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I

In the stylized versions of society analyzed by Mirrlees [2] in the context of optimal utilitarian taxation and by Arrow [1] in the context of elucidating the utilitarian meaning of equality in public expenditures, individuals are assumed to have identical preferences. Yet, even at the first-best optimum, asymmetries in individual characteristics other than preferences are seen to dictate unequal welfare outcomes. In other words, individuals with equal preferences are not treated equally; horizontal equity in these models applies only to individuals who are identical in every respect (i.e. with respect to all characteristics). Whether or not this result is deemed ethically appealing depends of course upon one's values.

In this note, by formulating a simple model of society which combines the features of those of Arrow and Mirrlees, utilitarianism (as well as maximization of any other concave and symmetric individualistic social welfare function) is shown to be conducive, at the first-best optimum, to egalitarian outcomes after all. Specifically, it is shown that under a natural way of incorporating Arrow's considerations into the first-best version of Mirrlees' optimal taxation model, the optimal distribution of welfare is egalitarian across all the individuals that are required to work at the first-best optimum. The problem of the proper tax base and that of the progressivity properties of the first-best-optimal tax structure are also clarified by remarking that, across this population, the equalization of potential income is called for. Whatever inequalities remain (within the nonworking population and between it and the working one) are explicitly shown to be due to the nontransferable nature of leisure-time. Finally, some implications of the present formulation insofar as the possible conflict between fairness (non-envy) and Pareto-efficiency is concerned are also discussed.

II

Recall first that in both Arrow's and Mirrlees' models all individuals have identical preferences. In Mirrlees' model individuals differ in the productivity of the time spent by them working. In Arrow's model, individuals differ in their ability to consume and this fact is reflected in their utility function. If we take the viewpoint that what individuals differ in is the productivity of time as such (whether spent at work, consumed as leisure proper or used as an input into consumption activities) we are led in a natural way to incorporate Arrow's considerations into Mirrlees' model.

Assume then that with each individual i is associated an ability parameter $a_i (i = 1,...,I)$ which denotes the productivity of his time. Per-unit of time worked the productivity of individual i in the production of the single produced commodity in the economy is thus, as in Mirrlees, assumed to be a_i. Regarding individual preferences assume as in Arrow that these are identical over a domain that includes a_i. In its most general formulation, this leads to specifying a utility function of the form $u(a_i,c_i,\ell_i)$ where c_i is the consumption by individual i of the produced commodity and ℓ_i is the amount of leisure-time "consumed" by him. In the present context, however, the differential capacity to benefit from consumption is assumed to be due entirely to differences in the effectiveness of individuals in their use of time. Therefore, it is plausible to further specify the utility function as $u(c_i,a_i\ell_i)$ with the relevant leisure-time units $a_i \ell_i$ expressed in efficiency units. Utility is assumed to be monotone increasing, continuous and strictly concave.

Normalizing the total amount of time (in its original units) available to any individual so that it is equal to one, each individual is thus assumed to be endowed with a_i efficiency units of time. The aggregate production (consumption) constraint of the economy is then given by

$$\sum_{i=1}^{I} a_i(1 - \ell_i) - \sum_{i=1}^{I} c_i \geqq 0$$

which, on grounds of efficiency, will be satisfied with equality at the first-best optimum.

The first-best utilitarian social welfare maximization problem is then:

$$\text{Maximize} \quad \sum_{i=1}^{I} u(c_i, a_i \ell_i)$$

$$\text{Subject to (1)} \sum_{i=1}^{I} a_i - \sum_{i=1}^{I} a_i \ell_i - \sum_{i=1}^{I} c_i = 0$$

$$\text{and (2)} \quad a_i \ell_i \leqq a_i \text{ for all } i.$$

Note that constraint (2) is due to the nontransferability of time.

Were it not for condition (2) the maximization problem would be perfectly symmetric in its $(c_i, a_i \ell_i)_{i=1}^{I}$ arguments. Since the utility function u is assumed to be strictly concave perfect equality would then follow. In other words, whenever constraint (2) is not binding (is satisfied with inequality), i.e. at an interior maximum, $c_i^* = c^*$ and $(a_i \ell_i)^* = (a\ell)^*$ for all i, where the * superscript denotes the optimizing values of the corresponding variables.

Noting that the economic meaning of $(a_i \ell_i)^* < a_i$ is that individual i is required to provide a positive amount of work at the optimum, it follows that perfect equality across all those individuals that are required to work at the optimum is called for. In other words, the distribution of welfare across actual workers no longer depends on their respective abilities. The only welfare implication of ability differentials is related to the cutoff level of ability below which society will deem it optimal for the corresponding individuals not to work at all. Among the working population, the less productive an individual's time the less time he will be required to spend at work; but since such an individual is assumed here to also be less able to "consume" time, utility across the working population is nevertheless equalized (with the more able required to relinquish part of the fruits of their labor so as to equalize consumption of the produced commodity in addition to that of efficiency-units of leisure). [1]

If we think in terms of the distribution of income required in order to implement this first-best optimum in a competitive market economy, observe the following. While the "net actual-market-incomes" (= c*) of the working population are of course equalized, the first-best solution calls in addition for the equalization of potential-income (i.e. the maximum income that an individual could realize were he to spend his entire free time working). Denoting the potential income of individual i by P_i, the first-best solution implies

$$P_i^* = c_i + a_i \ell_i = P^*$$

for all those i that are required to work. Noting that the aggregate potential income in the economy is fixed and equal to $\Sigma a_i = \Sigma c_i + \Sigma a_i \ell_i$ (from the production constraint), the problem is indeed best viewed as that of determining the optimal distribution of a fixed aggregate level of potential-income among the individuals in the society. When all the individuals are required to work at the optimum, the optimum solution calls for $\bar{a} = \frac{\Sigma a_i}{I}$, i.e. to reshuffle the initial endowments of potential income (a_i) in a perfectly egalitarian manner. In the more general case, the solution calls for

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[1]If the utility function is additively separable or if consumption and leisure are complements, the welfare of the nonworking population is lower, at the optimum, than that of the workers (and is an increasing function of ability). In all other cases additional assumptions are needed in order to make these welfare comparisons. Interestingly, work effort as such is not necessarily rewarded.

equalizing potential income across the working population.

By combining the features of the Arrow and Mirrlees models and by shifting our attention to the concept of potential income, it is thus possible to advance for production economies in which individual abilities differ an analytical counterpart to the egalitarian prescription that always applies in a world of fully identical individuals. It is to be noted, of course, that the egalitarian distribution of potential income obtains in the present formulation under the maximization of any symmetric and concave individualistic social welfare function. The utilitarian formulation was emphasized here only because it is the one used by both Arrow and Mirrlees.

I would like to suggest at this point that since, unlike Mirrlees' model, the first-best implications of the present formulation are clearcut (and perfectly egalitarian insofar as individuals who are required to work at the optimum are concerned) the second-best problem of designing an optimal (actual) income tax might also prove more tractable in the present framework than they turned out to be in Mirrlees' formulation. It would be interesting to see whether such is the case. Also it might be interesting to remark that in the only first-best example considered by Mirrlees ([2], p.201), the reader can easily verify that equality of potential income across all those who are required to work turns out to be optimal even though leisure-time units there are not expressed in efficiency units (implying that welfare levels are not distributed equally even across the working population since the solution favors the less able under the utility function considered by Mirrlees).[2] But this of course is

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[2]Under the standard formulation of identical preferences over uncorrected units of time, equalization of potential income implies differential welfare levels due to the differential efficiency prices of time. Under the present formulation of preferences this is of course no longer true.

a curiosum arising from the specific function used ($u = \log c_i + \log \ell_i$) since the only implication of translating leisure into efficiency units is to add a constant term ($\log a_i$) to utility in this special case.

It might also be worth mentioning that under the present formulation of preferencesthe possibility of inconsistency between the concept of fairness and that of Pareto-efficiency is somewhat mitigated under the kind of technology considered here. For, if preferences depend on efficiency units of leisure time, the problem of finding a meaningful concept of egalitarian endowments in production economies which lies at the heart of the above consistency question is significantly reduced. For instance, competitive trading under equal endowments of potential income leads to an allocation that is both envy-free and Pareto-efficient across all those individuals that will choose to work <u>even</u> when individual preferences differ since every individual faces the same price per-efficiency-unit of leisure. These results hold under any (convex) aggregate technology which is symmetric in all its individualistic arguments (a class of which the Mirrleesian technology used here is a special case). Finally, the serious equity problems raised by the nontransferability issue are illustrated here by the fact that the/fairness results are restricted to those individuals whose preferences/and ability are such that a transfer of leisure time to them would not be needed even if it were feasible. It should be noted though that the concept of potential-income fairness discussed in recent literature can be evaluated in a new light under the present formulation of preferences.

REFERENCES

[1] Arrow, K.J.(1971) "The Utilitarian Approach to the Concept of Equality in Public Expenditure," <u>Quarterly Journal of Economics</u>, August.

[2] Mirrlees, J.A. (1971), "An Exploration in the Theory of Optimum Income Taxation," <u>Review of Economic Studies</u>, April.