DISCUSSION PAPER NO. 171

AN INTERACTIVE PACKAGE OF PROGRAMS FOR UNCONSTRAINED OPTIMIZATION PURPOSES - UCNLP

by

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September 26, 1975

Preface

Development of the following interactive package would not have been possible without the help of the IBM Corporation which provided the funds and the support which made it feasible for me to attend the intensive APL course in UCLA in the Summer of 1975. Special thanks are due to Ephraim McLean, Ken Siler and Jim Schneck from UCLA who introduced me to the APL language and helped me develop UCNLP.

I am also indebted to Northwestern University's Graduate School of Management for covering my transportation expenses from Chicago to Los Angeles and awarding me a summer grant which enabled me to leave the Northwestern Campus for the period of the APL intensive seminar.

I. Background Information

UCNLP (Unconstrained Nonlinear Programming) is a package of computer programs coded in APL and designed for users who are mainly interested in experiments and tests involving statistical comparisons of nonlinear functions minimized under different conditions. The user provides the computer with information regarding the mathematical expressions of the function to be minimized. The number of iterations per one-one dimensional search. The starting point, and the name of the method to be used. The optional methods are:

1. Steepest Descent [5]
2. Newton's Method [5]
3. The Fletcher-Reeves Conjugate Gradient [4,5]
4. Perry's Modified Conjugate Gradient [6]
5. Davidon -Fletcher-Powell Variable Metric [33]
6. Broyden's Rank 1 Method [1, 5]

and the user may select the one he wishes to apply.

All the above methods are also known as gradient methods. They all require information regarding the mathematical expression of the gradient of the function in question. The only method which requires information regarding the mathematical expression of the hessian matrix is Newton's Method.

Gradient methods are procedures for minimizing a given differentiable unconstrained nonlinear function. The common denominator of all these methods is the fact that they all depend very heavily on the efficiency and the accuracy of the line-search used in the process. The main principle behind these methods is the fact that the function is evaluated at a given point, a direction of descent is then determined, and a search procedure locating the function minimum along that direction is performed. The new point is then used as the starting point for the next iteration. The process terminates at a local minimum where no direction of descent can be found.

While different gradient methods (in this package) vary with respect to the direction used in the process, they all use the same line search procedure. This procedure is divided into two major stages. In the first stage the program searches for the best unimodal region along the given direction. This is done by simultaneously evaluating the function at 10 arbitrary points along the given direction. If a unimodal region is determined, the interval of uncertainty is reduced to 1/9th of its original size, and the search moves on to its second stage. In the event that the simultaneous search fails to secure a unimodal region the process repeats itself until a unimodal region is located. Once a unimodal interval is located along the search direction, a sequential Fibonacci search [7] is performed. The accuracy of the search is determined by the user who specifies the number of sequential steps in the one-dimensional search. The final error in determining the minimum point along the original interval of uncertainty is $\pm 1/F_N$ where F_N is the Nth Fibonacci number specified by the user.

II. The Computer Application

After loading UCNLP the user is ready to start.

User: OPTIMIZE

Computer: ENTER FUNCTION TO BE MINIMIZED

User: The function is entered here. User should be familiar with APL arithmetic rules according to which the function is expressed. The variables in the function should be of the type X[J] where J = 1,2,...

Computer: ENTER GRADIENT OF FUNCTION

User: The gradient vector is entered here according to the same rules as above. If some elements of the gradient vector are mathematical expressions, they should be separated from each other by means of placing them within parentheses and separating them by a comma.

Computer: ENTER INITIAL STARTING POINT

User: The user may enter a numerical vector or a predefined variable name containing that vector.

Computer: ENTER NUMBER OF ITERATIONS PER ONE-ONE DIMENSIONAL SEARCH.

User: Enters the number which is used in the sequential part of the search after a unimodal region is determined.

Computer: DO YOU WISH TO SEE OUTPUT AFTER EVERY ITERATION? YES OR NO.

User: If user enters "YES" output will be printed at the end of every one dimensional search. The word "iteration" refers to a step consisting of a direction fixing and the line search along that direction. If user enters "NO" the only output consists of two statements regarding the optimal value of the function and the optimal point.

Computer: ENTER NAME OF ALGORITHM TO BE EXECUTED: CHOOSE AMONG: STEEP, NEW, CONGRD, DFPVM, MODCG, RANK.

User: The user must enter the full name (described above) of the algorithm he wishes to use.

STEEP = The Method of Steepest Descent

NEW = Newton's Method

CONGRD = Fletcher-Reeves: Conjugate Gradient Method

DFPVM = Davidon -Fletcher-Powell: Variable Metric Method

MODCG = Perry: Modified Conjugate Gradient Method

RANK = Broyden's: Rank-One Method

The computer now calculates the minimum value of the function and the corresponding n-dimensional point. After completing this assignment the user may restart by responding positively to the question:

Computer: DO YOU WISH TO RESTART? YES OR NO

The computer will then go on asking questions until all the required information is given and the process repeats itself. Upon a negative response to the above question the computer closes the "OPTIMIZE" function and gets ready for any new assignment.

Note that if the user wishes to use Newton's Method he must provide the computer with information on the hessian matrix. Therefore, upon responding "NEW" to the question concerning the algorithm's name the computer responds by first requesting that information. The Hessian matrix must be entered row-wise for example. Let $A = \{a_{ij}\}$ be the hessian of f then entering A is done in the following way: $a_{11}, a_{12}, \ldots, a_{1n}, a_{2n}, \ldots, a_{n1}, \ldots, a_{nn}$. (There is no need to dimension the matrix) if a_{ij} is a mathematical expression, it must be entered according to APL rules.

In the Appendix we provide the listing of the APL source programs followed by examples.

References

[1] Broyden, C. G., "Quasi-Newton Methods and Their Application to Function Minimization," Math. Comp., 21, 1967, pp. 368-381.

[2] Davidon, W. C., "Variable Metric Method for Minimization," Research and Development Report ANL-5990 (Ref.) U.S. Atomic Energy Commission, Argonne National Laboratories, 1959.

[3] Fletcher, R. and Powell, M. J. D., "A Rapidly Convergent Descent Method for Minimization," Computer J., 6, 1963, pp. 163-168.

[4] Fletcher, R. and Reeves, C. M., "Function Minimization by Conjugate Gradients," Computer J., 7, 1964, pp. 149-159.

[5] Himmelblau, D. M., "Applied Nonlinear Programming," McGraw-Hill, 1972, pp. 63-140.

[6] Perry, A., "Duality in Unconstrained Nonlinear Optimization," The Center for Mathematical Studies in Economics and Management Science, Northwestern University (forthcoming).

[7] Wilde, D. J. and Beightler, C. S., Foundations of Optimization, Prentice-Hall, 1967.

APPENDICES

Appendix 1

```
      &CGM[#]&
     & XBAR$IS CGM X;CØUNT;Z;DO;GO;Y;X1;G1;Q;P;S1;SO;D1
[1]    CØUNT$IS 0
[2]   START:Z$IS FUN X
[3]    DO$IS-GO$IS GR X
[4]    SO$IS($IØ@GO)$NL.=$IØ@GO
[5]    Y$ISX+DO
[6]   L1:X1$IS X FIBSCH Y
[7]    CØUNT$ISCØUNT+1
[8]    G1$IS GR X1
[9]    Q$IS((@G1),1)@(G1-GO)
[10]   P$IS((@G1),1)@(X1-X)
[11]   S1$ISSO+(P+.$ML($TP(P-Q)))$DV($TPP)+.$MLQ
[12]   D1$IS-S1+.$MLG1
[13]   Y$ISX1+D1
[14]   X$ISX1
[15]   DO$ISD1
[16]   GO$ISG1
[17]   $GØ L1$ML($IØ($ØR/($MDG1)>EPS))
[18]   'NØ. ØF ITERATIØNS = ';CØUNT
[19]   'MIN. FUNCTIØN VALUE = ';FUN X
[20]   'ØPTIMUM PØINT = ';X
     &


      &DFP[#]&
     & XBAR$IS DFP X;CØUNT;Z;DO;GO;Y;X1;G1;Q;P;S1;SO;D1
[1]    CØUNT$ISO
[2]   START:Z$IS FUN X
[3]    DO$IS-GO$ISGR X
[4]    SO$IS($IØ@GO)$NL.=$IØ@GO
[5]    Y$ISX+DO
[6]   L1:X1$ISXBAR$IS X FIBSCH Y
[7]    CØUNT$IS CØUNT+1
[8]    G1$IS GR X1
[9]    Q$IS((@G1),1)@(G1-GO)
[10]   P$IS((@G1),1)@(X1-X)
[11]   S1$IS(SO-((SO+.$MLQ+.$ML($TPQ)+.$MLSO)$DV(($TPQ)+.$MLSO+.$MLQ)))+((P
.$ML($TPP))$DV(($TPP)+.
       $MLQ))
[12]   D1$IS-S1+.$MLG1
[13]   Y$ISX1+,D1
[14]   X$ISX1
[15]   DO$ISD1
[16]   GO$ISG1
[17]   SO$ISS1
[18]   $GØ L1$ML($IØ($ØR/($MDG1)>EPS))
[19]   ' NØ. ØF ITERATIØNS = ';CØUNT
[20]   'MIN. FUNCTIØN VALUE = ';FUN X
[21]   ' ØPTIMUM PØINT = ';X
     &
```

```
     &FIBNUM[#]&
    & FN$IS FIBNUM K;R
[1]    FN$IS 1 1
[2]   END:$GØ(K>@FN$ISFN,+/↑2$TAFN)/END
    &
```

```
     &FIBSCH[#]&
    & PØINT$ISX FIBSCH Y;CØUNT;M1;FALPHA;C1;C2;X1;Y1;R;RN;FN;D;ND;RD;K;F;X
      0;N1
[1]   L20:D$ISY-X
[2]   L10:X1$ISX+EPS$MLD
[3]    $GØ(($MN/((FUN X),FUN X1))=((FUN X),FUN X1))/(L11,L12)
[4]   L11:Y$ISX+D$IS-GR X
[5]    $GØ L10
[6]   L12:ND$IS(D+.$MLD)*.5
[7]    RD$IS$DV$IØ9
[8]    X$IS(10,@D)@X,(,((9,@D)@X)+RD[$UGRD]$NL.$MLD)
[9]    K$IS1
[10]   F$IS$IØ0
[11]  L21:F$ISF,FUN X[K;]
[12]   K$ISK+1
[13]   $GØL21$ML$IØK$LE10
[14]   $GØ L22$ML$IØ(1$TA$UGF)=10
[15]   $GØ L25$ML$IØ(1$TA$UGF)=1
[16]   X1$IS(EPS$MLD)+,X0$ISX[1$TA$UGF;]
[17]   $GØ(((FUN X1)$GEFUN ,X0),(FUN X1)<FUN ,X0)/L23,L24
[18]  L22:X$IS,X[9;]
[19]   Y$ISY+D
[20]   $GØ L20
[21]  L23:Y$ISX1
[22]   X$IS,X[(1$TA$UGF)-1;]
[23]   $GØL0
[24]  L24:Y$IS,X[(1$TA$UGF)+1;]
[25]   X$IS,X0
[26]   $GØL0
[27]  L25:Y$IS,X[2;]
[28]   X$IS,X[1;]
[29]   $GØ L20
[30]  L0:N1$ISN
[31]   X1$IS,X
[32]   Y1$IS,Y
[33]   CØUNT$IS 2
[34]   D$ISY-X
[35]  L1:FN$IS FIBNUM N1
[36]   R$IS↑1$DRFN$DV↑1$TAFN
[37]   C2$ISX+(↑1$TAR)$MLD
[38]   C1$ISY-(↑1$TAR)$MLD
[39]   R$IS↑2$DRR
[40]  LØØP:$GØ(((FUN C1)>FUN C2),((FUN C1)=FUN C2),((FUN C1)<FUN C2))/(L2,L
```

```
      4,L3)
[41] L2:X$IS C1
[42]  C1$ISC2
[43]  C2$IS Y-(↑1$TAR)$MLD
[44]  $GØ RESTART
[45] L4:N1$ISN1-CØUNT
[46]  X$IS C1
[47]  Y$ISC2
[48]  D$IS Y-X
[49]  CØUNT$IS2
[50]  $GØL1$ML$IØ(N1>CØUNT)
[51]  $GØ RESTART
[52] L3:Y$ISC2
[53]  C2$ISC1
[54]  C1$ISX+(↑1$TAR)$MLD
[55] RESTART:R$IS↑1$DRR
[56]  CØUNT$ISCØUNT+1
[57]  $GØLØØP$ML$IØ(N1>CØUNT+1)
[58]  C1$ISC1-EPS$MLY-X
[59]  $GØLØØP$ML$IØ(N1=CØUNT+1)
[60]  FALPHA$IS($MN/(FUN X),(FUN C1),(FUN C2),FUN Y)=(FUN X),(FUN C1),(FUN
C
      2),FUN Y
[61]  M1$IS(4,@,X)@(X,C1,C2,Y)
[62]  PØINT$ISM1[FALPHA$IØ1;]
[63]  $GØL15$MLØUTPT
[64] L15:'THE FUNCTIØN EVALUATED AT THE MIN. IS ';FUN PØINT
[65]  ' THE PØINT MINIMIZING THE FUNCTIØN IS  ';PØINT
    &


      &FRCG[#]&
    & XBAR$IS FRCG X;CØUNT;D0;D1;G0;G1;Z
[1]   CØUNT$IS 1
[2]  START:Z$IS FUN X
[3]   D0$IS-G0$ISGR X
[4]   Y$ISX+D0
[5]   X$IS X FIBSCH Y
[6]   G1$IS GR X
[7]  L1:CØUNT$ISCØUNT+1
[8]   D1$IS(-G1)+((G1+.$MLG1)$DV(G0+.$MLG0))$MLD0
[9]   Y$ISX+D1
[10]  X$IS X FIBSCH Y
[11]  G0$ISG1
[12]  D0$ISD1
[13]  G1$ISGR X
[14]  $GØ L1$ML($IØ($ND/($MDG1)$LE EPS))
[15]  ' NØ. ØF ITERATIØNS = ';CØUNT
[16]  ' MIN. FUNCTIØN VALUE = '; FUN X
[17]  'ØPTIMUM PØINT = ';X
    &
```

```
      &FUN[#]&
    & F$IS FUN X
[1]   F$IS$EVFX
    &
      &GR[#]&
    & G$IS GR X
[1]   G$IS$EVGX
    &
      &HES[#]&
    & F$IS HES X
[1]   F$IS((@X),@X)@$EVHX
    &


      &NEWTØN[#]&
    & XBAR$IS NEWTØN X;Z;G;D;H;Y;CØUNT
[1]   CØUNT$IS 0
[2]  START:Z$ISFUN X
[3]   G$IS((@X),1)@GR X
[4]   $GØ L1$ML($IØ($AN/($MD,G)<EPS))
[5]   CØUNT$ISCØUNT+1
[6]   H$IS HES X
[7]   D$IS-($XDH)+.$MLG
[8]   Y$ISX+,D
[9]   X$ISX FIBSCH Y
[10]  $GØ START
[11] L1:'NØ. ØF ITERATIØNS = ';CØUNT
[12]  'MIN. FUNCTIØN VALUE = ';FUN X
[13]  'ØPTIMUM PØINT = ';X
    &
```

```
&ØPTIMIZE[#]&
& ØPTIMIZE
L1:'ENTER FUNCTIØN TØ BE MIMIMIZED'
 FX$IS$QP
L2:'ENTER GRADIENT VECTØR ØF THE ABØVE FUNCTIØN'
 GX$IS$QP
L3:'ENTER INITIAL STARTING PØINT'
 A$IS#
L4:'ENTER NUMBER ØF ITERATIØNS PER ØNE ØNE DIMENSIØNAL SEARCH'
 N$IS#
L6:'DØ YØU WISH TØ SEE ØUTPUT AFTER EVERY ITERATIØN? YES ØR NØ'
 ØUTPT$IS(1$TA$QP)='Y'
L5:'ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:'
 'STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK'
 $GØ$EV$QP
STEEP:SDT A
 $GØ FIN
NEW:'ENTER HESSIAN MATRIX ØF THE ABØVE FUNCTIØN'
 HX$IS$QP
 NEWTØN A
 $GØ FIN
CØNGRD:FRCG A
 $GØ FIN
MØDCG:CGM A
 $GØ FIN
RANK:RANK1 A
 $GØ FIN
DFPVM:DFP A
FIN:'DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ'
 ANS$IS1$TA$QP
 $GØ START$MLANS='Y'
START:'DØ YØU WISH TØ CHANGE FUNCTIØN? ANSWER YES ØR NØ'
 ANS$IS1$TA$QP
 $GØ L1$ML$IØANS='Y'
 'DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ'
 ANS$IS1$TA$QP
 $GØL3$ML$IØANS='Y'
 'DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIØNS PER ØNE ØNE-DIMENSIØNAL SEARCH? YES ØR NØ'
 ANS$IS1$TA$QP
 $GØ L4$ML$IØANS='Y'
 'DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ'
 ANS$IS1$TA$QP
 $GØ L5$MLANS='Y'
&
```

```
      &RANK1[#]&
     & XBAR$IS RANK1 X;CØUNT;Z;D0;D1;G0;Y;X1;G1;Q;P;S1;S0
[1]    CØUNT$IS0
[2]   START:Z$IS FUN X
[3]    D0$IS-G0$ISGR X
[4]    S0$IS($IØ@G0)$NL.=$IØ@G0
[5]    Y$ISX+D0
[6]   L1:X1$ISXBAR$ISX FIBSCH Y
[7]    CØUNT$ISCØUNT+1
[8]    G1$IS GR X1
[9]    Q$IS((@G1),1)@(G1-G0)
[10]   P$IS((@G1),1)@(X1-X)
[11]   S1$IS S0+((P-S0+.$MLQ)+.$ML($TP(P-S0+.$MLQ)))$DV($TP(P-S0+.$MLQ))+.$
LQ                              M
[12]   D1$IS-S1+.$MLG1
[13]   Y$ISX1+,D1
[14]   X$ISX1
[15]   D0$ISD1
[16]   G0$ISG1
[17]   S0$ISS1
[18]   $GØ L1$ML($IØ($ØR/($MDG1)>EPS))
[19]   ' NØ. ØF ITERATIØNS = ';CØUNT
[20]   ' MIN. FUNCTIØN VALUE = ';FUN X
[21]   ' ØPTIMUM PØINT = ';X
     &


      &SDT[#]&
     & XBAR$IS SDT X;Z;D;Y;CØUNT
[1]    CØUNT$IS0
[2]   START:Z$IS FUN X
[3]    D$IS GR X
[4]    $GØ L1$ML($IØ($AN/($MDD)$LEEPS))
[5]    Y$IS X-D
[6]    X$IS X FIBSCH Y
[7]    CØUNT$IS CØUNT+1
[8]    $GØ START
[9]   L1:'NØ. ØF ITERATIØNS = ';CØUNT
[10]   'MIN. FUNCTIØN VALUE = ';Z
[11]   'ØPTIMUM PØINT ';X
     &


      )VARS
A        ANS      EPS      FR1      FX       GR1      GX       HR1      HX
N        ØUTPT
      EPS
0.000001
      FR1
(100$ML(X[2]-X[1]*2)*2)+(1-X[1])*2
      FX
$EVFR1
      GR1
(((400$MLX[1]*3)-(400$MLX[1]$MLX[2]))+(2$MLX[1])-2),(200$ML(X[2]-X[1]*2))

      GX
$EVGR1
      HR1
(((1200$MLX[1]*2)-400$MLX[2])+2),(-400$MLX[1]),(-400$MLX[1]),200
      HX
$EVHR1
```

Appendix 2

```
      ØPTIMIZE
ENTER FUNCTIØN TØ BE MIMIMIZED
$EVFR1

ENTER GRADIENT VECTØR ØF THE ABØVE FUNCTIØN
$EVGR1
ENTER INITIAL STARTING PØINT
#:
      ↑1.2 1
ENTER NUMBER ØF ITERATIØNS PER ØNE ØNE DIMENSIØNAL SEARCH
#:
      20
DØ YØU WISH TØ SEE ØUTPUT AFTER EVERY ITERATIØN? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:
STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK
NEW
ENTER HESSIAN MATRIX ØF THE ABØVE FUNCTIØN
$EVHR1
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.73154706
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑1.175177674  1.382263817
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.045639354
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.9595197027  0.8752994427
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.160046149
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.6897928596  0.4206194213
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.954889045
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.333958907  0.06964267338
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.25388344
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.05087007794  ↑0.03608464387
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.6662845319
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.2388743903  0.02756990143
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.3241042274
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.4820798993  0.2087656752
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.10968867
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7072517458  0.484717253
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.02053228579
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.8842157548  0.7733957628
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.00006242121203
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.0030349  1.006808466
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.723432357E↑7
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9996244692  0.9992667769
THE FUNCTIØN EVALUATED AT THE MIN. IS 5.004856024E↑13
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.999999698  0.9999993321
THE FUNCTIØN EVALUATED AT THE MIN. IS 5.63422129E↑22
 THE PØINT MINIMIZING THE FUNCTIØN IS  1  1
NØ. ØF ITERATIØNS = 13
MIN. FUNCTIØN VALUE = 5.63422129E↑22
ØPTIMUM PØINT = 1  1
```

```
DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
YES
DØ YØU WISH TØ CHANGE FUNCTIØN? ANSWER YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIØNS PER ØNE ØNE-DIMENSIØNAL SEARC
      H? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:
STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK
CØNGRD
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.128101277
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑1.030025571  1.069377318
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.010183434
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.9876896931  0.9511848589
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.601940313
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.8211936621  0.6209554766
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.386261918
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.7070239084  0.4311564444
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.238243177
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.6166182964  0.301174562
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.126287847
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.5406214117  0.2055089893
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.036978248
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.4745132746  0.1322764198
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.96319986
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.4156882953  0.07486685993
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.900652678
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.3624651504  0.02918796243
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.846511903
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.3136710074  ↑0.007477414943
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.798815827
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.2684463449  ↑0.03701725006
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.756135641
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.2261347296  ↑0.06078847288
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.717387403
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.1861986648  ↑0.07979927274
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.681754192
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.148229187  ↑0.0947895712
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.64855315
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.1118561848  ↑0.1063296418
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.617246695
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.07677704204  ↑0.1148445875
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.58738084
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.04272125981  ↑0.120654003
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.558574948
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.009466284078  ↑0.1239891052
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.530473758
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.02321349277  ↑0.1250143845
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.502780236
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.05550318465  ↑0.1238329892
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.475206843
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.08758582501  ↑0.1204968635
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.447487845
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.1196329015  ↑0.1150108832
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.419371266
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 THE PØINT MINIMIZING THE FUNCTIØN IS  1.493878832  2.234517067
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.000006527763908
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.002513586  1.004987702
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.000006206209242
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.002488856  1.004994771
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.000005927883814
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.002399737  1.004764107
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.895894375E↑9
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000007092  1.00001848
THE FUNCTIØN EVALUATED AT THE MIN. IS 6.514123359E↑11
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000008069  1.000016156
THE FUNCTIØN EVALUATED AT THE MIN. IS 6.503300759E↑11
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000008048  1.000016147
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.211397662E↑12
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.00000022  1.000000332
THE FUNCTIØN EVALUATED AT THE MIN. IS 7.364359448E↑16
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000000027  1.000000054
 NØ. ØF ITERATIØNS = 58
 MIN. FUNCTIØN VALUE = 7.364359448E↑16
ØPTIMUM PØINT = 1.000000027  1.000000054
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 THE PØINT MINIMIZING THE FUNCTIØN IS  0.1517942861  ↑0.107339411
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.39058843
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.184231729  ↑0.0974021342
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.360878633
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.2170924165  ↑0.08508035092
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.32996354
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.2505357894  ↑0.07020801672
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.297567044
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.2846951892  ↑0.05258644599
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.263357725
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.3197355306  ↑0.03195555258
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.226996313
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.3558126825  ↑0.008008590538
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.188100793
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.3930932344  0.01962362634
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.146242483
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.4317526786  0.05137936184
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.100929744
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.4719823617  0.08778116097
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.051595151
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.5139949322  0.1294542242
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.997580631
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.5580249705  0.1771443834
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.938102693
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.6043449384  0.231757722
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.872227326
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.6532646528  0.294391497
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.798798831
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7051751092  0.3664332128
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.7164265
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7605098747  0.4495703402
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.623251309
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.8198908302  0.5460935777
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.516990607
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.8840456825  0.6589176347
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.394443629
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9540163456  0.7921501906
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.251157984
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.03127862  0.9517241611
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.080561686
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.118119649  1.146914764
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.872629344
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.218322423  1.39348196
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.6128575212
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.33862715  1.72134013
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.3184557171
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.475597928  2.147014532
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.2469437958
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.495507449  2.232779336
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.2449650529
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.494892075  2.235387057
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.2448859153
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.494609096  2.235430381
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.2447246239
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DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
YES
DØ YØU WISH TØ CHANGE FUNCTIØN? ANSWER YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIØNS PER ØNE ØNE-DIMENSIØNAL SEARC
      H? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:
STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK
DFPVM
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.128101277
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑1.030025571   1.069377318
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.849850871
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.9292133286   0.8276621666
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.532289332
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.7851689728   0.5577143794
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.881189587
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.3303949877   0.07580837214
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.805750468
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.3424666381   0.1113388264
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.454761675
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.1565763472  ↑0.009702676838
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.118841237
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.03019715963  ↑0.04131652107
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.8928892843
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.05656563833   0.008510876165
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.6918518279
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.2015731731   0.01731513417
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.6008832811
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.3028501993   0.0578264394
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.3604238432
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.4351964485   0.2097480473
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.2225880912
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.5420599557   0.2824804243
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.2025620443
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.5943418833   0.3337477784
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.07082108686
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7765607936   0.5885912146
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.05347983179
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7691162362   0.5902263184
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.0252599519
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.8636421911   0.7377129706
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.00052548464
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9866367256   0.9715894828
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.0002786455954
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9836218938   0.9671894895
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.00000226321 7993
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9999141729   0.9996781582
THE FUNCTIØN EVALUATED AT THE MIN. IS 5.472959736E↑8
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.999767203   0.9995321469
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.200698406E↑14
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9999999829   0.9999999806
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.10206391E↑19
 THE PØINT MINIMIZING THE FUNCTIØN IS  1   1.000000001
 NØ. ØF ITERATIØNS = 22
MIN. FUNCTIØN VALUE = 1.10206391E↑19
 ØPTIMUM PØINT = 1   1.000000001
1   1.000000001
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DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
YES
DØ YØU WISH TØ CHANGE FUNCTIØN? ANSWER YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIØNS PER ØNE ØNE-DIMENSIØNAL SEARC
      H? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:
STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK
MØDCG
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.128101277
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑1.030025571  1.069377318
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.849850782
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.9292113248  0.8276573746
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.532275169
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.7851616052  0.5577017769
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.880950734
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.3303426877  0.07578876472
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.805621297
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.3424164357  0.1112997209
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.454661692
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.1565297751  ↑0.009718390248
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.118604278
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.03028794762  ↑0.04130382137
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.892787072
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.0566124391  0.008503064048
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.6917714814
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.2016323047  0.0173359584
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.6007387278
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.3029645676  0.05789352772
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.04833889252
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7860151782  0.6127707134
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.04718751008
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.7834156684  0.6120705665
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.02301899187
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.8712791102  0.7510961458
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.00005305329812
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9960114992  0.9914294383
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.00002753941573
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9948852046  0.98967917
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.458011723E↑8
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000018323  1.000024711
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.05479106E↑11
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9999945787  0.9999890499
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.905018297E↑16
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9999999972  0.9999999957
NØ. ØF ITERATIØNS = 18
MIN. FUNCTIØN VALUE = 1.905018297E↑16
ØPTIMUM PØINT = 0.9999999972  0.9999999957
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DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
YES
DØ YØU WISH TØ CHANGE FUNCTIØN? ANSWER YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE INITIAL STARTING PØINT? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE NUMBER ØF ITERATIØNS PER ØNE ØNE-DIMENSIØNAL SEARC
      H? YES ØR NØ
NØ
DØ YØU WISH TØ CHANGE ALGØRITHM? YES ØR NØ
YES
ENTER NAME ØF ALGØRITHM TØ BE EXECUTED, CHØØSE AMØNG:
STEEP, NEW, CØNGRD , DFPVM, MØDCG, RANK
RANK
THE FUNCTIØN EVALUATED AT THE MIN. IS 4.128101277
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑1.030025571  1.069377318
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.849850871
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.9292133332  0.8276621778
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.532289685
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.7851666627  0.5577100201
THE FUNCTIØN EVALUATED AT THE MIN. IS 3.079141875
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.7526418437  0.5750653501
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.594523842
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.5687657336  0.2869570933
THE FUNCTIØN EVALUATED AT THE MIN. IS 2.395497261
 THE PØINT MINIMIZING THE FUNCTIØN IS  ↑0.4497160099  0.1480392545
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.0755142007 2
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.274319794  1.625512206
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.07543834475
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.274557857  1.62524825
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.03933280092
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.176880394  1.376077441
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.01635171155
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.087379081  1.173056988
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.007012927705
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.08368169  1.17468698
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.001874578518
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.032472849  1.063136459
THE FUNCTIØN EVALUATED AT THE MIN. IS 0.00005159673666
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.004959463  1.010463141
THE FUNCTIØN EVALUATED AT THE MIN. IS 6.733153324E↑7
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000661884  1.001275706
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.14356833E↑9
 THE PØINT MINIMIZING THE FUNCTIØN IS  0.9999812977  0.9999597784
THE FUNCTIØN EVALUATED AT THE MIN. IS 1.32057555E↑16
 THE PØINT MINIMIZING THE FUNCTIØN IS  1.000000009  1.000000018
 NØ. ØF ITERATIØNS = 16
 MIN. FUNCTIØN VALUE = 1.32057555E↑16
 ØPTIMUM PØINT = 1.000000009  1.000000018
1.000000009  1.000000018
DØ YØU WISH TØ RESTART? ANSWER YES ØR NØ
NØ
```

In Appendix 2 we provide examples of the UCNLP package in action. The function used for the testing is the well-known "banana-shaped" Rosenbrock [5] function. The procedure of entering the function, its gradient and hessian was somewhat different from the one described earlier. Here, instead of entering the function directly we first stored its alpha-numeric image in the variable names FR1, GR1, HR1 (function, gradient, hessian respectively) and then used the APL Evaluate operator with that variable name. This procedure is equivalent to entering the alpha-numeric image of the function directly. The user may elect to do it either way.

Ending Statement of

Appendix 3

Also note that several symbols on teletype 33 are used in place of the above Mneomonics.

Graphic	Mneomonics	Meaning
@	$RO	Rho
#	$QD	Quad
&	$DL	Del
↑	$NG	Neg