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"Limit Pricing, Potential Entry and Barriers to Entry"

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Limit Pricing, Potential Entry and Barriers to Entry

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ABSTRACT

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An established firm faced with potential entry is considered with the probability of entry being dependent on the limit price used by the established firm. The optimal limit price is less than the price that maximizes pre-entry profit and under reasonable conditions decreases as risk aversion increases. Barriers to entry are assumed to affect the probability of entry with higher barriers increasing the expected utility of the established firm. If the hazard rate is decreased by higher barriers to entry, the optimal limit price increases. American Economic Review,

Limit Pricing, Potential Entry, and Barriers to Entry David P. Baron*

A major advance in the theory of imperfect competition has been the recognition that established firms must take into account possible actions by potential as well as by existing competitors. The empirical research on potential entry has tended to focus on the effects of concentration and barriers to entry on industry profits and price levels, while theoretical study has emphasized the limit pricing behavior of established firms. The study of barriers to entry was pioneered by J. S. Bain (1956), who investigated barriers created by 1) product differentiation, 2) absolute cost advantages, and 3) economies of large scale operations. P. Sylos-Labini emphasized the effects of returns to scale as a barrier to entry, and F. Modigliani has provided an important interpretation of the early work of Bain and Sylos-Labini. Empirical studies by Bain and H. M. Mann (1966) indicate a positive relationship between concentration and profitability, while M. Hall and L. Weiss (p.327) found that "Concentration would seem to be less important than the capital requirements barrier as a determinant of profitability."

The theoretical study of the impact of potential entry was begun by Bain (1949) who first introduced the notion that potential competition may lead established firms to sacrifice current profit inorder to preclude entry. The deterministic theory of potential entry and limit pricing has been advanced by R. F. Harrod, P. W. S. Andrews, H. R. Edwards, J. R. Hicks, F. H. Hahn, P. B. Pashigian,

- G. Pyatt, D. W. Gaskins, and by J. D. Bhagwati, who has provided a survey and analysis of much of this work.
- O. E. Williamson first suggested a probabilistic approach to the issue of potential entry in which the established firms are thought of as assessing the probability that a potential entrant will actually enter the industry. M. I. Kamien and N. L. Schwartz have developed and interpreted a stationary, probabilistic model in which an established firm chooses a limit price so as to maximize discounted expected profit over an infinite horizon with the time until entry being uncertain. In an earlier paper this author analyzed the implications of alternative assumptions regarding the established firm's assessments of the probability of entry and its post-entry behavior. The purpose of this paper is to investigate, using a probabilistic model, the effects of potential entry and barriers to entry on the price and profitability of established firms in an industry faced with potential entry.

I. The Model and Optimality Conditions

The model represents an established firm in an industry composed of a number of firms selling differentiated but related products. Given K_n firms in the industry at the end of period n-1, an established firm chooses a period n limit price $p_n(K_n)$ conditional on that number of firms. Based on the limit price and the number of firms in the industry, k_n potential entrants decide to enter the market in period n. The established firm is assumed to be able to alter its price for the next period and hence chooses a price $p_{n+1}(K_{n+1})$ for period n+1 where $K_{n+1} = K_n + k_n$. The profit of the established firm in period n

is assumed to depend on the number of firms in the industry in that period with profit if no firms enter $(K_{n+1} = K_n)$ denoted by $\pi_n(p_n(K_n),K_n)$. If $k_n>0$ firms enter, period n profit is assumed to be reduced. The short-term reaction of the established firm to entry may be characterized in a variety of manners such as those discussed by Baron. Here the result of that short-term reaction will be reflected in a post-entry profit $\pi_n(K_{n+1}) < \pi_n(p_n(K_n), K_n)$ for all $K_{n+1} > K_n$ where $\pi_n(K_{n+1})$ does not depend on the pre-entry price and is assumed to be constant. The post-entry profit may be understood to represent the established firm's initial reaction to entry, while its complete reaction is to choose a price $p_{n+1}(K_{n+1})$ for the next period. The assumption that the profit after entry is less than that prior to entry is in the spirit of the empirical studies of Bain and Mann that indicate a positive relationship between profitability and concentration and agrees with the behavior of classical Cournot markets with entry as considered by C. R. Frank, for example. In addition to the profit earned in the industry in question the established firm may earn profit from operations in other industries and that profit is denoted by R_{n} .

The number of firms that will decide to enter the industry in a period is uncertain, so the established firm is assumed to assess a (subjective) probability distribution on the number of potential entrants that will actually enter the market in each period. The probability of entry is a function of the assessed profitability to the potential entrant from being in the industry. One possible assumption is that the probability of entry depends on the profit earned by the established firms. This assumption will not be

employed, however, since it is doubtful that a potential entrant would be able to observe the profit of an established firm, particularly for a multi-product firm. Also, the profit to an entrant may well be different from that of an established firm because of cost differences and the effect of product differentiation. A potential entrant, however, is able to observe the price charged by an established firm, and that price is an important indicator of the profitability to the potential entrant. No implication is made, however, that a potential entrant believes that the current price will be in effect after entry has taken place. The potential entrant also is likely to believe that the number of firms in the industry affects profitability with a decrease in concentration reducing the likelihood that entry will be profitable.

The probability $G_n\left(k_n\middle|p_n\left(K_n\right),K_n\right)$ that k_n firms enter in period n thus is a function of the price $p_n\left(K_n\right)$ and of the number K_n of firms in the industry. The probability of no more than a specified number m of firms entering the market is assumed to be decreasing in p_n and increasing in K_{n-1} or more formally

(1)
$$\sum_{k_n=0}^{m} G_n(k_n|p_n,K_n) < \sum_{k_n=0}^{m} G_n(k_n|p_n^*,K_n) \quad \text{for } p_n > p_n^* \quad \text{and}$$

for
$$m = 0, 1, 2, ..., n=1, 2, ...$$

(2)
$$\sum_{k_{n}=0}^{m} G_{n}(k_{n}|p_{n},K_{n}) > \sum_{k_{n}=0}^{m} G_{n}(k_{n}|p_{n},K_{n}^{*})$$
 for $K_{n} > K_{n}^{*}$ and

for
$$m = 0, 1, 2, ..., n=1, 2, ...$$

Consequently, as price increases entry is more likely, 7 and as the

number of firms in the industry increases entry is less likely. In the terminology of stochastic dominance (see J. Hadar and W. R. Russell and G. Hanoch and H. Levy) the distribution in (1) conditional on p_n^* is dominated in the first-degree by the distribution for p_n and similarly in (2) the distribution for K_n is dominated in the first degree by that for K_n^* . In order to avoid paradoxical results, it also is postulated that the industry is such that it is not optimal to encourage firms to enter in the current period in order to reduce the probability of future entry. Similarly, at the price that maximizes the current-period profit, the probability of entry is taken to be positive.

The established firms are assumed to maximize the expected utility of profit over an infinite horizon with a utility function that is temporally additive. Let U_n be a monotone increasing, concave, twice continuously differentiable, cardinal utility function for period n and let $\beta \in (0,1)$ reflect time preference. The optimal expected utility $F_n(K_n)$ for periods $n,n+1,n+2,\ldots$ depends only on the number of firms in the industry at the end of period n-1 and is defined as

(3)
$$F_{n}(K_{n}) = \max\{ (1 - \sum_{k_{n}=1}^{\infty} G_{n}(k_{n} | p_{n}(K_{n}), K_{n})) [U_{n}(\pi_{n}(p_{n}(K_{n}), K_{n}) + R_{n}) + \sum_{k_{n}=1}^{\infty} G_{n}(k_{n} | p_{n}(K_{n}), K_{n}) [U_{n}(\pi_{n}(K_{n} + k_{n}) + R_{n}) + R_{n}] + \sum_{k_{n}=1}^{\infty} G_{n}(k_{n} | p_{n}(K_{n}), K_{n}) [U_{n}(\pi_{n}(K_{n} + k_{n}) + R_{n}) + R_{n}] + \sum_{k_{n}=1}^{\infty} G_{n}(k_{n} | p_{n}(K_{n}), K_{n}) [U_{n}(\pi_{n}(K_{n} + k_{n}) + R_{n}) + R_{n}]$$

where F_{n+1} (K_{n+1}) is the optimal expected utility in periods n+1, n+2, ..., and is similarly defined. Given that profit is decreasing in the number of firms in the industry and that future entry is preferred to

present entry, $F_{n+1}(K_{n+1})$ may be shown to be decreasing in the number of firms in the industry. The profit function is assumed to be concave and continuously differentiable, and for $k_n \ge 1$, G_n is assumed to be continuously differentiable. The optimal price $\hat{p}_n(K_n)$ satisfies the first-order condition⁸

(4)
$$(1 - \sum_{k_{n}=1}^{\infty} G_{n})U_{n}^{\dagger}\pi_{n}^{\dagger} - \sum_{k_{n}=1}^{\infty} G_{n}^{\dagger}[U_{n}(\pi_{n}(\hat{p}_{n}(K_{n}), K_{n}) + R_{n}) + \beta F_{n+1}(K_{n})]$$

 $- U_{n}(\pi_{n}(K_{n} + k_{n}) + R_{n}) - \beta F_{n+1}(K_{n} + k_{n})] = 0,$

where $(1 - \sum_{k=1}^{\infty} G_{n})$ is the probability of no entry. The argument

of the functions will be omitted when the meaning is clear. The term in brackets, denoted by $\mathbf{M}_n(\mathbf{K}_n+\mathbf{k}_n)$ is the difference between the optimal expected utility without entry in period n and optimal expected utility with entry. Since $\pi_n(\hat{\mathbf{p}}_n(\mathbf{K}_n),\mathbf{K}_n) > \pi_n(\mathbf{K}_n+\mathbf{k}_n)$ and $\mathbf{F}_{n+1}(\mathbf{K}_n) > \mathbf{F}_n(\mathbf{K}_n+\mathbf{k}_n)$ for $\mathbf{k}_n \geq 1$, $\mathbf{M}_n(\mathbf{K}_n+\mathbf{k}_n)$ is positive and increasing in \mathbf{k}_n . The change in $\mathbf{M}_n(\mathbf{K}_n+\mathbf{k}_n)$ as the distribution of \mathbf{k}_n shifts to the right (in the sense of first-degree stochastic dominance) is

$$k_{n}^{\Sigma} = 1 + k_{n}^{G'_{n}} M_{n} (K_{n} + k_{n}) = \sum_{k_{n}=0}^{\infty} G'_{n} [U_{n} (\pi_{n} (\hat{p}_{n} (K_{n}), K_{n}) + R_{n}) + \beta F_{n+1} (K_{n})]$$

$$-U_{n}(\pi_{n}(K_{n}+k_{n})+R_{n}) - \beta F_{n+1}(K_{n}+k_{n})],$$

where $\pi_n(K_n+k_n)\equiv\pi_n(\hat{p}_n(K_n),K_n)$ and $F_{n+1}(K_n+k_n)\equiv F_{n+1}(K_n)$ for $k_n=0$. This implies that $\sum\limits_{k_n=1}^{\infty}G_nM_n(K_n+k_n)$ is increasing in $k_n=1$ of K_n and hence that $\sum\limits_{k_n=1}^{\infty}G_nM_n(K_n+k_n)>0$. To satisfy (4) the optimal price is such that the probability of no entry $(1-k_n)=1$ of K_n is positive

and since marginal utility is positive, marginal profit is positive. Consequently, the established firm prices below the price that maximizes pre-entry profit and prices less than the price that makes the probability of entry equal to one. These results hold for any monotone increasing utility function and thus are due to the probabilistic nature of entry and not to the risk preferences reflected by the utility function.

Harrod suggested that if entry is easy the established firm will price equal to average cost in order to prevent entry. If the firm follows this practice in every period, there is no difference between the utility with and without entry, so $M_n(K_n) = 0$. The optimal limit price thus is higher than the price that equals average cost unless, of course, marginal profit is zero at that price. The probabilistic equivalent of Harrod's entry preventing price may be defined as the price $p_n^e(K_n)$ that makes the probability of no entry equal to one or $G_n(k_n=0|p_n^e(K_n),K_n)=1$. The optimal limit price is at least as great as the entry-preventing price, since otherwise an increase in price would increase profit and leave the probability of entry equal to This result is a prediction of D. K. Osborne's finding that entry has occurred in industries using limit pricing. The deterministic entry-preventing price may also be thought of as the lowest price the established firm could set without finding it preferrable to leave the industry. One such price $p_n^{\star}(K_n)$ satisfies

$$(5) \quad (1 - \Sigma G_n) U_n (\pi_n (p_n^*(K_n), K_n) + R_n) + \Sigma G_n U_n (\pi_n (K_n + k_n) + R_n) = U_n (R_n).$$

The entry-preventing price may be less than $p_n^\star(\textbf{K}_n)\text{, so the established}$

firm may find it impossible to stay in the industry and prevent entry.

II. Risk Aversion and the Optimal Limit Price

The established firm's limit pricing decision involves the risk that profit will be reduced by entry, and the risk preferences reflected in the firm's utility functions \mathbf{U}_n affect the level of the limit price. Risk preferences will be measured by the Arrow-Pratt index of absolute risk aversion $\mathbf{r}_{\mathbf{U}_n}(\mathbf{y}) = -\mathbf{U}_n''(\mathbf{y})/\mathbf{U}_n'(\mathbf{y})$, where \mathbf{U}_n'' and \mathbf{U}_n' are the second and first derivatives, respectively. The interpretation of the index is that if $\mathbf{r}_{\mathbf{U}_n}(\mathbf{y})$ increases for all \mathbf{y} , as with a shift in risk preferences, the certainty equivalent associated with the risk increases. The certainty equivalent CE_n of period n profit is defined by

(6)
$$U_{n} (CE_{n}+R_{n})^{\pm} (1-\sum_{k_{n}=1}^{\infty} G_{n}) U_{n} (\pi_{n} (\hat{p}_{n}(K_{n}),K_{n})+R_{n}) + \sum_{k_{n}=1}^{\infty} G_{n} U_{n} (\pi_{n} (K_{n}+k_{n})+R_{n})$$

and is the minimum amount the firm would accept in exchange for the period n ex ante profit.

Given certain properties of the probability distribution, the optimal period n price may be shown to be a decreasing function of risk aversion. That is, let U_n^1 and U_n^2 be two utility functions such that $r_{U_n^1}(y) \geq r_{U_n^2}(y)$ for all y and $r_{U_n^1} > r_{U_n^2}(y)$ for some y with positive probability, and let $\hat{p}_n^1(K_n) > 0$ be optimal for U_n^1 and $\hat{p}_n^2(K_n) > 0$ be optimal for U_n^2 . Rewrite (4) for U_n^2 evaluated at $\hat{p}_n^2(K_n)$ as

(7)
$$(1-\Sigma G_n) + (-\Sigma G_n' \varphi_n^2 (k_n) = 0,$$

where
$$\varphi_n^2(k_n) = M_n^2(K_n + k_n) / U_n^2 \cdot (y^2)$$
 and $y^2 = \pi_n(\hat{p}_n^2(K_n), K_n) + R_n$. Pratt

(Eq. 22, p. 129) has shown that if $r_{U_n}^1(y) > r_{U_n}^2(y)$ for all y, then $(U_n^1(w) - U_n^1(v))/U_n^1(y) > (U_n^2(w) - U_n^2(v))/U_n^2(y)$ for $y \ge w > v$. To maintain the intertemporal relationship between U_n and U_j , $j \ne n$, scale the utility function U_n^1 so that $U_n^2(y^2) = U_n^1(y^2)$. An increase in risk aversion thus implies that $\phi_n^1(k_n) > \phi_n^2(k_n)$ for $k_n \ge 1$. A sufficient, but not necessary, condition for risk aversion to decrease the optimal limit price is that $G_n^1 \ge 0$ for all $k_n \ge 1$, 11 since then (7) for U_n^1 is negative evaluated at $\hat{p}_n^2(K_n)$, so $\hat{p}_n^2(K_n) > \hat{p}_n^1(K_n)$. A necessary condition is that $\Sigma G_n^1(\phi_n^1(k_n) - \phi_n^2(k_n))|\hat{p}_n^2(k_n)$. To simplify the following discussion, G_n^1 is assumed to be nonnegative which implies that an increase in price increases the probability that $k_n \ge 1$ firms enter.

The interpretation of this result is that as the established firm becomes more risk averse it prefers to exchange period n profit for a lower probability of entry and thus reduces price in period n. The risk to the established firm is that potential entrants will actually enter, and the more averse to risk the firm becomes the more willing it is to sacrifice current profit for future profit. The effect of risk aversion on the limit price makes intuitive sense and may help explain the differences in profit between industries found by Bain, Mann, and I. N. Fisher and G. R. Hall. Certainly many other factors such as those considered in the following section on barriers to entry are important as determinants of the limit price, but the analysis here indicates that risk aversion may act to lower the limit price and then has an effect similar to a barrier to entry in that both tend to make entry less likely. Increased risk aversion also reduces the certainty equivalent.

The effect of profits from other activities $\mathbf{R}_{\mathbf{N}}$ on the limit

price may be determined directly from the risk aversion result. A utility function is said to exhibit decreasing (increasing) (constant) absolute risk aversion if $r_{U}(y)$ is a decreasing (increasing) (constant) function of y. An increase in R_n for U_n exhibiting decreasing (increasing) (constant) absolute risk aversion consequently results in an increase (decrease) (no change) in the optimal limit price. L2 K. J. Arrow has argued that decreasing absolute risk aversion is a reasonable behavioral assumption. For U_n decreasingly absolute risk averse an increase in R_n decreases risk aversion, and the firm is willing to accept a higher probability of entry in exchange for greater profit if no entry occurs. L3

III. Barriers to Entry and Concentration

The principal barriers to entry considered by Bain are:

1) economies of large scale operations, 2) absolute cost advantages, and 3) product differentiation advantages. One general way to consider the effects of barriers to entry on the limit price is to assume that they are reflected by the established firm's assessment of the probability of entry. An increase in the height of barriers to entry will be represented by a shift to the left in the distribution function of k_n for all prices and for all K_n . For a given $p_n(K_n)$ and K_n , higher barriers to entry thus are assumed to increase the probability that fewer than m firms enter for m = 1,2,..., and hence result in a distribution that dominates in the first degree the distribution for lower barriers to entry. Such a change in the probability distribution of k_n affects both the established firm's certainty equivalent and its optimal price. Since period n utility

and future expected utility $F_{n+1}(K_{n+1})$ are decreasing in k_n , an increase in the height of the barriers to entry increases the expected utility in (3). The established firm would then be less likely to leave the industry and would require a greater payment for the sale of its activities in the industry. Empirical work by Bain and Mann supports the hypothesis that average profits are higher with "very high barriers" than with lower barriers. In the context of the probabilistic model considered here expected utility and the certainty equivalent profit increase as barriers become "higher."

While higher barriers to entry increase the expected utility of the established firm, the optimal price may increase or decrease. In (4) the probability of no entry $(1-\Sigma G_n)$ increases with higher barriers to entry but the G_n' terms also are affected. If $G_n'/(1-\Sigma G_n)$ is not increased by the higher barriers as would be the case if $G_n' \geq 0$, then (4) evaluated at the price optimal with lower barriers to entry is positive and the optimal price is increased. The term $G_n'/(1-\Sigma G_n)$ is referred to as the "hazard rate" in reliability theory, and if an increase in the barriers to entry reduces the hazard (of entry) rate, the optimal price increases. A Bain [1956] has argued that the higher are the barriers to entry the closer the limit price is to the profit maximizing price, and for the model considered here a reduction in the hazard rate for all $k_n \geq 1$ is a sufficient, but not a necessary, condition for this result.

Concentration in the industry affects the established firm by influencing profit and the probability of entry. Holding profit constant, an increase in concentration, such as would occur from firms leaving the industry, affects the probability of entry by

shifting the distribution function of the number of entrants to the right as indicated in (2). Such a shift decreases expected utility in agreement with the studies of Bain, Mann, and Rhoades and decreases the limit price if the hazard rate is increased. Changes in concentration also affect profit, however, so the net effect of exit from the industry is difficult to determine without further assumptions.

The effect of economies of large-scale operations on entry may also be reflected by changes in the probability of entry. Modigliani (p.220) concluded that "[the 'highest entry-preventing price'] will tend to be higher the steeper the cost curve, that is the greater the economies of scale." Similarly, M. Hall and L. Weiss found that profitability is positively related to the capital requirements barrier to entry. The effect of such a barrier is to shift to the left the distribution function of the number of firms entering. The shift increases the expected utility giving a theoretical prediction of Hall and Weiss' findings. If the shift reduces the hazard rate, the limit price increases as Modigliani suggested.

Absolute cost advantages may have two types of effects on the pricing policies of an established firm. First, the knowledge on the part of potential entrants that an established firm has achieved a cost reduction, through a technological advance, for example, may reduce the assessed probability of entry. Second, a reduction in cost to an established firm may affect the limit price independent of the probability distribution. For example, suppose that an established firm achieves a reduction in fixed costs in period n but that the potential entrants are unaware of it. A reduction in fixed costs in period n has the same effect as an increase in profits R_n from other

activities, so a reduction in fixed costs results in an increase (decrease) (no change) in the optimal limit price in period n if the utility function U_n exhibits decreasing (increasing) (constant) absolute risk aversion. A decrease in a constant marginal cost has both a risk aversion effect and a "cost effect." This risk aversion effect occurs because for a fixed output a reduction in marginal cost is equivalent to a decrease in fixed cost. As in the deterministic theory of the firm, the cost effect tends to decrease price by altering the relationship between marginal revenue (utility) and marginal cost. For nondecreasing absolute risk aversion, a decrease in a constant marginal cost results in a decrease in the limit price, 15 but with decreasing absolute risk aversion the limit price may increase if the risk aversion effect exceeds the cost effect.

Bain found that product differentiation, primarily in the form of advertising, played the most important role in building and sustaining barriers to entry in consumer industries. Utilizing assumptions similar to those for price, ¹⁶ the optimal product differentiation expenditures may be shown to be greater than the expenditures that maximize current period profit. The firm thus again is willing to exchange current profit for a lower probability of entry. If product differentiation expenditures decrease the hazard rate in (4), the optimal limit price increases.

The impact of industry growth on the behavior of both established firms and potential entrants is an important issue in industrial organization, and in the context of the model presented in this paper growth may affect both profit and the assessed probability of entry. Since shifts in the distribution function of $k_{\rm p}$ have already been

considered, only changes in future profit will be considered at this point with the probability distribution for a given price and K_n assumed fixed. If growth increases the difference in future expected utility with and without entry ($[F_{n+1}(K_n)-F_{n+1}(K_n+k_n)]$) and $G_n' \geq 0$ for $k_n \geq 1$, the term $\Sigma G_n'[F_{n+1}(K_n) - F_{n+1}(K_n+k_n)]$ increases and the optimal limit price in period n is decreased to reduce the probability of future entry. If growth lessens the difference, the optimal price will be increased, since entry causes less of a reduction in profitability.

IV. Conclusions

Potential entry has been represented in a probabilistic manner with the probability of entry dependent on the price charged by an established firm and on the number of firms in the industry. With this probabilistic view of entry the optimal limit price is less than the price that maximizes pre-entry profit and is greater than the price that equates price and average cost. Since the limit pricing decision involves uncertain profits, it seems natural to ask how risk preferences affect the limit price. Given certain properties of the probability distribution, an increase in risk aversion in period n as measured by the Arrow-Pratt index results in a reduction in the period n limit price, since the firm is willing to accept a reduced current period profit in exchange for a lower probability of entry. Barriers to entry may be reflected in the assessed probability of entry with higher barriers being represented by a shift to the left in the distribution function of the number of actual entrants. Higher barriers to entry result in an increased expected utility, and if the

hazard rate is reduced, the optimal limit price is increased.

The empirical research on barriers to entry, concentration, and risk adjusted profitability has been characterized by debates regarding what it is that is actually being measured. For example, see the recent papers by I. N. Fisher and G. R. Hall, Y. Brozen, H. M. Mann (1971), and R. E. Caves and B. S. Yamey. The results contained herein indicate the individual effects of concentration, barriers to entry, probability assessments, and risk aversion on profitability and price, but attempting to identify and isolate individual effects from empirical data is likely to be extremely difficult.

Footnotes

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- 1. The established firm also may be thought of as a single monopolist or as a cartel.
- 2. Firms may enter any time before the end of the period, but to simplify the notation and the analysis all entrants will be assumed to enter at the same point in time.
- 3. The length of the period may be considered to be defined by the time required for the established firm to respond to entry.
- 4. Kamien and Schwartz assume that post-entry profit is constant and the same in all periods after entry.
- 5. R. Sherman and T. D. Willett utilized an expected utility maximization model for determining if a potential entrant should or should not enter.
- 6. The assumption that entry depends on the pre-entry profit of established firms has been employed by Pyatt and Hahn. The sometimes paradoxical implications of such an assumption have been explored by Baron.
- 7. This probabilistic treatment of entry is equivalent to Hicks' case of oligopolistic expectations in which an increase in "close" period output was assumed to increase open period revenue by reducing the "amount" of entry. Hicks' polypolistic expectations may be represented by replacing the inequality in (1) by an equality.
- 8. The second-order condition is $(1-\Sigma G) \left(U'''''' + U'''''' \right) 2\Sigma G' U''''' \Sigma G'' M \left(K_- + k_- \right) < 0$

- where $\mathrm{M}_n(\mathrm{K}_n+\mathrm{k}_n)$ is defined below. A sufficient condition for $\hat{\mathrm{p}}_n(\mathrm{K}_n)$ to yield a maximum is that $\mathrm{G}_n' \geq 0$ and $\mathrm{G}_n'' \geq 0$. The second-order condition is assumed to be satisfied at $\hat{\mathrm{p}}_n(\mathrm{K}_n)$.
- 9. Kamien and Schwartz obtained the same result in a stationary, probabilistic model. They also demonstrate that the pre-entry profit is constant over time, and a similar result obtains for the model here if U_n , G_n , and R_n are the same in every period.
- 10. Hicks' polypolistic expectations implies that $G_n'=0$ for all k_n , and then the established firm maximizes pre-entry profit.
- 11. The condition that $G_n' \ge 0$ for $k_n \ge 1$ implies, but is not implied by, first-degree stochastic dominance.
- 12. This result obtains by letting $U_n^1(y) = U_n(y)$ and $U_n^2(y) = U_n(y+\Delta y)$, $\Delta y > 0$, and using the preceding analysis.
- 13. If a potential entrant must pay to an established firm an entry fee in the form of the purchase of the rights to a patented process, for example, R_n is increased and the firm with decreasingly absolute risk averse risk preferences will raise its limit price which increases the probability of further entry.
- 14. Kamien and Schwartz assume that the hazard rate is increasing in the price charged by the established firm.
- 15. This result requires that post-entry output is decreasing in k_n .
- 16. An increase in product differentiation expenditures is assumed to shift the probability distribution of the number of firms that enter to the left.

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