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EQUILIBRIUM IN AN ECONOMY WITH CHANGEABLE PREFERENCES*

by

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Preference orderings of individual agents in an economy are commonly taken to be exogenous not subject to change through the interaction between the agents or between an agent and the economic environment. Notable exceptions to this rule are studies by Gintis [1974] and von Weizsäcker [1971], who argue that past experiences, such as past consumption and education affect the preference orderings of the agents. Others, like Duesenberry [1949], Gaertner [1974], and Galbraith [1958], argue that community behavior influences individual behavior like the desire to "keep up with the Joneses".

In this study we construct a model of individual behavior where the agents actively try to alter each other's preferences to their own benefit. Examples of such behavior are political and religious propoganda and advertisement. In part I we define an (abstract) economy; in Part II, under suitable assumptions, we show the existence of a Nash equilibrium in the signals exchanged between the agents.

I. AN ECONOMY

An (abstract) economy is a collection

$$\mathfrak{E} \equiv (\mathcal{E}, \mathcal{T}, \mathcal{K}, \mathfrak{N})$$

where $\mathcal{E}$ is the economic environment, $\mathcal{T}$ is the technology, $\mathcal{K}$ is the constitution, and $\mathfrak{N}$ the set of economic agents. There are n agents and we will denote agents by a, b, c etc..

i. The Environment

The economic environment is the collection

$$\mathcal{E} \equiv (\Xi, \Omega, \Sigma, \Psi, \omega^o, \sigma^o)$$

where Ξ is the action space, Ω is the endowment space, Σ is the preference space, Ψ is the message space, ω^o is the initial endowment, and σ^o is the initial preference orderings.

α. The action space $\Xi \subset \mathbb{R}^k$ is the set of all possible (social) actions over which an agent has preferences. In the case of a private goods economy Ξ decomposes into the product of n sets; one for each agent.

$$\Xi = \Xi_1 \times \Xi_2 \times \cdots \times \Xi_n$$

Elements of Ξ and Ξ_a are denoted by ξ and ξ_a respectively.

β. The endowment space $\Omega \subset \mathbb{R}^{\ell}$ is the set of all possible (social) endowments. We will assume that Ω is decomposable into the product of private endowments Ω_a, one for each agent, and collective endowments $\Omega_{\{\ \}}$, over which the agents can only decide jointly:

$$\Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n \times \Omega_{\{\ \}}$$

Elements of Ω, Ω_a and $\Omega_{\{\ \}}$ are denoted by ω, ω_a and $\omega_{\{\ \}}$ respectively.

γ. The <u>preference space</u> $\Sigma \subset 2^{\Xi \times \Xi}$ is the set of all complete, representable preorderings on the action space Ξ. Representable is here taken to mean that for all $\sigma \in \Sigma$, the sets

$$L_\sigma(\xi_o) \equiv \{\xi \in \Xi \mid \xi_o \sigma \xi\} \subset \Xi$$

$$\text{and } U_\sigma(\xi_o) \equiv \{\xi \in \Xi \mid \xi \, \sigma \, \xi_o\} \subset \Xi$$

are closed in Ξ for all $\xi_o \in \Xi$ (cf. Debreu [1959]). An element $\sigma \in \Sigma$ is interpreted as "is not inferior to".

We define a (Hausdorf) topology on Σ by defining a metric ρ_Σ on Σ.

<u>Definition 1</u>: The Σ distance $\rho_\Sigma(\sigma_1,\sigma_2)$ between two preference orderings $\sigma_1, \sigma_2 \in \Sigma$ is defined to be

$$\text{l.u.b. } \{\rho(U_{\sigma_1}(\xi), U_{\sigma_2}(\xi)) \mid \xi \in \Xi\}$$

where, for $X_1, X_2 \subset \Xi$,

$$\rho(X_1,X_2) \equiv d(X_1,X_2) + d(X_2,X_1),$$

$$d(X_1,X_2) \equiv \text{l.u.b.}\ \{\text{g.l.b.}\{d(x_1,x_2) \mid x_2 \in X_2\} \mid x_1 \in X_1\}$$

and $d(x_1,x_2)$ is the Euclidean distance between x_1 and x_2 in Ξ

It is easily shown that ρ_Σ defines a metric, and hence a (Hausdorf) topology, on Σ (cf. Wickström[1975]).

δ. The message space (or language) $\Psi \subset \mathbb{R}^m$ is the set of all possible messages that the agents can transmit. We denote by $\gamma_{ab} \in \Psi$ a message dispatched by agent a to agent b. A vector of messages

$$\gamma_{a\cdot} \equiv (\gamma_{a1}, \gamma_{a2}, \ldots, \gamma_{an}) \in \Psi^n$$

is called a <u>signal dispatched</u> by agent a, and a vector

$$\gamma_{\cdot b} \equiv \begin{pmatrix} \gamma_{1b} \\ \gamma_{2b} \\ \vdots \\ \gamma_{nb} \end{pmatrix} \in \Psi^n$$

is called a <u>signal addressed</u> to agent b. A matrix

$$\gamma_{\cdot\cdot} \equiv \begin{pmatrix} \gamma_{1\cdot} \\ \gamma_{2\cdot} \\ \vdots \\ \gamma_{n\cdot} \end{pmatrix} \equiv (\gamma_{\cdot 1}, \gamma_{\cdot 2}, \ldots, \gamma_{\cdot n}) \in \Psi^{n2}$$

is called a <u>communication</u>. We will assume that $\gamma_{aa} = \emptyset$ for all agents a.

ϵ. The <u>initial endowment</u> $\omega^o \in \Omega$ is exogenously given.

ζ. The <u>initial preference orderings</u> $\sigma^o \in \Sigma^n$ is a vector of exogenously given preference orderings, one for each agent:

$$\sigma^o \equiv (\sigma_1^o, \sigma_2^o, \ldots, \sigma_n^o), \ \sigma_a^o \in \Sigma.$$

ii. <u>The Technology</u>

The technology is the collection,

$$\mathcal{T} \equiv (T),$$

of one mapping T, from the endowment space into the powerset of Ξ :

$$T: \Omega \twoheadrightarrow \Xi \ .$$

$T(\omega) \subset \Xi$ is the set of all technically possible actions if the initial endowment is ω.

iii. <u>The Constitution</u>

The constitution $\mathcal{K}$ of the economy is the collection of n mappings, one for each agent,

$$\mathcal{K} \equiv (K_1, K_2, \ldots, K_n),$$

each from the n-fold product of the message space into the power set of itself:

$$K_a : \Psi^n \twoheadrightarrow \Psi^n .$$

Thus, $K_a(\overline{\psi}_{.a}) \subset \Psi^n$ is the set of signals that agent a is allowed to dispatch if he receives signal $\overline{\psi}_{.a}$.

iv. <u>The Agents</u>

The set of agents $\mathfrak{N}$ is the collection $\mathfrak{N} \equiv (\eta_1, \eta_2, \ldots, \eta_n)$ where η_a , agent a, is the collection

$$\eta_a \equiv (B_a, h_a, f_a, q_a, i_a, e_a).$$

B_a is the agent's choice-set mapping, h_a his preference mapping, f_a his message filter, q_a his information-cost mapping, i_a his internal decision rule and e_a his external decision rule.

α. The <u>choice-set mapping</u> maps an agent's private endowment and a signal received by him into the power set of Ξ determining the agent's perceived choice set:

$$B_a : \Omega_a \times \Psi^n \twoheadrightarrow \Xi .$$

Thus, $B_a(\omega_a, \overline{\psi}_{.a}) \subset \Xi$ is agent a's perceived choice set if the initial endowment is ω_a and the signal received by him is $\overline{\psi}_{.a}$. We will assume that B_a is compact-valued.

β. The <u>preference mapping</u>, h_a, maps agent a's initial preference ordering and the signal received by him into the preference space:

$$h_a: \Sigma \times \Psi^n \to \Sigma.$$

Thus, $h_a(\sigma_a^o, \bar{\psi}_{\cdot a}) \in \Sigma$ is agent a's current preference ordering if his initial preference ordering is σ_a^o and signal $\bar{\psi}_{\cdot a}$ is received by him.

γ. The <u>message-filter mapping</u> f_a maps the n-fold product of the message space into itself:

$$f_a: \Psi^n \to \Psi^n.$$

Thus, $f_a(\psi_{\cdot a}) = \bar{\psi}_{\cdot a}$ is the signal received by agent a if the signal $\psi_{\cdot a}$ is addressed to him.

δ. The <u>information-cost mapping</u>, q_a, maps a signal dispatched and the initial endowment into the private endowment space:

$$q_a: \Omega \times \Psi^n \to \Omega_a$$

Thus, $q_a(\omega^o, \psi_{a\cdot}) = \omega_a \in \Omega_a$ is the private endowment agent a perceives if he sends out communication $\psi_{a\cdot}$ and the initial endowment is ω^o.

ε. The <u>internal decision rule</u> of agent a tells agent a which action in $B_a(\omega_a, \bar{\Upsilon}_{\cdot a})$ to choose if he sends out signal $\Upsilon_{a\cdot}$ and receives signal $\bar{\Upsilon}_{\cdot a}$. We will assume that, for any given $\Upsilon_{a\cdot}$, he chooses an action in $B_a(\omega_a, \bar{\Upsilon}_{\cdot a})$ that optimizes his preference ordering $h_a(\sigma_a^o, \bar{\Upsilon}_{\cdot a})$. We denote the set of such actions $C(B_a(\cdots), h_a(\cdots))$:

$$C_a(B_a(\omega_a, \bar{\Upsilon}_{\cdot a}), h_a(\sigma_a^o, \bar{\Upsilon}_{\cdot a})) \equiv \{\xi \in \Xi \mid \xi \in B_a(\omega_a, \bar{\Upsilon}_{\cdot a}) \text{ and}$$

$$\forall \xi' \in B_a(\omega_a, \bar{\Upsilon}_{\cdot a}), \ \xi \ h_a(\sigma_a^o, \Upsilon_{\cdot a}) \ \xi'\} \quad .$$

ζ. The <u>external decision rule</u> of agent a tells him which signal to dispatch to the other agents if he receives signal $\bar{\Upsilon}_{\cdot a}$. We will assume that he chooses a permitted signal that effects the economy in a manner beneficial to him. More specifically we note that his incoming signal $\bar{\Upsilon}_{\cdot a}$ will be indirectly dependent on his outgoing signal $\Upsilon_{a\cdot}$ in that $\Upsilon_{a\cdot}$ affects the other agents in several ways:

1^o, it helps determine the other's choice sets,

2^o, it helps determine the other's permitted signals,

3^o, it helps determine the other's preference orderings.

We assume, that, at each point in time, through some search process each agent, a, finds an optimal signal given the present signals of all the other agents and their reaction to a change in the signal transmitted by agent a.

η. We can represent an agent by a flow diagram as in Figure 1. Squares

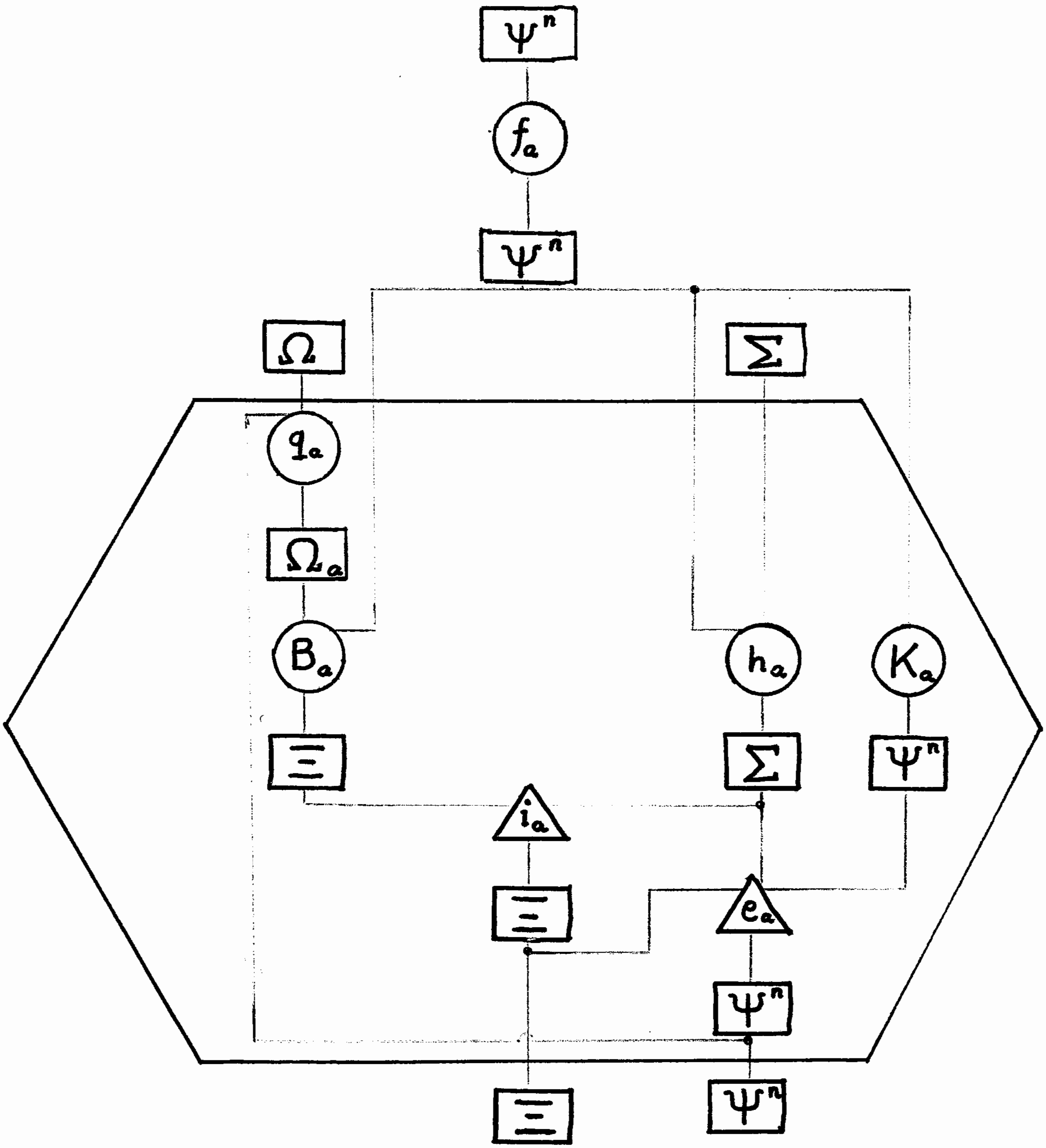

FIGURE 1.

here denote spaces, circles mappings, triangles decision rules and the hexagon denotes an agent. By making use of C_a an agent can be written as in Figure 2.

Agent a's interaction with the

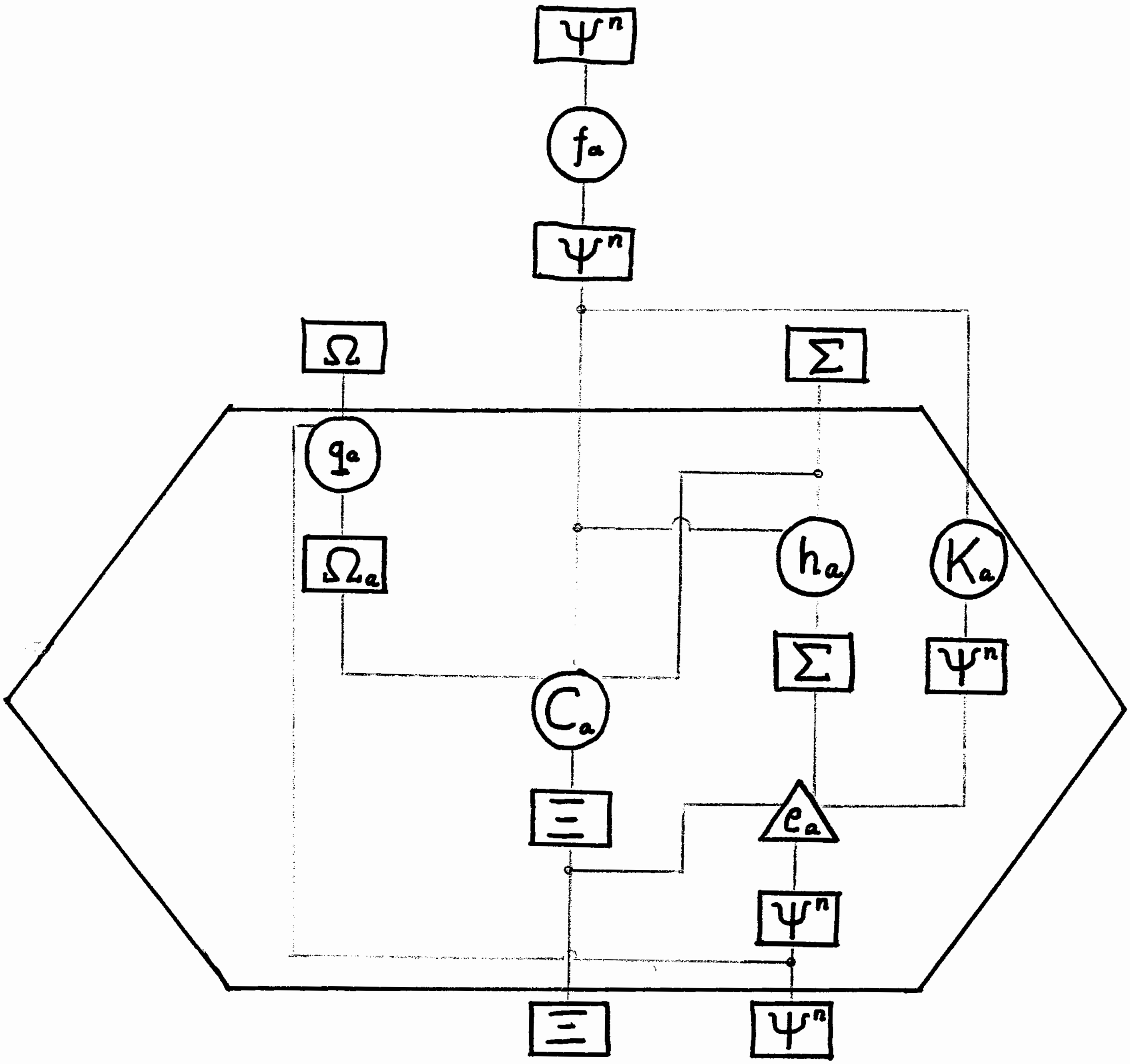

Figure 2.

rest of the economy can be represented as in Figure 3.

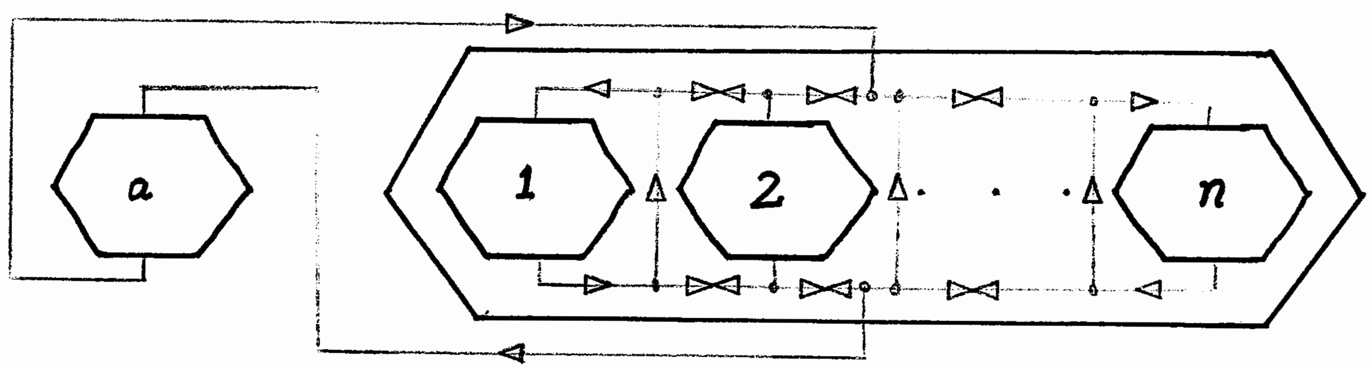

Figure 3.

The rest of the economy thus acts as a mapping, mapping a's outgoing signals into his incoming signals. Agent a's problem is thus to choose an outgoing signal $\psi_{a.}$ that will make his incoming signal $\bar{\psi}_{.a}$ as desirable as possible.

Thus the situation that we are faced with is that of a game (with limited information), where all agents try to find an optimal signal to transmit. In Part II we will show the existence of an equilibrium in the signals under a number of simplyfing assumptions.

II. EXISTENCE OF AN EQUILIBRIUM

i. We first make the following definitions:

Definition 1: A communication $\gamma^*_{..}$ is compatible if

$$C(\gamma^*_{..}, \sigma^o, \omega^o) \equiv \bigcap_{a=1}^{n} C_a(B_a(q_a(\omega^o, \gamma^*_{a.}), f(\gamma^*_{a.})), h_a(\sigma^o_a, f(\gamma^*_{a.}))) \neq \emptyset$$

Definition 2: A communication $\gamma^*_{..}$ is feasible if

$$C(\gamma^*_{..}, \sigma^o, \omega^o) \subset T\ ((q_a(\omega^o, \gamma^*_{a.})), \omega^o_{\{\ \}}).$$

Definition 3: A communication $\gamma^*_{..}$ is permitted if

$$\gamma^*_{a.} \in K_a(f_a(\gamma^*_{.a}))\ \forall a$$

Definition 4: A communication $\gamma^*_{..}$ is a potential equilibrium if it is compatible, feasible and permitted.

We denote the set of potential equilibria by $E(\sigma^o, \omega^o) \subset \Psi^{n^2}$.

ii. We decompose the message space into two components, Ψ_1 and Ψ_2, such that

$$\Psi^{n^2} = \Psi_1^{n^2} \times \Psi_2^{n^2},$$

and make the following assumptions:

<u>Assumption 1</u>: $\hat{\psi}_{\cdot a} \in \Psi^n$ and $\tilde{\psi}_{a\cdot} \equiv (\tilde{\psi}_{1a\cdot}, \tilde{\psi}_{2a\cdot}) \in K_a(f_a(\hat{\psi}_{\cdot a}))$

$$\Rightarrow (\psi_{1a\cdot}, \tilde{\psi}_{2a\cdot}) \in K_a(f_a(\hat{\psi}_{\cdot a}))\ \forall\ \psi_{1a\cdot} \in \Psi_1^n \ .$$

<u>Assumption 2</u>: $B_a(\omega_a, f_a(\hat{\psi}_{1\cdot a}, \hat{\psi}_{2\cdot a})) = B_a(\omega_a, f_a(\psi_{1\cdot a}, \hat{\psi}_{2\ a}))\ \forall\ \psi_{1\cdot a} \in \Psi_1^n.$

<u>Assumption 3</u>: $h_a(\sigma_a^o, f_a(\hat{\psi}_{1\cdot a}, \hat{\psi}_{2\cdot a})) =$

$$h_a(\sigma_a^o, f_a(\hat{\psi}_{1\cdot a}, \psi_{2\cdot a}))\ \forall\ \psi_{2\cdot a} \in \Psi_2^n \ .$$

<u>Assumption 4</u>: α) $q_a(\omega^o, (\hat{\psi}_{1a\cdot}, \hat{\psi}_{2a\cdot})) =$

$$q_a(\omega^o, (\hat{\psi}_{1a\cdot}, \psi_{2a\cdot}))\ \forall\ \psi_{2a} \in \Psi_2^n$$

β) $q_a(\omega^o, (0, \psi_{2a\cdot})) = \omega_a^o\ \forall\ \psi_{2a} \in \Psi_2^n$

γ) $q_a(\omega^o, (\psi_{1a\cdot}, \psi_{2a\cdot})) \leqq \omega_a^o\ \ \forall\ \psi_{a\cdot} \in \Psi^n$

δ) $(q_a(\omega^o, (\cdot\ , \psi_{2a\cdot})))_j$ is quasiconcave $\forall j$

ε) $(q_a(\omega^o, (\psi_{1a\cdot}, \psi_{2a\cdot})))_j \to -\infty$ as $\|\psi_{1a\cdot}\| \to \infty$ for some j.

<u>Assumption 5</u>: $\omega_a \ngeqq 0 \Leftrightarrow B_a(\omega_a, \bar{\psi}_{\cdot a}) = \emptyset$

<u>Assumption 6</u>: There exists a continuous, single-valued mapping (Walras map),

$$W\colon\ \Sigma^n \times \Omega \to \Psi_2^{n^2} \ ,$$

which determines the communication of type two in the economy, given the endowment $\omega \in \Omega$ and the preferences $\sigma \in \Sigma^n$, such that

$$\alpha)\ \emptyset \neq \bigcap_{a=1}^{n} C_a(B_a(\omega_a, f(\Upsilon_{1 \cdot a}, W_{\cdot a}(\sigma,\omega))), \sigma_a) \subset T(\omega)$$

$$\beta)\ (\Upsilon_{1a \cdot}, W_{a \cdot}(\sigma,\omega)) \in K_a(f_a(\Upsilon_{1 \cdot a}, W_{\cdot a}(\sigma,\omega)))$$

<u>Remark:</u> This assumption is plausible in view of assumptions 1 and 2.

<u>Assumption 7</u>: $B_a(\omega_a, \bar{\Upsilon}_{\cdot a})$ is a continuous correspondence.

<u>Assumption 8</u>: $h_a(\sigma_a^o, \bar{\Upsilon}_{\cdot a})$ is a continuous function.

<u>Assumption 9</u>: $q_a(\omega^o, \Upsilon_{a \cdot})$ is a continuous function.

<u>Assumption 10</u>: $f_a(\Upsilon_{\cdot a})$ is a continuous function.

Assumption 2 states that B_a only depends on one kind of signals, type two, we can call those coercive or pricelike; assumption 3 on the other hand states that h_a only depends on the other kind of signals, type one, we can call those persuasive or propaganda like. Assumption 1 tells us that the coercive signals are restricted by the constitution, whereas persuasive are not. On the other hand assumption 4 states that persuasive signals are costly whereas coercive are not. Assumption 4 also says that the cost is dependent on the intensity (or absolute value) of the persuasive signal. I.e. the interpretation we have in mind is that the louder or the more intensly we persuade, the costlier it will be. This together with assumption 5 implies that the set of signals compatible with a potential equilibrium is bounded.

Assumption 6 states that the Walras correspondence is single-valued and continuous, a very strong assumption. For a further discussion of this see Dierker [1974]. Assumptions 7 through 10 are straightforward and require no further discussion.

iii. We need the following lemmata:

<u>Lemma 1:</u> Under assumptions 4 and 9, the set

$$S(\sigma^o,\omega^o) \equiv \mathop{\times}\limits_{a=1}^{n} S_a(\sigma^o,\omega^o) \equiv$$

$$\mathop{\times}\limits_{a=1}^{n} \{\mathcal{T}_{1a\cdot} \in \Psi_1^n \mid q_a(\omega^o,(\mathcal{T}_{1a\cdot},\mathcal{T}_{2a\cdot})) \geqq 0\}$$

is well defined,compact and convex. Further,under assumptions 1,2,4 and 5 $S_a(\sigma^o,\omega^o)$ is the set of outgoing signals from agent a compatible with a potential equilibrium.

<u>Proof:</u> Trivial

<u>Lemma 2</u>: For any fixed σ^o,ω^o and $\mathcal{T}_{1\cdot\cdot}$, given assumptions 3 and 4, the composition $Wo(h_1,\ldots,h_n,q_1,\ldots,q_n) \equiv Wo(h,q)$ is constant and thus has a fixed point,

$$\Upsilon^*_{2..} = W_{..}(h(\sigma^o,\Upsilon_{1..},\Upsilon^*_{2..}),q(\omega^o,\Upsilon_{1..},\Upsilon^*_{2..}))$$

Proof: Trivial.

Lemma 3: This fixed point as a function of σ^o,ω^o and $\Upsilon_{1..}$ is continuous and single valued given assumption 6.

Proof: Trivial.

We write the mapping in lemma 3 as $\hat{W}$:

$$\hat{W}:\Sigma^n \times \Omega \times \Psi_1^{n^2} \rightarrow \Psi_2^{n^2}$$

Lemma 4: For given and fixed σ^o and ω^o the mapping from $\Psi_1^{n^2}$ into $\Psi_2^{n^2}$, $\hat{W}(\sigma^o, \omega^o)$ is such that $\hat{W}(\sigma^o, \omega^o) \subset E(\sigma^o, \omega^o)$ given assumptions 3, 4 and 6.

Proof: Trivial.

iv. We state the following proposition:

Proposition 1: The correspondence $\tilde{C}_a(\sigma^o, \omega^o, \Upsilon_{.a}, \Upsilon_{a.}) \equiv$

$$\equiv C_a(B_a(q_a(\omega^o, \Upsilon_{a.}), f_a(\Upsilon_{.a})), h_a(\sigma^o_a, f_a(\Upsilon_{.a})))$$

is upper hemi continuous given assumptions 7,8,9 and 10.

Proof: In appendix.

We define $\hat{C}_a(\sigma^o, \omega^o, \Upsilon_{1..}) \equiv$

$$\equiv \tilde{C}_a(\sigma^o, \omega^o, \Upsilon_{1.a}, \hat{W}_{.a}(\sigma^o,\omega^o,\Upsilon_{1..}), \Upsilon_{1a.}, \hat{W}_{a.}(\sigma^o, \omega^o, \Upsilon_{1..}))$$ and

make the following definition:

Definition 5: $F_a(\sigma^o,\omega^o,\gamma_{1)a(.}) \equiv \{\gamma_{1a.} \in S_a(\sigma^o, \omega^o) | \hat{C}_a(\sigma^o,\omega^o,\gamma_{1..})$

$h_a(\sigma^o_a, \gamma_{1\cdot a}, \hat{W}_{\cdot a}(\sigma^o,\omega^o,\gamma_{1..}))$

$\hat{C}_a(\sigma^o,\omega^o,(\gamma_{1)a(.},\hat{\gamma}_{1a.})) \;\forall \hat{\gamma}_{1a.} \in S_a(\sigma^o,\omega^o)\}$

Proposition 2: $F_a(\sigma^o,\omega^o,\gamma_{1)a(.})$ is an upper hemi continuous correspondence in $\gamma_{1)a(.}$ given assumptions 3,4,5,6,7,8,9 and 10.

Proof: In appendix.

$F_a(\cdots)$ gives agent a's optimal signal given all other agents' strategies. I.e.,agent a would under the external decision rule like to choose a signal in $F_a(\cdots)$. It is clear that $F_a(\cdots)$ is not in general single valued,nor is it convex valued. The existence of a Nash equilibrium in the persuasive signals requires the existence of a fixed point of the mapping $\overset{n}{\underset{a=1}{\times}} F_a(\cdots) \equiv F(\gamma_{1..})$. This is in general true if $F_a(\cdots)$ is convex valued. Therefore a hypothesis requiring the agents to choose signals from the convex hall of $F_a(\cdots)$is needed. A possible economic interpretation of such a policy is that the agent at all times chooses the optimal signal given his current preferences,but that the response to the persuasive signals is "slow" and therefore the coercive signals change "slowly" reacting to the "average" signal over a long time period.

It is clear that such a policy is non-optimal in general as the following example (figure 4) indicates:

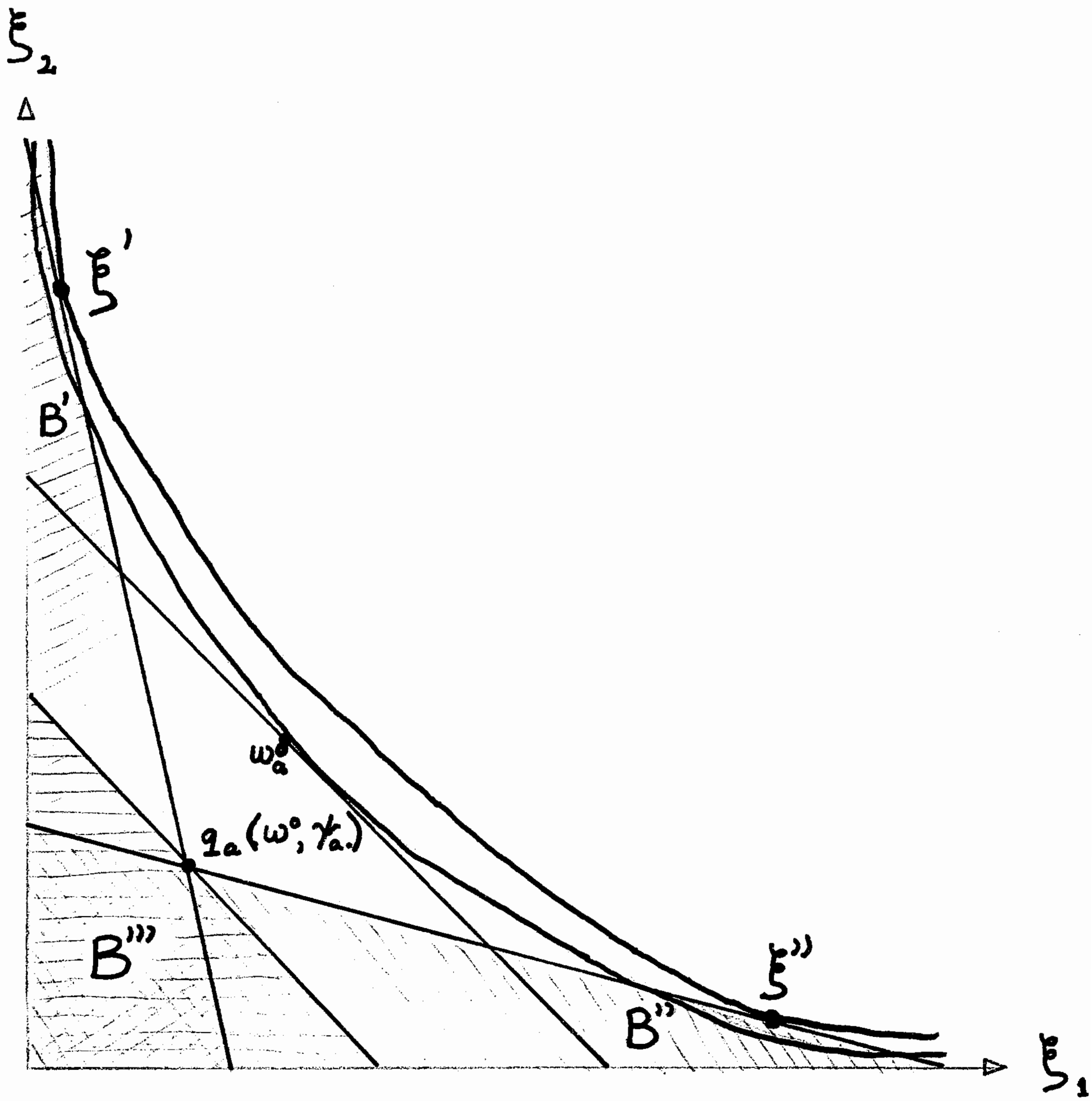

FIGURE 4.

ω_a^o is agent a's initial endowment; $q_a(\omega^o, \gamma_{a.})$ is his endowment if he transmits signal $\gamma_{a.}'$ or $\gamma_{a.}''$. If he transmits signal $\gamma_{a.}'$ he can in the long run count on encountering budget set B' and take action ξ'. If, on the other hand, he transmits signal $\gamma_{a.}''$ he can count on encountering budget set B'' and take action ξ''. He is at this point indifferent between ξ' and ξ''. Let us assume that he transmits signal $\gamma'_{a.}$; he will then in the beginning end up at $q_a(\omega^o, \gamma_{a.})$ with budget set B'''; however prices will slowly shift in the direction of budget set B' as a result of the preference changes in the agents receiving his signals. However, at the same time these other agents might alter their persuasive signals to agent a, thus shifting his preferences so that he now prefers ξ'' to ξ', whereupon agent a changes his strategy and transmits signal $\gamma''_{a.}$, which is now the longrun optimal signal. The price like signals will again change and agent a's budget set will change in direction B''. The other agents will again alter their persuasive signals changing a's preferences back so that he now prefers ξ' to ξ'' causing him to alter his strategy anew.

We thus encounter a situation where, from a's point of view, the optimal longrun strategy at any instant keeps shifting causing the agent to behave suboptimally; i.e. taking actions in the vicinity of $q_a(\omega^o, \gamma_{a.})$. It is importnat to note that a prerequisite for such a behavior is the assumption that the agent does not realize that his own preferences change as an indirect result of his actions. How well this picture fits real-life institutions is left for the reader to decide. We state the following theorem:

<u>Theorem 1</u>: If the persuasive signal transmitted by each agent a, $\gamma_{1a.}$ given the persuasive signals transmitted by all other agents, $\gamma_{1)a(.}$ is in the set

$$F_a(\sigma^o,\omega^o,\Upsilon_{1)a(\cdot}) \equiv \text{convex hull } (F_a(\sigma^o,\omega^o,\Upsilon_{1)a(\cdot}))$$

then there exists a Nash equilibrium in the persuasive signals.

Proof: Follows directly from Kakutani's fixed point theorem.

APPENDIX

Proof of proposition 1 (page 16) :

We need to show that $C_a(\sigma^o,\omega^o,\Psi_{\cdot a},\Psi_{a\cdot})$ has a closed graph. Denote $(\sigma^o,\omega^o,\Psi_{\cdot a},\Psi_{a\cdot})$ with x; with an obvious change in notation we write $\xi \in \tilde{B}_a(x)$, $\sigma = \tilde{h}_a(x)$, $\xi \in \tilde{C}_a(x)$. Let $\{x^\nu\}$, $\{\xi^\nu\}$ and $\{\sigma^\nu\}$ be sequences converging to x^*, ξ^* and σ^* such that

$$\sigma^\nu = \tilde{h}_a(x^\nu) \quad \text{and} \quad \xi^\nu \in \tilde{C}_a(x^\nu).$$

We need to show that

$$\xi^* \in \tilde{C}_a(x^*).$$

We know from continuity that $\xi^* \in \tilde{B}_a(x^*)$. Pick $\xi' \in \tilde{B}_a(x^*)$ such that $\xi' \sigma^* \xi^*$, i.e. $\xi' \in U_{\sigma^*}(\xi^*)$.

We need to show that $\xi^* \in U_{\sigma^*}(\xi')$ or $\xi' \in L_{\sigma^*}(\xi^*)$.

By continuity of $\tilde{B}_a$ we know that, for all $\epsilon > 0$, there exists a number K_ϵ such that, for all $\nu > K_\epsilon$, there exists a $\xi_{1/\epsilon} \in \epsilon(\xi')$ such that $\xi_{1/\epsilon} \in \tilde{B}_a(x^\nu)$. Here $\epsilon(\xi')$ is an ϵ-neighborhood of ξ'.

Since $\xi^\nu \in \tilde{C}_a(x^\nu)$ we know that $\xi^\nu \in U_{\sigma^\nu}(\xi_{1/\epsilon})$ for $\nu > K_\epsilon$. Continuity of $\tilde{h}_a$ implies that $\xi^* \in U_{\sigma^*}(\xi_{1/\epsilon})$ for all $\epsilon > 0$. I.e. $\xi_{1/\epsilon} \in L_{\sigma^*}(\xi^*)$ for all $\epsilon > 0$. Let $\epsilon = \frac{1}{n}$ for $n = 1,2,\ldots,$ then $\xi_{1/\epsilon}$ converges to ξ' and because of closedness of $L_{\sigma^*}(\xi^*)$

$$\xi' \in L_{\sigma^*}(\xi^*), \text{ q.e.d.}$$

Proof of Proposition 2 (page 17)

Neglecting the constant variables σ^o and ω^o we write with an obvious change in notation

$$y \in \tilde{F}_a(x) \quad \text{with} \quad x = \psi_{1)a(\cdot} \quad \text{and} \quad y = \psi_{1a\cdot} \quad ,$$

$$\sigma = \tilde{h}_a(x),$$

$$\xi \in \tilde{C}_a(x,y).$$

Let $\{x^\nu\}$, $\{\sigma^\nu\}$, $\{\xi^\nu\}$ and $\{y^\nu\}$ be sequences converging to x^*, σ^*, ξ^* and y^* respectively, such that

$$y^\nu \in \tilde{F}_a(x^\nu),\ \sigma^\nu = \tilde{h}_a(x^\nu) \ \text{and} \ \xi^\nu \in \tilde{C}_a(x^\nu, y^\nu).$$

We need to show that $y^* \in \tilde{F}_a(x^*)$. Let $y' \in \tilde{F}_a(x^*)$, i.e.

$$\tilde{C}_a(x^*, y')\ \tilde{h}_a(x^*)\ \tilde{C}_a(x^*, y^*),$$

(where $\tilde{C}_a(x,y)$ stands for $\xi \in \tilde{C}_a(x,y)$ as on page 17). We also know that

$$\tilde{C}_a(x^\nu, y^\nu)\ \tilde{h}_a(x^\nu)\ \tilde{C}_a(x^\nu, y').$$

By continuity of $\tilde{h}_a$ and upper semicontinuity of $\tilde{C}_a$ it is easily shown that

$$\tilde{C}_a(x^*, y^*)\ \tilde{h}_a(x^*)\ \tilde{C}_a(x^*, y')$$

Thus $y' \in \tilde{F}_a(x^*)$ implies that $y^* \in \tilde{F}_a(x^*)$,

q.e.d.

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