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A TECHNICAL NOTE ON
CARTEL STABILITY IN LARGE ECONOMIES

by

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Professor Johansen [1] has recently raised the issue of the stability of competitive behavior and outcomes in large economies when agents can form cartels and act cooperatively to manipulate price formation. However, such cartels would seem to be a serious threat only if they are themselves stable against agents who defect from the cartel and readopt competitive behavior. This raises the issue of the stability of cartels. The purpose of this note is to suggest one approach to this question based on our analysis in [2]. We argue here that, given certain continuity assumptions, individuals belonging to a cartel in large economies will find it advantageous to break from the cartel and adopt competitive behavior. This suggests that such cartels will be unstable in such situations and as such, do not present a serious threat.

We employ the framework of [2]. Thus, an economy E is defined by two mappings, a and s , on an index set T of agents, where $a_t = a(t)$ is the characteristics (preferences and endowments) of agent t and $s_t = s(t)$ is a "response correspondence" from prices into net trades such that any net trade $z = s_t(p)$ given by $s(t)$ at any price p satisfies $pz \leq 0$ and yields a consumption bundle in the agent's consumption set. We view s_t as being chosen by t and as indicating the trades he will make at each price. If s_t is not the competitive demand correspondence corresponding to a_t , then t

is behaving non-competitively. A cartel C is a subset of T whose members coordinate their choices of response correspondences in order to influence price formation. Given s_t for $t \in T \sim C$, we assume that the s_t , $t \in C$, are selected so that there is a unique, strictly positive price vector p at which the net trades given by the responses s_t , $t \in T$, balance. In the case of T finite, this is simply $\sum_t s_t(p) = 0$. We then say p is the price effected by C .

It is convenient to describe the maps a and s by the distributions they induce on the suitably topologized spaces of characteristics and response correspondences. A sequence $E^n = (a^n, s^n)$ of economies converges to E in responses (resp., characteristics), denoted $E^n \rightarrow_s E$ (resp., $E^n \rightarrow_a E$) if $\nu^n \Rightarrow \nu$ (resp., $\mu^n \Rightarrow \mu$) where " \Rightarrow " denotes weak convergence, ν^n (resp. μ^n) is the distribution on responses (resp. characteristics) induced by E^n via s^n (resp., a^n) and ν (resp., μ) is that induced by E . Given a sequence of economies with index sets T^n and $S^n \subset T^n$, if $E^n \rightarrow_a E$ we say S^n converges in characteristics to S , written $S^n \rightarrow_a S$, if $\mu^n(a^n(S^n)) \rightarrow \mu(a(S))$ and the sequence of distribution $\mu^n|_{S^n}$ induced by the restrictions of the a^n to S^n converge to that induced by the restriction of a to S . The notion of convergence of S_n in responses is defined analogously.

We take the choice of correspondences for the members of a cartel C to depend on the preferences and endowments of the members of C and on the response correspondences of the agents outside the cartel. We represent this in terms of distributions by considering

a map σ which assigns a distribution $\nu|_C$ of responses for the cartel C to each pair $(\mu|_C, \nu|_{E \sim C})$ consisting of a distribution of characteristics for the members of the cartel and a distribution of responses for those outside. We then say cartel behavior is continuous at C in E if the map σ is continuous at $(\mu|_C, \nu|_{E \sim C})$. Given $E^n \rightarrow_a E$, $C^n \rightarrow_a C$, $(E^n \sim C^n) \rightarrow_s (E \sim C)$ and $a \in \{[\cap[\text{support}(\mu^n|_{C^n})] \cap [\text{support}(\mu|_C)]]\}$, we say that a is a regular cartel member if $x^n(a)$, his consumption bundle when he belongs to C^n , converges to $x(a)$, his consumption bundle in C , and $x(a)$ differs from his competitive consumption $d(a)$ at the price p effected by C in E . A regular cartel member is thus one whose consumption bundle differs from the competitive in the limit: he is thus actually adopting non-competitive behavior. Finally, the Walras correspondence is that assigning to each distribution of response correspondences the market clearing prices.

Theorem: Let $E^n \rightarrow_a E$, $C^n \rightarrow_a C$, and $(E^n \sim C^n) \rightarrow_s (E \sim C)$, where E is atomless and where the Walras correspondence is continuous at ν , the distribution over response correspondences induced by E . Suppose cartel behavior is continuous at C in E . Then for any regular cartel member a there exists $N > 0$ such that for $n > N$, $d^n(a)$ is strictly preferred by a to $x^n(a)$, where $x^n(a)$ is his consumption in C^n and $d^n(a)$ is his competitive consumption at the price q^n effected by the coalition $D^n = C^n \sim \{a\}$ when a adopts competitive behavior in E^n .

Proof: Since E is atomless $C^n \rightarrow_a C$ implies $D^n \rightarrow_a C$. By the continuity of cartel behavior, $C^n \rightarrow_s C$ and $D^n \rightarrow_s C$. Thus, $E^n \rightarrow_s E$, so p^n and q^n , the prices effected by C^n and D^n respectively, converge to a common limit p , the price effected by C in E . Since p is strictly positive, this means that if U is any continuous utility representing the preferences of a , then $U(d^n(a)) \rightarrow U(d(a))$. But $U(d(a)) > U(x(a)) = \lim U(x^n(a))$. The result follows.

The key to the instability of cartels here is that the price vector an agent expects to prevail after he departs from the cartel will, in a sufficiently large economy, be close to that prevailing when he is in the cartel. In the context of a non-atomic continuum, in which each agent has a completely negligible impact on prices, this assumption seems eminently reasonable. In large but finite economies, the satisfaction of this hypothesis would seem to hinge on two continuity conditions. First is the continuity of the Walras correspondence, which insures that the shift in strategies by the individual in question to competitive behavior does not influence prices markedly. As well, one needs that the price changes which result from the readjustment of the behavior of the remaining members of the cartel after the individual has left should be small in large economies. This latter condition would hold if the cartel does not alter its behavior significantly when its composition and the behavior of the rest of the economy change slightly.

Given this, the theorem suggests that the competitive system is stable against the formation of cartels of large numbers of

individually small agents. That is, of course, very much in accord with both observed behavior and the arguments that have been made in the literature on industrial organization (see, e.g. [3]). Cartels involving many small agents do tend to fall apart unless government steps in to enforce the cartel agreement through marketing boards and similar institutions.

What does remain a problem for the stability of competition is the single agent or small group of agents which retains market power even in large economies. This may arise either because the agents in question do not become "small" relative to the economy, because the Walras correspondence is discontinuous, or because the cartel behaves discontinuously. The first of these corresponds to the situation in which a group of small agents can coalesce permanently, as under a regime of legally enforceable cartel agreements, or to the case in which the endowments of the agents do not become small relative to the total endowments for some good or goods. The implications of discontinuity of the Walras correspondence are discussed in [2], and we will not consider them further here except to note that discontinuity is a "rare" phenomenon. Regarding the continuity of behavior, note that what is in fact important is that the agent expects that the cartel will not react violently to his defection (and that he expects prices to depend continuously on behavior). These expectations would seem quite reasonable in the context of large economies. Thus, it is "only" the first case, the agent who stays "large," which threatens

the competitive system. Of course, it is exactly this case in encompasses the important manifestations of market power in actual economies.

REFERENCES

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- [3] Frederic M. Scherer, Industrial Market Structure and Economic Performance, Chicago: Rand-McNally, 1970, Chapters 6 and 7.