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EXCLUSION, EXTERNALITIES, AND PUBLIC GOODS

by

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INTRODUCTION

A focal point of recent literature in public economics has been characterization of the degree of "publicness" of a good in such manner that pure private goods and pure public goods constitute the two extremes of a spectrum spanning all commodities. The effort to fill the gap between Samuelson's 1954 polar cases [14] has met with some success, but the goal has yet to be fully realized.

Two distinguishable approaches to this problem have appeared. In what we may call the "consumption" approach, the location of any good along the spectrum is a technical datum. Those adopting this viewpoint, including Evans [6], Holtermann [7], Mishan [11] and Oakland [12], have concentrated principally on generalizing the Pareto optimality conditions obtained by Samuelson. In so doing, they have achieved a partial synthesis of the concepts of public good and consumption externality.

The other, "production", approach is represented by the work of Arrow [1], Buchanan [3], Davis and Whinston [4], Demsetz [5], Kamien and Schwartz [8], McGuire [9] and Millward [10]. In this viewpoint a good's location on the spectrum is an economic variable. Creation of markets, exclusion and property rights may be possible at a cost and the optimal allocation of resources to exclusion activities is sought.

In this paper we present a model encompassing both approaches and providing some new insights. The various sources of interrelationship among individuals including those attributable to utility or attitudes (consumption side), technical possibilities, and institutional arrangements (supply side) are distinguished. For example, three conceptually distinct circumstances are shown to lead to a good being a private good. Pure publicness, on the other hand, entails the coincidence of several conditions.

Unlike aforementioned contributions to this literature, our analysis is couched in terms of an infinite set of consumers, in particular, a continuum of infinitesimal individuals. The corresponding relations applicable for a finite set of consumers are readily obtained by reading "sum" for "integral" wherever the latter appears. While the continuum formulation is not necessary to obtain our results, it does enable us to demonstrate how the more familiar Samuelsonian conditions, and their progeny, can be derived and would appear under this supposition. As has been argued by Aumann [2], a continuum captures formally the notion that individuals are so inconsequential and numerous that no one can influence the outcome by his own actions. While the assumption of a many small consumers is more commonly associated with private goods economies, it has figured in the public goods literature as well. See, for example, Roberts [13] for a more formal treatment of pure public goods in an economy with a continuum of agents.

THE MODEL

We posit a continuum of consumers, with individual consumers indexed by the letter t where t lies in the unit interval [0,1].[1/] In the usual Samuelsonian approach, private goods are divided among consumers (the whole is the sum of the parts) but each individual receives or consumes the total amount of every public good (the whole is the same as the parts.) By contrast, in this paper the n goods are treated symmetrically without initial classification by degree of publicness or externality. Each good is fully allocated over the set of consumers [0,1] so that every unit has a $t \in [0,1]$ associated with it. Let $x_j(t) \geq 0$ denote the amount of good j assigned to consumer t, for $j=1,\ldots,n$ and $t \in [0,1]$, with X_j the total amount of good j in the economy. Then

$$(1) \qquad X_j = \int_0^1 x_j(t)dt, \quad j = 1,\ldots,n$$

We distinguish between the assignment or distribution of a good over the population and its impact upon the utility of individuals. For a private good, what matters to the individual is the level of his own consumption (allocation). Both one's own consumption and that of "neighbors" (in some sense) affect the utility of goods with consumption externalities. The particular assignment of public goods is irrelevant and only their total amounts affect utility. These possibilities can all be taken into account by allowing both the individual's own consumption of each good, its total amount and its manner of distribution to enter his utility function. This symmetric treatment of goods, with allocation of all goods among the consumers and the entire distribution of goods as utility function arguments, resemble Evans' formulation.

Tastes of individual t are represented by a function

$$U(x_1(t),\ldots,x_n(t),y_1(t),\ldots,y_n(t),t) \tag{2}$$

assumed differentiable in its first 2n arguments for each t [2/] and indicated in brief as U(t). The first n arguments in (2) are the individual's direct consumption of the n goods. The next n arguments reflect the impact on consumer t of the total assignment of the n goods. The partial derivatives of U(t) may be positive or negative according to the desirability or undesirability of a commodity.

We refer to $y_j(t)$ as individual t's "collective" consumption of good j and take this variable to be determined by the direct consumption of the good by other agents, the nature of the good, the interrelations among agents, and the degree of exclusion in supply of good j. Specifically, the impact $y_j(t)$ of the distribution of good j on consumer t is assumed proportional to a weighted sum of the assignment of good j over the unit interval;

$$y_j(t) = p_j \int_0^1 c_j(s,t)x_j(s)ds \qquad 0 \le t \le 1, \quad j=1,\ldots,n \tag{3}$$ [3/]

where both p_j and $c_j(s,t)$, to be discussed immediately below, take values between zero and one inclusive. The weighting function $c_j(\cdot,t)$ is given and reflects the influence of the distribution of good j on individual t. Alternatively, $c_j(s,\cdot)$ reflects the influence of s's consumption of j on the population. These functions are intended to represent spillover or externality phenomena in consumption. Since they need not be constant in either s or t, the distribution of the good may affect the level of collective consumption of the good by any individual and a particular distribution may affect individuals differently.

In particular, if $c_j(s,t)$ is not constant in s, the distribution of a given quantity of good j among the population affects the level of externality imposed on individual t. Further, if $c_j(s,t)$ is not constant in t, individual s's consumption of good j will vary in its affect on others. One interpretation of the functions is that "proximity" of one agent to others may influence the impact of their consumption upon him; the closer one is to the source of a spillover, the greater its effect.

We assume the weighting functions are scaled to satisfy for each j exactly one of the following two conditions:

$$(4) \qquad \begin{cases} \text{(i)} \quad c_j(s,t) = 0 \quad \text{for all } s,t \\ \text{(ii)} \quad \max\limits_{s,t} c_j(s,t) = 1 \end{cases}$$

Either there are no externalities associated with consumption of good j (case (i)); or there are some spillovers and they are scaled so the maximum impact exerted has weight 1.

The endogenous variable p_j represents the degree of exclusion with which good j is supplied. If good j is supplied as a purely private good, with no externality permitted, $p_j = 0$. According to this formulation, perfect exclusion means that the good must also be consumed in such manner that no spillover is generated. At the other extreme, $p_j = 1$ indicates that good j is supplied without any exclusivity. Supply without exclusivity is compatible with consumption without external effect, as will become evident. Intermediate values of p_j reflect supply of the good being neither completely exclusive nor freely available. Thus the variable p_j summarizes the various possibilities related to appropriability, exclusion, property rights, rationing, congestion, and institutional rules concerning the manner in which goods are to be consumed, and the manner of distributing the goods (here distinguished from the quantities to be distributed to each individual). Examples of rules include pollution emission standards for automobiles and regulation of fence construction and height, and of what classes of consumers may or must be served by business, health or educational establishments.

For collective consumption or externality $y_j(\cdot)$ to be nonzero, there must be potential consumption externalities $(c_j \not\equiv 0)$ and also institutional arrangements that do not completely suppress their impact $(p_j > 0)$. Thus, if consumption of j yields some form of pollution as a byproduct, so $c_j \not\equiv 0$, but the law makes reduction $(p_j < 1)$ or elimination $(p_j = 0)$ of the concommitant pollutants mandatory by the pollutor, then y_j is reduced (from its maximum potential value) or eliminated.

Since $p_j \leq 1$, $c_j \leq 1$, we have from (3) and (1) that $y_j(t) \leq X_j$, with equality holding only if $p_j = 1$ and $c_j(s,t) = 1$ for all s such that $x_j(s) > 0$. Thus t's collective consumption of j cannot exceed the total amount of j in the economy. Further, $y_j(t) = X_j$ only if the impact of external effects on t are nonnull, are independent of the distribution of the good among those receiving it, and the good is supplied without exclusion; in this situation we will say the good is purely public. The qualification that the distribution be irrelevant "among those receiving it" permits the possibility of the good being locally public. (If the qualification in quotes is not needed, then the good is globally public.) The classification of goods will appear later in more detail, but is mentioned now as further explication of y_j and its components.

The degree of exclusion in supply of a good is really a resource allocation decision. The resource cost may rise when exclusivity in supply is reduced, as in the case of bread and other items for which markets are well-suited [4]. In other instances, to increase exclusivity is costly, as in the case of parks and outdoor shows which must be fenced and ticket sellers provided. Laws regulating the manner of consumption or of conducting business are costly to enforce. Some goods may be so inherently public (national defense) or private that the corresponding degree of exclusion will optimally be unity or zero almost regardless of other parameter values.

The resource costs incurred in producing goods and distributing them and the technical possibilities available to the economy are summarized by a differentiable transformation function relating total production rates and degree of exclusivity in supply of all goods :

(5) $$F(X_1,\ldots,X_n,p_1,\ldots,p_n) = 0$$

WELFARE CONDITIONS

To obtain necessary conditions for Pareto optimality, we employ the artifice of choosing $x_j(t)$, $y_j(t)$, p_j for $t \in [0,1]$ and $j = 1,\ldots,n$ to

(6) $$\max \int_0^1 w(t)U(x_1(t),\ldots,x_n(t),y_1(t),\ldots,y_n(t),t)dt$$

subject to (1), (3), (5) and

$$x_j(t) \geq 0,\ 0 \leq p_j \leq 1 \quad j = 1,\ldots,n$$

where the function $w(t)$ is an arbitrary, given weighting of the consumers. 4/

Calculus of variations may be employed to solve the maximization problem posed. The resulting Euler equations, however, are degenerate since no derivatives with respect to t appear. We may also use ordinary calculus techniques to the same end. Let $\alpha_j(t)$, $\gamma_j(t)$ and β be the multipliers associated with constraints (3), (1), and (5) respectively. The Lagrangian is

$$L = \int_0^1 \{w(t)U(x_1(t),\ldots,x_n(t),y_1(t),\ldots,y_n(t),t) - \sum_{j=1}^{n} \alpha_j(t)[y_j(t) - p_j \int_0^1 c_j(s,t)x_j(s)ds]\}dt + \sum_{j=1}^{n} \gamma_j[X_j - \int_0^1 x_j(t)dt] - \beta F(X_1,\ldots,X_n,p_1,\ldots,p_n)$$

Values of p_j, X_j which maximize L must satisfy

(7) $$\int_0^1\int_0^1 \alpha_j(s)c_j(t,s)x_j(t)ds\,dt \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \beta F_{n+j} \quad \text{as} \quad \begin{Bmatrix} p_j = 0 \\ 0 \leq p_j \leq 1 \\ p_j = 1 \end{Bmatrix}$$

(8) $$\gamma_j = \beta F_j \quad \text{for } X_j > 0$$

for $j = 1,\ldots,n$, where F_k denotes the partial derivative of F with respect to its k^{th} argument. In addition, since derivatives with respect to t are absent, in order for the Lagrangian to be maximized, it must be maximized for each t in [0,1]. Hence differentiating with respect to $x_j(t)$, $y_j(t)$ yields respectively as further necessary conditions

(9) $$w(t)U_j(t) + p_j \int_0^1 \alpha_j(s)\, c_j(t,s)\, ds \leq \gamma_j \quad \text{with } = \text{ if } x_j(t) > 0$$

(10) $$w(t)U_{n+j}(t) = \alpha_j(t)$$

for $j = 1,\ldots,n$ and almost all t in [0,1] 5/, where U_k denotes the partial derivative with respect to the k^{th} argument of U. We consider only goods that are produced, i.e. $X_j > 0$ is assumed for $j = 1,\ldots,n$. Equality holds in (10) since no separate sign restriction on the $y_j(t)$ is necessary. Equations (8) and (10) may be used to eliminate the multipliers γ_j and $\alpha_j(t)$ respectively from (7) and (9), giving as necessary conditions

(11) $$\int_0^1\int_0^1 w(s)U_{n+j}(s)c_j(t,s)x_j(t)\,ds\,dt \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \beta F_{n+j} \quad \text{as} \quad \begin{Bmatrix} p_j = 0 \\ 0 \leq p_j \leq 1 \\ p_j = 1 \end{Bmatrix}$$

for $j = 1,\ldots,n$

(12) $$w(t)U_j(t) + p_j \int_0^1 w(s)U_{n+j}(s)c_j(t,s)\,ds \leq \beta F_j \quad \text{with } = \text{ if } x_j(t) > 0$$

for $j = 1,\ldots,n$ and almost all t in [0,1]

Let the n^{th} good be numeraire, purely private in consumption ($c_n(s,t) = 0$ for all s,t in [0,1]), desired by all ($U_n > 0$), and suppose that everyone optimally receives some of it ($x_n(t) > 0$ for all t in [0,1]). Then from (12) we obtain

(13) $$w(t)U_n(t) = \beta F_n \text{ for all } t \text{ in } [0,1]$$

Since (13) must hold for all t,

(14) $$w(t)U_n(t) = w(s)U_n(s) \quad \text{for all } s \text{ in } [0,1]$$

which is the familiar conclusion that marginal utilities for numeraire be inversely proportional to the welfare weights used in forming the maximand (6). Relations (13) and (14) may be employed to rewrite the necessary conditions one more time: (12) and (11) become

(15) $$u_j(t) + p_j \int_0^1 u_{n+j}(s)c_j(t,s)ds \leq f_j \text{ with } = \text{ if } x_j(t) > 0$$

$$\text{for } j = 1,\ldots,n \text{ and all in } t \text{ in } [0,1]$$

(16) $$\int_0^1\int_0^1 u_{n+j}(s)c_j(t,s)x_j(t)dsdt \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} f_{n+j} \quad \text{as} \begin{Bmatrix} p_j = 0 \\ 0 \leq p_j \leq 1 \\ p_j = 1 \end{Bmatrix}$$

$$\text{for } j = 1,\ldots,n$$

where

(17) $$u_m(t) \equiv U_m(t)/U_n(t) \quad \text{and} \quad f_m \equiv F_m/F_n\ , \quad m = 1,\ldots,2n$$

Before discussing these results, we mention briefly the analogous expressions for an economy that, apart from consisting of a finite number K of consumers indexed by $k = 1,\ldots,K$, is identical to the one described above. Let x_j^k be the amount of good j alloted consumer k, $X_j = \sum_k x_j^k$ be the total amount of good j in the economy, $y_j^k = p_j \sum_h c_j^{hk} x_j^h$ be k's collective consumption of j, and $U^k(x_1^k,\ldots,x_n^k,y_1^k,\ldots,y_n^k)$ be k's utility function. The transformation function for the economy is given by (5). Then instead of (15) - (17) respectively we would obtain for $j = 1,\ldots,n$

$$(15') \qquad u_j^k + p_j \sum_{h=1}^{K} u_{n+j}^h c_j^{kh} \leq f_j \quad \text{with} = \text{if } x_j^k > 0$$

$$(16') \qquad \sum_{h=1}^{K} \sum_{k=1}^{K} u_{n+j}^h c_j^{kh} x_j^k \begin{matrix} < \\ = \\ > \end{matrix} f_{n+j} \quad \text{as} \begin{cases} p_j = 0 \\ 0 \leq p_j \leq 1 \\ p_j = 1 \end{cases}$$

where

$$(17') \qquad u_m^k = U_m^k / U_n^k \quad \text{and } f_m = F_m / F_n, \quad m = 1,\ldots,2n$$

and subscripts on U^k, F indicate partial derivatives. While the subsequent discussion will be couched in terms of the continuum of consumers and (15) - (17), the reader may easily substitute (15') - (17') and the corresponding definitions for analogous results applicable to an economy with a finite number of consumers.

Our results may also be compared with those derived by Evans, Mishan, Oakland, and Holtermann by noting that k's marginal valuation in terms of numeraire of h's consuming good j is $p_j c_j^{kh} u_{n+j}^h$. (There is no analogous quantity in the continuum model as no single consumer has an impact on the economy or on others there.)

INTERPRETATION OF THE FIRST ORDER CONDITIONS

The benefit to the economy of allocating a marginal unit of good j to individual t is the sum of the direct marginal benefit to t, $u_j(t)$, and the indirect or external effects of his consumption on the populace. The indirect effect depends on the technical possibility of impact $c_j(t,\cdot)$, on the various individuals' marginal valuation $u_{n+j}(\cdot)$ of that impact, and the extent to which external effects are permitted in consumption of the good p_j. From (15) we see that, as usual, either the marginal benefit optimally falls short of marginal cost, both measured in terms of numeraire ,and the good is optimally not supplied to individual t, or else the good is allotted in such quantity that the sum of all marginal benefits equals marginal cost.

Various individuals may find the effect of collective consumption of a single good to be positive, neutral, or negative. Thus $u_{n+j}(s) > 0$ and $u_{n+j}(s') < 0$ for $s \neq s'$ are simultaneously compatible. Whether the external or collective impact of t's consumption of j is net "good" or "bad", that is, confers a positive or negative externality upon the economy, is reflected in the sign of the weighted sum $\int u_{n+j}(s)c_j(t,s)ds$.

Condition (16) may be interpreted as requiring that if possible, the marginal value of altering the degree of exclusion in supply of a good should equal the marginal cost of doing so, all evaluated in terms of numeraire. This condition is consistent with the intuitive notion the degree of exclusion in supply of a good is likely to be greater the higher the marginal cost of exclusion and the greater the benefit of publicness. In short, (15) and (16) are necessary conditions for the optimal production and distribution of a good and the optimal degree of exclusivity in its supply.

To clarify further the meaning of these conditions, we will show how a number of special cases, including the polar ones, appear in our formulation. The classification of a good when consumed by one individual may differ from its classification if consumed by another. We shall distinguish between publicness or privateness in consumption and publicness or privateness in supply. It will be shown that a good is purely private, as the term is typically employed, if it is private in either consumption or supply. On the other hand, for a good to be purely public, it must exhibit pure publicness in both consumption and supply. After further discussion of the two polar cases, we take up goods public in consumption that are not purely public, local public goods, and semipublic goods. Finally the case of private goods with externalities is considered.

Individual t's consumption of good j will be purely private and the familiar condition

(18) $u_j(t) = f_j$ for purely private consumption of j by t

obtains if any of the following situations holds:

(a) there is no technical possibility of external effect of t's consuming good j: $c_j(t,s) = 0$ for all s

(b) people do not care about others' consumption of j: $u_{n+j}(s) = 0$ for all s

(c) there is perfect exclusion in supply of good j so no external effects are permitted: $p_j = 0$

Recall (15). Good j is purely private for t if it is purely private <u>in consumption</u> ((a) or (b)), or if it is purely private <u>in supply</u> (c). Since the classification of a good can differ among individuals, good j may be purely private when consumed by some and yet have externality when consumed by others; case (a) for some but not all t. Good j will be purely private for all if externality is impossible, or people do not care about others' consumption of j, or externality is prohibited ((a),(b),(c) respectively).

Suppose good j is globally private <u>in consumption</u>: either (a) holds for all t or (b) obtains. In this situation (16) reduces to

$$(19) \qquad 0 \lesseqgtr f_{n+j} \quad \text{as} \quad \begin{cases} p_j = 0 \\ 0 \le p_j \le 1 \\ p_j = 1 \end{cases} \qquad \text{for j globally private in consumption}$$

The degree of exclusivity in supply does not affect utility in this case and p_j is chosen to minimize the cost of supplying the good. Thus the good will be supplied with no, partial, or complete exclusion according to which method is least costly. Goods globally private in consumption are often thought to be most economically provided through markets; in such cases (a), (b), and (c) may hold simultaneously. Note that market operation is not necessarily costless even though it may be the least costly option.

At the other pole, consider a commodity supplied without any exclusion and for which utility is derived from the total amount produced and available to the economy. This is the traditional definition of a pure public good. In our framework it corresponds to $p_j = 1$, $c_j(s,t) \equiv 1$ for all s,t, and $u_j(t) \equiv 0$ for all t. Then (15) yields

$$\int_0^1 u_{n+j}(t)dt = f_j \qquad \text{for j a pure public good} \tag{20}$$

Thus we view a public good (at least conceptually) as being distributed among consumers so the usual market balance condition of production equalling the total amount distributed obtains. For such a good, however, each individual derives satisfaction from the total amount produced. Its allocation among the various individuals (including himself) is a matter of complete indifference. Equation (20) corresponds to the Samuelsonian marginal condition for a public good, that the total of the various individuals' marginal valuation of the public good equal the good's marginal production cost, both valued in terms of numeraire.

Next, suppose only the weighted total allocation of j, $y_j(t)$, affects t's utility and his particular allocation of j does not matter to him. Following the taxonomy given earlier, we say good j is purely public <u>in consumption</u> for individual t in this case. Since $u_j(t) = 0$, we then have from (15) that

$$p_j \int_0^1 u_{n+j}(s)c_j(t,s)ds \leq f_j \text{ with } = \text{ if } x_j(t) > 0 \tag{21}$$

Since individual t gets no satisfaction from his personal allotment of j, he will receive j only if his consumption contributes enough to the general welfare. It follows from (21) that for any two individuals t, t' receiving positive amounts of good j, and any individual t'' alloted none of good j, with j purely public in consumption for t,t't''

$$\int_0^1 u_{n+j}(s)c_j(t,s)ds = \int_0^1 u_{n+j}(s)c_j(t',s)ds \geq \int_0^1 u_{n+j}(s)c_j(t'',s)ds \tag{22}$$

This means that if good j yields utility only through its effect on collective consumption, then an optimal assignment will allot good j among those individuals whose consumption of the good contributes the most to the aggregate enjoyment or least to total discomfort.

Suppose j is a public "good" in the sense that the total of individuals' marginal valuations of collective consumption is positive: $\int u_{n+j}(s)ds > 0$. Then from (4) and (22) it follows that among those individuals for whom j is purely public, the ones to receive j are indexed by t's such that $c_j(s,t) = 1$ for those s such that $u_{n+j}(s) > 0$. It follows that if good j is purely public in consumption for all t in [0,1],

$$\begin{cases} u_j(t) = 0 \text{ for all } t \\ c_j(s,t) = 1 \text{ for all } s \text{ and all } t \text{ for which } x_j(t) > 0 \end{cases} \tag{23}$$

Furthermore, with j public in consumption for all t, conditions (15) and (16) imply

$$p_j \int_0^1 u_{n+j}(s)ds = f_j \quad \text{for j purely public in consumption.} \tag{24}$$

and

$$\begin{cases} \text{either (i) } X_j f_j = p_j f_{n+j} \\ \text{or (ii) } X_j f_j \geq f_{n+j} \text{ and } p_j = 1 \end{cases} \tag{25}$$

where use has been made of (23) and (24) to obtain (25). If providing exclusion is costly, we have $f_{n+j} < 0$ so that $p_j = 1$ from (23) and (24) reduces to the usual condition (20) for a pure public good. A pure public good is purely public in consumption for all individuals and is supplied without exclusion.

If providing greater access to the benefits of the good is costly, $f_{n+j} > 0$, then (25i) might, but need not, obtain. If (25i) holds with $p_j < 1$, it is because there is a trade-off between supplying more of the good in total (larger X_j) and increasing the accessability or availability of the given supply to the population (larger p_j). The effect of this may be surmised from (24), where the total of the marginal potential benefit of the good is reduced by the extent to which there is exclusion in supply. A larger amount of good j supplied compensates for a reduced availability of the benefits from the given amount. Equation (25i) may be compared with Buchanan's breakeven requirement for a collectivity's consumption of a public good [3, expression (9)].

Following Evans, we may say that good j is a local public good without externality if it is a public good for all individuals in some subset (locality) S_j and if provision of the good to members of S_j does not affect the utility level of individuals not in S_j. In our model, good j is a local public good without externalities provided that

$$u_j(t) = 0 \quad \text{and} \quad c_j(t,s) = 1 \text{ for } s,t \text{ in } S_j$$

$$c_j(t,s') = 0 \text{ for all } t \text{ in } S_j \text{ and all } s' \text{ not in } S_j$$

A further possibility is Evans' concept of a semi-public good, a local public good exerting an externality on individuals outside the locality S_j.

For good j semi-public

$$u_j(t) = 0 \text{ and } c_j(t,s) = 1 \text{ for } s,t \text{ in } S_j$$

$$0 < c_j(t,s') < 1 \text{ for all } t \text{ in } S_j \text{ and some } s' \text{ not in } S_j$$

In view of (22), a semipublic good j (if it is a "good") will optimally be assigned among members of S_j only. These two cases do not exhaust all possibilities, as the interested reader can verify.

Finally, the composite case in which consumption of good j by individual t contributes directly to his own utility and also affects the utility of others through its effect on $y_j(\cdot)$ describes a private good with externalities. Assuming diminishing marginal utility of the good, its assignment to an individual would be enlarged or diminished (compared with its assignment if there were no external impact) according as the externality conferred is positive or negative. See again (15).

FOOTNOTES

1/ The interval [0,1] can be endowed with the usual Lebesgue measure. All the functions we consider are assumed Lebesgue measurable.

2/ The assumption of note 1 that U is measurable in t with respect to the usual Lebesgue measure on [0,1] is very weak.

3/ This means of introducing consumption of others into an individual's utility function is less general that that often employed when the number of consumers is finite. It allows an individual's own consumption to have particular significance to him while keeping the domain of U finite dimensional for each t in [0,1].

4/ This integral in (6) is to be taken in the sense of Lebesgue, rather than Riemann. Where both are defined, the two concepts of the integral agree, but in order for the Riemann integral to exist, U and w must be continuous almost everywhere in t. That is, we would have to assume that consumers indexed by close values of t have similar preferences.

5/ That is, except possibly for a set of values of t of Lebesgue measure zero. The qualifier is occasionally omitted in this paper and it should be understood that relations given for t or statements about individuals t are meant to hold for all such t except possibly for a set of Lebesgue measure zero. Likewise, a reference to "some individuals" means a set of values of t of positive Lebesgue measure.

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