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TOWARDS A CONSISTENT COMPARISON BETWEEN ALTERNATIVE EXCHANGE RATE SYSTEMS

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So far there has been no unified framework in which alternative exchange rate systems could be compared on an even basis. There are four elements which, we believe, constitute an essential part of such a framework. First, the economy's real overall resource constraint should be independent of the exchange rate system. Second, demand functions should be derived from intertemporal utility maximization, and these utility functions should be used for welfare evaluations. Third, when discussing a floating exchange rate system one should consider only exchange rate patterns which fulfill an appropriate market clearing condition. This means that one should not assume a given distribution of exchange rates, because this distribution is endogenous to the economy. Fourth, for a small country, the economy's transaction opportunities with the rest of the world should not depend on the exchange rate system.

In this paper we construct and analyze simple models of a small open economy which contain these ingredients. These models include money, self-fulfilling expectations, uncertainty elements which are both internal and external to the economy, and international (financial) capital movements.

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We find that some comparisons between fixed and floating exchange rate systems that were done in the literature, implicitely assumed different opportunities to transact with the outside world, and that this accounts for the differences in outcomes that were found in the two systems. Moreover, when the differences in opportunities are removed, the resulting differences in outcomes disappear.

We also find that a permanently fixed exchange rate system is dominated by a system in which the exchange rate declines at a constant rate which is equal to a sure rate of return on a foreign asset, independent of the existence and source of uncertainty. In a world of floating exchange rates among the foreign countries, we find that it does not matter with respect to which currency the interest parity result applies.

We relate the interest parity result to the optimum quantity of money rule. Then, we identify an externality effect in the financing of deficits (surpluses) in the balance of payments, which prevents the attainment of this rule in a fixed exchange rate regime.

The exchange rate pattern which satisfies the interest parity rule is shown to be an equilibrium pattern in a floating exchange rate system. However, this is not the unique equilibrium pattern, which suggests that -- for a small country--a managed exchange rate system in which the exchange rate is changed according to the interest parity rule may be superior to both a permanently fixed and a floating exchange rate regime.

Our work should be considered as an attempt to compare alternative exchange rate systems in a systematic way. We have not taken into account important aspects like monetary disturbances, monetary and fiscal policy, and price rigidities, and we do not know to what extent our results will be affected by these considerations.

I

Consider a simple one-consumer three-period model of an open small economy without production. Each period foreign prices and the country's commodity endowments are given. Domestic residents may hold both domestic money and a foreign asset (liability). The two types of assets serve only as stores of value; i.e., they enable transfers of purchasing power from one period to another.

We begin by assuming that the foreign asset is a zero interest bearing bond, and we allow both positive and negative holdings. Negative holdings mean that the economy borrows from abroad while positive holdings mean that the economy lends to foreigners. Holdings of local currency have to be non-negative.

We assume that the economy starts with a given quantity of domestic money and that it has to end up without foreign debt and with the same quantity of domestic money. The terminal condition on domestic money is imposed in order to make meaningful the comparison between a fixed and a floating exchange rate system. To see this observe that under a fixed exchange rate without this restriction, the economy gets rid of all its local money in the third period, because then money has no future value. This is possible because the exchange rate authority (which buys and sells foreign currency at a fixed exchange rate) is willing to provide foreign currency in exchange for domestic currency, and the foreign currency can be used to purchase commodities abroad. Under a floating exchange rate this is impossible, and the attempts to get rid of local currency drive the third period exchange rate to infinity. However, second period expectations of an infinite exchange rate in the third period make people try to get rid of local currency in the second period. This drives the second period exchange rate also to infinity. By a similar argument the first period exchange rate is also driven to infinity. Hence, in order to avoid exchange rate indeterminancy, we need a terminal condition. [1/] (A terminal condition of this kind is needed in every finite-horizon model. We have chosen to use a finite-horizon model for expositional purposes. This terminal condition assures an intertemporal balanced budget for the economy.)

We assume that the consumer knows in the first period the correct prices, exchange rates, and endowments for every period. Hence, we assume self-fulfilling expectations. The consumer (private sector) solves in the first period the following standard intertemporal optimization problem:

(1) Choose $c^1, c^2, c^3, M^1, M^2, A^1, A^2$ to maximize

$$u(c^1, c^2, c^3)$$

subject to

$$\text{(i)}\quad e^1p^1c^1 + e^1A^1 + M^1 = e^1p^1y^1 + M$$

$$\text{(ii)}\quad e^2p^2c^2 + e^2A^2 + M^2 = e^2p^2y^2 + M^1 + e^2A^1$$

$$\text{(iii)}\quad e^3p^3c^3 + M = e^3p^3y^3 + M^2 + e^3A^2$$

$$\text{(iv)}\quad c^1, c^2, c^3, M^1, M^2 \geqq 0$$

where

1/ The above terminal condition is not the only possibility. We could assume, for example, that under both a fixed and a floating exchange rate the local government buys in the last period domestic money at a predetermined exchange rate. Our main results do not change if we adopt this assumption.

c^i = consumption vector in period i

M = domestic stock of money at the beginning of period 1; has to be returned in the last period

M^i = domestic money holdings at the end of period i and the beginning of period $i + 1$

A^i = foreign asset holdings at the end of period i and the beginning of period $i + 1$

e^i = exchange rate in period i

p^i = period i price vector in terms of foreign currency; exogeneous to the small country

y^i = period i endowment vector

We do not present the second and third period problems. Basically, they do not differ from the first period problem. The consumer arrives at the second period with M^1 and A^1 that were chosen in the first period. Then he chooses the second period consumption vector, a plan for third period consumption, and a portfolio, so as to maximize his utility, given c^1 that was chosen in the first period. In the third period a consumption vector is chosen, and the money stock M as well as foreign debt are returned. Since expectations are self-fulfilling, the second and third period decisions coincide with the first period plans.

In the fixed exchange rate regime, we assume that the exchange rate authority (or the government) chooses to stabilize the exchange rate at a fixed level e (i.e., $e^i = e$ for $i = 1,2,3$). This is done by means of purchase and sale of foreign exchange at e units of local currency per unit of foreign exchange.

In a floating exchange rate regime the vector (e^1, e^2, e^3) is said to be an equilibrium exchange rate pattern if there exists a solution to (1) such that $M^i = M$ for $i = 1,2$.

Since we assume a passive monetary policy, the quantity of money does not change in a floating exchange rate regime. It can however change in a fixed exchange rate regime by means of trading with the foreign exchange rate authority; i.e., by means of deficits and surpluses in the balance of payments.

Since foreign asset holdings can be both positive and negative, the three single period budget constraints, (i) - (iii), in (1) can be reduced to a single budget constraint:

$$(2) \qquad \sum_{i=1}^{3} p^i(c^i - y^i) = M/e^1 - M^1/e^1 + M^1/e^2 - M^2/e^2 + M^2/e^3 - M/e^3$$

On the left hand side of (2) we have the accumulated deficit in the balance of trade in terms of foreign currency, while on the right hand side we have the accumulated deficit in the balance of payments in terms of foreign currency. This means that the accumulated deficit in the capital account is always zero, which stems from the fact that all foreign debt is repaid.

Now, in a fixed exchange rate regime; i.e., $e^i = e$ for $i = 1,2,3$, the RHS of (2) is zero, independent of the exchange rate level and the levels of domestic money holdings. This means that the accumulated loss of reserves is also zero. 2/ Hence, the economy is

2/ Note that this results from the assumption that the initial stock of money has to be returned at the end of the third period.

constrained to choose its consumption schedule from a set in which the accumulated deficit in the balance of trade equals zero. This is indeed the real constraint on the economy.

In a floating exchange rate regime the private sector holds in equilibrium the initial stock of money in every period; i.e., in equilibrium $M^i = M$ for $i = 1,2,.$ Therefore, in this case the deficit in the balance of payments is zero in every period (but not the balance of trade), and so is the accumulated deficit in the balance of payments. Hence, the accumulated deficit in the balance of trade is also zero.

It is therefore clear that the real constraint on the choice of consumption schedules is the same in both exchange rate regimes, which implies that in this case the economy attains the same welfare level in both systems. (This result holds also in the presence of uncertainty.)

Observe now that a fixed exchange rate; i.e., e^i = constant for $i = 1,2,3,$ is the only equilibrium exchange rate pattern in a floating exchange rate regime. For, if the exchange rate is expected to increase the demand for local money drops to zero,while if the exchange rate is expected to decrease the demand for local money goes to infinity. This stems from the fact that in the former case the foreign asset dominates domestic money as a store of value, while in the latter case local money dominates the foreign asset as a store of value, and both assets perform here only the role of stores of value. In equilibrium the two assets become perfect substitutes as stores of value.

In a floating exchange rate system the private sector makes all its borrowing and lending transactions directly with foreigners, while in a fixed exchange rate system part of these transactions is done with the exchange rate authority (provided that the quantity of money is not the same in all periods). Since the exchange rate authority needs foreign currency in order to stabilize the exchange rate, then if it is assumed that it borrows from foreigners whenever it needs to buy local currency and it lends to foreigners whenever it needs to sell local currency, the private sector's transactions with the exchange rate authority are in fact indirect transactions with foreigners. In this case the exchange rate authority serves only as an intermediary between the private sector and foreigners. [3/]

This argument points out a difficulty in some comparisons of fixed versus floating exchange rate systems that were done in the literature. Suppose, as it is often done, that the private sector cannot borrow and lend abroad (there are no international capital movements). This means that we add the constraint $A^1 = A^2 = 0$.

Suppose now that the solution to (1) (without the constraint $A^1 = A^2 = 0$) is such that for the fixed exchange rate case $M^i + eA^i \geq 0$ for $i = 1,2$; i.e., the private sector's financial wealth is non-negative in each period. (This assumption is not needed for the main argument that follows, but it helps to sharpen it.) Then, the additional constraint $A^1 = A^2 = 0$ does not effect the optimal consumption schedule in the fixed exchange rate regime. The only difference is that with capital movements the private sector can hold

3/ This argument brings out a possible advantage of a fixed exchange rate over a floating exchange rate system. Suppose that the local government can borrow abroad cheaper than the private sector. This is often the case, and especially for developing countries. In this case a fixed exchange rate system is preferred to a floating one. Nevertheless, if the government borrows abroad cheaper than the private sector and it provides the private sector with these loans on a break-even basis, then again, there is no advantage of one exchange rate system over the other.

part of its wealth in terms of foreign assets while now it has to hold it only in terms of domestic currency. Therefore, if in the case of free capital flows the private sector does not wish to spend in each period his entire income on goods, the quantity of money will change from period to period in the case of no capital mobility.

Now,in a floating exchange rate system without capital mobility,the exchange rate pattern which makes the private sector hold in each period the initial stock of money makes also the balance of trade equal to zero in every period. To achieve this, the exchange rate has to depreciate at the rate of time preference. In this case the welfare level will be lower if in the case of free capital mobility some trade imbalance is desired.

The difference between the exchange rate systems that emerges in the case of no capital movements results from the fact that in the floating system no foreign borrowing and lending is allowed, while in the fixed system indirect foreign borrowing and lending is allowed by means of the foreign exchange rate authority. This is why in this case a fixed exchange rate system provides a higher welfare level than a floating one. Here, we obtain also the often mentioned distinction between the systems; in one the quantity of money fluctuates,while in the other the exchange rate fluctuates.

We maintain that this type of comparison between the exchange rate systems is not appropriate, since foreign borrowing and lending is prohibited in one case and implicitly permitted in the other. The above described differences in outcomes should not be attributed to differences in exchange rate regimes, but rather to the assymetric assumptions regarding foreign borrowing and lending.

Consider, for example, Fischer's comparison of the fixed versus floating exchange rate regimes in section I.B of Fischer (1976). He uses the following model:

$$L_t = kP_tY_t$$

$$P_t = P^*e_t$$

$$B_t = M_t - M_{t-1}$$

$$M_t - M_{t-1} = \alpha(L_t - M_{t-1}), \qquad 0 < \alpha < 1$$

$$C_t = P_tY_t - B_t$$

$$Y_t = Y + u_t$$

where L is the demand for money, P the domestic price level, Y domestic income and output, P^* the foreign price level, e the exchange rate, B the balance of payments (in domestic currency), α an adjustment coefficient, C consumption, and u is a random variable with zero mean, variance σ_u^2 and is serially uncorrelated.

Analyzing the steady state of this model for fixed and floating exchange rate regimes, Fischer concludes:

> "...the variance of consumption in the fixed rate regime is less than that in the floating rate regime. This result reflects the shock absorber role of the balance of payments under fixed rates. Since there is also no variance of prices under fixed rates, it is clear that fixed rates are preferable if disturbances are real." (p. 7)

In this model, a floating exchange rate equilibrium is one in which $M_t = M$ for all t. This implies that real consumption equals real income $Y + u_t$ for all t. Hence, there is no possibility to transfer purchasing power from one period into another; variations in income are fully reflected in variations in consumption.

In the steady state equilibrium of the fixed exchange rate system purchasing power can be transferred from one period into another by means of the exchange rate authority, which amounts to indirect foreign borrowing and lending. The exchange rate authority's budget is balanced on <u>average</u>; i.e., the expected value of B_t is zero, because in the steady state M_t has the same distribution, and thus the same expected value, as M_{t-1}.

If the demand for money reflects in this example intertemporal maximizing behaviour, then irrespective of variance comparisons one can argue that the fixed exchange rate regime is preferred to the floating regime, simply because the feasible (intertemporal) consumption set of the former includes that of the latter. This does not result from differences in the exchange rate regimes but rather from differences in the implicit assumptions regarding foreign borrowing and lending. Once we introduce the possibility of foreign borrowing and lending into the floating exchange rate regime, these differences disappear.

II

Let us now modify our model to include positive interest, $r > 0$,on the foreign asset,and liquidity services of domestic money holdings. We assume, therefore, that the utility level depends, in addition to consumption, on average real money balances in every period. In order to avoid index-number problems, we write the utility level as a function of consumption, average nominal money balances, and nominal prices, assuming that this function is homogenous of degree zero in average money holdings of period i and nominal prices of period i, for every $i = 1,2,3$.

Now the first-period consumer's problem is: [4/]

(3) Choose $c^1, c^2, c^3, M^1, M^2, A^1, A^2$ to maximize

$$u\left(c^1, c^2, c^3; \frac{1}{2}\,\frac{M+M^1}{e^1}\,,\ \frac{1}{2}\,\frac{M^1+M^2}{e^2}\,,\ \frac{1}{2}\,\frac{M^2+M}{e^3}\,;\ p^1, p^2, p^3\right)$$

subject to

(i) $e^1p^1c^1 + e^1A^1 + M^1 = e^1p^1y^1 + M$

(ii) $e^2p^2c^2 + e^2A^2 + M^2 = e^2p^2y^2 + M^1 + e^2RA^1$

(iii) $e^3p^3c^3 \qquad + M = e^3p^3y^3 + M^2 + e^3RA^2 - T$

(iv) $c^1, c^2, c^3, M^1, M^2 \geqq 0$

where $R = 1 + r$, and T stands for taxes imposed in the last period,whose role we shall explain later.

4/ In writing the utility function, we have taken advantage of the homogeneity assumption.

Since there is no sign restriction on foreign asset holdings, we can again reduce the single period budget constraints (i) - (iii) to one constraint:

(4) $$\sum_{i=1}^{3} R^{3-i}p^i(c^i - y^i) = R^2M/e^1 - R^2M^1/e^1 + RM^1/e^2 - RM^2/e^2 + M^2/e^3 - M/e^3 - T/e^3$$

The left hand side represents the third period value (in terms of foreign currency) of the accumulated deficit in the balance of trade. The right hand side is the third period value of the accumulated deficit in the balance of payments. Hence, if the foreign exchange authority borrows and lends abroad at the going interest rate in order to stabilize the exchange rate, the RHS of (4) is also the third period accumulated deficit in the foreign exchange authority's budget. We assume that the foreign exchange authority does indeed engage in foreign borrowing and lending.

It is clear from (4) that in a fixed exchange rate system the foreign exchange authority need not end up with a balanced budget unless some type of absorption policy is introduced. For suppose that $T = 0$, then it may happen that the private sector will choose to hold in the first and second period money balances which are lower than M. In this case the foreign exchange authority will end up with a deficit,since it borrows abroad at a positive interest rate and it lends to local residents at zero interest. Its deficit equals interest payments. If, on the other hand, the private sector chooses to hold money balances above M, the foreign exchange authority will end up with a surplus.

In order to make meaningful the comparison between the two exchange rate systems, we assume that taxes T (positive or negative) are chosen so as to balance the foreign exchange authority's budget. (We have chosen to impose taxes in the third period, but this choice is not restrictive because of perfect capital mobility.)

In the fixed exchange rate regime constraint (4) can be written as:

$$(4') \qquad \sum_{i=1}^{3} R^{3-i}p^i c^i + (R^2 - R)\frac{M^1}{e} + (R-1)\frac{M^2}{e} = \sum_{i=1}^{3} R^{3-i}p^i y^i + \frac{1}{e}(R^2-1)M - \frac{1}{e}T$$

The right hand side of (4') is the third period value of the consumer's wealth in terms of foreign currency; it is exogenous to him. The price of M^1/e in this budget constraint is $R^2 - R > 0$, and the price of M^2/e is $R - 1 > 0$, which means that the consumer will not attain a saturation level in money holdings; i.e., in the consumer's optimal program the marginal utilities of M^1/e and M^2/e are both positive.

The consumer's plan has an additional feature. Since taxes are chosen so as to assure a balanced budget of the exchange rate authority (which means in fact a balanced budget of the economy), his consumption plan satisfies

$$(5) \qquad \sum_{i=1}^{3} R^{3-i}p^i(c^i - y^i) = 0$$

Consider now a floating exchange rate equilibrium. Here $T = 0$. If we have an equilibrium exchange rate pattern, then $M^i = M$ for $i = 1,2$, and the right hand side of (4) equals zero. Therefore, the consumption pattern that was chosen in the fixed exchange rate regime is affordable in the floating exchange rate regime and vise-versa, but we do not know how to generally compare real balance holdings in the two regimes.

However, there exists an equilibrium pattern of exchange rates in a floating exchange rate regime that leads to an allocation of resources which is preferred to the fixed exchange rate allocation. This is the exchange rate pattern which satisfies the interest parity rule. Namely, suppose that we have an exchange rate pattern which satisfies:

$$(6) \qquad e^1/e^2 = e^2/e^3 = R$$

i.e., the exchange rate declines at the rate of interest (the value of local currency appreciates at that rate). Then, it is immediately seen from (4) that the price of money holdings is zero in every period, and that the RHS of (4) is zero, independent of money holdings. Hence, if e^1 is sufficiently low so that M/e^i provides saturation in money holdings in every period, given the optimal choice of consumption, then the consumer is willing to hold the initial stock of money in every period. In this case the consumer attains a higher welfare level than in a fixed exchange rate regime, since he can afford to buy the same consumption bundles and he has more real money balances (remember that in a fixed exchange rate regime the consumer does not attain bliss in money holdings).

The last result has an obvious relation to the optimum quantity of money argument (see Friedman (1969) and Samuelson (1968)). The declining exchange rate provides the same rate of return on domestic money as on foreign assets. As a result, local money has a perfect substitute for its role as a store of value, and can therefore be fully utilized to provide other services. 5/

The source of inferiotity of a permanently fixed exchange rate as compared to a declining one at the rate of interest can be explained as follows. Suppose that every individual is taxed according to his contribution to the foreign exchange authority's deficit, and that he knows it. This means that he knows that the RHS of his single budget constraint (4) is zero, independent of his choice of money holdings. Then, his price of money holdings is zero, and he will choose money holdings so as to attain saturation in every period. In this case he attains the same welfare level as with a declining exchange rate at the rate of interest.

However, in a many person economy, even if all individuals are identical,

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5/ This result, which is in line with the optimum quantity of money rule, is excluded by assumption from many models of flexible exchange rates. Take as an example the recent paper by Calvo and Rodriguez (1976). They use a flexible exchange rate model in which local residents hold both domestic and foreign exchange (both bearing zero interest). They postulate a monotonically decreasing functional relationship between the ratio of domestic to foreign currency holdings and the expected (= actual) rate of depreciation of the exchange rate. Since the rate of depreciation along their equilibrium path (not necessarily steady state path)is not necessarily zero, the optimal monetary rule is not satisfied. Moreover, their analysis is incorrect for those segments of the equilibrium path on which the exchange rate appreciates, because then domestic money dominates foreign currency as a store of value, so that if it provides also other services, no foreign currency will be willingly held. Because of lack of uncertainty in their model, the assumption that local money provides services in excess of those provided by foreign exchange is necessary to make sense of their asset demand function for an depreciating exchange rate. For if local money and foreign exchange provide exactly the same services, then only a constant exchange rate is consistent with positive holdings of both currencies. This difficulty does not exist in Dornbush (1976), since in his paper safe bonds always command a premium over local money. He could however do without the local bond, since he assumed that it is a perfect substitution for the foreign bond.

one cannot expect this relationship to be realized, because people do not expect to be taxed according to their own contribution to the country's loss of reserves. Take, for example, the case of identical individuals who do not behave (as it is reasonable to assume) in a Kantian way as defined by Laffont (1975). I.e., assume that every individual believes that if he increases his contribution to the deficit of the exchange rate authority by one dollar other individuals will not do the same, and that it will cause an increase in taxes by one dollar, but that the additional taxes will be spread also over other individuals. In this case his price of money holdings is positive, and in the resulting (Nash) equilibrium individuals will not be totally satiated with money balances, which is inefficient. Hence, the financing of deficits in the balance of payments generates an externality (through the required absorption policy) which creates an inefficiency in resource allocation.

Observe, however, that the absorption policy per se was assumed to be non-distorting, since we employed lump-sum taxation. It is, of course, clear that if distroting means are employed to finance balance of payments deficits (like tariffs, for example), then the loss of welfare will be even larger.

This does not mean, of course, that a centrally controlled exchange rate system is inferior to a floating one. If the government stabilizes the exchange rate, but it reduces its value at the rate of interest on foreign assets, then no absorption policy is needed and an optimal resource allocation is obtained. Since the above exchange rate pattern is not necessarily the unique equilibrium exchange rate pattern in a floating system, a point can be made in support of a crawling peg. We shall come back to this point in the next section when considering uncertainty.

III

So far we considered an intertemporal economy in a certain environment. It has, however, been argued that the source of random disturbances to which an economy is subjected constitutes a major element in optimal exchange rate system considerations. It is, therefore, important to examine the effect of uncertainty on our previous results. For this purpose we extend our model to take into account random elements both internal and external to the economy.

Now we assume that the first period foreign prices are given and so is the country's commodity endowment. Foreign prices and the country's endowment in the second and third period are unknown in the first period. In the second period there are S possible states and we denote by α a state in the second period. In the third period there are K possible states and we denote by β a state in the third period. A state of the world is described by a state in the second period and a state in the third period.

In the first period there is a subjective probability distribution over α and a subjective conditional probability distribution over β given α. $\pi(\alpha)$ denotes the probability of α ($\sum_{\alpha=1}^{S} \pi(\alpha) = 1$) and $\pi(\beta/\alpha)$ denotes the conditional probability of β given α ($\sum_{\beta=1}^{R} \pi(\beta/\alpha) = 1$ for all $\alpha = 1,2,\ldots,S$).

The first period problem can now be stated as follows:

(7) Choose c^1, M^1, A^1,

$c^2(\alpha)$, $M^2(\alpha)$, $A^2(\alpha)$, $\alpha = 1,2,\ldots,S$

$c^3(\beta/\alpha)$, $\alpha = 1,2,\ldots,S;\ \beta = 1,2,\ldots,K$

to maximize

$$E_{(\alpha,\beta)}u[c^1, c^2(\alpha), c^3(\beta/\alpha); \frac{1}{2}\frac{M+M^1}{e^1}, \frac{1}{2}\frac{M^1+M^2(\alpha)}{e^2(\alpha)}, \frac{1}{2}\frac{M^2(\alpha)+M}{e^3(\alpha)};$$

$$p^1, p^2(\alpha), p^3(\beta/\alpha)]$$

subject to

(i) $e^1p^1c^1 + e^1A^1 + M^1 = e^1p^1y^1 + M$

(ii) $e^2(\alpha)p^2(\alpha)c^2(\alpha) + e^2(\alpha)A^2(\alpha) + M^2(\alpha) = e^2(\alpha)p^2(\alpha)y^2(\alpha) + M^1 + e^2(\alpha)RA^1$

(iii) $e^3(\beta/\alpha)p^3(\beta/\alpha)c^3(\beta/\alpha) + M = e^3(\beta/\alpha)p^3(\beta/\alpha)y^3(\beta/\alpha) + M^2(\alpha) + {} + e^3(\beta/\alpha)RA^2(\alpha) - T(\beta/\alpha)$

(iv) $c^1, c^2(\alpha), c^3(\beta/\alpha), M^1, M^2(\alpha) \geqq 0,\ \alpha = 1,2,\ldots,S;\ \beta = 1,2,\ldots,K$

where

c^1 = consumption vector in period 1

$c^2(\alpha)$ = consumption vector in period 2 if state α realizes

$c^3(\beta/\alpha)$ = consumption vector in period 3 if state β realizes given that state α realized in period 2

M^1 = domestic money holdings at the end of period 1 and at the beginning of period 2

$M^2(\alpha)$ = domestic money holdings at the end of period 2 and at the beginning of period 3 if state α realizes

$M^3(\beta/\alpha)$ = domestic money holdings at the end of period 3 if state β realizes and if α realized in period 2

A stands for foreign assets. The superscripts and within parenthesis notation have the same interpretation as for M.

Similarly, for e, p, and y

$T(\beta/\alpha)$ = taxes imposed in the third period in state β given that state α realized in the second period

$u(\cdot)$ = von Neumann-Morgenstern strictly concave utility function, and

$$E_{(\alpha,\beta)}u[\cdot] = \sum_{\alpha=1}^{S} \pi(\alpha) \sum_{\beta=1}^{K} \pi(\beta/\alpha)u[\cdot]$$

This is a regular expected utility maximization problem under a set of constraints. We assume self-fulfilling expectations in the following sense. In the first period the consumer does not know the second period variables which are exogenous to him, because he does not know the state that will prevail then. However, he knows the correct values of these variables in every state. Similarly, for third period variables. This rule applies not only to variables which are exogenous to the country, but also to variables which are endogenous to the country and exogenous to the consumer. The last statement refers in particular to the exchange rate. This can be interpreted as meaning that the consumer knows the model of exchange rate determination.

The second and third period problems do not differ significantly from the first period problem. The consumer arrives at the second period with A^1 and M^1 that were chosen in the first period. Then, after a state α realizes, he maximizes his expected utility $\sum_{\beta=1}^{K} \pi(\beta/\alpha)u[\cdot]$, where c^1, M^1 were chosen in the first period, subject to constraints (ii) - (iv), but for a given α. He may revise, of course, his expectations for the third period, but he will not do so because he has self-fulfilling expectations. For the second period he <u>knows</u> the relevant variables (since a state α has realized). To the third period he arrives with $A^2(\alpha)$ and $M^2(\alpha)$ that were chosen in the second period. Then, after a state β realizes, he maximizes u[$\cdot$], where c^1, M^1 were chosen in the first period, and $c^2(\alpha)$, $M^2(\alpha)$ were chosen in the second period, subject to constraints (iii) - (iv), but now for given α and β.

Analogously to the single constraint (4) we can reduce now the single period budget constraints (i) - (iii) of (7) to one constraint for <u>each</u> state of the world (α,β):

(8) $$R^2 p^1(c^1 - y^1) + Rp^2(\alpha)[c^2(\alpha) - y^2(\alpha)] + p^3(\beta/\alpha)[c^3(\beta/\alpha) - y^3(\beta/\alpha)] =$$

$$R^2 M/e^1 - R^2 M^1/e^1 + RM^1/e^2(\alpha) - RM^2(\alpha)/e^2(\alpha) + M^2(\alpha)/e^3(\beta/\alpha) - M/e^3(\beta/\alpha) - T(\beta/\alpha)/e^3(\beta/\alpha),$$

$$\alpha = 1,2,\ldots,S;\ \beta = 1,2,\ldots,K$$

In the present model we require the foreign exchange authority, which chooses $e^1 = e^2(\alpha) = e^3(\beta/\alpha) = e$ for all α,β , to have a balanced budget in every state of the world (α,β). This implies that taxes $T(\beta/\alpha)$ have to be chosen so as to make the RHS of (8) equal to zero in every state of the world. In a floating exchange rate equilibrium, $T(\beta/\alpha) = 0$ and $M^1 = M^2(\alpha) = M$ for all α,β, which also implies that the RHS of (8) is zero in every state of the world. This means that the consumption vectors that are chosen in a fixed exchange rate system are feasible in a floating system, and vise-versa.

By rearranging (8) in the form of (4'), we find that in a fixed exchange rate regime the prices of M^1/e and $M^2(\alpha)/e$ are positive in every state of the world. This implies again that the private sector will not attain saturation in real balance holdings. However, if the exchange rate declines with certainty at the rate of interest;i.e.,

$$(9) \qquad e^1/e^2(\alpha) = e^2(\alpha)/e^3(\beta/\alpha) = R \qquad \text{for } \alpha = 1,2,\ldots,S: \quad \beta = 1,2,\ldots,K$$

then the price of money holdings is zero in <u>every</u> state of the world and the private sector attains saturation in real balance holdings in <u>every</u> state of the world. This is also an equilibrium of a floating exchange rate system if e^1 is sufficiently low relative to M. Hence, we have identified a floating exchange rate equilibrium which is superior to every fixed exchange rate equilibrium. Unfortunately, there are many other floating exchange rate equilibria which we are not able to compare to fixed exchange rate equilibria. These equilibria include cases in which the exchange rate is state and time dependent. 6/

The source of this result is exactly the same as that in Section II. When the exchange rate appreciates at the rate of interest, foreign assets become a perfect substitute for domestic money as a store of value, independent of the structure of uncertainty. Hence, domestic money can be fully utilized in the provision of other services. On the other hand, in the fixed exchange rate system, we have the same externality in the financing of deficits in the balance of payments, and this upsets the optimal quantity of money rule.

In a world in which currencies are floating, a certain rate of return

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6/ The fact that an equilibrium self-fulfilling expectations pattern of exchange rates is not unique has been overlooked in the literature. Dornbush (1976) and Calvo and Rodriguez (1976), for example, <u>choose</u> to analyze only one pattern from those possible in their models. See the Appendix for more on this point.

in terms of one currency means an uncertain rate of return in terms of other currencies. It is therefore legitimate to ask to what extent do our results hold in this type of a world.

Suppose therefore that there are two foreign countries--Country A and Country B--and that the rate of exchange between their currencies is a random variable, exogenous from the point of view of the home country. Suppose that B^i is the quantity of B's asset bought by the home country in period i, and that r_B is the sure rate of return on this asset in terms of B's currency. Then, it is easy to see that our result concerning the superiority over a fixed exchange rate of a declining exchange rate at the rate r (the first country's sure rate of interest) still holds. Obviously, in this case the home country's exchange rate vis-a-vis the currency of Country B is random. Symmetrically, it can be seen that a declining exchange rate at the rate r_B vis-a-vis the second country's currency provides the same welfare level as the declining exchange rate at rate r vis-a-vis the first country's currency. 7/ This result stems from the fact that in either case local money has a perfect substitute for its role as a store of value, and can therefore be fully utilized to provide other services. A similar result holds when there also exist risky foreign assets, as long as there exists at least one asset with a safe rate of return in terms of a foreign currency, because then we can choose to appreciate the exchange rate vis-a-vis the safe asset's currency at the rate of return on this asset. This supports our proposition that a managed exchange rate system in which the exchange rate is appreciated according to the interest parity rule may be prefered to both a fixed and a floating exchange rate system.

7/ Provided, of course, that in both cases the level of the exchange rate is sufficiently low relative to M.

APPENDIX

Consider a simplified version of the model from Dornbush (1976) by assuming that commodity prices adjust instantaneously so as to clear the market for goods. In this case the purchasing parity rule holds at all points of time; i.e., the domestic price level equals the foreign price level P^* multiplied by the exchange rate E. Given self-fulfilling expectations, the exchange rate pattern satisfies

$$\text{(A.1)} \qquad \frac{M(t)}{P^*E(t)} = m[r^* + x(t)] \equiv e^{-[r^* + x(t)]}$$

where $M(t)$ is the stock of money at time t, r^* is the rate of interest on foreign assets, and $x(t)$ is the expected and the actual rate of depreciation of the exchange rate at time t.

$$\text{(A.2)} \qquad x(t) = \dot{E}(t)/E(t)$$

$m(\cdot)$ is the demand function for real balances, which is assumed to be exponential. We have suppressed real income in the demand function because it is assumed to be constant.

Consider now the case in which the quantity of money is constant; i.e. $M(t) = M$ for all t. By substituting (A.2) into (A.1) and solving for $\dot{E}$, we obtain a homogenous differential equation:

$$\text{(A.3)} \qquad \dot{E} = [-r^* - \log(M/P^*)]E + E \log E$$

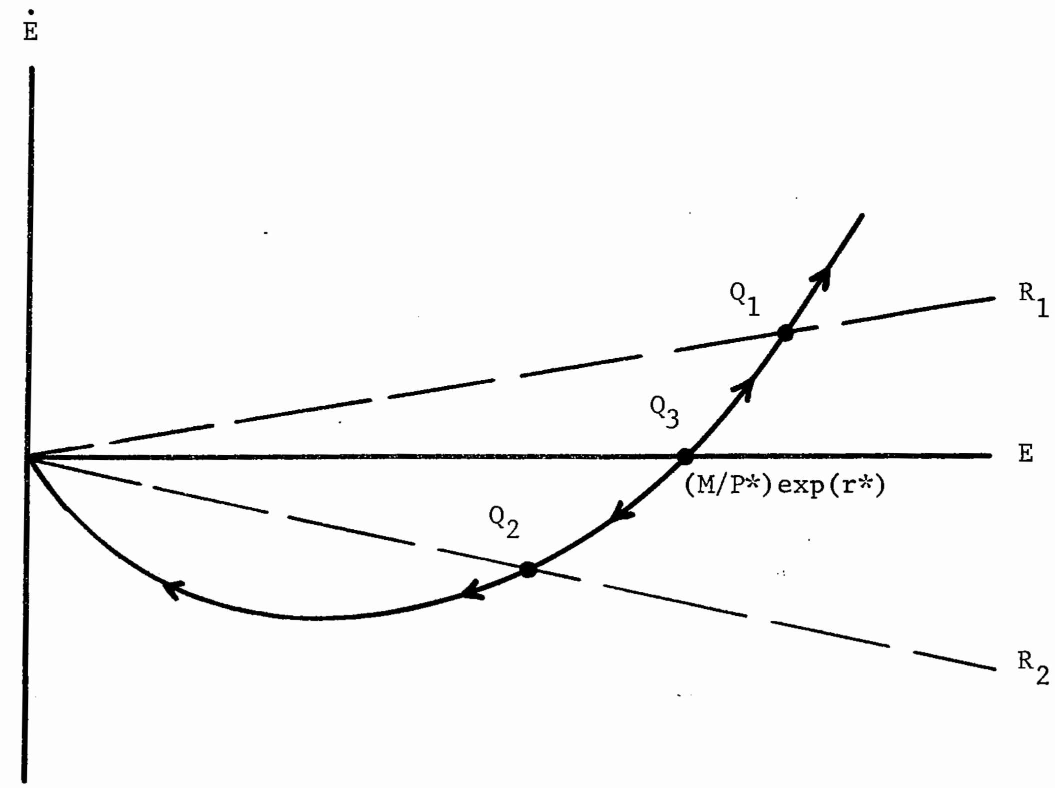

FIGURE 1

The differential equation (A.3) is described in Figure 1. Suppose that the initial expected rate of depreciation of the exchange rate is given by the slope of R_1. Then, assuming that $E \neq 0$, the economy starts at Q_1 and moves upwards. If, on the other hand, initially the exchange rate is expected to appreciate at the rate given by the slope of R_2, the economy starts at Q_2 and moves towards the origin. Both paths are consistent with self-fulfilling expectations. If initially the exchange rate is expected to be constant, then it will indeed be constant at the level $(\frac{M}{P^*})e^{r^*}$ (point Q_3). It is therefore clear that even in this simple case there are many patterns of exchange rates which are consistent with self-fulfilling expectations.

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