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SOME LIMITATIONS OF DEMAND
REVEALING PROCESSES

by

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1. Introduction

The proposition that, due to the free-rider phenomenon, there is no decentralized mechanism yielding a Pareto-optimal allocation of resources in the presence of public goods has recently been challenged by Clarke[2], Groves and Loeb [8], Groves and Ledyard [7], and Tideman and Tullock [12] among others. It is now believed by some that, through the use of "demand revealing mechanisms", allocations can be achieved which are efficient or, at the very least, superior to those arrived at by markets or other means. While we agree that these mechanisms are potential candidates for implementation, we feel that there are a number of limitations which they suffer which must be seriously considered before one is adopted in practice.

In this paper, we present five warnings intended to dampen any premature urge to adopt a constitutional amendment to institute one of these demand revealing mechanisms. To coherently discuss the various limitations, we briefly survey (in Sections 3 - 7) the state of knowledge about these mechanisms, in the context of a simple model. We then present five specific properties of these mechanisms (Sections 8 - 11) which could hinder success-ful implementation. These potential problems are summarized in the next section. We conclude this paper with a brief discussion (Section 12) of some progress made in overcoming a few of these difficulties.

2. Summary of Limitations

We have been able to identify five main reasons why a "demand revealing mechanism" may not entirely solve the problem of efficiently allocating

resources in the presence of public goods. There are undoubtedly other reasons; however, these five are sufficient to raise doubts about the efficacy of these procedures.

The first, and possibly most damaging limitation, is that although these mechanisms lead individual consumers to correctly reveal their demands for public goods (in the absence of income effects) and therefore lead to the "correct" level of public good production, they do not lead to Pareto-optimal allocations. In fact, they typically waste resources through the necessity for the collection of a budgetary surplus. To emphasize this point, we show (in our simple model) that an alternative procedure based on majority rule voting may lead to an allocation of resources which is Pareto-superior (preferred by everyone) to that produced by a demand revealing mechanism.

Second, in order to create demand-revealing behavior these mechanisms rely on taxing rules which are potentially confiscatory in nature. That is, any consumer's ownership rights to his endowments are limited. Thus, since there are minimum levels of private goods consumption below which the consumer cannot survive (an economic fact of life), it is possible for the demand revealing mechanism to drive some consumers into bankruptcy. This would prevent the attainment of equilibrium unless some method of compensating those who would be bankrupt were devised. However, once such potential compensation is introduced demand revelation will be lost if any compensation actually occurs.

Third, if income effects are present in the demand for public goods,

i.e. if the income elasticity of demand for public goods is not <u>identically</u> equal to zero, <u>then the demand revealing mechanisms may be dynamically unstable</u> and if implemented might never settle down to an equilibrium but cycle explosively. Obviously, efficient equilibrium allocations are of use only if some equilibrium can be found.

Fourth, in the presence of income effects, <u>a consumer will usually be able to strategically manipulate a demand revealing mechanism</u> in a way such that he is better off than he would be if he behaved competitively. Such manipulation would be similar to that which a Stackleberg quantity-setter engages in when the rest of the market behaves like a Cournot oligopolist. If such strategic manipulation arises, the resulting allocation produced by a demand revealing mechanism may very likely be inefficient, thus defeating the original justification for the use of the mechanism.

Fifth, even if income elasticities of demand for the public good are identically zero, <u>coalitions of consumers will be able to strategically manipulate a demand revealing mechanism</u> not because they may be able to exploit their market power but rather because the mechanism provides incentives to some coalitions to misrepresent their collective demand. Clearly, as in the case of one consumer, if such manipulation occurs then the resulting allocation will generally be inefficient and the hopes for the mechanism defeated.

3. A Simple Illustrative Model

Throughout this paper, we employ a simple model with only one private
and one public good. [1] There are I consumers, indexed i = 1,...,I,
with concave utility functions $u_i(x_i,y)$ where x_i denotes i's consump-
tion of the private good, y is the total amount of the public good
provided collectively to all consumers, and $u_i : \mathbb{R}^2 \to \mathbb{R}^1$ is assumed to have
continuous second derivatives. We assume the marginal utility of the private
good is strictly positive at all non-negative bundles (x_i,y) . Each con-
sumer is initially endowed with ω_i units of the private good, $\omega_i \in \mathbb{R}^1$.

Production possibilities are defined by assuming that private goods
may be transformed into public goods at a constant fixed rate of transfor-
mation. Thus, measuring public good units in units of the private good
needed to produce it, society's marginal rate of transformation is normalized
to unity. Furthermore, any allocation consisting of strictly positive net
outputs of both goods will be consistent with competitive profit maximizing
only if relative prices are also equal to unity.

For this simple economy, efficient resource allocations are easily
characterized: [2]

Theorem 1: An allocation $(x_1^*,...,x_I^*;y^*)$ is Pareto-optimal _only if_ it
satisfies:

(3.1)   Lindahl-Samuelson Condition:

$$\Sigma_{i=1}^{I} \left. \frac{\partial u_i/\partial y}{\partial u_i/\partial x_i} \right|_{(x_i^*,y^*)} = 1,$$

and

(3.2) Non-wastefulness Condition:

$$\Sigma_{i=1}^{I} x_i^* + y^* = \Sigma_{i=1}^{I} \omega_i^* \ .$$

4. Decision Mechanisms and Evaluative Criteria

In our simple economy, public goods decisions are made by a "special agent -- a computer or, as we have called it elsewhere, a government. The computer receives messages from each consumer; it uses these messages to calculate according to fixed rules (a) the quantity of public good to provide and (b) each consumer's tax. [3/] Thus, we define a mechanism by:

Mechanism : A mechanism m consists of

(i) a language M, a set of messages m_i that any consumer may choose;

(4.1) (ii) an allocation rule, $y(\cdot)$; a function of message I-tuples
$m = (m_1, \ldots, m_I)$, $y : M^I \to \mathbb{R}^1$

(iii) I tax rules, $\{T_i(\cdot)\}_{i=1}^{I}$; each a function of m, $T_i : M^I \to \mathbb{R}^1$

All consumers are assumed to know the mechanism used.

Although many different mechanisms exist, we are interested in finding ones that lead to efficient resource allocations. To specify how allocations are determined by a mechanism, consider a consumer's decision problem given the mechanism m in use:

Consumer's Decisions Problem. [4/]   Given $\mathcal{m}$, choose a message m_i^* in M to maximize utility:

(4.2) $v_i(m'/m_i;\mathcal{m}) \equiv u_i[\omega_i - T_i(m'/m_i), y(m'/m_i)]$

where $(m'/m_i) \equiv (m_1', \ldots, m_{i-1}', m_i, m_{i+1}', \ldots, m_I')$.

Now, for a given mechanism $\mathcal{m}$, the consumer's problem may not have an unambiguous solution. But, if every consumer's problem does, then we say the mechanism satisfies the:

(4.3)   (Strong) Dominant Equilibrium Condition: A mechanism $\mathcal{m}$

satisfies this condition if, for every consumer i, there

exists a message m_i^* in M that maximizes the (indirect)

utility $v_i(m'/m_i;\mathcal{m})$ regardless of the messages m_j' sent

by all the other consumers $j \neq i$.

This equilibrium condition requires that a best message for i be independent of the others' messages. In the language of n-person (non-cooperative) game theory, the joint message $m^* = (m_1^*, \ldots, m_I^*)$ is a dominant strategy equilibrium since m_i^* is best against any $m_j, j \neq i$, and hence dominates (weakly) any other possible strategy m_i.

If a mechanism $\mathcal{m}$ satisfies this condition, then if used it would seem reasonable to expect each consumer to play his dominant strategy and thus that the allocation $[\{x_i^* = \omega_i - T_i(m^*)\}_i, y(m^*)]$ would result. Unfortunately, for many economies and mechanisms dominant strategy

equilibria do not exist. In general situations, since consumer i's
tax and the allocation of public goods will depend on the other consumers'
messages, his best message will vary with the others' messages and hence
not be independent of them. We return to this point below in Section 10.

Now, since we desire efficient resource allocations, we seek mechanisms
such that when consumers are in equilibrium at, say, $m^* = (m_1^*, \ldots, m_I^*)$,
the resulting allocation $[\{x_i^*(m^*)\}_i, y(m^*)]$ where $x_i^*(m^*) \equiv \omega_i - T_i(m^*)$
is Pareto-optimal. From Theorem 1, such a mechanism must satisfy the two
conditions:

Lindahl-Samuelson Condition: A mechanism $\mathcal{m}$ satisfies this condition if,
at any joint message equilibrium m^*,

$$(4.4) \qquad \Sigma_i \left. \frac{\partial u_i / \partial y}{\partial u_i / \partial x_i} \right|_{(x_i^*(m^*), y^*(m^*))} = 1 ,$$

Budget Balance Condition: A mechanism $\mathcal{m}$ satisfies this condition if,
at any joint message equilibrium m^*,

$$(4.5) \qquad \Sigma_i T_i(m^*) = y(m^*).$$

5. An Impossibility Theorem

Since a consumer's message is independent of others' messages when a dominant equilibrium exists, a strong result would be to find some mechanisms that, for a reasonably wide class of economic environments (here, utility functions $u_i(x_i,y)$), satisfies the Dominant Equilibrium, Lindahl-Samuelson, and Budget Balance Conditions. However, as has been shown by Hurwicz [9], and Green and Laffont [3].

Theorem 2: There exists no mechanism m that satisfies the Dominant Equilibrium, Lindahl-Samuelson, and Budget Balance Conditions simultaneously for a sufficiently wide class of possible utility functions.

In particular, the impossibility exists if the class of utility functions considered includes only those of the form:

$$(5.1) \qquad u_i(x_i,y) = x_i + \psi_i(y)$$

where $\psi_i(\cdot)$ is any increasing function of the public good with continuous third derivatives.

6. The Demand Revealing Mechanisms

In view of Theorem 2, if one seeks mechanisms leading to Pareto-optimal resource allocations, the Dominant Equilibrium Condition must be weakened since the other two are required for Pareto-optimality. This was our approach (Groves and Ledyard [7]) which is discussed below in Section 12. However, historically a class of mechanisms we may call [5] Demand Revealing Mechanisms were first discovered [6] in the context of partial

equilibrium models with utility functions of the special form (5.1) and were shown to satisfy the Dominant Equilibriums and the Lindahl-Samuelson Conditions. (Of course, they do not satisfy, in general, the Budget Balance Condition.)

Now, utility functions of the form (5.1) have the special property that the marginal rate of substitution between the two goods is independent of the quantity of private good consumed,

$$(6.1) \qquad MRS^i(x_i,y) = (\partial u_i/\partial y)/(\partial u_i/\partial x_i) = d\psi_i(y)/dy \equiv \psi'(y).$$

Thus, the consumer's "demand" or marginal "willingness-to-pay" function for the public good is indepdent of his income; i.e. there are no "income effects" on the demand for public goods. In the language of game theory, utility is freely transferrable among the players (through the medium of income or purchasing power).

In such transferrable utility environments, the Lindahl-Samuelson equation (4.4) is independent of the distribution of the private good and hence is an implicit function in the public good only. Thus, if consumers could be induced to report honestly their true marginal rate of substitution schedule it would be an easy matter to solve the Lindahl-Samuelson equation for the unique [7] efficient quantity of the public good. Furthermore, since in this special case the true MRS schedule is independent of the distribution of private goods, if the consumer is induced to report his MRS schedule honestly then it will be a dominant strategy for him to do so.

It was for such environments that mechanisms were discovered to induce honest demand revelation and to select the efficient quantity of

the public good. The following theorem of Groves [4,5] defines a class
of such mechanisms:

Theorem 3: Let all utility functions be of the transferrable utility

form, $u_i(x_i,y) = x_i + \Psi_i(y)$. Consider the class $\mathcal{B}$ of mechanisms
$\hat{\mathcal{M}} = [\hat{M}, \hat{y}(\cdot), \{\hat{T}_i(\cdot)\}_i]$ defined by:

(6.2a) $\hat{M} = \{$all functions $\Phi_i: \mathbb{R}_+ \to \mathbb{R} \}$;

(6.2b) $\hat{y}(\Phi) = \hat{y}$ maximizes $\Sigma_{i=1}^{I} \Phi_i(y) - y$ subject to $y \geq 0$; and

(6.2c) $\hat{T}_i(\Phi) = \hat{y}(\Phi) - \Sigma_{j \neq i} \Phi_j[\hat{y}(\Phi)] + R_i(\Phi)_{i}()$

where R_i is any arbitrary function of $\Phi)_{i}(\equiv (\Phi_1, \ldots, \Phi_{i-1}, \Phi_{i+1}, \ldots, \Phi_I)$.

Every mechanism of $\hat{\mathcal{M}}$ in the class $\mathcal{B}$ satisfies the Dominant

Equilibrium and Lindahl-Samuelson Condition. Furthermore, $\Phi^* = (\Psi_1, \ldots, \Psi_I)$

is a dominant joint strategy; i.e. "telling the truth" -- sending Ψ_i is

a dominant strategy for every consumer. 8/

Since reporting Ψ_i is equivalent to reporting the consumer's true

demand for the public good, 9/ the class $\mathcal{B}$ is called the class of

Demand Revealing Mechanisms.

An important theorem of Green and Laffont [3] establishes that any

mechanism satisfying the two conditions of Theorem 3 is equivalent to a

mechanism $\hat{\mathcal{M}}$ in $\mathcal{B}$:

Theorem 4: Let all utility functions be of the transferrable utility form, $u_i(x_i,y) = x_i + \psi_i(y)$. If $\overset{o}{m} = [\overset{o}{M}, \overset{o}{y}(\cdot), \{\overset{o}{T_i}(\cdot)\}_i]$ is any mechanism satisfying the Dominant Equilibrium and Lindahl-Samuelson Conditions, then there is a demand revealing mechanism $\hat{m}$ in $\mathcal{B}$ [i.e. defined by (6.2a-c)] that is equivalent to $\overset{o}{m}$ in the sense that they both lead to the same essential allocation. $\underline{10/}$

7. The Surplus Revelation Mechanism

   Since no demand revealing mechanism can guarantee that the Budget Balance Condition is satisfied, a serious question is whether or not feasibility can be guaranteed, that is, does any $\hat{m}$ in $\mathcal{B}$ satisfy:

(Weak) Feasibility Condition: A mechanism $\hat{m}$ satisfies this condition if, at any equilibrium joint-message m^*,

(7.1) $\quad \Sigma_i x_i^*(m^*) + y(m^*) \leq \Sigma_i \omega_i$

where $x_i^*(m^*) = \omega_i - T_i(m^*)$, or, equivalently,

(7.2) $\quad y(m^*) \leq \Sigma_i T_i(m^*)$.

Thus, feasibility is assured if the mechanism can guarantee a non-negative budgetary surplus.

   It is actually easy to exhibit a demand revealing mechanism that guarantees a budgetary surplus. This specific mechanism was first given by Clarke [2] and later by Groves and Loeb [8].

<u>Surplus Revelation (SR) Mechanism:</u> Let α_i, $i = 1,\ldots,I$, be any positive real numbers summing to unity. Let $\hat{m}$ be the demand revealing mechanism defined by (6.2a-c) with:

$$(7.3) \qquad \hat{R}_i(\Phi)_i() \equiv \underset{y \geq 0}{\text{Max}} \ \Sigma_{j \neq i}[\Phi_j(y) - \alpha_j \cdot y] \quad \text{for every } i.$$

It is straight forward to verify that this mechanism generates a non-negative budget surplus and hence satisfies the (Weak) Feasibility Condition.

8.   The Non-optimality Problem:

Although the Surplus Revelation mechanism satisfies the three conditions -- Feasibility, Dominant Equilibrium, and Lindahl-Samuelson (in transferrable utility environments), it does not guarantee Pareto-optimality. With transferrable utility, the optimal quantity of the public good will be provided, but because a budget surplus is generated, an optimal distribution of the private good will not obtain -- some will be wasted. It is important to note that <u>any</u> deterministic mechanism to redistribute the surplus to consumers will, in general, destroy the Lindahl-Samuelson condition -- i.e. it will not be best for consumers to communicate their true demand. Thus, our first caveat:

W A R N I N G   N O. 1

Since the Surplus Revelation mechanism generally wastes resources (i.e. is not efficient), it <u>may</u> lead to worse (i.e. Pareto-inferior) resource allocations than yielded by some other mechanisms.

To show that this warning is not empty, we present an alternative
mechanism based on majority voting and show for a class of economies
that the resulting allocations are Pareto-superior to those generated by
the SR mechanism.

Consider the special transferrable utility function of the form:

(8.1) $\quad u_i(x_i,y) = x_i + \theta_i \ln y, \; \theta_i > 0, \; i = 1,\ldots,I.$

The alternative mechanism's tax rules are defined by:

(8.2a) $\quad T_i^+(y) = t_i \cdot y \qquad$ where $\; t_i > 0, \; \Sigma_i t_i = 1$

and y is the quantity of the public good to be determined by an allocation
rule. The parameter t_i is consumer i's share of total taxes and might
be set equal to $1/I$ if equal taxation is desired or equal to $\omega_i / \Sigma_j \omega$
if taxation proportional to wealth is desired. Now, with these tax rules,
the consumer's utility as a function of y is:

(8.3) $\quad v_i(y) = \omega_i - t_i \cdot y + \theta_i \ln y.$

Since all consumer's utility functions v_i are concave, their preferences
over alternative quantitites of the public good are single-peaked. Thus,
if the allocation is chosen by majority voting, then the only quantity
preferred by some majority over every other alternative is the median
voter's most preferred quantity. Hence, we may specify the alternative

allocation rule as follows: Every consumer communicates a proposed quantity of the public good, denoted $y_i (= m_i)$. The allocation rule selects the median value of the consumers' proposals:

(8.2b) $\quad y^+(m) = \text{median } \{y_1, \ldots, y_I\} \quad$ where $\quad m = (y_1, \ldots, y_I)$.

It is easy to verify the following properties for the Majority Voting (MV) mechanism $\mathcal{m}^+ = [\, \mathbb{R}_+, y^+(\cdot), \{T_i^+(y^+(\cdot))\}_i\,]$:

Property 1: Under utility functions of form (8.1), the MV mechanism $\mathcal{m}^+$ satisfies the Dominant Strategy Equilibrium and the Budget Balance Condition. Furthermore, each consumer has as his dominant strategy his true most desired allocation $m_i^+ = y_i^+ = \theta i/t_i$.

Property 2: Under the above conditions, the MV mechanism produces the allocation

(8.4a) $\quad y^+ = y^+(m^+) = \text{median } \{\theta 1/t_1, \ldots, \theta I/t_I\}$

(8.4b) $\quad x_i^+ = w_i - t_i \cdot y^+$.

This allocation is generally not Pareto-optimal.

It is also easy to show:

Property 3: Under the above conditions, the SR mechanism produces the allocation.

(8.5a) $y^* = y^*(m^*) = \Sigma_{i=1}^{I} \theta_i$

(8.5b) $x_i^* = \omega_i - y^* - \nu_i \ln(\nu_i/(1-\alpha_i)y^*) + \nu_i$

where $\nu_i = \Sigma_{j \neq i} \theta_j$,

and the budgetary surplus:

(8.6) $\text{Surplus} = \Sigma_i T_i^*(m^*) - y^* = \Sigma_i \nu_i \ln(\nu_i/(1-\alpha_i)y^*) \geq 0$

We now show Warning 1 is not empty:

<u>Proposition 1</u>: Let $u_i(x_i,y) = x_i + \theta_i \ln y, \theta_i > 0$, all i. Let
$\alpha = (\alpha_1,\ldots\alpha_I) = (t_1,\ldots,t_I) = t$ be the parameters of the SR and MV
tax rules $T_i^*(\cdot)$ and $T_i^+(\cdot)$.

The allocation $(\langle x_i^+ \rangle_i, y^+)$ produced by the MV mechanism <u>is preferred
by a majority of the consumers</u> to the allocation $(\langle x_i^* \rangle_i, y^*)$ produced by
the SR mechanism.

Furthermore, if the median value of $\left\{ \theta_1/\alpha_1, \ldots, \theta_I/\alpha_I \right\}$ equals
$\Sigma_{i=1}^{I} \theta_i$, then $(\langle x_i^+ \rangle_i, y^+)$ is Pareto-optimal and also Pareto-superior
to $(\langle x_i^* \rangle_i, y^*)$.

<u>Proof</u>: See appendix.

<u>Corollary</u>: Under the assumptions of Proposition 1, if $t_i = \alpha_i = \theta_i / \Sigma_j \theta_j$ for all $i = 1, \ldots, I$, then $(\langle x_i^* \rangle_i, y^*) = (\langle x_i^+ \rangle_i, y^+)$ and is Pareto-optimal. That is, the surplus under the SR mechanism will be zero since these values of the parameters α_i are equal to the Lindahl prices.

<u>Proof:</u> See appendix.

The result of the corollary is more general; for any transferrable utility functions $u_i(x_i, y) = x_i + \psi_i(y)$, if $\alpha_i = t_i$ equals the Lindahl price $[\psi_i'(y^*) / \Sigma_j \psi_j'(y^*)]$ where y^* solves $\Sigma_j \psi_j'(y) = 1$ (i.e. is the optimal quantity of the public good), then the conclusion holds. But, of course, if these α_i's were known, there would be no need to communicate with the consumers since the optimal quantity would also be known.

Now, although Proposition 1 <u>may</u> be interpreted as showing that the MV mechanism is "better" generally than the SR mechanism, it would be inappropriate in our opinion to hold this view. Proposition 1 was established for a very special class of utility functions in a model with only one public and one private good. The extent to which it generalizes is an open question. Our intent in presenting the MV mechanism was to show (i) that the demand revealing mechanisms are not the only ones for which it is a dominant strategy for a consumer to reveal his true demand, and (ii) that since the demand revealing mechanisms do not yield Pareto-optimal allocations, satisfying the Lindahl-Samuelson Condition may not be as important as satisfying the Balanced-Budget Condition.

9. The Bankruptcy Problem

An implicit assumption of the analysis to this point has been that every consumer's utility function is well-defined over all possible quantities, <u>negative</u> as well as positive, of the private good. In other words, we have been assuming that no consumer could be bankrupted, no matter how large his taxes might be. If there are, indeed, limits on how large a tax burden a consumer is able to sustain, then we would want a mechanism to satisfy:

<u>(Strong) Feasibility Condition</u>: A mechanism $\mathcal{m}$ satisfies this condition if, at any equilibrium joint-message m^*

$$(9.1a) \qquad \Sigma_i x_i^*(m^*) + y(m^*) \leq \Sigma_i \omega_i \qquad \text{(weak aggregate feasibility)}$$

and

(9.1b)   for every i, $(x_i^*(m^*), y(m^*)) \in X_i$ = consumption set of consumer i.

Now, if the consumer's consumption set is bounded from below in the private good, then a demand revealing mechanism may not satisfy the (Strong) Feasibility Condition; i.e. may bankrupt some consumer:

W A R N I N G   N O. 2:

If every consumer's consumption of the private good is bounded below and the demand revealing mechanism satisfies the Weak Feasibility Condition, then it __may__ not satisfy the Strong Feasibility Condition since the taxes levied may bankrupt some consumer, i.e. force him below his minimal consumption level.

To demonstrate this possibility, let $I = 3$, $u_i(x_i, y) = x_i + \theta_i y - \frac{1}{6} y^2$ for all non-negative (x_i, y) and $\omega_i = 1$ for all i. Since weak feasibility is guaranteed only by ensuring a budgetary surplus, we will consider the Surplus Revelation mechanism with $\alpha_i = \frac{1}{3}$ for all i, since initial endowments are identical.

For this example, each consumer's best message is of the form $m_i^*(y) = \theta_i y - \frac{1}{6} y^2 + C_i$ (where C_i is any constant) as long as this message is individually feasible given the other two consumers' messages. Now if $\theta_1 = \frac{1}{2}$ and $\theta_2 = \theta_3 = 3/2$, then $m_2^*(\cdot)$ is feasible and thus dominant for consumer 2 if 1 and 3 are sending $m_1^*(\cdot)$ and $m_3^*(\cdot)$. Similarly $m_3^*(\cdot)$ is feasible and dominant for 3 given $m_1^*(\cdot)$ and $m_2^*(\cdot)$. However, if 2 and 3 send $m_3^*(\cdot)$ then consumer 1's taxes will be:

$$(9.2) \qquad T_1(m^*/m_1) = \frac{1}{3} y(m^*/m_1) + \underset{y}{\text{Max}} \ [\frac{7}{3} y - \frac{1}{3} y^2]$$

$$- [\frac{7}{3} y (m^*/m_1) - \frac{1}{3} y(m^*/m_1)^2]$$

and consumer 1's minimum tax will be:

$$(9.3) \qquad T_1(m^*/\hat{m}_1) = \underset{m_1}{\text{Min}} \ T_1(m^*/m_1) = \frac{13}{12} > 1 \ .$$

Thus consumer 1 is bankrupt when 2 and 3 send their best messages $m_2^*(\cdot)$ and $m_3^*(\cdot)$. For this example, then, a dominant strategy equilibirium does not exist.

Now it might be noted for this example that if consumer 2 sends $m_2^*(\cdot)$ he is, in effect, professing positive marginal valuations at levels of y that he cannot possibly afford under the Surplus Revelation mechanism. Since the mechanism charges him at least $\alpha_2 = \frac{1}{3}$ per unit (his proportion of total wealth), he could never afford any level of public good y greater than 3. Thus, it might seem that if no consumer is allowed to profess positive marginal valuations at levels of y greater than 3, then the bankruptcy problem may be avoided. [11]/

Unfortunately this modification of the Surplus Revelation mechanism does not solve the problem satisfactorily. With this modification, each consumer's best message is of the form:

$$(9.4) \qquad m_i^o(y) = \begin{cases} \theta_i y - \frac{1}{6} y^2 + C_i \ , & y \le 3 \\ 3\theta_i - \frac{3}{2} + C_i & y > 3 \end{cases}$$

(where C_i is any constant) as long as this message is individually feasible. But here too, given $m_2^o(\cdot)$ and $m_3^o(\cdot)$, consumer 1 cannot afford to send $m_1^o(\cdot)$. In this case though he can avoid bankruptcy by over-reporting his valuations. For the particular numerical example here, if consumer 1 also

sends $\hat{m}_1(\cdot) = m_2^o(\cdot)$, then no consumer is bankrupt and the allocation provided is $x_1^o = x_2^o = x_3^o = 0$, $y^o = 3$; i.e., all resources are devoted to the public good. Interestingly enough this allocation is Pareto-optimal since $x_1^o = 0$ and the message triple $(\hat{m}_1(\cdot), m_2^o(\cdot), m_3^o(\cdot))$ is a Nash equilibrium (although not a dominant equilibrium). But, the optimality of this allocation is a fluke. By modifying consumer 1's lower bound on consumption of the public good x_1 to any arbitrarily small positive number (instead of zero), consumer 1 will again be unable to avoid bankruptcy when 2 and 3 send $m_2^o(\cdot)$ and $m_3^o(\cdot)$.

The problem of bankruptcy is not merely a technical difficulty, but is related to the confiscatory nature of the demand revealing mechanism. Regardless of a consumer's initial endowment, under a demand revealing mechanism he is not guaranteed to be as well-off as he could be if he lived on his initial endowment alone. In the language of game theory, the outcomes generated by the demand revealing mechanism are not necessarily individually rational. This implies that under these mechanisms a consumer's ownership rights over his initial endowment of private goods are limited and may, in fact, be essentially non existent. For this reason alone, one might expect a certain amount of reluctance on the part of consumers to accept such a mechanism as a method of allocating public goods.

10. The No Income Effects Assumption

Theorem 3 of Section 6 establishes that any demand revealing mechanism satisfies the Dominant Equilibriums and the Lindahl-Samuelson Conditions when the utility functions are of the transferrable utility form, i.e. when consumers income elasticity of demand for public goods is identically zero. Since such an assumption is mildly preposterous and barely respectable in modern economic theory, it is of interest to investigate the properties of the demand revealing mechanism in environments not characterized by freely transferrable utility. In such environments, under a demand revealing mechanism, no dominant strategy equilibrium will exist. Since any consumer's valuations of public goods depends on their income, their true valuation function will have to depend on the messages of the other consumers.

Now although a Dominant Strategy Equilibrium will not exist in these cases, a Nash or non-cooperative equilibrium may. That is, the Demand Revealing mechanism may satisfy:

(10.1)   (Weak) Nash Equilibrium Condition: A mechanism $\mathcal{m}$ satisfies this condition if there exists a joint message $m^* = (m_1^*, \ldots, m_I^*)$ in M^I such that m_i^* maximizes the (indirect) utility $v_i(m^*/m_i, \mathcal{m})$ for each i.

This equilibrium condition requires that the strategy m_i^* be best only against m_j^*, $j \neq i$; not against every m_j, $j \neq i$. In the language of game theory, such an m^* is a Nash Equilibrium.

We have previously provided the following theorem (Groves and Ledyard [7]):

<u>Theorem 5:</u> Let $u_i(x_i,y)$ be arbitrary concave continuously differentiable utility functions such that $\partial u_i/\partial x_i > 0$. Then any Demand Revealing Mechanism $\hat{m}$ defined by (6.2a-c) satisfies the (Weak)Nash Equilibrium and the Lindahl-Samuelson conditions, but not, generally, the (Strong) Dominant Equilibrium Condition. Furthermore, $\Phi^* = (\Phi^*_1,\ldots,\Phi^*_I)$ where

$$(10.2) \qquad \frac{\partial \Phi^*_i}{\partial y} \equiv \left. \frac{\partial u^i/\partial y}{\partial u^i/\partial x_i} \right|_{x^*_i(\Phi^*)} \qquad \text{for each } y$$

is a Nash Equilibrium. That is, correctly revealing the true marginal rate of substitution at every y is a best strategy for a consumer, given the other consumers' messages.

However, in general, the mechanism $\hat{m}$ will not satisfy the Budget Balance condition, although the SR mechanism of Section 7 will satisfy the (Weak)Feasibility Condition.

Now, although a demand revealing mechanism will satisfy the (Weak) Nash Equilibrium Conditions when utility functions are not of the transferrable utility form, the question of how such an equilibrium is to be arrived at becomes important. When a dominant strategy equilibrium exists, each consumer's best message is independent of the others'; but, in the general case, the best message of a consumer will depend on the others' messages. Thus, in principle at least, every consumer must know the messages of all the other consumers in order to be able to compute his best message. Thus, it seems that some type of adjustment process would be required in order to arrive at the Nash Equilibrium. But given any type of adjustment process, two difficulties appear. First of all, for a wide class of environments. the adjustment process may not converge. Secondly, it follows from Hurwicz's

Impossibility Theorem [9] that given any adjustment process,

a sophisticated consumer could strategically manipulate the outcome

from the Nash Equilibrium of Theorem 5 to a more advantageous outcome

for him.

We state these two difficulties as caveats 3 and 4:

W A R N I N G   N O. 3

If consumers' preferences for public goods are not independent of

their income, then the Nash Equilibrium joint-message associated with

a Demand Revealing Mechanism may be unstable and hence an adjustment

process to find the equilibrium may not converge.

W A R N I N G   N O. 4

If consumers' preferences for public goods are not independent of

their income, then the Nash Equilibrium joint-message associated with a

Demand Revealing Mechanism is subject to strategic manipulation. That is,

given any adjustment process, a sophisticated consumer could play in such

a way that, if it converged at all, the process would converge to an outcome

not satisfying the Lindahl-Samuelson equations, but an outcome prefereable

for the consumer.

While we believe these two difficulties are not exclusively of theore-

tical interest, how important they would be in practice remains to be

examined further. Vernon Smith has reported [11] on some experiments

based on Demand Revealing Mechanisms and his results show rapid convergence to the Nash Equilibrium.

11. The Coalition Problem

Having suggested that the Nash Equilibrium may not be a stable solution, we might reconsider the stability of the Dominant Strategy Equilibrium. Specifically we can ask if the Dominant Strategy Equilibrium for any Demand Revealing Mechanism is immune from group or coalitional manipulation. An answer is suggested by the following theorem of Bennett and Conn [1]:

Theorem 6: Let all utility functions be of the transferrable utility form, $u_i(x_i,y) = x_i + \Psi_i(y)$. Given any Demand Revealing mechanism $\hat{m}$, for any coalition C of two or more consumers (C is a non-empty subset of I) the vector of dominant strategies for the members of the coalition $\Phi_c^* = \{\Psi_i, i \in C\}$ is not a dominant strategy for the coalition.

This theorem suggests that, contrary to the case for individuals, it is not optimal for coalitions to correctly reveal their true joint valuations for the public good and thus, if such coalitions form, the outcome will not generally satisfy the Lindahl-Samuelson Condition, i.e. the resulting public goods allocation $y(m^*)$ will not be optimal. Thus, caveat five

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W A R N I N G   N O. 5

The Demand Revealing Mechanisms are not immune from group m If coalitions form, then under these mechanisms optimal alloca goods would not be expected to result.

12. Epilogue: An Alternative Mechanism

As we have seen in Section 8 the failure of Demand Revealing Mechanisms to produce Pareto-optimal allocations (ignoring for now all other potential problems) is a serious flaw of such mechanisms and should give pause to any sudden inclination to institute such mechanisms on a grand scale. Furthermore by the Impossibility Theorem (Theorem 2, Section 5) there is no mechanism which simultaneously satisfies the Dominant Equilibrium Condition and yields Pareto-optimal allocations. Thus, we are inescapably confronted with a second-best choice as to which property (if either) is preferable.

As we have also seen in Section 10, if consumers' preferences are such that all individual income elasticities of demand for public goods are not identically zero, then the Demand Revealing Mechanisms fail to satisfy both the Dominant Equilibrium condition and Pareto-optimality (by violating the Balanced Budget condition). However, they do satisfy the (weak) Nash Equilibrium and the Lindahl-Samuelson conditions. Whether income effects are or are not important is strictly an empirical question(to date unanswered). However, if there is a mechanism for allocating resources in the presence of public goods which satisfies (in environments with income effects or without) the (weak) Nash Equilibrium, the Lindahl-Samuelson, and the Budget Balance Conditions, then such a mechanism may be preferable to Demand Revealing Mechanisms in all environments and certainly preferable in environments where income effects are important.

That such a mechanism exists was recently demonstrated by us (Groves and Ledyard [7]). The mechanism M^*, which we called an Optimal Mechanism,

is defined by:

(12.1a) $M^* = \mathbb{R}$

(12.1b) $y^*(m) = \Sigma_i m_i$

(12.1c) $T_i^*(m) = \alpha_i y^*(m) + \frac{\gamma}{2}[\frac{I-1}{I}(m_i - \mu(m)_{i()})^2 - \sigma(m)_{i()}^2]$

where

(12.1d) $\gamma > 0, \ \Sigma_i \alpha_i = 1, \ \mu(m)_{i()} = \frac{1}{I-1}\Sigma_{j \neq i} m_j,$ and

$\sigma(m)_{i()}^2 = \frac{1}{I-2}\Sigma_{j \neq i}(m_j - \mu(m)_{i()})^2$

We showed:

Theorem 7: Let $u_i(x_i,y)$ be arbitrary concave utility functions such that $\partial u_i/\partial x_i > 0$. Then the Optimal Mechanism M^* defined by (12.1 a-d) satisfies the (Weak) Nash Equilibrium, Lindahl-Samuelson, and Balanced Budget Conditions.

The reason that the M^* mechanism is able to satisfy all three desirable conditions is that it is not, strictly speaking, a Demand Revealing Mechanism. However, it is equivalent to one for which the language space $\hat{M}$ [see (6.1a)] is restricted to the set:

(12.2) $\hat{M} = \{\phi_i : \mathbb{R}_+ \to \mathbb{R} \ | \ \phi_i(y) \equiv \beta_i y - \frac{\gamma}{2I} y^2, \ \beta_i \in \mathbb{R} \}.$

This restricts consumers to send only linear "demand" functions with a preassigned slope rather than any arbitrary function. Besides simplifying the communication process (a message is now just a number instead of a function), this restriction allows Budget Balance to be achieved without

loss of the Lindahl-Samuelson condition. Thus, this mechanism is not subject to the first limitation of Demand Revealing Mechanisms discussed in Section 8.

On the other hand, the Optimal Mechanism, M^*, may well be subject to the other four limitations of Demand Revealing Mechanisms although further research seems necessary before this is known with certainty.

APPENDIX: PROOFS OF PROPOSITION 1 AND COROLLARY

Proof of Proposition 1:

Let $\Delta_i \equiv u_i(x_i^+, y^+) - u_i(x_i^*, y^*) = x_i^+ + \theta_i \ln y^+ - x_i^* - \theta_i \ln y^*$

$$= \alpha_i[\theta_i/\alpha_i - y^+] + [y^* - \theta_i] \ln \left[\frac{y^* - \theta_i}{y^*(1 - \alpha_i)} \right] + \theta_i \ln \left(\frac{y^+}{y^*} \right)$$

$$= \alpha_i(y_i - y^+) + (y^* - \alpha_i y_i) \ln \left(\frac{1 - \alpha_i(\, y_i/y^*)}{1 - \alpha_i} \right) + \alpha_i y_i \ln \left(\frac{y^+}{y^*} \right)$$

where $y_i = \theta_i/\alpha_i$. Now suppose $y_i = y^+$, i.e. i is the median voter. Then

$$\Delta_i \Big|_{y_i = y^+} = y^* \left[(1 - \alpha_i \frac{y^+}{y^*}) \ln \left(\frac{1 - \alpha_i y^+/y^*}{1 - \alpha_i} \right) + \alpha_i \frac{y^+}{y^*} \ln \left(\frac{y^+}{y^*} \right) \right].$$

Now setting $\epsilon = y^+/y^*$

$$F(\epsilon) \equiv \Delta_i \Big|_{y_i = y^+} = y^* \left[(1 - \alpha_i \frac{y^+}{y^*}) \ln \left(\frac{1 - \epsilon \alpha_i}{1 - \alpha_i} \right) + \alpha_i \epsilon \ln \epsilon \right]$$

It is easy to verify that for $0 < \epsilon < 1/\alpha_i$, $F(\epsilon)$ is a strictly convex function with a mimimum at $\epsilon = 1$. Then, since $F(1) = 0$, $F(\epsilon) > 0$ for all $0 < \epsilon < 1/\alpha_i$, $\epsilon \neq 1$. Thus $\Delta_i > 0$ for i = median voter.

Now, it can be verified that

$$\frac{\partial \Delta_i}{\partial y_i} = \alpha_i \ln \left[\frac{y^+(1 - \alpha_i)}{y^* - \alpha_i y_i} \right] \begin{cases} \geq 0 & \text{if } y_i \geq y^+ \geq y^* \\ \leq 0 & \text{if } y_i \leq y^+ \leq y^* \end{cases} .$$

FOOTNOTES

1/ The problems that arise when there are more than a single private and/a public good will be indicated when relevant; basically, nearly all results mentioned hold for the more general case.

2/ See Samuelson [10].

3/ The role of the computer ("government") is similar in spirit to that of the Walrasian auctioneer in models of competitive market mechanisms.

4/ Note that since the marginal utility of the private good $\partial u_i / \partial x_i$ is always strictly positive, the consumer will always choose to be on the budget line given any tax; i.e. $x_i^* = \omega_i - T_i(m)$ will be the utility maximizing choice of the private good given after-tax income $\omega_i - T_i(m)$.

5/ The term is due to Tideman and Tullock [12].

6/ The earliest paper containing one of these mechanisms seems to be by Vickrey [13], although Vernon Smith reports, in [11], that Jacob Marschak had the essential idea in the early 1950's. The recent revival of interest is due to the independent rediscovery of these mechanisms by Groves [4,5] and Clarke [2].

7/ Up to equivalence in preference. If $\psi_i(\cdot)$ is strictly monotonic increasing for all i, then the Lindahl-Samuelson equation has a unique solution.

8/ The strategy $\Phi_i = \Psi_i$ is not the unique dominant strategy; all dominant strategies are of the form $\Phi_i = \Psi_i +$ constant. See Groves [6]. Thus, all dominant strategies convey the true marginal rate of substitution schedule, $\partial \Psi_i / \partial y$, or the true marginal willingness to pay function.

9/ See footnote 8/.

10/ Up to some lump-sum redistribution; see Green and Laffont [3] .

11/ This suggestion is due to T. N. Tideman.

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