Discussion Paper No. 253

# Straightforward Allocation Mechanisms*

by

Mark A. Satterthwaite

October 11, 1976

Northwestern University
Graduate School of Management
Evanston, Illinois 60201

*Allan Gibbard, Ehud Kalai, and Hugo Sonnenschein have given me substantial aid in the development of this paper. All responsibility for the remaining errors is mine.

Straightforward Allocation Mechanisms

by

Mark A. Satterthwaite

## I. Introduction

A mechanism for allocating resources among several individuals is called straightforward if each individual's optimal strategy depends only on his own preferences and not on the strategies that he expects other individuals to play. Most allocation mechanisms are not straightforward. The one, major exception known to economic theory is the competitive exchange economy that contains a large number of atomless consumers and a central price setting authority. The strategies available to individuals are to report an excess demand correspondence from prices to net trades. The central authority calculates a market clearing vector of prices and, based on the reported demand correspondences, assigns each individhal a final allocation of goods. Agents' optimal strategies are to act as price takers and report their true excess demand correspondences. Because the excess demand correspondence of an agent depends only on his initial endowment and his preferences, the mechanism is straightforward.[1]

The general question with which I am concerned here is why more mechanisms are not straightforward. The specific means by which I answer this question is identification of two conditions that guarantee that an allocation mechanism is not straightforward. An informal statement of the result that I derive is this. Within the classical economic environment where the consumption set is a subset of $R^n$ and preferences are strictly monotonic and continuous, two conditions are jointly sufficient to rule out the possibility of constructing a straightforward allocation mechanisms. The first condition is that the mechanism allows pairs of individuals to interact and affect each other. For example, such interaction takes place within a competitive economy where individuals are sufficiently few that each can affect prices and thereby affect the welfare of the other individuals in the economy. The second condition is that the class of orderings that are admissible as individuals' preferences be broad enough. In particular, if an individual's income level and his consumption pattern at that income level is not sufficient information to make an accurate prediction of how he will change his consumption pattern in response to an income increase, then the class of admissible orderings is broad enough. This breadth requirement, of course, is almost always satisfied within economic situations; different individuals who have identical incomes and who have chosen identical consumption bundles may, for specific commodities, have widely varying income elasticities of demand. The conclusion that I draw, which answers the general question I posed above, is that the two conditions for nonstraightforwardness are almost always satisfied. Therefore construction of a straightforward allocation mechanism within an economic environment is impossible except in the special circumstance where a large number of individuals, all of whom are economically small, exist.

This paper's theorem is a generalization of two theorems, the first by Hurwicz [9] and the second by Gibbard [4] and, independently, by me [12,13]. Hurwicz's result, like this paper's result, is within the context of the classical economic model. Individuals have convex, monotonic, and continuous preferences over an n-dimensional consumption set. They report their preferences to the central authority who calculates the allocation each receives. The preference map that an individual reports need not be his true preference map; the only requirement is that he reports preferences that are convex, monotonic, and continuous. This type of mechanism, Hurwicz showed, cannot always (a) grant each individual the option to retain his original endowment, (b) generate Pareto satisfactory outcomes, and (c) provide each individual with an optimal, straightforward strategy.

Gibbard and I have shown a similar result in the context of voting situations. A group of individuals must select one alternative from a set of three or more alternatives. Individuals have preferences over the set of alternatives and no a priori restriction, such as continuity or convexity,is placed on the order in which an individual may rank the several alternatives. Each individual casts a ballot that consists of a ranking (indifference is permitted) of the alternatives. Individuals may or may not report a ranking that agrees with their true preferences. The ballots are counted by a voting procedure that selects one alternative as the group's choice. The theorem states that no voting procedure exists that is both nondictatorial and straightforward.

Together these two theorems suggest that construction of a straightforward allocation mechanism may be impossible within a wide variety of environments, but neither theorem adequately characterizes these environments. Hurwicz's result depends importantly on the requirement that the mechanism allow each individual the option to retain his original endowment and thus does not focus squarely on straightforwardness. Gibbard and I in our proofs depend on the restrictive assumption that preferences have no a priori structure such as convexity, monotonicity, and continuity. Thus our result, while focusing on straightforwardness, is not applicable to classical economic environments.

This paper consists of seven sections including this introduction. I formulate the model in Section 2 and state and prove the theorem in Section 3. Sections 4 and 5 each contain an application of the theorem. The example of Section 4 is a simple, explicitly specified, exchange economy while that of Section 5 is a monopoly market. In Section 6 I argue, in more depth than I have here in the introduction, my conclusion that construction of straightforward mechanisms is impossible within economic environments that are not composed of atomless agents. Moreoever I also argue that straightforwardness, if it were attainable, would be a desirable property. Finally, in the last section I show with three short examples that my two sufficient conditions for nonstraightforwardness are in a weak, informal sense also necessary conditions.

## 2. Formulation

Let $I = \{1,2,\ldots,\ell\}$ be the group of $\ell$ ($\ell \geq 2$) individuals who must allocate a quantity of resources among themselves. The consumption set of each individual $i$ is $C_i$, either the m-dimensional non-negative orthant $R^m_+$, $m \geq 2$, or some m-dimensional subset of $R^m_+$. Over $C_i$ each individual $i$ has a utility function $u_i$ that is monotonic, continuously differentiable, and is an element of some set $U_i$ of admissible utility functions.[2] Every individual $i \in I$ has an n-dimensional compact strategy space: $S_i \subset R^n$ where $n \geq 1$.

The outcome function $F$ is from the strategy space $S = \Pi^{\ell}_{i=1} S_i$ into the consumption sets $C = \Pi^{\ell}_{i=1} C_i$ of the individuals. The function $F$ breaks into components:

$$(2.01) \qquad F(s_1,\ldots,s_\ell) = \{F_1(s_1,\ldots,s_\ell),\ F_2(s_1,\ldots,s_\ell),\ \ldots\ ,\ F_\ell(s_1,\ldots,s_\ell)\} = \{x_1,\ldots,x_\ell\}$$

where $s_i = (s_{i1},\ldots,s_{in}) \in S_i$ is individual i's n-dimensional strategy vector and $x_i = (x_{i1},\ldots,x_{im}) \in C_i$ is individual i's m-dimensional final allocation. Each component $F_i$, $i \in I$, of $F$ itself breaks into components:

$$(2.02) \qquad F_i(s_1,\ldots,s_\ell) = \{F_{i1}(s_1,\ldots,s_\ell),\ F_{i2}(s_1,\ldots,s_\ell),\ \ldots\ ,\ F_{im}(s_1,\ldots,s_\ell)\} = (x_{i1},x_{i2},\ldots,x_{im}) = x_i$$

where $x_{ij}$ is the allocation person i's allocation of good j.

The one assumption I make concerning F is this. Let $s_{)i(} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_\ell)$. If for any $i \in I$, for some $[s_{)i(},s_i'] \in S$, and for some $[s_{)i(},s_i''] \in S$

$$F_i[s_{)i(},s_i'] = F_i[s_{)i(},s_i''], \tag{2.03}$$

then

$$F_j[s_{)i(},s_i'] = F_j[s_{)i(},s'']\tag{2.04}$$

for all $j \in I$. This assumption means that person i through a change in his strategy can affect person j's final allocation $x_j$ only if his change in strategy also affects his own final allocation $x_i$. The justification for this assumption is that if an allocation function did not satisfy it, then the potential for coercion would be unbounded. I call this assumption the <u>nonzero cost assumption</u>.[3]

Competitive mechanisms, for example, always satisfy this assumption because the only manner in which one person can affect another person's allocation is to cause a change in price. If a person changes his strategy in such a manner that his final allocation remains unchanged, then the original prices remain the market clearing prices. Therefore, because no price change is caused by the first person's change in strategy, no change in a second person's final allocation is caused. All other mechanisms of which I am able to conceive also satisfy the nonzero cost assumption. For example, consider mechanisms that involve fixing levels of public goods' production. If one person changes his strategy in a manner that leaves his allocation of public goods unchanged, then by definition that change also leaves all other people's allocation of public goods unchanged, which is exactly the result that the nonzero cost assumption requires.

These basic definitions enable me to define the entity that is an allocation mechanism and the property that is straightforwardness. An allocation mechanism $\Omega$ is the $3\ell+1$ tuple $\langle C_1,\ldots,C_\ell,U_1,\ldots,U_\ell,S_1,\ldots,S_\ell,F\rangle$. I define straightforwardness with respect to specific pairs of individuals. Let $\xi\in I$ and $\eta\in I$ be the indices of these two distinct individuals and let the strategies of all the other individuals be fixed at $\bar{s}_{)\xi,\eta(} = (\bar{s}_1,\ldots,\bar{s}_{\xi-1}, s_{\xi+1},\ldots,\bar{s}_{\eta-1},\bar{s}_{\eta+1},\ldots,\bar{s}_\ell) \in \prod_{\eta\neq i\neq\xi} S_i$. The allocation mechanism $\Omega$ is <u>straightforward</u> <u>for</u> $\xi$ <u>and</u> $\eta$ <u>at</u> $\bar{s}_{)\xi,\eta(}$ if and only if single-valued functions $\sigma_\xi$ and $\sigma_\eta$, with domains $U_\xi$ and $U_\eta$ and ranges contained in $S_\xi$ and $S_\eta$, exist such that, for all $u_\xi\in U_\xi$, $s_\xi\in S_\xi$, $u_\eta\in U_\eta$, and $s_\eta\in S_\eta$:

$$u_\xi\{F_\xi[\sigma_\xi(u_\xi),s_\eta,\bar{s}_{)\xi,\eta(}]\} = \max_{s_\xi\in S_\xi} u_\xi\{F_\xi[s_\xi,s_\eta,\bar{s}_{)\xi,\eta(}]\} \tag{2.05}$$

and

$$u_\eta\{F_\eta[s_\xi,\sigma_\eta(u_\eta),\bar{s}_{)\xi,\eta(}]\} = \max_{s_\eta\in S_\eta} u_\eta\{F_\eta[s_\xi,s_\eta,\bar{s}_{)\xi,\eta(}]\}. \tag{2.06}$$

The strategy $s_\xi = \sigma_\xi(u_\xi)$ is $\xi$'s <u>straightforward</u> <u>strategy</u> <u>at</u>

$\bar{s}_{)\xi,\eta(}$ and the strategy $s_\eta = \sigma_\eta(u_\eta)$ is person $\eta$'s straightforward strategy at $\bar{s}_{)\xi,\eta(}$. Thus person $\xi$'s straightforward strategy at $\bar{s}_{)\xi,\eta(}$ is a pure dominant strategy vis-a-vis person $\eta$'s choice of strategy and similarly, $\eta$'s straightforward strategy at $\bar{s}_{)\xi,\eta(}$ is a pure dominant strategy vis-a-vis $\xi$'s choice of strategy.

This particular concept of straightforwardness is closely related to, but less demanding than the concept of straightforwardness that Gibbard [4] formulated. His definition is that an allocation mechanism $\Omega$ is straightforward if and only if, for each person $i \in I$, a singlevalued function $\sigma_i^*$ with domain $U_i$ and range contained in $S_i$ exists such that, for any $u_i \in U_i$ and any $s_{)i(} = [s_1,\ldots,s_{i-1}, s_{i+1},\ldots,s_\ell) \in \prod_{j \neq i} S_j$,

$$(2.07) \qquad u_i\{F_i[\sigma_i^*(u_i), s_{)i(}]\} = \max_{s_i \in S_i} u_i \{F_i[s_i, s_{)i(}]\} .$$

The strategy $s_i = \sigma_i^*(u_i)$ is called person i's straightforward strategy. Thus each person's straightforward strategy is a pure dominant strategy vis-a-vis every one else's choice of strategy and, consequently, every $\ell$-tuple of straightforward strategies is a Nash equilibrium. Evidently a mechanism that is straightforward is straightforward for any pair of people $\xi$ and $\eta$ at any point $\bar{s}_{)\xi,\eta(}$. The converse, however, is not true. I use this weaker concept because it suffices and because, as I show in Section 5, it permits the analysis of situations that can not be analyzed using the stronger concept.

Suppose an allocation mechanism $\Omega = \langle C_1,\ldots,C_\ell,U_1,\ldots,U_\ell, S_1,\ldots,S_\ell,F\rangle$ is straightforward for persons $\xi$ and $\eta$ at $\bar{s}_{)\xi,\eta(}$. A reference quintuple $V = \langle \bar{u}_\xi,\bar{u}_\eta,\bar{s}_{)\xi,\eta(}, S^*_\xi, S^*_\eta\rangle$ for $\Omega$ is constructed as follows. Define functions $f_\xi$ and $f_\eta$ to be:

$$f_\xi(s_\xi,s_\eta) = F_\xi[s_\xi,s_\eta,\bar{s}_{)\xi,\eta(}], \tag{2.08}$$

$$f_\eta(s_\xi,s_\eta) = F_\eta[s_\xi,s_\eta,\bar{s}_{)\xi,\eta(}]. \tag{2.09}$$

Pick utility functions $\bar{u}_\xi \in U_\xi$ and $\bar{u}_\eta \in U_\eta$ for individuals $\xi$ and $\eta$. They have straightforward strategies $\bar{s}_\xi = \sigma_\xi(\bar{u}_\xi)$ and $\bar{s}_\eta = \sigma_\eta(\bar{u}_\eta)$ respectively. Finally pick open neighborhoods $S^*_\xi \subset \text{int } S_\xi$ and $S^*_\eta \subset \text{int } S_\eta$ that contain $\bar{s}_\xi$ and $\bar{s}_\eta$ respectively.[4]

My reason for picking the reference quintuple $V = \langle \bar{u}_\xi,\bar{u}_\eta, \bar{s}_{)\xi,\eta(},S^*_\xi,S^*_\eta\rangle$ is to make statement and proof of the theorem possible in terms of $\Omega$'s local properties. Assume that $\Omega$ is straightforward for persons $\xi$ and $\eta$ at $\bar{s}_{)\xi,\eta(}$. I show that if the outcome function F satisfies certain properties in the region $S^*_\xi \times S^*_\eta$ surrounding $(\bar{s}_\xi,\bar{s}_\eta,\bar{s}_{)\xi,\eta(})$ and if person $\eta$'s class $U_\eta$ of admissible utility functions has sufficient variety, then, contrary to the definition of straightforwardness, person $\xi$ has an incentive to change his strategy from $\bar{s}_\xi$ as person $\eta$'s preferences (and therefore strategy) vary from $\bar{u}_\eta$. Consequently the selection of the reference quintuple V is guided by the question: does a V exist at which $\Omega$ locally satisfies the aforesaid conditions on F and $U_\eta$?

If so, then the assumption that $\Omega$ is straightforward for persons $\xi$ and $\eta$ at $\bar{s}_{)\xi,\eta(}$ is contradicted.

Formal statement of these requirements on F and $U_\eta$ require several additional, preparatory defnitions. Let the functions $\bar{g}_\xi$ and $\bar{g}_\eta$ be:

(2.10) $$\bar{g}_\xi(s_\eta) = \{x_\xi | x_\xi = f_\xi(s_\xi, s_\eta), \quad s_\xi \in S_\xi\},$$

(2.11) $$\bar{g}_\eta(s_\xi) = \{x_\eta | x_\eta = f_\eta(s_\xi, s_\eta), \; s_\eta \in S_\eta\} .$$

These functions are the feasible sets for $\xi$ and $\eta$ respectively, given the other's strategy. Since each person has a monotonic utility function he chooses only from the frontier of his feasible set. These frontiers, which may be regarded as offer curves, are:

(2.12) $$g_\xi(s_\eta) = \{x_\xi | x_\xi \in \bar{g}_\xi(s_\eta) \; \& \; [y_\xi \in \bar{g}_\xi(s_\eta) \Rightarrow (y_\xi - x_\xi) \text{ is not strictly monotonic}]\},$$

(2.13) $$g_\eta(s_\xi) = \{x_\eta | x_\eta \in \bar{g}_\eta(s_\xi) \; \& \; [y_\eta \in \bar{g}_\eta(s_\xi) \Rightarrow (y_\eta - x_\eta) \text{ is not strictly monotonic}]\}.$$

A vector $\theta \in R^m$ is strictly monotonic if and only if each of its components is positive. An allocation mechanism is called <u>continuously differentiable at the reference quintuple</u> V if and only if $f_\xi$ and $f_\eta$ are both continuously differentiable within the region $S^* = S^*_\xi \times S^*_\eta$.

Finally, for the last preliminary definition, suppose F is continuously differentiable at V. Let A{[0,1]} and B{[0,1]} be smooth, rectifiable, simple curves contained respectively in $S_\xi^*$ and $S_\eta^*$.[5] As such they break into components: for all $p \in [0,1]$,

(2.14) $$A(p) = \{A_1(p),\ldots,A_n(p)\} = \{s_{\xi 1},\ldots,s_{\xi n}\} = s_\xi \in S_\xi^*,$$

(2.15) $$B(p) = \{B_1(p),\ldots,B_n(p)\} = \{s_{\eta 1},\ldots,s_{\eta n}\} = s_\eta \in S_\eta^*.$$

Define, for all $p \in (0,1)$:

(2.16) $$v_\xi[\bar{s}_\xi, B(p)] = \{\sum_{j=1}^{n} \frac{\partial f_{\xi 1}}{\partial s_{\eta j}} \frac{\partial B_j}{\partial p},\ldots,\sum_{j=1}^{n} \frac{\partial f_{\xi m}}{\partial s_{\eta j}} \frac{\partial B_j}{\partial p}\};$$

(2.17) $$v_\eta[A(p), \bar{s}_\eta] = \{\sum_{j=1}^{n} \frac{\partial f_{\eta 1}}{\partial s_{\xi j}} \frac{\partial A_j}{\partial p},\ldots,\sum_{j=1}^{n} \frac{\partial f_{\eta m}}{\partial s_{\xi j}} \frac{\partial A_j}{\partial p}\}.$$

The vector $v_\xi$ is the velocity with which person $\xi$'s allocation changes as person $\eta$'s strategy varies along the curve B. The vector $v_\eta$ has the same interpretation, _mutatis mutandis_.

The requirement on F and the requirement on $U_\eta$, which in the presence of straightforwardness are inconsistent with each other, may now be stated. The requirement on F is that smooth, rectifiable, and simple curves $A\{[0,1]\} \subset S_\xi^*$ and $B\{[0,1]\} \subset S_\eta^*$ exist that are suitable at V. The two curves A and B are _suitable at_ V if and only if: (a) F is continuously differentiable at V; (b)

(b) $A(\frac{1}{2}) = \sigma_\xi(\bar{u}_\xi) = \bar{s}_\xi$ and $B(\frac{1}{2}) = \sigma_\eta(\bar{u}_\eta) = \bar{s}_\eta$; and (c), for all $p \in [0,1]$, (2.18) $$\frac{d\bar{u}_\eta}{dp} = \nabla\, \bar{u}_\eta\{f_\eta[A(p),\bar{s}_\eta]\}\cdot v_\eta[A(p),\bar{s}_\eta] > 0$$

and

(2.19) $$\frac{d\bar{u}_\xi}{dp} = \nabla\, \bar{u}_\xi\{f_\xi[\bar{s}_\xi,B(p)]\}\cdot v_\xi[\bar{s}_\xi,B(p)] > 0.$$

Inequality (2.18) states that if person $\xi$ changes his strategy along the curve A, then each increment of change favorably affects person $\eta$. Inequality (2.19) reverses the players roles. Thus this requirement that suitable curves exist is the requirement that individuals $\xi$ and $\eta$ interact and affect each other both favorably and unfavorably. As such it plays the role that the non-dictatorship and citizens' sovereignty requirements play in the impossibility results that Arrow [2] and others have proven within social choice theory.

The second requirement is that the set $U_\eta$ of admissible utility functions be broad at V with respect to the suitable curve A. Suppose F is continuously differentiable at V and suppose that the curve A{[0,1]} is suitable with respect to V. Let $\bar{x}_\eta = f_\eta(\bar{s}_\xi,\bar{s}_\eta) = f_\eta[A(\tfrac{1}{2}),B(\tfrac{1}{2})] = f_\eta[\sigma_\xi(\bar{u}_\xi),\ \sigma_\eta(\bar{u}_\eta)]$. The set $U_\eta$ is <u>A-V broad</u> if and only if a scalar $\mu > 0$ exists such that the following condition is satisfied for every $p_\xi' \in (\tfrac{1}{2},1)$. If $z_\eta = f_\eta[A(p_\xi'),\bar{s}_\eta]$ and if $y_\eta$ is any allocation such that both

(2.20) $$y_\eta \in \{g_\eta(\bar{s}_\xi) \cap f_\eta(\bar{s}_\xi,S_\eta^*)\}$$

and

(2.21) $$\frac{|y_\eta-\bar{x}_\eta|}{|z_\eta-\bar{x}_\eta|} < \mu,$$

then a $u'_\eta \in U_\eta$ exists such that

(2.22) $$f_\eta[\bar{s}_\xi, \sigma_\eta(u'_\eta)] = y_\eta$$

and

(2.23) $$f_\eta[A(p'_\xi), \sigma_\eta(u'_\eta)] = z_\eta .$$

Recall that $g_\eta(\bar{s}_\xi)$ is the offer curve that faces $\eta$ when $\xi$ plays strategy $\bar{s}_\xi$.

Figure 1 shows the type of situation that must always be constructible if $U_\eta$ is A-V broad. The definition of $\sigma_\eta$ implies that $\bar{x}_\eta$ and $z_\eta$ are points of maximum utility for person $\eta$ given that his true utility function is $\bar{u}_\eta$ and given that person $\xi$'s strategy is, respectively, $\bar{s}_\xi$ and $A(p'_\xi)$. In other words, the indifference curves generated by $\bar{u}_\eta$ are tangent to $g_\eta(\bar{s}_\xi)$ at $\bar{x}_\eta$ and to $g_\eta[A(p'_\xi)]$ at $z_\eta$. Similarly the indifference curves generated by $u'_\eta$ are tangent to $g_\eta(\bar{s}_\xi)$ at $y_\eta$ and to $g_\eta[A(p'_\xi)]$ at $z_\eta$. Thus broadness requires that $U_\eta$ contain enough variety of admissible preferences to insure that at points such as $z_\eta$ several possible trajectories of allocations exist and cross.

The purpose of the requirement that $y_\eta$ may be any point such that (2.20) and (2.21) are satisfied is to guarantee that the angle at which the trajectories through $z_\eta$ may cross does not approach zero as $z_\eta$ is chosen increasingly close to $\bar{x}_\eta$. In other words, (2.20) and (2.21) guarantee that no neighborhood of $\bar{x}_\eta$ exists in which only degenerate crossings of trajectories occur.

Intuitively the broadness requirement may be considered as a requirement on $\eta$'s income elasticity of demand at $z_\eta$. On Figure 1 the shifting out of the offer curve from $g_\eta(\overline{s}_\xi)$ to $g_\eta[A(p')]$ is primarily an income increase for person $\eta$. If a $u'_\eta$ does not exist such that the trajectories $f_\eta[A\{[0,1]\}, \sigma_\eta(\overline{u}_\eta)]$ and $f_\eta[A\{[0,1]\}, \sigma_\eta(u'_\eta)]$ cross at $z_\eta$, then, for all $u_\eta \in U_\eta$ such that $f_\eta[A(p'), \sigma_\eta(u_\eta)] = z_\eta$ person $\eta$'s income elasticity of demand at $z_\eta$ is a constant no matter what his preferences are. If on the other hand a $u'_\eta$ does exist such that the two trajectories cross at $z_\eta$, then $\eta$'s income elasticity at $z_\eta$ varies depending on whether his preferences are $\overline{u}_\eta$ or $u'_\eta$.[6] Consequently, within economic contexts where a priori restrictions on individual's tastes that go beyond monotonicity and concavity of utility functions are difficult to justify, the requirement that $U_\eta$ is A-V broad is quite weak.

## 3. The Theorem and Proof

Theorem. Consider an allocation mechanism $\Omega = \langle C_1,\ldots,C_\ell,\ U_1,\ldots,U_\ell,\ S_1,\ldots,S_\ell,\ F\rangle$ and suppose that it is straightforward for persons $\xi$ and $\eta$ at $\overline{s}_{)\xi,\eta(}$. Pick a reference quintuple $V = \langle \overline{u}_\xi, \overline{u}_\eta, \overline{s}_{)\xi,\eta(}, S^*_\xi, S^*_\eta\rangle$. If F is continuously differentiable at V, then the existence of curves A and B that are suitable with respect to V is inconsistent with $U_\eta$ being A-V broad.

<u>Proof</u>. <u>Step 1</u>. Suppose that an allocation mechanism $\Omega$ is straightforward and continuously differentiable at some reference quintuple $V = \langle \overline{u}_\xi, \overline{u}_\eta, \overline{s}_{)\xi,\eta(}, S^*_\xi, S^*_\eta\rangle$. For convenience let $\xi$=1 and $\eta$=2. Suppose also that A and B are two curves that are suitable with respect to V. Finally suppose that, contrary to the theorem, $U_2$ is A-V broad. I shall show that this hypothesis leads to a contradiction.

<u>Step 2</u>. Define $u_1^*(p_1,p_2) = \overline{u}_1\{f_1[A(p_1),B(p_2)]\}$ over the domain $P^2 = [0,1]\times[0,1]$. Three properties are of interest:

(3.01) $u_1^*$ is continuously differentiable over $P^2$,

(3.02) $$\frac{\partial u_1^*(\tfrac{1}{2},p_2)}{\partial p_1} = 0 \text{ per all } p_2 \in (0,1);$$

(3.03) $$\frac{\partial u_1^*(\tfrac{1}{2},p_2)}{\partial p_2} > 0 \text{ for all } p_2 \in (0,1).$$

Property (3.01) is a consequence of $\Omega$ being continuously differentiable at V and of curves A and B being smooth. Property (3.02) is a consequence of $\Omega$ being straightforward, of $A(\frac{1}{2}) = \overline{s}_1 = \sigma_1(\overline{u}_1)$ being person one's straightforward strategy, and of $u_1^*$ being continuously differentiable. Property (3.03) is a consequence of curve B being suitable -- see inequality (2.19).

Because $u_1^*$ is continuously differentiable on $P^2$, it has a differential h at $\overline{p} = (\overline{p}_1, \overline{p}_2) = (\frac{1}{2}, \frac{1}{2})$:

$$(3.04) \quad h(\overline{p}, p) = \nabla u_1^*(\overline{p}) \cdot p$$

where $p \in P^2$. The definition of the differential states that, for every $\varepsilon > 0$, a neighborhood of $\overline{p}$, denoted by $N(\varepsilon, \overline{p}) \subset P^2$, exists such that[7]

$$(3.05) \quad |u_1^*(p) - u_1^*(\overline{p}) - \nabla u_1^*(\overline{p}) \cdot (p - \overline{p})| < \varepsilon |p - \overline{p}|$$

for all $p = (p_1, p_2) \in N(\varepsilon, \overline{p})$.

<u>Step 3</u>. Pick an $\varepsilon > 0$. Let $N(\varepsilon, \overline{p})$ be the neighborhood such that (3.05) is satisfied. Pick points $p^z = (p_1^z, \frac{1}{2}) \in N(\varepsilon, \overline{p})$ and $p^y = (\frac{1}{2}, p_2^y) \in N(\varepsilon, \overline{p})$ such that, for some admissible utility function $u_2' \in U_2$,

$$(3.06) \quad p_1^z > \tfrac{1}{2} \text{ and } p_2^y < \tfrac{1}{2};$$

(3.07) $z_2 = f_2[A(p_1^z), \bar{s}_2]$ and $y_2 = f_2[\bar{s}_1, B(p_2^y)]$

(3.08) $$\frac{|y_2 - \bar{x}_2|}{|z_2 - \bar{x}_2|} > \tfrac{1}{2}\mu$$

(3.09) $z_2 = f_2[A_1(p_1^z), \sigma_2(u_2')]$ and $y_2 = f_2[A(\tfrac{1}{2}), \sigma_2(u_2')]$.

where $\mu$ is the positive scalar constant that appears in the definition of A-V broad classes of preferences and where $\bar{x}_2 = f_2(\bar{s}_1,\bar{s}_2)$. If $p_\xi'$ is allowed to represent $p_1^z$, then Figure 1 shows the configuration these points have within person two's consumption set.

The existence of such a set of points is not immediately clear. Nevertheless, they do exist and can be found by the following algorithm. Pick a point ${}_{(1)}p^z = ({}_{(1)}p_1^z, \tfrac{1}{2}) \in N(\varepsilon,\bar{p})$ such that $\tfrac{1}{2} < {}_{(1)}p_1^z < 1$. This determines ${}_{(1)}z_2 = f_2[A({}_{(1)}p^z), \bar{s}_2]$. Pick a point ${}_{(1)}y_2 \in f_2[\bar{s}_1, B\{(0,\tfrac{1}{2})\}]$ such that

(3.10) $$\mu > \frac{|{}_{(1)}y_2 - \bar{x}_2|}{|{}_{(1)}z_2 - x_2|} > \tfrac{1}{2}\mu \, .$$

This point ${}_{(1)}y_2$ will not exist if

(3.11) $$\tfrac{1}{2}\mu > \frac{|f_2[\bar{s}_1, B(0)] - \bar{x}_2|}{|{}_{(1)}z_2 - \bar{x}_2|} \, .$$

In that event pick a point ${}_{(2)}p^z = ({}_{(2)}p_1^z, \tfrac{1}{2})$ such that $\tfrac{1}{2} < {}_{(2)}p_1^z < {}_{(1)}p_1^z$. If ${}_{(2)}p_1^z$ is made close enough to $\tfrac{1}{2}$, then a ${}_{(2)}y_2$ will exist such that

$$(3.12)\quad \mu > \frac{|{}_{(2)}y_2 - \bar{x}_2|}{|{}_{(2)}z_2 - \bar{x}_2|} > \tfrac{1}{2}\mu$$

where ${}_{(2)}z_2 = f_2[A({}_{(1)}p^z),\ \bar{s}_2]$. This is because $f_2[A(p_1),\ \bar{s}_2]$ is continuous. Therefore, as $p_1$ approaches $\frac{1}{2}$ from above,

$$(3.13)\quad |z_2 - \bar{x}_2| = |f_2[A(p_1), \bar{s}_2] - \bar{x}_2|$$

is a quantity that approaches zero from above.

Requirements (3.06) through (3.08) are therefore satisfied; the only remaining requirement to check is (3.09). It is satisfied because requirement (3.08) and the assumption that $U_2$ is A-V broad together imply a $u_2' \in U_2$ exists such that:

$$(3.14)\quad z_2 = f_2[A(p_1^z),\ \sigma_2(u_2')]$$

$$(3.15)\quad y_2 = f_2[\bar{s}_1,\ \sigma_2(u_2')]$$

Therefore associated with each $\epsilon > 0$ are points $p^z, p^y \in N(\epsilon, \bar{p})$ that satisfy (3.06)-(3.09).

This algorithm for finding the points $(z_2, y_2)$ makes clear the existence of the following infinite sequence. Given the two points ${}_{(1)}p^z$, ${}_{(1)}p^y \in N(\epsilon, \bar{p})$ satisfying (3.06)-(3.09), an infinite sequence of paired points

$$(3.16) \quad \{[{}_{(1)}p^z, {}_{(2)}p^y], [{}_{(2)}p^z, {}_{(2)}p^y], \ldots, [{}_{(n)}p^z, {}_{(n)}p^y], \ldots\}$$

exists such that each pair $[{}_{(n)}p^z, {}_{(n)}p^y]$ satisfies (3.06)-(3.09), ${}_{(n)}p_1^z$ converges to $\frac{1}{2}$ from above, and ${}_{(n)}p_2^y$ converges to $\frac{1}{2}$ from below. Note that the construction of this sequence, together with (3.02) and (3.03), implies that, for all n:

$$(3.17) \quad \nabla u_1^*(\bar{p}) \cdot [{}_{(n)}p^z - \bar{p}] = \frac{\partial u_1^*(\bar{p})}{\partial p_1} \cdot ({}_{(n)}p_1^z - \tfrac{1}{2}) = 0;$$

$$(3.18) \quad \nabla u_1^*(\bar{p}) \cdot [{}_{(n)}p^y - \bar{p}] = \frac{\partial u_1^*(\bar{p})}{\partial p_2} \cdot ({}_{(n)}p_2^y - \tfrac{1}{2}) < 0.$$

The sequence (3.16), the equality (3.17), and the inequality (3.18) play key roles later in the proof.

<u>Step 4</u>. Because $f_2[A(p_1), B(p_2)]$ is continuously differentiable on $P^2$ it satisfies a Lipschitz condition at $\bar{p} = (\frac{1}{2}, \frac{1}{2})$: for some scalar $M > 0$ and some neighborhood $N(\bar{p})$ of $\bar{p}$

$$(3.19) \quad |f_2[A(p_1), B(p_2)] - \bar{x}_2| < M|p - \bar{p}|$$

for all $p = (p_1, p_2) \in N(\bar{p})$. Recall, from (3.06) through (3.09), that

(3.20) $\quad y_2 = f_2[\overline{s}_1, B(p_2^y)]$ and $z_2 = f_2[A(p_1^z), \overline{s}_2]$

(3.21) $\quad p_2^y < \frac{1}{2}$

(3.22) $\quad |y_2 - \overline{x}_2| > \frac{1}{2}\mu|z_2 - \overline{x}_2|$.

Assume that $p^y = (\frac{1}{2}, p_2^y) \in N(\overline{p})$. Substitution into (3.19) gives

(3.23) $\quad |y_2 - \overline{x}_2| < M|p^y - \overline{p}| = M(\frac{1}{2} - p_2^y)$.

Finally substitution of (3.23) into (3.22) gives

(3.24) $2\,\frac{M}{\mu}(\frac{1}{2} - p_2^y) > |z_2 - \overline{x}_2|$.

This becomes, after simplification,

(3.25) $\quad 0 < p_2^y < \frac{1}{2}\{1 - \frac{\mu}{M}|f_2[A(p_1^z), \overline{s}_2] - \overline{x}_2|\}$,

for all $p^y = (\frac{1}{2}, p_2^y) \in N(\overline{p})$. This is a useful upper bound on $p_2^y$ that is used in step 7.

<u>Step 5</u>. The assumption that B is a suitable curve, which means it satisfies (2.19), together with my choice of $p_2^y$ such that it is less than $\frac{1}{2}$ implies that

(3.26) $\quad \overline{u}_1\{f_1[\overline{s}_1, \overline{s}_2]\} > \overline{u}_1\{f_1[\overline{s}_1, B(p_2^y)]\}$.

Because $\Omega$ is straightforward for person one

$$(3.27)\quad \bar{u}_1\{f_1[\bar{s}_1,\sigma_2(u_2')]\} \geq \bar{u}_1\{f_1[A(p_1^z),\ \sigma_2(u_2')]\}.$$

From (3.07) and (3.08)

$$(3.28)\quad z_2 = f_2[A(p^z{}_1),\bar{s}_2] = f_2[A(p_1^z),\ \sigma_2(u_2')]$$

and

$$(3.29)\quad y_2 = f_2[\bar{s}_1,B(p_2^y)] = f_2[\bar{s}_1,\ \sigma_2(u_2')].$$

The nonzero cost assumption on F therefore implies that

$$(3.30)\quad f_1[A(p_1^z),\ \bar{s}_2] = f_1[A(p_1^z),\ \sigma_2(u_2')]$$

and

$$(3.31)\quad f_1[\bar{s}_1,B(p_2^y)] = f_1[\bar{s}_1,\ \sigma_2(u_2')].$$

Substitution of (3.30) and (3.31) into (3.27) and comparison with (3.26) gives

$$(3.32)\quad \bar{u}_1\{f_1[\bar{s}_1,\bar{s}_2]\} > \bar{u}_1\{f_1[\bar{s}_1,B(p_2^y)]\} \geq \bar{u}_1\{f_1[A(p_1^z),\bar{s}_2]\}.$$

Restated this is:

$$(3.33)\quad u_1^*(\bar{p}) > u_1^*(p^y) \geq u_1^*(p^z).$$

This inequality is used in step 6.

<u>Step 6</u>. Relation (3.05) applies to both $p^z \in N(\varepsilon,\bar{p})$ and $p^y \in N(\varepsilon,\bar{p})$:

$$(3.34)\quad |u_1^*(p^y) - u_1^*(\bar{p}) - \nabla u_1^*(\bar{p})\cdot(p^y-\bar{p})| < \varepsilon|p^y-\bar{p}|;$$

$$(3.35)\quad |u_1^*(p^z) - u_1^*(\bar{p}) - \nabla u_1^*(\bar{p})\cdot(p^z-\bar{p})| < \varepsilon|p^z-\bar{p}|.$$

Substitution of (3.17) into (3.35) gives

$$(3.36)\quad |u_1^*(p^z) - u_1^*(\bar{p})| < \varepsilon|p^z-\bar{p}|.$$

The inequality (3.33) implies that (3.36) may be written as

$$(3.37)\quad |u_1^*(\bar{p}) - u_1^*(p^y)| < \varepsilon|p^z-\bar{p}|.$$

Inequalities (3.34) and (3.37) may be added:

$$(3.38)\quad |u_1^*(p^y)-u_1^*(\bar{p}) - \nabla u_1^*(\bar{p})\cdot(p^y-\bar{p})|+|u_1^*(\bar{p}) - u_1^*(p^y)|$$
$$< \varepsilon \{|p^y-\bar{p}|+|p^z-\bar{p}|\}.$$

The triangle inequality simplifies the left hand side:

$$(3.39)\quad |\nabla u_1^*(\bar{p})\cdot(p^y-\bar{p})| < \varepsilon \{|p^y-\bar{p}| + |p^z-\bar{p}|\}.$$

Relation (3.18) and the definitions of the points $\overline{p}$, $p^y$, and $p^z$ allows further simplification:

$$(3.40) \quad \frac{\partial u_1^*(\overline{p})}{\partial p_2} (\tfrac{1}{2}-p_2^y) < \epsilon\{(\tfrac{1}{2}-p_2^y)+(p_1^z-\tfrac{1}{2})\}$$

or, rewriting in a particular manner,

$$(3.41) \quad \frac{\partial u_1^*(\overline{p})}{\partial p_2} < \epsilon\{1 + \frac{p_1^z-\frac{1}{2}}{\frac{1}{2}-p_2^y}\}.$$

Inequality (3.41) is true for any $(p^z,p^y) \in N(\overline{p},\epsilon) \times N(\overline{p},\epsilon)$ pair that satisfies (3.06) through (3.09). Finally from (3.03) I know that

$$(3.42) \quad \frac{\partial u_1^*(\overline{p})}{\partial p_2} > 0.$$

The last step of the proof is to show that (3.41) leads to a contradiction of (3.42).

<u>Step 7</u>. Recall from step 3 that, given the arbitrarily chosen $\epsilon>0$, an infinite sequence of pairs $\{_{(n)}p^z, _{(n)}p^y\}$ exists such that each pair satisfies (3.06) through (3.09), $_{(n)}p_1^z$ approaches $\frac{1}{2}$ from above, and $_{(n)}p_2^y$ approaches $\frac{1}{2}$ from below. Since I may choose $\epsilon$ arbitrarily close to zero, if I can show that in equation (3.41) the term

$$(3.43) \quad \frac{_{(n)}p_1^z-\frac{1}{2}}{\frac{1}{2}-_{(n)}p_2^y}$$

does not explode as n goes to infinity, then I have succeeded in contradicting (3.42). The calculation that (3.43), in fact, does not explode is this:

$$(3.44)\qquad 0 < \frac{{}_{(n)}p_1^z - \frac{1}{2}}{\frac{1}{2} - {}_{(n)}p_2^y}$$

$$< \frac{{}_{(n)}p_1^z - \frac{1}{2}}{\frac{1}{2} - \frac{1}{2}\{1 - \frac{\mu}{M}|f_2[A({}_{(n)}p_1^z),\bar{s}_2] - \bar{x}_2|\}}$$

$$= \frac{2M}{\mu}\left\{\frac{{}_{(n)}p_1^z - \frac{1}{2}}{|f_2[A({}_{(n)}p_1^z),\bar{s}_2] - \bar{x}_2|}\right\}$$

where the second line is a substitution of (3.25). That substitution is valid provided that n is picked large enough that ${}_{(n)}p^y = (\frac{1}{2}, {}_{(n)}p_2^y) \in N(\bar{p})$. The last line of (3.44) is of the indeterminate form 0/0 as n goes to infinity (or equivalently as $p_1^z$ goes to $\frac{1}{2}$ from above). When evaluated by L'Hopital's rule the result is:

$$(3.45)\qquad \lim_{n\to\infty} \frac{2M}{\mu}\left\{\frac{{}_{(n)}p_1^z - \frac{1}{2}}{|f_2[A({}_{(n)}p_1^z),\bar{s}_2] - x_2|}\right\}$$

$$= \frac{2\,M}{\mu|v_2[A(\frac{1}{2}),\bar{s}_2]|}\ .$$

Because the curve A is suitable, $|v_2[A(\frac{1}{2}),\overline{s}_2]| \neq 0$. If it did equal zero, then (2.17) would be violated and A would not be a suitable curve. Consequently, because M, $\mu$, and $|v_2[A(\frac{1}{2}),\overline{s}_2]|$ are positive, finite numbers, the limit (3.45) is finite.

Therefore, based on (3.42), (3.44) and (3.45), I can rewrite relation (3.41) as:

$$(3.46) \quad 0 < \frac{\partial u_1^*(\overline{p})}{\partial p_2} < \lim_{n\to\infty} \epsilon\{1 + \frac{(n)p_1^z - \frac{1}{2}}{\frac{1}{2} - (n)p_2^y}\}$$

$$< \epsilon\{1 + \frac{2M}{\mu|v_2[A(\frac{1}{2}),\overline{s}_2]|}\}.$$

The right hand side of (3.46) can be made arbitrarily close to zero because, in step 3, $\epsilon$ may be selected arbitrarily close to zero and because the value of the term

$$(3.47) \quad \{1 + \frac{2M}{\mu|v_2[A(\frac{1}{2}),\overline{s}_2)|}\}$$

is not dependent on the choice of $\epsilon$. This creates the contradiction I have been seeking: $\partial u_1^*(\overline{p})/\partial p_2$ can not be a constant that is simultaneously greater than zero and arbitrarily close to zero. Therefore the proof is complete.

## 4. Nonstraightforwardness of a Simple Exchange Economy

In this section I illustrate the theorem by showing that a simple competitive exchange economy is not straightforward.[8] The group consists of two people who are trading two goods. Both begin with initial endowments of one unit of good x and one unit of good y. Each individual has preferences that can be described by a utility function that is a member of the class of utility functions that Stone [14] used in his econometric studies of demand.

Specifically I define $U_i$, $i \in \{1,2\}$, to consist of every utility function of the form

$$(4.01) \qquad u_i(x_i,y_i) = \alpha_i \log_e(x_i-a_i) + (1-\alpha_i)\log_e(y_i-b_i)$$

where the parameters are subject to the bounds

$$(4.02) \qquad \begin{aligned} .4 &\leq \alpha_i \leq .6, \\ .0 &\leq a_i \leq .3, \\ .0 &\leq b_i \leq .3. \end{aligned}$$

The pair $(x_i,y_i)$ is person i's allocation of goods x and y. For convenience, let $U_i$ be represented by the set of triples $(\alpha_i,a_i,b_i)$ that satisfy (4.02). The interpretation of $a_i$ is that it is the minimum quantity of good x that person i needs for survival. Note that if both $a_i = 0$ and $b_i = 0$, then $u_i$ has the familiar Cobb-Douglas form.

Person i has true preferences $\bar{u}_i \in U_i$; these preferences are fully described by a triple $(\bar{\alpha}_i,\bar{a}_i,\bar{b}_i)$ that satisfies the bounds of

(4.02), i.e. $(\alpha_i, a_i, b_i) \in U_i$. Let the consumption sets be $C_i = [.4,1.6] \times [.4,1.6]$. Given the bounds of (4.02), the utility functions $\bar{u}_i \in U_i$ are defined for every $(x_i, y_i) \in C_i$.

The strategy $s_i$ for each person is to report an admissible utility function $u_i \in U_i$. He reports it by specifying a triple $(\alpha_i, a_i, b_i)$ that satisfies (4.02). Thus $S_i = [0.4,0.6] \times [0.0,0.3] \times [0.0,0.3]$. He may state whatever parameters he calculates are to his advantage; they need not coincide with his true values $(\bar{\alpha}_i, \bar{a}_i, \bar{b}_i)$.

The outcome function F picks the reallocation of goods among the two individuals that is a competitive equilibrium with respect to the utility functions they reported. To be specific, let, for i=1,2, $(\alpha_i, a_i, b_i)$ be the parameters that are reported and $(\bar{\alpha}_i, \bar{a}_i, \bar{b}_i)$ be the true parameters. Let good x have price p and let good y be the <u>numeraire</u>. Recall that the initial endowments for each are one unit of x and one unit of y. Taking the price p as given, utility maximization implies the following demand curves:

$$x_i = \alpha_i + a_i(1-\alpha_i) + \frac{\alpha_i(1-b_i)}{p} \tag{4.03}$$

$$y_i = (1-\alpha_i)\,[1 + (1-a_i)p] + \alpha_i\, b_i \tag{4.04}$$

for $i \in \{1,2\}$. From these the market clearing price is derived as a function of the strategies $s_1 = (\alpha_1, a_1, b_1)$ and $s_2 = (\alpha_2, a_2, b_2)$; it is:

$$p = \frac{\alpha_1(1-b_1)+\alpha_2(1-b_2)}{(1-\alpha_1)(1-a_1)+(1-\alpha_2)(1-a_2)} \quad . \tag{4.05}$$

Substitution of (4.05) into (4.03) and (4.04) therefore gives the components of the outcome function:

$$F_{i1}(s_1,s_2) = x_i = \alpha_i + a_i(1-\alpha_i) + \alpha_i(1-b_i)\ \frac{(1-\alpha_1)(1-a_1)+(1-\alpha_2)(1-a_2)}{\alpha_1(1-b_1)+\alpha_2(1-b_2)} \tag{4.06}$$

$$F_{i2}(s_1,s_2) = y_i = (1-\alpha_i)\left[1 + (1-a_i)\ \frac{\alpha_1(1-b_1)+\alpha_2(1-b_2)}{(1-\alpha_1)(1-a_1)+(1-\alpha_2)(1-a_2)}\right] + \alpha_i\ b_i \tag{4.07}$$

where $i \in \{1,2\}$.

Equations (4.06) and (4.07) complete the description of this allocation mechanism. Now I shall apply the theorem and show that the mechanism is not straightforward between persons one and two. Suppose that it is straightforward. Therefore, for $i \in \{1,2\}$, a function $\sigma_i$ exists such that person i's optimal strategy is $\tilde{s}_i = (\tilde{\alpha}_i,\tilde{a}_i,\tilde{b}_i) = \sigma_i(\bar{\alpha}_i,\bar{a}_i,\bar{b}_i)$. Suppose further that the true utility function $\bar{u}_1 = (\bar{\alpha}_1,\bar{a}_1,\bar{b}_1) \in \text{int } U_1$ and $\bar{u}_2 = (\bar{\alpha}_2,\bar{a}_2,\bar{b}_2) \in \text{int } U_2$ exist such that (a) trade is potentially mutually beneficial[9] and (b) $\tilde{s}_1 = \sigma_1(\bar{u}_1) \in \text{int } S_1$ and $\tilde{s}_2 = \sigma_2(\bar{u}_2) \in \text{int } S_2$. Let the reference quintuple be $V = \langle \bar{u}_1,\bar{u}_2,\emptyset,S_1,S_2\rangle$. The third component of V is the null set because the group consists of only two people. Setting $S_1^*$ and $S_2^*$ as identical to $S_1$ and $S_2$ respectively causes no problem because F is continuously differentiable over its entire domain.

The first step in applying the theorem is to ascertain that suitable curves A and B exist. That they do is seen by noting two facts. First each individual can affect the price p either positively or negatively by appropriately changing his strategy from his straightforward strategy $\tilde{s}_i$. This follows from (4.05) and the assumption that $\tilde{s}_i \in \text{int } S_i$. Second, a change in price generally leads to either a positive or negative change in their utility. This follows by direct substitution of the demand curves (4.03) and (4.04) into the utility function (4.01) and differentiating with respect to p, e.g. for person one

$$(4.08) \qquad \bar{u}_1(s_1,p) = \bar{\alpha}_1 \log_e\{\alpha_1 + a_1(1-a_1) + \frac{\alpha_1(1-b_1)}{p} - \bar{a}_1\}$$

$$+ (1-\bar{\alpha}_1) \log_e\{(1-\alpha_1)[1+(1-a_1)p] + \alpha_1 b_1 - \bar{b}_1\}$$

$$= \bar{\alpha}_1 \log_e(w-\bar{a}_1) + (1-\bar{\alpha}_1) \log_e(z-\bar{b}_1)$$

and

$$(4.09) \qquad \frac{d\bar{u}_1}{dp} = - \frac{\bar{\alpha}_1}{(w-\bar{a}_1)} \frac{\alpha_1(1-b_1)}{p^2} + \frac{(1-\bar{\alpha}_1)}{(z-\bar{b}_1)} (1-\alpha_1)(1-a_1) .$$

Thus except at the unique value of p such the right hand side of (4.09) equals zero any change in price leads to a change in person one's utility. Consequently suitable curves A and B generally exist at $V = \langle \bar{u}_1, \bar{u}_2, \emptyset, S_1, S_2 \rangle$.

The second step in applying the theorem is to ascertain that the set $U_2$ of admissible utility functions is A-V broad. Figure 2 diagrams the problem. Let point $\overline{x}_2$ be person two's optimal allocation given that his utility function is $\overline{u}_2$ and person one's strategy is $\tilde{s}_1$. If person one changes his strategy to $A(p')$, then let person two's new optimal point be $z_2$. Therefore

$$(4.10) \qquad \overline{x}_2 = f_2[A(\tfrac{1}{2}), \sigma_2(\overline{u}_2)] = f_2[\tilde{s}_1, \tilde{s}_2]$$

and

$$(4.11) \qquad z_2 = f_2[A(p'), \sigma_2(\overline{u}_2)]$$

because the definition of straightforwardness states that person i's straightforward strategy $\sigma_2(\cdot)$ always results in selection of that feasible allocation which person i most prefers. Reference back to the definition and accompanying diagram, Figure 1, shows that A-V broadness essentially requires that a utility function $u_2' = (\alpha_2', a_2', b_2') \in U_2$ exist such that $z_2$ remains optimal, $x_2$ becomes suboptimal, and $w_2$ becomes optimal. If such a $u_2'$ exists, then, for the same reasons (4.10) and (4.11) are true, $z_2 = f_2[A(p'), \sigma_2(\alpha_2', a_2', b_2')]$ and $w_2 = f_2[\tilde{s}_1, \sigma_2(\alpha_2', a_2', b_2')]$. Therefore existence of $u_2'$ implies that the trajectories that $\sigma_2(\overline{u}_2)$ and $\sigma_1(u_2')$ generate as $s_1$ varies from $A(\tfrac{1}{2})$ to $A(1)$ cross at $z_2$.

Thus it is sufficient to find a triple $u_2' = (\alpha_2', a_2', b_2') \in U_2$ such that, first, the marginal rate of substitution for $u_2'$ (denoted by MRS$'$) at $z_2$ equals the marginal rate of substitution for $\overline{u}_2$ (denoted by $\overline{\text{MRS}}$) at $z_2$ and, second, the MRS$'$ at $\overline{x}_2$ does not equal the $\overline{\text{MRS}}$ at $x_2$. This is easily done because $U_2$ is a three parameter family of utility functions. Specifically let $\overline{M}_z$ be the $\overline{\text{MRS}}$ at $z_2$

and let $\overline{M}_x$ be the $\overline{MRS}$ at $\overline{x}_2$. Appropriate differentiation of (4.01) gives the $\overline{MRS}$ at any point $(x_2, y_2)$:

$$(4.12) \qquad \overline{MRS} = - \frac{\overline{\alpha}_2 (y_2 - \overline{b}_2)}{(1-\overline{\alpha}_2)(x_2 - \overline{a}_2)} \quad .$$

Let $z_2 = (x_z, y_z)$ and $\overline{x}_2 = (\overline{x}_x, \overline{y}_x)$. Consequently $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2)$ satisfy the equations

$$(4.13) \qquad \overline{M}_z = - \frac{\overline{\alpha}_2 (y_z - \overline{b}_2)}{(1-\overline{\alpha}_2)(x_z - \overline{a}_2)} \quad ,$$

$$(4.14) \qquad \overline{M}_x = - \frac{\overline{\alpha}_2 (\overline{y}_x - \overline{b}_2)}{(1-\overline{\alpha}_2)(\overline{x}_x - \overline{a}_2)} \quad .$$

Let $M_z' = \overline{M}_z$ and pick a $M_x'$ such that $M_x' \neq \overline{M}_x$. If a $u_2' \in U_2$ exists such that its MRS$'$ at $z_2$ is $M_z'$ and its MRS$'$ at $\overline{x}$ is $M_x'$, then the requirements of A-V broadness are met. The parameter for such a $u_2'$ can be found by picking a triple $(\alpha_2', a_2', b_2') \in U_2$ that satisfies the equations

$$(4.15) \qquad M_z' = - \frac{\alpha_2' (y_z - b_2')}{(1-\alpha_2')(x_z - a_2')}$$

$$(4.16) \qquad M_x' = - \frac{\alpha_2' (y_x - b_2')}{(1-\alpha_2')(x_x - a_2')} \quad .$$

Such a triple exists and is contained in $U_2$ provided that the difference between $M_x'$ and $\overline{M}_x$ is made small. This follows from the continuity of the equations and the existence of $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2) \in \text{int } U_2$ that satisfy equations (4.13) and (4.14). In other words, if $\overline{M}_x$ and $M_x'$ are picked so that their difference is small, then a perturbation of the point $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2)$ exists such that (4.15) and (4.16) are satisfied. Therefore $U_2$ is A-V broad.

This creates a contradiction. Because suitable curves A and B exist at V and because $U_2$ is A-V broad, the mechanism, with one qualification, can not be straightforward. The qualification stems from the dependence of this demonstration on the existence of straightforward strategies that lie within the interior of each person's strategy space $S_i$. Therefore the possibility remains that $\sigma_i$, $i \in \{1,2\}$, exist and always pick strategies that lie on the frontier of $S_i$.

## 5. Straightforwardness and Nonstraightforwardness within Monopoly Markets

A monopolist who sells to a continuum of atomless consumers acts straightforwardly towards each individual consumer in the following sense. He analyses the market demand curve and his cost curves and sets his price such that marginal cost equals marginal revenue. He does not need to know, in order to act optimally, what any particular consumer's preferences are. Only the aggregate quantity that he expects to sell is significant. Individual consumers also act straightforwardly towards the monopolist. Each acts as a price taker, calculates his demand curve, and, when the monopolist announces his price, buys the quantity specified on his demand curve. The consumer does not modify his behavior depending on the cost functions of the monopolist; the only information a consumer needs is the price and his own preferences. Behavior, however, is no longer straightforward if consumers are not atomless. The monopolist needs to know each consumer's true demand curve in order to determine his optimal strategy and consumers need to know the monopolist's costs in order to determine their optimal strategies.

My purpose in this section is twofold. First, I show that the straightforward behavior that exists between the monopolist and each atomless consumer is consistent with the theorem. Second, I show that if consumers are not atomless, then the theorem applies and the behavior between each consumer and the monopolist can not be straightforward. Throughout this discussion the definition of straightforwardness that I use is the narrow definition (see Section 2) of straightforwardness between a pair of individuals -- the monopolist and a specific consumer -- where the strategies of all other individuals are held fixed. The monopolist's behavior is not globally straightforward in Gibbard's sense; he must alter his strategy (the price) in response to the preferences held by groups of individuals because, even in the atomless case, groups of individuals may affect the market demand curve.

Consider first the case of a monopolized market with a continuum of atomless consumers. Let the consumer be person $\xi$ and let the monopolist be person $\eta$. The monopolist's consumption set is all possible price-quantity pairs; his class of admissible utility functions consists of all possible profit functions that are consistent with the usual assumptions of production theory; his strategy space consists of all nonnegative prices. The consumer's consumption set consists of all price-quantity pairs that his income allows him to afford; his class of admissible preferences over the consumption set are all weak orderings that can be derived from the usual assumptions of consumer behavior; his strategy space is the set of all demand curves that are consistent with the simple maximization of admissible utility functions. Let the reference quintuple V be chosen such that (a) the monopolist's straightforward strategy, as shown in Figure 3, is to set the price P and (b) the consumer's straightforward strategy, as shown in Figure 4, is to report demand curve DD.

Given this model, how does it relate to the theorem? The answer is that a suitable curve A does not exist. In other words, the consumer through a change in his strategy is unable to affect the monopolist's utility level and therefore the theorem's hypothesis is not satisfied. Suppose, as shown on Figure 4, the consumer changes his strategy from reporting the demand curve DD = $A(\frac{1}{2})$ to reporting the demand curve $D''D''$ = A(1). This increases the quantity he purchases from the monopolist, but, because consumers are assumed atomless, does not perceptibly shift the monopolist's optimal point on Figure 3 outward from B. Every consumer would have to change his reported demand curve from DD to $D''D''$ in order to shift the monopolist's market demand cruve from dd to $d''d''$. The individual consumer can not affect, as the definition of suitability requires, the utility level of the monopolist. Consequently the straightforwardness of this market is consistent with the theorem.

Consider now the second case where the assumption that a continuum of atomless consumers exists is replaced with the polar assumption that a bilateral monopoly exists. In this situation the theorem's requirements are met and, consequently, bilateral monopoly is not straightforward. The analysis is this. Again let the consumer be person $\xi$ and the monopolist be person $\eta$. Assume that both have straightforward strategies. A suitable curve A may be constructed as follows. Let the consumer shift his strategy, as shown on Figure 4, from reporting demand curve $D'D'$ = A(0) to reporting DD = $A(\frac{1}{2})$ to reporting $D''D''$ = A(1). These shifts, since the market is a bilateral monopoly, are shifts in the monopolist's market demand curve as shown on Figure 3. Because they are outward shifts and because, by definition, the monopolist's straightforward strategy always results in attainment of the feasible point that is optimal, these shifts in the demand curve result in the monopolist's equilibrium moving in sequence from a low profit level at point A to a high profit level at point C. The requirement that a suitable curve A exists is thus met. A suitable curve B may be similarly constructed. If on Figure 3 the monopolist changes his strategy from $P'' = B(0)$ to $P = B(\frac{1}{2})$ to $P' = B(1)$, then the consumer's utility continuously increases, which is precisely the requirement for curve B. The last requirement of the theorem is that the monopolist have a set of admissible preferences that is A-V broad. That it is A-V broad is easily seen by reference to Figure 5 and by reference back to the definition of broadness and its accompanying diagram, Figure 1. The crossing of the marginal cost curves cc and CC in Figure 5 is all that is necessary to produce the crossing at z of the trajectories of the monopolist's most preferred points. Consequently the theorem's requirements are met and bilateral monopoly is not straightforward.

## 6. The Desirability and Impossibility of Straightforwardness

If an allocation mechanism is nonstraightforward, then it should be analyzed as a game of incomplete information. To see this, recall the situation of the monopolist and the consumer in the previous section's bilateral monopoly illustration. If each knows the other's preferences, then the problem is a two-person, nonzero sum, complete information game for which the core is an appropriate solution concept. This, however, is an understatement of the actual problem because the model with which this paper is concerned and which gives rise to the problem of nonstraightforwardness implicitly assumes that each individual only uncertainly knows the true preferences of the other individuals. In the bilateral monopoly illustration the monopolist is uncertain what the consumer's true, price-taking demand curve is and the consumer is uncertain what the monopolist's marginal cost curve is. Consequently it is an incomplete information game: individuals are uncertain concerning other individuals payoff functions.

This allows me to state,very briefly, my argument in favor of straightforwardness. Economic allocation mechanisms, as shown above, are games of incomplete information. Games of incomplete information have undesirable properties. The only known way in which the problems that are attendant to games of incomplete information may be solved is to redesign the game to be straightforward. Consequently straightforwardward allocation mechanisms are desirable.

The second and third steps in this argument need elaboration. If the allocation mechanism is not straightforward, then it is a nontrivial game of incomplete information which gives rise to a host of problems. These problems are only dimly understood because game theory does not have fully satisfactory concepts of what rational behavior consists of within incomplete information games and what types of equilibriums may result.[11] Since even in simple, theoretical examples the analysis is very difficult, actual players of complicated, real world, incomplete information games may justifiably act in accordance with any of a wide variety of rules of thumb. Consequently the outcomes and optimality of incomplete information games is very difficult to predict or evaluate.

Nevertheless, despite the problems within the theory, the nature of certain phenomena within games of incomplete information are clear. First is the problem of the infinite regress of expectations. Returning to the bilateral monopoly example, the monopolist and the consumer each wants to play the strategy that is optimal given the strategy the other is likely to play. In order to calculate what the consumer is likely to play, the monopolist needs knowledge of what the consumer's true preferences are and of what the consumer's expectations concerning his own strategy are. For the consumer to form reasonable expectations concerning the monopolist's expected strategy he needs knowledge of what the monopolist's true preferences are and of what the monoplist's expectations concerning his, the consumer's,choice of strategy is. Therefore the monopolist, in order to choose his strategy, needs to know the consumer's true preferences, the consumer's perception of his own true preferences, etc. The result is an infinite regress.[12]

That each person's choice of strategy depends on uncertain estimates of other people's true preferences makes possible a type of strategic manipulation that is not available within games of complete information. Individual one, for example, may have an incentive to mislead individual two concerning what his, one's, preferences truly are.[13] The reason is that if individual two falls for the ploy, then individual two bases his choice of strategy on incorrect information and, if individual one has been clever, may be led to choose a strategy that is more favorable to individual one than the strategy that he would choose if individual one had not mislead him. An example of this is the negotiations between the monopolist and consumer within bilateral monopoly. If the consumer proposes that the monopolist sell at a particular price, the monopolist is likely to respond, perhaps untruthfully, that the price is too low to meet his costs and therefore must be raised. Symmetrically the consumer tries to convince the monopolist that the product has low value to him. If he succeeds, then the monopolist will be willing to sell at a price just above cost in order to make at least some profit. This process has the danger that if one or both individuals push their misrepresentations too stubbornly, then either no trade or a drastically inefficient trade may take place. Thus, to summarize, nonstraightforward allocation mechanisms are nontrivial games of incomplete information that appear to consume substantial resources in bargaining and produce final allocations that are quite unpredictable and perhaps objectively unsatisfactory.

These problems that are associated with games of incomplete information evaporate when the allocation mechanism is straightforward. The fact that individuals are playing a game of incomplete information is irrelevant if the allocation mechanism is straightforward. The players have no need for the information they lack because they can select their optimal strategies on the basis of their preferences and the outcome function alone.

Therefore, if they exist, straightforward allocation mechanisms are desirable. Unfortunately within economic environments they do not exist except in the case of an infinite number of atomless agents. This assertion -- that straightforward allocation mechanisms do not exist within economic environments -- follows directly from this paper's theorem and identification of two, essentially universal characteristics of economic allocation mechanisms. The first characteristic, which reflects the variability of people's tastes, is that an individual's preferences may be any one of a wide variety of admissible orderings. The second characteristic is that within an economy with a market sector the demand of each individual feeds back through the price system and affects all other individuals. These two characteristics are exactly the two properties that the requirements of broadness and suitability formalize. Consequently, according to the theorem, the only means by which economic allocation mechanisms can be made straightforward is to enforce the requirement that individuals be permitted only negligible economic power. This, of course, is precisely the means by which the perfectly competitive model with a large number of atomless consumers achieves straightforwardness.

7. Comments on the Necessity of the Suitability and Broadness Requirements

This section consists of three examples whose point is this.[14] The theorem shows that two conditions -- suitability and broadness -- are sufficient to guarantee that a resource allocation mechanism is not straightforward. These conditions are not necessary for a mechanism to be nonstraightforward. Yet in an informal, weak sense they do contain an element of necessity. Mechanisms do exist that are straightforward and that satisfy only one or the other of the suitability and broadness conditions. Consequently neither broadness nor suitability can completely be discarded and still retain a valid theorem.

The first example is a mechanism that proves, by counter-example, that the theorem does not specify necessary as well as sufficient conditions for nonstraightforwardness. This mechanism, which does not satisfy the broadness requirement, is derived from the exchange economy model presented in Section 4 by fixing the parameters $a_1, a_2, b_1, b_2$, $\bar{a}_1, \bar{a}_2, \bar{b}_1, \bar{b}_2$ equal to zero. Under this restriction only Cobb-Douglas utility functions are admissible as individual's true preferences and as their reported preferences. This class can not satisfy the broadness assumption because a change in the one parameter $\alpha_i$ causes a change in the marginal rate of substitution at every point in the consumption set.[15]

Consequently the necessary crossing of trajectories that is shown on Figure 1 is impossible to obtain. Inspection of the outcome function's components (4.06) through (4.07) shows that suitable curves A and B exist.

That this modified mechanism is not straightforward is demonstrated by maximizing each person's utility function and the other person's choice of strategy. For person one this results in a quadratic equation:

$$(7.01)\quad (1-\bar{\alpha}_1-\alpha_2)\alpha_1^2 + (\alpha_2-\alpha_2^2+2\bar{\alpha}_1\alpha_2)\alpha_1 + \bar{\alpha}_1 a_2^2 - 2\bar{\alpha}_1\alpha_2 = 0.$$

Person one's optimal strategy is the root $\alpha_1$ of this equation that satisfies the second order conditions for a maximum. Clearly its value depends not only on the value of a $\bar{\alpha}_1$, person one's true perferences, but also on $\alpha_2$, person two's reported preferences. Therefore this mechanism is not straightforward, despite the lack of broadness within the sets of admissible utility functions.

My second example is a mechanism Hurwicz [10] discovered. Its set of admissible utility functions does not satisfy the broadness condition, it admits suitable curves A and B, and it is straightforward. Its existence shows that some version of the broadness condition is necessary to guarantee nonstraightforwardness. The example is this. Three individuals, $i \in \{1,2,3\}$, are jointly deciding on allocation of one public good y and one private good x. Each individual begins with an initial endowment of $w_i$

units of the private good and has a utility function

$$(7.02) \qquad u_i(x_i, y_i) = x_i + 2\theta_i \, y^{\frac{1}{2}}$$

where $x_i$ is person i's private good allocation and $\theta_i$ is a positive parameter. Each person's true preferences are characterized by a parameter value $\overline{\theta}_i$ and each person's strategy is to report a value for the parameter $\theta_i$. The outcome functions are the "basic quadratic" of Groves and Ledyard [5]:

$$(7.03) \qquad F_i[\theta_1, \theta_2, \theta_3] = x_i = w_i - \theta_i^2 - 2\theta_j \, \theta_k$$

$$F_y[\theta_1, \theta_2, \theta_3] = y = (\theta_1 + \theta_2 + \theta_3)^2$$

where $i \neq j \neq k \neq i$. Person i's utility, given the strategies $\theta_j$ and $\theta_k$ of the other two people, is therefore

$$(7.04) \qquad u_i = w_i - \theta_i^2 - 2\theta_j \, \theta_k + 2\overline{\theta}_i \, (\theta_1 + \theta_2 + \theta_3).$$

Maximization with respect to person i's reported preferences $\theta_i$ gives

$$(7.05) \qquad \frac{du_i}{d\theta_i} = -2\theta_i + 2\overline{\theta}_i = 0$$

or

$$(7.06) \qquad \theta_i = \overline{\theta}_i,$$

i.e. the mechanism is straightforward because the optimal strategy of person i is to report a parameter value that is identical to his true parameter.[16] Inspection of (7.04) shows that suitable curves A and B exist. The broadness requirement is violated for the same reason that it is violated by the family of Cobb-Douglas utility functions: a change in the single parameter changes the marginal rate of substitution at every point within the consumption set.

My third example, which I already discussed from one point of view in Section 5, is the monoplist who sells to a continuum of atomless consumers. My purpose for reintroducing it here is that it is a straightforward mechanism for which the broadness requirement is satisfied.[17] Therefore it demonstrates that some version of the suitability condition is necessary to guarantee that a mechanism is not straightforward. By itself broadness of the set of admissible utility functions is not sufficient.

Nevertheless, in addition to the theorem, one general result does suggest itself. If a mechanism with a continuously differentiable outcome function does not admit suitable curves, then it is straightforward between pairs of individuals. An informal constructive proof of this assertion is as follows. Suppose, as in the monopoly example where the consumer is person $\xi$ and the monopolist is person $\eta$, a suitable curve A can not be constructed, i.e. person $\xi$ can not affect person $\eta$. Therefore person $\eta$'s choice of strategy is straightforward -- he can ignore $\xi$ because $\xi$ can not affect him. Person $\eta$, however, may affect person $\xi$. Therefore, generally, person $\xi$'s choice of strategy must be dependent on person $\eta$'s choice of strategy. This, however, poses no problem. Because $\eta$'s choice of strategy is not influenced by

$\xi$'s choice of strategy, person $\xi$'s optimal strategy is a list of actions whose implementation is conditional on $\eta$'s choice of strategy. The consumer's demand curve in the monopoly example is an illustration of this approach. Such a list is a straightforward strategy because person $\xi$ can optimally prepare it without access to any information concerning person $\eta$'s true preferences. Thus if suitable curves A and B do not exist, the mechanism is straightforward between the pair of individuals in question.

## Footnotes

1. Roberts and Postlewaite [11] have discussed in detail the incentive properties of large exchange economies.
2. A monotonic utility function $u_i$ has the property that if $x_i, y_i \in C_i$ is a pair of allocations such that $x_i > y_i$, then $u_i(x_i) > u_i(y_i)$. The notation $x_i > y_i$ means that every component of $x_i$ is strictly greater than every component of $y_i$.
3. This assumption shares with Arrow's independence of irrelevant alternatives assumption a common justification. Arrow [2,p.109] stated in support of that assumption: "The essential point ... is the application of Leibniz's principle of the identity of indiscernables. Only observable differences can be used as a basis for explanation." This principle applies equally well to my nonzero cost assumption.
4. The notation "int $S_\eta$" represents the interior of the set $S_\eta$.
5. A curve $A\{[0,1]\}$ is smooth if its components $A_i$ are differentiable, it is rectifiable if it has finite length, and it is simple if it has no closed loops, i.e. a smooth, rectifiable, simple curve is well behaved.

6. Groves and Ledyard [6, Section 10] have observed that if the income elasticity of demand is not constant as preferences vary, then no allocation mechanism exists that is straightforward and satisfies the Lindahl-Samuelson optimality condition.
7. See Apostol [1,p.107] for the definition of a differential.
8. That this mechanism is not straightforward can also be determined by direct calculation.
9. I want to rule out the possibility that the two individuals' initial endowments are on the contract curve implied by their true utility functions.
10. That F is continuously differentiable and that $(\bar{s}_1,\tilde{s}_1,\bar{s}_2,\tilde{s}_2) \in \text{int } [S_1 \times S_1 \times S_2 \times S_2]$ assures that first order conditions are satisfied at person two's optimal points. Satisfaction of first order conditions in turn implies that the optimal points are tangencies.
11. Harsanyi [8] has developed the most complete theory of such games.
12. Harsanyi [7] has discussed in detail the possibility of an infinite regress arising in this manner.
13. Within the context of voting Blin and I [3] have discussed in detail the possibilities of profitably misleading others concerning one's true preferences.

14. One interesting and important example of a straightforward mechanism that I do not discuss here is majority rule with single-peaked preferences. Analysis of it shows that it satisfies neither the suitability nor broadness requirements.

15. This assertion may be checked by differentiating equations (4.10) with respect to $\alpha$.

16. Hurwicz [10] shows that this mechanism, in addition to being straightforward, gives optimal outcomes and that $u_i(x_i,y) = x_i + 2\theta_i\, y^{\frac{1}{2}}$ is the only class of utility functions that is consistent with a three person, one public good, and one private good economy being straightforward.

17. A dictatorial allocation mechanism is a second example of a straightforward mechanism that satisfies the broadness re-requirement, but not to suitability requirement. A dictatorial mechanism is one where a single individual has the power to specify fully the final allocation.

## REFERENCES


[1] Apostol, T., Mathematical Analysis, (Reading, Mass: Addison-Wesley, 1957).

[2] Arrow, K.J., Social Choice and Individual Values, 2nd Edition, (New Haven: Yale University Press, 1963).

[3] Blin, J.M. and M. Satterthwaite, "On Preferences, Beliefs, and Manipulation within Voting Situations," Discussion Paper No. 212, Center for Mathematical Studies in Economics and Management Science, Northwestern University, March 1976. Forthcoming in Econometrica.

[4] Gibbard, A., "Manipulation of Voting Schemes: A General Result," Econometrica, 41 (1973), pp. 587-602.

[5] Groves, T. and J. Ledyard, "Optimal Allocation of Public Goods: A Solution to the Free Rider Problem," forthcoming in Econometrica.

[6] Groves, T. and J. Ledyard, "Some Limitations of Demand Revealing Processes," Discussion Paper No. 219, Center for Mathematical Studies in Economics and Management Science, Northwestern University, May 1976.

[7] Harsanyi, J., "Bargaining in Ignorance of the Opponents Utility Function," J. of Conflict Resolution, 6 (March 1962), pp. 29-38.

[8] Harsanyi, J., "Games with Incomplete Information Played by Bayesian Players; Part I: The Basic Model; Part II: Bayesian Equilibrium Points; Part III: The Basic Probability Distribution of the Game," Management Science, 14 (1967-68), pp. 159-82, 320-34, 486-502.

[9] Hurwicz, L., "On Informationally Decentralized Systems," in Decision and Organization, ed. by C. B. McGuire and R. Radner (Amsterdam: North-Holland, 1972), pp. 297-336.

[10] Hurwicz, L., "On the Existence of Allocation Systems whose Manipulative Nash Equilibria are Pareto-Optimal: The Case of One Public Good and One Private Good." Duplicated notes for presentation given at the 5th World Congress of the Econometric Society, Toronto, August 23, 1975.

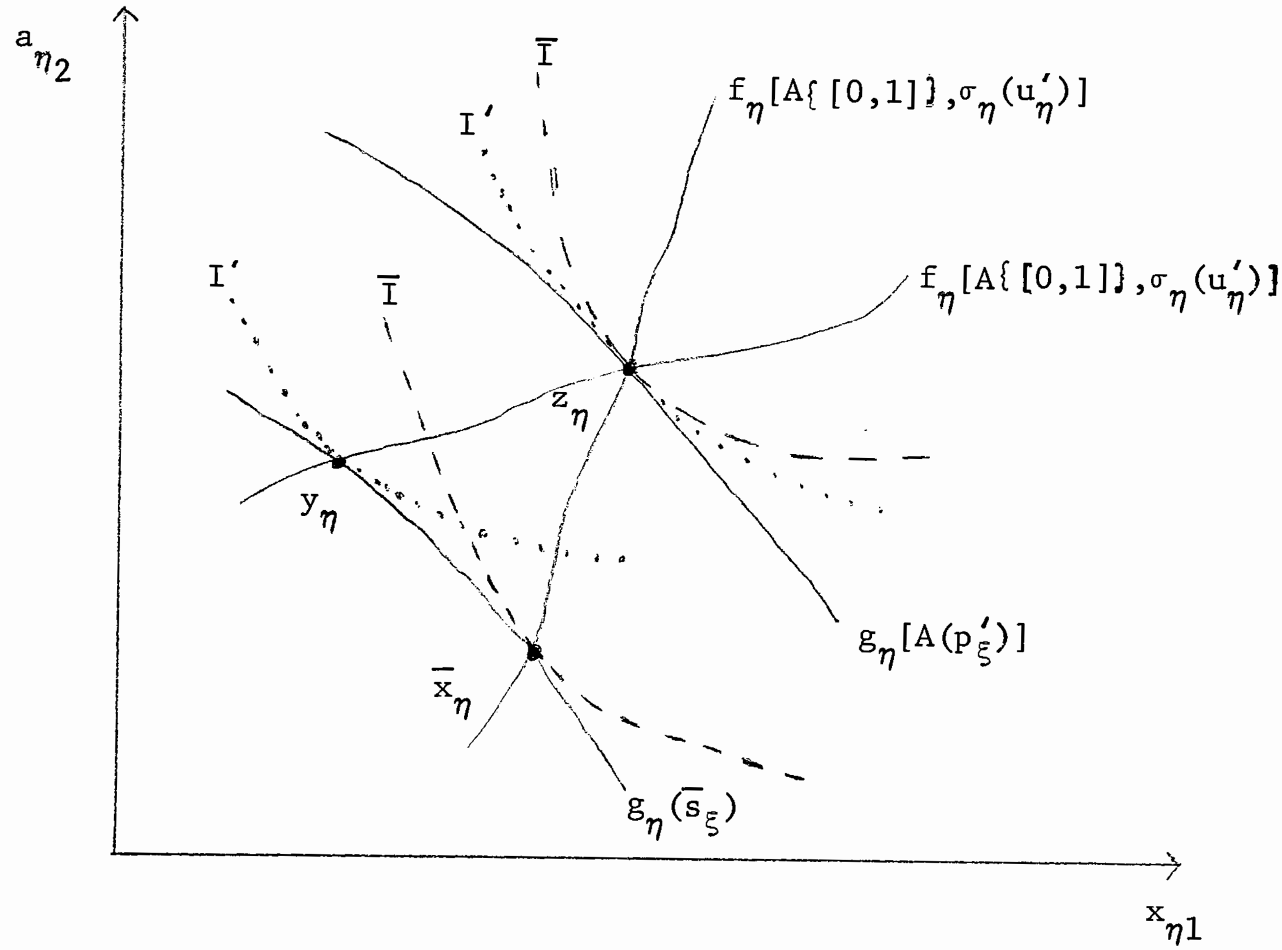
[11] Roberts, D. J. and A. Postlewaite, "The Incentive for Price-Taking Behavior in Large Exchange Economies," Econometrica, 44 (1976), pp. 115-128.

[12] Satterthwaite, M. A., "Existence of a Strategy Proof Voting Procedure: A Topic in Social Choice Theory," (unpublished Ph.D. dissertation, Univ. of Wisconsin, Madison, 1973).

[13] Satterthwaite, M. A., "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions," J. of Econ. Theory, 10 (1975), pp. 187-217.

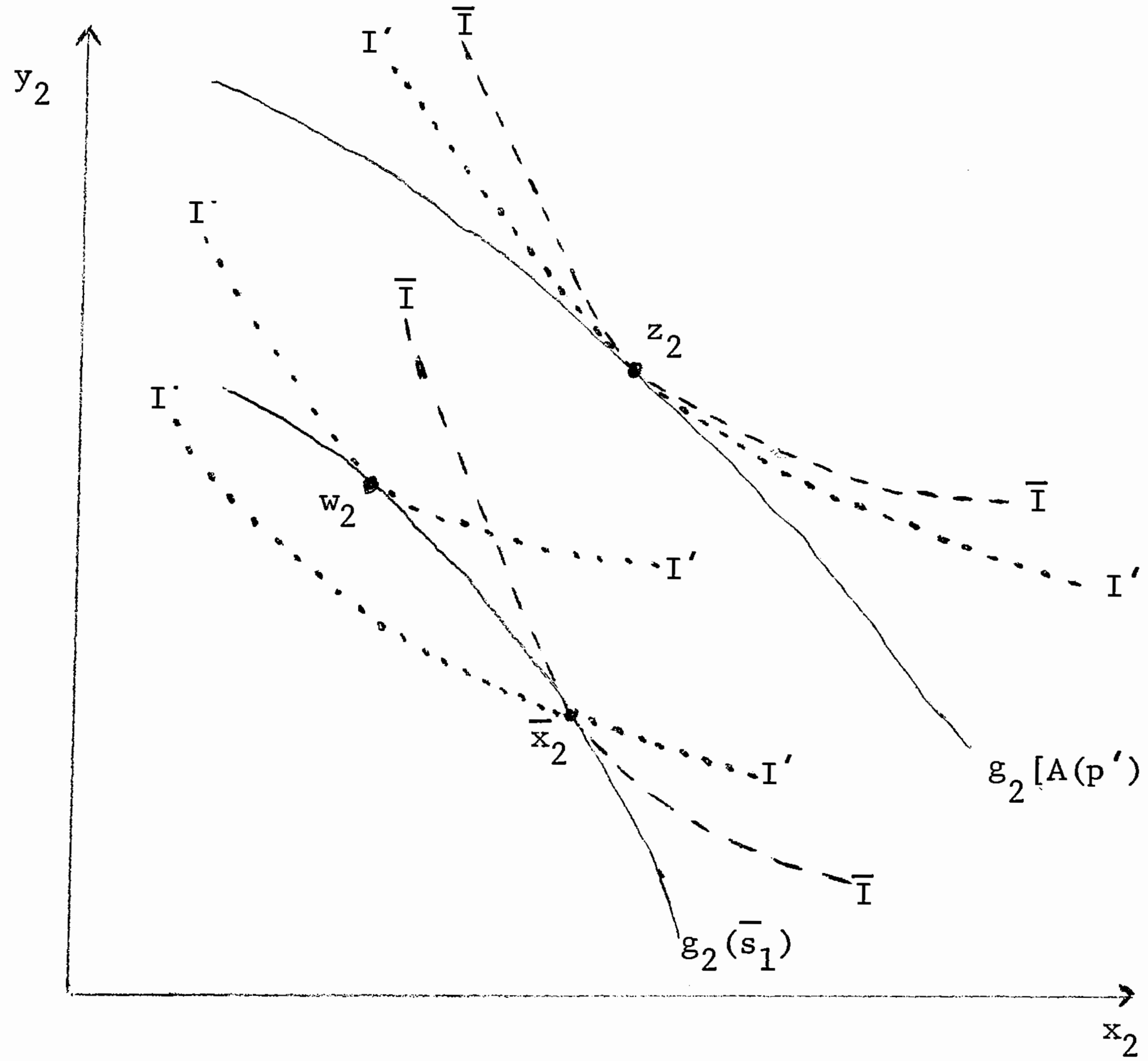
[14] Stone, R., "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand," Economic Journal, 64 (1954), pp. 511-27.

Figure 1



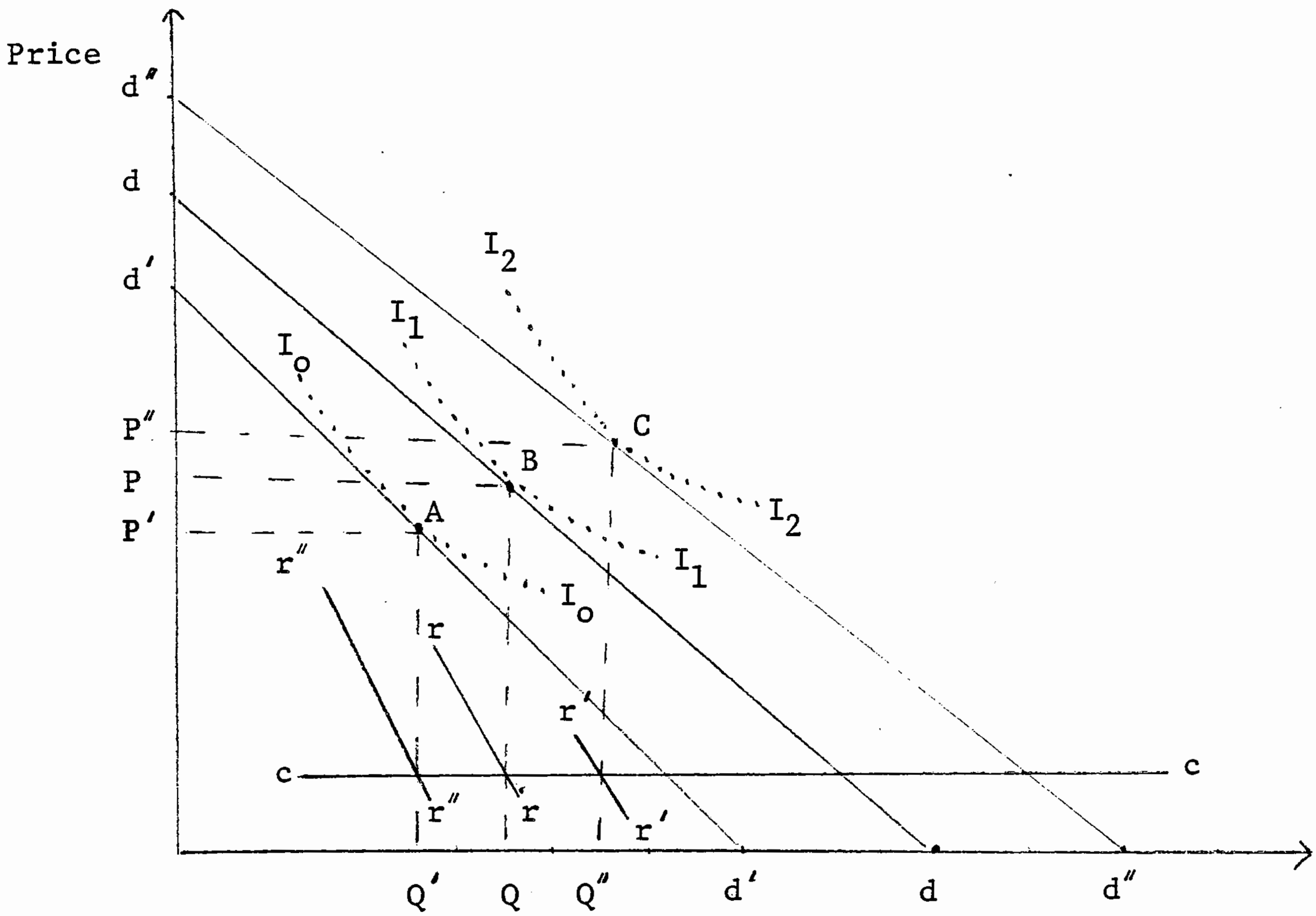

Explanation. The curve $f_\eta[A\{[0,1]\},\sigma_\eta(\overline{u}_\eta)]$ is the trajectory person $\eta$'s allocation follows if he plays his straightforward strategy $\overline{s}_\eta = \sigma_\eta(\overline{u}_\eta)$ and person $\xi$ varies his strategy along the suitable curve $A\{[0,1]\} \subset S^*_\xi$. The dashed curves $\overline{I}$ represent two of the indifference curves that $\overline{u}_\eta$ generates. The curve $f_\eta[A\{0,1]\},\sigma_\eta(u'_\eta)$ has a similar interpretation as a trajectory except that person $\eta$ is playing that strategy which is straightforward for utility function $u'_\eta$. The dotted curves I′ represent the indifference curves generated by $u'_\eta$. The curves $g_\eta(\overline{s}_\xi)$ and $g_\eta[A(p'_\xi)]$ represent the offer curves facing $\eta$ when $\xi$'s strategy is, respectively, $\overline{s}_\xi$ and $A(p'_\xi)$.

Figure 2



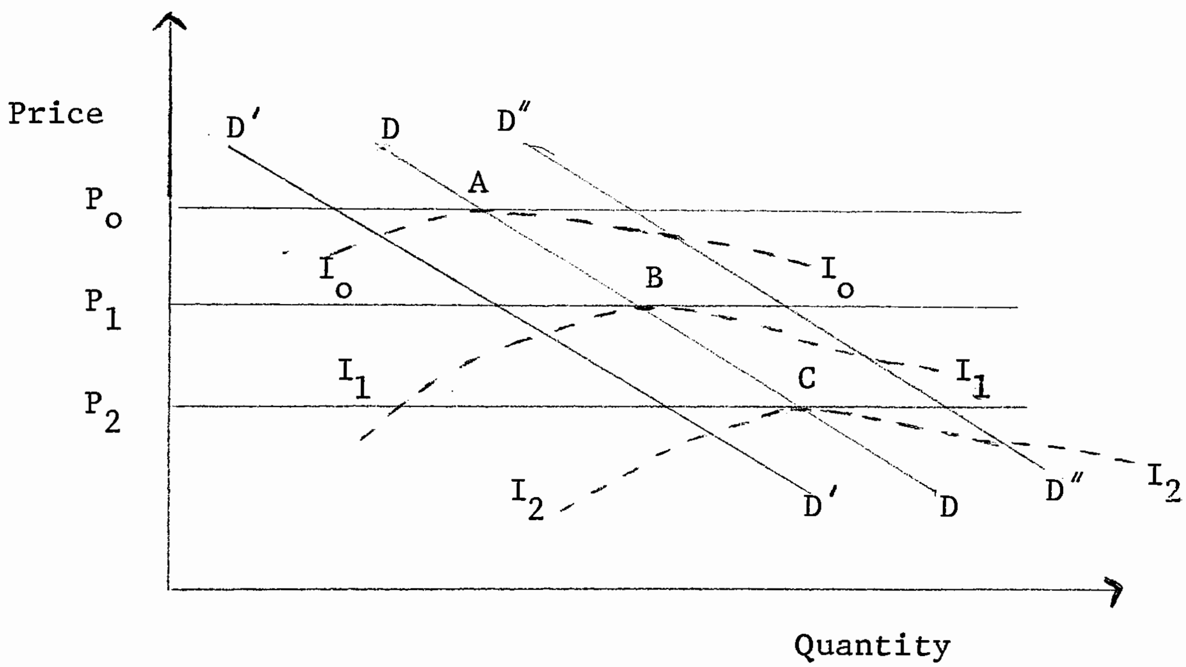

Explanation. The dashed lines $\bar{I}\ \bar{I}$ represent indifference curves generated by $(\bar{\alpha}_2, \bar{a}_2, \bar{b}_2)$. The dotted lines $I'I'$ represent indifference curves generated by $(\alpha_2', a_2', b_2')$.

Figure 3



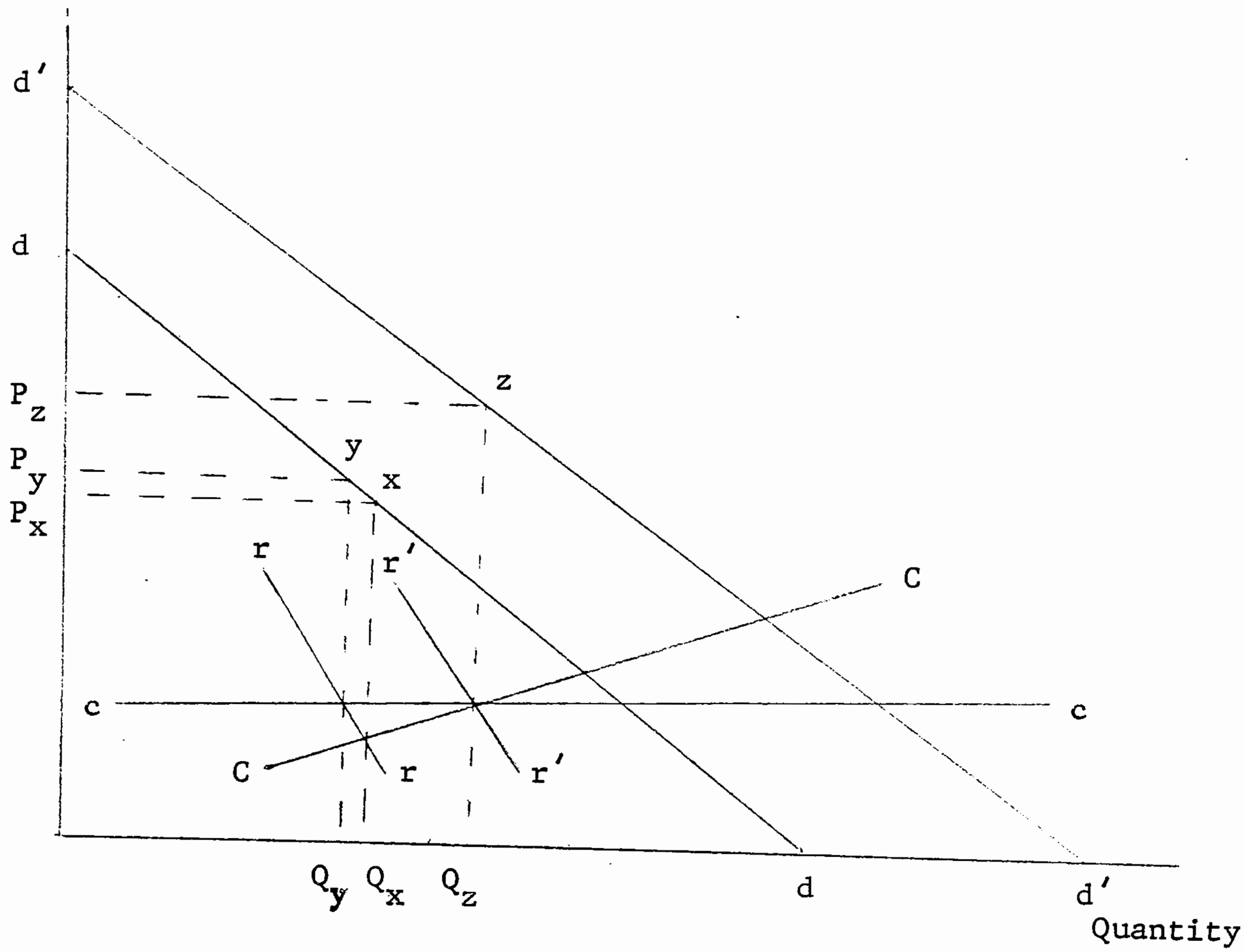

Explanation. Lines r″r″, rr, and r′r′ are the marginal revenue curves associated with the market demand curves d″d″, dd, and d′d′ respectively. Line cc is the firm's marginal cost curve. The dotted curves $I_oI_o$, $I_1I_1$, and $I_2I_2$ are the monopolist's indifference (isoprofit) curves.

Figure 4




Explanation. The dotted lines $I_oI_o$, $I_1I_1$ and $I_2I_2$ are the consumers indifference curves among price-quantity pairs. $I_oI_o$ represents lower satisfaction than $I_1I_1$, etc.

Figure 5




Explanation. Lines rr and r'r' are marginal revenue curves for demand curves dd and d'd' respectively. Lines CC and cc are marginal cost curves.