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"Federal Directives, Local Discretion and the Majority Rule"

Key words: Federalism, Local Discretion, Directive, Partial Decentralization, Majority rule

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Federal Directives, Local Discretion and the Majority Rule[1]

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Abstract

We consider a federation in which citizens determine by federal majority rule a discretionary policy space which partially restricts the sovereignty of member states. Citizens first vote on the size of the discretionary space (the degree of local discretion), and then on its location on the policy space (the federal directive). Finally, each state votes on its respective policy within the discretionary space. This federal mechanism allows voters to express directly their trade-off between flexibility and policy harmonization.

We show that at the voting equilibrium, the federal directive is negatively sensitive to the preferences of nonmedian voters. Moreover, the degree of local discretion is too limited and insufficiently sensitive to the magnitude of externalities. Hence, the model shows that inadequate and excessively rigid federal interventions can emerge from a neutral and democratic decision process without agency costs or informational imperfections.

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1 Introduction

In its most simple formulation, federalism is about optimally allocating public responsibilities between the local and federal levels in order to exploit their comparative advantages. Economic models typically derive conditions under which a policy domain is better handled entirely at the local or at the central level. However, the functioning of actual federal systems does not fit this dichotomic description. Decision rights are often shared by different layers of governments. In the E.U., most state laws are transpositions of European directives. These directives impose some constraints but leave member states with a certain amount of leeway as to their implementation. Likewise, the Stability and Growth Pact boils down to a set of bounds on states' fiscal policies. In the U.S., the Sentencing Reform Act imposes sentencing ranges on state courts but grants them discretion within those ranges. In many countries, local taxes are subject to minima and maxima set by the central level. These interventions essentially define a *discretionary space* within which members keep residual sovereignty and can set the policy that best fits their specific needs.

The presumed advantage of partially restricting local discretion through federal constraints is that it allows to combine the comparative advantages of decentralization (responsiveness to local circumstances) and centralization (policy coordination). However, citizens and local policy makers often complain that federal directives are insufficiently sensitive to local needs and that the discretion devolved to the local level is too limited, even when the gains from policy coordination are negligible. In what follows, we shall refer to the former bias as the *preferences-matching* problem and to the latter as the *federal-encroachment* problem. From a public choice perspective, the usual suspects for these inefficiencies are the vested interests or the lack of information of federal bureaucrats. This has led some observers to conclude that a more democratic, bottom-up decision process could remedy these biases.

To investigate this claim, we analyze a model in which voters choose directly by majority rule the orientation and flexibility of the federal directives. The federation is composed of a finite number of jurisdictions which we call states for concreteness. In each state, a unidimensional policy has to be implemented. Voters have heterogeneous preferences but also care about the harmonization of policies across states. The federal intervention consists in imposing an interval $[L, R]$ within which states can choose the policies that best meet the needs of their constituents. This class of federal interventions encompasses complete decentralization (if $[L, R]$ include the ideal policies of every state) and unitarian centralization (if $L = R$). The size of the interval $|R - L|$ allows for different degrees of local discretion while a size-preserving translation of the interval allows for different policy orientations.

Citizens first vote at the federal level on the discretionary interval $[L, R]$ and then vote at the state level on their respective policies. Since we rule out institutional or informational imperfections, this federal intervention boils down to a simple *preferences-aggregation* mechanism. Hence, this model provides a transparent framework for investigating whether majoritarian decision making at the federal level leads to a satisfactory trade-off between coordination and flexibility.

We first analyze a voting game in which citizens vote separately on the left bound (L) and the right bound (R) of the discretionary interval. We show that irrespective of the voting sequence, the only voting equilibrium entails $L = R$, and the federal intervention is equivalent to complete centralization. The reason is that the vote on each bound opposes leftist and rightist voters,[1] so the median voters are pivotal at both stages and can thus impose their most preferred policy across the federation.

To avoid the tyranny of the median voters, we consider an—arguably more sensible—alternative voting game which generates different coalition structures at each voting round: citizens vote first on the size of the interval $\Delta = \frac{|R-L|}{2}$ (the degree of local discretion) and then on its location $\Gamma = \frac{L+R}{2}$ (the policy orientation). The first stage

[1] In this paper, rightist and leftist voters are defined as voters whose ideal policy is to the right and the left, respectively, of the ideal policy of the median voter. This distinction do not necessarily corresponds to the liberal and conservative categories.

can be interpreted as a constitutional referendum on the degree of decentralization while the second stage can be interpreted as a vote on the federal directive, i.e., a guideline from which states should not depart by more than Δ. The vote on the degree of decentralization pits moderate versus extreme voters while the vote on the federal directive opposes rightist and leftist voters.

For this voting game, the main results are the following: first, consistent with the aforementioned preferences-matching problem, the equilibrium federal directive Γ varies negatively with the preferences of nonmedian states. The magnitude of this bias depends on the *skewness* of the preference distribution: when the preference distribution is sufficiently skewed, the federal intervention is socially detrimental irrespective of the magnitude of externalities. Second, consistent with the aforementioned federal-encroachment problem, when the magnitude of externalities is small, the equilibrium degree of discretion Δ is too limited: the federal intervention is socially detrimental and leaves a majority of voters worse off than they would have been without the federal intervention. Third, when externalities are more severe, the federal intervention always receives the support of a majority of voters but its welfare effect depends on the *polarization* of preferences: when preferences are not too polarized, state discretion is still too limited and the federal intervention is socially detrimental. Conversely, when preferences are sufficiently polarized, state discretion is too broad but the federal intervention is Pareto improving.

These results show that rigid and inadequate federal directives, traditionally blamed on dysfunctional federal institutions or the neglect of local specificities by federal bureaucrats, can also emerge from a neutral and democratic decision process. One reason is that moderate voters have an incentive to impose most of the harmonization effort on the states that need the most flexibility: by doing so, they maximize policy coordination across the federation without restricting their own sovereignty.

However, this problem goes beyond the usual tyranny of the majority (of moderate voters) on the minorities (of extreme voters) because the federal intervention can make a majority of voters worse off. The reason is that the vote on the degree of discretion and the vote on the federal directive are driven by two different sets of pivotal voters with conflicting incentives. It turns out that this equilibrium feature further reduces local discretion in equilibrium. To see why, observe that the proponents of coordination (moderate voters pushing for a smaller Δ) have more homogeneous preferences than the proponents of local discretion (extreme right and extreme left voters pushing for a greater Δ). Indeed, the latter have diametrically opposite preferences when voting on Γ at the second stage. This lack of cohesion makes their induced preferences on Δ at the first stage less congruent than those of the proponents of coordination. Hence, our model formalizes the idea that in a federation composed of a homogeneous group of "core" states and a group of "peripheral" states with different centrifugal motives, the latter have difficulties forming a cohesive opposition to the centripetal influence of the former.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 lays out the model. Section 4 analyzes the case in which voters vote on each bound separately. Section 5 analyzes the case in which voters vote on the degree of discretion and on the federal guideline. Section 6 contains the welfare analysis. Section 7 illustrates our results in the case of a federation of three states and section 8 concludes. All proofs are relegated to the appendix.

2 Related Literature

Since the seminal work of Oates (1972), a large normative literature has analyzed the costs and benefits of complete centralization and decentralization.[2] We depart from it in two respects: decision rights are shared between the local and the central levels and the degree of decentralization is decided by a popular vote.

[2]See, e.g., Oates (1999) or Epple and Nechyba (2004) for a review, and Seabright (1996), Lockwood (2002), or Besley and Coate (2003) for a more political-economy approach.

A number of papers have analyzed federal systems in which fiscal responsibilities are shared between the local and the central level.[3] Most closely related to ours are the voting models of federal mandates (Kanbur and Keen 1993, Crémer and Palfrey 2000, 2006, and Monheim-Helstroffer and Obidzinski 2010) and of dual provision of public goods (Epple and Romano 2003, Hafer and Landa 2007, Lulfesman 2008). Since these papers consider free-riding rather than harmonization issues, the federal intervention takes the form of a unidirectional constraint. For this reason, the conflicts of interest between voters at the federal level have a different flavor. However, the federal-encroachment phenomenon is reminiscent of the "delimitation problem" highlighted by Crémer and Palfrey (2000)

Janeba (2006) considers a model of ideological externalities in which federalism is modelled as an interval of discretion (see his section 4.5). Because he considers only two types of voters, the voting equilibrium always entails complete uniformity.

Crémer and Palfrey (1996, 1999) propose a model in which, as in ours, voting on the architecture of the federation pits moderate versus extreme voters. Preferences for centralization stem from the voters' uncertainty about the location of the local and federal median voter rather than externalities. To allow for various degrees of centralization, they assume that the policy in each district is a weighted sum of the ideal policy of the representative voter at the federal and at the local level.

Hatfield and Padro-i-Miquel (2008) endogenize the architecture of a federation through a vote on the vertical allocation of public-good provision. They show that an intermediate degree of decentralization allows the capital-poor median voter to commit not to tax capital too heavily.

Finally, the literature on direct democracy has shown that referenda and public initiatives can increase the risk that the majority oppresses minority groups (Gamble 1997, Eule 1998). This model shows further that federal interventions governed by direct democracy may actually make a majority of voters worse off. Redoano and

[3] See, e.g., Schwager (1999), Wilson and Janeba (2005), Brueckner (2009), or Joanis (2009).

Scharf (2004) have shown that political integration is more likely under representative democracy because strategic delegation allows voters to commit to a small federal government (see also Feld et al. 2008).

3 The Model

We first introduce some notations. If x is a vector of $\mathbb{R}^N$, $\overline{x}$ denotes its mean $\frac{1}{N}\sum_n x_n$ and $med\,(x)$ denotes its median coordinate. For all $n \in \{1, .., N\}$ and $x_o \in \mathbb{R}$, (x_o, x_{-n}) denotes the vector x in which the n^{th} coordinate has been replaced by x_o.

3.1 The Federation

We consider a federation composed of an odd number N of jurisdictions that we call states for concreteness. For all $n \in \{1, .., N\}$, the policy of state n is denoted by x_n and the welfare of its residents is given by:

$$U_n(x) = -\left|x_n - \theta_n\right|^2 - \frac{\beta}{N}\sum_{m \neq n}\left|x_n - x_m\right|^2 . \tag{1}$$

The first term in (1) corresponds to the intrinsic preferences of state n, i.e., whether its policy x_n meets the specific needs θ_n of its constituents. The profile of state types $\theta \in \mathbb{R}^N$ allows for heterogeneity across states. For simplicity, we rule out heterogeneity within state.[4] The second term captures the gains from policy coordination. Depending on the policy considered, it can embody the legal uncertainty, litigation costs, or sense of unfairness generated by heterogeneous laws; the transaction costs and barriers to trade caused by a fragmented regulatory system; the fixed-cost duplication due to the lack of standardization of public services; or the barriers to mobility

[4] Intrastate preferences heterogneity would not change the results of the paper if we assume that votes are aggregated at the federal level through the "one state one vote" rule, i.e., if votes are aggregated first at the state level via intrastate majority rule and then at the federal level via interstate majority rule (see, e.g., Crémer and Palfrey 1996). The median voter in each state would simply become its representative voter.

generated by incompatible school curricula.[5] The parameter $\beta > 0$ determines the magnitude of these externalities. For welfare comparison, we use the usual utilitarian social welfare function $W = \sum_n U_n$.

By convention, $\theta_1 \leq ... \leq \theta_N$, and $\mu = \frac{N+1}{2}$ refers to the state with median preferences. The states n such that $\theta_n < \theta_\mu$ ($\theta_n > \theta_\mu$) are called *leftist* (*rightist*) states, although the type space should not necessarily be interpreted as a liberal/conservative ideological spectrum. We assume that no majority of voters have the same type so all voters agree that some degree of harmonization is desirable but no majority agree on the direction of harmonization.

Finally, a remark is in order about the setting of the model. We depart from the standard case of positive or negative spillovers and consider instead coordination externalities, i.e., externalities that are driven by the differences and incompatibilities between local policies (as in Garoupa and Ogus 2006, Carbonara and Parisi 2007, Baniak and Grazl 2009, or Loeper 2010), for three reasons. First, coordination externalities are at the center of many economically relevant issues in federal systems (e.g., legal and regulatory harmonization, labor mobility, and standardization of technical norms). Second, the case of spillovers has already received some attention in the federalism literature.[6] Finally, with either positive or negative externalities, the conflict of interest at the federal level typically pits low- versus high-demand voters.[7] This unidirectional conflict fails to capture a fundamental aspect of federalism: the voters who push for more centralization are usually more homogeneous than the voters who

[5] On the cost of legal and regulatory heterogeneity, see, e.g., Ribstein and Kobayashi (1996), Kox et al (2005) for an empirical analysis of the cost of regulatory fragmentation, and Marciano and Josselin (2002) for an edited volume of contributions on the topic. See Loeper (2010) for more references on the cost of policy heterogeneity in federal systems.

[6] See the references in section 2.

[7] It should be noted that in the case of joint provision of the public good (as opposed to unfunded federal mandates), if the federal provision is financed by progressive taxation, rich voters will also be larger contributor. In this case, coalitions of poor and rich voters versus moderate income voters might emerge in equilibrium. See Epple and Romano (2003) and Hafer and Landa (2008).

push for more decentralization, the reason being that the centrifugal pressures of the latter group have typically diverse motives.[8]

On the contrary, in this model, as we will see, the voters who prefer more coordination are residents of *core* states (loosely defined as states whose type is relatively close to θ_μ) while the voters who prefer more local discretion are residents of *peripheral* states (loosely defined as states whose type is relatively distant from θ_μ).

3.2 Centralization and Decentralization

Under decentralization, each state has complete sovereignty on its policy and maximizes the welfare of its constituents, taking the other policies as given. The corresponding equilibrium x^{dec} is given by

$$\text{for all } n = 1..N,\ x_n^{dec} = \frac{\theta_n + \beta\overline{\theta}}{1+\beta}. \tag{2}$$

One can easily show that x^{dec} is Pareto inefficient whenever states' types are not uniform: voters do not internalize interstate externalities and choose policies which are too heterogeneous. For instance, the policy x^* which maximizes W is given by

$$\text{for all } n = 1..N,\ x_n^{*} = \frac{\theta_n + 2\beta\overline{\theta}}{1+2\beta}, \tag{3}$$

which is a mean-preserving contraction of x^{dec}. Hence, a federal intervention could improve on decentralization by imposing some degree of coordination.

Typically, decentralization is compared to a centralized regime in which a uniform policy vector is chosen by federation-wide majority rule. Since induced preferences on uniform policies are single-peaked, the centralized voting equilibrium is

$$x^c = (\theta_\mu, ..., \theta_\mu)\,. \tag{4}$$

[8] The case of Europe and its various international treaties illustrate this point nicely. The countries pushing for more political integration are typically rich social democracies which share a long common history (France, Germany, Luxembourg, Belgium, Netherlands) while the countries opposing further political integration or refusing to adopt the Euro or to join the E.U. include countries as diverse as the U.K., ex-soviet satellites, Sweden, and Switzerland.

3.3 The Federal Coordination Mechanism

In this simple setup, as shown in Loeper (2010), unitarian centralization is never socially better than decentralization. The present paper analyzes a more flexible coordination mechanism which limits states' policy to an interval $[L, R]$ within which states have residual control. Citizens first vote at the federal level on the discretionary interval—the details of the voting scheme will be specified in the following sections—and then vote at the state level on their respective policy within the federal bounds $[L, R]$. This class of federal mechanism can accommodate different degrees of local discretion (via the range of the interval) and different policy orientations (via the location of the interval on the preferences spectrum).

Modeling the federal intervention as a discretionary interval captures the idea of a federal mechanism which coordinates the states by imposing broad constraints rather than micromanaging their policies. This model does not attempt to describe any real-world institution in detail. Instead, its main goal is to provide a tractable framework for analyzing how democratic forces trade off the need for coordination and for flexibility.

This type of federal intervention is, however, inspired by existing institutions. For instance, the European Commission leaves member states some leeway in the transposition and implementation of European directives, and often specify an interval of time for implementation.[9] In the U.S., the Sentencing Reform Act provides a grid of sentencing ranges for fines and jail times for each offense category. Its goal is to "provide certainty and fairness" while "avoiding unwarranted sentencing disparities" and "maintaining sufficient flexibility".[10] Likewise, several nations have a core school curriculum specifying a set of goals for student achievement but granting some degree

[9] Article 249 of the Treaty establishing the European Community: "A directive shall be binding, as to the result to be achieved, upon each Member State to which it is addressed, but shall leave to the national authorities the choice of form and methods."

[10] U.S. Code, Chapter 58, section 991 : United States Sentencing Commission; establishment and purposes.

of discretion to localities and schools as to organization of course system, hiring and training of teachers, choice of the textbooks, and the like. In Germany, the supreme court's decision in 2003 gives some discretion to the Länder on the regulation of religious signs in public schools (see Janeba 2006). In taxation, many central governments impose lower and upper bounds on the tax rates set by subnational governments.[11]

3.4 The State Equilibrium

We first characterize the equilibrium policies at the state level once the federal bounds have been set. A state equilibrium is a policy vector x such that for all n, x_n maximizes the welfare of the voters of state n on $[L, R]$ taking x_{-n} as given.

Proposition 1 *For all $L \leq R$, there is a unique state equilibrium denoted by $x(L, R)$. It is characterized by the number $l(L, R)$ and $r(L, R)$ of states constrained by the left and right bound, respectively:*

$$\begin{aligned} \textit{for } n &\leq l(L,R),\ x_n = L \textit{ and } L > \frac{\theta_n + \beta\overline{x}}{1+\beta}, \\ \textit{for } l(L,R) &< n \leq N - r(L,R),\ x_n = \frac{\theta_n + \beta\overline{x}}{1+\beta}, \\ \textit{for } n &> N - r(L,R),\ x_n = R \textit{ and } R < \frac{\theta_n + \beta\overline{x}}{1+\beta}. \end{aligned}$$

4 Voting on the Federal Bounds

At the federal level, a natural way to aggregate votes on the discretionary space $[L, R]$ is to vote separately on each bound. For instance, if the federation votes first on L and then on R, the voting game is as follows:

[11] Revelli (2010) documents the effect of federal minima and maxima on vehicle registration tax, electricity taxation, and waste management surcharge in Italy. See Joumard and Kongsrud (2003) for further discussion on this issue.

1. The federation votes on the left bound L,
2. The federation votes on the right bound R,
3. Each state votes on its respective policy within $[L, R]$.

The game is solved by backward induction, and votes are aggregated at the federal level by simple majority rule (henceforth SMR). At the third stage, for any (L, R), the subgame equilibrium is the state equilibrium $x(L, R)$ characterized in proposition 1. At the second stage, for any first-stage outcome L, $R^e \in [L, +\infty[$ is a subgame equilibrium if for all $R \geq L$, $x(L, R)$ is not preferred by SMR to $x(L, R^e)$. Finally, a federal equilibrium is a pair (L^e, R^e) such that R^e is a subgame equilibrium following L^e, and for any L and R such that R is a subgame equilibrium following L, $x(L, R)$ is not preferred by SMR to $x(L^e, R^e)$. It turns out that this issue-by-issue decomposition is equivalent to complete centralization.

Proposition 2 *If L and R are voted upon sequentially in any order or simultaneously,*[12] *the unique equilibrium is given by $L = R = \theta_\mu$.*

Observe that the equilibrium entails complete uniformity irrespective of the severity of externalities, and the uniform policy is independent of nonmedian preferences. This two features can be viewed as a form of federal encroachment and preference matching, respectively. The intuition behind proposition 2 is that both voting stages oppose rightist to leftist voters. This makes the voters of the median state pivotal at both stages and allows them to secure their most preferred policy vector.

However, complete rigidity is not an inevitable consequence of majoritarian decision-making. As there is no Condorcet winner on $F = \{(L, R) \in \mathbb{R}^2 : L \leq R\}$, the outcome of a sequential voting mechanism depends on how the space of alternative F is divided

[12] By simultaneously, we mean that L and R are voted upon simultaneously but separately as in Schepsle (1979): an equilibrium is a pair (L^e, R^e) such that for all $L \leq R^e$, $x(L, R^e)$ is not SMR-preferred to $x(L^e, R^e)$, and for all $R \geq L^e$, $x(L^e, R)$ is not SMR-preferred to $x(L^e, R^e)$.

into issues, i.e., how the referendum questions are framed. For this reason, the next section analyzes a different issue-by-issue decomposition.[13]

5 Voting on Directives and Local Discretion

In order to avoid the tyranny of the median voters and obtain a more flexible outcome, the preceding result suggests a referendum design that induces different coalitional structures, and thus different pivotal voters, at each voting stage. For this reason, we consider a natural alternative to the previous voting sequence:

1. The federation votes on the size of the discretionary interval $\Delta = \frac{R-L}{2}$,
2. The federation votes on the location of the discretionary interval $\Gamma = \frac{L+R}{2}$,
3. Each state votes on its respective policy within $[\Gamma - \Delta, \Gamma + \Delta]$.

As we will see, the first stage will oppose coalitions of extreme versus moderate voters, while at the second stage, leftist and rightist coalitions will emerge. Each stage has a meaningful interpretation: the first stage can be viewed as a constitutional referendum on the degree of decentralization.[14] The second stage can be interpreted as a popular vote on the federal directive, a guideline from which states should not depart by more than Δ. The timing of the voting game reflects the fact that the degree of local discretion implied by the constitution is more resistant to change than the federal directive.

For notational convenience, for any function f of (L, R), we will write $f(\Delta, \Gamma)$ as a shortcut for $f(\Gamma - \Delta, \Gamma + \Delta)$. The game is solved by backward induction: at the

[13] Kramer (1972) and Schepsle (1979) first argued that the details of the institutional rules used to aggregate votes may influence the outcome of the voting game. Ferejohn and Krehbiel (1987) made a similar point to ours in the case of a vote on a two-dimensional budget.

[14] The degree of coordination that a federal administration can impose can be interpreted more broadly as the institutional features that determine its strength, such as the efficacy of its coercive levers on member states or its financial and legal resources.

third stage, for all (Γ, Δ), the subgame equilibrium is the state equilibrium $x(\Gamma, \Delta)$ defined in proposition 1. At the second stage, for any $\Delta \geq 0$, the set of subgame equilibria $\mathbf{\Gamma}(\Delta)$ is the set of Γ such that for all Γ', $x(\Gamma', \Delta)$ is not preferred by SMR to $x(\Gamma, \Delta)$. A federal equilibrium is a pair (Γ^e, Δ^e) such that $\Gamma^e \in \mathbf{\Gamma}(\Delta^e)$ and for all $\Delta \geq 0$ and all $\Gamma \in \mathbf{\Gamma}(\Delta)$, $x(\Gamma, \Delta)$ is not preferred by SMR to $x(\Gamma^e, \Delta^e)$.

5.1 Federal Directives

The state equilibrium defines an induced utility function $V_n(\Gamma, \Delta)$ for each state n. Note that for a given Δ, $l(\Gamma, \Delta)$ and $r(\Gamma, \Delta)$ as defined in proposition 1 have discontinuous jumps in Γ, and this induces a kink in $V_n(\Gamma, \Delta)$, so $V_n(\Gamma, \Delta)$ may have multiple peaks in Γ. Nevertheless, a Condorcet winner always exists at the second stage.

Proposition 3 *For all $\Delta \geq 0$, the second-stage equilibria $\mathbf{\Gamma}(\Delta)$ are the most preferred Γ of the voters of the median state. For all $\Gamma \in \mathbf{\Gamma}(\Delta)$, the policy of the median state is unconstrained, i.e., $l(\Gamma, \Delta) < \mu < r(\Gamma, \Delta)$, and*

$$\Gamma = \theta_\mu + \frac{l(\Gamma, \Delta) - r(\Gamma, \Delta)}{l(\Gamma, \Delta) + r(\Gamma, \Delta)} \Delta. \tag{5}$$

Proposition 3 suggests that, in equilibrium, the federal directive is not positively responsive to the preferences of peripheral states. Indeed, the farther from θ_μ the types of the rightist states are, relative to the types of the leftist states, the greater r relative to l, so, from (5), the more leftist the federal directive. The intuition for this bias is the following: when choosing the federal directive, the voters of the median state dictate which states will be constrained or unconstrained by the federal bounds. This has two consequences. First, they will vote for a directive which lets them choose the policy they prefer. Roughly speaking, this motive corresponds to the term θ_μ in (5). Second, conditional on being unconstrained by the federal bounds, their most preferred directive is the one that minimizes policy heterogeneity. For this reason, they will choose a Γ which constrains the most extreme states. This motive explains the term $\frac{l-r}{l+r}\Delta$ in (5).

By imposing most of the harmonization effort on the states that need the most flexibility, the voters of the median state maximize policy coordination without restricting their discretion, more so than under the same nominal degree of local discretion Δ but with a socially optimal federal guideline Γ^*. Indeed, Γ^* is increasing in the type of constrained states and thus leans towards the most extreme side of the preferences distribution.[15] As the next proposition shows, the equilibrium directive does exactly the opposite.

Definition 1 *A distribution of type θ is skewed to the right (left) if for all $n \neq \mu$, $\frac{\theta_n+\theta_{2\mu-n}}{2} > \theta_\mu$ ($\frac{\theta_n+\theta_{2\mu-n}}{2} < \theta_\mu$).*

Proposition 4 *The correspondence $\mathbf{\Gamma}(\Delta)$ is weakly decreasing in nonmedian types for the strong set order $\succeq$:[16] for all θ, θ' such that $\theta_\mu = \theta'_\mu$ and $\theta \leq \theta'$, $\mathbf{\Gamma}^\theta(\Delta) \succeq \mathbf{\Gamma}^{\theta'}(,\Delta)$.*
Moreover, if θ is skewed to the right (left), then for all $\Gamma \in \mathbf{\Gamma}(\Delta)$, $\Gamma \leq \theta_\mu$ ($\Gamma \geq \theta_\mu$).

In words, a distribution of preferences is skewed to the right if rightist states are more extreme than leftist states. In this case, the median voters will bias the directive towards the moderate (i.e., left) side of the preferences spectrum so as to reduce the leeway of the most extreme (i.e., rightist) states and force their policies to be more aligned with their own preferences. Hence, proposition 4 shows that the preference-matching problem mentioned in the introduction can emerge in a neutral and open democratic decision process without institutional imperfections.

[15] For instance, for any $n \leq l(\Gamma^*, \Delta)$, $x(\Gamma^*, \Delta)$ is locally constant in θ_n, so $\frac{\partial^2 W(U(x(\Gamma^*,\Delta)))}{\partial\theta_n \partial\Gamma} = \frac{\partial}{\partial\Gamma}\left(\frac{\partial U_n}{\partial\theta_n}\right)(x(\Gamma^*, \Delta))$. We show in the appendix (lemma 2) that $\frac{\partial U_n}{\partial\theta_n}$ is increasing in Γ. Therefore, from Topkis theorem, Γ^* must be increasing in θ_n.

[16] For all $X, Y \subset \mathbb{R}$, $X \succeq Y$ if for all $(x, y) \in (X, Y)$, $\max(x, y) \in X$ and $\min(x, y) \in Y$.

The next proposition shows further that when preferences are sufficiently skewed, this bias can make the federal intervention socially detrimental irrespective of the magnitude of externalities and the degree of local discretion.

Proposition 5 *If the preference distribution is sufficiently skewed, i.e., if* $|\theta_n - \theta_\mu| \gg |\theta_m - \theta_\mu|$ *for all* $n > \mu > m$ *or for all* $n < \mu < m$*, then for all* $\Delta \geq 0$ *and all* $\Gamma \in \mathbf{\Gamma}(\Delta)$*,* $x(\Gamma, \Delta)$ *is socially worse than decentralization.*

This result contrasts with the literature on fiscal federalism which argues that the welfare effect of centralization hinges on the heterogeneity of local preferences and the magnitude of externalities.

5.2 State Discretion

Let us now consider the first stage of the voting game. Notice that as Δ varies, the number of left- and right-constrained states l and r changes, and (5) implies that $\mathbf{\Gamma}(\Delta)$ jumps discontinuously, possibly nonmonotonically in Δ. As a consequence, induced preferences on Δ are neither single-peaked, nor order-restricted, nor continuous, so a Condorcet winner may not exist at the first stage.[17] In section 7, we show that a federal equilibrium always exists in the specific case of three states. In the general case, we restrict our attention to local majority rule equilibria as introduced by Kramer and Klevorick (1973).

Definition 2 (Γ^e, Δ^e) *is a local federal equilibrium (henceforth LFE) if* $\Gamma^e \in \mathbf{\Gamma}(\Delta^e)$*, if there exists a neighborhood* N *of* Δ^e *such that* $X^N = \{x(\Gamma, \Delta) : \Delta \in N, \Gamma \in \mathbf{\Gamma}(\Delta)\}$ *is not a singleton and if for all* $x \in X^N$*,* x *is not preferred by SMR to* $x(\Gamma^e, \Delta^e)$*.*[18]

[17] Denzau and Mackay (1981) and Cremer and Palfrey (2006) provide examples of nonexistence of a voting equilibrium in a similar setup.

[18] The requirement that X^N is not a singleton rules out trivial equilibria in which the policies are locally constant on both sides of Δ, which could arise if Δ is sufficiently large.

Since this equilibrium concept considers only local deviations, it is more permissive than the standard Condorcet winner requirement, and equilibrium multiplicity might be a problem. Fortunately, we will see that for our purpose, the concept is sufficiently discriminating to derive clear welfare results.

Proposition 6 *A LFE exists.*[19] *At any LFE* (Γ^e, Δ^e), $\Delta^e > 0$ *and at least a majority of states are constrained:* $l(\Gamma^e, \Delta^e) + r(\Gamma^e, \Delta^e) \geq \frac{N+1}{2}$.

Hence, opposing moderate to extreme voters at the first stage guarantees that the equilibrium degree of local discretion is positive, and that the voters of the median state cannot impose their most preferred policy. To see why a majority of states must be constrained at any LFE, it is helpful to notice the following: If $\mathbf{\Gamma}(\Delta)$ is single-valued and continuous on some interval I, we show in the appendix (see lemma 4) that for all $\Delta \in I$,

$$\begin{aligned} \text{for } n &\leq l, \frac{\partial [x_n(\mathbf{\Gamma}(\Delta), \Delta)]}{\partial \Delta} < 0, \\ \text{for } n &> N-r, \frac{\partial [x_n(\mathbf{\Gamma}(\Delta), \Delta)]}{\partial \Delta} > 0, \\ \text{for } l &< n \leq N-r, \frac{\partial [x_n(\mathbf{\Gamma}(\Delta), \Delta)]}{\partial \Delta} = 0. \end{aligned} \tag{6}$$

Thus, as the degree of local discretion decreases, the policies of the unconstrained states are unaffected by Δ, and the harmonization efforts are borne entirely by the constrained states. For this reason, the unconstrained states unanimously prefer less local discretion. As long as they are a majority, they will form a winning coalition of free riders pushing for less local discretion.

[19] Observe that since induced preferences are not continuous, we cannot resort to the existence theorem of Kramer and Klevorick (1973).

6 Welfare Analysis

6.1 Small Externalities

Proposition 6 implies that in equilibrium, the pivotal voters are necessarily from a state constrained by the federal bounds. When voting on Δ, they internalize neither the benefits of harmonization for the unconstrained states nor the benefits of flexibility for the other constrained states. For small coordination costs, the former is negligible compared to the latter, so the pivotal voters, together with the voters of unconstrained states, have an incentive to excessively constrict the leeway of peripheral states to secure negligible coordination gains. The next proposition confirms this intuition.

Proposition 7 *For all θ, there exists $\underline{\beta} > 0$ such that for all $\beta < \underline{\beta}$,*

a) any LFE is socially worse than decentralization and makes a majority of voters worse off.

b) for any LFE (Γ^e, Δ^e), fixing $\Gamma = \Gamma^e$, the welfare of a majority of voters is strictly increasing in Δ around Δ^e.

However, proposition 7 part a) suggests that the above intuition—a majority of moderate voters restricting the leeway of a minority of extreme voters—is not the only cause of the excessive rigidity of the federal intervention. Indeed, this intuition cannot explain why—contrary to Crémer and Palfrey (2000), Hafer and Landa (2007), or Lulfesman (2008)—the federal intervention makes a majority of voters worse off.

That result is somewhat counter-intuitive since the class of federal interventions we consider encompasses decentralization and all decisions are taken by majority rule. The reason is that the pivotal voter at the first stage is not the median voter as in the second stage, and since the two types of pivotal voters have conflicting incentives, their choices may leave a majority of voters worse off.

To understand why having two different pivotal voters further restricts local discretion, notice that the pivotal voters at the first stage take into account not only the *direct* effect of Δ on the trade-off between policy coordination and flexibility, but also the *strategic* effect of Δ on the second-stage equilibrium, i.e., its effect on the incentives of the median state when voting on Γ. With respect to the direct effect, peripheral states unanimously prefer more discretion. With respect to the strategic effect, their interests collide since at the second stage, rightist and leftist states have diametrically opposing preferences. To fix ideas, suppose that, locally, $\mathbf{\Gamma}(\Delta)$ is leaning to the left as Δ decreases. In this case, the leftist voters will strategically vote for less discretion. Together with the voters from core states—who always prefer less discretion—they will form a majority pushing for more coordination. In words, the heterogeneity of peripheral states makes their incentives at the first voting stage less aligned than that of core states, so the former cannot form a cohesive opposition to the centripetal influence of the latter.

6.2 Large Externalities

When the magnitude of externalities increases, the coordination gains from the federal intervention might counterbalance its inefficiencies so its welfare effect is ambiguous and depends on the preferences distribution. We already know from proposition 5 that the federal intervention is socially detrimental when the preference distribution is sufficiently skewed. In what follows, we analyze the opposite case of symmetric preference distribution, i.e., for all n, $\frac{\theta_{2\mu-n}+\theta_n}{2}=\theta_\mu$. We show in the appendix that in this case, at any LFE (Γ^e, Δ^e), $\mathbf{\Gamma}(\Delta^e)=\{\theta_\mu\}$, which is the socially optimal directive for any $\Delta \geq 0$. Since the second-stage equilibrium is optimal, we can focus on the first-stage vote. As we will see, the main determinant of the equilibrium degree of decentralization is then the polarization of preferences:

Definition 3 *The degree of polarization $\pi(\theta)$ of a preference distribution θ is*

$$\pi(\theta)=\frac{med_n\left|\theta_n-\theta_\mu\right|}{\frac{1}{N}\sum_n\left|\theta_n-\theta_\mu\right|}. \tag{7}$$

A preference distribution is maximally polarized if there exists $\delta > 0$ such that for $n < \left\lceil \frac{N+1}{4} \right\rceil$, $\theta_n = \theta_\mu - \delta$, for $n > n - \left\lceil \frac{N+1}{4} \right\rceil$, $\theta_n = \theta_\mu + \delta$ and $\theta_n = \theta_\mu$ otherwise.[20,21]

Roughly speaking, a preference distribution is polarized if it comprises a homogeneous group of rightist states and a homogenous group of leftist states at similar distances from the median type, and these two groups command a majority which can oppose the centripetal influence of moderate voters.[22]

Proposition 8 *Let θ be a symmetric preference distribution, (Γ^e, Δ^e) be a LFE, and Δ^* be the socially optimal degree of local discretion given $\Gamma = \theta_\mu$.*

a) If $\pi(\theta) \leq \frac{1+\frac{3}{2}\beta}{1+2\beta}$, then $\Delta^e < \Delta^*$.

b) If θ is maximally polarized and $N > 3$, then there exists $\overline{\beta}$ such that for all $\beta \geq \overline{\beta}$, $\Delta^e > \Delta^*$.

c) For all β, there exists $\underline{\pi}$ such that if $\pi(\theta) < \underline{\pi}$, any LFE is socially worse but majority preferred to decentralization.

d) If θ is maximally polarized, then there exists $\overline{\beta}$ such that for all $\beta \geq \overline{\beta}$, any LFE Pareto dominates decentralization.

Part a) implies that if $\pi(\theta) < \frac{3}{4}$, then for all β, the equilibrium degree of local discretion is too small:[23] a low degree of polarization exacerbates the federal-encroachment problem. Conversely, part b) shows that when preferences are suffi-

[20] $\left\lceil \frac{N+1}{4} \right\rceil$ denotes the smallest integer weakly greater than $\frac{N+1}{4}$.

[21] One can show that a maximally polarized preferences profile indeed maximizes $\pi(\theta)$ among symmetric profiles of type, and is a global maximizer when $\frac{N+1}{4}$ is an integer.

[22] Notice that the notion of polarization is orthogonal to the notion of heterogeneity since $\pi(\theta)$ is invariant by affine transformation of θ. See Esteban and Ray (1994) for a related notion of polarization.

[23] To see this, notice that for all $\beta \geq 0$, $\frac{1+3/2\beta}{1+2\beta} > 3/4$. To see what $\pi(\theta) < 3/4$ means, notice that if each θ_n is i.i.d., normally distributed, then as $N \to \infty$, $\pi(\theta) \to 0.84$, while if they are chi-square distributed on each side of their mean, $\pi(\theta) \to 0.46$.

ciently polarized and externalities are sufficiently large, the equilibrium entails an inefficiently low degree of policy harmonization.

Comparing proposition 8 part b) with proposition 7, we see that when preferences are polarized, the equilibrium degree of decentralization Δ^e is insufficiently sensitive to the magnitude of externalities: the federal intervention entails too much coordination when the gains from coordination are small and too little coordination when it is crucial.

Part c) and d) show that when the gains from coordination are large, the federal intervention receives the support of majority of voters, but it is not necessarily socially beneficial. Observe that contrary to the common wisdom, the federal intervention can be socially detrimental even when externalities are arbitrarilly large and preferences are homogeneous.

Combining propositions 5 and 8, we see that skewness exacerbates the perverse incentive of the median voters at the second stage—the preferences-matching problem—while polarization counterbalances the centripetal pressures of moderate voters at the first stage—the federal-encroachment problem. Hence, unless the distribution of preferences is sufficiently symmetric and polarized and externalities are sufficiently large, even a perfectly democratic federal system can be excessively rigid and poorly responsive to local preferences.

7 Triadic Federations

In this section, we illustrate our results in the case of a federation composed of three homogeneous groups of leftist, moderate, and rightist states. Formally, a preference profile is triadic when

$$\theta_1 = .. = \theta_\lambda < \theta_{\lambda+1} = .. = \theta_\mu = .. = \theta_{N-\rho} < \theta_{N-\rho+1} = .. = \theta_N,$$

for some $\lambda, \rho \in \left\{1, .., \left\lfloor \frac{N+1}{2} \right\rfloor\right\}$ such that $\rho + \lambda > \frac{N+1}{2}$. The latter condition means that no single group commands a majority but any coalition of two groups does.

Proposition 9 *If θ is triadic, the unique LFE (Γ^e, Δ^e) is a (global) federal equilibrium. It is given by*

$$\Delta^e = \min\left\{D^o_\lambda, D^o_\rho, D_\lambda, D_\rho\right\} \ \textit{and}\ \Gamma^e = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta^e,$$

where

$$D^o_\lambda = \sqrt{\frac{(\lambda+\rho)(N+\beta\lambda+\beta\rho)}{(\beta+1)^2\rho(N+\beta\rho)}}\frac{\theta_\mu-\theta_1}{2}, \quad D^o_\rho = \sqrt{\frac{(\lambda+\rho)(N+\beta\lambda+\beta\rho)}{(\beta+1)^2\lambda(N+\beta\lambda)}}\frac{\theta_N-\theta_\mu}{2},$$
$$D_\lambda \equiv \frac{\lambda+\rho}{\rho+\frac{\beta}{N}\left(\lambda^2+\rho\lambda+N\rho\right)}\frac{\theta_\mu-\theta_1}{2}, \qquad D_\rho \equiv \frac{\lambda+\rho}{\lambda+\frac{\beta}{N}(\rho^2+\lambda\rho+N\lambda)}\frac{\theta_N-\theta_\mu}{2}.$$

The intuition behind proposition 7 is the following. D^o_λ and D^o_ρ are the degrees of local discretion below which leftist and rightist states are constrained, respectively. As shown in (6), $\Delta > \min\left\{D^o_\lambda, D^o_\rho\right\}$ cannot be a LFE because at Δ, the voters of unconstrained states form a majority pushing for less discretion. D_λ and D_ρ are the ideal degrees of discretion of the citizens of leftist and rightist states, respectively, conditional on both being constrained by the federal bounds. Therefore, $\Delta > \min\{D_\lambda, D_\rho\}$ cannot be a LFE because the voters of median-type states always prefer a lower Δ and either rightist or leftist states are constrained but their voters prefer less discretion. Likewise, at $\Delta < \min\left\{D^o_\lambda, D^o_\rho, D_\lambda, D_\rho\right\}$, both rightist and leftist states are constrained and their voters prefer more discretion. Therefore, the only LFE candidate is $\Delta = \min\left\{D^o_\lambda, D^o_\rho, D_\lambda, D_\rho\right\}$.

When β is sufficiently small, we know from proposition 7 that the federal intervention excessively restricts local discretion. In the case of a triadic federation with $\lambda = \rho$, the rigidity of the federal intervention takes the following form: As $\beta \to 0$, $D^o_\lambda < D_\lambda$ and $D^o_\rho < D_\rho$ so $\Delta^e = \min\left\{D^o_\lambda, D^o_\rho\right\}$. Moreover, $\lim_{\beta\to 0}\left(D^o_\lambda, D^o_\lambda\right) = \left(\frac{\theta_\mu-\theta_1}{\sqrt{2}}, \frac{\theta_N-\theta_\mu}{\sqrt{2}}\right)$ so

$$\lim_{\beta\to 0}\Delta^e = \frac{1}{\sqrt{2}}\min\left\{\theta_\mu-\theta_1, \theta_N-\theta_\mu\right\},$$

while conditional on $\Gamma^e = \theta_\mu$, the socially optimal degree of decentralization is

$$\lim_{\beta\to 0}\Delta^* = \max\left\{\theta_\mu-\theta_1, \theta_N-\theta_\mu\right\}.$$

By comparing the two expressions above, we see that the excessive rigidity of the federal intervention is increasing in the skewness of the preference distribution.

When β is sufficiently large, proposition 8 shows that the federal intervention can be Pareto improving. In the case of a triadic federation, it can even be Pareto optimal. To see this, observe that for β sufficiently large, $D_\lambda^o > D_\lambda$ and $D_\rho^o > D_\rho$ so $\Delta^e = \min\{D_\lambda, D_\rho\}$. If we assume further that the preference distribution is symmetric, i.e., $|\theta_\mu - \theta_1| = |\theta_N - \theta_\mu|$ and $\lambda = \rho$, then $D_\lambda = D_\rho$. In this case, both rightist and leftist states are pivotal at the first stage. Since the second-stage equilibrium $\Gamma^e = \theta_\mu$ is optimal, the equilibrium outcome is Pareto optimal.

The next proposition characterizes the welfare effect of the federal intervention when $\lambda \neq \rho$ or $|\theta_\mu - \theta_1| \neq |\theta_N - \theta_\mu|$.

Proposition 10 *For β sufficiently large,*

a) if $|\theta_\mu - \theta_1| = |\theta_N - \theta_\mu|$ and $\lambda = \rho$, the federal equilibrium Pareto dominates decentralization and is Pareto optimal, while if $\lambda|\theta_\mu - \theta_1| \neq \rho|\theta_N - \theta_\mu|$, some states strictly prefer decentralization,

b) if $|\theta_\mu - \theta_1| = |\theta_N - \theta_\mu|$ and $\lambda \neq \rho$, the federal equilibrium is socially better than decentralization,

c) if $\lambda = \rho$ and $|\theta_\mu - \theta_1| \neq |\theta_N - \theta_\mu|$, decentralization is socially better than the federal equilibrium,

d) for all preference profiles, the federal equilibrium is majority preferred to decentralization.

The intuition behind part a) and b) is the following: if $|\theta_\mu - \theta_1| = |\theta_N - \theta_\mu|$, rightist and leftist voters trade off policy coordination and flexibility in a similar manner. This mitigates the gap between the private cost and the social cost of the pivotal voters, and the federal intervention is socially beneficial. Parts c) and d) are in line with proposition 8: the federal intervention always receive a majority support independently of its welfare effect.

8 Conclusion

This paper considers a class of federal coordination mechanisms that impose some degree of harmonization while granting states some degree of sovereignty. We investigate whether a neutral and democratic decision process at the federal level can lead to a satisfactory trade-off between flexibility and policy coordination. To do so, we let citizens determine by federal majority rule the degree of discretion granted to the states and the policy orientation of the federal intervention.

We show that in equilibrium, the policy orientation of the federal directive is negatively responsive to voters' preferences and the degree of local discretion is typically too limited. The excessive rigidity of the federal intervention is due not only to the usual problem of the majority (of moderate voters) enslaving the minority (of extreme voters) but also to the fact that extreme voters are less homogenous than moderate voters so the former cannot form a cohesive opposition to the centripetal influence of the latter.

9 Appendix

Throughout, V^{dec} denotes the profile of welfare under decentralization. Lemma 1 proves proposition 1. Lemma 2 gives conditions under which SMR preferences on $\{L, R\}$ aggregate transitively.

Lemma 1 *For all Γ, Δ, the state equilibrium $x(L,R)$ characterized in proposition 1 is unique and $x_n(L,R)$ is weakly increasing in θ, L, R and n. For any affine map $L(.)$ and $R(.)$, $x(L(\lambda), R(\lambda))$ is continuous, piecewise affine in λ and*

$$\frac{\partial \overline{x}(L,R)}{\partial L} = \frac{(1+\beta)\,l}{N+\beta(l+r)} \text{ and } \frac{\partial \overline{x}(L,R)}{\partial R} = \frac{(1+\beta)\,r}{N+\beta(l+r)}. \tag{8}$$

Proof. The first-order condition of 1) immediately gives that a state equilibrium x is a fixed point of the best-response function:

$$\forall n,\ f_n(x) = \max\left(L, \min\left(R, \frac{\theta_n + \beta\overline{x}}{1+\beta}\right)\right). \tag{9}$$

Since f is a contraction for the sup norm on $[L,R]^N$, the state equilibrium exists and is unique. From (9), $x_n(L,R)$ is weakly increasing in n. Since f is weakly increasing in x and in (θ, L, R), Villas-Boas (1997, theorem 4) implies that its fixed point $x(L,R)$ is weakly increasing in (θ, L, R).

Observe that for all $A, B \subset N$, the set of $\{L, R\}$ such that the constraint $L \leq \frac{\theta_n + \beta\overline{x}}{1+\beta}$ is binding for $n \in A$ and the constraint $\frac{\theta_n+\beta\overline{x}}{1+\beta} \leq R$ is binding for $n \in B$ is a convex subset of $\mathbb{R}^2$, since if x and x' are solutions of (9) for (L,R) and (L', R'), respectively, then $\alpha x + (1-\alpha)x'$ is solution of (9) for $\alpha(L,R) + (1-\alpha)(L',R')$. The implicit function theorem implies that $x(L,R)$ is differentiable on the interior of these convex sets, and since there is a finite number of subsets A and B, for all affine maps $L(.)$ and $R(.)$, $x(L(\lambda), R(\lambda))$ is piecewise affine in λ. Differentiating (9) wrt L and R, summing over n, and solving for $\frac{\partial\overline{x}(L,R)}{\partial L}$ and $\frac{\partial\overline{x}(L,R)}{\partial R}$, we get (8). ■

Lemma 2 *For all $L \leq L', R \leq R'$, $V_n(L',R') - V_n(L,R)$ is weakly increasing in n. In particular, SMR preferences between $[L,R]$ and $[L',R']$ coincide with the preferences of the voters of the median state.*

Proof. Observe that the induced utility function $V_n(L,R)$ of state n can be written as

$$W_n(t_n, x) = \max_{y \in [L,R]} \left(-|y - t_n|^2 - \frac{\beta}{N} \sum_{m=1}^{N} |y - x_m|^2 \right), \tag{10}$$

for $x = x(L,R)$ and $t_n = \theta_n$. Let $y^*(t_n, x)$ be the maximizer of (10). From the envelope theorem, for all $t_n \in \mathbb{R}$, $\frac{\partial W_n}{\partial t_n} = 2(y^*(t_n,x) - t_n)$, so

$$V_n(L,R) - V_m(L,R) = \int_{\theta_n}^{\theta_m} 2\left(y^*(t, x(L,R)) - t\right) dt,$$

which in turns implies

$$V_n(L',R') - V_n(L,R) - (V_m(L',R') - V_m(L,R)) = 2 \int_{\theta_n}^{\theta_m} \begin{pmatrix} y^*(t, x(L',R')) \\ -y^*(t, x(L,R)) \end{pmatrix} dt. \tag{11}$$

Observe that if $W(t_n, x, y)$ denotes the maximand of (10), for all m, $\frac{\partial^2 W}{\partial y \partial x_m} > 0$. Hence W is supermodular in (y, x_m) and Topkis theorem implies that y^* is weakly increasing in x. From lemma 1, if $L' \geq L$ and $R' \geq R$ then $x(L', R') \geq x(L, R)$, so the integrand in (11) is non negative, which proves the first part of the lemma.

To prove the second part, notice that if the median voters strictly prefer (L', R') to (L, R), so do the voters of the states $n \geq \mu$. ■

Lemma 3 *For all $L \leq R$, the state equilibrium $x(L, R)$ is equal to $x^{dec}(t)$ (see (2)) where t is given by $t_n = \max\left(\underline{t}, \min\left(\bar{t}, \theta_n\right)\right)$ for all n with*

$$\begin{aligned} \underline{t} &= (1+\beta) L - \beta \overline{x}(L, R), \\ \bar{t} &= (1+\beta) R - \beta \overline{x}(L, R). \end{aligned} \tag{12}$$

At the decentralized equilibrium, for all $m \neq n$,

$$\frac{\partial V_n^{dec}}{\partial \theta_p} = \frac{2\beta}{N(1+\beta)} \left(\theta_n - x_p^{dec}\right). \tag{13}$$

Proof. The map $\theta \to x^{dec}(\theta)$ defined in (2) can be inverted as follows: $\theta_n = (1+\beta) x_n^{dec} - \beta \overline{x^{dec}}$. Substituting $x^{dec} = x(L, R)$ in the previous expression and using (9), we get the "as if" profile of type t.

Observe that for all n, the welfare under decentralization is given by $V_n^{dec} = W_n\left(\theta_n, x^{dec}(\theta)\right)$ where W is defined in (10) with $[L, R] = [\theta_1, \theta_N]$. Using the envelope theorem, we have that for $p \neq n$,

$$\begin{aligned} \frac{\partial V_n^{dec}}{\partial \theta_p} &= \frac{2\beta}{N} \sum_{m \neq p} \left(x_n^{dec} - x_m^{dec}\right) \frac{\partial x_m^{dec}}{\partial \theta_p} + \frac{2\beta}{N} \left(x_n^{dec} - x_p^{dec}\right) \frac{\partial x_p^{dec}}{\partial \theta_p}, \\ &= \frac{2\beta}{N(1+\beta)} \left[\sum_{m \neq p} \left(x_n^{dec} - x_m^{dec}\right) \frac{\beta}{N} + \left(x_n^{dec} - x_p^{dec}\right) \left(1 + \frac{\beta}{N}\right)\right], \end{aligned} \tag{14}$$

where $\frac{\partial x_m^{dec}}{\partial \theta_p}$ and $\frac{\partial x_p^{dec}}{\partial \theta_p}$ are calculated from (2). Substituting the first-order condition of (10, i.e., $\frac{\beta}{N} \sum_m \left(x_n^{dec} - x_m^{dec}\right) = \theta_n - x_n^{dec}$, in (14), we get (13). ■

9.1 Proofs in Section 4

Proof of proposition 2. Suppose for concreteness that citizens vote first on L and then on R. From lemma 2, the Condorcet winners at the second stage will be the most preferred R of the median state. which exists by continuity of $x(L, R)$.

At the first stage, for any $L \geq \theta_\mu$, one can easily see from (1) that the most preferred L of the median state is $L = R$. Hence, from lemma 2, the median states and all leftist states will prefer $L = \theta_\mu$ to any $L > \theta_\mu$ at the first stage.

Suppose now that $L < \theta_\mu$. We first show that in this case, the most preferred R of the median state at the second stage will be such that $L < R \leq \theta_\mu$. For all L, R such that $L < \theta_\mu \leq R$, the median state is unconstrained at the state equilibrium and from lemma 3, for almost all R,

$$\begin{aligned}
\frac{\partial V_\mu (L, R)}{\partial R} &= \frac{\partial \left[V_\mu^{dec} \left(\left(\max \left(\underline{t}, \min \left(\bar{t}, \theta_n\right)\right)\right)_n\right)\right]}{\partial R} \\
&= \sum_{n > N - r(\theta, L, R)} \frac{\partial \left[V_\mu^{dec} (t)\right]}{\partial t_n} \frac{\partial \bar{t}}{\partial R} + \sum_{n \leq l(\theta, L, R)} \frac{\partial \left[V_\mu^{dec} (t)\right]}{\partial t_n} \frac{\partial \underline{t}}{\partial R}, \\
&= \frac{2\beta}{N (1 + \beta)} \left[r (L, R) (\theta_\mu - R) \frac{\partial \bar{t}}{\partial R} + l (L, R) (\theta_\mu - L) \frac{\partial \underline{t}}{\partial R}\right].
\end{aligned}$$

From (9), $0 \leq \frac{\partial \overline{x}(L,R)}{\partial R} \leq 1$ so from (12), for almost all R, $\frac{\partial \bar{t}}{\partial R} > 0$ and $\frac{\partial \underline{t}}{\partial R} < 0$. Therefore, if $L < \theta_\mu$, for almost all $R \geq \theta_\mu$, $\partial V_\mu / \partial R \leq 0$. Hence, given $L < \theta_\mu$, the most preferred R of the median state is such that $R \leq \theta_\mu$. Clearly, the median state prefers $L = R = \theta_\mu$ to $L \leq R \leq \theta_\mu$, and from lemma 2, so do all rightist states.

In the simultaneous case, from lemma 3, the median voters are pivotal both on L and R so for any $(L, R) \neq (\theta_\mu, \theta_\mu)$, they can increase their payoff by changing either L or R. ■

9.2 Proofs in Subsection 5.1

Proof of proposition 3. *Step 1: the voters of the median state are pivotal.*
Since $\Gamma - \Delta$ and $\Gamma + \Delta$ are increasing in Γ, lemma 2 implies that the majority preferences on Γ coincide with the preferences of the median state, so $\mathbf{\Gamma}(\Delta)$ are the most preferred Γ of the voters of the median state.

Step 2: at their most preferred Γ of *the voters of the median state, their policy is unconstrained.*
Using the notation of lemma 3, for all θ, Δ and for almost all Γ at which the median

state is unconstrained,

$$\begin{aligned}\frac{\partial V_\mu}{\partial \Gamma}(\Gamma,\Delta) &= \frac{\partial\left[V_\mu^{dec}\left(\left(\max\left(\underline{t},\min\left(\overline{t},\theta_n\right)\right)\right)_n\right)\right]}{\partial \Gamma} \\ &= \sum_{n>N-r(L,R)} \frac{\partial\left[V_\mu^{dec}(t)\right]}{\partial t_n}\frac{\partial \overline{t}}{\partial \Gamma} + \sum_{n\leq l(L,R)} \frac{\partial\left[V_\mu^{dec}(t)\right]}{\partial t_n}\frac{\partial \underline{t}}{\partial \Gamma}. \qquad (15)\end{aligned}$$

Substituting (8) in (12), we get

$$\frac{\partial \overline{t}}{\partial \Gamma} = \frac{\partial \underline{t}}{\partial \Gamma} = 1+\beta-\beta\frac{(1+\beta)(l+r)}{N+\beta(l+r)} = \frac{(1+\beta)N}{N+\beta(l+r)}. \qquad (16)$$

Substituting (13) and (16) in (15), we have that for almost all Γ at which the median state is unconstrained,

$$\begin{aligned}\frac{\partial V_\mu}{\partial \Gamma}(\Gamma,\Delta) &= \frac{\beta}{N(1+\beta)}\left[l\times(\theta_\mu-\Gamma+\Delta)\frac{\partial \underline{t}}{\partial \Gamma} + r\times(\theta_\mu-\Gamma-\Delta)\frac{\partial \overline{t}}{\partial \Gamma}\right] \\ &= \frac{\beta\left[l\times(\theta_\mu-\Gamma+\Delta)+r\times(\theta_\mu-\Gamma-\Delta)\right]}{N+\beta(l+r)}. \qquad (17)\end{aligned}$$

From (17), for almost all such Γ, $\frac{\partial V_\mu}{\partial \Gamma} > 0$ whenever $\Gamma+\Delta<\theta_\mu$ and $\frac{\partial V_\mu}{\partial \Gamma} < 0$ whenever $\Gamma-\Delta>\theta_\mu$. The same is true a fortiori when the median state is constrained by the federal bounds.[24] So the most preferred directive Γ^* of the voters of the median state is such that $\theta_\mu \in [\Gamma^*-\Delta,\Gamma^*+\Delta]$. Since for all Γ, $\overline{x}(\Gamma,\Delta)\in[\Gamma-\Delta,\Gamma+\Delta]$, necessarily $\frac{\theta_\mu+\beta\overline{x}(\Gamma^*,\Delta)}{1+\beta} \in [\Gamma^*-\Delta,\Gamma^*+\Delta]$, so the median state is unconstrained.

Step 3: for all $\Gamma \in \mathbf{\Gamma}(\Delta)$, $\Gamma = \theta_\mu + \frac{l-r}{l+r}\Delta$.

Observe that $l(\Gamma,\Delta)$ and $r(\Gamma,\Delta)$ are weakly increasing and decreasing, respectively. So from (17), at any point Γ^d of discontinuity of l or r, $\frac{\partial V_\mu}{\partial \Gamma}$ has an upward jump. Hence, the kinks of V_μ are all convex kinks so V_μ is maximized at a differentiability point. To conclude, observe that $\frac{\partial V_\mu}{\partial \Gamma}=0$ implies $\Gamma = \theta_\mu+\frac{l-r}{l+r}\Delta$. ■

Proof of proposition 4. *Step 1:* $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ *is weakly decreasing in* θ_n *for all* $n\neq\mu$ *and for almost all* Γ *such that at* (Γ,Δ)*, the median state is unconstrained.*

[24] For instance, if the constraint $x_\mu \leq \Gamma+\Delta$ is binding, then $\Gamma+\Delta<\theta_\mu$ so (17) is positive. Using the envelope theorem on the program (10) for $x=x(\Gamma,\Delta)$, $\frac{\partial V_\mu}{\partial \Gamma}$ is given by (17) plus the Lagrange multiplier of the constraint $x_\mu\leq\Gamma+\Delta$, which is positive. Therefore, $\frac{\partial V_\mu}{\partial \Gamma}$ is positive.

From (17), for all $n \neq \mu$, $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ depends on θ_n only through r or l. Therefore, $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ is weakly decreasing in θ_n if (17) is decreasing in r and increasing in l. If we call $A(l,r)$ the right hand-side of (17),

$$A(l,r+1) - A(l,r) = \beta \frac{N(\theta_\mu - \Gamma - \Delta) - 2\beta l \Delta}{(N + \beta(l+r))(N + \beta(l+r+1))}. \tag{18}$$

For a fixed (Γ, Δ) and $\theta_{-\mu}$, since l is decreasing in θ_μ, the numerator of (18) is increasing in θ_μ. Let t_μ be the largest θ_μ at which state μ is unconstrained. Since the median state is unconstrained in equilibrium, from what precedes, to prove that $A(l,r)$ is weakly decreasing in r, it suffices to prove that the numerator of (18) is negative at $\theta_\mu = t_\mu$. From (9), at $\theta_\mu = t_\mu$, $x_\mu = \frac{\theta_\mu + \beta \overline{x}}{1+\beta} = \Gamma + \Delta$, so

$$t_\mu - \Gamma - \Delta = \beta(\Gamma + \Delta - \overline{x}). \tag{19}$$

Moreover, we always have that

$$\overline{x} \leq \frac{(N-l) \times (\Gamma + \Delta) + l \times (\Gamma - \Delta)}{N}. \tag{20}$$

Substituting (20) in (19), we get $t_\mu - \Gamma - \Delta \leq 2\beta \frac{l\Delta}{N}$, which shows that (18) is negative. Similar algebra shows that $A(l,r)$ is decreasing in l.

Step 2: $\mathbf{\Gamma}^\theta(\Delta)$ *is weakly decreasing in* θ_n *in the strong set order sense for* $n \neq \mu$. This follows directly from step 1, Topkis theorem and proposition 3.

In what follows, t is a profile of type which is skewed to the right and t^s is defined by $t_n^s = \frac{t_n + (2t_\mu - t_{2\mu - n})}{2}$ for all n.

Step 3: if there exists $G \in \mathbf{\Gamma}^t(\Delta)$ *such that* $G > t_\mu$*, then* $2\theta_\mu - G \in \mathbf{\Gamma}^t(\Delta)$.
By construction, t^s is symmetric around t_μ (i.e., for all n, $t_n^s = 2t_\mu^s - t_{2\mu-n}^s$), $t_\mu^s = t_\mu$ and $t_n^s < t_n$ for $n \neq \mu$. By symmetry, $\mathbf{\Gamma}^{t^s}(\Delta)$ is symmetric around t_μ so there exists $G' \in \mathbf{\Gamma}^{t^s}(\Delta)$ such that $G' \leq t_\mu$. Since $G' < G$ and $t^s \leq t$, from step 2, $G' \in \mathbf{\Gamma}^t(\Delta)$ and $G \in \mathbf{\Gamma}^{t^s}(\Delta)$. By symmetry, $2\theta_\mu - G \in \mathbf{\Gamma}^{t^s}(\Delta)$ and since $2\theta_\mu - G \leq G$, step 2 implies that $2\theta_\mu - G \in \mathbf{\Gamma}^t(\Delta)$.

Step 4: if there exists $G \in \mathbf{\Gamma}^t(\Delta)$ *such that* $G > t_\mu$*, then for almost all* Γ *in* $[2\theta_\mu - G, G]$*,* $l^{t^s}(\Gamma, \Delta) = l^t(\Gamma, \Delta)$ *and* $r^{t^s}(\Gamma, \Delta) = r^t(\Gamma, \Delta)$.

From (17), $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ depends on $\theta_{-\mu}$ only through l and r. As shown in step 1, $\frac{\partial V_\mu^\theta}{\partial \Gamma}$ is increasing in l and decreasing in r. Since $t^s \leq t$,

$$l^{t^s}(\Gamma, \Delta) \geq l^t(\Gamma, \Delta) \text{ and } r^{t^s}(\Gamma, \Delta) \leq r^t(\Gamma, \Delta), \tag{21}$$

which implies that for almost all Γ, $\frac{\partial V_\mu^{t^s}}{\partial \Gamma}(\Gamma, \Delta) \leq \frac{\partial V_\mu^t}{\partial \Gamma}(\Gamma, \Delta)$, with a strict inequality when one of the inequalities in (21) is strict. From step 3, the median state is indifferent between $\Gamma = G$ and $\Gamma = 2\theta_\mu - G$ at $\theta = t$ and $\theta = t^s$, i.e. $\int_{2\theta_\mu - G}^{G} \frac{\partial V_\mu^\theta}{\partial \Gamma}(\Gamma, \Delta)\, d\Gamma = 0$ for $\theta \in \{t, t^s\}$. From what precedes, this implies that the inequalities in (21) hold with equality for almost all Γ in $[2\theta_\mu - G, G]$.

Step 5: for all $G \in \mathbf{\Gamma}^t(\Delta)$, $G \leq t_\mu$.

Suppose $G > t_\mu$. For all $\theta \in \mathbb{R}^N$ and $n < \mu$, let $\Gamma_n^\theta(\Delta)$ be a value of Γ at which l jumps from $n-1$ to n, i.e., $\frac{\theta_n + \beta \overline{x}(\Gamma_n, \Delta)}{1+\beta} = \Gamma_n - \Delta$. Since $\overline{x}$ and θ_n are greater at $\theta = t$ than at $\theta = t^s$, strictly so for θ_n, $\Gamma_n^{t^s}(\Delta) < \Gamma_n^t(\Delta)$. Likewise, one can show that the value of Γ at which r jumps from $n-1$ to n is strictly greater at $\theta = t$ than at $\theta = t^s$. Together with step 4, this implies that l and r are constant on $[2\theta_\mu - G, G]$. From proposition 4, $G = 2\theta_\mu - G = \theta_\mu + \frac{l-r}{l+r}\Delta$ so $G = \theta_\mu$. ■

Proof of Proposition 5. Substituting (2) in (1) and summing over N, the welfare under decentralization is $W^{dec} = -N\frac{2\beta+\beta^2}{1+2\beta+\beta^2} var(\theta)$. When $|\theta_n - \theta_\mu| \gg |\theta_m - \theta_\mu|$ for $n > \mu > m$, $W^{dec} \approx -\frac{2\beta+\beta^2}{1+2\beta+\beta^2}\sum_{n>\mu}(\theta_n - \theta_\mu)^2$.

Let $\theta' \in \mathbb{R}^N$ be triadic preference profile defined by: $\theta'_n = \theta_1$ for $n < \mu$, $\theta'_\mu = \theta_\mu$, and $\theta'_n = \theta_{\mu+1}$ for $n > \mu$. Using the notations of proposition 9, for all $\Delta < D_\lambda^o$, $\mathbf{\Gamma}^{\theta'}(\Delta) = \{\theta_\mu - \Delta\}$. Since $\theta \geq \theta'$, from proposition 4, $\mathbf{\Gamma}^{\theta'}(\Delta) \succeq \mathbf{\Gamma}^\theta(\Delta)$ so for all $\Delta > D_\lambda^o$, $\max\left(\mathbf{\Gamma}^\theta(\Delta)\right) + \Delta \leq \theta_\mu$. Simple algebra yields that $D_\lambda^o \leq \theta_\mu - \theta_1$. Together with proposition 3, and what precedes, this implies that for all $\Delta \leq D_\lambda^o$, $\max\left(\mathbf{\Gamma}^\theta(\Delta)\right) + \Delta \leq \theta_\mu + 2(\theta_\mu - \theta_1)$, from what precedes, this inequality holds for all $\Delta \geq 0$. Therefore, when $|\theta_n - \theta_\mu| \gg |\theta_m - \theta_\mu|$ for $n > \mu > m$, for all $\Delta \geq 0$ and all $\Gamma \in \mathbf{\Gamma}^\theta(\Delta)$, $V(\Delta, \Gamma) \approx \sum_{n>\mu}(\theta_n - \theta_\mu)^2$, so $V(\Delta, \Gamma) < W^{dec}$. ■

9.3 Proofs in Subsection 5.2

In the sequel, $\boldsymbol{L}(\Delta) = \{l(\Gamma,\Delta) : \Gamma \in \mathbf{\Gamma}(\Delta)\}$ and $\boldsymbol{R}(\Delta) = \{r(\Gamma,\Delta) : \Gamma \in \mathbf{\Gamma}(\Delta)\}$. Lemma 4 proves the inequalities in (6). Lemma 3 shows that $\mathbf{\Gamma}(\Delta)$ has a finite number of discontinuity in Δ. Lemma 6 and 7 characterize properties of $\mathbf{\Gamma}(\Delta)$ at its continuity and discontinuity points.

Lemma 4 *The induced preferences of the voters of the median state on $x(\mathbf{\Gamma}(\Delta),\Delta)$ are decreasing in Δ. If $\mathbf{\Gamma}(\Delta)$ is single valued on $]D,D'[$ the same is true for the voters of the states $\{l+1,..,N-r\}$. Moreover, for all $\Delta \in]D,D'[$,*

$$\begin{aligned} \text{for } n &\leq l,\ x_n(\mathbf{\Gamma}(\Delta),\Delta) = \theta_\mu - \frac{2r}{l+r}\Delta, \\ \text{for } n &> N-r,\ x_n(\mathbf{\Gamma}(\Delta),\Delta) = \theta_\mu + \frac{2l}{l+r}\Delta, \\ \text{for } l &< n \leq N-r,\ \frac{\partial[x_n(\mathbf{\Gamma}(\Delta),\Delta)]}{\partial\Delta} = 0. \end{aligned} \tag{22}$$

Proof. Suppose that $\mathbf{\Gamma}(\Delta)$ is single valued on $]D,D'[$. From (5), l and r are constant on $]D,D'[$ and (6) for $n \leq l$ and $n > N-r$ is immediate from (5). For $m \in \{l+1,..,N-r\}$,

$$x_m\left(\theta_\mu + \frac{l-r}{l+r}\Delta,\Delta\right) = \frac{\theta_m + \beta\overline{x}\left(\theta_\mu + \frac{l-r}{l+r}\Delta,\Delta\right)}{1+\beta}.$$

From (8),

$$\frac{\partial\left[x_m\left(\theta_\mu + \frac{l-r}{l+r}\Delta,\Delta\right)\right]}{\partial\Delta} = \frac{\beta}{1+\beta}\frac{(1+\beta)(l+r)\frac{l-r}{l+r} + (1+\beta)(r-l)}{N+\beta(l+r)} = 0.$$

Therefore, one can see from (1) that the preferences of unconstrained states are decreasing on $]D,D'[$.

From proposition 3, the median voters are indifferent between all elements of $\mathbf{\Gamma}(\Delta)$ and their policy is unconstrained at (Γ,Δ) for any $\Gamma \in \mathbf{\Gamma}(\Delta)$. From what precedes, this implies that the median voters always prefer less discretion. ■

Lemma 5 *There exists $D_1, ..., D_I$ (with the convention that $D_0 = 0$, $D_{I+1} = +\infty$) such that for all $i = 0, .., I$, $(\boldsymbol{L}, \boldsymbol{R})(\Delta)$ is constant on $]D^i, D^{i+1}[$ and for all $i = 0, .., I$,*

$$\left\{ \lim_{\Delta \nearrow D^i} (\boldsymbol{L}, \boldsymbol{R})(\Delta), \lim_{\Delta \searrow D^i} (\boldsymbol{L}, \boldsymbol{R})(\Delta) \right\} \subset (\boldsymbol{L}, \boldsymbol{R})\left(D^i\right). \tag{23}$$

Proof. From proposition 3, $(\boldsymbol{L}, \boldsymbol{R})(\Delta) = \arg\max_{(l,r)} V_\mu\left(\theta_\mu + \frac{l-r}{l+r}\Delta, \Delta\right)$. From lemma 1, for all (l, r), $V_\mu\left(\theta_\mu + \frac{l-r}{l+r}\Delta, \Delta\right)$ is piecewise quadratic in Δ. Since (l, r) can only take a finite number of values, $(\boldsymbol{L}, \boldsymbol{R})(\Delta)$ must be piecewise constant. The Berge maximum theorem implies that $(\boldsymbol{L}, \boldsymbol{R})(\Delta)$ is upper hemi-continuous in Δ, which implies (23). ■

Lemma 6 *If (Γ^e, Δ^e) is a LFE, then $\boldsymbol{\Gamma}(\Delta)$ is single valued on $]\Delta^e - \varepsilon, \Delta^e[$ for some positive ε and $\Gamma^e = \lim_{\Delta \nearrow \Delta^e} \boldsymbol{\Gamma}(\Delta)$.*

Proof. Suppose that $\boldsymbol{\Gamma}(\Delta)$ is not single-valued on $]\Delta^e - \varepsilon, \Delta^e[$. From (23), $\boldsymbol{\Gamma}(\Delta^e)$ is not single-valued. Let $\Gamma^o \in \boldsymbol{\Gamma}(\Delta^e)$ such that $\Gamma^o \neq \Gamma^e$, say $\Gamma^o < \Gamma^e$ for concreteness. From proposition 3, $\Gamma^o = \theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^e$ where $(l^o, r^o) = (l, r)(\Gamma^o, \Delta^e)$ and the voters from the median state are indifferent between (Γ^o, Δ^e) and (Γ^e, Δ^e). From lemma 2, the voters from leftist states strictly prefer (Γ^o, Δ^e) to (Γ^e, Δ^e). By continuity, they strictly prefer $\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon'), \Delta^e - \varepsilon'\right)$ for some small $\varepsilon' > 0$. From lemma 4, so do the median voters. From lemma 5, $\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon') \in \boldsymbol{\Gamma}(\Delta^e - \varepsilon')$, hence $\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon'), \Delta^e - \varepsilon'\right)$ is a valid deviation from (Γ^e, Δ^e) and (Γ^e, Δ^e) cannot be a LFE.

If $\Gamma^e \neq \lim_{\Delta \nearrow \Delta^e} \boldsymbol{\Gamma}(\Delta)$, the same reasoning shows that a majority of voters will prefer $\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}(\Delta^e - \varepsilon), \Delta^e - \varepsilon\right)$ to (Γ^e, Δ^e) where $(l^o, r^o) = (l, r)(\Gamma^o, \Delta^e)$ and $\Gamma^o = \lim_{\Delta \nearrow \Delta^e} \boldsymbol{\Gamma}(\Delta)$. ■

Lemma 7 *Using the notations of lemma 5, (Γ^e, Δ^e) is a LFE if and only if there exists $i > 0$ such that one of the following is true:*
(a) $\Delta^e \in]D^{i-1}, D^i[$, $\boldsymbol{\Gamma}(\Delta)$ is single-valued on $]D^{i-1}, D^i[$, $\boldsymbol{\Gamma}(\Delta^e) = \{\Gamma^e\}$ and majority preferences on Δ are strictly single peaked on $]D^{i-1}, D^i[$ with a peak at Δ^e

(*b*) $\Delta^e = D_i$, *$\mathbf{\Gamma}(\Delta)$ is single-valued on* $]D^{i-1}, D^i[$, $\Gamma^e = \lim_{\Delta \nearrow D_i} \mathbf{\Gamma}(\Delta)$ *and majority preferences on* Δ *are increasing on* $]D^{i-1}, D^i[$.

Proof. If (a) is satisfied, (Γ^e, Δ^e) is clearly a LFE. Reciprocally, suppose that (Γ^e, Δ^e) is a LFE and $\Delta^e \in]D^{i-1}, D^i[$. From lemma 6 and 5, $\mathbf{\Gamma}(\Delta)$ is single-valued on $]D^{i-1}, D^i[$. Therefore, the induced preferences of all voters on $x(\mathbf{\Gamma}(\Delta), \Delta)$ are well-defined, quadratic and concave, and not flat by definition of a LFE. Therefore, majority preferences are strictly quasi-concave on $]D^{i-1}, D^i[$, and the conclusion follows from the median voter theorem.

If (Γ^e, Δ^e) is a LFE and $\Delta^e = D^i$, from lemma 6 and $\mathbf{\Gamma}(\Delta)$ is single-valued on $]D^{i-1}, D^i[$. Suppose majority preferences are not increasing on $]D^{i-1}, D^i[$. As argued earlier, majority preferences are quasi-concave on $]D^{i-1}, D^i[$, so they must be decreasing on $]D^i - \varepsilon, D^i[$ for some $\varepsilon > 0$. From lemma 6, $\Gamma^e = \lim_{\Delta \nearrow D^i} \mathbf{\Gamma}(\Delta)$ and from what precedes, (Γ^e, Δ^e) cannot be a LFE.

Reciprocally, suppose (b) is satisfied, then by assumption, there exists $\varepsilon > 0$ such that for all $\Delta \in]\Delta^e - \varepsilon, \Delta^e[$, $(\mathbf{\Gamma}(\Delta), \Delta)$ is not preferred by SMR to (Γ^e, Δ^e). For all $\Gamma \in \mathbf{\Gamma}(\Delta^e)$, the median voter is indifferent between (Γ^e, Δ^e) and (Γ, Δ^e) so from lemma 2, (Γ, Δ^e) is not preferred by SMR to (Γ^e, Δ^e). Finally, let $\Delta^k \searrow \Delta^e$ and $\Gamma^k \in \mathbf{\Gamma}(\Delta^k)$. One can restrict attention to sequences such that $(l, r)(\Gamma^k, \Delta^k) = (l^o, r^o)$ for all k and some (l^o, r^o). Since $\Delta^k > \Delta^e$, lemma 4 implies that the median state strictly prefer (Γ^e, Δ^e) to (Γ^k, Δ^k). From (5), $\Gamma^o \equiv \lim \Gamma^k$ exists. Suppose to fix ideas that $\Gamma^e < \Gamma^o$.[25] For k sufficiently large, $\Gamma^e \pm \Delta^e < \Gamma^k \pm \Delta^k$ so from lemma 2 and what precedes, (Γ^e, Δ^e) is strictly preferred to (Γ^k, Δ^k) by leftist states. Finally (Γ^e, Δ^e) is preferred by SMR to (Γ^k, Δ^k) for k sufficiently large. ■

Proof of proposition 6.

As $\Delta \to 0$, from proposition 3, all states n such that $\theta_n \neq \theta_\mu$ must be constrained and $\mathbf{\Gamma}(\Delta)$ is single valued. In this case, one can easily see from (1) that $V_n(\mathbf{\Gamma}(\Delta), \Delta)$

[25] From the proof of lemma 5, it should be clear that since $\mathrm{G}(\theta, \Delta)$ is single-valued on $]\Gamma^e - \varepsilon, \Gamma^e[$, necessarily $\Gamma^e \neq \Gamma^o$.

must be increasing in a neighborhood of $\Delta = 0$.[26] Since a majority of states have a type different from θ_μ, $\Delta = 0$ cannot be a LFE. Since $\mathbf{\Gamma}(\Delta)$ is single valued on $]D^0, D^1[$, majority preferences are quasi-concave on $]D^0, D^1[$. If they are increasing on $]D^0, D^1[$, lemma 7 implies that D^1 is a LFE. If they are not increasing, from what precedes, they must be single-peaked and lemma 7 implies that there exists a LFE in $]D^0, D^1[$.

From lemma 7, at any LFE (Γ^e, Δ^e), a majority of voters have preferences which are weakly increasing in Δ on $]\Delta^e - \varepsilon, \Delta^e[$. From lemma 4, these voters are from states which are constrained by the federal bounds, which proves that a majority of states must be constrained at (Γ^e, Δ^e). ■

9.4 Proofs in Section 6

Proof of proposition 7. We will show by contradiction that as $\beta \to 0$, for any selection of LFE $\left(\Gamma^\beta, \Delta^\beta\right)$, for all constrained states n, $\left|x_n^\beta\left(\Gamma^\beta, \Delta^\beta\right) - \theta_n\right|$ is bounded away from 0. One can easily see from (1) and (2) that this implies that all constrained states are strictly better-off under decentralization than at the LFE $\left(\Gamma^\beta, \Delta^\beta\right)$ (part a) and that the welfare of their voters is strictly increasing in Δ conditional on $\Gamma = \Gamma^\beta$ (part b). Since the welfare of unconstrained states under both regime is asymptotically equivalent, decentralization is also socially better.

In what follows, β^k is a sequence such that $\beta^k \to 0$ and $\left(\Gamma^k, \Delta^k\right)$ is a sequence of LFE for the profile of preferences $\left(\theta, \beta^k\right)$ such that $x_m^{\beta^k}\left(\Gamma^k, \Delta^k\right) \to \theta_m$ for some state m (independent of k) which is constrained for all k, and $(\boldsymbol{L}, \boldsymbol{R})$ refer to the map defined before lemma 5. We assume wlog that for all k, m is constrained by the right bound.

Step 1: there exists a subsequence of β^k *such that* $\left(\Gamma^k, \Delta^k\right) \to (\Gamma^o, \Delta^o)$, *for all* k, $l^{\beta^k}\left(\Gamma^k, \Delta^k\right) = l^o$ and $r^{\beta^k}\left(\Gamma^k, \Delta^k\right) = r^o$ for some Γ^o, Δ^o, l^o and r^o, and $\theta_m = \Gamma^o + \Delta^o$.

[26] The first order effect on $-\left|x_n - \theta_n\right|^2$ is positive while the first order effect on $-\frac{\beta}{N}\sum_{m \neq n}\left|x_n - x_m\right|^2$ is zero.

The existence of the limit is immediate from the boundedness of $\left(\Gamma^k, \Delta^k\right)$ and the fact that $(\boldsymbol{L}, \boldsymbol{R})$ can only take a finite number of values. To see the last point, observe that for all k, $\bar{x}^{\beta^k}\left(\Gamma^k, \Delta^k\right) \leq \Gamma^k + \Delta^k$. So proposition 1 implies that $\theta_m \geq x_m^{\beta^k}\left(\Gamma^k, \Delta^k\right)$, so $\theta_m \geq \Gamma^o + \Delta^o$. Since m is constrained by the right bound, proposition 1 implies further that $\theta_m > \Gamma^k + \Delta^k$, so $\theta_m \geq \Gamma^o + \Delta^o$.

Step 2: there exists a subsequence of β^k such that $l^{\beta^k}\left(\Gamma, \Delta^k\right)$ and $r^{\beta^k}\left(\Gamma, \Delta^k\right)$ converge pointwise to two step functions $l\left(\Gamma\right)$ and $r\left(\Gamma\right)$ such that $\lim_{\Gamma \searrow \Gamma^o} r\left(\Gamma\right) \leq r^o - 1$ and $\lim_{\Gamma \searrow \Gamma^o} l\left(\Gamma\right) \geq l^o$.

Observe that for a given k, $l^{\beta^k}\left(\Gamma, \Delta^k\right)$ and $r^{\beta^k}\left(\Gamma, \Delta^k\right)$ are functions of Γ which are piecewise constant, bounded and have a bounded number of points of discontinuities as $k \to \infty$. The existence of $l\left(\Gamma\right)$ and $r\left(\Gamma\right)$ follows from Bolzano-Weierstrass. From step 1, for all $\Gamma > \Gamma^o$, for k sufficiently large, $\Gamma^k < \Gamma$ so $\lim_{k\to\infty} l^{\beta^k}\left(\Gamma, \Delta^k\right) \geq l^o$. From step 1 again, $\theta_m = \Gamma^o + \Delta^o$ so for k sufficiently large, $r^{\beta^k}\left(\Gamma, \Delta^k\right) \leq N - m$. Since m is constrained for all k, $N - m + 1 \leq r^o$.

Step 3: there exists $\epsilon > 0$ and $c > 0$ such that

$$V_\mu^{\beta^k}\left(\Gamma^k - \epsilon, \Delta^k\right) - V_\mu^{\beta^k}\left(\Gamma^k, \Delta^k\right) = c\beta^k + o\left(\beta^k\right) \tag{24}$$

From (17), for all $\epsilon > 0$,

$$V_\mu^{\beta^k}\left(\Gamma^k + \epsilon, \Delta^k\right) - V_\mu^{\beta^k}\left(\Gamma^k, \Delta^k\right) = \frac{\beta^k}{N} \int_{\Gamma^k}^{\Gamma^k+\epsilon} \left(\begin{array}{c} l^{\beta^k}\left(\Gamma, \Delta^k\right)\left(\theta_\mu - \Gamma + \Delta^k\right) \\ +r^{\beta^k}\left(\Gamma, \Delta^k\right)\left(\theta_\mu - \Gamma - \Delta^k\right) \end{array} \right) d\Gamma + o\left(\beta^k\right). \tag{25}$$

From proposition 3, $\Gamma^k \to \theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^o$. Together with Step 2 and the dominated convergence theorem, this implies that

$$\begin{aligned}
&\lim_{k\to\infty} \int_{\Gamma^k}^{\Gamma^k+\epsilon} \left[l^{\beta^k}\left(\Gamma, \Delta^k\right)\left(\theta_\mu - \Gamma + \Delta^k\right) + r^{\beta^k}\left(\Gamma, \Delta^k\right)\left(\theta_\mu - \Gamma - \Delta^k\right)\right] d\Gamma \\
= &\int_0^\epsilon \left[\begin{array}{c} l\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^o + \varepsilon\right) \times \left(\frac{2r^o}{l^o + r^o}\Delta^o - \varepsilon\right) \\ +r\left(\theta_\mu + \frac{l^o - r^o}{l^o + r^o}\Delta^o + \varepsilon\right) \times \left(-\frac{2l^o}{l^o + r^o}\Delta^o - \varepsilon\right) \end{array} \right] d\varepsilon \\
\geq &\int_0^\epsilon \left(-\left(l^o + r^o + 1\right)\varepsilon + \frac{2l^o}{l^o + r^o}\Delta^o \right) d\varepsilon.
\end{aligned} \tag{26}$$

Simple calculus shows that (26) is positive for ϵ sufficiently small, which, together with (25), completes the proof of step 3.

To conclude, observe that step 3 implies that $V_{\mu}^{\beta^k}\left(\Gamma^k-\epsilon,\Delta^k\right) > V_{\mu}^{\beta^k}\left(\Gamma^k,\Delta^k\right)$ for some k and some $\epsilon>0$, which contradicts proposition 3. ■

The next lemma will be used in the proof of proposition 8.

Lemma 8 *If $c(\Delta)$ denotes the number of states constrained by the left bound at $(\Gamma=\theta_\mu,\Delta)$, then at any continuity point of $c(.)$,*

$$\text{for } n \le c(\Delta)\,, \frac{\partial V_n(\Gamma=\theta_\mu,\Delta)}{\partial\Delta} = 2\left[(\theta_\mu-\theta_n)-\left(1+\beta\frac{N+2c(\Delta)}{N}\right)\Delta\right], \quad (27)$$

$$\text{for } n > N-c(\Delta)\,, \frac{\partial V_n(\Gamma=\theta_\mu,\Delta)}{\partial\Delta} = 2\left[(\theta_n-\theta_\mu)-\left(1+\beta\frac{N+2c(\Delta)}{N}\right)\Delta\right],$$

$$\text{and for } c(\Delta) < n \le N-c(\Delta)\,, \frac{\partial V_n(\theta_\mu,\Delta)}{\partial\Delta} = -4\frac{\beta}{N}\Delta c(\Delta)\,.$$

and if $\Delta^* \in \arg\max_\Delta \sum_n V_n(\Gamma=\theta_\mu,\Delta)$,

$$\Delta^* = \frac{\sum\limits_{n\le c(\Delta^*)}|\theta_n-\theta_\mu| + \sum\limits_{n>N-c(\Delta^*)}|\theta_n-\theta_\mu|}{2(1+2\beta)\,c(\Delta^*)} \ge \frac{\sum\limits_{n}|\theta_n-\theta_\mu|}{(1+2\beta)\left((N-1)\right)}. \quad (28)$$

Proof. By symmetry, $c(\Delta)$ is also the number of states constrained by the right bound. When no ambiguity arises, the argument of $c(.)$ will be omitted. From (1), for $n \le c$,

$$V_n(\theta_\mu,\Delta) = -(\theta_\mu-\theta_n-\Delta)^2 - \frac{\beta}{N}\sum_{m=c+1}^{N-c}(\theta_\mu-x_m-\Delta)^2 - \frac{4\beta\Delta^2 c}{N}.$$

Since θ is symmetric, $\sum_{m=c+1}^{N-c}(\theta_\mu-x_m)=0$. Moreover, from lemma 4, at any points of continuity of $c(.)$, for all $m\in[c+1,N-c]$, $\frac{\partial x_m}{\partial\Delta}=0$ so

$$\frac{\partial V_n(\theta_\mu,\Delta)}{\partial\Delta} = 2\left[(\theta_\mu-\theta_n-\Delta)+\frac{\beta}{N}\sum_{m=c+1}^{N-c}(\theta_\mu-x_m-\Delta)\right]-\frac{8\beta\Delta c}{N},$$

which implies (27) for $n\le c$, and for $n>N-c$ by symmetry. For $n\in\{c+1,N-c\}$, from lemma 4, for almost all Δ, $\frac{\partial x_n}{\partial\Delta}=0$ so

$$\frac{\partial V_n(\theta_\mu,\Delta)}{\partial\Delta} = -2\frac{\beta}{N}(x_n-\theta_\mu+\Delta)\,c+2\frac{\beta}{N}(x_n-\theta_\mu-\Delta)\,c = -4\frac{\beta}{N}\Delta c.$$

If $W(\Delta) = \sum_n V_n(\theta_\mu, \Delta)$, summing (27) for all n, we get at any continuity point of $c(.)$,

$$\frac{\partial W}{\partial \Delta} = 2c\left[\frac{1}{c}\sum_{n\leq c}(\theta_\mu - \theta_n) + \frac{1}{c}\sum_{n>N-c}(\theta_n - \theta_\mu) - (2+4\beta)\Delta\right]. \tag{29}$$

One can see from (29) that a discontinuity point of $c(.)$ corresponds to a convex kink of W. So if $\Delta^* \in \arg\max_\Delta W(\Delta)$, $W(\Delta)$ is differentiable at Δ^* and (28) is derived from (29) by setting $\frac{\partial W}{\partial \Delta} = 0$. ■

Proof of proposition 8. *Part a:* if (Δ^e, Γ^e) is a LFE, from lemma 6, $\mathbf{\Gamma}(\Delta)$ must be single-valued on $]\Delta^e - \varepsilon, \Delta^e[$ for some $\varepsilon > 0$. Since θ is symmetric, this means that for all $\Delta \in]\Delta^e - \varepsilon, \Delta^e[$, $\mathbf{\Gamma}(\Delta) = \{\theta_\mu\}$ and lemma 6 implies $\Gamma^e = \{\theta_\mu\}$. So $x(\mathbf{\Gamma}(\Delta), \Delta)$ and $x(\theta_\mu, \Delta)$ coincide on $]\Delta^e - \varepsilon, \Delta^e[$. Moreover, from lemma 7, majority preferences must be increasing on $]\Delta^e - \varepsilon', \Delta^e[$ for some $\varepsilon' > 0$. States which voters have increasing preferences on $]\Delta^e - \varepsilon', \Delta^e[$ necessarily include the $\frac{N+1}{2}$ states with the largest $|\theta_n - \theta_\mu|$ and from proposition 6, all such states are constrained. For the voters of these states, at $\Delta = \Delta^e$, $\frac{\partial U_n(x(\Gamma=\theta_\mu,\Delta))}{\partial \Delta} \geq 0$. From (27), this implies that for a majority of states,

$$|\theta_n - \theta_\mu| \geq \left(1 + \beta\frac{N + 2c(\Delta^e)}{N}\right)\Delta^e.$$

Therefore,

$$\Delta^e \leq \frac{med_n(|\theta_\mu - \theta_n|)}{1+\beta\left(1+2\frac{c(\Delta^e)}{N}\right)} \leq \frac{med_n(|\theta_\mu - \theta_n|)}{1+3/2\beta}, \tag{30}$$

which, together with the inequality in (28), proves part a).

Part b: Notice that if θ is maximally polarized, necessarily $c(\Delta^*) = \lceil \frac{N+1}{4} \rceil$.[27] So the equality in (28) implies $\Delta^* = \frac{\delta}{1+2\beta}$. Using the notation of proposition 9, the symmetry of θ implies $D_\lambda = D_\rho$, $D^o_\lambda = D^o_\rho$ and for β sufficiently large, $D_\lambda < D^o_\lambda$ so

$$\Delta^e = D_\lambda = \frac{\delta}{1+\frac{\beta}{N}\left(2\lceil\frac{N+1}{4}\rceil + N\right)} > \frac{\delta}{1+2\beta}$$

[27] If θ is maximally polarized, $c(\Delta)$ can only take three values: 0, $\lceil \frac{N+1}{4} \rceil$ and $\frac{N-1}{2}$. A close inspection of the sign of $\frac{\partial W}{\partial \Delta}$ at the two discontinuity points of $c(\Delta)$ shows that $c(\Delta^*) = \lceil \frac{N+1}{4} \rceil$.

since for $N \geq 5$, $\left\lceil \frac{N+1}{4} \right\rceil \leq \frac{N+4}{4} < \frac{N}{2}$.

Part c: let θ^k be a sequence of profile of type such that $\pi\left(\theta^k\right) \to 0$ and let $\left(\Gamma^k, \Delta^k\right)$ be a corresponding sequence of LFE. Wlog, we re-normalize θ^k by an affine transformation so that for all k, $\theta_\mu^k = 0$ and $med_n\left(\left|\theta_n^k - \theta_\mu^k\right|\right) = 1$. By definition of π, necessarily, $var\left(\theta^k\right) \to \infty$. From what precedes, $\Gamma^k = 0$ and from (30), Δ^k is bounded. This implies that $x^{\theta^k}\left(\Gamma^k, \Delta^k\right)$ is bounded so $W\left(\Gamma^k, \Delta^k\right) = Nvar\left(\theta^k\right) + O(1)$. From (2), simple algebra shows that $W^{dec} = \frac{2\beta+\beta^2}{1+2\beta+\beta^2} Nvar\left(\theta^k\right)$, which shows that decentralization is socially better as $k \to \infty$. Under our assumptions, a majority of states have a bounded type, so since $x^{\theta^k}\left(\Gamma^k, \Delta^k\right)$ is bounded and $x^{dec}\left(\theta^k\right)$ is unbounded, a majority of voters strictly prefer $\left(\Gamma^k, \Delta^k\right)$ to decentralization as $k \to \infty$.

Part d: follows from proposition 10 parts a) and d). ■

9.5 Proofs in Section 7

The next lemma characterizes the second stage equilibrium $\mathbf{\Gamma}(\Delta)$ for all Δ.

Lemma 9 *Under the notations of proposition 9:*

$$\begin{aligned}
\Delta &> \min\left\{D_\lambda^o, D_\rho^o\right\} \Rightarrow \mathbf{\Gamma}(\Delta) = \begin{cases} \{\theta_\mu - \Delta\} \text{ if } D_\lambda^o < D_\rho^o \\ \{\theta_\mu + \Delta\} \text{ if } D_\lambda^o > D_\rho^o \\ \{\theta_\mu - \Delta, \theta_\mu + \Delta\} \text{ if } D_\lambda^o = D_\rho^o. \end{cases}, \\
\Delta &< \min\left\{D_\lambda^o, D_\rho^o\right\} \Rightarrow \mathbf{\Gamma}(\Delta) = \left\{\theta_\mu + \frac{\lambda - \rho}{\lambda + \rho}\Delta\right\}.
\end{aligned}$$

For $\Delta \in \left]0, \min\left\{D_\lambda^o, D_\rho^o\right\}\right[$, $U_1\left(\theta, \mathbf{\Gamma}(\theta, \Delta), \Delta\right)$ *is single-peaked in* Δ *with a peak at* D_λ *while* $U_N\left(\theta, \mathbf{\Gamma}(\theta, \Delta), \Delta\right)$ *has a peak at* D_ρ.

Proof. From proposition 3, for all Δ, there are three possible equilibria at the second stage: either $(l, r) = (0, \rho)$ and $\Gamma = \theta_\mu - \Delta$, or $(l, r) = (\lambda, 0)$ and $\Gamma = \theta_\mu + \Delta$, or $(l, r) = (\lambda, \rho)$ and $\Gamma = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta$.

Step 1: derivation of the state equilibrium when $(l, r) = (0, \rho)$ *and* $\Gamma = \theta_\mu - \Delta$*:* In this case, $x(\theta_\mu - \Delta, \Delta)$ as characterized in (9) is given by

$$\begin{pmatrix} x_1 \\ x_\mu \\ x_N \end{pmatrix} = \begin{pmatrix} \frac{\theta_1 + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} \\ \frac{\theta_\mu + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} \\ \theta_\mu \end{pmatrix} = \begin{pmatrix} \frac{(N+\beta\lambda+\beta\rho)\theta_1 + \left(\beta^2\rho + N\beta - \beta\lambda\right)\theta_\mu}{(\beta+1)(N+\beta\rho)} \\ \frac{\left(N+\beta^2\rho+N\beta-\beta\lambda+\beta\rho\right)\theta_\mu + \beta\lambda\theta_1}{(\beta+1)(N+\beta\rho)} \\ \theta_\mu \end{pmatrix}, \quad (31)$$

where the second equality is derived by solving the system described by the first equality. This solution is possible at Δ iff the leftist states are indeed unconstrained at $(\Gamma = \theta_\mu - \Delta, \Delta)$, i.e.

$$x_1 \geq \theta_\mu - 2\Delta \Leftrightarrow \Delta \geq D' \equiv \frac{N + \beta\lambda + \beta\rho}{2(\beta+1)(N+\beta\rho)}(\theta_\mu - \theta_1).$$

In this case, substituting (31) in (1), simple algebra yields

$$V_\mu(\theta_\mu - \Delta, \Delta) = -\frac{\beta\lambda(N + \beta\lambda + \beta\rho)}{N(\beta+1)^2(N+\beta\rho)}(\theta_\mu - \theta_1)^2. \quad (32)$$

Step 2: derivation of the state equilibrium when $(l, r) = (\lambda, 0)$ *and* $\Gamma = \theta_\mu + \Delta$*: In this case,* $x(\theta, \theta_\mu + \Delta, \Delta)$ is given by

$$\begin{pmatrix} x_1 \\ x_\mu \\ x_N \end{pmatrix} = \begin{pmatrix} \theta_\mu \\ \frac{\theta_\mu + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} \\ \frac{\theta_N + \frac{\beta}{N}(\lambda x_1 + (N-\lambda-\rho)x_2 + \rho x_N)}{1+\beta} \end{pmatrix} = \begin{pmatrix} \theta_\mu \\ \frac{\left(N+\beta^2\lambda+N\beta-\beta\rho+\beta\lambda\right)\theta_\mu + \beta\rho\theta_N}{(\beta+1)(N+\beta\lambda)} \\ \frac{(N+\beta\rho+\beta\lambda)\theta_N + \left(\beta^2\lambda+N\beta-\beta\rho\right)\theta_\mu}{(\beta+1)(N+\beta\lambda)} \end{pmatrix}. \quad (33)$$

Substituting (33) in (1), simple algebra yields

$$V_\mu(\theta_\mu + \Delta, \Delta) = -\frac{\beta\rho(N + \beta\lambda + \beta\rho)}{N(\beta+1)^2(N+\beta\lambda)}(\theta_N - \theta_\mu)^2. \quad (34)$$

From proposition 3, for any Δ, $\mathbf{\Gamma}(\Delta)$ are the most preferred Γ of the voters of the median state. From (32) and (34), they strictly prefer $\Gamma = \theta_\mu - \Delta$ to $\Gamma = \theta_\mu + \Delta$ if and only if

$$\lambda(N + \beta\lambda)(\theta_\mu - \theta_1)^2 < \rho(N + \beta\rho)(\theta_N - \theta_\mu)^2 \Leftrightarrow D^o_\lambda < D^o_\rho. \quad (35)$$

Step 3: derivation of the state equilibrium when $(l,r)=(\lambda,\rho)$ *and* $\Gamma=\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta$*:*, In this case, $x\left(\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta,\Delta\right)$ is given by

$$\begin{pmatrix} x_1 \\ x_\mu \\ x_N \end{pmatrix}=\begin{pmatrix} \theta_\mu-\frac{2\rho}{\lambda+\rho}\Delta \\ \theta_\mu \\ \theta_\mu+\frac{2\lambda}{\lambda+\rho} \end{pmatrix}, \tag{36}$$

Notice that this solution is possible at Δ if and only if states 1 and N are indeed constrained at $(\Gamma,\Delta)=\left(\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta,\Delta\right)$, i.e.

$$\begin{aligned}\frac{\theta_1+\frac{\beta}{N}\left(\lambda x_1+(N-\lambda-\rho)\,x_\mu+\rho x_N\right)}{1+\beta} &\leq \Gamma-\Delta \text{ and}\\ \frac{\theta_N+\frac{\beta}{N}\left(\lambda x_1+(N-\lambda-\rho)\,x_\mu+\rho x_N\right)}{1+\beta} &\geq \Gamma+\Delta,\end{aligned}$$

which is equivalent to

$$\Delta\leq D''\equiv\frac{\lambda+\rho}{2\,(1+\beta)}\min\left\{\frac{\theta_\mu-\theta_1}{\rho},\frac{\theta_N-\theta_\mu}{\lambda}\right\}.$$

In this case, substituting (36) in (1), we get that at $(\Gamma,\Delta)=\left(\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta,\Delta\right)$,

$$\begin{pmatrix} V_1 \\ V_\mu \\ V_N \end{pmatrix}=\begin{pmatrix} -\left(\theta_1-\theta_\mu+\frac{2\rho}{\lambda+\rho}\Delta\right)^2-\frac{\beta}{N}\left((N-\lambda-\rho)\left(\frac{2\rho}{\lambda+\rho}\right)^2+4\rho\right)\Delta^2 \\ -4\frac{\beta}{N}\frac{\lambda\rho}{\lambda+\rho}\Delta^2 \\ -\left(\theta_N-\theta_\mu-\frac{2\lambda}{\lambda+\rho}\Delta\right)^2-\frac{\beta}{N}\left((N-\lambda-\rho)\left(\frac{2\rho}{\lambda+\rho}\right)^2+4\lambda\right)\Delta^2 \end{pmatrix} \tag{37}$$

Step 4: Derivation of the second stage equilibrium: from (32) and (37), the voters of the median state prefer $\Gamma=\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta$ to $\Gamma=\theta_\mu-\Delta$ if and only if

$$\Delta\leq\sqrt{\frac{(\lambda+\rho)\,(N+\beta\lambda+\beta\rho)}{4\rho\,(\beta+1)^2\,(N+\beta\rho)}}\,(\theta_\mu-\theta_1)=D^o_\lambda.$$

A symmetric reasoning shows that the voters of the median state prefer $\Gamma=\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta$ to $\Gamma=\theta_\mu-\Delta$ if and only if $\Delta\leq D^o_\rho$. Together with (35), this proves that if $D^o_\lambda\leq D^o_\rho$, for $\Delta>D^o_\lambda$, $\mathbf{\Gamma}(\Delta)=\{\theta_\mu-\Delta\}$ and for $\Delta<D^o$, $\mathbf{\Gamma}(\Delta)=\left\{\theta_\mu+\frac{\lambda-\rho}{\lambda+\rho}\Delta\right\}$. Likewise, if

$D^o_\lambda \geq D^o_\rho$, for $\Delta > D^o_\rho$, $\mathbf{\Gamma}(\Delta) = \{\theta_\mu + \Delta\}$ and for $\Delta < D^o_\rho$, $\mathbf{\Gamma}(\Delta) = \left\{\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta\right\}$. Simple algebra shows that the feasibility constraints, i.e. $D'' \geq \min\left\{D^o_\lambda, D^o_\rho\right\} \geq D'$, are always satisfied,[28] which completes the proof of the first part of the lemma.

Step 5: derivation of the induced preferences of state 1 *and* N *on* Δ*:* If $\Delta < \min\left\{D^o_\lambda, D^o_\rho\right\}$, from (37) $\frac{\partial U_1\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)}{\partial \Delta} = 0$ iff

$$-\frac{4\rho}{\lambda+\rho}\left(\theta_1 - \theta_\mu + \frac{2\rho}{\lambda+\rho}\Delta\right) - \frac{2\beta}{N}\left((N-\lambda-\rho)\left(\frac{2\rho}{\lambda+\rho}\right)^2 + 4\rho\right)\Delta = 0.$$

Solving for Δ yields $\Delta = D_\lambda$. Likewise, $\frac{\partial V_N\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)}{\partial \Delta} = 0$ iff $\Delta = D_\rho$. ■

Proof of proposition 9. For the sake of brevity, we assume throughout the proof that $D^o_\lambda > D^o_\rho$. The cases $D^o_\lambda < D^o_\rho$ and $D^o_\lambda = D^o_\rho$ can be derived identically. On $\left]D^o_\rho, +\infty\right[$, from lemma 9, the state equilibrium is constant (see. (31)) and equal to $x\left(\theta_\mu + D^o_\rho, D^o_\rho\right)$. Since $\lim_{\Delta \nearrow D^o_\rho} \mathbf{\Gamma}(\Delta) \neq \theta_\mu + D^o_\rho$, from lemma 6, $(\Gamma, \Delta) = \left(\theta_\mu + D^o_\rho, D^o_\rho\right)$ is not a LFE. From lemma 6, the only LFE candidates are therefore $\left\{\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right) : \Delta \leq D^o_\rho\right\}$. On $\left[0, D^o_\rho\right[$, the preferences of all voters on $x\left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)$ are quadratic and concave in Δ. Therefore, majority preferences are quasi-concave on $\left[0, D^o_\rho\right[$. Moreover, they are either single-peaked or strictly increasing on $\left[0, D^o_\rho\right[$.[29]

Case 1: majority preferences are increasing on $\left[0, D^o_\rho\right[$. From lemma 6 $(\Gamma^e, \Delta^e) \equiv \left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}D^o_\rho, D^o_\rho\right)$ is the only LFE. By assumption, (Γ^e, Δ^e) is preferred by SMR to

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[28] Suppose to fix ideas that $D^o_\lambda \leq D^o_\rho$. If $\frac{|\theta_\mu - \theta_1|}{\rho} \leq \frac{|\theta_N - \theta_\mu|}{\lambda}$, $D'' \geq D^o_\lambda \geq D'$ can be rewritten

$$\sqrt{\frac{(\lambda+\rho)(N+\beta\rho)}{(N+\beta(\lambda+\rho))\rho}} \geq 1 \geq \sqrt{\frac{(N+\beta(\lambda+\rho))\rho}{(\lambda+\rho)(N+\beta\rho)}},$$

which is satisfied since $x \to \frac{N+\beta x}{x}$ is decreasing and $\lambda + \rho > \rho$. If $\frac{|\theta_\mu - \theta_1|}{\rho} > \frac{|\theta_N - \theta_\mu|}{\lambda}$, $D'' \geq D^o_\lambda \geq D'$ holds whenever

$$\frac{|\theta_N - \theta_\mu|}{|\theta_\mu - \theta_1|} \geq \sqrt{\frac{(N+\beta(\lambda+\rho))\lambda^2(N+\beta\lambda)}{(\lambda+\rho)(N+\beta\lambda)\rho(N+\beta\rho)}},$$

which is satisfied since $D^o_\lambda \leq D^o_\rho$ implies $\frac{|\theta_N - \theta_\mu|}{|\theta_\mu - \theta_1|} \geq \sqrt{\frac{\lambda(N+\beta\lambda)}{\rho(N+\beta\rho)}}$, and from what precedes, $\sqrt{\frac{(N+\beta(\lambda+\rho))\lambda}{(\lambda+\rho)(N+\beta\lambda)}} < 1$.

[29] They canot be decreasing to the right of $\Delta = 0$ from (37).

any $(\Gamma, \Delta) = \left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)$ for $\Delta < D^o_\rho$. From proposition 3, the voters of the median state are indifferent between (Γ^e, Δ^e) and $\left(\theta_\mu + D^o_\rho, D^o_\rho\right)$, from lemma 2 the rightist and leftist voters have opposite preferences between these two alternatives, so $\left(\theta_\mu + D^o_\rho, D^o_\rho\right)$ is not preferred by SMR to (Γ^e, Δ^e). This shows that (Γ^e, Δ^e) is a Condorcet winner among all (Γ, Δ) such that $\Gamma \in \mathbf{\Gamma}(\Delta)$

Case 2: majority preferences are single-peaked on $\left[0, D^o_\rho\right[$. From lemma 9, $\Delta^e = \min\{D_\lambda, D_\rho\}$ and $\Delta^e \leq D^o_\rho$. From lemma 6, $\left(\Gamma^e = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta^e, \Delta^e\right)$ is the unique LFE. By assumption, it is preferred by SMR to any $(\Gamma, \Delta) = \left(\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)$ for $\Delta \leq D^o_\rho$. From lemma 4, the voters of the median state strictly prefer (Γ^e, Δ^e) to $\left(\theta_\mu + D^o_\rho, D^o_\rho\right)$. Moreover, since $\Delta^e \leq D^o_\rho$,

$$\theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta^e \pm \Delta^e < \theta_\mu + D^o_\rho \pm D^o_\rho,$$

so from lemma 2 (Γ^e, Δ^e) is strictly preferred by SMR to $\left(D^o_\rho, \theta_\mu + D^o_\rho\right)$. Hence, (Γ^e, Δ^e) is a Condorcet winner among all (Γ, Δ) such that $\Gamma \in \mathbf{\Gamma}(\Delta)$. ■

Proof of proposition 10. As $\beta \to \infty$, $D_\lambda < D^o_\lambda$ and $D_\rho < D^o_\rho$ so $\Delta^e = \min\{D_\lambda, D_\rho\}$. Suppose to fix ideas that $D_\lambda \leq D_\rho$. From (2), as $\beta \to \infty$, for all n, $x^{dec}_n = \overline{\theta} + \frac{\theta_n - \overline{\theta}}{\beta} + o\left(\frac{1}{\beta}\right)$ so $\beta var\left(x^{dec}\right) \to 0$ and from (1),

$$\lim_{\beta\to\infty} V^{dec}_n = -\left(\theta_n - \overline{\theta}\right)^2. \tag{38}$$

Notice that as $\beta \to \infty$, $D_\lambda = \frac{N(\lambda+\rho)(\theta_\mu - \theta_1)}{2\left(\lambda^2+\rho\lambda+N\rho\right)}\frac{1}{\beta} + o\left(\frac{1}{\beta}\right)$ so $\beta\left(D_\lambda\right)^2 \to 0$. Substituting $\Delta^e = D_\lambda$ in (37), we get

$$\lim_{\beta\to\infty} V_n = -\left(\theta_n - \theta_\mu\right)^2. \tag{39}$$

Let W^{dec} and W^e denote the welfare under decentralization and at the federal equilibrium, respectively. Combining (38) and (39),

$$\lim_{\beta\to\infty}\left(W^{dec} - W^e\right) = \frac{\left(\lambda\left|\theta_1 - \theta_\mu\right| - \rho\left|\theta_N - \theta_\mu\right|\right)\begin{pmatrix} (2N - 3\lambda)\left|\theta_1 - \theta_\mu\right| \\ -(2N - 3\rho)\left|\theta_N - \theta_\mu\right| \end{pmatrix}}{N^2}. \tag{40}$$

If $|\theta_1 - \theta_\mu| = |\theta_N - \theta_\mu|$ and $\lambda \neq \rho$, the left-hand side of (40) is $-\frac{3(\lambda-\rho)^2|\theta_1-\theta_\mu|^2}{N^2} < 0$, which proves part b). If $\lambda = \rho$ and $|\theta_1 - \theta_\mu| \neq |\theta_N - \theta_\mu|$, the left-hand side of (40) is $\frac{\lambda(2N-3\lambda)(|\theta_N-\theta_\mu|-|\theta_1-\theta_\mu|)^2}{N^2} > 0$, which proves part c).

The voters of the median state are always strictly better-off at the federal equilibrium than under decentralization.[30] If $\lambda\,|\theta_1 - \theta_\mu| > \rho\,|\theta_N - \theta_\mu|$, then $\overline{\theta} < \theta_\mu$ so (38) and (39) imply that as $\beta \to \infty$, $U_1^{dec} > V_1$ and $U_N^{dec} < V_N$. Likewise, if $\lambda\,|\theta_1 - \theta_\mu| < \rho\,|\theta_N - \theta_\mu|$, then $U_1^{dec} < V_1$ and $U_N^{dec} > V_N$, which proves the necessary part of part a). If $\lambda\,|\theta_1 - \theta_\mu| = \rho\,|\theta_N - \theta_\mu|$, then simple algebra shows that $x\left(\Gamma = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta, \Delta\right)$ is equal to x^{dec} for $\Delta = \left|\theta_\mu - x_1^{dec}\right| \frac{\lambda+\rho}{2\rho}$ and the induced preferences of leftist voters on Δ conditional on $\Gamma = \theta_\mu + \frac{\lambda-\rho}{\lambda+\rho}\Delta$ are quadratic, with a maximum at $\Delta = D_\lambda$ by definition of D_λ. Therefore, leftist voters are better-off at this maximum than under decentralization, which proves part d). By symmetry, if $|\theta_1 - \theta_\mu| = |\theta_N - \theta_\mu|$ and $\lambda = \rho$, all voters are better-off, which completes the proof of part a). ∎

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