DISCUSSION PAPER NO. 196

ON EXPECTED PRESENT VALUE VS. EXPECTED FUTURE VALUE: FURTHER REMARKS

by

Elisha A. Pazner*/ and Assaf Razin**/

January 30, 1976

*/ Northwestern University, Evanston, Illinois

**/ Tel Aviv University, Tel-Aviv, Israel

January 1976

ON EXPECTED PRESENT VALUE VS. EXPECTED FUTURE VALUE: FURTHER REMARKS

by

Elisha A. Pazner and Assaf Razin

I. INTRODUCTION

The purpose of this note is to elaborate on our casual claim in [5] that the expected present value (EPV) criterion is more investment promoting than the expected future value (EFV) criterion. While the major purpose in [5] was to illustrate that in the presence of interest rate uncertainty the ranking of investment projects is not invariant with respect to the criterion used, we wish now to specify general conditions under which the above mentioned claim is valid. Such clarification seems to be called for in light of a recent comment on our earlier paper (see [1]).

II. COMPARING THE INVESTMENT PROMOTING NATURE OF THE TWO CRITERIA

Consider a standard project evaluation model with a stream of net-benefits $\{B_t\}$, $t = 0,\ldots,T$ where T is the terminal (endogenously determined or not) date of the project. Letting the interest rate r be a random variable, we now define the EPV and EFV concepts:

$$(1) \qquad EPV = E\left[\sum_{t=0}^{T}\prod_{i=o}^{t}\frac{1}{1+r_i}B_t\right]$$

$$(2) \qquad EFV = E\left[\sum_{t=0}^{T}\prod_{i=t+1}^{T}(1+r_i)B_t\right]$$

where the expectation operator E is applied to the probability distribution of r_i.

Denoting by $\bar{r}_i$ the mean interest rate in period i, i =1,...,T, observe that whenever the random variables r_i, i = 1,...,T, <u>are independently distributed</u> it is always true that:

$$(3) \qquad EFV = \sum_{t=0}^{T}\prod_{i=t+1}^{T}(1+\bar{r}_i)B_t$$

implying that an investor guided by the EFV criterion is risk-neutral with respect to interest rates uncertainty. On the other hand, as shown in [4] under the same condition of independence of the random interest rates, and whenever the present value is a decreasing function of the interest rates, an expected present value maximizer will display risk-love with respect to interest rate uncertainty. In other words, any mean preserving increase in the riskiness of interest rates will never lead to a decrease in the number of investment projects selected by either EFV or EPV maximizers. But while the EFV maximizer's investment decisions will be guided solely by the mean interest rates (regardless of riskiness), the EPV maximizer will <u>increase</u> investment in response to increases in riskiness in interest rates whenever those are significant enough (in the sense of rendering profitable, according to the EPV criterion, projects that under lower degrees of riskiness were nonprofitable).

As is well known, an implication of the property that the present value of a project is a nonincreasing function of the interest rates is the

uniqueness of the internal rate of return. We wish to stress however that what is significant for our present purposes is the monotonic relationship as such. This obtains whenever, as in [4], the terminal date T of a project is optimally chosen (see [2] and [3]), implying that rational EPV maximizers are more investment oriented than EFV maximizers (whenever confronted by time-independent probability distributions of the interest rate).

An additional by-product of the analysis is that the truncation period (i.e. the optimal choice of T) is different under the two criteria. In [2] and [3] it is shown that the desired truncation period is a nonincreasing function of the interest rates. Since the present value deterministic equivalent marginal rate of interest is always smaller than the mean interest rate for that period (see [4]) and sine the future value deterministic equivalent marginal rate of interest is equal to the mean interest rate (see equation (3)) it also follows that the lifetime of projects chosen under the EPV criterion is never shorter and often longer than that chosen under the EFV criterion. This can be looked at as being an additional aspect of interest relating to the investment promoting nature of the EPV criterion in the presence of interest rate uncertainty.

Our final remark relates to the significance or the independence assumption with respect to the random interest rate variables. This point is illustrated by some of the examples given in [1] in which interest rates in different periods are not independent. As a result the EPV criterion turns out in some of the examples to be less investment promoting than the EFV criterion. Our above discussion should make it clear that this is not in conflict with our claim but rather illustrates how significant the independence assumption really is.

REFERENCES

[1] _____, "On Expected Present Value Vs. Expected Future Value: Comment", mimeograph, 1975.

[2] Arrow, K. J. and D. Levhari, "Uniqueness of the Internal Rate of Return with Variable Life Investment," Economic Journal, LXXIX (September, 1969), 560-566.

[3] Flemming, J. S. and J. F. Wright, "Uniqueness of the Internal Rate of Return: A Generalization," Economic Journal, LXXXI (June, 1971), 256-263.

[4] Pazner, E. A. and A. Razin, "A Model of Investment Under Interest Rate Uncertainty," International Economic Review 15 (October, 1974), 798-802.

[5] _____, "On Expected Present Value Vs. Expected Future Value," Journal of Finance, XXX (June, 1975), 875-877.