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THE OPTIMAL RESOURCE-CAPITAL RATIO AND MARKET STRUCTURE

by

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I. Introduction

At least since Hotelling's analysis [2], a major issue regarding exhaustible resources has been the influence of market structure on its rate of extraction. Hotelling concluded that "monopolistic exploitation of an exhaustible asset is likely to be protracted immensely longer than competition would bring about." He attributed this difference to "part of the general tendency for production to be retarded under monopoly." More recently Weinstein and Zeckhauser [8], Kay and Mirrlees [4], Stiglitz [6], and Sweeney [7] have pointed out that if extraction costs are zero and the elasticity of demand for the resource is constant, then the monopolistic and competitive resource extraction rates will be identical. It has also been shown that if the elasticity of demand is rising (falling) through time, then the monopolist will be more (less) conservationist than the competitive industry. While extraction costs are seldom zero, these results approximate situations where they are negligible. Further, the modifications required in the presence of such costs are straightforward.

The source of demand for the exhaustible resource is not explicitly identified in the analyses cited. One may suppose that resource demand is largely derived from its role as a factor of production. Moreover, in Solow's discussion [5] of the constraint imposed on economic growth by the irreplenishability of an essential resource, the possibility of substitution of capital is stressed as a source of relief. These observations suggest that a deeper look at the demand for the resource as a derived demand is warranted. We especially wish to understand the role of market structure on the course of the factor ratio through time. This investigation requires a general equilibrium framework rather than the partial equilibrium methodology used in studies of resource extraction alone. We employ a simple decentralized version of an aggregative growth model (with an exhaustible resource) studied by Dasgupta and Heal [1] and others to compare the time profiles of the resource-capital ratio under competitive supply and monopolistic supply of the exhaustible resource.

Our major finding is that if production is impossible without employing the exhaustible resource, then the economy's behavior would be identical under monopolistic supply and competitive supply of the resource if and only if the production function is Cobb-Douglas. If production can be conducted without the exhaustible resource, then the economy's behavior would be independent of market structure under a slightly larger class of production functions. For all other production functions, we characterize the influence of market structure upon the rate of decline of the resource-capital ratio in terms of the elasticity of substitution between the factors of production and its dependence on the factor ratio. For instance, if the production function is CES with elasticity of substitution less (greater) than one, then the resource-capital ratio will decline less (more) rapidly under monopolistic supply of the resource than under competitive supply. These results can also be stated in terms of the temporal behavior of the elasticity of demand for the resource. Specifically we show that the elasticity of derived demand for the resource will rise or fall through time according as the elasticity of substitution is less or greater than one. The link between the elasticity of demand and the elasticity of substitution provides the basis for relating our major findings to the results in the literature cited earlier: The price of the exhaustible resource will be proportional to its marginal revenue if and only if the production function is Cobb-Douglas.

These results are of both theoretical and empirical interest. From a theoretical standpoint it is interesting, even surprising, that identical behavior can occur under monopolistic or competitive supply of the resource only if the production function is of a precise familiar form. It is of empirical interest to know if the conditions for invariance with respect to market structure are in fact met. Further, empirical estimates of the elasticity of substitution could, for example, be combined with our results to evaluate proposals for modifying the market structure of the resource industry. More broadly, our characterization of the difference in behavior of the resource-capital ratio under the polar market structures in terms of the elasticity of substitution provides a link through Solow's analysis to the question of whether economic growth can be maintained by substitution of capital for the exhaustible resource in a decentralized economy.

II. The Model

We posit an economy with a constant population of n individuals who have identical utility-of-consumption functions. The single produced good is manufactured from capital and a nonreplenishable resource. It can be consumed or used to augment the stock of productive capital. The economy is organized into three sectors: production, capital, and resource. Each is described below.

Production Sector

The single multipurpose manufactured good is produced using the services of a stock K of productive capital and a (flow of) exhaustible resource at rate R according to a linearly homogeneous, twice differentiable production function F(K,R). Both factors are essential. Hence

$$(1)\quad F(K,R)=Kf(y)$$

where

$$(2)\quad y\equiv R/K,\ f(y)\equiv F(1,y)$$

and further

$$(3)\quad F(0,R)=F(K,0)=0 \text{ or } f(0)=0, f'(0)=\infty, f'>0, f''<0, \lim_{y\to\infty} f(y)<\infty$$

We suppose the manufacturing sector acts as a price taker, with factor and product markets always in equilibrium. Taking the produced good as numeraire, the rental price r(t) of capital equals the value of its marginal product:

(4) $r(t) = F_K(K(t),R(t)) = f(y(t))-y(t)f'(y(t))$

The exhaustible resource price is likewise the value of its marginal product:

(5) $p(t)=F_R(K(t),R(t))=f'(y(t))$

In sum, the manufacturing sector behaves competitively, purchasing services of the two factors and selling its output. Although the sector experiences constant returns to scale and behaves competitively, its scale is governed by the quantities of factors made available to it. With constant returns to scale and price taking behavior, there are no profits above factor payments.

Capital Sector

Suppose that n_1 identical individuals hold equal shares of the productive capital stock earning at known rate $r(t)$ (see (4)), but no exhaustible resource. These individuals may also lend to or borrow from the resource sector at the same rate $r(t)$. Let $A(t)$ be the net asset holdings of the individuals of the capital sector. If $A(t)$ exceeds $K(t)$, then they are creditors while when A is less than K, they are debtors. The equilibrium conditions for the economy will determine $K(t)$; however, these people are concerned only with their net assets A since productive capital and bonds have identical earning capacity.

The instantaneous individual utility $u(c_1(t))$ is an increasing strictly concave, twice differentiable function of individual consumption $c_1(t)$ with

(6) $u'(0)=\infty$, $u'>0$, $u''<0$

The sector's income $r(t)A(t)$ in the form of the manufactured good is divided between consumption $n_1c_1(t)$ and investment

$A'(t)$ to maximize discounted utility

$$(7) \quad \int_0^\infty e^{-\delta t} n_1 u(c_1(t))dt$$

subject to

$$(8) \quad A'(t)=r(t)A(t)-n_1c_1(t), \; A(0)=K_o, \; \lim_{t\to\infty} A(t)\geq 0$$

Since all individuals of this sector share their initial endowment K_o of capital equally and are otherwise identical, we have aggregated to write the problem for the whole sector rather than for a single individual. Sector utility is just n_1 times individual utility, so the problems are equivalent. In (7)-(8), c_1 is a control variable, A a state variable, and r is assumed to be a known exogenous function of time.

Necessary conditions for solution can be quickly stated. The current value Hamiltonian is

$$H_1=n_1u(c_1)+\lambda_1[rA-n_1c_1]$$

where $\lambda_1(t)$ is the current value multiplier associated with (8). Then

$$(9) \quad \partial H_1/\partial c_1=n_1[u'(c_1)-\lambda_1]=0 \text{ so } u'(c_1)=\lambda_1$$

$$(10) \quad \lambda_1' = \delta\lambda_1-\partial H_1/\partial A = -\lambda_1(r-\delta)$$

Assumption (6) insures consumption will be positive; this is reflected in the equality of (9).

Resource Sector: Competitive

There are $n_2(=n-n_1)$ identical individuals equally sharing the initial endowment S_o of exhaustible resource. Under competitive supply, the resource can be sold at any rate R(t) at a known price p(t)((5) in equilibrium).

We assume resource extraction is costless and common property externalities are absent.

Resource owners may lend to or borrow from the capital sector at equilibrium interest rate $r(t)$. Let $B(t)$ be the net bond holdings of the resource sector; it may be either positive (creditor) or negative (debtor). The income of the sector from sales of resource and from interest is divided between current consumption and changed lending. Thus individuals choose the amount of resource to sell $R(t)$ and their consumption $c_2(t)$ to maximize

(11) $\int_0^{\infty} e^{-\delta t} n_2 u(c_2(t))dt$

where $c_2(t)$ is the individual consumption rate in this sector at t, subject to

(12) $S'(t)=-R(t), \quad S(0)=S_o>0, \quad S(t)\geq 0$

(13) $B'(t)=p(t)R(t)+r(t)B(t)-n_2c_2(t), \quad B(0)=0, \quad \lim_{t\to\infty} B(t)\geq 0$

where $S(t)$ is the remaining stock of exhaustible resource at t.

Associate current value multipliers $\mu(t)$ and $\lambda_2(t)$ with (12) and (13) respectively. Then the current value Hamiltonian for (11)-(13) is

$$H_2=n_2u(c_2)-\mu R+\lambda_2[pR+rB-n_2c_2].$$

In order that a positive finite amount of resource be offered for sale at each t, it is necessary that

(14) $\partial H_2/\partial R = \lambda_2 p-\mu = 0.$

Also, in view of (6), consumption will be positive so

(15) $\partial H_2/\partial c_2 = n_2[u'(c_2)-\lambda_2] = 0.$

The multipliers obey

(16) $\mu' = \delta\mu - \partial H_2/\partial S = \delta\mu \qquad$ so $\mu(t) = \mu(0)e^{\delta t}$

(17) $\lambda_2' = \delta\lambda_2 - \partial H_2/\partial B = -\lambda_2(r-\delta)$

Extension of the finite time transversality conditions suggests that either

$$\lim_{t\to\infty} S(t)=0$$

or else

$$\lim_{t\to\infty} e^{-\delta t}\mu(t)=\lim_{t\to\infty} \mu(0)=0$$

Since the latter alternative would mean that the exhaustible resource is not scarce, we assume the former holds.

Resource Sector: Monopolistic

The alternative supposition is that the resource sector is monopolistic. Then it recognizes the dependence of the unit resource price on the amount offered:

(18) $p(t)=f'(R(t)/K(t))$

where $K(t)$ is viewed as a known exogenous function. Substituting (18) into (13) gives the modified state equation

(19) $B'(t)=R(t)f'(R(t)/K(t))+r(t)B(t)-n_2c_2(t)$

The revised Hamiltonian is

$$H_2=n_2u(c_2)-\mu R+\lambda_2[Rf'(R/K)+rB-n_2c_2].$$

Instead of (14) we have

(20) $\partial H_2/\partial R = \lambda_2(f'+yf'')-\mu = 0.$

The forms of the other first order conditions for the sector's maximization problem are the same as before when the resource sector was regarded as competitive and so we do not rewrite them. The other two sectors' behavior is unaffected. The actual level of the variables will in general differ in the two cases as will the values of the multipliers.

Balance Condition

The total amount of physical productive capital in the economy is

(21) $K(t)=A(t)+B(t)$

the sum of the assets held by all the individuals. For an equilibrating solution, we require that all agents act as though they correctly forecast the actions of all other agents (and these forecasts are realized). With (21) specifying K, (2)(together with the resource sector's actions R) gives y, and then factor prices must also satisfy (4)-(5). Thus a solution involves simultaneous satisfaction of all the necessary conditions for each sector, with correct forecasts between sectors, and with the additional balancing condition specified here. Our next task will be to see what behavior of the economy emerges from simultaneous satisfaction of this myriad of conditions.

Before undertaking this next task, however, we note that the terminal conditions on A(t) and B(t) together with (21) imply

(22) $\lim_{t\to\infty} K(t)\geq 0$

as desired. To see that the capital stock will then be nonnegative for all t, we observe that since (21) holds through time,

$K'=A'+B'$

Substitute from (8) and (13)

$$K'=rA-n_1c_1+qR+rB-n_2c_2 \ .$$

Collect terms, recalling that the production function is homogeneous of degree one, so factor payments just exhaust the product:

$$(23) \quad K' = F(K,R)-n_1c_1-n_2c_2.$$

We assumed that $F(0,R)=0$, so from (23), once capital has been depleted it is impossible to increase it. Hence nonnegativity of K for all t is assured by (22). It will happen "automatically" so long as each group satisfies the nonnegativity restriction on the limiting value of its own assets. (While K must always be nonnegative, it is possible for either A or B to be negative for a finite period of time.)

III. Analysis of Solution Behavior

The course of consumption in each sector can be determined from the conditions (9) and (15) that specify equality between marginal utility of consumption and marginal utility of wealth along the optimal path. These are independent of the structure of the resource market. Differentiation of (9) and (15) totally with respect to time, substitution from (10) and (17) for λ_i', then from (9) and (15) for λ_i, and division by $u'(c_i)$, yields

$$(24) \quad -u_i''c_i'/u_i' = r(t)-\delta, \ i=1,2$$

where

$$(25) \quad u_i''\equiv u''(c_i), \ u_i'\equiv u'(c_i)$$

It follows from the assumed properties of the individual utility functions in (6) that the behavior of consumption through time is determined by the difference between the rental rate on capital and the discount rate (the Ramsey condition). Moreover, division of (9) by (15) yields

(26) $u'(c_1)/\lambda_1 = u'(c_2)/\lambda_2$ for all $t \geq 0$

Thus along an optimal path the marginal rates of substitution between consumption and wealth for the two groups are equal. Of course equality of the marginal rates of substitution does not imply equality of per capita consumption between resource owners and capital owners. However this equalization could be accomplished by redistribution of initial endowment or, equivalently, by redistribution of individuals between sectors.

We turn now to our main concern, namely the behavior of the resource-capital ratio y(t) through time under different market structures. First, suppose the resource is supplied competitively so (14) must hold. Differentiating (14) with respect to t, using (16) to eliminate μ' from the result, then substituting from (14) for μ, and finally dividing through by λ_2 p yields $\lambda_2'/\lambda_2 + p'/p = \delta$. Substituting from (17) for the first term and simplifying yields

(27) $p'(t)/p(t) = r(t)$

Recalling (4) and (5), (27) is equivalent to

(28) $y_c'(t) = f'(f - yf')/f'' < 0$

where negativity follows from (3)-(5) and the subscript c indicates that (28) obtains under competitive supply of the resource. The differential equation followed by y_c depends only on the production function. The exhaustible resource intensity of production falls through time while capital intensity rises. We also note that, in view of (4) and (28)

(29) $r'(t) = -yf''y_c' < 0.$

This combined with (24) suggests that consumption will eventually, if not immediately, decline.

Several remarks regarding the behavior of the natural resource price are in order at this point. First, according to (27) the percent change in the price of the exhaustible resource always equals the rental rate of capital along the optimal path. Second, since μ is the shadow value of the resource in the ground in terms of utility, it follows from (14) that $\lambda_2 p$ is the shadow value of the resource in the market place in terms of utility, along the optimal path. Third, it follows from (14) and (16) that the price or value of the resource in utility units $\lambda_2 p$ rises exponentially at the discount rate δ. This, of course, is the familiar result regarding the price rise of an exhaustible resource when it is supplied competitively. The conclusion that the marginal revenue of the exhaustible resource rises exponentially at the rate of discount when it is supplied monopolistically follows by the same line of argument from (20).

Next we seek an equation analogous to (28) giving the behavior of the resource-capital ratio y_m under monopolistic supply of the resource. Differentiate (20) with respect to t, use (16) and (20) to eliminate μ' and μ respectively, substitute from (17) for λ_2'/λ_2 and rearrange the result to get finally

(30) $y_m'(t) = (f-yf')(f'+yf'')/(2f''+yf''').$

To sign y_m', we note that $f'+yf''$ is just marginal revenue, the derivative of total revenue $Rf'(R/K)$ with respect to R, and so is positive. Further $(2f''+yf''')$ is $1/K$ times the derivative of marginal revenue with respect to R and so is negative. Hence, from (30) we have

$$(31) \quad y_m'<0$$

It is readily established as before that $r'(t)<0$ in this case also. Hence the qualitative characteristics of consumption and resource-capital ratio profiles are independent of market structure. Of course, the levels of these variables will differ in general.

There may, however, be circumstances under which the two models agree not only qualitatively but quantitatively as well. Our aim now is to indicate exactly when this can occur. Recalling that for a linear homogeneous production function, the elasticity of substitution between factors is

$$(32) \quad \sigma=-f'(f-yf')/yff''$$

we write

$$(33) \quad f'+yf''=f'[1-(f-yf')/\sigma f]$$

so that

$$(34) \quad d(f'+yf'')/dy=2f''+yf''' =$$

$$-\frac{f'(f-yf')}{\sigma fy}\left[1-\frac{f-yf'}{\sigma f}+\frac{yf'}{\sigma f}-y\left(\frac{\sigma'}{\sigma}+\frac{f'}{f}\right)\right]$$

where

$$(35) \quad \sigma'(y)=d\sigma/dy.$$

Then substitution of (33) and (34) into (30) gives (after simplification)

$$(36) \quad y_m' = -\sigma f y \left\{ \frac{1-(f-yf')/\sigma f}{1-(f-yf')/\sigma f - y[f\sigma' + f'(\sigma-1)]/\sigma f} \right\}$$

The corresponding expression when the resource is supplied competitively is obtained by substitution from (32) into (28)

$$(37) \quad y_c' = -\sigma y f.$$

The comparison of the competitive expression (37) with the monopoly expression (36) for given y rests on whether the curly bracketed expression in (36) is greater or less than one. This in turn depends on the sign of

$$(38) \quad f\sigma' + f'(\sigma-1)$$

If (38) is positive, then the curly bracket exceeds one and for given y, $|y_m'| > |y_c'|$. Conversely if (38) is negative, then $|y_m'| < |y_c'|$.

There are other interesting results derivable from (36)-(38). First, if substitution of capital for the resource becomes increasingly difficult as the resource-capital ratio declines, then, since $y' < 0$ under either market structure, (38) will remain negative once it becomes negative. Second, if the production function is CES, then the sign of (38) is constant, governed by whether $\sigma \gtrless 1$. Third, if the production function is CES with $\sigma < 1$ so the resource is essential, the resource-capital ratio will decline less rapidly when the resource is supplied monopolistically than when it is sold competitively. Moreover, since (38) is of constant sign in the CES case, the time profiles of the resource-capital ratio under monopolistic supply and competitive supply of the resource will cross at most once.

It is also apparent from (36)-(38) that if the production function is Cobb-Douglas so $\sigma=1$, then $y_c'\equiv y_m'$. More surprisingly, this is the only case under which $y_c'\equiv y_m'$. We highlight this result as a theorem.

<u>Theorem</u>: The rate of decline of the optimal resource-capital ratio with monopolistic supply of the resource coincides with its rate of decline under competitive supply, $y_m'\equiv y_c'$, if and only if the production function $F(K,R)$ is Cobb-Douglas.

<u>Proof</u>: The "if" part follows immediately from employing in (36) and (37) the fact $\sigma=1$ for the Cobb-Douglas function. The "only if" part involves showing that if (38) is identically zero, then $F(K,R)$ is Cobb-Douglas. Thus we set (38) identically equal to zero and substitute from (33) for σ and $\sigma'(y)$:

$$(39)\quad f\sigma(f''/f'-f'''/f''-1/y)=0$$

Since $f\sigma\neq 0$

$$(40)\quad (f'f'''-(f'')^2)/f'f''=-1/y$$

which can be rewritten after separation of variables in a form suitable for integration

$$(41)\quad d(-f''/f')/(-f''/f')=-dy/y.$$

This leads to

$$(42)\quad -f''/f'=(1-a)/y \text{ where } (1-a)>0 \text{ is the constant of integration}$$

and upon separation of variables to integration of

$$(43)\quad -df'/f'=(1-a)dy/y$$

which yields

(44) $f'=ky^{(a-1)}$, where $k>0$

A final separation of variables and integration yields

(45) $f(y)=ky^{a}/a + b$

But since by assumption (3), $f(0)=0$ we have $b=0$, so

(46) $f(y)=Ay^{a}$, where $A=k/a$

Substituting (46) into (1) gives

(47) $F(K,R)=AK^{(1-a)}R^{a}$

as claimed.

The clue to intuitive explanation for this result is provided by rewriting (42) as

(48) $af' = f'+yf''$

which, recalling (5) and (20), says that the price of the resource is proportional to its marginal revenue. This proportionality is, of course, the source of the conclusion reached in the partial equilibrium analysis referred to in the introduction; the rate of resource extraction is independent of market structure in just those cases that its elasticity of demand is constant.

If the production function is Cobb-Douglas, then not only the resource-capital ratio will be independent of the resource market structure; indeed all the variables will be invariant to that structure. To see this, recall that the sole formal difference in the sets of conditions induced by the difference in the organization of resource sale lies in (14) and (20). Suppose (48) holds and that we have the solution to the competitive case. Denote the resulting function $\mu=\mu_c$.

Then the competitive solution with μ_c replaced by $\mu_m = a\mu_c$ satisfies all the conditions for the monopoly economy. Thus the theorem may be strengthened to say that the behavior of the economy is the same under competitive or monopolistic resource supply if and only if the production function is Cobb-Douglas.

If we drop the condition that $f(0)=0$, then the class of linear homogeneous production functions that yield $y_c' \equiv y_m'$ is enlarged to those specified by (45). These functions can be rewritten as

$$(49) \quad F(K,R) = bK + AK^{(1-a)}R^a$$

If $b>0$, the elasticity of substitution between capital and the resource exceeds one and is decreasing in y. While the resource is not essential in that $f(0)\neq 0$, (45) still has the property that $f'(0)=\infty$. This latter property assures that a positive level of R will always be optimally provided.

If the production function does not satisfy (49), the difference between y_c' and y_m' for given y is characterizable in terms of changes in the elasticity of demand through time. The elasticity of derived demand for the resource is

$$(50) \quad -(p/R)/(dp/dR) = -f'/yf'' = \sigma f/(f-yf')$$

where the right side was obtained by substituting from (32). We compute

$$(51) \quad d(\sigma f/(f-yf'))/dy = [(\sigma-1)f' + f\sigma']/(f-yf').$$

Since $y'<0$, we know that the temporal movement of the elasticity of demand has the opposite sign of (51). Thus the elasticity of demand for R is rising when (38) is negative and falling when (38) is positive. These results indicate that when elasticity of demand is rising (falling) through time, $|y_m'|$ is smaller (larger) than $|y_c'|$ for given y. They are similar to those obtained by Weinstein and Zeckhauser and by Stiglitz except for referring to differences in the rate of change of the optimal resource-capital ratio rather than in the resource extraction rate alone. Thus in their analyses, a temporally rising elasticity of demand causes the monopolist to exhaust the resource less rapidly than a competitive industry, while in our analysis it leads to slower substitution of capital for the resource under monopoly than under competition. This, of course, is the expected difference between a partial equilibrium result and a general equilibrium result. The partial equilibrium result would obtain in our model if the level of capital employed were held fixed.

IV. Summary Remarks

We have shown in the context of a simple general equilibrium growth model that the behavior of the economy through time will be the same under monopolistic or competitive supply of the resource if and only if the production function is Cobb-Douglas. This result rests on the assumption that the exhaustible resource is a costlessly extracted, essential factor of production. It extends the earlier conclusion that the resource extraction rate is independent of market structure if and only if its demand function is isoelastic. We also isolated the broader class of linear homogeneous production functions that is both necessary and sufficient for the time profile of the economy to be independent of the resource industry market structure if the exhaustible resource need not be an essential productive factor.

Regardless of the production function, the qualitative temporal behavior of consumption and of the resource-capital ratio is independent of the resource market structure. However, the rate of decline in the latter ratio is generally dependent on that structure. We showed how the elasticity of factor substitution governs the comparison and related our findings to those of earlier investigators using a partial equilibrium analysis.