DISCUSSION PAPER NO. 171

AN INTERACTIVE PACKAGE OF PROGRAMS FOR UNCONSTRAINED OPTIMIZATION PURPOSES - UCKLP

by

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Preface

Development of the following interactive package would not have been possible without the help of the IBM Corporation which provided the funds and the support which made it feasible for me to attend the intensive APL course in UCLA in the Summer of 1975. Special thanks are due to Ephraim McLean, Ken Siler and Jim Schneck from UCLA who introduced me to the APL language and helped me develop UCHLP.

I am also indebted to Northwestern University's Graduate School of Management for covering my transportation expenses from Chicago to Los Angeles and awarding me a summer grant which enabled me to leave the Northwestern Campus for the period of the APL intensive seminar.
I. Background Information

UCNLP (Unconstrained Nonlinear Programming) is a package of computer programs coded in AFL and designed for users who are mainly interested in experiments and tests involving statistical comparisons of nonlinear functions minimized under different conditions. The user provides the computer with information regarding the mathematical expressions of the function to be minimized. The number of iterations per one-one dimensional search. The starting point, and the name of the method to be used. The optional methods are:

1. Steepest Descent [5]
3. The Fletcher-Reeves Conjugate Gradient [4,5]
5. Davidon-Fletcher-Powell Variable Metric [33]
6. Broyden's Rank 1 Method [1, 5]

and the user may select the one he wishes to apply.

All the above methods are also known as gradient methods. They all require information regarding the mathematical expression of the gradient of the function in question. The only method which requires information regarding the mathematical expression of the Hessian matrix is Newton's Method.

Gradient methods are procedures for minimizing a given differentiable unconstrained nonlinear function. The common denominator of all these methods is the fact that they all depend very heavily on the efficiency and the accuracy of the line-search used in the process. The main principle behind these methods is the fact that the function is evaluated at a given point, a direction of descent is then determined, and a search procedure locating
the function minimum along that direction is performed. The new point is then used as the starting point for the next iteration. The process terminates at a local minimum where no direction of descent can be found.

While different gradient methods (in this package) vary with respect to the direction used in the process, they all use the same line search procedure. This procedure is divided into two major stages. In the first stage the program searches for the best unimodal region along the given direction. This is done by simultaneously evaluating the function at 10 arbitrary points along the given direction. If a unimodal region is determined, the interval of uncertainty is reduced to 1/9th of its original size, and the search moves on to its second stage. In the event that the simultaneous search fails to secure a unimodal region the process repeats itself until a unimodal region is located. Once a unimodal interval is located along the search direction, a sequential Fibonacci search [7] is performed. The accuracy of the search is determined by the user who specifies the number of sequential steps in the one-dimensional search. The final error in determining the minimum point along the original interval of uncertainty is $\pm \frac{1}{F_N}$ where $F_N$ is the $N$th Fibonacci number specified by the user.
II. The Computer Application

After loading UCNL the user is ready to start.

User: OPTIMIZE

Computer: ENTER FUNCTION TO BE MINIMIZED

User: The function is entered here. User should be familiar with APL arithmetic rules according to which the function is expressed. The variables in the function should be of the type X[J] where J = 1, 2, ...

Computer: ENTER GRADIENT OF FUNCTION

User: The gradient vector is entered here according to the same rules as above. If some elements of the gradient vector are mathematical expressions, they should be separated from each other by means of placing them within parentheses and separating them by a comma.

Computer: ENTER INITIAL STARTING POINT

User: The user may enter a numerical vector or a predefined variable name containing that vector.

Computer: ENTER NUMBER OF ITERATIONS PER ONE-DIMENSIONAL SEARCH.

User: Enters the number which is used in the sequential part of the search after a unimodal region is determined.

Computer: DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO.

User: If user enters "YES" output will be printed at the end of every one dimensional search. The word "iteration" refers to a step consisting of a direction fixing and the line search along that direction. If user enters "NO" the only output consists of two statements regarding the optimal value of the function and the optimal point.

Computer: ENTER NAME OF ALGORITHM TO BE EXECUTED: CHOOSE AMONG: STEEP, NEW, CORDND, DFPUN, NDCGC, RANK.
User: The user must enter the full name (described above) of the algorithm he wishes to use.

STEER = The Method of Steepest Descent
NEW = Newton's Method
CONGRD = Fletcher-Reeves: Conjugate Gradient Method
DFPWM = Davidson-Fletcher-Powell: Variable Metric Method
MODCC = Perry: Modified Conjugate Gradient Method
RANK = Broyden's: Rank-One Method

The computer now calculates the minimum value of the function and the corresponding n-dimensional point. After completing this assignment the user may restart by responding positively to the question:

Computer: DO YOU WISH TO RESTART? YES OR NO

The computer will then go on asking questions until all the required information is given and the process repeats itself. Upon a negative response to the above question the computer closes the "OPTIMIZE" function and gets ready for any new assignment.

Note that if the user wishes to use Newton's Method he must provide the computer with information on the hessian matrix. Therefore, upon responding "NEW" to the question concerning the algorithm's name the computer responds by first requesting that information. The Hessian matrix must be entered row-wise for example. Let \( A = [a_{ij}] \) be the hessian of \( f \) then entering \( A \) is done in the following way: \( a_{11}, a_{12}, \ldots, a_{1n}, a_{22}, \ldots, a_{2n}, \ldots, a_{nn} \) (There is no need to dimension the matrix) if \( a_{ij} \) is a mathematical expression, it must be entered according to APL rules.

In the Appendix we provide the listing of the APL source programs followed by examples.
References


*Appendix 1*

```
ACM#16
XBRA716  CSX XICOUNT+1400 BY1(1111011D)150:DI
(1) COUNT16 0
(2) START16  FUN X
(3) D015S-0315S ER X
(4) S315S(+13856)XL.x13856
(5) YS15S+10
(6) YX15S X FDSCY Y
(7) COUNT16COUT+1
(8) S15S ER X
(9) Q15S(0315S,1)*(G1-GQ)
(10) S15S(~G1)+10(01-X)
(11) S15S(59)({(6)K(3P-0)-(3)} D1' T1E1)+.AXL)
(12) Q15S=51-31.91
(13) YS15S:.QD
(14) YS15S
(15) D015S01
(16) D015S13
(17) B03 L1EML(31K<275/2MD31>32PS)
(18) NM. OF ITERATIONS = *COUNT
(19) MIN. FUNCTION VALUE = *FUN X
(20) OPTIMUM POINT = *X

& XBRA716
DPX XICOUNT125409411:131115151551
(1) COUNT159
(2) START159  FUN Y
(3) D015S-0315S ER Y
(4) S015S(+13856)XL.x13856
(5) YS15S+10
(6) YX15S X FDSCY Y
(7) COUNT159 COUT+1
(8) S15S ER Y
(9) Q15S(G1),10(01-GQ)
(10) S15S(0315S,1)*(01-X)
(11) S15S(59)({(6)K(3P-0)-(3)} D1' T1E1)+.AXL)
(12) Q15S=51-31.91
(13) YS15S:.QD
(14) YS15S
(15) D015S01
(16) D015S13
(17) B03 L1EML(31K<275/2MD31>32PS)
(18) NM. OF ITERATIONS = *COUNT
(19) MIN. FUNCTION VALUE = *FUN X
(20) OPTIMUM POINT = *X
```
4.53)
(41) LIMITS C1
(42) C1:ISCO
(43) ORDER Y-(*1STAR)*3LDDL
(44) $G RESTART
(45) L4IS1I(S:COUNT)
(46) X1IS C1
(47) Y1IS0
(48) $S1IS Y X
(49) C1N1=1I2
(50) $S1I:1COUNT(N1;COUNT)
(51) $S1I:1RESTART
(52) L5IS1ISCO
(53) C5ISCL
(54) C1ISX:11STAR)*3LDDL
(55) RESTAR1*M1LS+11ERN
(56) COUNTS:COUNT:1
(57) SPOLO:34MS(L6IN X COUNT=1)
(58) C5ISCL-EMS+1LY-X
(59) SPOLO:34MS(L6IN X COUNT=1)
(60) FALPHA:1S2MN(FUN X),(FUN C1),(FUN C21),(FUN Y),(FUN C1),(FUN C21),(FUN C
(61) Y1IS1(X,A3,X;C1,;C21,Y)
(62) PRINTI1SMIFALPHA:1I1
(63) $S1I:1SM1LWITFT
(64) L5**THE FUNCTION EVALUATED AT THE MIN. IS \"FUN POINT \" 
(65) * THE POINT MINIMIZING THE FUNCTION IS \"POINT \"

APROX[@]4
XBAR=10 FREQ X1COUNT1D0:DI0000; 11
C1
(1,1)
STARTS(YH), X1 Y
(31) DI1S IS:ISHERE X
(4) Y1ISX:DM
(5) Y1IS X FINISH Y
(51) DI4S ON X
(71) L1:COUNTS:COUNT:1
(51) DISX-1I-((C11.*SHGD)0.3503+$NL0)+31MDDL
(9) Y1ISX:DI
(10) Y1IS X FINISH Y
(11) TNS1SS
(15) TNS1SS
(12) IS1SS1X
(14) SIS LIML(M1L($NLD/$SHGD)$SLE EP5))
(15) * NB. 3F ITERATIONS = \"COUNT \"
(16) * MIN. FUNCTION VALUE = \"FUN X \"
(17) * MINIMUM POINT = \"YX \"
$\text{FUN} \#1$

$\text{FEIS FUN X}$

(1) $\text{FEIS} \oplus \text{PX}$

$\text{AND} \#1$

$\text{GSI} \oplus \text{Y}$

(1) $\text{GSI} \oplus \text{EVX}$

$\text{HEX} \#1$

$\text{FEIS RES X}$

(1) $\text{FEIS} (\#X, \#X) \oplus \text{EVX}$

$\text{NEWTON} \#1$

$\text{XABSIS NEWTON XIZIGIDHIVICOUNT}$

(1) $\text{COUNT} \oplus \text{IS} \#0$

(2) $\text{START} \oplus \text{IS} \oplus \text{FUN} \oplus \text{X}$

(3) $\text{GSI} \oplus (\#X, 1) \oplus \text{EVX} \oplus \text{X}$

(4) $\text{GSI} \oplus \text{ML} \oplus (\text{SIN} \oplus (\text{SUM} \oplus (\text{SUM} \oplus (\text{EPS})))$

(5) $\text{COUNT} \oplus \text{IS} \oplus \text{COUNT} \#1$

(6) $\text{HEIS RES X}$

(7) $\text{GSI} \oplus (\text{SYNC} \oplus \text{MLOG}$

(8) $\text{YSIX} \oplus \#$

(9) $\text{YFSK FIBSCH Y}$

(10) $\text{SICK STANT}$

(11) L) $\oplus \text{NA. OF ITERATIONS} \oplus \text{COUNT}$

(12) $\text{MIN. FUNCTION VALUE} \oplus \text{IS} \oplus \text{FUN} \oplus \text{X}$

(13) $\text{OPTIMUM POINT} \oplus \text{IX}$
OPTIMISE
ENTER FUNCTION TO BE MINIMIZED
SCFM1
ENTER GRADIENT VECTOR OF THE ABOVE FUNCTION
SEPARATE
ENTER INITIAL STARTING POINT
#1
1.2 1
ENTER NUMBER OF ITERATIONS PER ONE ONE DIMENSIONAL SEARCH
#2
20
DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO
YES
ENTER NAME OF ALGORITHM TO BE EXECUTED, CAUSE ABORT:
STEP, MII, CANON, FDDSF, ABSDG, RANS
HST
ENTER HESSIAN MATRIX OF THE ABOVE FUNCTION
SCFM
THE FUNCTION EVALUATED AT THE MIN. IS 4.73154776
THE PRINT MINIMIZING THE FUNCTION IS 1.175176794 1.382253217
THE FUNCTION EVALUATED AT THE MIN. IS 4.046832354
THE PRINT MINIMIZING THE FUNCTION IS 1.05955197057 0.3752994457
THE FUNCTION EVALUATED AT THE MIN. IS 3.160446449
THE PRINT MINIMIZING THE FUNCTION IS 1.06979853596 0.4926194513
THE FUNCTION EVALUATED AT THE MIN. IS 4.954858945
THE PRINT MINIMIZING THE FUNCTION IS 1.3332595597 0.46964263733
THE FUNCTION EVALUATED AT THE MIN. IS 1.25350344
THE PRINT MINIMIZING THE FUNCTION IS 1.462097307794 10.53888454387
THE FUNCTION EVALUATED AT THE MIN. IS 0.8662945319
THE PRINT MINIMIZING THE FUNCTION IS 0.2348743903 0.0275699143
THE FUNCTION EVALUATED AT THE MIN. IS 0.3241045874
THE PRINT MINIMIZING THE FUNCTION IS 0.4302098993 0.1087656792
THE FUNCTION EVALUATED AT THE MIN. IS 0.103696767
THE PRINT MINIMIZING THE FUNCTION IS 0.07076517458 0.43471723
THE FUNCTION EVALUATED AT THE MIN. IS 0.2055323997
THE PRINT MINIMIZING THE FUNCTION IS 0.8540415768 0.7733957687
THE FUNCTION EVALUATED AT THE MIN. IS 0.00765242120013
THE PRINT MINIMIZING THE FUNCTION IS 1.000032449 1.38686456
THE FUNCTION EVALUATED AT THE MIN. IS 1.783432352167
THE PRINT MINIMIZING THE FUNCTION IS 0.2936846692 0.9932665769
THE FUNCTION EVALUATED AT THE MIN. IS 5.3485600245713
THE PRINT MINIMIZING THE FUNCTION IS 0.999297908 0.999999321
THE FUNCTION EVALUATED AT THE MIN. IS 5.53428212821822
THE PRINT MINIMIZING THE FUNCTION IS 1
NO. OF ITERATIONS = 13
MIN. FUNCTION VALUE = 5.634261292128
OPTIMUM POINT = 1 1
DO YOU WISH TO RESTART? ANSWER YES OR NO
YES
DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO
YES
DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO
YES
DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE ONE-DIMENSIONAL SEARCH AT YES OR NO
YES
DO YOU WISH TO CHANGE ALGORITHM? YES OR NO
YES
ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:
STE EK, XEN, CONVEX, DFP/M, RANK CONVEX
THE FUNCTION EVALUATED AT THE MIN. IS 4.128101277
THE PRINT MINIMIZING THE FUNCTION IS 3.030055791 1.09377318
THE FUNCTION EVALUATED AT THE MIN. IS 4.010183434
THE PRINT MINIMIZING THE FUNCTION IS 1.9786896931 0.951848599
THE FUNCTION EVALUATED AT THE MIN. IS 3.651940313
THE PRINT MINIMIZING THE FUNCTION IS 4.291193682 1.6299554766
THE FUNCTION EVALUATED AT THE MIN. IS 3.386361919
THE PRINT MINIMIZING THE FUNCTION IS 1.7072399084 0.4311544444
THE FUNCTION EVALUATED AT THE MIN. IS 3.23824177
THE PRINT MINIMIZING THE FUNCTION IS 6.185189264 0.301176562
THE FUNCTION EVALUATED AT THE MIN. IS 3.19287847
THE PRINT MINIMIZING THE FUNCTION IS 3.5406514417 0.2055589893
THE FUNCTION EVALUATED AT THE MIN. IS 3.036785204
THE PRINT MINIMIZING THE FUNCTION IS 0.4745132746 0.3227641978
THE FUNCTION EVALUATED AT THE MIN. IS 2.96319996
THE PRINT MINIMIZING THE FUNCTION IS 1.0416882953 0.0768685993
THE FUNCTION EVALUATED AT THE MIN. IS 2.903652676
THE PRINT MINIMIZING THE FUNCTION IS 0.3524651904 0.0291876243
THE FUNCTION EVALUATED AT THE MIN. IS 2.546151903
THE PRINT MINIMIZING THE FUNCTION IS 0.3133610074 0.06747714943
THE FUNCTION EVALUATED AT THE MIN. IS 2.798645627
THE PRINT MINIMIZING THE FUNCTION IS 0.8084664349 0.0374275006
THE FUNCTION EVALUATED AT THE MIN. IS 2.756135641
THE PRINT MINIMIZING THE FUNCTION IS 0.2661347296 0.0978347288
THE FUNCTION EVALUATED AT THE MIN. IS 2.71738403
THE PRINT MINIMIZING THE FUNCTION IS 0.1561986448 0.07799227274
THE FUNCTION EVALUATED AT THE MIN. IS 2.681754192
THE PRINT MINIMIZING THE FUNCTION IS 0.1489629187 0.0967895712
THE FUNCTION EVALUATED AT THE MIN. IS 2.64855315
THE PRINT MINIMIZING THE FUNCTION IS 0.1118561048 0.1063296418
THE FUNCTION EVALUATED AT THE MIN. IS 2.617246995
THE PRINT MINIMIZING THE FUNCTION IS 0.076177708294 0.1148048787
THE FUNCTION EVALUATED AT THE MIN. IS 2.58730854
THE PRINT MINIMIZING THE FUNCTION IS 0.4275125991 0.12654003
THE FUNCTION EVALUATED AT THE MIN. IS 2.56074948
THE PRINT MINIMIZING THE FUNCTION IS 0.009466684075 0.1238910592
THE FUNCTION EVALUATED AT THE MIN. IS 2.530477359
THE PRINT MINIMIZING THE FUNCTION IS 0.23293219477 0.1250143645
THE FUNCTION EVALUATED AT THE MIN. IS 2.502780336
THE PRINT MINIMIZING THE FUNCTION IS 0.05550318665 0.1238926992
THE FUNCTION EVALUATED AT THE MIN. IS 2.475206543
THE PRINT MINIMIZING THE FUNCTION IS 0.3878956501 0.1204926535
THE FUNCTION EVALUATED AT THE MIN. IS 2.474578945
THE PRINT MINIMIZING THE FUNCTION IS 0.1196339015 0.1150108832
THE FUNCTION EVALUATED AT THE MIN. IS 2.419371266
THE POINT MINIMIZING THE FUNCTION IS 1.493878832 2.234517061
THE FUNCTION EVALUATED AT THE MIN. IS 0.0000006527763908
THE POINT MINIMIZING THE FUNCTION IS 1.002513586 1.004967702
THE FUNCTION EVALUATED AT THE MIN. IS 0.0000000020320242
THE POINT MINIMIZING THE FUNCTION IS 1.0002488556 1.004994771
THE FUNCTION EVALUATED AT THE MIN. IS 0.0000000028883814
THE POINT MINIMIZING THE FUNCTION IS 1.00002399737 1.004764107
THE FUNCTION EVALUATED AT THE MIN. IS 1.69958943754E9
THE POINT MINIMIZING THE FUNCTION IS 1.00000000002 1.00001848
THE FUNCTION EVALUATED AT THE MIN. IS 6.514123559E11
THE POINT MINIMIZING THE FUNCTION IS 1.00000000006 1.000016156
THE FUNCTION EVALUATED AT THE MIN. IS 6.50320007561E11
THE POINT MINIMIZING THE FUNCTION IS 1.00000000008 1.000016147
THE FUNCTION EVALUATED AT THE MIN. IS 1.211397662E12
THE POINT MINIMIZING THE FUNCTION IS 1.00000000008 1.000000332
THE FUNCTION EVALUATED AT THE MIN. IS 7.36439448E16
THE POINT MINIMIZING THE FUNCTION IS 1.00000000007 1.00000000054
NO. OF ITERATIONS = 58
MIN. FUNCTION VALUE = -7.36439448E16
OPTIMUM POINT = 1.00000000007 1.00000000005
<table>
<thead>
<tr>
<th>Function</th>
<th>Minimum Value</th>
<th>Function Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function 1</td>
<td>1.0206391819</td>
<td>1.000000001</td>
</tr>
<tr>
<td>Function 2</td>
<td>1.0206391819</td>
<td>1.000000001</td>
</tr>
<tr>
<td>Function 3</td>
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<td>1.000000001</td>
</tr>
<tr>
<td>Function 4</td>
<td>1.0206391819</td>
<td>1.000000001</td>
</tr>
</tbody>
</table>

**Optimum Point =** | **1.000000001**

 **MIN.** **Function Value =** | **1.0206391819**

**Optimum Point =** | **1.000000001**

**Min. ** **Function Value =** | **1.0206391819**

**Optimum Point =** | **1.000000001**
THE FUNCTION EVALUATED AT THE MIN. IS 4.128151277
THE POINT MINIMIZING THE FUNCTION IS 1.030465571 1.069377318
THE FUNCTION EVALUATED AT THE MIN. IS 3.8429850782
THE POINT MINIMIZING THE FUNCTION IS 0.9292113243 0.8276573746
THE FUNCTION EVALUATED AT THE MIN. IS 2.532275169
THE POINT MINIMIZING THE FUNCTION IS 1.7851616052 0.5577017769
THE FUNCTION EVALUATED AT THE MIN. IS 1.889250734
THE POINT MINIMIZING THE FUNCTION IS 0.3303426577 0.0758876472
THE FUNCTION EVALUATED AT THE MIN. IS 1.805621297
THE POINT MINIMIZING THE FUNCTION IS 0.3625614357 0.112997209
THE FUNCTION EVALUATED AT THE MIN. IS 1.454661692
THE POINT MINIMIZING THE FUNCTION IS 1.0156529751 1.009718390248
THE FUNCTION EVALUATED AT THE MIN. IS 1.116006278
THE POINT MINIMIZING THE FUNCTION IS 0.030329794762 0.04103082137
THE FUNCTION EVALUATED AT THE MIN. IS 0.8927807072
THE POINT MINIMIZING THE FUNCTION IS 0.0566124391 0.008503064048
THE FUNCTION EVALUATED AT THE MIN. IS 0.6917714814
THE POINT MINIMIZING THE FUNCTION IS 0.20132323947 0.0173595856
THE FUNCTION EVALUATED AT THE MIN. IS 0.6007363728
THE POINT MINIMIZING THE FUNCTION IS 0.3029645676 0.057893352772
THE FUNCTION EVALUATED AT THE MIN. IS 0.04933899252
THE POINT MINIMIZING THE FUNCTION IS 0.7860151782 0.612707134
THE FUNCTION EVALUATED AT THE MIN. IS 0.44718751008
THE POINT MINIMIZING THE FUNCTION IS 0.7834155668 0.6120705665
THE FUNCTION EVALUATED AT THE MIN. IS 0.0230199957
THE POINT MINIMIZING THE FUNCTION IS 0.8712791102 0.7510961458
THE FUNCTION EVALUATED AT THE MIN. IS 0.00005303529512
THE POINT MINIMIZING THE FUNCTION IS 0.9960114092 0.9914294383
THE FUNCTION EVALUATED AT THE MIN. IS 0.00002753941573
THE POINT MINIMIZING THE FUNCTION IS 0.9948852346 0.98267917
THE FUNCTION EVALUATED AT THE MIN. IS 1.458011736845
THE POINT MINIMIZING THE FUNCTION IS 1.0300113323 1.00002471
THE FUNCTION EVALUATED AT THE MIN. IS 3.05479106611
THE POINT MINIMIZING THE FUNCTION IS 0.9999945787 0.999989049
THE FUNCTION EVALUATED AT THE MIN. IS 1.905018297216
THE POINT MINIMIZING THE FUNCTION IS 0.999999957 0.999999957
NO. OF ITERATIONS = 18
MIN. FUNCTION VALUE = 1.905018297216
OPTIMUM POINT = 0.999999957 0.999999957
DO YOU WISH TO RESTART? ANSWER YES OR NO
YES
DO YOU WISH TO CHANGE FUNCTION? ANSWER YES OR NO
NO
DO YOU WISH TO CHANGE INITIAL STARTING POINT? YES OR NO
NO
DO YOU WISH TO CHANGE NUMBER OF ITERATIONS PER ONE ONE-DIMENSIONAL SEARCH
YES OR NO
NO
DO YOU WISH TO CHANGE ALGORITHM? YES OR NO
YES
ENTER NAME OF ALGORITHM TO BE EXECUTED, CHOOSE AMONG:

STEP, NEW, CONJUG, DFPSOL, MQGGS, RANK

THE FUNCTION EVALUATED AT THE MIN. IS 4.16810177
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761 1.06937738
THE FUNCTION EVALUATED AT THE MIN. IS 3.849607271
THE POINT MINIMIZING THE FUNCTION IS 1.06937738
THE FUNCTION EVALUATED AT THE MIN. IS 1.076543197
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761
THE FUNCTION EVALUATED AT THE MIN. IS 2.8345219548
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761
THE FUNCTION EVALUATED AT THE MIN. IS 2.395692861
THE POINT MINIMIZING THE FUNCTION IS 1.0467110579
THE FUNCTION EVALUATED AT THE MIN. IS 2.076415072
THE POINT MINIMIZING THE FUNCTION IS 1.06937738
THE FUNCTION EVALUATED AT THE MIN. IS 1.429538475
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761
THE FUNCTION EVALUATED AT THE MIN. IS 1.0766503394
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761
THE FUNCTION EVALUATED AT THE MIN. IS 1.087379081
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761
THE FUNCTION EVALUATED AT THE MIN. IS 1.087651469
THE POINT MINIMIZING THE FUNCTION IS 1.0360285761
THE FUNCTION EVALUATED AT THE MIN. IS 0.01874575518
THE POINT MINIMIZING THE FUNCTION IS 1.036478249 1.06313849
THE FUNCTION EVALUATED AT THE MIN. IS 0.0000542973686
THE POINT MINIMIZING THE FUNCTION IS 1.04095463 1.010453851
THE FUNCTION EVALUATED AT THE MIN. IS 0.9731539477
THE POINT MINIMIZING THE FUNCTION IS 1.036061884 1.00007589
THE FUNCTION EVALUATED AT THE MIN. IS 1.1436530327
THE POINT MINIMIZING THE FUNCTION IS 0.9999197977 0.9999197977
THE FUNCTION EVALUATED AT THE MIN. IS 1.32687356777516
THE POINT MINIMIZING THE FUNCTION IS 1.0000000009 1.0000000009
NO. OF ITERATIONS = 16
MIN. FUNCTION VALUE = 1.32687356777516
OPTIMUM POINT = 1.0000000009 1.0000000009

1.0000000009 1.0000000009
Do YOU WISH TO RESTART? ANSWER YES OR NO
In Appendix 2 we provide examples of the UCRIP package in action. The function used for the testing is the well-known "banana-shaped" Rosenbrock [5] function. The procedure of entering the function, its gradient and hessian was somewhat different from the one described earlier. Here, instead of entering the function directly we first scored its alpha-numeric image in the variable names FNL, GNL, HNL (function, gradient, hessian respectively) and then used the APL Evaluate operator with that variable name. This procedure is equivalent to entering the alpha-numeric image of the function directly. The user may elect to do it either way.
Also note that several symbols on teletype 39 are used in place of the above mnemonics.

<table>
<thead>
<tr>
<th>Graphic</th>
<th>Mnemonics</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>@</td>
<td>$RO</td>
<td>Rho</td>
</tr>
<tr>
<td>@</td>
<td>$QD</td>
<td>Quad</td>
</tr>
<tr>
<td>&amp;</td>
<td>$DL</td>
<td>Do l</td>
</tr>
<tr>
<td>?</td>
<td>$NG</td>
<td>Neg</td>
</tr>
</tbody>
</table>