \nu s] \quad (47)$$

Given the first order conditions for the general case (39), the partial derivatives of g(.) are representable as follows by virtue of (47):

$$g_w(.) = \lambda^o(1 - \ell^o - s^o) + s^o r_w(.) \quad (48a)$$

$$g_y(.) = \lambda^o + s^o r_y(.) \quad (48b)$$

$$g_\theta(.) = s^o r_\theta(.) \quad (48c)$$

$$g_k(.) = s^o r_k(.) . \quad (48d)$$

Note that if no search is optional, the indirect utility function is independent of the market condition parameters k and θ. Indeed, g(.) reduces to the indirect utility function in the standard neo-classical case. By virtue of (38b) we have

$$r_w(.) = -\theta g_w(.)[1 - F(w)] \quad (49a)$$

$$r_y(.) = \theta \sum_{\tilde{w} > w} [g_y(\tilde{w},.) - g_y(w,.)]f(\tilde{w},k) \quad (49b)$$

$$r_\theta(.) = r(.)/\theta + \sum_{\tilde{w} > w} [g_\theta(\tilde{w},.) - g_\theta(w,.)]f(\tilde{w},k) \quad (49c)$$

$$r_k(.) = \theta \sum_{\tilde{w} > w} [g(\tilde{w},.) - g(w,.)] \frac{\partial f(\tilde{w},k)}{\partial k} \quad (49d)$$

The equation of (48) and (49) are used to establish the following characterization of the marginal return to search function:

Proposition 5: At the highest possible wage w_1, the largest solution to $F(w,k) = 1, r(w_1,y,\theta,k) = 0$. For all $w < w_1$, (a) $r_w(.) < 0$ if $w > w*$ and $r_w(.) = 0$ if $w \leqq w*$, (b) $r_y(.) < 0$ if leisure is normal when employed but not searching, (c) $r_\theta(.) > 0$, and (d) $r_k(.) > 0$.

Proof. That $r(w_1,.) = 0$ is implied by the definition (38b). Property (a) is implied by (48a), (49a) and the fact that the worker supplies time to work if and only if the wage is acceptable. Formally,

$$r_w(.) = -\theta[1 - F(w,k)]\lambda^\circ(1 - \ell^\circ - s^\circ)/(1 + s^\circ\theta[1 - F(w,k)]) < 0 \Leftrightarrow w > w* \quad (50a)$$

given $w < w_1$.

Equations (48b) and (49b) in combination yield

$$(1 + \theta s^\circ(w,.)[1 - F(w,k)])\, r_y(w,.) \quad (50b)$$

$$= \theta \sum_{w > \tilde{w}} [\lambda^\circ(\tilde{w},.) - \lambda^\circ(w,.)]f(\tilde{w},k) + \theta \sum_{w > \tilde{w}} s^\circ(\tilde{w},.)\, r_y(\tilde{w},.)f(\tilde{w}).$$

The first term on the right is negative if $\lambda^\circ(.)$ is decreasing in the wage rate. Under the hypothesis, the equations of (44) imply that this condition holds when the worker is employed but not searching. When the worker does allocate time to search while employed, $\lambda^\circ(w,.) = r(w,.)/w$ from (39b) and ~~(39c) because $\nu^\circ = 0$. Hence, property (a) implies that $\lambda^\circ(.)$ is decreasing in~~ w when searching. Since $r(w_1,.) = 0$ and $u_2(.) > 0$, (39c) and (40c) imply $s^\circ(w_1,.) = 0$. In otherwords, when employed at his highest possible wage, w_1, the worker does not search. Hence, (50b) implies $r_y(w_2,.) < 0$ where w_2 is the second highest possible wage offer. Property (b) then follows by induction for all $w \in W = \{w_1, w_2, \ldots, w_m\}$.

By virtue of (48c) and (49c),

$$(1 + s^{\circ}(w,.)\ [1 - F(w,k)])r_{\theta} \qquad \text{(50c)}$$
$$= r(w,.)/\theta + \sum_{\tilde{w} > w} s^{\circ}(\tilde{w},.)\ r_{\theta}(\tilde{w},.)f(\tilde{w},k).$$

Since $r(w,.)/\theta > 0$ for all $w < w_1$ and $s^{\circ}(w_1,.) = 0$, an analogous proof by induction implies (c). Finally, (d) is implied by the fact that $g(\tilde{w},.) - g(w,.)$ is an increasing function of $\tilde{w}$ and by the definition of the shift parameter k given (49d). Q.E.D.

The following result is implied by (46), Proposition 4 and Proposition 5.

<u>Proposition</u> <u>6</u>: When unemployed, demand for search time increases with θ and k in general. It decreases with non-labor income if income and leisure are complements in household production.

The somewhat stronger condition is needed to explain the observed relationship between participation rates and non-labor income because non-labor income generally affects the marginal cost of search, $u_2(y,1)$, as well as the marginal return. However, the theory does explain discouraged worker effects. Specifically, reductions in the expected rate at which offers are received per unit of search time α, in the expected job duration $1/\delta$, and in the probability of ~~finding a high wage offer k all "discourage" participation as measured by~~ the time allocated to search while unemployed.

If the worker allocates time to both work and search, then $\eta = \nu = 0$ from (40b) and (40c). Consequently, a complete differentiation of (39) yields the partial derivatives of (x,ℓ,s) with respect to (w,y,r) that follow. Although the marginal return to search is also a function of the wage and non-labor income, these structural relationships are useful for expository purposes.

$$\frac{\partial x}{\partial w} = w\lambda^{o}u_{22}/J \tag{51a}$$

$$\frac{\partial x}{\partial y} = 0 \tag{51b}$$

$$\frac{\partial x}{\partial r} = w[wu_{12} - u_{22}]/J \tag{51c}$$

$$\frac{\partial \ell}{\partial w} = -w\lambda^{o}u_{11}/J + w\lambda^{o}[wu_{11} - u_{21}]/J \tag{52a}$$

$$\frac{\partial \ell}{\partial y} = 0 \tag{52b}$$

$$\frac{\partial \ell}{\partial r} = w[u_{21} - wu_{11}]/J \tag{52c}$$

$$\frac{\partial s}{\partial w} = \lambda^{o}J_{o}/J + w\lambda^{o}[wu_{11} - u_{12}]/J + (1 - \ell^{o} - s^{o})/w \tag{53a}$$

$$\frac{\partial s}{\partial y} = 1/w \tag{53b}$$

$$\frac{\partial s}{\partial r} = -J_{o}/J \tag{53c}$$

where

$$J = -w^{2}[u_{11}u_{22} - u_{12}u_{21}] < 0$$

is the Jacobian of the system (39) and $J_o = -[u_{22} + w^2u_{11} - w(u_{12}+u_{21})] > 0$.

The most important difference between these results and those for the standard case, (42) - (44), is the absence of "full income" effects on the demand for income and leisure except through its effect on the marginal return to search time. These are absent because $u_1(.) = \lambda = r/w$ and $u_2(.) = r$ when the worker searches while employed.

Holding the marginal return constant, the right side of (51a) is the cross substitution effect of a real wage increase on the demand for income which is positive. The two terms on the right side of (52b) are first the own substitution effect, which is negative, and second the cross substitution effect of an increase in the price of search time. This interpretation arises as a consequence of the fact that the wage is the market price of both search time and leisure time when the worker is searching and working. Since the sum of the two terms is $-w\lambda^{\circ}u_{21}/J$, the sign of the total effect is positive if and only if income and leisure are complements in household production.

Because the demand for search time increases with income from (53b), the income effect of an increase in the market value of the worker's time endowment, the last term on the right side of (53c), is positive. The first two terms on the right of (53b) are the own substitution effect and the cross substitution effect of an increase in the price of leisure in order. Because the sum of the own and cross substitution effects, $\lambda^{\circ}(wu_{21} - u_{22})/J$, is negative if income and leisure are complements, the total effect of a wage increase on search time is ambiguous even holding the return search constant.

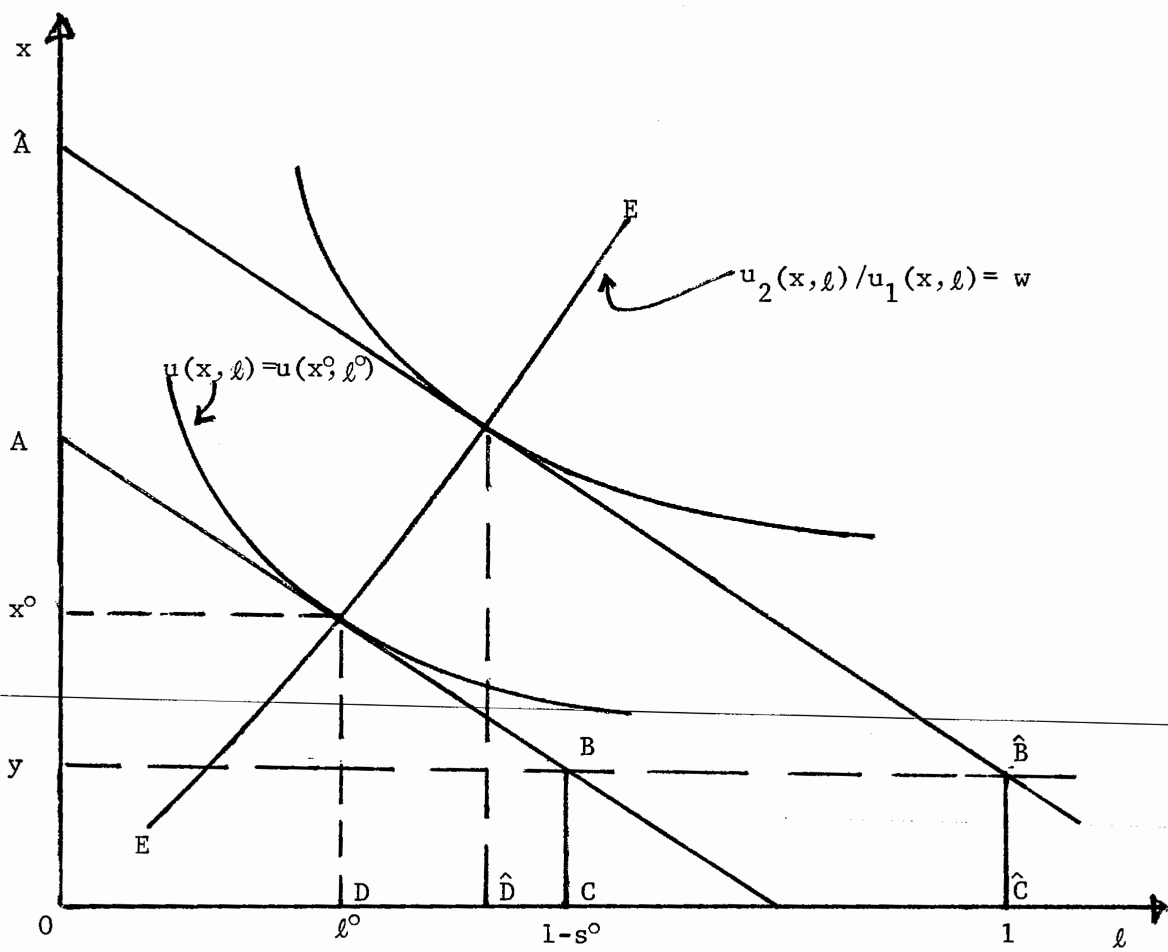
The effect of an increase in the marginal return to search on either the demand for income (51c) or the demand for leisure (52c) is identical to that of a reduction in non-labor income in the no search case. Of course, ceteris paribus, the time allocated to search increases with its marginal return by virtue of (53c). That the first result is implied by the second is illustrated in Figure 3.

~~The determinants of the optional income -- leisure combination~~ $(x^{\circ}, \ell^{\circ})$ given that it is optional to allocate the fraction s° of the current period to search is illustrated in Figure 3. Feasible combinations are those that satisfy the budget constraint, $x \leqq y + w(1 - \ell - s^{\circ})$, the time constraint $\ell \leqq 1 - s^{\circ}$ and the non-negativity condition $(x, \ell) \geqq 0$. The feasible choices, given s°, are then the combinations enclosed by the figure labelled OABC, where the segment AB has a slope equal to $-w$. The optimal combination

$(x^{\circ}, \ell^{\circ})$ lies at a point of tangency of this segment to the highest possible indifference curve. Note that the worker could achieve a higher current utility flow by setting $s^{\circ} = 1$. As $s^{\circ} > 0$ is optimal, the gain in future utility attributable to that search compensates for the opportunity loss in the realized current utility flow.

FIGURE 3:

OPTIMAL TIME ALLOCATION



Let EE represent the Engel's curve through the optimal combination; the locus of points such that the marginal rate of substitution equals the given wage rate. In the figure the Engel's curve is drawn with a positive slope, which is the case if both income and leisure are normal. In the no search case, $s^{\circ} = 0$, a small increase in non-labor income, holding the wage constant, shifts the segment AB upward but does not change its slope. In response, the optimal conbination $(x^{\circ}, \ell^{\circ})$ moves up along the Engel's curve in the familiar manner.

However, if $s^{\circ} > 0$, an increase in non-labor income, holding the wage and marginal return to search constant, only induces an increase in the time allocated to search. Geometrically, the line segment AB shortens as the point B moves toward A by virtue of (51b), (52b) and (53b). However, a decrease in the marginal return to search, holding y and w constant, decreases s°. Hence, AB shifts up with no change in slope and $(x^{\circ}, \ell^{\circ})$ moves up along the Engel's curve as in the case of an increase in non-labor given no search.

The reduced form comparative static results follow. They are implied by Proposition 5 and the equations of (51), (52) and (53).

Proposition 7: When an employed worker searches, then (a) a wage increase induces an increase in the demand for income if income and leisure are both normal and an increase in the demand for leisure if income and leisure are complements in household production, (b) income and leisure are normal if they are complements in household production, (c) the demands for both income and leisure decrease with θ and with k if they are both normal.

Proof. Given (51) and (52),

$$\frac{\partial x^{\circ}}{\partial y} = \frac{\partial x}{\partial y} + \frac{\partial x}{\partial r} r_y(.) = w\, r_y(.)(wu_{12} - u_{22})/J \tag{54a}$$

and

$$\frac{\partial \ell^\circ}{\partial y} = \frac{\partial \ell}{\partial y} + \frac{\partial \ell}{\partial r} r_y(.) = wr_y(.)(u_{21}-wu_{11})/J. \tag{54b}$$

Risk aversion implies $u_{11} < 0$, $u_{22} < 0$ and $J < 0$, and income and leisure are complements if and only if $u_{12} = u_{21} > 0$. The hypothesis, Proposition 4(b), and Proposition 5(b) imply $r_y(.) < 0$. Hence (b) holds.

The equations of (51) and (52) also imply

$$\frac{\partial x^\circ}{\partial w} = \frac{\partial x}{\partial w} + \frac{\partial x}{\partial r} r_w(.) = w\lambda^\circ u_{22}/J + wr_w(.)(wu_{12} - u_{22})/J \tag{55a}$$

and

$$\frac{\partial \ell^\circ}{\partial w} = \frac{\partial \ell}{\partial w} + \frac{\partial \ell}{\partial r} r_w(.) = -w\lambda^\circ u_{12}/J + wr_w(.)(u_{21} - wu_{11})/J . \tag{55b}$$

Since $r_w(.) < 0$ from Proposition 5(a) and since $u_{22} < 0$ and $J < 0$ from risk aversion, both terms of (55a) are positive if $(wu_{12} - u_{22}) > 0$. Since leisure normal implies $r_y(.) < 0$, income normal as well is sufficient given (54a). Finally, because $u_{22} < 0$ as well from risk aversion and $u_{12} = u_{21} > 0$ when complements, both terms of (55b) are positive under the hypothesis. Assertion (c) is implied by (51c) and (52c) given Propositions 5(c) and 5(d) and the equations of (54). Q.E.D.

We conclude the section by considering the implications of the analysis for the supply of labor time. By definition, the supply function is

$$h^\circ(w,.) = 1 - \ell^\circ(w,.) - s^\circ(w,.) = (x^\circ(w,.) - y)/w. \tag{56}$$

Consequently, the supply of time to work depends on the state of the worker's labor market as well as on the wage earned and on non-labor income in general.

Since $h^\circ(w,.) > 0$ if and only if $w > w^*$, continuity implies $h^\circ(w^*,0) = 0$. Equivalently,

$$w^* = u_2(y,1 - s^*)/u_1(y,1 - s^*) \tag{57}$$

where s^* is the fraction of any interval spent searching when unemployed; i.e. , the solution to equation (46). Since $s^* > 0$ by definition in the case of a participant, it follows that any participant searches while employed when earning a wage near his reservation wage. As a corollary to Proposition 5 (see the proof) we also know that no worker search when employed at a wage equal or close to the maximum wage offer available w_1. Hence, the interval of wage rates at which the worker could be employed and would accept employment $(w^*, w_1]$ can be partitioned into two subsets. On the first, which includes a neighborhood of w^*, the worker searches while employed. On the second, which includes a neighborhood of w_1, the worker does not search. Unfortunately, one cannot establish that each of these two subsets is connected because the response in the demand for search time to an increase in the wage is ambiguous due to the full income effect of an increase in the wage. However, as a corollary to Proposition 7(a), it does follow that the demand for search time decrease with the wage if the supply of labor time increase with the wage when income and leisure are complements in household production. In this case, a critical wage rate $w^* < \hat{w} < w$, exists such that the worker searches when $w \in (w^*, \hat{w})$ and does not search when $w \in [\hat{w}, w_1]$.[20/]

It is of interest to note that the intercept of the general supply curve w^* is stricly greater than w_o , the marginal rate at which the worker is willing to substitute income for labor at $(x, \ell) = (y, 1)$, when income and labor are both normal. With reference to Figure 3, w^*, the absolute value of the slope of the indifference curve through the point B, exceeds the absolute value of the slope at $\hat{B}$ because the Engel's curve through either is upward sloping. Indeed, the Figure also implies that the supply of labor time given no search, the line segment $\hat{D}\hat{C}$, exceeds the supply of labor given that some search is optional, the line segment DC, at every wage when EE has a positive slope. In otherwords,

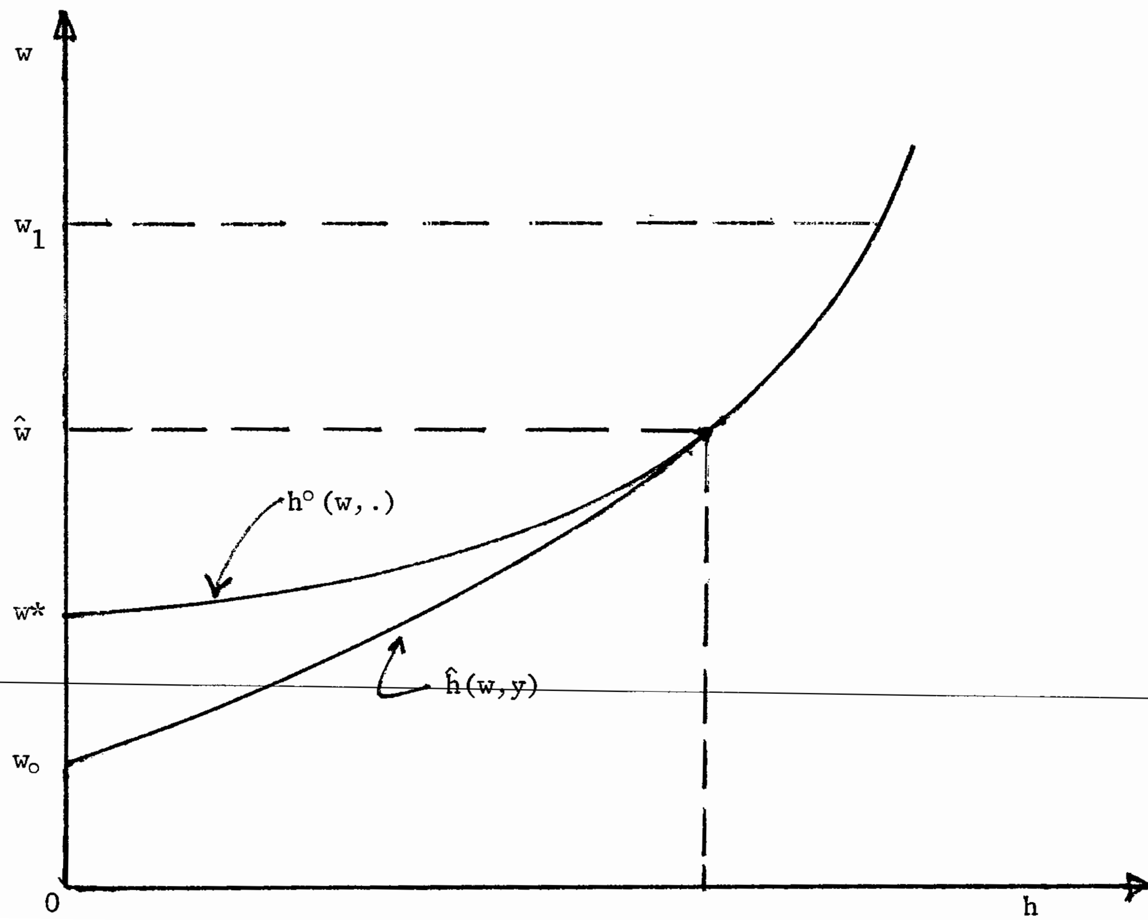
$$h^{\circ}(w,.) \begin{cases} = \hat{h}(w,y) \text{ if } s^{\circ}(w,.) = 0 \\ < \hat{h}(w,y) \text{ if } s^{\circ}(w,.) > 0 \end{cases} \quad (58)$$

where $\hat{h}(w,y)$ is the labor supply function in the received theory. Because $\hat{h}(w_o,y) = 0, h^{\circ}(w^*, .) = 0$, and $s^{\circ}(w^*,.) > 0$ and because both $\hat{h}$ and h° increase with w at their intercepts (there are no income effects when h = 0), $w^* > w_o$ is implied by (58).

The inequality (58) is represented in Figure 4 for the case in which the set of wage rates at which the worker searches is connected, although the comments that follow do not depend on this representation. One interpretation of (58) is based on the fact that the time spent working is equal to $\hat{h}(w,y)$ when the wage offer distribution is degenerate, i.e., the set of wage offers includes only the wage w. The worker allocates generally less time to work at each wage rate when the distribution of offers is disperse than when it is degenerate. The reason is clearly illustrated in Figure 3. The worker spends time looking for a higher wage offer in general when one exists and no time when one does not. Moreover, the additional leisure taken when the wage offer is degenerate is less than the time spent searching when it is not if income and leisure are both normal.

FIGURE 4:

LABOR SUPPLY



By virtue of the definition (56),

$$\frac{\partial h^o}{\partial w} = \frac{1}{w} \left[\frac{\partial x^o}{\partial w} - h^o\right] \tag{59a}$$

$$\frac{\partial h^o}{\partial y} = \frac{1}{w} \left[\frac{\partial x^o}{\partial y} - 1\right] \tag{59b}$$

$$\frac{\partial h^o}{\partial \theta} = \frac{1}{w} \frac{\partial x^o}{\partial \theta} = \frac{1}{w} \frac{\partial x}{\partial r} r_\theta(.) \tag{59c}$$

$$\frac{\partial h^o}{\partial k} = \frac{1}{w} \frac{\partial x^o}{\partial k} = \frac{1}{w} \frac{\partial x}{\partial r} r_k(.) \; . \tag{59d}$$

When no search is optional, the response to a wage increase is ambiguous, the response to an increase in non-labor income is negative if leisure is normal and there is no response to a change in either market condition parameter.

When the worker is searching, the sign of the reponse to a wage increase is ambiguous although (55a), risk aversion, $r_w(w^*,.) = 0$ and $h^o(w^*,.) = 0$ imply $\frac{\partial h^o}{\partial w} > 0$ at $w = w^*$. In otherwords, the supply curve is upward sloping at its intercept in general as drawn in Figure 4. Especially when income and leisure are both normal, it need not be the case that the marginal propensity to consume from non-labor income, $\partial x^o/\partial y$, is less than unity by virtue of (54a). Because the return to search is decreasing in non-labor income when leisure is normal, an increase can decrease the time spent searching. In this case, the time freed will be reallocated to both work and leisure if income is normal. If the decrease in search time is large enough, the supply of labor time increases. Figure 3 illustrates the point given an increase in y big enough to induce the worker to stop searching. Nevertheless, empirical evidence suggests that $\partial x^o/\partial y < 1$ is the relevant case.

Finally, because the return to search increase with both k and θ, the equations of (51) imply that the supply of labor decreases given an improvement of either type in the worker's labor market if income and leisure are both normal. In other words, the supply curve in Figure 4 shifts to the left as w^* and $\hat{w}$ both increase given an increase in either the rate at which offers arrive per unit of search, job duration, or the probability of obtaining higher wage rates. The worker invests income as well as leisure in search when his labor market improves. Of course, the investment is made in the hope of spending more time in the future working at a higher wage.

We conclude our discussion of the supply of labor time with the following corollary of the results just presented.

<u>Proposition</u> <u>8</u>: In the case of a participant, the reservation wage w^* is such that

$$w^* > w_o \tag{60a}$$

$$\frac{\partial w^*}{\partial y} > 0 \Leftrightarrow \frac{\partial x^o}{\partial y} < 1 \tag{60b}$$

$$\frac{\partial w^*}{\partial \theta} > 0 \tag{60c}$$

$$\frac{\partial w^*}{\partial k} > 0 \tag{60d}$$

if income and leisure are normal.

Proof. The results are implied by $h^o(w^*,.) = 0$ and the properties of $h^o(w,.)$ at $w = w^*$.

The reader may also wish to verify that $w^* \to w_1$ as $\alpha \to \infty$ in general. Hence, the received theory which implies that a worker participates if and only if $w_1 > w_o$ is simply the special case in which the highest wage offer can be located instantaneously. In this extension, both the participation and the labor supply decisions depend on the parameters characterizing the worker's labor market because the return to search time is finite.

G. A Two Person Model

The purpose of this section is to preview the implications of our formulation when the household is composed of more than one person. The effects of one member's wage on the others' supply of labor time has been studied in the literature.[21/] However, there is no study that investigates the dependence of one person's optimal search strategy on the employment status of other household members. In this section we take a step toward filling that gap.

Consider a two person household that acts as a single decision unit. The household's utility function is defined as in (16). However, now x is interpreted as the household income and ℓ is a vector (ℓ_1, ℓ_2) where ℓ_1 represents the fraction of the time period devoted to leisure by member #1, and ℓ_2 is the fraction devoted to leisure by member #2. In an analogous manner $s = (s_1, s_2)$ and $w = (w_1, w_2)$ where s_i is the fraction of the period devoted to search by member i and w_i is the wage earned by member i when employed. Let $\Gamma(w,y)$ denote the set of possible choices of (x,ℓ,s) open to the household given a wage vector w and a household non-labor income flow y. It is the subset of R^5_+ such that

$$x + w_1\ell_1 + w_2\ell_2 + w_1 s_1 + w_2 s_2 \leqq y + w_1 + w_2 \qquad (61a)$$

$$\ell_1 + s_1 \leqq 1 \qquad (61b)$$

$$\ell_2 + s_2 \leqq 1 \ . \qquad (61c)$$

The participation states for each member are essentially the same as those described in the one person household case. However for simplicity we rule out temporary layoffs as in Section F. Let α_i denote the expected rate at which member i receives offers per unit of search, δ_i represent the expected rate at which he or she separates from any job and $f_i(w_i)$ be the probability that an offer received is equal to w_i. By extending the state space to include the employment status of both members, one can again represent the household's labor market history as the realization of a Markov process.

The probability that member i will receive an offer w_i during the short time interval $[t, t + \Delta t)$ is $\alpha_i s_i f(w_i)\Delta t + 0(\Delta t)$. Given employment, member i is separated with probability $\delta_i \Delta t + 0(\Delta t)$. Note that the probability that both members receive offers, equal say to w_1 and w_2 respectively, is

$$[\alpha s_1 f(w_1)\Delta t + 0(\Delta t)][\alpha_2 s_2 f(w_2)\Delta t + 0(\Delta t)] = 0(\Delta t).$$

Given both employed, the probability that both lose their jobs during a short interval of time equal in length to Δt is also of order $0(\Delta t)$. Similarly, all other joint events have negligible probability when Δt is small.

We assume that the household's choice of strategy maximized the mathematical expectation of U at each date. Let $V(w_1, w_2, t)$ denote the household's expected lifetime utility given an optimal strategy when the two members are both employed at wage rates (w_1, w_2) during the short time future interval $[t, t+\Delta t)$. Because the constraint set is $\Gamma(w_1, w_2, y)$ with $w_i = 0$ when member i is unemployed during the interval $[t, t + \Delta t)$, $V(0,0,t)$ is the expected utility when both are unemployed, $V(0, w_2, t)$ is the expected utility when member #1 is unemployed and $V(w_1, 0, t)$ is the expected utility when member #2 is unemployed. Since each member has a free choice of the time he or she allocates to work, the value function is non-decreasing in both wage rates.

Because joint events (both obtain an offer, one receives an offer but the other is laid off and both are laid off) occur with negligible probability when Δt is small and because $V(w_1, w_2, t)$ is non-decreasing in (w_1, w_2), Bellman's principle of dynamic optimality implies that the value functions satisfy

$$(1 + \rho\Delta t)\, V(w_1,w_2,t) = \max_{(x,\ell,s)\epsilon\Gamma(w,y)} \Big[u(x,\ell_1,\ell_2)\Delta t$$

$$+ \alpha s_1 \Delta t \sum_{\tilde{w}_1 \epsilon W_1} f_1(\tilde{w}_1) \max[V(\tilde{w}_1,w_2,t+\Delta t),\ V(w_1,w_2,t+\Delta t)]$$

$$+ \alpha s_2 \Delta t \sum_{\tilde{w}_2 \epsilon W_2} f_2(\tilde{w}_2) \max[V(w_1,\tilde{w}_2,\ t+\Delta t), V(w_1,w_2,t+\Delta t)] \qquad (62)$$

$$+ \delta_1 \Delta t\ V(0,w_2,t+\Delta t) + \delta_2 \Delta t\ V(w_1,0,\ t+\Delta t)$$

$$+ (1 - \delta_1\Delta t - \delta_2\Delta t - \alpha_1 s_1 \Delta t - \alpha_2 s_2 \Delta t)\ V(w_1,w_2,t+\Delta t) + 0(\Delta t) \Big] \quad .$$

Of course, W_i is the set of possible wage offers for member i so that $\sum_{w_i \epsilon W_i} f_i(\tilde{w}_i) = 1.$ As in the single person model, the value function is stationary ; i.e. $V(w_1,w_2) = V(w_1,w_2,t) = V(w_1,w_2,t+\Delta t)$ for all $\Delta t > 0$. Hence, by subtracting $V(w_1,w_2)$ from both sides, by dividing the result by Δt and then by taking the limit as $\Delta t \to 0$, we obtain

$$\rho V(w_1,w_2) = \max_{(x,\ell,s)\ \epsilon\ \Gamma(w,y)} \Big[u(x,\ell_1,\ell_2)$$

$$+ \alpha_1 s_1 \sum_{\tilde{w}_1 > w_1} [V(\tilde{w}_1,w_2) - V(w_1,w_2)] f(\tilde{w}_1) \qquad (63)$$

$$+ \alpha_2 s_2 \sum_{\tilde{w}_2 > w_2} [V(w_1,\tilde{w}_2) - V(w_1,w_2)] f(\tilde{w}_2)$$

$$+ \delta_1 [V(0,w_2) - V(w_1,w_2)] + \delta_2 [V(w_1,0) - V(w_1,w_2)] \Big] \quad .$$

In the two person case, there is a total expected utility gain attributable to the fraction of any time interval that each member allocates to search. These are the second and third terms on the right side of (63). The household's optimal choice of current income, leisure, and search time maximizes the sum of these plus the current utility flow, $u(x,\ell,s)$, realized by the choice.

An optimal solution $(x^\circ,\ell^\circ,s^\circ)$ satisfies the following Kuhn-Tucker conditions:

$$u_x(x,\ell_1,\ell_2) - \lambda = 0 \tag{64a}$$

$$u_{\ell_1}(x,\ell_1,\ell_2) - w_1\lambda - \eta_1 = 0 \tag{64b}$$

$$u_{\ell_2}(x,\ell_1,\ell_2) - w_2\lambda - \eta_2 = 0 \tag{64c}$$

$$\alpha_1 \sum_{\tilde{w}_1 > w_1} [V(\tilde{w}_1,w_2) - V(w_1,w_2)]f_1(\tilde{w}_1) - w_1\lambda - \eta_1 + \nu_1 = 0 \tag{64d}$$

$$\alpha_2 \sum_{\tilde{w}_2 > w_2} [V(w_1,\tilde{w}_2) - V(w_1,w_2)]f_2(\tilde{w}_2) - w_2\lambda - \eta_2 + \nu_2 = 0 \,. \tag{64e}$$

The multipliers $(\lambda,\eta_1,\eta_2,\nu_1,\nu_2)$ are non-negative and such that

$$\lambda[y + w_1(1 - \ell_1 - s_1) + w_2(1 - \ell_1 - s_2) - x] \geqq 0 \tag{65a}$$

$$\eta_i[1 - \ell_i - s_i] \geqq 0 \quad , \quad i = 1,2 \tag{65b}$$

$$\nu_i s_i \geqq 0 \quad , \qquad i = 1,2. \tag{65c}$$

Again, λ is imputed marginal utility of "full income". Of course, η_i and ν_i are respectively the Kuhn-Tucker multipliers associated with the non-negativity constraints on the work time and the search time of member i. In the remainder of the section we assume that the instantaneous utility function u(.) is strictly concave. Again, the assumption can be interpreted as risk aversion in income and leisure.

As in the one person case, the marginal rate at which income is substituted for leisure must equal the wage if the worker allocated time to labor and the marginal return to search equals the marginal utility of leisure if the worker allocates time to search. However, these conditions now hold for each member of the household. In addition, if both work, ($\eta_1 = \eta_2 = 0$), then the marginal rate at which leisure of one member is substituted for leisure of the other must equal their relative wage rates. This condition is well known from the standard theory of household time allocation. Because the assumption of risk aversion (u(.) strictly concave) implies that the utility function is strictly - quasi concave in ℓ_1 and ℓ_2, there always exists some range of relative wage rates at which both will work if one does. In otherwords, the two types of leisure are not perfect substitutes except as a limiting case.

The conditions (64d) and (64e) are original to our formulation. Note that these require that the ratio of marginal returns to search be equal to the ratio of wage rates when both member's work and search; i.e., when $\eta_i = \nu_i = 0$ for both i = 1 and 2. But, this condition can hold only by accident. Indeed, because we assume that the total return to the search of each member is linear in the time allocated to search by that member and because the cost of search for each member is the utility value of the time were it spent working λw_i, the search times of the two members when both work are perfect substitutes. As a consequence, only the member with comparative advantage in search, the higher return to search relative to the wage, searches when both are employed. Although hardly ever will both search when both are employed, the conditions allow both to search when either both or one member is unemployed because η_1 and η_2 generally take on different values.

In the remainder of the section we study the effect of one household member's participation state on the other's decision to participate in the labor market and to supply labor time. Specifically, we answer the following questions. What is the set of wage offers such that:

(a) both members supply time to work?

(b) only one member supplies time to work?

(c) neither supplies time to work?

(d) one member does not participate given that the other is working?

For this purpose we assume that both are participants in the sense that both allocate time to search when neither is employed. Were this not the case, the two person model simply reduces to one person version already studied. That the marginal utility of leisure for each given no search by either is less than the marginal return to search is necessary by virtue of (64d) and (64e).

Because $\Gamma(0,0,y)$, $\Gamma(w_1,0,y)$ and $\Gamma(0,w_2,y)$ are all subsets of $\Gamma(w_1,w_2,y)$ one can easily use (63) to establish that $V(w_1,w_2)$ is constant as the set of wage rates at which neither member chooses to work. Formally

$$V(w_1,w_2) = V(w_1,0) = V(0,w_2) = V(0,0) \Leftrightarrow 1 - \ell_i^o - s_i^o = 0 \text{ for } i = 1 \text{ and } 2. \tag{66a}$$

Furthermore, the equations of (64) and (65) and the envelope theorem imply

$$V_1(w_1,w_2) > 0 \Leftrightarrow 1 - \ell_i^o - s_i^o > 0 \tag{66b}$$

and

$$V_2(w_1,w_2) > 0 \Leftrightarrow 1 - \ell_2^o - s_2^o > 0 \tag{66c}$$

where $V_i(.)$ is the partial derivative of $V(.)$ with respect to w_i. Finally, since $V(.)$ is non-decreasing, it follows that

$$1 - \ell_1^{\circ} - s_1^{\circ} > 0 \Leftrightarrow w_1 > w_1^*(w_2) \tag{67a}$$

and

$$1 - \ell_2^{\circ} - s_2^{\circ} > 0 \Leftrightarrow w_2 > w_2^*(w_1) \tag{67b}$$

where $w_i^*(.)$, $i = 1$ and 2, are respectively the largest solutions to

$$V(w_1^*(w_2),w_2) = V(0,w_2) \tag{68a}$$

and

$$V(w_1,w_2^*(w_1)) = V(w_1,0). \tag{68b}$$

Of course, $w_i^*(.)$ is the reservation wage of member i, the lower bound of his or her set of acceptable wage rates. As the notation of (68) indicates, the reservation wage of each member depends on whether or not the other is employed and, if employed, on the wage earned.

Clearly, (66a) implies that $V(w_1,w_2) = V(0,0)$ for all $(w_1,w_2) \leq (w_1^*(0),w_2^*(0))$. For all wage combinations satisfying this inequality, neither member of the household allocates time to work - they are all unacceptable to both. By virtue of (64), (65) and (67)

$$w_1^*(0) = u_{\ell_1}(y,1-s_1^*, 1-s_2^*)/u_x(y, 1-s_1^*, 1-s_2^*)$$

and

$$w_2^*(0) = u_{\ell_2}(y,1-s_1^*, 1-s_2^*)/u_x(y, 1-s_1^*, 1-s_2^*)$$

where (s_1^*,s_2^*) are the search times per period allocated to search given that both are unemployed. Hence, $(w_1^*(0),w_2^*(0)) > 0$ and finite given non-satiation.

The set of wage rate combinations at which both members allocate time to work is

$$\Omega_{22} = \{(w_1,w_2) \mid V(w_1,w_2) > \min[V_1(w_1,0),V_2(0,w_2)]\}$$

$$= \{(w_1,w_2) \mid w_1 > w_1^*(w_2) \text{ and } w_2 > w_2^*(w_1)\} .$$

Our first task is to prove that this set is non empty. Consider the wage for member #2, $\Phi(w_1)$, given a wage of w_1 for member #1, such that

$$V(w_1,0) = V(0,\Phi(w_1)) . \tag{69}$$

From the argument above the reservation wage combination given that both are unemployed is a solution; i.e., $\Phi(w_1^*(0)) = w_2^*(0)$. Continuity implies that a solution exists for all $w_1 > w_1^*(0)$ is some neighborhood of $w_1^*(0)$ as well. Since $V(w_1,0) > V(w_1^*(0),0)$ when $w_1 > w_1^*(0)$ and $V(w_1^*(0),0) = V(0,w_2^*(0))$, we know that $V(0,\Phi(w_1)) > V(0,w^*(0))$ and, consequently, $\Phi(w_1) > w_2^*(0)$ if $w_1 > w_1^*(0)$. Hence, the household is indifferent between member #1 working at w_1 given #2 is unemployed and member #2 working at $\Phi(w_1)$ given member #1 is unemployed.

Suppose that $V(w_1,\Phi(w_1)) = V(w_1,0) = V(0,\Phi(w_1))$. In words, the household is also indifferent between the two options even when faced with the opportunity for both to be employed at the combination $(w_1,w_2) = (w_1,\Phi(w_1))$. Since this supposition and the equations of (64) imply that the leisure time of the two members are perfect substitutes, it contradicts the fact that risk aversion (u(.) strictly concave) implies the u(.) is strictly quasi-concave is (ℓ_1,ℓ_2). Hence, $(w_1,\Phi(w_1))$ is an element of Ω_{22}. Indeed, by ~~virtue of the continuity of $V(w_1,w_2)$, Ω_{22} is an open dense set containing~~ every $(w_1,\Phi(w_1))$ in its interior for all $w_1 > w_1^*(0)$.

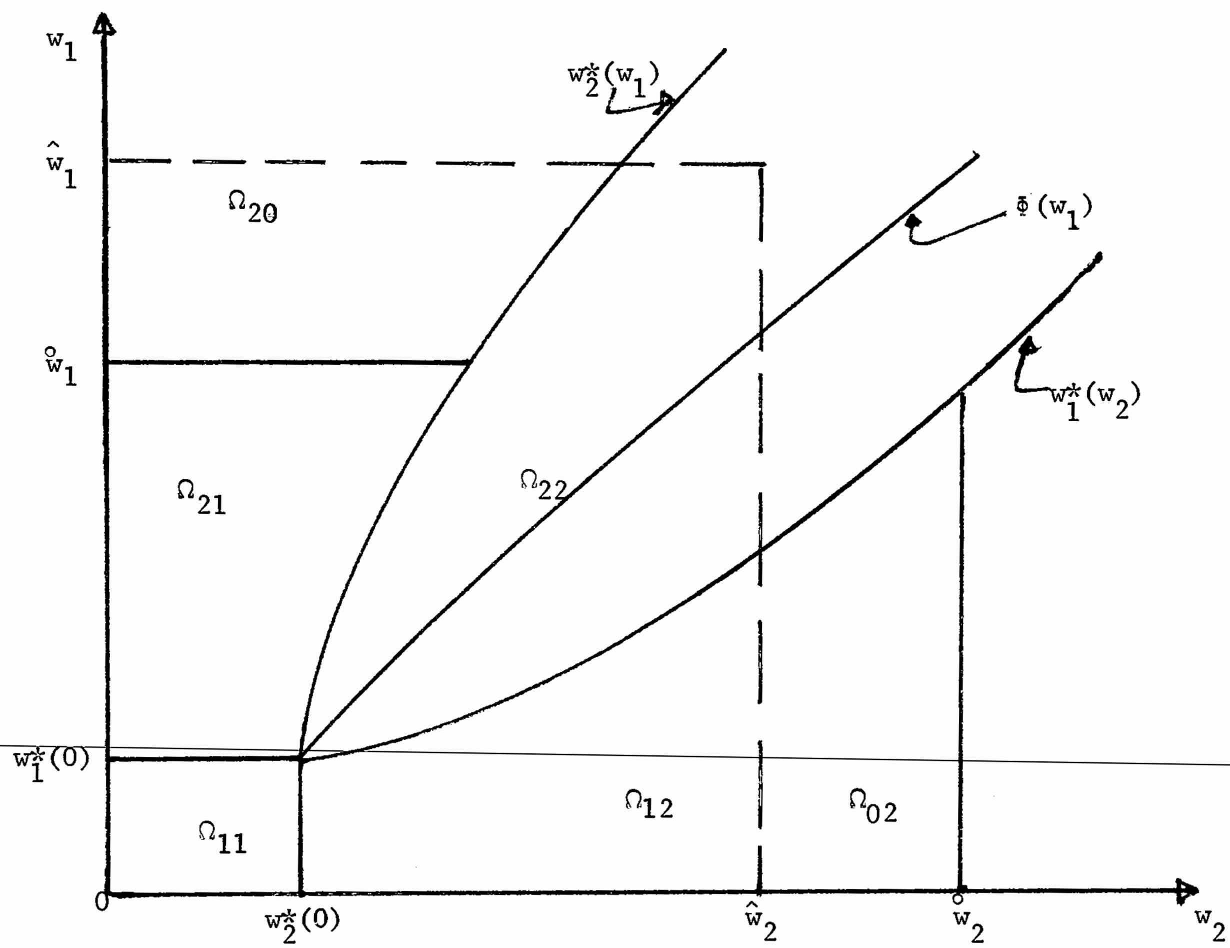
The fact that $V(w_1,0)$ is strictly increasing in w_1 on $w_1 > w_1^*(0)$ and $V(0,w_2)$ is strictly increasing in w_2 on $w_2 > w_2^*(0)$, implies that $\Phi(w_1)$ is increasing given (69). The curve $w_2 = \Phi(w_1)$ is appropriately represented in Figure 5. Finally, since $V(w_1,\Phi(w_1)) > V(w_1,0)$ and $V(w_1,w_2)$ is increasing in w_2 given w_1, the largest solution to $V(w_1,w_2^*(w_1)) = V(w_1,0)$ is such that $w_2^*(w_1) < \Phi(w_1)$. By an analogous argument $w_1^*(w_2) < \Phi^{-1}(w_2)$. The combinations at which both work, Ω_{22}, is the set of points bounded by the curves $w_1 = w_1^*(w_2)$ and $w_2 = w_2^*(w_1)$ in Figure 5.

Although we have assumed that both members are participants if both are unemployed, we have not ruled out the possibility that one member does not participate given that the other member is employed. In the standard two person model this situation typically arises when the leisure times of the two members are reasonably close substitutes and both are normal goods. In this case, the cross substitution effect and the income effect of an increase in the wage of one member both act to increase the demand for the other's leisure. Given a sufficiently large wage for one, the other does not participate.

Essentially the same result holds in the extended model. Specifically, the demand for search time by the unemployed member decreases with the wage of the employed members given that the employed member is not searching, if income and leisure are complements in household production, if the leisure time of the two member are substitutes in the sense that an increase in the leisure of one decreases the marginal product of the other, and if an increase in the wage of the working member reduces the return to the other member's search.[22/] Given the first two conditions, the last condition is likely to hold as well. The income effect of an increase in the other's wage reduces the return to search by the unemployed member for the same reason that an increase in non-labor income reduces the return to search in the one person household case. When the leisure times are substitutes in the sense defined, one expects that the cross substitution effect on the return to search is also negative. In any case, given the conditions stated each member of the household does not participate if the other is employed as a sufficiently large wage. In Figure 5, $\overset{\circ}{w}_1$ and $\overset{\circ}{w}_2$ represent these critical wage rates.

FIGURE 5:

DYNAMIC INTERACTIONS



In sum, the set of all non-negative wage combinations (w_1,w_2) can be partitioned into the following subsets:

$$\Omega_{11} = \{(w_1,w_2)|w_1 \leqq w_1^*(0) \text{ and } w_2 \leqq w_2^*(0)\} \quad (70a)$$

$$\Omega_{22} = \{(w_1,w_2)|w_1 > w_1^*(w_2) \text{ and } w_2 > w_2^*(w_1)\} \quad (70b)$$

$$\Omega_{21} = \{(w_1,w_2)|\mathring{w}_1 \geqq w_1 > w_1^*(w_2) \text{ and } w_2 < w_2^*(w_1)\} \quad (70c)$$

$$\Omega_{12} = \{(w_1,w_2)|\mathring{w}_2 \geqq w_2 > w_2^*(w_1) \text{ and } w_1 < w_1^*(w_2)\} \quad (70d)$$

$$\Omega_{20} = \{(w_1,w_2)|w_1 > \mathring{w}_1 \text{ and } w_2 < w_2^*(w_1)\} \quad (70e)$$

$$\Omega_{02} = \{(w_1,w_2)|w_2 > \mathring{w}_2 \text{ and } w_1 < w_1^*(w_2)\} \quad (70f)$$

For the purpose of interpreting these subsets one can think of w_i, i = 1 or 2, as either the wage currently earned if member i is employed or as a wage offer.

Combinations in Ω_{11} are unacceptable to both members under all circumstances. If the two members are both employed at wage rates (w_1,w_2) then the combination is an element of Ω_{22}. Furthermore, if one is employed and the other receives an offer such that the combination is in Ω_{22}, then both are employed subsequently. If member #1 is employed and member #2 receives an offer while unemployed such that $(w_1,w_2) \in \Omega_{21}$, then #2 rejects it but continues to search while member #1 remains employed. However, given the same combination but with member #2 initially employed, member #1 will accept the offer w_1 and member #2 will quit and search for a better job while unemployed. If member #1 is employed at a wage in excess of $\mathring{w}_1$, then member #2 is not a participant if unemployed. Finally, if member #2 is employed and member #1 receives an offer such that the combination is in Ω_{20}, then member #2 quits and does not serch for another job. The sets Ω_{12} and Ω_{02} has the same interpretations given a reversal of member roles.

The dependence of the decision to participate and to accept employment by each household on the employment state of the other member is illustrated by the interpretations of the regions designated in Figure 5. An even better understanding of these interactions is acquired by constructing possible household labor market histories. Our first example illustrates the so called "added worker" effect.

Suppose that initially member #1 is employed at $\hat{w}_1$ and that member #2 is unemployed. Member #2 does not participate because the marginal utility of his or her leisure exceeds the expected future utility gain attributable to search time given $w_1 = \hat{w}_1 > \mathring{w}_1$. But, if member #1 is laid off, member #2 enters the market as a searching unemployed worker. The change in the status of member #1 has reversed the relationship between the marginal cost and return attributable to the second member's search time.

To continue the story, we suppose that member #2 is the first to find an acceptable job and that it pays $\hat{w}_2$, as illustrated in Figure 5. Shortly thereafter member #1 is reemployed at a job paying his old wage $\hat{w}_1$. Since $(w_1, w_2) \varepsilon \Omega_{22}$, member #1 accepts but member #2 does not quit. The two examples together illustrate that one must take into account the household's past employment history in order to explain the participation behavior of a single member. Specifically, it is possible for both to be employed at $(\hat{w}_1, \hat{w}_2)$ only if member #2 is employed first. Other examples exist that illustrate the point. We encourage the reader to construct them.

H. Summary

A frame work is developed that permits an analysis of the effects of uncertainty in the wage and in the durations of employment and unemployment spells on the household's decisions concerning the allocation of time to leisure, work and search activities. The core of the formulation is a synthesis of labor supply theory under certainty and of job search theory in which search is viewed as a time using alternative to leisure and work. Given the formulation, the opportunity cost of search is derived as the utility value of leisure, on the one hand, and the opportunity costs of income and leisure depend on the marginal utility return to search time. As a consequence, the supply of labor time depends on parameters that characterize the worker's labor markets and on the employment states of other household members, and the demand for search time depends on non-labor income, the wage rates of other household members and the worker's own wage rate if employed.

A number of avenues for future research suggest themselves. First, the multi-person version of the model needs to be elaborated further. Given the availability of panel data on household employment histories, a model of this type is needed as a tool of interpretation.

Second, one can easily incorporate multi-dimensional differences in job characteristics. For example, let layoff, recall and separation rates differ across jobs as well as the wage rate. The acceptance decision in this generalization is characterized by a derived reservation indifference surface in the job characteristics space that reflects the worker's willingness to trade one characteristic for another.

Third, investment in education, training and other forms of human capital can be incorporated in the model. The effects of changes in the employment states of other members of a household on the extent of and timing of such investment by another is an interesting issue in such a model. The effects of employment duration uncertainty on investment in training is another.

FOOTNOTES

1/ Classic theoretical and empirical work in this tradition include Robbins [32], Becker [2], Mincer [27] and Bowen and Finegan [8]. For more recent treatments see Ashenfelter and Heckman [1], Ghez and Becker [17], and Heckman [20].

2/ The job search literature is surveyed in Lippman and McCall [25].

3/ For examples see Katz [22], Parsons [31], and Mortensen [29].

4/ The literature in this tradition is less extensive. See Holt [21], Hall [19], and Marston [26].

5/ The terminology is taken from Çinlar [14]. This work also includes proofs of the assertions made in this section.

6/ See Wilde [33] and Mortensen [28] for other applications of the assumption.

7/ This idea is exploited in other papers by the authors. See [9], [11], [29], and [30].

8/ Employment in two different jobs at the same time is also ruled out by the structure assumed.

9/ An excellent model of this type is formulated and analyzed by Blinder and Weiss [7]. See also Ghez and Becker [17].

10/ Although only the value of $V_i(w,t)$ at the wage rates that are possible, the discrete elements of W, are of interest to the worker, the value function is defined for all non-negative real values of w given (19).

11/ See Beckman [3] for a proof of this assertion. The general problem of existence and uniqueness of a solution is treated in Blackwell [6].

12/ The value function $V_i(w)$ is continuous in w as well.

13/ This notion of risk aversion implies risk aversion in the Arrow-Pratt sense. For a discussion, see Kihlstrom and Mirman [23].

14/ Because $\sum_{w \epsilon W_i^*(w)} f(\tilde{w})$ is continuous except at each finite element of W, the derivatives of $g_i(w)$ and $V_i(w)$ are continuous except at each $w \epsilon W$. At these points $g_i'(w)$ and $V_i'(w)$ as defined below are the right hand derivatives because the set $W_i^*(w)$ is open.

15/ In his recent work on temporary layoffs, Feldstein [16] assumes that workers who are on temporary layoff don't search. This result provides some theoretical support for his assumption.

16/ As Chipman [13] points out, both Edgeworth and Pareto define complementary goods in this way.

17/ See Lewis [24].

18/ This literature includes Mincer [27], Cain [12], Ashenfelter and Heckman [1], and Gronau [18].

19/ Recently Chipman [13] has shown that (b) holds in the general n-good model of household demand.

20/ For a more extensive discussion of search while employed, see Burdett [10].

21/ For a theoretical analysis of the received two person model see Gronau [18] and Ashenfelter and Heckman [1].

22/ See Felder [15] for some relevant empirical evidence on the effect of the wage rates of other household members on search behavior.

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