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## Ranking Friends

Willemien Kets  
Northwestern University

Yossi Feinberg  
Stanford University

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NORTHWESTERN  
UNIVERSITY



# Ranking Friends\*

Yossi Feinberg<sup>†</sup>      Willemien Kets<sup>‡</sup>

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## Abstract

We investigate the scope for cooperation within a community engaged in repeated reciprocal interactions. Players seek the help of others and approach them sequentially according to some fixed order, that is, a ranking profile. We study the ranking profiles that are most effective in sustaining cooperation in equilibrium, that is, profiles that support full cooperation in equilibrium under the largest set of parameters. These are the profiles that spread the costs of helping others equally among the members of the community. We show that, generically, these socially optimal ranking profiles correspond to Latin squares – profiles in which each player appears in a given position exactly once in other players' list. In addition, we study equilibria with bilateral enforcement in which only the victims punish non-cooperating deviators. We show that the Latin squares in which every two players rank each other at the same position can sustain cooperation for the widest range of parameters in this case.

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<sup>†</sup>Graduate School of Business, Stanford University. E-mail: yossi@gsb.stanford.edu.

<sup>‡</sup>Kellogg School of Management, Northwestern University. E-mail: w-kets@kellogg.northwestern.edu.

# 1 Introduction

Among the characteristics of any social structure is the nature of reciprocal interaction among its members. This paper is concerned with a particular type of interactions, namely, one in which the members of a community repeatedly face problems that require the assistance of others. Consider a group of friends, say Alice, Bob, and Carol. Alice is moving to a new apartment and needs a friend to help her carry boxes. Helping Alice is costly to Bob and Carol, as they will have to take a day off from work. Assuming that Alice needs only one friend to help, it will be inefficient to ask both friends simultaneously, because there is the possibility that they both show up to help and miss a day from work. If Alice asks one friend, say Carol, first, and she is unable to help Alice move—perhaps because her boss does not approve, or because she has prior commitments—, then Alice can ask Bob.

Suppose that over time, the three friends encounter similar problems on a regular basis. As long as the value of the assistance that each member of the community receives exceeds the average costs of helping, the socially optimal outcome is one in which every player helps all their friends. It may be more difficult to attain this maximum level of cooperation when players approach others sequentially, i.e., when they rank their friends.

To see this, suppose both Alice and Bob always ask Carol for help first, as in Figure 1a, and assume that each player receives the same benefits of being helped, and incurs the same costs when assisting others; also assume that each player needs help with the same frequency.

In the structure in Figure 1a, Carol is asked for help each and every time Alice and Bob have a problem. If the probability that her boss will let her take time off is high enough, Carol will be spending much more time helping her friends than the average time they spend helping their friends. To see this, consider Bob’s expected cost of helping. Bob is only called upon to help Alice (resp., Carol) if Carol (resp., Alice) was unable to. Hence, he is asked to help less frequently than Carol, that is, the costs are distributed unevenly. On the other hand, in expectation, Alice, Bob and Carol each receive the same amount of help if all players help whenever asked to do so: each has two friends they can go to for help. That is, while the expected cost for a player depends on the exact structure, her expected benefit only depends on the number of players who are willing to help her. This suggests that it may be hard to incentivize Carol to help her friends when they ask her for assistance.

On the other hand, suppose Alice asks Bob for help first, Bob asks Carol first, and Carol asks Alice first, as in Figure 1b. In this case, the expected costs of helping friends are identical across players. Hence, the ranking profiles—the profiles of ordered lists that specify the order in which players approach each other for help—in Figure 1 determine players’ expected costs.

The question we address is what ranking profiles are best at supporting the socially optimal outcome. Our first result states that ranking profiles in which players have identical

<table style="border-collapse: collapse; margin: auto;"> <tr><td style="border-right: 1px solid black; padding: 5px;">Alice</td><td style="padding: 5px;">Carol</td><td style="padding: 5px;">Bob</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">Bob</td><td style="padding: 5px;">Carol</td><td style="padding: 5px;">Alice</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">Carol</td><td style="padding: 5px;">Alice</td><td style="padding: 5px;">Bob</td></tr> </table> <p style="text-align: center; margin-top: 5px;">(a) An example of a ranking profile with different expected costs</p>	Alice	Carol	Bob	Bob	Carol	Alice	Carol	Alice	Bob	<table style="border-collapse: collapse; margin: auto;"> <tr><td style="border-right: 1px solid black; padding: 5px;">Alice</td><td style="padding: 5px;">Bob</td><td style="padding: 5px;">Carol</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">Bob</td><td style="padding: 5px;">Carol</td><td style="padding: 5px;">Alice</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">Carol</td><td style="padding: 5px;">Alice</td><td style="padding: 5px;">Bob</td></tr> </table> <p style="text-align: center; margin-top: 5px;">(b) An example of a ranking profile with equal expected costs</p>	Alice	Bob	Carol	Bob	Carol	Alice	Carol	Alice	Bob
Alice	Carol	Bob																	
Bob	Carol	Alice																	
Carol	Alice	Bob																	
Alice	Bob	Carol																	
Bob	Carol	Alice																	
Carol	Alice	Bob																	

Figure 1: Two possible ranking profiles. The first column indicates the player seeking help. Each corresponding row lists the other players by the order in which they are approached.

expected costs can sustain full cooperation for the largest range of parameters. In other words, the incentive compatibility constraint is the least binding when expected costs for help are equal among all players. We then turn to our main result which gives a characterization of the ranking profiles that induce equal expected costs (for generic values of the parameters). These ranking profiles are precisely the Latin squares, that is, ranking profiles in which every player appears in a certain position in the list of exactly one other player, as in Figure 1b. Graphically, these are rankings such that every column is a permutation of the list of players.

We then turn to the scope for bilateral enforcement in sustaining cooperation. That is, we restrict attention to a class of equilibria in which only the victim punishes the deviator. This is motivated by empirical research that shows that in many social situations, agents are more concerned with maintaining balance in their own relations than with correcting imbalances within a larger group (Blau, 1964; Fiske, 1992). In addition, it may simply be infeasible for a society to monitor the interactions between its members and communicate defections for the community to collectively punish deviators. In other words, individuals will often prefer bilateral enforcement over community enforcement.

We find that bilateral enforcement is more successful in sustaining cooperation in some ranking profiles than in others. Intuitively, because bilateral enforcement operates at the level of pairs of individuals, what matters is the balance in expected costs between two individuals, rather than the overall expected costs. We show that under bilateral enforcement, no ranking profile can outperform a Latin square in which every pair of players has the same expected costs of helping each other. This is in line with empirical evidence that shows that relationships that are characterized by high but similar levels of mutual obligations are especially productive (Shore and Barksdale, 1998).

There is of course an extensive literature on reciprocity in strategic settings, going back to the first days of repeated games. Some notable contributions include papers such as Ali and Miller (2009), Jackson et al. (2012), Lippert and Spagnolo (2011), Mihm et al. (2009),

and Raub and Weesie (1990) which study favor exchange and prisoners' dilemma games on networks; see Jackson et al. (2012) for a more extensive survey of the literature. The focus of this literature is on the effect of network structure on sustaining equilibria that are socially optimal.<sup>1</sup>

While we share the objective of characterizing social structures that are most conducive to sustaining cooperation, we focus on ranking profiles rather than on networks. There are two important differences between our approach and the existing literature on cooperation on networks. First, in our setting, the structural relationship between two players is independent of the lists of all other players, i.e., how Alice ranks Bob on her list is not constrained by any other player ranking (including Bob's). That is, Alice may rank Carol first, and Carol may rank Bob first, but there is no restriction on how Alice ranks Bob, or, for that matter, how Carol ranks Alice. By contrast, in a network, the access, or distance to members of a community, is constrained by the network structure. Second, the *behavior* of others crucially impacts the relationship between players, e.g., if Alice ranks Carol after Bob, Bob's tendency to help Alice will impact how often Alice asks Carol for help.

## 2 Model

There is a community  $N$  of players, labeled  $1, \dots, n$ , who occasionally encounter problems that require others' help. Time is discrete, and indexed by  $t = 1, 2, \dots$ . In every period, a player is chosen uniformly at random to receive a problem.<sup>2</sup> Let player  $i$  be the player facing a problem in period  $t$ . Any player  $j \neq i$  is equally qualified to help player  $i$ . In the situations we have in mind, a player can either solve another player's problem or not. Player  $j$  may refuse or fail to help player  $i$ , in which case player  $i$  may ask others for help. However, if  $j$  succeeds in helping  $i$ , then  $i$ 's problem is solved and he does not need to approach other players. We assume  $i$  will not approach a player more than once with the same problem.

Each player  $i \in N$  has a list of other players, which specifies the order in which  $i$  approaches them for help. We denote by  $r_i(k) \in \{1, \dots, i-1, i+1, \dots, n\}$  the player who is in the  $k$ th position on  $i$ 's list. We call the collection of players' lists a *ranking profile* and denote it by  $R = (r_1, \dots, r_n)$ .

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<sup>1</sup>Another strand of the literature deals with anonymous random matching, as in the seminal contributions of Ellison (1994) and Kandori (1992). In those papers, there is no structural restriction on the relationship between players since players are randomly paired.

<sup>2</sup>We assume that a player cannot solve the problem himself. Assuming that a player can solve his own problem with some probability does not change the results qualitatively.

## 2.1 Stage game

The stage game proceeds as follows. At the beginning of the period, one of the players, say  $i$ , receives a problem. Player  $i$  then goes down his list  $r_i$ , asking the other players for help. Each player  $j$  can decide whether to try and help  $i$  or not. Write  $e_j^i = 1$  if  $j$  tries to solve  $i$ 's problem, and  $e_j^i = 0$  otherwise. If  $j$  attempts to solve  $i$ 's problem, then the problem is solved with probability  $p \in (0, 1)$ . In that case, the payoff to  $i$  is  $v$  while  $j$ 's cost of helping  $i$  is  $c$ . Once the problem is solved, the stage game ends. If  $j$  does not try to help  $i$ , or tries and fails (which happens with probability  $1 - p$ ),  $i$  approaches the next player on his list. If the problem is not solved, then the payoff to all players is 0 in this period. Note that the cost of helping is borne only if the problem is solved.<sup>3</sup>

Hence, conditional on the event that player  $i$  needs help, his per-period expected benefit is

$$pv \sum_{\ell=1}^{n-1} e_{r_i(\ell)}^i \prod_{k=1}^{\ell-1} (1 - pe_{r_i(k)}^i),$$

and conditional on  $j \neq i$  needing help, the per-period expected cost for  $i$  is

$$pce_i^j \prod_{k=1}^{\rho_j(i)-1} (1 - pe_{r_j(k)}^j),$$

where  $\rho_j(i)$  is the position of  $i$  on  $j$ 's list. (Thus,  $\rho_j$  is just the inverse of  $r_j$ .) Throughout, we assume  $v > c$ , i.e., helping a player is efficient.

## 2.2 Repeated interactions

The stage game defined above is repeated in every period. Note that the ranking profiles are fixed throughout. In each period  $t$ , players observe the identity of the player who needs help, as well as the effort choices of the players who are asked for help. A (pure) *strategy*  $s_i$  for player  $i$  maps the history into an action for that period: if  $j$  is the player who has a problem in period  $t$ , then  $s_i(h) = e_i^j \in \{0, 1\}$  specifies whether  $i$  will try to help  $j$  in case  $j$  comes to him for help in that period, conditional on players' actions up to that point.

Payoffs are discounted by  $\delta \in (0, 1)$ , so that the normalized expected payoff of a strategy profile  $s = (s_j)_{j \in N}$  to player  $i$  is given by

$$U_i(s) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}_{h|t} [u_i(s(h))]$$

where  $u_i(s(h))$  is the stage-game payoff to  $i$ , and the expectation is taken over the histories  $h$  given  $t$ .

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<sup>3</sup>Assuming that helping is costly even if the attempt was unsuccessful does not change the results in a substantive way.

### 3 Community enforcement

We are interested in maximizing social welfare in the utilitarian sense. In the present context, social welfare is maximized if all players help whenever another player asks them for assistance (given that  $v > c$ ). That is, we investigate the conditions for the existence of a subgame-perfect equilibrium in which every player  $i$  chooses  $e_i^j = 1$  on the equilibrium path, for every  $j \neq i$ , whenever  $j$  asks  $i$  for help. In that case, we say that full cooperation can be sustained. In this section, we allow any player to participate in the punishment of a player who deviated, that is, we allow for community enforcement. In the next section, we consider equilibria in which only the player who was refused help punishes the deviator.

As a preliminary result, we observe that for any ranking profile, there is a minimum threshold such that full cooperation can be sustained if and only if the discount factor meets this threshold.

**Lemma 3.1** *If full cooperation can be sustained in a ranking profile with a given discount factor  $\delta < 1$ , then full cooperation can be sustained in that ranking profile for any discount factor  $\delta' > \delta$ .*

**Proof.** Fix a ranking profile and consider the following grim-trigger profile  $s$ . At the beginning of the first period, if  $j$  asks  $i \neq j$  for help, then  $i$  chooses  $e_i^j = 1$ . At later points in the game, if each player  $m$  has chosen  $e_m^k = 1$  for all  $k \neq m$  in the past, then any player  $i$  who is asked for help by some  $j \neq i$  chooses  $e_i^j = 1$ . Otherwise, player  $i$  chooses  $e_i^j = 0$  for all  $j \neq i$  whenever he is approached for help by player  $j$ .

We now check the conditions under which this is a subgame-perfect equilibrium. Assume that no player has deviated so far, i.e., every player who was approached by another player provided help in the past; and suppose player  $i$  is approached by player  $j$  for help in the present period. Then, the maximum he can gain by deviating from  $s$  is  $c(1 - \delta)$ , while he loses the expected future net gains from cooperation. Hence, player  $i$  cannot gain by deviating if and only if

$$c(1 - \delta) \leq \frac{\delta p}{n} \left[ v(1 + (1 - p) + \dots + (1 - p)^{n-2}) - c \sum_{j \neq i} (1 - p)^{\rho_j^{(i)} - 1} \right], \quad (1)$$

where we recall that  $1/n$  is the probability that an arbitrary player receives a problem. The incentive constraint (1) cannot be satisfied if the right-hand side is zero or negative (given that  $\delta$  lies between 0 and 1); in that case, cooperation cannot be sustained under  $s$ . If the right-hand side is positive, on the other hand, then there is a minimum discount factor  $\underline{\delta}$  for which (1) is satisfied, and full cooperation can be sustained with  $s$  if and only if the discount factor lies in  $[\underline{\delta}, 1)$ ; the critical discount factor  $\underline{\delta}$  may depend on the ranking profile.

The proof is complete by noting that if full cooperation cannot be sustained under the grim-trigger profile  $s$ , then it cannot be sustained under *any* strategy profile, as grim-trigger strategies offer the strongest possible form of punishment.  $\square$

We now turn to examining the factors that make it possible to sustain cooperation. Consider the two ranking profiles with four players in Figures 2a and 2b. The first column specifies the players, and the row following a player corresponds to its ranking. For example, in Figure 2a, player 3 first asks player 2 for help, then player 4, and ultimately player 1.

$i$	$r_i(1)$	$r_i(2)$	$r_i(3)$	$i$	$r_i(1)$	$r_i(2)$	$r_i(3)$
1	2	3	4	1	2	3	4
2	3	4	1	2	3	4	1
3	2	4	1	3	4	1	2
4	1	2	3	4	1	2	3
(a) A profile with heterogeneity in expected costs				(b) A profile with identical ex- pected costs			

Figure 2: Two ranking profiles with four players.

We compare the two ranking profiles in terms of the distribution of the (per-period) expected costs across players, assuming full cooperation. In the ranking profile in Figure 2a, player 2 is ranked first on the lists of both players 1 and 3, so helping these two players gives an expected cost of  $2cp/n$ . Player 2 is also the second player on player 4's list. This gives an additional expected cost of  $(1-p)cp/n$ , because player 4 approaches 2 only if player 1 was unable to solve player 4's problem, which happens with probability  $1 - e_1^4 p = 1 - p$ . On the other hand, player 4 is ranked second on the list of two players, and third on the list of another one, and has an expected cost of  $2(1-p)cp/n + (1-p)^2 cp/n$ . Hence, the costs for player 4 are strictly lower than those for player 2 in this ranking profile. By contrast, in Figure 2b, all players face the same expected cost of  $c/n + (1-p)c/n + (1-p)^2 c/n$ .

While the expected costs depend on the ranking profile, the expected benefit for a player is always  $pv(1 + (1-p) + (1-p)^2)/n$  under full cooperation, regardless of the ranking profile. Intuitively, the expected benefits depend only on the number of players who can help, and this is the same across players. In particular, it is immaterial for a player which player on his list solves his problem.

This suggests that full cooperation can be sustained whenever the incentive constraint for the player with the highest expected cost is satisfied. It turns out that the incentive constraints are weakest when players' expected costs are equal.



**Proposition 3.2** *For every  $p \in (0, 1)$ ,  $v, c > 0$ , and  $\delta < 1$ , if there exists a ranking profile in which full cooperation can be sustained in equilibrium, then full cooperation can be sustained in a ranking profile in which players have equal expected costs. Furthermore, there exist discount factors for which full cooperation can be sustained only if players' expected costs are equal.*

**Proof.** In any ranking profile, the total expected cost (per period) is

$$pc \sum_{i \in N} \sum_{j \neq i} (1-p)^{\rho_j^{(i)}-1} = pc(1 + (1-p) + \dots + (1-p)^{n-2}).$$

Fix a ranking profile. If players have different expected costs, then there is a player  $i$  for whom the expected cost under full cooperation exceeds  $1/n$  of the total expected costs, i.e.,

$$\frac{pc}{n} \sum_{j \neq i} (1-p)^{\rho_j^{(i)}-1} > \frac{pc}{n} (1 + (1-p) + \dots + (1-p)^{n-2}).$$

Recall from the proof of Lemma 3.1 that player  $i$  cannot gain by deviating if and only if

$$c(1-\delta) \leq \frac{\delta p}{n} \left[ v(1 + (1-p) + \dots + (1-p)^{n-2}) - c \sum_{j \neq i} (1-p)^{\rho_j^{(i)}-1} \right], \quad (2)$$

If the right-hand side of (2) is positive for each player  $i$  for some ranking profile, then it is positive for each player in a ranking profile in which players have identical expected costs. Moreover, the minimum discount factor  $\underline{\delta}$  identified in Lemma 3.1 is the same for any ranking profile in which players have equal expected costs. This discount factor satisfies

$$c(1-\underline{\delta}) = \frac{p\underline{\delta}}{n} (1 + (1-p) + \dots + (1-p)^{n-2})(v-c).$$

Finally, for any ranking profile in which some players differ in their expected costs, either full cooperation cannot be sustained, or the minimum discount factor for which full cooperation can be sustained is strictly greater than  $\underline{\delta}$ .  $\square$

The next result shows that for generic values of the parameters, the ranking profiles in which players have identical expected costs are the so-called Latin squares. A *Latin square* is a ranking profile in which every player  $i$  appears in the  $k$ th place in the list of exactly one other player, for every  $k$ , as in Figure 2b.<sup>4</sup> In other words, in a Latin square, each player is approached first by exactly one other player, approached second by exactly one other player, and so on.

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<sup>4</sup>Formally, for each player  $i = 1, 2, \dots, n$  and for each  $k = 1, 2, \dots, n-1$ , there is precisely one player  $j_k \neq i$  such that  $\rho_{j_k}^{(i)} = k$ .

**Proposition 3.3** *For a generic set of parameter values for  $p, v, c$ , and  $\delta$ , every player has the same expected costs in a ranking profile if and only if the ranking profile is a Latin square.*

It follows from Propositions 3.2 and 3.3 that Latin square rankings are the socially optimal ranking profiles (generically), in the sense that they sustain full cooperation for the widest range of parameters.

**Proof.** Under full cooperation, the expected per-period cost for player  $i$  in a given ranking profile is

$$\frac{pc}{n} \sum_{j \neq i} (1-p)^{\rho_j^{(i)}-1}.$$

If player  $i$  appears in every position only once in other players' lists, as in a Latin square, then this term is equal to

$$\frac{pc}{n} (1 + (1-p) + \dots + (1-p)^{n-2}),$$

independent of  $i$ . Hence, every player has the same expected costs in a Latin square.

For the other direction, fix  $v, c$ , and  $\delta$ . Suppose that each player has the same expected cost. Since the sum of the expected costs (across players) is  $pc(1 + (1-p) + \dots + (1-p)^{n-2})$ , the average expected cost is

$$\frac{pc}{n} (1 + (1-p) + \dots + (1-p)^{n-2}).$$

Assume by contradiction that the ranking profile is not a Latin square. Hence, we must have at least one player (in fact, at least two) who does not appear exactly once in every position. Assume without loss of generality that this is player  $n$ . Then, we have:

$$\frac{pc}{n} \sum_{k=1}^{n-1} (1-p)^{(\rho_k^{(n)}-1)} = \frac{pc}{n} (1 + (1-p) + \dots + (1-p)^{n-2}).$$

Since the ranking profile is not a Latin square,  $\rho_k^{(n)}$  does not run from 0 to  $n-1$  in the summation. We thus obtain a non-trivial polynomial equation in the variable  $p$ . In addition, the finite collection of ranking profiles yields at most a finite set of values  $p$  (for any given set of parameters  $v, c$ , and  $\delta$ ) for which there is a ranking profile that is not a Latin square and that has equal expected costs.  $\square$

As the proof suggests, for some parameters there may be ranking profiles in which players have equal expected costs that are not Latin squares. Indeed, consider the ranking profile in Figure 3. This ranking profile does not form a Latin square (for example, player 1 appears in the first position of both player 6 and 7), but there exists parameter values such that players' expected costs are equal: one can check that players have identical expected costs if and only

$i$	$r_i(1)$	$r_i(2)$	$r_i(3)$	$r_i(4)$	$r_i(5)$	$r_i(6)$
1	7	6	4	2	3	5
2	3	4	5	6	7	1
3	4	5	7	2	6	1
4	5	3	2	7	1	6
5	6	7	2	3	1	4
6	1	2	3	5	4	7
7	1	2	6	4	5	3

Figure 3: A ranking profile in which players have equal expected costs if and only if  $1 = (1 - p) + (1 - p)^2$ .

if  $1 = (1 - p) + (1 - p)^2$ , which has a solution for  $p \in (0, 1)$ . Hence, the characterization of ranking profiles with equal expected costs as Latin squares can hold only in the generic sense.

Community enforcement can be effective at sustaining cooperation, especially when the costs are helping are spread equally among players. However, it comes at a high cost for society: if a player deviates, cooperation breaks down entirely. In the next section, we consider the case of bilateral enforcement, where only the victim punishes the deviator.

## 4 Bilateral enforcement

In this section, we restrict attention to a class of equilibria in which only the victim punishes the deviator. As before, every player provides full assistance to the other players on the equilibrium path. If a player deviates, then only the player who did not receive assistance punishes the deviator, by ceasing to help the deviator in the future. Given that, the deviator will not assist the victim in any future period. All other players continue to help each other. However, we allow players to adjust the probability with which they help the victim and the deviator, because the breakdown of cooperation between the victim and the deviator may increase their expected costs in the absence of such an adjustment.

For example, suppose Alice approaches Bob first and Carol second. In equilibrium, each player helps every other player with probability 1 whenever they are approached for help. Now assume that Bob deviates in a given period and does not help Alice when she asks him for help. Under the strongest form of bilateral punishment—a bilateral grim-trigger strategy—, Alice and Bob do not help each other in any future period. If Carol were to continue to help Alice with probability 1 (as she did before Bob’s deviation), then her (discounted) expected cost of helping Alice increases from  $p(1 - p)c/n$  to  $pc/n$ . To correct for this, the strategy profile dictates that Carol helps Alice with probability  $1 - p$  after Bob’s deviation, so that her

expected cost of helping Alice equals  $p(1-p)c/n$ , as it did before Bob’s deviation. We assume no player will be punished for making such an adjustment.<sup>5</sup> In other words, punishment is bilateral, and players adjust their behavior to maintain the same expected cost in the face of deviations and punishments that do not concern them.

The scope for such a strategy profile to be a (subgame-perfect) equilibrium depends on the pairwise expected costs that players incur and the pairwise expected benefits that they provide each other. As such, of special interest are the ranking profiles in which every two players have symmetric expected costs and benefits. For example, consider the ranking profile in Figure 4. In this ranking profile, every player has the same expected costs, as in the ranking profile in Figure 2b, but, moreover, the expected cost for player  $i$  of helping player  $j$  is equal to the expected cost for player  $j$  of helping  $i$ . Whenever player 1 is faced with a problem, he first approaches player 2 for help, so that player 2’s expected cost of helping 1 is equal to  $pc$ . If player 2 receives a problem, she goes to player 1 first, so player 1’s expected cost of helping player 2 equals  $pc$  as well. It can be checked that the same holds for every other pair of players. The key is that every pair of players occupies exactly the same position in each other’s list. We say that a ranking profile is a *Bilateral Friendship Form* (BFF) if for each pair of distinct players  $i, j \in N$ , the position of  $j$  on  $i$ ’s list is the same as  $i$ ’s position on  $j$ ’s list, i.e.,  $\rho_i(j) = \rho_j(i)$ . Note that every BFF is a Latin square.

$i$	$r_i(1)$	$r_i(2)$	$r_i(3)$
1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

Figure 4: A ranking profile for four players that forms a BFF.

We want to allow players to follow a mixed (behavioral) strategy, to adjust the probability with which they help other players after a deviation.<sup>6</sup> A mixed strategy  $\sigma_i$  for player  $i$  maps each history in which player  $i$  is asked for help by some player  $j$  into a probability  $\alpha_i^j \in [0, 1]$  with which she helps  $j$  in that period.

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<sup>5</sup>Note that players following Alice on Bob’s list will also adjust their probabilities of helping accordingly to keep their expected cost at the same level as on the equilibrium path. Moreover, if there are more than three players, a player who adjusts the probability with which she helps other players to keep her expected costs constant has to take into account that the players preceding her on the victim’s or deviator’s list will adjust their behavior as well. See the specification of the bilateral enforcement profile  $\sigma$  below for details.

<sup>6</sup>We thus assume that mixed strategies are (publicly) observable. We could restrict attention to pure strategies if the action set  $A_i$  of each player  $i$  is taken to be some interval, say  $A_i = [0, 1]$ , with the interpretation that  $a_i \in A_i$  is the effort that  $i$  exerts in helping the player that asked her for help.

A strategy profile  $\sigma$  is a *bilateral enforcement profile* if at each history  $h$  such that player  $j$  asks player  $i$  for help,  $\sigma_i(h)$  is given by:

- If in some past period  $t'$ , player  $i$  approached player  $j$  for help, and  $j$  helped  $i$  with a probability different than the prescribed probability, i.e., with probability  $\alpha_j^i$  that does *not* satisfy

$$\alpha_j^i = \frac{\prod_{k=0}^{\rho_i(j)-1} (1-p)}{\prod_{k=0}^{\rho_i(j)-1} (1 - \alpha_{r_i(k)}^i p)}, \quad (3)$$

then  $\sigma_i(h) := 0$  (where  $\alpha_\ell^m$  is the probability with which player  $\ell$  helped player  $m$  in  $t'$ );

- If in some past period  $t'$ , player  $j$  approached player  $i$  for help, and  $i$  helped  $j$  with a probability different than the prescribed probability, i.e., with probability  $\alpha_i^j$  that does *not* satisfy

$$\alpha_i^j = \frac{\prod_{k=0}^{\rho_j(i)-1} (1-p)}{\prod_{k=0}^{\rho_j(i)-1} (1 - \alpha_{r_j(k)}^j p)}, \quad (4)$$

then  $\sigma_i(h) := 0$ ;

- otherwise, if in every past period  $t'$  player  $i$  and player  $j$  helped each other with probabilities  $\alpha_j^i$  and  $\alpha_i^j$  that satisfy (3) and (4), respectively, then  $i$  helps  $j$  with the prescribed probability, that is, the probability with which  $i$  helps  $j$  is given by

$$\sigma_i(h) := \frac{\prod_{k=0}^{\rho_j(i)-1} (1-p)}{\prod_{k=0}^{\rho_j(i)-1} (1 - \alpha_{r_j(k)}^j p)},$$

where  $\alpha_m^j$  is the effort level of a predecessor  $m = r_j(k)$  of  $i$  on  $j$ 's list in the current period.

If a bilateral enforcement profile  $\sigma$  is a subgame-perfect equilibrium, then we say that *bilateral enforcement can sustain full cooperation*. We have the following preliminary result:

**Lemma 4.1** *Bilateral enforcement can sustain full cooperation in a BFF of  $n$  players if and only if*

$$c(1-\delta) \leq \left(\frac{\delta p}{n}\right) (1-p)^{n-2}(v-c). \quad (5)$$

The proof follows directly from the observation that player  $i$ 's incentive constraint for helping player  $j$  is hardest to satisfy if the interaction between  $i$  and  $j$  is not very frequent. Hence, the strongest incentive constraint for player  $i$  is the one that describes his incentive of helping the player who is the last player on his list. The (discounted) expected benefits and costs of interacting with that player are  $(1-p)^{n-2}v/n$  and  $(1-p)^{n-2}c/n$ , respectively, which gives Equation (5).

Not surprisingly, it is more difficult to sustain full cooperation with bilateral enforcement than with community enforcement, given that the deviator is punished only by the victim, not by the whole community. A similar phenomenon of course arises in network settings (e.g., Jackson et al., 2012). However, bilateral enforcement is more effective in sustaining cooperation in some ranking profiles than in others. The next result shows that no ranking profile is more effective at sustaining cooperation under bilateral enforcement than a BFF.

**Proposition 4.2** *If full cooperation can be sustained by bilateral enforcement in any ranking profile, then bilateral enforcement can sustain full cooperation in a BFF.*

**Proof.** Assume that bilateral enforcement sustains cooperation in an arbitrary ranking profile  $R$ . Consider an arbitrary BFF  $R_{BFF}$  and two distinct players  $k, \ell \in N$ . Let  $i, j$  be players in  $N$  such that  $i$  has the same position on  $j$ 's list in  $R$  as the position that  $k$  and  $\ell$  occupy in each other's list in  $R_{BFF}$  (i.e.,  $\rho_j^R(i) = \rho_k^{R_{BFF}}(\ell) = \rho_\ell^{R_{BFF}}(k)$ ).

If player  $j$  appears at an earlier position on  $i$ 's list than  $j$  does on  $i$ 's list (i.e.,  $\rho_i^R(j) < \rho_j^R(i)$ ), then  $j$ 's incentive constraint for helping  $i$  in  $R$  is tighter than the incentive constraint that  $k$  and  $\ell$  face in  $R_{BFF}$  for helping each other. Alternatively, if  $j$  appears later on  $i$ 's list than  $i$  does, then  $i$ 's incentive constraint for helping  $j$  in  $R$  is tighter than the corresponding constraints for  $k$  and  $\ell$  in  $R_{BFF}$ . Finally, if  $i$  and  $j$  have the same position on each other's list (i.e.,  $\rho_i^R(j) = \rho_j^R(i)$ ), then the incentive constraint that  $i$  and  $j$  face in helping each other is of course the same as the constraint for  $k$  and  $\ell$ . We conclude that for every pair of players in the BFF, there is a corresponding constraint in  $R$  that is at least as tight. This implies that cooperation can be sustained in the BFF under bilateral enforcement, as required.  $\square$

Thus, some ranking profiles are more conducive to sustaining cooperation through bilateral enforcement than others. The intuition is that the ranking structure induces interdependencies between bilateral relationships. This is not the case in networks and random matching models of cooperation, where bilateral relations can be treated in isolation.

In general, ranking profiles capture settings where the members of a society interact directly with many other members but with varying degrees of intensity. This fits the setting of sequentially approaching friends for help. Sequentially approaching friends for help can be socially beneficial because there are no unnecessary attempts to help. There are many potential variants of this model that may fit different real-life settings. In a world where connecting with others becomes easier we find that the fine shades of relationships can be crucial.

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