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ENVIRONMENTAL MANAGEMENT IN GENERAL EQUILIBRIUM: A NEW INCENTIVE COMPATIBLE APPROACH

by

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A B S T R A C T

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Pollution is introduced into a microeconomic general equilibrium model as a set of detrimental pure public consumers' goods. Its emergence is explained by the discharge of the economy's aggregate excess supply of private goods (Mäler) into the environment. Not only the generation of (useless) by-products in consumption and production is considered but also recycling in production.

In addition to consumers and producers there is a government which enforces an effluent charge-transfer scheme in order to restore Pareto-efficiency. But in contrast to the well-known Lindahl-equilibrium solutions (Foley, Tietenberg, Mäler), in the present paper the free rider problem is not defined away. Instead, a solution to this problem is offered: The government's environmental management is shown to be incentive compatible in the sense that in an equilibrium of the economy relative to this government it is in every consumer's own self interest to reveal his preferences on pollution (or on environmental quality).

The pollution allocation mechanism (government) is based on some recent joint work by Groves and Ledyard: There is a communication process between the government and the consumers which helps to determine the "socially desired" allocation of pollution. Furthermore, the government redistributes its effluent charge revenues to the consumers according to specific functions (rules) whose value depends on prices, on the state of the communication process (messages), and on the consumer's own proposed pollution allocation. The equilibrium concept for the pollution economy relative to this government is a Nash equilibrium.

The main results of the paper are an existence theorem whose relevance is to establish the non-vacuousness of the government and the two Fundamental Theorems of Welfare Economics.

I. Introduction

Production and consumption activities generate waste products which cause pollution when they are discharged into the environment. If the negative feedback of pollution on the economic activities is considered analytically as a detrimental externality (or "public bad"), the competitive equilibrium allocation of resources will generally be inefficient in a polluted economy without environmental management. T.H. Tietenberg [23] and K.G. Mäler [19, pp. 19-46] studied this externality problem in a microeconomic general equilibrium model. They showed, as could be expected in the Pigouvian tradition, that there exist effluent charges and a redistribution scheme which restore Pareto-efficiency.[1] But their models are unsatisfactory (1) with respect to the concept of waste products applied and (2) with respect to the informational requirements for achieving Pareto-efficiency.

The two authors define waste products (or residuals) as a set of private commodities which is disjoint from the set of "regular" or "useful" private goods. However, since generally the demand and supply for private goods, hence also that of waste products, depend on technologies, the consumers' characteristics and on prices, it seems to be unduely restrictive to exclude by assumption that in equilibrium (a) "regular" goods are not discharged into the environment or (b) "residuals" may be used as "regular" goods, i.e. for consumption or production purposes. Mäler himself admits that the a priori distinction between residuals and regular goods "... is not a logical approach..." [19, p. 36]. Moreover, he briefly sketches an alternative model [19, pp. 46-48] which circumvenes these difficulties by defining residuals as private goods in total excess supply. This natural approach to the generation of waste products appears to be a satisfactory microeconomic basis for the explanation of the pollution phenomenon. It will be followed and elaborated in this paper.

The second more important issue is to increase the informational efficiency of environmental management. In Tietenberg's and Mäler's models - as generally for Pigouvian tax-subsidy-schemes - the informational requirements are extremely high. The government must have at its disposal all the information needed to find a Pareto-efficient allocation [23, p. 513], that is, all private agents must be assumed to report truthfully and completely their technologies and preferences to the government. In particular, the consumers have to reveal their preferences for the environmental quality or pollution, respectively, i.e. at given (personalized) prices they report their true demand for environmental quality. However, it has been argued by many economists since Wicksell [25, p. 100][2] and is rigorously shown by Ledyard and Roberts [16] that it is not in a consumer's self-interest to reveal his preferences for public goods.[3] The consumer's incentive to be a free rider on (governmental) public goods projects is strengthened by the fact that for the government there is hardly any possibility to discover (plausible) misrepresentations of preferences. This so-called free rider problem is the reason why public goods cannot be efficiently allocated by decentralized methods (markets) in the same way as private goods. Therefore many economists (see, for example Buchanan [5 , p. 87]) have apparently been convinced that the free rider problem is conceptually unsolvable. This pessimistic view also prevailed in the field of environmental economics.[4]

The first attempts to attack the free rider problem were made in the 1940s. Subsequent major contributions are those of Vickery [24], Clarke [6], and Groves [11], who finally exhibited the whole class of direct revelation mechanisms which are strongly individually rational [10, Corrollary 2, p. 25]. All these results were obtained in partial equilibrium models. In particular, the central agency whose policy ensures incentive compatibility cannot balance its budget [10, Theorem 9, p. 39]. For this reason such incentive mechanisms could not be incorporated into general equilibrium models until recently when Groves and Ledyard [13] proposed a revelation mechanism with the balanced budget property.[5)]

In the present paper we apply the Groves-Ledyard incentive mechanism to the issue of environmental management. Since the allocation problem in the case of pollution differs substantially from that in the Groves-Ledyard public goods model, the application of their incentive mechanism is not straightforward. It had to be modified adequately in order to fit into the pollution model, but on the other hand its decicive incentive and balanced budget properties are preserved.

The main results are an existence theorem of competitive equilibrium relative to the incentive compatible environmental management and the two Fundamental Welfare Theorems for the pollution economy.

II. The Model

There are L <u>private commodities</u>, indexed $l = 1,\ldots,L$. If an economic agent discharges a commodity l into the environment rather than using it in private, i.e. for consumption or production purposes, this commodity is said to be the residual l. A bund le of <u>residuals</u> is denoted $r \in \mathbb{R}^L$.

If residuals are released into the environment, they may either be completely absorbed and neutralized due to the environmental assimilative capacity, they may remain unchanged, or they may be transformed into different (detrimental) "objects". A famous example for residuals interacting in the environment and turning into more obnoxious substances is the smog in the Los Angeles area. All these various processes are described by the pollution function $S: \mathbb{R}^L_+ \to \mathbb{R}^K_+$, where $S(r) \in \mathbb{R}^K_+$ is a vector of K pollutants, indexed $k = 1,\ldots,K$, being generated by the residuals r. The components of S(r) are quantity indices for pollutants which may be measured by some concentration index of (detrimental) "end-products" remaining ambient in the environment as result of the residuals discharge. If the k^{th} entry in S(r) is zero, then the state of the environment with respect to the pollutant k is the same as if $r = 0$.

There are I consumers, indexed $i = 1,\ldots,I$. $x^i \in \mathbb{R}^L_+$ is consumer i's consumption bundle of private goods. The consumers' utility not only depends on the L private goods, but also on the K pollutants which are analytically treated as pure public goods (or rather public "bads"). Hence consumer i's preference relation $\preccurlyeq_i$ is defined on his consumption set $\mathfrak{X}^i \subset \mathbb{R}^L_+ \times \mathbb{R}^K_+$. His initial endowment of private goods is $\omega^i \in \mathbb{R}^L$.

Since environmental issues are essentially concerned with the disposal of private goods , the traditional free disposal assumption on production and consumption seem to be conceptionally inadequate in environmental economics. In order to exclude free disposal in consumption we will regard consum ption as an activity transforming consumption bundles x^i into different private goods bundles of approximately the same total weight (consumption wastes)[6]. For simplicity we assume that the unique consumption transformation function[7] is a linear mapping from $\mathbb{R}^L_+$ into $\mathbb{R}^L_+$ such that $A^i x^i \in \mathbb{R}^L_+$ is the "output" vector of i's consumption activity when his (input) consumption bundle is x^i. The generic element a^i_{ln} of the (L×L)-matrix A^i defines the amount of commodity l generated in the process of consuming one unit of commodity n.

Each of the J producers, indexed j = 1,...,J, is characterized by his production set $Y^j \subset \mathbb{R}^L$ whose elements y^j are technologically feasible input output vectors. Negative components of y^j are inputs, positive components are outputs. Associated with each producer j is a profit share distribution θ^{ij}, i= 1,...,I such that $\theta^{ij} \in [0,1]$ and $\Sigma_i \theta^{ij} = 1$ for every j, where θ^{ij} is the i^{th} consumer's share of the j^{th} producer's profit.

For each consumption allocation $(x^i) \equiv (x^1, x^2,...,x^I)$ and production allocation $(y^j) \equiv (y^1, y^2,...,y^J)$, we define the unique vector of residuals by

$$r \equiv \Sigma_j y^j - \Sigma_i (x^i - A^i x^i - \omega^i).$$

r may also be called the supply of residuals since by definition it is equivalent to the familiar concept of an economy's aggregate excess supply when (x^i) and (y^j) are consumption and production plans, respectively. Note that S(r) is not defined if $r \notin \mathbb{R}^L_+$. Also, residuals and consumption wastes have to be strictly distinguished, since $\Sigma_i A^i x^i > 0$ is obviously compatible with a non-positive supply of residuals. The reason is that other consumers or the producers may have non-zero demands for a consumer's consumption wastes. In particular the production technologies Y^j (j = 1,...,J) may include various kinds of recycling activities.

An allocation $((x^i), (y^j))$ will be said to be attainable, if

(i) $y^j \in Y^j$ for every j

(ii) $(x^i, S(r)) \in \mathfrak{X}^i$ for every i, where $r = \Sigma_j y^j - \Sigma_i (x^i - A^i x^i - \omega^i)$.

This definition clarifies the analytical treatment of pollutants as pure public goods. By condition (ii) above non-negativity of r is a necessary condition for an allocation to be attainable (and hence also for every equilibrium allocation).

All private commodities $l = 1,\ldots,L$ are marketable, whereas the K pollutants are not. We assume that there is a competitive market for each private commodity and we denote by $p \in \mathbb{R}^L$ the associated price system. Negative prices are not excluded and in fact, when they exist, they will play a decicive role in the model.

In addition to consumers and producers there is a special agent in the economy, the government (or the environmental protection agency). Between the government and the consumers a communication process is taking place for the purpose to determine the social demand for residuals (and hence - via the pollution function - also for pollutants). Every consumer sends a message to the government by which he communicates his most preferred residuals allocation relative to given prices and the given messages of the other consumers. The government informs each consumer about the messages of the others and on the basis of all messages received it determines the social demand for residuals. In equilibrium, the government carries out this social demand for residuals by buying the residuals demanded at prevailing prices for no other reason than to discharge them into the environment. It also monitors effectively all markets, in particular those with negative prices. Thus, when private agents sell some commodities to the government, we can equivalently say that they discharge residuals into the environment after having paid the prevailing price (effluent charge) to the government. Finally, the government imposes a distribution rule on each consumer by which it redistributes the revenues from its market activity among the consumers.

The communication between the government and the consumers is formalized by the language $M \subset \mathbb{R}^L$. $m^i \in M$ is called consumer i's message. The government's <u>(social) demand rule for residuals</u>, $r^g: M^I \to \mathbb{R}^L$, is defined by $r^g(m) = \Sigma_h m^h$, where $r^g(m)$ denotes a vector of private goods which the government demands in order to release them into the environment, when it receives the messages

$$m \equiv (m^1, m^2, \dots, m^I) \in M^I \equiv \underbrace{M \times \dots \times M}_{I \text{ times}}$$

When consumer i demands (proposes, votes for) the residuals allocation $r^i \in \mathbb{R}^L_+$ at given messages $m^{)i(} \equiv (m^1, \dots, m^{i-1}, m^{i+1}, \dots, m^I) \in M^{I-1}$ of the other consumers he reports this choice to the government by sending the message

$$m^i = r^i - \Sigma_{h \neq i} m^h$$

This <u>message rule</u> is derived from, but is not identical to the government's demand rule for residuals. At given $(\bar{m}, p) \in M^I \times \mathbb{R}^L$ the consumers may choose residuals demands r^i and hence messages m^i, such that $r^g \neq r^i$ for some i and $r^i \neq r^h$, $i \neq h$. But it is also obvious that if for every i, given $(\bar{m}, p)$, the message $\bar{m}^i$ is consumer i's best response, then $r^i = r^g = \Sigma_h \bar{m}^h$ for every i.

The particular form of the message rule suggests the following interpretation: With his message m^i consumer i communicates how much more or less of each residual he would like to have provided given the messages, i.e. the differential amounts of residuals requested by the other consumers.

The government's <u>distribution rule</u> for consumer i is a function $c^i: \mathbb{R}^L_+ \times M^{I-1} \times \mathbb{R}^L \to \mathbb{R}$ such that [8)]

$$c^i(r^i, m^{)i(}, p) = \alpha^i p \cdot r^i + \frac{\gamma}{2} \left[\frac{I-1}{I}[r^i - I\mu^i(m^{)i(})]^2 - \sigma^i(m^{)i(})^2\right]$$

where $\gamma > 0$, $\alpha^i = 1 - \Sigma_{h \neq i} \alpha^h > 0$,

$\mu^i(m^{)i(}) \equiv \mu^i \equiv \frac{1}{I-1} \Sigma_{h \neq i} m^h$

$\sigma^i(m^{)i(})^2 \equiv \sigma^{i2} \equiv \frac{1}{I-2} \Sigma_{h \neq i}(m^h - \mu^i)^2$

$C^i(r^i, m^{)i(}, p)$ is the transfer which consumer i receives from or has to pay to the government, if $(m^{)i(}, p) \in M^{I-1} \times \mathbb{R}^L$ is given and if he demands the residuals allocation r^i. $C^i(r^i, m^{)i(}, p)$ is the sum of three components: $\alpha^i p \cdot r^i$ is the α^i-th part of the government's revenue when it realizes consumer i's demand for residuals. This transfer (which will turn out to be nonpositive in equilibrium) is diminished by the (non-negative) amount $\frac{\gamma(I-1)}{2I}(r^i - I\mu^i)^2$. Since this term increases progressively with the difference $(r^i - I\mu^i)$ it may be interpreted as a "penalty cost" for deviations of consumer i's demand r^i from some "guide line" demand $I\mu^i$ depending on the mean of the other consumers' messages. The last (non-negative) factor $-\frac{1}{2}\gamma\sigma^{i2}$ increases consumer i's total transfer the more, the larger the variance of the other consumers' messages. Thus consumer i gains from diverting messages of the others' whereas by the second term he looses when his own message m^i deviates from the average message μ^i of the other consumers.

When the consumer selects his best consumption plan he does not only take prices as given but also the other consumers' messages. Furthermore, he must send a message himself. So he is asked to choose a commodity bundle $(x^i, r^i) \in \mathbb{R}^{2L}$ and a message $m^i \in M$ out of his <u>budget set</u> [9)]

$$B^i(m^{)i(}, p) = \{(\bar{x}^i, \bar{r}^i, \bar{m}^i) \in \mathbb{R}^{3L} \mid (\bar{x}^i, S(\bar{r}^i)) \in \mathfrak{X}^i \text{ and}$$
$$p \cdot (\bar{x}^i - A^i \bar{x}^i) + C^i(\bar{r}^i, m^{)i(}, p) \leqslant w^i(p) \equiv$$
$$\equiv p \cdot \omega^i + \Sigma_j \theta^{ij} \operatorname{Max} p \cdot Y^j ;\ \bar{m}^i = \bar{r}^i - \Sigma_{h \neq i} m^h\}$$

where $\operatorname{Max} p \cdot Y^j \equiv \operatorname{Max}_{y^j \in Y^j} p \cdot y^j$

We can now introduce the definition of competitive (Nash) equilibrium for this economy:

A <u>competitive equilibrium relative to the government</u> $G \equiv \{M, r^g(\cdot), (m^i(\cdot), c^i(\cdot))\}$ <u>in the economy</u> $\mathcal{E} \equiv \{(\bar{x}^i, \succcurlyeq_i, \omega^i, A^i), (Y^j), (\theta^{ij}), S(\cdot)\}$ is a (I+J+1)-tuple $((x^i, r^i, m^i), (y^j), p)$ of consumer decisions, producer decisions and a price system such that

(i) y^j is a solution to Max $p\cdot Y^j$ for every j

(ii) for every i $(x^i, r^i, m^i) \in B^i(m)^i(, p)$ and $(x^i, S(r^i)) \succcurlyeq_i (\bar{x}^i, S(\bar{r}^i))$, if $(\bar{x}^i, \bar{r}^i, \bar{r}^i - \Sigma_{h\neq i}m^h) \in B^i(m)^i(, p)$

(iii) $\Sigma_j y^j - \Sigma_i(x^i - A^i x^i - \omega^i) = r^g(m)$

It is obvious that in equilibrium for every i $r^i = r^g(m) \in \mathbb{R}^L_+$. Therefore equilibrium condition (iii) implies that, for every l, the supply of good l for private use is equal to its demand for private use (i.e. for consumption or production purposes). It is also important to note that equilibrium condition (ii) implies $\Sigma_i c^i(r^i, m)^i(, p) \leqslant p\cdot r^g(m)$. If, moreover, all consumers' best decisions satisfy their budget constraint with equality (as is guaranteed by some standard nonsatiation assumption on preferences) equilibrium requires a balanced budget (Walras' Law). In fact, the distribution functions defined above exhibit this important property (for $I \geqslant 3$): for every $(m, p) \in M^I \times \mathbb{R}^L$ one obtains $\Sigma_i c^i(r^i, m)^i(, p) = p\cdot r^g(m)$, if for every i $m^i = r^i - \Sigma_{h\neq i}m^h$, or equivalently, if $r^i = r^h$ for every $i,h = 1,\dots,I$, $i \neq h$ [13, p. 25].

Finally it can be shown (see also Claim 2 in the proof of Lemma 2 below) that the equilibrium price system has the following economically interesting property: If $p_l > 0$, then $r^g_l = 0$, where r^g_l is the l^{th} component of $r^g(m)$. But if $p_l < 0$ $(p_l = 0)$ then

$r_1^g \leq 0$ ($r_1^g \leq 0$). Hence in this model negative prices can be interpreted consistently as effluent charges. This property of the equilibrium price system also implies that (in equilibrium) $\Sigma_i C^i(r^g, m)^{i}(\ , p) \leq 0$, which does not exclude, however, that $C^h(r^g, m)^{h}(\ , p) > 0$ for some h. It is exactly this possibility, which is the reason for the "γ^*-condition" in the existence theorem of the next section.

III. The Existence of Equilibrium

Since various existence theorems for equilibria of economies with public goods are already presented (see, for example, [9]), difficulties to establish an equilibrium for $\mathcal{E}$ relative to G (under some set of prior conditions on $\mathcal{E}$) must be due to the special form of the government G, if on $\mathcal{E}$ standard assumptions are introduced. In fact, the main problem in the existence proof is to compactify the language M adequately and to prevent the consumers' bankruptcy caused by excessively high "taxes" $C^i(\cdot) > 0$.[10] Hence the subsequent existence theorem should be primarily viewed as a theorem on the existence (non-vacuousness) of the government G. In order to state and provethe existence theorem we now summarize and complete the list of assumptions already discussed in the previous section:

(a) S: $\mathbb{R}^L_+ \to \mathbb{R}^K_+$ satisfies $S(0) = 0$; S is continuous, non-decreasing and convex

(b.1) $\mathfrak{X}^i = X^i \times \mathbb{R}^K_+$ is closed and convex; $\mathbb{R}^L_+ \supset X^i \supset \{0\}$

(b.2) $\omega^i \in \text{Int } T^i$ and $\Sigma_i \omega^i \in \mathbb{R}^L_+$; Int denotes interior, $T^i \equiv \{t^i \in \mathbb{R}^L \mid t^i = x^i - A^i x^i,\ x^i \in X^i\}$, where A^i is a non-negative (L×L)-matrix

(c.1) for every $(x^i, s) \in \mathfrak{X}^i$ there exists some $(\bar{x}^i, s) \in X^i \times S(\mathbb{R}^L_+)$ such that $(\bar{x}^i, \bar{s}) >_i (x^i, s)$ (non-satiation) (c.2) for every $(x^i, s) \in \mathfrak{X}^i$, the sets $\{(\bar{x}^i, \bar{s}) \in \mathfrak{X}^i \mid (\bar{x}^i, \bar{s}) \succsim_i (x^i, s)\}$ and $\{(\bar{x}^i, \bar{s}) \in \mathfrak{X}^i \mid (x^i, s) \succsim_i (\bar{x}^i, \bar{s})\}$ are closed (continuity)

(c.3) $(x^i, s) \succ_i (\bar{x}^i, \bar{s})$ implies $(\lambda x^i + (1-\lambda)\bar{x}^i, \lambda s + (1-\lambda)\bar{s}) \succ_i (\bar{x}^i, \bar{s})$ for every $\lambda \in (0,1) \subset \mathbb{R}$ (convexity of preferences)

(c.4) for every $s, \bar{s} \in \mathbb{R}^K_+$ with $s_k \geqslant \bar{s}_k$ for every k, if (x^i, s), $(x^i, \bar{s}) \in \mathfrak{X}^i$, then $(x^i, \bar{s}) \succsim_i (x^i, s)$ (monotonicity of preferences in pollutants)

(d.1) $0 \in Y^j$ (possibility of inaction)

(d.2) Y^j is closed and convex

(d.3) $Y \cap \mathbb{R}^L_+ \subset \{0\}$, where $Y \equiv \Sigma_j Y^j$ (no free production)

(d.4) $Y \cap (-Y) \subset \{0\}$ (irreversibility of production)

(e) $x^i \in X^i$ for every i, $v \in [0, \Sigma_i A^i x^i]$, and $\bar{v} \geqslant \Sigma_i x^i$ implies $(\bar{v} - v) \notin Y$ ("irreversibility of consumption")

(f) $M = \mathbb{R}^L$ (language)

(g) $r^g(m) = \Sigma_h m^h$ (social demand rule for residuals)

(h) $m^i(r^i, m^{)i(}) = r^i - \Sigma_{h \neq i} m^h$ (message rule)

(k) $c^i(r^i, m^{)i(}, p) = \alpha^i p r^i + \frac{\gamma}{2}[\frac{I-1}{I}(r^i - I\mu^i)^2 - \sigma^{i2}]$ (distribution rule) where $\gamma > 0$, $\alpha^i = 1 - \Sigma_{h \neq i}\alpha^h > 0$, $\mu^i = \frac{1}{I-1}\Sigma_{h \neq i} m^h$ and $\sigma^{i2} = \frac{1}{I-2}\Sigma_{h \neq i}(m^h - \mu^i)^2$

The assumptions (f) - (k) on the government have already been discussed. The assumptions (d) on the production technologies are standard except that the free disposal assumption is omitted. It can easily be demonstrated that in economies with the free disposal option in production environmental management is always able to achieve zero pollution at zero (resource) costs. By (e) consumption bundles (x^i) cannot be re-produced with the consumption wastes, which are generated by (x^i). Hence (e) is an irreversibility assumption similar as (d.4), which is needed to ensure that the set of attainable allocations is bounded. The assumption (a) on the pollution function S considers the widely accepted hypothesis that the environment's capacity to absorb and neutralize residuals diminishes with increasing levels of residuals discharged.

Both, the assumptions (b.1) and (b.2) are restrictive. They are used to prevent the emptiness of the consumers' budget sets. Finally, the specification of preferences in (c) is almost standard. Since the consumer's decision is made in the "derived" consumption set $X^i \times \mathbb{R}^L_+$ and not in $\mathfrak{X}^i$, a corresponding transformation of preferences $\succcurlyeq_i$ from $\mathfrak{X}^i$ to $X^i \times \mathbb{R}^L_+$ is necessary. To ensure that these induced preferences are "well-behaved" (see Lemma 1γ) assumption (c.4) cannot be relaxed unless S is assumed to be a linear function.

<u>Theorem 1</u>: (Existence) Let γ be the parameter in assumption (k). There exists $\gamma^* \in \mathbb{R}_+$ such that under the assumptions (a) - (k) the economy $\mathcal{E}$ (with I > 2) has a competitive equilibrium relative to the government G_γ for every $\gamma \geqslant \gamma^*$.

The proof of Theorem 1 will proceed in several steps.

(1) The first step is to compactify the economy. Applying Bergstrom's technique [4] we restrict prices to the non-empty, convex, compact hyper-ball

$$P \equiv \{p \in \mathbb{R}^L \mid \|p\| \leqslant 1\}$$

The set of attainable allocations is non-empty, since by (b.1) and (b.2) the allocation $((x^i = 0,\ S(\Sigma_i \omega^i)), (y^j = 0))$ is attainable. Its boundedness can be proved as Proposition (2) of Section 5.4 in [8, p. 77n.] with the help of assumption (e).

We denote by $\hat{R}$, $\hat{X}^i$, $\hat{Y}^j$ the projections of the set of attainable allocations on the spaces $\mathbb{R}^L$ which contain the residuals vectors, X^i, and Y^j, respectively. Clearly, $\hat{R}$, all $\hat{X}^i$ and $\hat{Y}^j$ are bounded sets. By (a) $\hat{S} \equiv S(\hat{R})$ is also bounded. Let $\hat{r}_l$ be the greatest l^{th} component of every $r \in \hat{R}$ and define $\hat{r} \equiv \max_l \hat{r}_l$. Further select two real numbers $\underline{n}$ and $\bar{n}$ such that

$$\bar{n} > \hat{r} + \frac{2I\alpha^i}{\gamma} \quad \text{and} \quad \underline{n} < -(I-1)\bar{n},$$

and define a "truncated" message space as

$$\widehat{M} \equiv \{m^i \in \mathbb{R}^L \mid \text{for every } l,\ \underline{n} \leqslant m^i_l \leqslant \bar{n}\}.$$

$\widehat{M}$ is clearly non-empty, convex, compact. Let $\hat{K}$ be a closed cube in $\mathbb{R}^L$ with center zero which contains in its interior the sets $\hat{X}^i$, $\hat{Y}^j$, and $\{\Sigma_h m^h \in \mathbb{R}^L \mid m^h \in \widehat{M}\}$. Denote $\widehat{X}^i \equiv X^i \cap \hat{K}$, $\widehat{R} \equiv$ $\hat{K}$, $\widehat{S} \equiv S(\widehat{R})$, $\widehat{\mathfrak{X}}^i \equiv (\widehat{X}^i \times \widehat{S})$, and $\widehat{Y}^j \equiv Y^j \cap \hat{K}$. All sets $\widehat{X}^i$, $\widehat{R}$, and $\widehat{Y}^j$ are convex and compact. Replacing everywhere the original (unbounded) sets by their compact ("$\frown$")- subsets we obtain a compactified economy $\widehat{\mathcal{E}}$ and a compactified government $\widehat{G}_\gamma$.

(2) In the second step the compactified government $\widehat{G}_\gamma$ is substituted by a "compact" government $\widehat{G}^*_\gamma \equiv \{r^g(\cdot),\ (m^{*i}(\cdot),\ c^{*i}(\cdot)), \widehat{M}\}$ which is defined as $\widehat{G}_\gamma$ except that the assumptions (h) and (k) are replaced by

(h^*) $\quad m^i = m^{*i}(r^i, m^{)i(}, p) = \min\,[\bar{n},\ r^i - (I-1)\bar{\mu}^i(m^{)i(}; p)]$

where $\bar{\mu}^i(m^{)i(}, p) \equiv (\bar{\mu}^i_1(m^{)i(}, p), \ldots, \bar{\mu}^i_L(m^{)i(}, p))$ and for every l $\bar{\mu}^i_l(m^{)i(}, p) \equiv \max\,[\delta_l(p)\,\hat{\mu}^i_l(p),\ \ \mu^i_l(m^{)i(})]$

with $\delta_l(p) = 0$, if $p_l > 0$ and $\delta_l(p) = 1$, if $p_l \leqslant 0$,

$\hat{\mu}_l(p) \equiv \dfrac{\alpha^i p_l}{\gamma(I-1)}$ and $\mu^i_l(m^{)i(}) \equiv \dfrac{1}{I-1} \Sigma_{h \neq i}\, m^h_l$

$$(k^*)\quad c^{*i}(r^i, m^{)i(},p) \equiv D^i(r^i, m^{)i(},p) - \Delta^i(m^{)i(},p) + \min\left[\tfrac{1}{2} W^i(p), \Delta^i(m^{)i(},p)\right],$$

where

$$W^i(p) \equiv p\cdot\omega^i - \mathrm{Min}\; p\cdot\hat{T}^i + \Sigma_j \theta^{ij}\, \mathrm{Max}\; p\cdot Y^j$$

$$\hat{T}^i \equiv \{t \in \mathbb{R}^L \mid t = x^i - A^i x^i \text{ and } x^i \in \hat{X}^i\},$$

$$\mathrm{Min}\; p\cdot\hat{T}^i \equiv \{p\cdot t \mid t \in \hat{T}^i \text{ and } p\cdot t \leq p\cdot t' \text{ for every } t' \in \hat{T}^i\};$$

$$D^i(r^i, m^{)i(},p) = \alpha^i p r^i + \frac{\gamma}{2}\left[\frac{I-1}{I}\,[r^i - I\bar{\mu}^i(m^{)i(},p)]^2 - \bar{\sigma}^i(m^{)i(},p)^2\right].$$

where

$$\alpha^i = 1 - \sum_{h\neq i}\alpha^h > 0,\quad \gamma > 0,\quad \bar{\sigma}^i(m^{)i(})^2 \equiv \frac{1}{I-2}\sum_{h\neq i}[m^h - \bar{\mu}^i(m^{)i(},p)]^2,$$

$$\Delta^i(m^{)i(},p) \equiv \Sigma_l \Delta^i_l(m^{)i(},p), \text{ and for every } l$$

$$\Delta^i_l(m^{)i(},p) \equiv D^i_l(m^{)i(},p)_{\min} + d^i_l(m^{)i(},p)\,\bar{D}^i_l(m^{)i(},p);$$

$$D^i_l(m^{)i(},p)_{\min} \equiv \min_{r^i_l \in \mathbb{R}} D^i_l(r^i_l, m^{)i(},p)$$

$$(\text{where } \Sigma_l D^i_l(r^i_l, m^{)i(},p) = D^i(r^i, m^{)i(},p))$$

$$d^i_l(m^{)i(},p)\quad \begin{cases} = 0, \text{ if } \mu^i_l(m^{)i(}) \equiv \Sigma_{h\neq i} m^h_l \geq \dfrac{\alpha^i p_l I}{\gamma(I-1)} \\[2ex] = 1, \text{ if } \mu^i_l(m^{)i(}) < \dfrac{\alpha^i p_l I}{\gamma(I-1)} \end{cases}$$

$$\bar{D}^i_l(m^{)i(},p) \equiv D^i_l(0, m^{)i(},p) - D^i_l(m^{)i(},p)_{\min}$$

(3) Step 3 in the proof of Theorem 1 consists in converting the economy $\hat{\mathcal{E}}$ with government $\hat{G}^*_\gamma$ into an abstract economy ([7], [1]) and to prove that this abstract economy has an equilibrium. An <u>abstract economy</u> $\Gamma = (Z^n, K^n, U^n)_{n=1}^N$ is defined by N ordered triples (Z^n, K^n, U^n), where $K^n: \Pi_{v\neq n} Z^v \to Z^n$ are correspondences and $U^n: \Pi_v Z^v \to \mathbb{R}$ are functions.

There are $N = I + J + 1$ agents in the abstract economy. The first I agents are consumers, the agents $n = I + j$, $j = 1,\ldots,J$, are producers, and the N-th agent is the market player. For convenience we substitute the superscript n by $i = 1,\ldots,I$, if $n = 1,\ldots,I$, and by $j = 1,\ldots,J$, if $N = I + j$. We define $Z^i = \hat{X}^i \times \hat{R} \times \hat{M}$, $Z^j = \hat{Y}^j$, and $Z^N = P$. Hence the generic element of $Z = \Pi_n Z^n$ is $z = ((x^i, r^i, m^i), (y^j), p)$. The constraint correspondences $K^i(z^{)i(})$ are defined by[12]

$$K^i(z^{)i(}) = \{(\bar{x}^i, \bar{r}^i, \bar{m}^i) \in Z^i \mid \bar{m}^i = m^{*i}(\bar{r}^i, m^{)i(}, p),\ p\cdot(\bar{x}^i - A^i\bar{x}^i) + C^{*i}(\bar{r}^i, m^{)i(}, p) \leq w^i(z^{)i(}) + 1 - \|p\| \},$$

where $z^{)i(} = (z^1, z^2,\ldots,z^{i-1}, z^{i+1},\ldots,z^N)$. Further we assign

$$K^j(z^{)j(}) = Z^j = \hat{Y}^j \text{ for every } z^{)j(} \in Z^{)j(} = \Pi_{n\neq j} Z^n$$

$$K^N(z^{)N(}) = Z^N = P \text{ for every } z^{)N(} \in Z^{)N(}.$$

In order to specify the functions U^n for $n = 1,\ldots,I$, we define the preference relation $\succcurlyeq_{z_i}$ on Z for every consumer: For every pair $z, \bar{z} \in Z$ one has $z \succcurlyeq_{z_i} \bar{z}$ if and only if $(x^i, S(r^i)) \succcurlyeq_i (\bar{x}^i, S(\bar{r}^i))$, where x^i and r^i are components of z while $\bar{x}^i$ and $\bar{r}^i$ are components of $\bar{z}$. Now we take $U^i\colon Z \to \mathbb{R}$ to be a (utility) function representing the preference relation $\succcurlyeq_{z_i}$ on Z. Obviously, for every $z^i \in Z^i$, the function U^i satisfies $U^i(z^{)i(}, z^i) = U^i(\bar{z}^{)i(}, z^i)$ for arbitrary $z^{)i(}, \bar{z}^{)i(} \in Z^{)i(} = \Pi_{v\neq i} Z^v$. For the producers we introduce the function $U^j\colon Z \to \mathbb{R}$, such that $U^j(z^{)j(}, z^j) = p\cdot y^j$ (where p is a component in $z^{)j(}$). The function U^N assigned to the market player is $U^N(z^{)N(}, z^N) = p\cdot e$, where $e = \Sigma_i(x^i - A^i x^i - \omega^i) - \Sigma_j y^j + \Sigma_i m^i$. Again, x^i, y^j and m^i are here understood to be components of $z^{)N(}$.

An equilibrium for the abstract economy Γ is defined by a vector $z^* \in Z$, satisfying for each n $z^{n*} \in K^n(z^{)n(*})$ and $U^n(z^*) \geq U^n(z^{)n(*}, z^n)$ for every $z^n \in K^n(z^{)n(*})$.

<u>Lemma 1</u>: (Arrow and Debreu [1, p. 274][13]) Let $\Gamma = ((Z^n, K^n, U^n)_{n=1}^N$ be an abstract economy satisfying, for each i,

(α) Z^n is nonempty compact convex,

($\beta 1$) for every $z^{)n(} \in \Pi_{v \neq n} Z^v$, $K^n(z^{)n(})$ is non-empty, compact and convex,

($\beta 2$) K^n is a continuous correspondence,

(γ) U^i is continuous on Z and quasi-concave in z^i.

Then Γ has an equilibrium.

<u>Proof</u>: (α) follows from step 1 in the proof of Theorem 1.

(β): For $n > I$, (β) is straightforward.

($\beta 1$) for $n \leq I$: (non-emptyness) It follows easily from (b.2) that for every $z^{)i(} \in Z^{)i(}$ there exists $\hat{x}^i \in \hat{X}^i$ such that $p \cdot (\hat{x}^i - A^i \hat{x}^i) < p \cdot \omega^i \leq w^i(z^{)i(}) \leq w^i(z^{)i(}) + 1 - \|p\|$. Further, by the definitions of $\hat{M}$ and $\bar{\mu}^i(m^{)i(},p)$ the set $R^i(m^{)i(},p) \equiv \{r^i \in \hat{R} \mid [r^i - (I-1)\bar{\mu}^i(m^{)i(},p)] \in \hat{M}\}$ is non-empty. By construction of C^{*i} in (k^*) for every $z^{)i(} \in Z^{)i(}$ we obtain $\underset{r^i \in R^i(m^{)i(},p)}{\text{Min}} C^{*i}(r^i, m^{)i(},p) \leq \frac{1}{2} W^i(p)$,

since $\underset{r^i \in R^i(m^{)i(},p)}{\text{Min}} [D^i(r^i, m^{)i(},p) - \Delta^i(m^{)i(},p)] = 0$. Hence for

every $z^{)i(} \in Z^{)i(}$ there is $\hat{r}^i \in \hat{R}$ and $\hat{x}^i \in \hat{X}^i$ such that $z^i = (\hat{x}^i, \hat{r}^i, m^{*i}(\hat{r}^i, m^{)i(},p)) \in \text{Int } K^i(z^{)i(})$.

(compactness) $K^i(z^{)i(})$ is bounded since it is contained in Z^i. It is also closed, if the functions S, m^{*i}, and C^{*i} are continuous. S is continuous by (a). Continuity of m^{*i} follows from (h^*). It remains to show that C^{*i} is continuous on Z.

Continuity of $D^i(r^i, m^{)i(},p)$, $D^i_1(m^{)i(},p)_{min}$, $\bar{D}^i_1(m^{)i(},p)$ and $W^i(p)$ is straightforward. Hence $C^{*i}(r^i, m^{)i(},p)$ is continuous, if $\Delta^i(m^{)i(},p)$ is continuous. Obviously $\Delta^i_1(m^{)i(},p)$ is continuous for all (m,p) such that $\mu^i_1 \neq \dfrac{\alpha^i p_1 I}{\gamma(I-1)}$. Consider now a sequence

$\{(m)^{i}(,p)^{q}\} \subset \hat{M}^{I-1} \times P$ such that

(i) for every q, $(m)^{i}(,p)^{q}$ satisfies $\frac{\alpha^{i} p_{l}^{q} I}{\gamma(I-1)} > \mu_{l}^{iq}$ for every l, and

(ii) $(m)^{i}(,p)^{q} \rightarrow (m)^{i}(,p)^{o}$ where $\frac{\alpha^{i} p_{l}^{o} I}{\gamma(I-1)} = \mu_{l}^{io}$ for every l.

Associated with $\{(m)^{i}(,p)^{q}\}$ is a sequence $\{\Delta_{l}^{i}(m)^{i}(,p)^{q}\}$ which satisfies $\Delta_{l}^{i}(m)^{i}(,p)^{q} = D_{l}^{i}(0, m)^{i}(q, p^{q})$ for every q. $D_{l}^{i}(0, m)^{i}(q, p^{q})$ converges to $D_{l}^{i}(m)^{i}(o, p^{o})_{min}$ when $(m)^{i}(,p)^{q} \rightarrow (m)^{i}(,p)^{o}$, since $r_{l}^{io} \equiv \mu_{l}^{io} - \frac{\alpha^{i} p_{l}^{o} I}{\gamma(I-1)} = 0$ is the solution to $\min_{r_{l}^{i} \in \mathbb{R}} D_{l}^{i}(r_{l}^{i}, m)^{i}(o, p^{o})$. This proves the continuity of C^{*i} on $\hat{R} \times \hat{M}^{I-1} \times P$. Thus $K^{i}(z)^{i}()$ is compact.

(convexity) Convexity of C^{*i} in r^{i} is straightforward. Hence the convexity of $K^{i}(z)^{i}()$ can be shown as in Lemma 3.9 in [12, p. 42].

(β2) for $n \leqslant I$: The continuity of K^{i} can be proved as in Proposition (1) of Section 4.8 in [8, p. 64 n.]

(γ) is obvious for $n > I$.

(γ) for $n \leqslant I$: It can be shown that the induced preference relation $\succcurlyeq_{z_i}$ satisfies (c.1) - (c.4) (with appropriately adjusted notation), if $\succcurlyeq_{i}$ satisfies these assumptions. Hence U^{i}, the representation of $\succcurlyeq_{z_i}$, is a continuous function on Z [8, p. 56] which is quasi-concave in z_i by (c.3).

Q.E.D.

(4) To show the link between an equilibrium for the abstract economy and the compactified economy $\hat{\mathcal{E}}$ with government $\hat{G}_{\gamma}$, we define a function γ: $Z \rightarrow \mathbb{R}_{+}$ such that

$$\gamma(z) = \bar{\gamma}(p) = \frac{1}{W(p)}$$

where $W(p) = \underset{i}{\text{Min}}\ W^{i}(p)$ (with $W^{i}(p)$ as defined in (k^{*})). Clearly,

γ is continuous on Z. Since Z is non-empty compact, by the Weierstrass Theorem there exists $\gamma^* = \underset{p \in \bar{P}}{\text{Max}}\ \bar{\gamma}(p)$ where $\bar{P} \equiv \{p \mid \|p\| = 1\}$ It follows from the proof of Lemma 1 (β1), that for every $p \in P$, $p \neq 0$, $W(p) > 0$. Hence $\gamma^* > 0$.

Lemma 2: If z is an equilibrium for the abstract economy Γ, then z is an equilibrium for the economy $\hat{\mathcal{E}}$ relative to the government $\hat{G}_\gamma$ for every $\gamma \geq \gamma^*$.

Proof: Lemma 2 is proved in several steps.

Claim 1: If z is an equilibrium for Γ, then $r^i = \Sigma_h m^h \geq 0$ for every $i = 1,\ldots,I$ (where r^i and m^h are components of z).

$z^i \in K^i(z^{)i(})$ implies $r^i \geq 0$ for every i. $r^i = \Sigma_h m^h$ is satisfied, if $\bar{\mu}^i(m^{)i(},p) = \mu^i(m^{)i(})$. Suppose that for some l for which $p_l \geq 0$ this equality does not hold. Then by (h^*) $\bar{\mu}^i_l(m^{)i(},p) = 0$. But $r^i \geq 0$ implies $m^i_l \geq 0$ for every i and hence $\mu^i_l(m^{)i(}) \geq 0$. This contradiction proves $r^i_l = \Sigma_h m^h_l$ for all l with $p_l \geq 0$.

Suppose now that for some l such that $p_l < 0$ $\mu^i_l(m^{)i(}) < \hat{\mu}^i_l(p) < 0$. In this case $\hat{r}^i_l = 0$ minimizes $D^i_l(r^i_l, m^{)i(},p)$. By (c.4) $\hat{r}^i_l = r^i_l$ so that $m^i_l = -(I-1)\hat{\mu}^i_l(p) > 0$. From $\mu^i_l(m^{)i(}) < \hat{\mu}^i_l(p)$ it follows that $\Sigma_h m^h_l < 0$. Hence there exists some $k \neq i$ such that $m^k_l < \frac{1}{I}\Sigma_h m^h_l < 0$. Now ($h^*$) requires that $\bar{\mu}^k_l(m^{)i(},p) = \mu^k_l(m^{)k(}) > 0$. But on the other hand $r^k_l \geq 0$ requires that $m^k_l \geq -(I-1)\mu^k_l(m^{)k(})$, i.e. $\Sigma_h m^h_l \geq 0$. This contradiction completes the proof of claim 1.

Claim 2: If z is an equilibrium for Γ then (i) for every l such that $p_l > 0$ $m^i_l = 0$ for every $i = 1,\ldots,I$, and (ii) for every l such that $p_l = 0$ $m^i_l = m^h_l$ for every $i,h = 1,\ldots,I$, $i \neq h$.

ad (i): From the proof of claim 1 we know that for all l with $p_l > 0$ we have $m^i_l \geq 0$ for every i. Suppose that $\mu^i_l(m^{)i(}) \geq 0$ for

some i. Then by (c.4) i's best choice r^i_ℓ satisfies $r^i_\ell \leq \hat{r}^i_\ell \equiv$
$\equiv I\mu^i_\ell(m^{)i(}) - \dfrac{\alpha^i p_\ell I}{\gamma(I-1)} < I\mu^i_\ell(m^{)i(})$, where $\hat{r}^i_\ell$ minimizes $D^i_\ell(r^i_\ell, m^{)i(}, p)$.
Since $\mu^i_\ell > m^i_\ell$ and $r^i = r^g$ for every i, there exists $k \neq i$ such that $m^k_\ell > \frac{1}{I}\Sigma_h m^h_\ell$ and hence $m^k_\ell > \mu^k_\ell$. But from the same argument as for consumer i it follows that k's best choice r^k_ℓ implies $r^k_\ell < I\mu^k_\ell$ and therefore $m^k_\ell < \mu^k_\ell$. Contradiction.

ad (ii): Suppose not. Then there is some i such that $m^i_\ell > \mu^i_\ell \geq 0$. But since $\hat{r}^i_\ell = I\mu^i_\ell$ minimizes $D^i_\ell(r^i, m^{)i(}, p)$, by (c.4) i's best choice is $r^i_\ell \leq I\mu^i_\ell$ which implies $m^i_\ell \leq \mu^i_\ell$. This contradiction completes the proof of claim 2.

<u>Claim 3</u>: If z is an equilibrium for Γ, p satisfies $\|p\| = 1$.

It is shown in [13, p. 25] that if $r^i = r^g(m)$ for every i then $\Sigma_i D^i(r^i, m^{)i(}, p) = \Sigma_i C^i(r^i, m^{)i(}, p) = p \cdot r^g(m)$. By (b.2) we have $W^i(p) \geq 0$ for every $p \in P$. Hence $\Sigma_i C^{*i}(\cdot) \leq p \cdot r^g(\cdot)$ and $p \cdot r^g(\cdot) \leq 0$ by claim 2. Further, it can be shown by standard arguments that in equilibrium the equality $p(x^i - A^i x^i) + C^{*i}(\cdot) = w^i(\cdot) + 1 - \|p\|$ is satisfied for every i. Summing over i yields

$$p \cdot [\Sigma_i (x^i - A^i x^i - \omega^i) + r^g(\cdot) - \Sigma_j y^j] \equiv p \cdot e =$$
$$= p \cdot r^g(\cdot) - \Sigma_i C^{*i}(\cdot) + 1 - \|p\| > 0$$

Suppose now, $\|p\| < 1$. Then there exists $p' \in P$ such that $p' \cdot e > p \cdot e$ contrary to the equilibrium definition. Hence $\|p\| = 1$.

<u>Claim 4</u>: If z is an equilibrium for Γ and $\gamma \geq \gamma^*$, then $C^{*i}(r^i, m^{)i(}, p) = C^i(r^i, m^{)i(}, p)$ for every i.

Consider ℓ such that $p_\ell \geq 0$. From claim 2 it follows easily that $\Delta^i_\ell(r^i, m^{)i(}, p) = 0$. Hence we obtain

$$C^{*i}(r^i, m^{)i(}, p) = D^i(r^i, m^{)i(}, p) - \sum_{\ell \in L(p)} \Delta^i_\ell(m^{)i(}, p) +$$
$$\min\,[\tfrac{1}{2}W^i(p),\ \sum_{\ell \in L(p)} \Delta^i_\ell(m^{)i(}, p)]$$

where $L(p) = \{\mathit{l} \mid p_{\mathit{l}} < 0\}$. The next step is to prove that $\sum_{\mathit{l} \in L(p)} \Delta_{\mathit{l}}^{i}(m)^{i}(\,,p) \leq \frac{1}{2} W^{i}(p)$. For every $\mathit{l} \in L(p)$ we have $\Delta_{\mathit{l}}^{i}(m)^{i}(\,,p) =$

$$= D_{\mathit{l}}^{i}(m)^{i}(\,,p)_{\min} = \alpha^{i} I p_{\mathit{l}} \bar{\mu}_{\mathit{l}}^{i}(m)^{i}(\,,p) - \frac{\alpha^{i2} I p_{\mathit{l}}^{2}}{2\gamma(I-1)} - \frac{\gamma}{2} \bar{\sigma}_{\mathit{l}}^{i}(m)^{i}(\,,p)^{2}.$$

Obviously, the maximum of $D_{\mathit{l}}^{i}(m)^{i}(\,,p)_{\min}$ is obtained, if $\bar{\sigma}_{\mathit{l}}^{i}(m)^{i}(\,,p)^{2} = 0$ and $\bar{\mu}_{\mathit{l}}^{i}(m)^{i}(\,,p) = \hat{\mu}_{\mathit{l}}^{i}(p) := \frac{\alpha^{i} p_{\mathit{l}} I}{\gamma(I-1)}$ (Note that by claim 1 $\hat{\mu}_{\mathit{l}}^{i}(p) = \mu_{\mathit{l}}^{i}(m)^{i}(\,)$). Hence for every $p \in P$

$$\sum_{\mathit{l} \in L(p)} \Delta_{\mathit{l}}^{i}(m)^{i}(\,,p) \leq \frac{I(I-\frac{1}{2})\, \alpha^{i2} \sum_{\mathit{l} \in L(p)} p_{\mathit{l}}^{2}}{\gamma(I-1)} < \frac{I^{2}}{\gamma(I-1)} .$$

By claim 3 and the definition of γ^{*} we obtain $\frac{I^{2}}{\gamma(I-1)} \leq \frac{1}{2} W^{i}(p)$ for every $\gamma \geq \gamma^{*}$. This proves claim 4.

From claim 3 and 4 we know that $p \cdot e = 0$. The proof of Lemma 2 is completed, if it is shown that $e = 0$. By the definition of equilibrium for Γ it follows that $0 = p \cdot e \geq p' \cdot e$ for every $p' \in P$. Suppose that $e \neq 0$. Then there is $\bar{p} \in P$ such that $\bar{p} \cdot e < 0$. But $(-\bar{p}) \in P$ and $(-\bar{p})e > 0$. Contradiction.

Q.E.D.

(5) Finally, it remains to show that z is an equilibrium for the (original and unbounded) economy $\mathcal{E}$ relative to the government G_{γ}, if z is an equilibrium for $\hat{\mathcal{E}}$ relative to $\hat{G}_{\gamma}$. In the proof of Lemma 2 it is established that $e = 0$. Hence $((x^{i}, S(r^{g})), (y^{j}))$ is an attainable allocation for $\mathcal{E}$. It is also easy to see from claim 1 in Lemma 2 and from the definition of $\hat{M}$ that for every i $m^{i} \in \text{Int}\, \hat{M}$. With these results the proposition of step 5 follows by standard arguments.

This completes the proof of Theorem 1.

Some remarks concerning the significance of the γ^{*}- condition in Theorem 1 seem to be in order: Techniqually speaking, the lower bound on γ is (only) a sufficient condition to ensure existence, which need not indicate a basic property of the government. At least immediate economic interpretations could not be found. In order to choose an appropriate value of γ the government must be assumed to have information on the set of attainable allocations. It is important, however, that the government need not have any information on consumers' preferences;[14)] the lack of this information is, of course, the fundamental presupposition for the free rider problem and the construction of the incentive mechanism. Furthermore, even without information on the set of attainable allocations the government can select a suitable γ by a trial-and-error procedure. Choosing any arbitrary increasing sequence of parameters γ the government will eventually satisfy Theorem 1.

IV. Optimality and Unbiasedness

We define a competitive allocation in $\mathcal{E}$ relative to the government G as an allocation $((x^i), (y^j))$ when there exist messages (m^i) and a price system p such that $((x^i, r^i, m^i), (y^j), p)$ is a competitive equilibrium in $\mathcal{E}$ relative to the government G . Further, an allocation $((x^i), (y^j))$ in $\mathcal{E}$ is called Pareto-efficient if (i) it is attainable and (ii) there does not exist another attainable allocation $((\hat{x}^i),(\hat{y}^j))$ such that $(\hat{x}^i, S(\hat{r})) \succsim_i (x^i, S(r))$ for $i = 1,\dots,I$ and $(\hat{x}^h, S(\hat{r})) \succ_h (x^h, S(r))$ for some consumers h. (r and $\hat{r}$ are the residuals allocations associated to $((x^i), (y^j))$ and $((\hat{x}^i), (\hat{y}^j))$, respectively).

The two Fundamental Theorems of Welfare Economics assert for a private ownership economy (without public goods) that under suitable conditions (i) every competitive allocation is Pareto-efficient and (ii) every Pareto-efficient allocation is competitive for some initial distribution of endowments and profit shares (See [8, p.90n]). It will be shown in this section that these two Fundamental Welfare Theorems hold for the pollution economy $\mathcal{E}$ with the government G .

Theorem 2: (Optimality) Let $\mathcal{E}$ be an economy satisfying the following conditions for every i = 1,...,I:

(a) (continuity of preferences) for every $(x^i,s) \in \mathfrak{X}^i$ the sets $\{(\bar{x}^i,\bar{s}) \in \mathfrak{X}^i \mid (\bar{x}^i, \bar{s}) \succcurlyeq_i (x^i, s)\}$ and $\{(\bar{x}^i, \bar{s}) \in \mathfrak{X}^i \mid (x^i,s) \succcurlyeq_i (\bar{x}^i,\bar{s})\}$ are closed in $\mathfrak{X}^i$,

(b) (convexity of the derived consumption set $\mathfrak{X}^i_R$ and preferences) $\mathfrak{X}^i_R \equiv X^i \times \mathbb{R}^L_+$ is convex, and if $(\bar{x}^i,\bar{s}),\ (x^i,s) \in \mathfrak{X}^i,\ (\bar{x}^i,\bar{s}) \succ_i (x^i,s)$, then $(\lambda\bar{x}^i + (1-\lambda)x^i,\ \lambda\bar{s} + (1-\lambda)s) \succ_i (x^i,s)$ for every $\lambda \in (0,1)$.

If $((x^i, r^i, m^i), (y^j),p)$ is a competitive equilibrium relative to the government $G = \{M, r^g(\cdot), (m^i(\cdot),c^i(\cdot))\}$ such that, for every i

(c) (non-satiation) there exists $(\bar{x}^i,\bar{s}) \in [X^i \times S(\mathbb{R}^L_+)] \cap \mathfrak{X}^i$ such that $(\bar{x}^i,\bar{s}) \succ_i (x^i,S[r^g(m)])$, and

(d) (no minimum wealth) there exists $(\bar{x}^i, \bar{r}^i, \bar{m}^i)$ such that $(\bar{x}^i, S(\bar{r}^i)) \in \mathfrak{X}^i$ and $p\cdot(\bar{x}^i - A^i\bar{x}^i) + c^i(\bar{r}^i, m^{)i(},p) < p\cdot(x^i - A^ix^i) + c^i(r^i,m^{)i(},p)$,

then the competitive allocation $((x^i), (y^j))$ is a Pareto-efficient allocation for $\mathcal{E}$.

Proof: Theorem 2 is proved as Theorem 4.1 in [13, p. 27 n.] using in addition the relationship between $\succcurlyeq_i$ and the derived preference relation $\succcurlyeq_{z_i}$ as introduced in the proof of Theorem 1.

Q.E.D.

The Second Fundamental Theorem of Welfare Economics will be proved with the help of Lemma 3. In many, but not all aspects the procedure will be similar than that in [13, pp. 29-36].

<u>Lemma 3</u>: Let $((x^i),(y^j))$ (with the residuals allocation $r = \Sigma_j y^j - \Sigma_i(x^i - A^i x^i - \omega^i))$ be a Pareto-efficient allocation for the economy $\mathcal{E}$. Let $\mathcal{E}$ satisfy, for every i, conditions (a) (continuity) and (b) (convexity) of Theorem 2, (c) $Y \equiv \Sigma_j Y^j$ is non-empty and convex, (d) for some consumer h there is $(\hat{x}^h,\hat{s}) \in [X^h \times S(\mathbb{R}^L_+)] \cap \mathfrak{X}^h$, such that $(\hat{x}^h,\hat{s}) >_h (x^h,S(r))$ and (e) (x^i,s), $(x^i,\bar{s}) \in \mathfrak{X}^i$ and $s \geqslant \bar{s}$ implies $(x^i,\bar{s}) \succcurlyeq_i (x^i,s)$ (monotonicity of preferences in pollutants).

(i) Then there is a vector $\tau^* = (p^*,t^{1*},\ldots,t^{I*}) \in \mathbb{R}^{L+IL}$, $\tau^* \neq 0$, such that

(1) $\Sigma_i t^{i*} = p^*$, where for every i and l, $t^i_l = 0$, if $p^*_l = 0$,

(2) y^j maximizes $p^* \bar{y}^j$ on Y^j

(3) $(\bar{x},S(\bar{r})) \succcurlyeq_i (x^i,S(r))$ implies $p^* \cdot (\bar{x}^i - A^i \bar{x}^i) + t^{i*} \cdot \bar{r}$ $\geqslant p^* \cdot (x^i - A^i x^i) + t^{i*} \cdot r$ for every i.

(ii) Then there exist messages $m^{i*} \in M$ for every i such that

(1) $\Sigma_h m^{h*} = r$

(2) $(\bar{x}^i,S(\bar{r}^i)) \succcurlyeq_i (x^i,S(r))$ implies $p^* \cdot (\bar{x}^i - A^i \bar{x}^i) + C^i(\bar{r}^i, m)^{i(*},p^*) \geqslant p^* \cdot (x^i - A^i x^i) +$ $+ C^i(r,m)^{i(*},p^*)$ for every i.

(iii) Let (m^*,p^*) be as in Lemma 3 (i) and (ii). Then there exist profit share distributions (θ^{ij*}) and initial endowments (ω^{i*}) such that $\Sigma_i \omega^{i*} = \Sigma_i \omega^i$ with the property that
$p^* \cdot (x^i - A^i x^i) + C^i(r,m)^{i(*},p^*) = p^* \cdot \omega^* + \Sigma_j \theta^{ij*} p^* \cdot y^j$.

<u>Proof</u>: (i) We define, similarly as in [9] and [13],

$$F^f \equiv \{(\Sigma_j \bar{y}^j - z, z^1,\ldots,z^I) \in \mathbb{R}^{L+IL} \mid z^i = z^h = z \in \mathbb{R}^L_+ \text{ for every } i,h = 1,\ldots,I, \text{ and } \Sigma_j \bar{y}^j \in Y\} \text{ and}$$

$F^c \equiv \{(\Sigma_i(\bar{x}^i - A^i\bar{x}^i), \bar{r}^1,\ldots,\bar{r}^I) \in \mathbb{R}^{L+IL} \mid \bar{r}^i = \bar{r}^h \equiv \bar{r} \in \mathbb{R}^L_+$

for every $i,h = 1,\ldots,I$; for every i $(\bar{x}^i, S(\bar{r})) \in \mathfrak{X}^i$,

$(\bar{x}^i, S(\bar{r})) \succsim_i (x^i, S(r))$ and for some h, $(\bar{x}^h, S(\bar{r})) \succ_h (x^h, S(r))\}$.

F^f is convex and non-empty by (d). F^c is convex by (a),(b), (c) and by the linearity of the function $\hat{z} \equiv x^i - A^i x^i$. F^c is non-empty by (b). Hence $G \equiv F^c - F^f$ is a non-empty convex subset of $\mathbb{R}^{L+IL}$. Define $\bar{\omega} \equiv (\Sigma_i \omega^i, 0,\ldots,0) \in \mathbb{R}^{L+IL}$. Since $((x^i), (y^j))$ is Pareto--efficient, $\bar{\omega} \notin G$. But by (a) and (b) $\bar{\omega} \in$ clos ure G. Thus, by Minkowski's Theorem (see [8, p. 25]) there is a hyperplane through $\bar{\omega}$ and bounding for G. Hence there exists a vector $\tau \equiv (p^*, t^1,\ldots,t^I) \in \mathbb{R}^{L+IL}$, $\tau \neq 0$, such that $\tau \cdot g > \tau \cdot \bar{\omega}$ for every $g \in G$. It follows from $\bar{\omega} \in$ closure G, that τ satisfies $\tau \cdot g \geqslant \tau \cdot \bar{\omega}$ for every $g \in$ closure G. Therefore $\bar{\omega}$ minimizes $\tau \cdot g$ on the closure G, and by (1) of Section 3.4 in Debreu [8, p. 45] we obtain

(α) $\tau \cdot (\Sigma_j y^j - r, r^1,\ldots,r^I) = \text{Max } \tau \cdot F^f$

and

(β) $\tau \cdot (\Sigma_i (x^i - A^i x^i), r^1,\ldots,r^I) = \text{Min } \tau \cdot \text{closure } F^c$.

Let T^α and T^β be the set of all τ' such that $p' = p^*$ and (α) and (β), respectively, hold for τ'. Clearly $\tau \in T^\alpha \cap T^\beta$. It is to show, however, that there is $\tau^* \in T^\alpha \cap T^\beta$ satisfying Lemma 3 (i.1).

We first prove that $\tau^* \in T^\alpha$. By (α) we have $\tau \cdot (\Sigma_j y^j - r, r^1,\ldots,r^I) = p^* \cdot \Sigma_j y^j + (\Sigma_i t^i - p^*) \cdot r$. We want to show that $(\Sigma_i t^i - p^*) \leqslant 0$. Suppose not. Then for some l $(\Sigma_i t^i_l - p^*_l) > 0$. But since for arbitrarily large vectors $\bar{r} > r$, $(\Sigma_j y^j - \bar{r}, \bar{r}^1,\ldots,\bar{r}^I) \in F^f$, Max $\tau \cdot F^f$ would not exist. Hence $(\Sigma_i t^i - p^*) \leqslant 0$. Now we consider the case where for some l $(\Sigma_i t^i_l - p^*_l) < 0$. It is easy to see that condition (α) and $\Sigma_i t^i_l - p^*_l < 0$ implies $r_l = 0$. Therefore

$\tau \cdot (\Sigma_j y^j - r, r^1,\ldots,r^I) = \tau' \cdot (\Sigma_j y^j - r, r^1,\ldots,r^I) = \text{Max } \tau' \cdot F^f$

for every $\tau' \in \hat{T}^{\alpha} \equiv \{(p',t^{1'},\ldots,t^{I'}) \in \mathbb{R}^{L+IL} \mid p' = p^{*}$ and for every l $\Sigma_i t_l^{i'} = p_l^{*}\} \subset T^{\alpha}$. Clearly, $\hat{T}^{\alpha} \neq \emptyset$.

We now consider the set T^{β}. By assumption (e) there is $\tilde{\tau} \in T^{\beta}$ such that $\tilde{t}^{i} \leq 0$ for every i. Hence by definition of T^{β} for every $\tau' \in T^{\beta}$ and every l it follows that "$r_l > 0$ implies $p_l^{*} \leq 0$" and "$p_l^{*} = 0$, $r_l > 0$ implies $t_l^{i'} = 0$ for every i". Since $\tau \in T^{\alpha} \cap T^{\beta}$ we have for every l such that $r_l > 0$ $(\Sigma_i t_l^i - p_l^{*}) = 0$. So it remains to consider the subset L^{o} of private goods for which $r_l = 0$ and $\Sigma_i t_l^i < p_l^{*}$ (where the components t_l^i belong to $\tau \in T^{\alpha} \cap T^{\beta}$). Let $\hat{T}^{\beta} \equiv \{\hat{\tau} \in \mathbb{R}^{L+IL} \mid$ for every i, $\hat{t}_l^i = t_l^i$, if $l \notin L^{o}$, and $\hat{t}_l^i \geq t_l^i$, if $l \in L^{o}$; $\hat{p} = p^{*}\}$. By (e) $\hat{T}^{\beta} \subset T^{\beta}$, and it is straightforward that $\hat{T}^{\alpha} \subset \hat{T}^{\beta}$. Hence there is $\tau^{*} \in (\hat{T}^{\alpha} \cap \hat{T}^{\beta}) \subset (T^{\alpha} \cap T^{\beta})$ satisfying Lemma 3 (i.1). Since (α) and (β) hold for $\tau = \tau^{*}$, the Lemmata 3 (i.2) and (i.3) follow easily.

(ii) For each i, let $m^{i*} = \frac{1}{I} r + \frac{1}{\gamma} (t^{i*} - \alpha^{i} p^{*})$. Then $\Sigma_i m^{i*} = r$, since $\Sigma_i t^{i*} = p^{*}$ by Lemma 3 (i.1). Lemma 3 (ii.2) follows from 3 (ii.1), if $p^{*} \cdot (\bar{x}^{i} - A^{i}\bar{x}^{i}) + t^{i*} \cdot \bar{r} \geq p^{*} \cdot (x^{i} - A^{i}x^{i}) + t^{i*} \cdot r$ implies $p^{*} \cdot (\bar{x}^{i} - A^{i}\bar{x}^{i}) + C^{i}(\bar{r},m)^{i(*}, p^{*}) \geq p^{*} \cdot (x^{i} - A^{i}x^{i}) + C^{i}(r,$ $m)^{i(*}, p^{*})$. , This statement is proved as in Lemma 4.1 in [13, p. 31 n].

(iii) Let $\theta^{ij*} = \frac{1}{I}$ for all i,j. Let $\omega^{i*} = x^{i} - A^{i}x^{i} - \frac{1}{I} \Sigma_j y^{j} + \frac{1}{I} r + [C^{i}(r,m)^{i(*}, p^{*}) - \frac{1}{I} p^{*} \cdot r] \cdot e$, where $e \equiv (e_1,\ldots,e_L)$, $e_l = 1/(p_l^{*} L_1)$ if $p_l^{*} \neq 0$, $e_l = 0$ if $p_l^{*} = 0$ and where L_1 is the number of private goods $l = 1,\ldots,L$ with non-zero prices. Since Lemma 3 (i.1) implies $p^{*} \neq 0$ we obtain $p^{*} \cdot e = 1$ and hence finally

$$p^{*} \cdot \omega^{i*} + \Sigma_j \theta^{ij*} p^{*} y^{j} =$$

$$= p^* \cdot (x^i - A^i x^i) - p^* \cdot \frac{1}{I} \Sigma_j y^j + p^* \cdot \frac{1}{I} r + [C^i(r,m)^{i(*}, p^*) - p^* \frac{1}{I} r]$$

$$p^* \cdot e + p^* \cdot \frac{1}{I} \Sigma_j y^j$$

$$= p^* \cdot (x^i - A^i x^i) + C^i(r,m)^{i(*}, p^*).$$

Q.E.D.

Lemma 3 (i.1) establishes an interesting property of the pollution economy which has no counterpart in the public goods model of Foley [9] and Groves-Ledyard [13]. In their models, any set of supporting individualized Lindahl prices $(t^1, \ldots, t^I)$ for public goods can be taken in order to define the desired market prices q for public goods as $q = \Sigma_i t^i$. This procedure is possible since private and public goods are different commodities. In the present model, however, every commodity l can simultaneously be a residual and a "regular" good and must have the same "value" (market price) in all uses.

<u>Theorem 3</u>: (Unbiasedness) Let $\mathcal{E}$ be an economy satisfying conditions (a), (b), (c) and (e) of Lemma 3. Let $((x^i), (y^j))$ be a Pareto-efficient allocation for $\mathcal{E}$. If there exists some private commodity, say $l = 1$, such that

(d.1) (monotonicity in commodity 1) for all i, $(\hat{x}^i, s) \in \mathfrak{X}^i$, $\bar{x}^i_1 > \hat{x}^i_1$, $\bar{x}^i_l = \hat{x}^i_l$ for $l = 2, \ldots, L$ imply $(\bar{x}^i, s) \in \mathfrak{X}^i$ and $(\bar{x}^i, s) \succ_i (\hat{x}^i, s)$, and

(d.2) (no minimum consumption) for all i, there is $(\hat{x}^i, \hat{s}) \in \mathfrak{X}^i$, such that $\hat{s} = S(r)$ and $\hat{x}^i_1 < x^i_1$, $\hat{x}^i_l = x^i_l$ for $l = 2, \ldots, L$, then there exist messages $m^{i*} \in M$ for every i and prices p^* such that $((x^i, r, m^{i*}), (y^j), p^*)$ is a competitive equilibrium relative to the government G, following if necessary a redistribution of initial endowments and profit shares.

<u>Proof</u>: 1. (d.1) implies (d) of Lemma 3. Hence the conclusions of Lemma 3 hold.

2. (d.1) implies that $(p_1^* - \Sigma_l p_l^* a_{l1}^i) > 0$ for every i. Let $F_i^c \equiv \{(\hat{x}^i,\hat{r}) \in (X^i \times \mathbb{R}_+^L) \mid (\hat{x}^i, S(\hat{r})) \succ_i (x^i, S(r))\}$. From the proof of Lemma 3 (1.3) it follows that $(p^*,t^{i*})\cdot(\hat{x}^i,\hat{r}) > (p^*,t^{i*})\cdot(x^i,r)$ for every $(\hat{x}^i,\hat{r}) \in F_i^c$. Let $(\bar{x}^i, S(\bar{r})) \in \mathfrak{X}^i$ such that $\bar{x}_l^i = x_l^i$ for $l = 2,\dots,L$, $\bar{x}_1^i > x_1^i$, and $S(\bar{r}) = S(r)$. Since $(\bar{x}^i,\bar{r}) \in F_i^c$ we have $p^*(\bar{x}^i - A^i\bar{x}^i) - p^*(x^i - A^i x^i) = (p_1^* - \Sigma_l p_l^* a_{l1}^i) > 0$.

3. Let $w^{i*}(p^*) = p^*(x^i - A^i x^i) + C^i(r,m)^{i(*},p^*)$ be the value of i's wealth after the redistribution according to Lemma 3 (iii). We want to show that $(\tilde{x}^i,\tilde{s}) \in \mathfrak{X}^i$, $(\tilde{x}^i,\tilde{s}) \succ_i (x^i,S(r))$, $\tilde{s} = S(\tilde{r})$ implies $p^*(\tilde{x}^i - A^i\tilde{x}^i) + C^i(\tilde{r},m)^{i(*},p^*) > w^{i*}(p^*)$. Suppose not. Then by Lemma 3 (ii.2) $p^*\cdot(\tilde{x}^i - A^i\tilde{x}^i) + C^i(\tilde{r},m)^{i(*},p^*) = w^{i*}(p^*)$. Further, (d.2) and 2 above imply that there is $\bar{x}^i$ such that $(\bar{x}^i, S(r)) \in \mathfrak{X}^i$, $p^*\cdot(\bar{x}^i - A^i\bar{x}^i) + C^i(r,m)^{i(*},p^*) < w^{i*}(p^*)$ and $r = \Sigma_h m^{h*}$. But by (d.1) $(\tilde{x}^i,\tilde{s}) \succ_i (\bar{x}^i,S(r))$. Let $F^i \equiv \{(x^{i'},r') \in \mathfrak{X}^i \mid (x^{i'},r') = (\lambda\bar{x}^i + (1-\lambda)\tilde{x}^i, \lambda r + (1-\lambda)\tilde{r})$ for all $\lambda \in [0,1]\}$. By (b) $F^i \subset (X^i \times \mathbb{R}_+^L)$ and by (a) there is a neighborhood N of $(\tilde{x}^i,\tilde{r})$ such that $(x^{i'},r') \in N \cap (X^i \times \mathbb{R}_+^L)$ implies $(x^{i'},S(r')) \succ_i (x^i,S(r))$. But $N \cap F^i \neq \emptyset$. This leads easily to a contradiction. Thus $(x^i,S(r))$ is maximal for $\succcurlyeq_i$ on $\{(\bar{x}^i,\bar{s}) \in \mathfrak{X}^i \mid \bar{s} = S(\bar{r})$ and $p^*\cdot(\bar{x}^i - A^i\bar{x}^i) + C^i(\bar{r},m)^{i(*},p^*) \leqslant p^*\cdot\omega^{i*} + \Sigma_j \theta^{ij*} p^*\cdot y^j\}$. This result when combined with Lemma 3 (i.2) and (ii.1) implies the desired conclusion.

Q.E.D.

Footnotes

1) Their models belong to the body of literature on the theory of public goods characterized by the contributions of Lindahl [18], Samuelson [21], and Foley [9].

2) For a comprehensive historic review on the problem of preference revelation for public goods seen Green and Laffont [10].

3) Hurwicz [14] showed that it is also not in a consumer's self-interest to reveal his true preferences for private goods. This additional aspect of the free rider problem will be ignored here.

4) See, for example, the different approaches to the revelation of preferences for environmental services in [19] and Mäler's negative conclusions. One group of economists while still in favour for the effluent charge solution sacrificed the efficiency objective in favour of informationally less demanding environmental price and standard systems, [3].

5) Groves and Ledyard achieved their allocation system at the cost of weakened incentive properties (see [13, p. 22], in particular footnote 20). Hurwicz [15] calls the Nash equilibrium in the Groves-Ledyard model nonmanipulative and the associated allocation incentive compatible in a wide sense. For the case which he interpretes to be the "classical issue of truthful revelation of preferences" (incentive compatibility in the narrow sense) he states impossibility results for economies with 3 agents [15, p.4]. For a limit theorem on preference revealing mechanisms with a balanced budget see Green and Laffont [10, pp. 51 n.]. A different approach to balance the budget is proposed by d'Aspemont and Gerard-Varet [2].

6) The relevance of material balance considerations for environmental economics is demonstrated in [16].

7) In general, consumer i may be able to transform a given consumption bundle in various different ways, when the goods contained in this consumptions bundle are distinguished only by their physical characteristics. We can, however, admit this possibility and still maintain uniqueness of the consumption transformation function by treating physically identical consumption inputs as different private goods if they are used differently in some consumers' consumption process. A similar function is applied by Mäler [19, p. 30], but he incorrectly argues that in a model without an a priori participation of "useful" goods and residuals such a consumption transformation function is unnecessary [19,p.47]

8) The function C^i is equal to the function $\hat{C}^i$ in Groves and Ledyard [13, p. 27] (defined in the proof of Theorem 4.1). Note, however, that the domain of C^i contains quantities and prices of (all) private goods, whereas in the domain of $\hat{C}^i$ we have quantities and prices of public goods.

9) Since the consumer's preferences are defined on private consumption goods and pollutants the definition of the budget set in terms of residuals presupposes that every consumer has complete information on the pollution function S. To avoid this severe informational requirement (see also [19,]) the Groves-Ledyard mechanism would have to be directly used for the allocation of pollutants instead of residuals (with the hope that only the government needs information on S). However, the additional difficulties of such an approach could not be satisfactorily solved.

10) The existence proof which is presented by Groves and Ledyard in [12], confirms these difficulties, although their tax rule in [12] is different from that in [13]. In their comments on the existence problem in [13, pp. 37-39] which are based on an unpublished manuscript, the authors describe the same difficulties. But their remarks also indicate that their approach to the existence question differs considerably to that presented here.

11) In this sense all preference revealing governments are vacuous which do not exhibit the balanced budget property. Furthermore, Groves and Ledyard [12, p. 20n.] demonstrate that even when this condition holds there are preference revealing governments which are compatible with equilibrium only in exceptionally rare and uninteresting economies. Thus they correctly emphasize that optimality results should be accepted only if a demonstration of non-vacuousness is made.

12) The device to distort the budget constraint in the definition of $K^{i}(z^{)i(})$ by $(1 - \|p\|)$ is due to Bergstrom [4]. It is shown in Claim 3 of Lemma 2 that this term vanishes in an equilibrium of Γ.

13) Lemma 1 is a special case of the Debreu's lemma in [7]. Recently, Debreu's lemma has been modified by Shafer and Sonnenschein [22] such that it can be used to prove the existence of equilibrium in economies with interdependent preferences, price dependent preferences, and preferences which may be both nontransitive and noncomplete.

14) In [13, p. 37-39], Groves and Ledyard indicate that they introduced two restrictions on preferences to ensure existence in the case of unrestricted positive parameters γ.

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