Discussion Paper No. 29

INCENTIVES AND PUBLIC INPUTS

by

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Revised

July, 1974

1. Introduction

1.1 The problem of incentives has arisen in a number of economic contexts, including most recently, as a general issue in the design and analysis of resource allocation mechanisms [c.f. Hurwicz (1973)]. Most simply, the incentive problem concerns the rationale there is (or can be provided) for economic agents to follow prescribed rules of behavior designed to achieve some objective, usually an efficient or optimal allocation of resources. Typically the prescribed behavioral rules are some type of competitive market behavior such as, the communication of one's (net) demand for goods at prices that are announced by a central agent, e.g. a central planning bureau, market custodian, or auctioneer. The incentive problem concerns whether or not it is in an agent's interest to behave according to these rules and truthfully reveal his demand. Although it has been shown by Hurwicz (1972) that even in the most classical, no externality, private goods context it is typically in an agent's interest to violate the rules, the incentive problem is most familiar to economists as it arises with reference to public goods. Furthermore, it appears widely believed that no mechanism can be devised to optimally allocate public goods that avoids the incentive problem. The reason for this belief is easy to see.

In the private goods, no externality case, if equilibrium prices are known and announced, an efficient allocation will result, under general convexity conditions, when agents maximize preferences or profits at these prices and demand or supply the maximizing quantities, i.e. behave competitively. However, in the public goods case, or more generally when externalities exist, no price system with a single price for every good will lead to an efficient allocation.

INCENTIVES AND PUBLIC INPUTS

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Abstract:

Considered in this paper is a mechanism to coordinate the decision to provide a public input to a group of firms designed to overcome the "free rider" problem. The coordinating agent relies on information communicated by the firms and it is shown that the mechanism provides an incentive for each firm to send truthful information so that an optimal quantity of the public input will be provided. An interpretation of the mechanism is discussed in which the information sent are the firms' demands for the public input and thus the mechanism guarantees the truthful revelation of demand.

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profit maximum under general conditions. In fact, typically every firm would be better off if each purchased a larger quantity of the public input. [2] Traditional economic analysis suggests that centralizing the decision over the quantity of the public input to provide may lead to an improved solution. However, any scheme for centrally determining the quantity of the public input creates an incentive problem of inducing the firms to reveal their true preferences or demands for the input.

It has been argued by Kaizuka (1965, p. 118) that "the benefiting firms have every incentive not to reveal their exact benefit from (the public input)...An attempt to use decentralized pricing will here, too, invite game-theoretic dissembling on the part of producers. Hence in the absence of detailed social planning and fiats, any system of voting and taxing can be expected not to achieve social or even Pareto optimality."

In section 2, we formulate the model and develop a general procedure for solving the incentive problem, Kaizuka's comment notwithstanding. The procedure is defined by rules that specify the quantity of the public input to be provided and the share of the costs to be borne by each firm given all the firm's messages. It is shown that the procedure provides the firms an incentive to send the "correct" messages.

Section 3 discusses an interpretation of the procedure in which the "correct" messages is a truthful revelation of demand and further interprets the procedure in terms of a typical tâtonnement adjustment process in which prices are announced, firms respond with demands, and then prices are adjusted repeatedly until convergence is achieved.

In order to achieve efficiency through competitive behavior, different agents must face different prices for those commodities for which an externality in use exists [c.f. Arrow (1969)]. Furthermore, these <u>individualized prices</u> will depend directly on the individual characteristics, primarily the <u>preferences</u>, of each agent. Thus, in the absence of general knowledge of the agents' characteristics, any procedure for determining these prices must rely on the agents to truthfully reveal their characteristics. But as Samuelson (1954, p. 389) states, although "one could imagine every person...indoctrinated to...<u>reveal</u> his preferences,...by departing from his indoctrinated rules, any one person can hope to snatch some selfish benefit..."[1] Thus, Kamien and Schwartz (1970, p. 19) argue that "...there is no market mechanism to determine either appropriate price or quantity" of a public good. And Buchanan (1968, p. 87) goes further, in asserting that "it may prove almost impossible, however, to secure agreement among a large number of persons, and to enforce such agreements as are made. The reason for this lies in the 'free rider' (problem)...Even if an individual should enter into...(an) agreement, he will have a strong incentive to break his own contract, to chisel on the agreed terms".

1.2. In this paper we consider a group of firms, all of which use in their production processes a good that is non-exclusionary in use -- a public input. Examples of such goods might be weather forecasts available to a group of commercial farmers, general research available to all firms, or advertising by an industry wide trade association.

It is well known that if each firm purchases the quantity of the public input that maximizes its own profits, given the quantities purchased by the other firms, the resulting (non-cooperative) equilibrium will not be a joint

any K, the familiar free rider problem exists. Since firm i will benefit from the total purchases made by the firms, it will add to this total only if the marginal benefit of doing so (the marginal revenue π_i' evaluated at the level of K provided by the other firms) exceeds the marginal cost to the firm (the market price p). Thus, at any aggregate quantity K where each firm's marginal benefit is less than the price, none of the firms will purchase more of the input. This holds even if, in addition, the aggregate marginal benefit (the sum of the n firm's marginal benefits) exceeds the price indicating that the firms could all profit by purchasing more of K and sharing the cost. [4]

Since independent purchases will typically lead to such a suboptimal purchase of the public input, there is the recognized advantage in coordinating decisions over the purchase of the public input. One method for accomplishing this coordination is to centralize the decision. We therefore consider an agent, hereafter called the center, whose task it is to determine the quantity of the public input to be provided the firms and the share of its costs to be borne by each firm. The center may be thought of either as an agent hired by the firms to perform this task in accordance with some agreed upon rules, or alternatively as a central authority (a government or the administration of a large conglomerate organization composed of the n firms as divisions) with the power to purchase the public input and to levy charges against the firms to finance its costs.

The center's objective in determining the quantity of K to provide is to maximize joint profits:

Choose $K^* \geq 0$ to maximize $\sum_i \pi_i(K) - p \cdot K$.   (4)

2. The Public Input Model

2.1 Consider a collection of n firms, indexed i = 1,...,n, which use a particular commodity K in their production processes. The commodity K is a <u>public input</u> to the n firms since the total quantity is available to all firms and the use of it by one firm does not diminish its availability to the other firms. The net revenue r_i of each firm i (gross revenues less costs of inputs other than K) is given by a function R_i of the total available quantity K, however provided, and some local decisions L_i, e.g. a vector of other inputs, variables specifying the choice of technique, etc.:

$$r_i = R_i(K,L_i), \quad i = 1,..,n. \tag{1}$$

Assuming (a) the total available quantity of K is known by all firms at the time they make their decisions L_i, (b) for every quantity K there exists a value of L_i that maximizes net revenues, (c) the firms are profit maximizing, and (d) the difference between profits and net revenues is independent of the firm's choice of L_i; we may restrict attention to the (maximum net) revenue functions π_i defined by:

$$\pi_i(K) = \underset{L_i}{Max} \ R_i(K,L_i), \quad i = 1,...,n. \tag{2}$$

The revenue functions π_i are assumed to satisfy the regularity condition:

For every i, $\pi_i(K)$ is monotonic increasing, strictly concave, and every- (3) where differentiable for $K \geq 0$ with $\underset{K \to \infty}{\lim} \pi_i'(K) = 0$. Let Π denote the set of all such functions.

2.2 Suppose the public input K is available to the n firms in any quantity at a (positive) market price p.[3] If the firms must independently purchase

is not constrained to satisfy the budget constraint:

$$\sum_i C_i(M) = p \cdot \hat{K}(M) \tag{6}$$

In other words, the center is permitted to run a surplus or a deficit (see below, Sections 2.6-2.8).

Given the center's purchasing rule $\hat{K}(\cdot)$ and cost share functions $C_i(\cdot)$, the firms' profits are defined by:

$$\omega_i(M; C_i) = \pi_i[\hat{K}(M)] - C_i(M), \quad i = 1, \ldots, n. \tag{7}$$

Since the center's rule $\hat{K}(\cdot)$ gives the optimal quantity K^* when the firms send their true revenue functions, the incentive problem asks if there exist cost share functions C_i^* such that each firm will maximize its own profits $\omega_i(M; C_i^*)$ by sending its true revenue function:

<u>Incentive Problem</u>: Find cost share functions C_i^*, $i = 1, \ldots, n$, such that, for every i, $M_i^* = \pi_i$ maximizes $\omega_i(M, C_i^*)$ over all M_i in Π for any $M \setminus M_i = (M_1, \ldots, M_{i-1}, M_{i+1}, \ldots, M_n)$ that is:

$$\omega_i(M/M_i^*; C_i^*) \geq \omega_i(M, C_i^*) \tag{8}$$

for all M_i in Π where $M_i M_i^* = (M_1, \ldots, M_i^*, \ldots, M_n)$.

Any n-tuple $C^* = (C_1^*, \ldots, C_n^*)$ of cost share functions solving the incentive problem is called an <u>optimal incentive structure</u>.

Several important properties of an optimal incentive structure deserve emphasis. First, although both the quantity $\hat{K}(M)$ and the cost share $C_i^*(M)$ depend, in general, on <u>all</u> the messages received by the center, under an optimal incentive structure the true revenue function maximizes the firm's

If the center knows the revenue functions π_i, this problem is trivial. If, however, the center does not know these functions, as we assume to be the case, in order to calculate the optimal or joint profit maximizing quantity of K, it must acquire information from the firms regarding the functions π_i.

We suppose, therefore, that each firm i communicates information to the center in the form of a function M_i from the set Π of all revenue functions satisfying the regularity condition (3) which the center interprets as the firm's revenue function π_i. Thus, given any n-tuple of messages $M = (M_1, \ldots, M_n)$, the center then maximizes "reported" joint profits:

Choose $\hat{K}(M) \geq 0$ to maximize $\sum_i M_i (K) - p \cdot K.$ (5)

Since each message M_i must satisfy the regularity condition (3), $\hat{K}(M)$ exists and is unique if $p > 0$. Furthermore, comparing (4) and (5), if the firms send the center their <u>true</u> revenue functions, i.e. $M_i^* = \pi_i$, the center's rule will select the optimal quantity, i.e. $\hat{K}(M^*) = K^*$.

2.3. However, depending on the method by which the purchase of the public input is financed, it may not be in the interest of the individual firms to send their <u>true</u> revenue functions. The incentive problem asks if it is possible to induce the firms to send their true revenue functions by levying charges in an appropriate manner.

Formally, the center assesses the firms by choosing functions C_i of the messages $M = (M_1, \ldots, M_n)$ it receives from the firms -- $c_i = C_i(M)$ is the ith firm's cost share for the public input $\hat{K}(M)$ purchased by the center. Although it would be natural to do so, the choice of cost share functions

$$\omega_i(M/M_i^*,C_i^*) + A_i(M\backslash M_i) = M_i^*[\hat{K}(M/M_i^*)] + \sum_{j\neq i} M_j[\hat{K}(M/M_i^*)] - p \cdot \hat{K}(M/M_i^*) \quad (10)$$

$$\geq M_i^*(K) + \sum_{j\neq i} M_j(K) - p \cdot K$$

for all K. In particular, the inequality holds for $\hat{K}(M/M_i)$, for all M_i. Thus

$$\omega_i(M/M_i^*,C_i^*) + A_i(M\backslash M_i) \geq \omega_i(M/M_i,C_i^*) + A_i(M\backslash M_i) \quad (11)$$

or, since $A_i(M\backslash M_i)$ is independent of M_i,

$$\omega_i(M/M_i^*,C_i^*) \geq \omega_i(M/M_i,C_i^*) \quad \text{for all } M_i, \quad (12)$$

which is the requirement for an optimal incentive structure, [see (8)].

2.5 The optimal incentive structure C^* is interpreted most easily by defining the cost shares independent of the particular rule used to determine the quantity of K. Specifically, for every quantity K, define functions $\hat{C}_i$ of K and the messages M by:

$$\hat{C}_i(K,M) = p \cdot K - \sum_{j\neq i} M_j(K) + A_i(M\backslash M_i), \quad i = 1,\ldots,n. \quad (13)$$

When the center uses the rule $\hat{K}(\cdot)$ to determine the quantity of K to purchase, the cost shares defined by $\hat{C}_i$ are the same as those given by the optimal incentive structure C^*:

$$\hat{C}_i[\hat{K}(M),M] = C_i^*(M) \quad \text{for all M}, \quad i = 1,\ldots,n. \quad (14)$$

To interpret the cost share functions $\hat{C}_i$, recall that the center interprets each firm's message M_i as the firm's revenue function. Thus $\hat{C}_i$ assesses firm i for the full cost of the public input, but offsets this by the full amount of "reported" revenues.[7] Additionally, the firm is

profits independent of the messages sent by the other firms.[5]

Second, since the cost share rules C_i^* are functions only of the messages received by the center, the center need not know either the true revenue functions or the actual revenue received by the firms in order to determine the cost shares. An optimal incentive structure provides a justification for the center to assume the firms are sending their true revenue functions.

Third, under an optimal incentive structure, the cost share of a firm is independent of how well or poorly the other firms behave in implementing their best local decisions L_j. Although the ith firm's profit depends on the others firms' **messages** M_j, it is independent of their decisions L_j and their actual revenues $R_j[\hat{K}(M), L_j]$, $j \neq i$, [see (2)].

Fourth, under an optimal incentive structure, each firm's actual profits depend on all the "local" decisions L_i, since its actual profits are given by:

$$R_i(\hat{K}(M), L_i) - C_i^*(M)$$

Thus, each firm has an incentive to be as efficient as possible in selecting its own local decisions.

2.4 It is actually quite easy to solve the incentive problem stated in (8). Consider the cost share functions C_i^* defined by:

$$C_i^*(M) = - \sum_{j \neq i} M_j[\hat{K}(M)] + p \cdot \hat{K}(M) + A_i(M \backslash M_i), \quad i = 1, \ldots, , \tag{9}$$

where $A_i(M \backslash M_i)$ is a number that may depend on the $(n-1)$-tuple of messages $M \backslash M_i$. Note that although the messages M_j are functions of K, the quantity $A_i(M \backslash M_i)$ is independent of K as well as, of course, the message M_i.[6]

To show that C^* is an optimal incentive structure it is only necessary to recall that $\hat{K}(M/M_i^*)$ maximizes the joint-profits $M_i^*(K) + \sum_{j \neq i} M_j(K) - p \cdot K$. Thus, from (7),

2.6. Since the center's choice of cost share functions C_i is not constrained by the budget constraint (6):

$$\sum_i C_i(M) = p \cdot \hat{K}(M),$$

under an optimal incentive structure C^* the center may run a surplus or a deficit. The magnitude of the surplus or deficit depends on the functions $A_i(\cdot)$ chosen; under C^* the _net surplus_ may be defined as:

$$\text{Net Surplus} = \sum_i C_i^*(M) - p \cdot \hat{K}(M) = \sum_i A_i(M\backslash M_i) - (n-1)\{\sum_i M_i[\hat{K}(M)] - p \cdot \hat{K}(M)\} \quad ($$

Although in special cases it is possible to find functions $A_i(\cdot)$ that will ensure a zero net surplus, in general such functions do not exist.[8]

An example where it is possible to balance the center's budget is the quadratic model in which each firm's gross profit function $\pi_i(K)$ is given by:

$$\pi_i(K) = \alpha_i K - \tfrac{1}{2}K^2, \quad i = 1,\ldots,n. \tag{19}$$

In this simple example, the message M_i for each firm may be taken to be the linear coefficient α_i and, thus, if the firms send the messages a_i, $i = 1,\ldots,n$, the optimal rule for the center to use to determine the quantity of the public input is:

$$\hat{K}(a) = \frac{\sum_i a_i - p}{n} \qquad \text{for } \sum a_i - p > 0. \tag{20}$$

It is straightforward to verify that if the functions $A_i(\cdot)$ are defined by:

$$A_i(a\backslash a_i) = \frac{1}{2n} \left(\sum_{j \neq i} a_j - p\right)^2 + \frac{1}{2n(n-2)} \sum_{\substack{j \neq k \\ j,k \neq i}} a_j a_k - \frac{1}{2n^2} p^2 \tag{21}$$

then C_i^*, $i = 1,\ldots,n$ is an optimal incentive structure where

assessed an amount $A_i(M \setminus M_i)$ which to this point is given by an arbitrary function of $M \setminus M_i$. Since this later amount is independent of M_i, it has no effect in influencing firm i's choice of its best message. It's role is examined in the next section.

Now writing each firm's profits in terms of the functions $\hat{C}_i$, they are functions $\hat{\omega}_i$ of the quantity K, the messages M, and the cost share function $\hat{C}_i$:

$$\hat{\omega}_i (K,M,\hat{C}_i) = \pi_i(K) - \hat{C}_i(K,M), \quad i = 1,\ldots,n. \tag{15}$$

The marginal profitability of the public input for the ith firm and hence the value to firm i of the marginal unit of K is

$$\frac{\partial \hat{\omega}_i}{\partial K} = \pi_i'(K) + \sum_{j \neq i} M_j'(K) - p. \tag{16}$$

However, when the center receives the messages M_i from the firms and endeavors to maximize joint profits, the marginal joint profitability for the public input as perceived by the center is:

$$\sum_{j=1}^{n} M_j'(K) - p. \tag{17}$$

Thus, if firm i reports truthfully, ie. sends $M_i^* = \pi_i$; the center will value the marginal unit at every level of K the same as firm i. Also, when all firms are communicating truthfully, each firm's profit $\hat{\omega}_i(K,M^*; \hat{C}_i)$ is maximized at the same quantity $\hat{K}(M^*) = K^*$ -- the true joint profit maximizing quantity -- although there is no reason for all the firms' profits to be equal at this quantity since the quantities $A_i(M^* \setminus M_i^*)$ need not be identical for all i.

By this discussion, the optimal incentive structure may be interpreted as a scheme to induce each firm to evaluate the public input in terms of its true marginal social benefit, $\sum_i \pi_j'(K)$, and its full marginal social

were zero, since aggregate profits would be increased by the center's coordin-ation in providing the public input presumably entrants would be attracted to the group. But analysis of entrants is outside our model since we consider a group of n firms, not necessarily all in the same industry, and do not specify what potential entrants there might be in the complete economy. Fur-thermore, although the center's surplus, whether positive, negative, or zero, may have an effect on entry, it is not a priori obvious that the total allocation of resources - both private and public - resulting from the imple-mentation of an optimal incentive structure will be less efficient than the initial allocation of resources or even non-Pareto Optimal.

An evaluation of the total resource allocation depends on many issues outside the scope of the partial equilibrium model presented here. In parti-cular, one would need to know the structure of the industries to which the n firms belong - whether they are competitive, oligopolistic, or monopolistic. For example, the n firms might be a group of electric power companies, each a natural monopoly in its own region and perhaps publically regulated. The public input in question might be, say, research and development of nuclear power technology. In this example, entry effects would be nonexistent.

Additionally, the effect of the surplus on entry depends on the relative importance of the public input to the firms. The loss in joint profits from not coordinating the provision of the public input may be large absolutely yet small relative to the total profits of the n firms.

More generally, the entry issue depends on the full dynamic general equilibrium context of the problem. If it is the government that is serving as the center and is raising a surplus from the firms or running a deficit, one would need to know how the surplus is being disposed of or how the deficit

$$
\begin{aligned}
C_i^*(a) &= p \cdot \hat{K}(a) - \sum_{j \neq i} M_j[\hat{K}(a)] + A_i(a \backslash a_i) \\
&= \frac{1}{2n}[a_i^2 - \frac{1}{n}(\sum_j a_j - p)^2] + \frac{1}{2n(n-2)} \sum_{\substack{j \neq k \\ j, k \neq i}} a_j a_k - \frac{1}{2n^2} p^2
\end{aligned}
\tag{22}
$$

and $M_j(K) = a_j K - \frac{1}{2}K^2$. With a little algebra it may also be verified that this optimal incentive structure has the budget balance property:

$$
\sum_i C_i^*(a) - p \cdot \hat{K}(a) = 0.
\tag{23}
$$

Thus, for this special example, if the center uses the rules $\hat{K}(\cdot)$ and $C_i^*(\cdot)$, every firm will have an incentive to send it's "true" message, i.e. $a_i^* = \alpha_i$, regardless of the messages sent by the other firms and furthermore the center is assured of having a zero net surplus regardless of the messages sent by the firms.

2.7 The impossibility of balancing the center's budget in general with an optimal incentive structure of the form C^* given in (9) raises two problems. First, if the center's surplus is negative (a budget deficit), a subsidy is provided the n firms which must be raised by the center somehow and which also suggests that new firms that could not survive in the absence of the subsidy might enter and seek to be included among the original n firms. Second, if the center's surplus is positive, in aggregate the group of firms is being taxed which raises the possibility that some firms which in the absence of the tax would enter or remain in the group might stay out of or exit from the group.

Since our model is a static, partial equilibrium model, we are unable to analyze the effects of the surplus on entry. However, even if the surplus

Or, $\langle \widetilde{R}_j \rangle$ may be the quantities the n firms jointly agree each firm will be responsible for providing. Or, $\langle \widetilde{R}_j \rangle$ may be the quantities the firms will purchase if some of the firms agree among themselves and the others do not cooperate and remain free riders. In any case, the n-tuple $\langle \widetilde{R}_j \rangle$ will be called the _initial situation_ in which the total amount of the public input provided, the _initial quantity_, is, of course, $\widetilde{R} = \sum_j \widetilde{R}_j$ which may be greater than, less than, or equal the true joint-profit maximizing quantity R^*.

Now, consider an optimal incentive structure C^* of the form [repeating (9)]

$$C_i^*(M) = p \cdot \hat{R}(M) - \sum_{j \neq i} M_j[\hat{R}(M)] + A_i(M \diagdown M_i)$$

in which the functions $A_i(M \diagdown M_i)$ are defined in terms of the initial situation $\langle \widetilde{R}_j \rangle$ and non-negative weights θ_i, summing to unity by:

$$A_i(M \diagdown M_i; \langle \widetilde{R}_j \rangle, \theta_i) = p \cdot \widetilde{R}_i - \theta_i p \cdot \widetilde{R} + \underset{K}{Max} \left[\sum_{j \neq i} M_j(K) - (1 - \theta_i) p \cdot K \right] \tag{24}$$

The cost functions $C_i^*(\cdot)$ may then be written as:

$$C_i^*(M; \langle \widetilde{R}_j \rangle, \theta_i) = \theta_i p[\hat{R}(M) - \widetilde{R}] + p \cdot \widetilde{R}_i$$

$$- \sum_{j \neq i} [M_j(\hat{R}(M)) - \theta_j p \cdot \hat{R}(M)] + \underset{K}{Max} \sum_{j \neq i} [M_j(K) - \theta_j p \cdot K] \tag{25}$$

Since $C_i^* \geq \theta_i p \cdot \hat{R}(M) + p \cdot \widetilde{R}_i - \theta_i p \cdot \widetilde{R}$, $\sum_j \widetilde{R}_j = \widetilde{R}$, and $\sum_j \theta_j = 1$, the center's net surplus is always non-negative:

$$C_j^* - p\hat{R}(M) \geq \sum_j \theta_j p \cdot \hat{R}(M) - p\hat{R}(M) + p \sum_j \widetilde{R}_j - \sum_j \theta_j p\widetilde{R} = 0 \tag{26}$$

This incentive structure may be given the following description: Let the weights $\langle \theta_j \rangle \equiv (\theta_1, \ldots, \theta_n)$ be called the _normal cost share_ distribution

is raised and then how this affects the intertemporal behavior of other economic agents. It is possible, however, to construct a static general equilibrium model - an Arrow-Debreu economy with public inputs - and prove that a _competitive_ equilibrium with central coordination of the public input decision using an optimal incentive structure gives a Pareto Optimal allocation of resources.[9] This, of course, would not answer entirely the entry question, but no static, partial or general equilibrium model can.

2.8 Concerning the possible effects of the surplus on the exit of some of the n original firms, the question is deeper than just whether in aggregate the firms are being taxed or not. Even if the surplus is negative, indicating that in aggregate the n firms are receiving a subsidy, whether any one firm receives a subsidy or is taxed to such an extent that it leaves the group is yet to be examined. To answer this question we will exhibit a family of optimal incentive structures [a subset of all those defined by (9)] that a) guarantee the center a positive surplus (thus avoiding the question of how a deficit is to be raised), and b) leave each firm better off than it would be in the absence of the center's coordination or in a reference initial situation. Also, we suggest a plausible method by which the n firms could agree to adopt one member of this family of optimal incentive structures.

To begin let $\langle \widetilde{K}_j \rangle \equiv (\widetilde{K}_1, \ldots, \widetilde{K}_n)$ denote an n-tuple of non-negative quantities of the public input that would be provided by the n firms in the absence of the center's coordination. For example, $\langle \widetilde{K}_j \rangle$ might be the quantities the n firms will individually purchase if there is no centralized coordination of the public input decision - formally, a Nash or non-cooperative equilibrium of the n-person game with payoff functions $\pi_i(\sum_j K_j) - pK_i$.

$$\pi_i(K^*) - C_i^*(\pi;\langle\tilde{K}_j\rangle,\theta_i) > \pi_i(\tilde{K}) - p\cdot\tilde{K}_i \qquad \text{for all } i \qquad (27)$$

for any $\langle\theta_j\rangle \in \Theta(\tilde{K})$.

<u>Proof</u>: Consider the cost shares $\hat{\theta}_i \equiv \pi_i'(K^*) / \sum_j \pi_j'(K^*)$, $i = 1,\ldots,n$. Under the regularity condition (3), $\hat{\theta}_i > 0$ and clearly $\sum \hat{\theta}_j = 1$. Furthermore, it is easily verified that:

$$C_i^*(\pi;\langle\tilde{K}_j\rangle,\hat{\theta}_i) = \frac{p}{\sum_j \pi_j'(K^*)} \cdot \pi_i'(K^*)(K^* - \tilde{K}) + p\cdot\tilde{K}_i \quad . \qquad (28)$$

Thus,

$$\pi_i(K^*) - C_i^* - \pi_i(\tilde{K}) + p\cdot\tilde{K}_i = [\pi_i'(K^i) - \frac{p}{\sum_j \pi_j'(K^*)} \pi_i'(K^*)](K^* - \tilde{K})$$

for some K^i between K^* and $\tilde{K}$.

If $K^* - \tilde{K} > 0$, then $K^* > 0$, $\sum_j \pi_j'(K^*) = p$, and since $\pi_i(\cdot)$ is strictly concave, $\pi_i'(K^i) > \pi_i'(K^*)$. Thus, if $\theta_i = \hat{\theta}_i$,

$$\pi_i(K^*) - C_i^* > \pi_i(\tilde{K}) - p\cdot\tilde{K}_i \quad \text{if } K^* > \tilde{K}.$$

If $K^* < \tilde{K}$, then $\pi_i'(K^i) < \pi_i'(K^*)$ and

$$\pi_i'(K^i) - \frac{p}{\sum_j \pi_j'(K^*)} \pi_i'(K^*) < \pi_i'(K^*)(1 - p/\sum_j \pi_j'(K^*)) \leq 0$$

since $\pi_i'(K^*) > 0$ and $\sum_j \pi_j'(K^*) \leq p$. Thus, if $\theta_i = \hat{\theta}_i$

$$\pi_i(K^*) - C_i^* > \pi_i(K^*) - p \cdot \tilde{K}_i \quad \text{if } K^* < \tilde{K}$$ also and we have established that $\langle\hat{\theta}_j\rangle$ is in $\Theta(\tilde{K})$.

The proposition follows by verifying that the cost $C_i^*(\pi; \langle\tilde{K}_j\rangle, \theta_i)$ as a function of θ_i is continuous and monotonic increasing in θ_i.

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and $M_j(K) - \theta_j p \cdot K$ the (reported) net <u>normal profit</u> of firm j. The incentive structure C^* of (25) assesses each firm (a) its cost in the initial situation, $p \cdot \widetilde{K}_i$; (b) plus (minus) its normal cost share of the increased (decreased) quantity of public input provided, $\theta_i p[\hat{K}(M) - \widetilde{K}]$; (c) plus the total difference in the (reported) net normal profits caused by the center's choice of the joint (reported) profit maximizing quantity $\hat{K}(M)$ rather than the quantity maximizing the other firms' (reported) net normal profits. In other words, this incentive structure assesses each firm, in addition to (a) and (b), the full (reported) impact that it has on all the other firms.

Equation (25) defines an optimal incentive structure for <u>any</u> normal cost share distribution $\langle \theta_j \rangle$. Furthermore, each incentive structure in this family guarantees the center a non-negative surplus. However, given any particular member of the family, i.e. a specific cost share distribution, any one firm might be better off in the initial situation $\langle \widetilde{K}_j \rangle$ than with the optimal quantity $\hat{K}(\pi) = K^*$ when it is charged $C_i^*(\pi, \langle \widetilde{K}_j \rangle, \theta_i)$. This depends on the particular normal cost share θ_i chosen. If $\theta_i = 0$, then firm i is is better off with the optimal quantity K^* then in the initial situation $\langle \widetilde{K}_j \rangle$, but if θ_i is close to unity, it is better off in the initial situation. The following proposition establishes, however, that there any many possible choices of the cost share distribution such that every firm is better off with K^* when charges $C_i^*(\pi; \langle \widetilde{K}_j \rangle, \theta_i)$ then it is in the initial situation.

<u>Proposition:</u> Under the regularity condition (3), if the optimal quantity K^* is not equal to the initial quantity $\widetilde{K}$, then there exists a non-empty open convex set depending only on $\widetilde{K}$, $\Theta(\widetilde{K})$, in the unit simplex such that for every cost share distribution $\langle \theta_j \rangle$ in $\Theta(\widetilde{K})$, each firm is better off with $K^* = \hat{K}(\pi)$ then in the initial situation; i.e.,

providing $\hat{K}(M)$ to the firms and billing each firm $C_i^*(M; \langle \tilde{K} \rangle, \vartheta_i)$."

The results established above imply:

1. The n firms have an incentive to accept this arrangement since there are many normal cost share distribution $\langle \vartheta_j \rangle$ that will leave them all better off than they are or would be in the initial situation. (Proposition).

2. The center is assured of not running a deficit. (26)

3. The center's surplus will be strictly positive when the firms report truthfully, i.e. send $M_i^* = \pi_i$, for any cost share distribution $\langle \vartheta_j \rangle$ except when $\vartheta_i = \hat{\vartheta}_i$ (defined in proof of proposition) for all i, in which case the surplus is zero. [Follows from summing (28) over all i.]

4. Given any cost share distribution, each firm has an incentive to report truthfully its revenue function π_i to the center, regardless of what any of the other firms are communicating. [By (12) since C^* is optimal].

5. Given any cost share distribution, if the firms respond to the incentives and communicate truthfully (as is in each firm's own interest), the center will provide the true joint-profit maximizing quantity of the public input K^*. (By definition of the rule $\hat{K}(\cdot)$ since $K^* = \hat{K}(\pi)$).

Conclusion (4) has the further implication that any agreement a group of firms might reach while bargaining over $\langle \vartheta_j \rangle$ as to the subsequent messages M_i that they will send is not stable. As Buchanan (1968, p. 87) has stated: "Even if an individual should enter into...(an) agreement, he

This proposition only establishes that, given any initial situation $\langle \widetilde{K}_j \rangle$ where $\widetilde{K} \neq K^*$, there exist many normal cost share distributions $\langle \theta_j \rangle$ such that each firm is better off with the center's coordination than it is in the initial situation. How any particular distribution with this property may be chosen has not been specified. A simple possibility would be for the firms to bargain among themselves until they agree on the distribution and then announce it to the center.

In detail, the entire process of implementing the central coordination of the public input decision might be envisioned as follows:

A group of n firms use in common some public input. Let us suppose that, whatever prior arrangements have or have not been made to coordinate their purchases of the input, their current purchases are $\langle \widetilde{K}_j \rangle$, the initial situation, and that the total amount provided $\widetilde{K}$ is recognized or suspected to be inefficient or not joint-profit maximizing, although the precise optimal quantity K^* is, of course, unknown. Now, suppose some agent, who might be the manager of one of the n firms or an outside agent or the government, proposes the following arrangement to the firms:

"The agent, hereafter called the center, will provide the service of centrally purchasing and providing the public input to the n firms. The center will use the rule $\hat{K}(\cdot)$ defined by (5) to calculate the quantity of the public input to provide and will charge each firm $c_i^*(M; \langle \widetilde{K}_j \rangle, \theta_i)$ where $\langle \theta_j \rangle$ is a normal cost share distribution agreed upon by the firms. Upon acceptance of this arrangement, the firms will jointly communicate their choice of $\langle \theta_j \rangle$ to the center. The center will then ask each firm for its message M_i and execute its decisions,

3.1. Previous discussions of incentive problems with public goods mentioned in Section 1 have concerned the inability of decentralized prices to elicit true preferences from consumers of public goods. For the public input model of Section 2 the issue may be posed as follows.

Under the regularity condition (3), the joint profit maximizing or optimal quantity of the public input K^* is uniquely characterized by the Kuhn-Tucker conditions:

$$\sum_i \pi_i' (K^*) = p \text{ unless } \sum_i \pi_i' (0) < p \text{ which implies } K^* = 0. \tag{29}$$

Thus, it would be sufficient for the center to know each firm's marginal revenue schedule $\pi_i' (\cdot)$ since the inverse of the sum evaluated at p gives K^*:

$$K^* = \begin{cases} 0 & \text{if } \sum_i \pi_i' (0) < p \\ [\sum_i \pi_i' (\cdot)]^{-1} (p) & \text{otherwise} \end{cases} \tag{30}$$

The marginal revenue schedule $\pi_i'(\cdot)$ is informationally equivalent to to the firm's demand function $\hat{K}_i(\cdot)$ defined by:

$$\hat{K}_i(p_i) \text{ maximizes } \pi_i(K) - p_i K \tag{31}$$

since under the regularity condition (3) they are inverse functions of each other:

$$\hat{K}_i (p_i) = [\pi_i' (\cdot)]^{-1} (p_i) \text{ and } [\hat{K}_i (\cdot)]^{-1} (K) = \pi_i'(K). \tag{32}$$

Therefore, if each firm would truthfully communicate its demand function $\hat{K}_i(\cdot)$ for the public input (i.e. correctly reveal its demand), the center

will have a strong incentive to break his own contract, to chisel on the agreed terms." However, Buchanan advanced this argument as a reason why agreements to provide optimal quantities of public goods are not stable. We use the same argument as a reason why the mechanism proposed here will lead to the optimal provision of the public input. Once the agreed upon $\langle \theta_j \rangle$ is announced to the center, each firm does best for itself by reporting its true revenue function π_i, regardless of the other firms messages and regardless of what agreements might have been concluded as to the messages they would send the center.

Conclusion (3) also shows that in aggregate the best agreement for the n firms to conclude is for $\theta_i = \hat{\theta}_i$ for all i since then, the aggregate net profits of the firms (revenues less costs paid to the center) equals the maximum joint-profits when they reveal their true revenue functions. However, even if the firms are unable to discover this particular cost share distribution and agree instead on some other $\langle \theta_j \rangle$, they each will have an incentive to report truthfully. (Conclusion (4)). Thus, the result that each firm will have an incentive to report truthfully and consequently that the true optimal or joint-profit maximizing quantity of the public input will be provided does not depend on the firms' abilities to discover through bargaining the particular cost share distribution $\langle \theta_j \rangle$.

The bargaining is merely a device to insure every firm's willing participation in the centralized coordination arrangement. If the center is the government and the firms are not allowed to escape the arrangement, then the bargaining can be dispensed with and $\langle \theta_j \rangle$ selected arbitrarily. Since it would be reasonable to permit the government to know the actual realized revenues of the firms (but not their revenue functions, of course), it would be an easy matter to select the $\langle \theta_j \rangle$ such that no firm would be bankrupt by the cost rules.

fact, it is possible to reinterpret the communications process of section 2 and the optimal incentive structure C^* [see (9)] as a method for providing an incentive for the truthful revelation of the firms' demands for the public input.

Specifically the communication process is reinterpreted by supposing that each firm's message to the center is a <u>demand</u> function $d_i(\cdot)$ instead of a <u>revenue</u> function $M_i(\cdot)$. Corresponding to the regularity condition (3) on the set Π of allowable revenue functions is the regularity condition:

The set D of allowable messages (demand functions $d_i(\cdot)$) consists of $\qquad$ (35) functions $d(\cdot)$ that are inverse functions of strictly monotonic decreasing integrable real-valued functions $d^{-1}(\cdot)$ on the non-negative real line R_+ with $\lim_{K \to \infty} d^{-1}(K) \leq 0$

There is, of course, an obvious relation between the sets D and Π:

<u>Lemma 1</u>: M_i is an element of Π if and only if d_i is an element of D, where

$$M_i(K) = \int_0^K d_i^{-1}(x) d_x \qquad \text{(if)}$$

$$(36)$$

$$d_i(p) = [M_i'(\cdot)]^{-1}(p) \qquad \text{(only if)}$$

<u>Proof</u>: Follows readily from the facts that every concave function on an interval is absolutely continuous on each closed subinterval and that every absolutely continuous function is the indefinite integral of its derivative [c.f. Royden, (1968, p. 107 and 109)].

Next, given any n-tuple of messages (demand functions) $d = (d_1, \ldots, d_n)$, let the center's rule for determining the quantity of the public input K to

could compute K^* by taking the _vertical_ sum of these demand functions and evaluating it at the market price p:

$$K^* = \begin{cases} 0 & \text{if } [\sum_i \hat{K}_i^{-1}(0)] < p \\ [\sum_i \hat{K}_i^{-1}(\cdot)]^{-1}(p) & \text{otherwise} \end{cases} \tag{33}$$

Corresponding to the optimal quantity K^* is a set of prices p_i^*, one for each firm, such that, each firm's demand $\hat{K}_i(p_i^*)$ is equal to the optimal quantity K^* and such that the sum of these prices is the market price p. Furthermore, these prices p_i^* are equal to (greater than) the firms' marginal revenues evaluated at the optimal quantity K^* when K^* is positive (zero): That is:

There exist p_i^*, $i = 1,\ldots,n$ such that $\sum_i p_i^* = p$ and $\hat{K}_i(p_i^*) = K^*$ for all i

$$\tag{34}$$

and

$$p_i^* \geq \pi_i'(K^*) \quad \text{with equality if } K^* > 0.$$

These prices are called the _optimal decentralized prices_ since if each firm i were charged p_i^* for the entire quantity of public input provided, each firm would demand or desire the optimal quantity K^*.

However, if a firm is charged for the public input using a price p_i^* calculated in this manner, it will in general have an incentive to under-state or misreveal its demand by communicating a false demand function such that at every price p_i, the quantity demanded is less than the firm's true demand at that price, $\hat{K}_i(p_i)$. This observation has been the substance of many previous discussions of public goods.

On the other hand, if the firms' charges are not calculated in this manner, they may or may not have an incentive to reveal their demands. In

(a) $\qquad \hat{K}(M) = \tilde{K}(d)$ $\quad$ when $M_j(K) = \int_0^K d_j^{-1}(x)dx + \text{constant}$, for all j.

(b) $\qquad \hat{K}(M^1) = \hat{K}(M^2)$ $\quad$ when $M_j^1(K) = M_j^2(K) + \text{constant}$, for all j.

Thus, (a) and (b) imply:

$$\tilde{w}_i(d;\tilde{C}_i) = w_i(M;C_i^*) \text{ where } M_j(K) = \int_0^K d_j^{-1}(x)dx \tag{41}$$

and

$$w_i(M/\tilde{M}_i; C_i^*) = w_i(M/\pi_i; C_i^*) \text{ where } \tilde{M}_i(K) = \int_0^K \pi_i'(x)\,dx = \pi_i(K) + \text{constant.} \tag{42}$$

Now, suppose $\tilde{C}$ is not optimal. Then for some i, given $d \setminus d_i$, there exists some $d_i^0 \in D$ such that

$$\tilde{w}_i(d/d_i^0; \tilde{C}_i) > \tilde{w}_i(d/d_i^*; \tilde{C}_i)$$

or using (41) and (42)

$$w_i(M/M_i^0; C_i^*) > w_i(M/\tilde{M}_i; C_i^*) = w_i(M/\pi_i; C_i^*)$$

which contradicts the fact that C^* is optimal. $\qquad$ Hence $\tilde{C}$ is optimal.

3.2. Since the communication of an entire function -- either a revenue or demand function -- may involve complicated coding problems, it is of interest to inquire if equivalent information could be communicated using less complicated messages. This issue has been encountered previously in the context of the Walrasian tâtonnement procedure for finding a competitive equilibrium. For our purposes here, it is sufficient to note that the tâtonnement procedure is a method of calculating equilibrium prices alternative to obtaining from every economic agent his demand or supply function, taking the appropriate

purchase be:

$$\bar{K}(d) = \begin{cases} 0 & \text{if } \sum_i d_i^{-1}(0) < p \\ [\sum_i d_i^{-1}(\cdot)]^{-1}(p) & \text{otherwise} \end{cases} \qquad (37)$$

Also, let $\bar{C} = (\bar{C}_1, \ldots, \bar{C}_n)$ be the incentive structure defined by:

$$\bar{C}_i(d) = -\sum_{j \neq i} M_j[\bar{K}(d)] + p \cdot \bar{K}(d) + A_i(M \setminus M_i), \quad i = 1, \ldots, n \qquad (38)$$

where $M_j(K) = \int_0^K d_j^{-1}(x)dx$ for every j.

Given these rules, the firms' profits are defined by

$$\tilde{\omega}_i(d; \bar{C}_i) = \pi_i[\bar{K}(d)] - \bar{C}_i(d), \quad i = 1, \ldots, n \qquad (39)$$

Finally, since the optimal quantity K^* is given by the rule $\bar{K}(d)$ when the firms communicate their true demand functions: $d_i^* = \hat{K}_i$, the incentive structure $\bar{C}$ is optimal if

$$\tilde{\omega}_i(d/d_i^*; \bar{C}_i) \geq \omega_i(d/d_i; \bar{C}_i) \quad \text{for all } d_i \in D. \qquad (40)$$

<u>Corollary</u>   The incentive structure $\bar{C}$ defined by (38) is optimal.

<u>Proof</u>: Since the rules $\hat{K}(M)$ and $\bar{K}(d)$ are defined by

$$\hat{K}(M) = \begin{cases} 0 & \text{if } \sum_i M_i'(0) < p \\ \sum_i (M_i'(\cdot))^{-1}(p) & \text{otherwise} \end{cases}$$

$$\bar{K}(d) = \begin{cases} 0 & \text{if } \sum_i d_i^{-1}(0) < p \\ \sum_i (d_i^{-1}(\cdot))^{-1}(p) & \text{otherwise} \end{cases}$$

we have:

a knowledge of an infinite number of different price - demand pairs $\{[\hat{K}_i(p_i(t)), p_i(t)], t = 1, 2, \ldots\}$ is <u>exactly</u> equivalent of a knowledge of the complete demand function $\hat{K}_i(\cdot)$.

<u>Lemma 2</u>: If $f: R \rightarrow R$ is an analytic function and $s = \{(x_t, f(x_t)), t = 1, 2, \ldots\}$ is a convergent sequence where $x_t \neq x_{t'}$, for all $t \neq t'$, then a knowledge of s is equivalent to a knowledge of f in the sense that $y = f(x)$ can, in principle, be calculated for any x.

<u>Proof</u>: Since f is analytic, f is representable by a Taylor's series expansion about any $\bar{x}$:

$$f(x) = \sum_{r=0}^{\infty} \frac{1}{r!} f^{(r)}(\bar{x})(x-\bar{x})^r \text{ for all } x \in A$$

Suppose $(x_t, f(x_t))$ converges to $(\bar{x}, \bar{y})$. The derivatives $f^{(r)}(\bar{x})$ can then be calculated for every r using the method of divided differences [see Fröberg (1965, p. 148)].

Define

$$f(x_t, x_{t+1}) \equiv \frac{y_t - y_{t+1}}{x_t - x_{t+1}} \text{ for all } t = 1, 2, \ldots$$

and

$$f(x_t, \ldots, x_{t+r}) \equiv \frac{f(x_t, \ldots, x_{t+r-1}) - f(x_{t+1}, \ldots, x_{t+r})}{x_t - x_{t+r}}$$

for all $t = 1, 2, \ldots$, and $r = 2, 3, \ldots$

The derivatives in the Taylor's series representation of f can be calculated, in principle, by:

$$\frac{1}{r!} f^{(r)}(\bar{x}) = \begin{cases} \bar{y} & \text{for} \quad r = 0 \\ \lim_{t \to \infty} f(x_t, \ldots, x_{t+r}) & \text{for all } r = 1, 2, \ldots \end{cases}$$

sum of these functions to form the excess demand function, equating it to zero and solving directly for the equilibrium prices.

For the public input model of Section 2 it is possible to specify the firm-center communication in terms of a traditional tâtonnement adjustment process rather than in terms of messages consisting of entire functions. Suppose, for example, that a communication process is defined in stages indexed by $t = 1,2,\ldots$. At the beginning of each stage t, the center communicates a price $p_i(t)$ to each firm i and at the end of the stage each firm communicates a message $K_i(t)$ interpreted by the center as its demand for K at the price $p_i(t)$.

If the firms send their true demands at each stage, ie. $\hat{K}_i(t) = \hat{K}_i[p_i(t)]$, under the regularity conditions (3), the center can suitably adjust the prices $p_i(t)$ from one stage to the next and arrive at a quantity K arbitrarily close to the optimal quantity K^* after sufficient number of iterations. Since the objective of the price adjustment is to find the decentralized prices p_i^* [see (34)], one method of proceding from one stage to the next is to raise the prices of those firms demanding the maximum quantity of K and lower the prices of those firms demanding the minimum quantity, keeping the sum of all prices $\sum_i p_i(t)$ equal to the market price p. Under the regularity condition (3), this process will converge to the decentralized prices p_i^* of (34) as $t \to \infty$.

This type of tâtonnement adjustment process is informationally quite similar to the communication of a complete demand function since in the course of the adjustment the center will learn the demand functions $\hat{K}_i(\cdot)$, at least in the neighborhood of the optimal quantity K^*. In fact, under the additional assumption that the revenue function $\pi_i(K)$ is sufficiently smooth,

to compute the cost shares as defined by the incentive structure $\tilde{C}$ [see (38)]. But since this incentive structure is optimal, each firm therefore cannot do better (i.e. receive higher net profit) then it will if it sends its "true" demand $\hat{K}_i[p_i(t)] = K_i(t)$ at each stage t.

3.3. To briefly summarize the results of this section, it has been shown that it is possible to use a price - demand tâtonnement procedure to determine the optimal quantity of the public input. This result follows since using an appropriate incentive structure, the firms will have an incentive to truthfully reveal their demands for the public input. A crucial aspect of this model that distinguishes it from previous treatment in the literature is that the firms are not actually charged for the public input in accordance with the limit prices (i.e. the optimal decentralized prices), but rather more complicated cost sharing rules are required that may not permit the center's budget to be balanced in every instance.

In conclusion we note some directions for extensions of the methods and models of this paper. First of all, although the commodity K was interpreted as a public input, it could just as easily be interpreted as an input used by one (or many firms) which cause a negative externality (diseconomy) on the other firms (see Groves, (1974)]. More abstractly, the methods are applicable to n-person games with freely transferable utility in which decisions affecting more than one player (i.e. externalities) are centralized.[12]

In addition, a recent paper by Groves and Ledyard (1974) show that these methods may also be used for solving the incentive problem for the general public goods model where the public good is a consumption good entering each consumer's utility function at the same level. The cost or compensation functions $C_i(\cdot)$ for this model, enter the consumers' budget equations instead of defining direct transfers of utility.

Corollary: If $\pi_i(K)$ is an analytic function for $k \geq 0$ and $\{[\hat{K}_i(p_i(t)), p_i(t)], t = 1, 2, \ldots\}$ is a sequence of demand - price pairs converging to $(\overline{K}_i, \overline{p}_i)$ where $\overline{K}_i > 0$ with $p_i(t) \neq p_i(t')$ for all $t \neq t'$, then $\hat{K}_i(p)$ can be calculated, in principle, for any p in the interval

$$\left(\lim_{K \to \infty} \pi_i'(K), \quad \pi_i'(0) \right).$$

Proof: Since the demand - price pair sequence converges to $(\overline{K}_i, \overline{p}_i)$ with $\overline{K}_i > 0$, without loss of generality we can assume $p_i(t) < \pi_i'(0)$ for all t and thus $\hat{K}_i(p_i(t)) > 0$ for all t. Then, under the regularity condition (3), $K_i(p) = [\pi_i'(\cdot)]^{-1}(p)$ for all p in the interval $(\pi_i'(\infty), \pi_i'(0))$. Furthermore, since π_i is analytic for $K \geq 0$ so is π_i' and hence so is $[\pi_i'(\cdot)]^{-1} = \hat{K}_i$ on $(\pi_i'(\infty), \pi_i'(0))$. The conclusion then follows from Lemma 2.

In view of the Corollary, since the communication of demand - price pairs is equivalent to the communication of the entire demand function, the results of Section 3.1 are applicable. Thus, the incentive structure $\tilde{C}$ defined by (38) can be reinterpreted to provide an incentive to each firm to communicate its true demands in a tâtonnement procedure.

Specifically, suppose the center announces prices that are adjusted in such a way that if the firms respond with demands calculated in accordance with any fixed set of demand functions $d_i \in D$, the prices converge to the optimal decentralized prices for these demand functions.[10] Next, suppose the "demands" $K_i(t)$ that the firms respond with enable the center to calculate a demand curve $d_i \in D$ for each firm using the method of divided differences discussed in Lemma 2.[11] This procedure will then enable the center

7. Of course, if firm j communicates a false revenue function; i.e. $M_j \neq \pi_j$, the cost share function $\hat{C}_i$ will calculate i's cost using the reported revenue schedule instead of the true one since the center would not know the true one in this case.

8. If the set Π of allowable revenue functions is parameterized by some real variable so that each message M_i is just the parameter defining the particular revenue function, and if the net surplus is a polynomial function of the messages M of degree less than or equal $n - 1$, then, the budget can be balanced always. This suggests that if the number of firms is sufficiently large, the net surplus can be made small in an approximation sense.

9. See Groves and Ledyard (1974) for an indication of how such a model might be constructed and such results proved. Their model, however, concerns the more difficult case of public consumption goods and not public inputs, but the issues are similar.

10. For example, if n is even, for all $i = 1,\ldots,n$, start with $p_i(1) = \frac{1}{n} p$ and then for all $t = 2,3,\ldots$ Let $p_i(t) = p_i(t-1) \pm \frac{1}{t}$ as $K_i(t-1) \gtrless$ median $\{K_1(t-1),\ldots,K_n(t-1)\}$.

11. This can be assured by enabling the center to assess a sufficiently large penalty against any firms whose "demand" messages are inconsistent with the calculations of a demand function in D.

12. This would include "team" problems and for a discussion of incentive problems in team models, see Groves (1974).

FOOTNOTES

* This research was supported, in part, by a grant from the National Science Foundation. The authors would like to thank Professors Kamien, Ledyard and Hurwicz, the editor, and an anonymous referee for helpful comments and suggestions. Responsibility for errors, of course, remains with the authors.

1. Although Samuelson was referring to the public goods case here, the statement is equally true for the private goods case as shown by Hurwicz (1972). The difference between the two cases is that although the efficient price for a private good depends on all agents' demands, if there are large numbers of agents this price will not be as sensitive to any one agent's demand as that agent's individualized price for the public good will be in the public good case.

2. This is by no means universally true; in the case when then the public good is perfectly complementary in use to other inputs, the non-cooperative equilibrium (if it exists) will be joint-profit maximizing.

3. The assumption of a constant market-price could be relaxed at the expense of additional notational complexity but without the benefit of additional insight.

4. Under the regularity condition (3), this will occur for some $K > 0$ as long as the price p is less than the sum of the firms' marginal revenue evaluated at zero, i.e. $p < \sum_i \pi_i' (0)$.

5. This property is stronger than the Nash equilibrium property which requires only that M_i^* maximize $\omega_i (M^*/M_i, C_i^*)$ for every i, or that sending the true message is best when the other firms are also sending their true messages.

6. For example, $A_i(M \backslash M_i)$ might be defined by $A_i(M \backslash M_i) = \sum_{j \neq i} M_j(\bar{K}) - p \cdot \bar{K}$ where $\bar{K}$ is some fixed, predetermined, quantity. In section 2.8 another example is given.

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