Signalling with Career Concerns*

Kim-Sau Chung
School of Economics and Finance, The University of Hong Kong

Péter Eső
MEDS Department, Kellogg School, Northwestern University

January 2007

Abstract

Consider an agent (manager, artist, etc.) who has imperfect private information about his productivity. At the beginning of his career (period 1, "short run"), the agent chooses among publicly observable actions that generate imperfect signals of his productivity. The actions can be ranked according to the informativeness of the signals they generate. The market observes the agent's action and the signal generated by it, and pays a wage equal to his expected productivity. In period 2 (the "long run"), the agent chooses between a constant payoff and a wage proportional to his true productivity, and the game ends. We show that in any equilibrium where not all types of the agent choose the same action, the average productivity of an agent choosing a *less informative* action is *greater*. However, the types choosing that action are not uniformly higher. In particular, we derive conditions for the existence of a tripartite equilibrium where low and high types pool on a less informative action while medium (on average, lower) types choose to send a more informative signal.

JEL classification: D82, D86. Keywords: signalling, career concerns

*We thank seminar participants at Northwestern University and the 2004 SED Meeting in Budapest, and in particular Yeon-koo Che, Hanming Fang, Sven Feldman, Drew Fudenberg, Alessandro Pavan, Kane Sweeney, Jeroen Swinkels for comments, and Renato Gomes for comments and excellent research assistance.

1 Introduction

Many economic activities involve agents generating public information about their qualities; these signals are often *informative for the agents themselves* as well as for the outside world. For example, drug companies carry out or pay for experiments in order to convince regulators and customers that their products are safe and effective. However, the outcome of a preclinical trial is also important for the company for determining if more investment in the drug is worthwhile. Another often-studied example is that of individuals who join organizations, choose certain activities, or even participate in higher education in order to reveal their abilities to possible future employers (or simply to the rest of society). Note, however, that the grades received at school are useful for the individual, too, in evaluating his or her career options.

In this paper we study a game where an agent chooses among overt actions that generate public signals regarding his productivity (a payoff-relevant state of nature), which he is imperfectly informed about. The agent's actions are ranked according to the associated signal's informativeness regarding his true productivity.[1] In the first period (the "short run"), after the agent's action and the generated signal are observed, the market pays the agent his expected productivity. Then, in period 2 (the "long run"), the agent chooses between a constant payoff and a wage proportional to his true productivity, and the game ends. Notice that the first-period signal regarding the state of nature informs not only the market but also the agent's second-period decision because it updates his beliefs about his talent. However, the precision of this signal is determined by the agent's action, which, by being observed by the market, may also affect his first-period wage. We look for sequential equilibria in this game, in particular, equilibria where the agent may choose different actions depending on his information.

For a concrete example that corresponds to this game think of the agent as the product manager at a pharmaceutical company planning to test the effectiveness of a new drug. (The effectiveness of the drug is the manager's "productivity" in this application.) The manager has private information about the drug, and he can choose either an in-house experiment, or to provide a grant to a university-affiliated research team. It is reasonable to assume that the signal generated by the outside investigators is more informative than that of an in-house test.[2]

[1] In order to focus on the effects generated by information transmission we disregard potential differences in the direct cost of the actions.

[2] It is also probable that outsourcing the tests costs less than doing it in-house. While we formally do not represent this possibility in our model, a modification where the more precise

The manager's pay is tied to the company's stock price as long as he stays with the company. While in the long run, the company's stock price will reflect the drug's eventual effectiveness, in the short run, it is set according to the market's expectation of the drug's quality based on both the manager's choice of experiment and the outcome of the experiment. The question is which experiment the manager would choose in equilibrium. Would he choose the more informative test to signal that he is "not afraid of the truth"? Or would he choose the less informative one to signal that he has little doubt about the quality of the drug? While a more informative experiment allows the manager to make a better decision regarding whether or not to stay on and continue to pursue the drug, the mere act of choosing that test may have an adverse impact on the short-run stock price. More generally, are there any *testable implications* of this model for the manager's behavior?

Another example is that of a budding artist who is uncertain, but not completely uninformed, about her talent. She launches her career with a project that can either be a painting or an installation. The quality and reception of her first artwork is informative regarding her artistic talent. The artist's short-run payoff (e.g., the price of her artwork or the prize she wins with it) is correlated with the market's expectation of her talent given the project choice and the signal (buzz) generated by the piece. The quality of her first work also informs the artist whether she should continue her career in art or become, say, a decorator at a department store. If she remains an artist then, in the long run, the world learns her talent and appreciates (pays) her accordingly; if she quits art then her payoff is independent of her talent. The crucial, but reasonable, assumption is that a traditional painting gives a more informative signal regarding the artist's talent than an installation does. This is so because the latter is an experimental art form, and it is more difficult to evaluate the artist's talent based on an experimental piece. Again, the question is which project the artist will choose in equilibrium, and what the market can infer from that choice.

In the formal analysis of this problem we obtain three main results. First, in any equilibrium where not all types of the agent choose the same action, the average productivity of the agent choosing a *less* informative action is *higher*. That is, the choice of a less-informative action signals strength, not the agent's fear from the truth. However, the equilibrium is never a "threshold-equilibrium": the agent-types that choose a more informative action are *not* uniformly higher than those that choose a less informative action. In other words, there exist

signal costs less would not alter the results.

types of the agent—very optimistic and very pessimistic ones—that choose the same, less informative action, while an agent-type with moderate beliefs about his productivity chooses a more informative one. Finally, we provide conditions for the existence of an informative equilibrium where low and high types of the agent pool on a less informative action while medium (but, on average lower) types choose a more informative one.

In the examples seen above, our results imply that the average talent of a young artist choosing an experimental project for her debut is greater. However, young creative professionals who choose non-traditional projects at the beginning of their careers are not uniformly more talented than those choosing traditional projects. In fact, according to our model, we would expect to find truly gifted and also utterly untalented individuals among those who choose "the road less travelled". This seems to agree with our casual observation of talent markets. Similarly, in the drug company example, an in-house test (which is less informative by assumption) would be an indication that the manager is either very optimistic or very pessimistic about the product's quality, but also that, on average, the drug's prospects are better than they would be had the manager opted for outside testing.

The first result—that a less informative action is associated with a higher average productivity agent—follows from the interaction of two effects. On the one hand, all else equal, higher types of the agent should prefer a more informative action because that increases the chance that the market observes a correct and favorable signal about the agent's true productivity. In other words, a more informative action may signal that the agent is not afraid of the truth. On the other hand, a less informative action is costly for (almost) all types because it decreases the value of the option to "get out," that is, to choose the payoff that is independent of the agent's productivity in the long run. Therefore, the opportunity cost of a less-informative action must be compensated by the market's perception that it is taken by, on average, higher productivity agents. It turns out that the latter effect is stronger. The key step in the argument is to show that if the market's beliefs regarding the average types choosing each action were equal, then this average type would get the same short-run payoff from either action, and higher types would gain more from the more informative one. After figuring in the second-period option value, all types at or above the average would still strictly prefer the more informative action. This contradicts the assumption that the market's beliefs (that the average types choosing either action are equal) are rational. In order to restore equilibrium, the market's expectation of the average type choosing the less-informative action must be raised.

The intuition for the second result—that the productivity of the types of the agent choosing a less informative action is *not* uniformly greater—is also somewhat subtle. The fundamental tradeoff that each type of the agent faces is that the less informative action is perceived by the market as a signal of "strength"; however, it is less likely to generate a favorable signal in case the agent's true productivity is high, and it is also less valuable for the agent for the purpose of learning about his productivity. The tradeoff disappears for the most pessimistic agent (the worst type, the one that *knows* that his productivity is low)—he does not care about learning, and he is actually glad that the signal is less likely to reveal his true, low productivity. Therefore the lowest type of the agent chooses the least informative action, which is associated with on average the most productive agents. Hence the types choosing a less informative action cannot be uniformly higher than the types playing a more informative action.

Our third result shows that informative equilibria—ones where at least two different actions are chosen by different types of the agent—indeed exist in our model for a non-trivial set of parameter values. In this type of equilibrium, the most pessimistic and most optimistic types pool on a less informative action, while medium type(s) trade off the short-run "stigma" associated with sending a more informative signal for the long-run benefits of learning. This type of non-monotonic equilibrium is not present in standard signalling models.

Mainstream explanations of signalling phenomena usually rely on variants of Spence's (1973) model.[3] The starting point is an adverse selection situation; in addition, the privately informed agent can engage in a certain costly activity interpreted as a "signal". The key assumption is that the signalling activity is relatively less costly for an agent that has higher quality. This sorting (or single-crossing) condition enables high-quality agents to separate themselves from low-quality ones by choosing a sufficiently high level of the signal so that imitation is not worthwhile.

It has been argued that in many signalling situations we only see intermediate types sending the costly signals, while very high productivity agents seem not to engage in such activity.[4] For example, college dropouts include some of the

[3]For a textbook exposition, see Fudenberg and Tirole (1991), Chapter 7. For the earliest examples of signaling models, see Spence (1973) on education, Nelson (1974) on advertising, Ross (1977) on the choice of a firm's financial structure, and Zahavi (1975) on mate selection in the animal kingdom.

[4]This observation dates back to Veblen (1899).

most talented (not to mention richest) members of society. Feltovich, Harbaugh and To (2002) cite other examples as well: the truly rich do not flaunt their wealth by spending on symbols of status, only the "nouveau rich" do; a person of the highest character does not bother to disprove accusations, only people with average reputations do; and so on.

Feltovich, Harbaugh and To (2002) model these "countersignalling" phenomena in a variant of a Spencian signalling game where the market receives an additional, objective signal about the agent's type besides observing his action. Under certain conditions, medium types find it worthwhile to differentiate themselves from low types by traditional wasteful signalling, while high types—confident that in the end the exogenous piece of information will separate them from the low types—can afford not to signal in the traditional sense. In effect, in the equilibrium studied by the authors, the high type relies on an exogenous technology to credibly reveal his productivity, while the intermediate and low types play a Spencian signalling game.[5]

Our motivation, model, and results differ from those in this line of research in many ways. First of all, our signalling game is not a Spencian signalling game as the single-crossing condition does not hold; that is, in our model it is not inherently cheaper for a higher type to choose a more informative action. This is so because the agent's short-run payoff depends on the market's beliefs about which types choose each action, while his long-run payoff depends on the value of learning about his productivity from the signal generated by his action. In fact, the latter "learning benefit" from a more informative action is small for very low and very high types (ones that are almost sure about their productivity), therefore the cost of a less informative action is not even monotonic.[6] A second, related difference is that in our model, *all* informative equilibria have the property that some low and some high types pool on an action different from the one chosen by intermediate types. In the modified Spencian models cited above, some type

[5] Another explanation of why talented individuals may skip higher education is that of Hvide (2003). He assumes that there are two sectors for employment: one where the wage depends on talent, and one where it does not. An individual who is privately informed about his ability may enter either sector right away, or get more education (=private signals about his talent) before making his choice. Education is relatively cheaper for more talented people. A fully separating equilibrium (whose existence depends on parameter values) is where low types enter the flat-wage sector, high types choose the talent-based sector, and medium types get more education before making a choice. Notice that this, too, is a Spencian signalling game, and in equilibrium, extreme types are not pooling on any action—in fact, all types separate.

[6] The single-crossing property cannot be re-established even by transforming the type space (i.e., by relabeling types). This will become clear as we describe the model.

of equilibrium refinement is needed to get a similar prediction; in general, other types of equilibria also exist (for example, a fully separating one).

The inefficiency result of our model (i.e., some types of the agent endogenously choose an inefficient method to learn about their own talents) is closely related to the inefficiency result of Brandenburger and Polak (1996). In both models, the agent cares about not only his productivity in the long run, but the market's current perception of his future productivity as well. In our model, this "short-term reputational concern" distorts the agent's incentive to learn about his own talent; in Brandenburger and Polak's it induces a manager to make the decision that the market wants to see (instead of the decision that maximizes the firm's long term profitability). One crucial difference, however, is that in Brandenburger and Polak's model, the "short-term reputational concern" eliminates every possibility of separating equilibrium. In our model, separating equilibria are possible, and necessarily take a non-monotonic form.

While the agent's short-term reputational concern distorts his incentives to do the "right" thing, his long-term career concerns determine the manner in which these incentives are being distorted. Our model is hence a contribution to the literature on career concerns, which studies various implications of an agent's long-term career concerns on his short-term behavior. In Holmström (1999), an agent's career concerns help motivate him to exert effort, which otherwise cannot be rewarded with an enforceable incentive contract. In Morris (2001), an informed advisor, who otherwise would have current incentive to truthfully reveal her information to her advisee, may refrain from doing so because she is concerned of her long-term reputation as an unbiased advisor.

In Ottaviani and Sorensen (2006a,b), the advisor is concerned of her reputation as being accurate (instead of being unbiased), and this concern in turn reduces the credibility of her short-term advice, so much so that truthful revelation becomes impossible. In Prendergast and Stole (1996), career concerns have opposite effects on young and old investors. Young investors tend to exaggerate their reactions to new information in order to signal that they are fast learners. On the contrary, old investors are more conservative in order to signal that they have always been fast learners and hence have already learned enough in the past. In Avery and Chevalier (1999), young investors who know little about their own ability herd in their investment behavior as in Scharfstein and Stein (1990). But as they get older and learn more about their abilities, they choose to "anti-herd" in order to signal that they are confident in themselves.

Finally, our model is also marginally related to the cheap talk literature.

Cheap talk games can be interpreted as an extreme form of non-Spencian signalling games, where cost differentials across different actions (messages) are type-independent as all messages are costless. Nevertheless, separating equilibria are still possible, because the receiver's (or the market's) reactions to different messages are different, and this creates endogenous type-dependent cost differentials across messages. In a clever twist of the standard setup of cheap talk games, Fang (2001) allows those cost diffentials to be stochastic, while maintaining the assumption that they are type-independent. Endogenous type-dependent cost differentials can arise as in standard cheap talk models, and separating equilbria exist where different actions result in different market reactions. Fang (2001) interprets these different actions as different cultural activities, and uses this model to explain why productivity-unrelated cultural activities would nevertheless be rewarded differently by the market.

The rest of the paper is structured as follows: We set up the model in Section 2, present our results in Section 3, and conclude in Section 4. Omitted proofs are collected in an Appendix.

2 The Model

In this section we formally describe our model of signalling with career concerns. First, a partially-informed, risk-neutral agent chooses among observable actions that generate public signals about his true productivity. The market observes his action and the signal generated by it, and pays him a wage equal to his expected productivity. Upon observing all this, the agent chooses between an additional fixed payoff and a payoff that is proportional to his true productivity.

Denote the agent's productivity (the unobservable state of the world) by ω, and assume that it can take one of two values, H (high) or L (low), $H > L$. The prior distribution of ω is commonly known. Before the game starts, the agent observes a private signal regarding the state of nature. The signal generates a posterior distribution of ω; indeed, without any loss of generality, we can identify the agent's private information with his updated belief that the state of nature is H. That is, the agent's type, denoted by θ, is simply $\theta = \Pr(\omega = H)$. From an outside observer's perspective, the agent's type is drawn according to a commonly known distribution F with full support on $[0, 1]$. The ex ante expectation of θ is simply the commonly known prior probability that the state of nature is H.

There are two periods, and for simplicity no discounting. The agent is assumed to be risk neutral. In the first period, the agent undertakes a publicly observable action. In order to simplify the exposition we assume that there are two alternatives available to him, a_1 and a_2. (All our results go through with an arbitrary number of actions.) Each action generates a random signal conditional on ω that is observable to the agent and the market alike. The realization of the public signal is denoted by $y \in \{H, L\}$. The restrictions that y is binary and that realizations of y correspond to realizations of ω are imposed purely for convenience and do not affect the results. The distribution of y conditional on a_i is characterized by $\pi_i \equiv \Pr(y = \omega | \, \omega, a_i)$ for $i = 1, 2$. Without loss of generality, let $\pi_i \geq 1/2$ for $i = 1, 2$. Our key assumption is that action a_1 generates a *more informative signal* about ω than a_2 does, that is, $\pi_1 > \pi_2$. The parameters π_1 and π_2 are commonly known.

After action a_i and signal value y are publicly observed, the agent is paid the expectation of his true productivity (the expectation of ω) given all publicly available information, including a_i, y, and the agent's equilibrium strategy, $\theta \mapsto a(\theta)$. This wage can be thought of as a "credence wage" for the agent's first-period performance (or services), which the market values according to the agent's yet unobservable productivity. In our earlier example, the budding artist's debut project was rewarded by the market (art speculators) according to their expectation of the artist's talent given the type and quality of her first art piece.

In the second period, the agent again chooses between two actions, labeled "in" and "out". If he stays in then he gets a payoff proportional to his true productivity, ω. If he chooses to get out then he gets a fixed payment, K. One may interpret the second period as the "long run", and the agent's choice between "in" and "out" as the reduced form of some more complex continuation game: If the agent continues with his activity then his productivity is eventually learned by the market, and he gets rewarded accordingly. However, he can also choose an outside option whose value is independent of his talent.[7]

Denote the agent's period-2 updated belief that his productivity is high (given that he knows θ and observes y generated by a_i) by θ_i^y, that is,

$$\theta_i^y(\theta) = \Pr\left(\omega = H \mid \theta, y, a_i\right). \tag{1}$$

Note that $\theta_i^L(\theta) \leq \theta \leq \theta_i^H(\theta)$ with $E_y\left[\theta_i^y(\theta)\right] = \theta$, that is, the second-period belief is a mean-preserving spread of the first period belief, θ. (The spread is wider if π_i is larger.) In the second period, the agent chooses "in" whenever θ_i^y exceeds a certain threshold that depends on the value of the outside option, K.

[7] Allowing K to depend on the agent's productivity would not alter our results, as long as the outside option is less sensitive to ω than the agent's payoff when he stays "in".

To summarize, the order of moves in the game and the payoffs are as follows.

0. Nature chooses $\theta \in [0, 1]$ according to c.d.f. F, and picks either $\omega = H$ or $\omega = L$ with probabilities θ and $(1-\theta)$, respectively. The risk-neutral agent privately learns his type, θ, while his productivity, ω, remains unknown.
1. The agent chooses a publicly observable action from $\{a_1, a_2\}$. Nature generates a publicly observable signal y where $y = \omega$ with probability π_i for action a_i ($i = 1, 2$), and $\pi_1 > \pi_2 \geq 1/2$. The agent is paid a wage that equals $E[\omega|a(\cdot), a_i, y]$, his expected productivity given the equilibrium strategy, the action taken, and the signal generated by the action.
2. The agent chooses between staying "in" and getting "out". The former yields a payoff proportional to ω while the latter yields a payoff of K.

In the second period, the agent chooses "in" if and only if $\theta_i^y(\theta)$, given θ, the choice of a_i, and the realization of y, exceeds a certain threshold that depends on K. In the first period, a rational-expectations equilibrium is characterized by the agent's choice of action conditional on his type, $a(\theta) \in \{a_1, a_2\}$ for all $\theta \in [0, 1]$, and the market's belief that the agent's productivity is high given his action, $x_i \in [0, 1]$ for $i = 1, 2$. In equilibrium, the market's beliefs must be consistent with the agent's strategy, which in turn has to be an optimal choice for the agent given his type and the market's beliefs. In the next section, we will analyze separating equilibria, that is, rational-expectations equilibria where in period 1 both actions are taken with positive probabilities.

3 The Structure of Signaling Equilibria

In this section we establish three results. First, we show that in any equilibrium, if both actions are chosen in equilibrium, then the average productivity of agents playing a *less* informative action (in the model, a_2) is *greater*. However, we also show that an agent that is nearly sure that his productivity is low chooses action a_2, hence the types that choose a less informative action do not dominate those playing a more informative one. Finally, we derive conditions for the existence of equilibria where very low and very high types choose a less informative action, while medium type(s) choose a more informative action. At the end of the section we discuss the robustness of the results by examining variants of the model.

3.1 Preliminary analysis of payoffs

Before we establish the results we introduce some notation and derive certain properties of the agent's payoff function.

The agent's payoff (in expectation, at the beginning of the first period) consists of two terms: his expected wage in the first period, and his future expected payoff from being able to choose between "in" and "out" in period two. The first-period expected wage is a function of his type, the action that he chooses, and the market's belief about his productivity that is associated with the action. The agent's expectation at the beginning of the game of the "option value" that he will enjoy in the second period also depends on his type and the action that he chooses in period one, but it does not depend on the market's perception of his productivity based on his initial choice. We formally define and derive these two parts of the agent's total payoff in turn.

Recall that the market's belief (estimated probability) regarding $\omega = H$ when the agent takes action a_i is denoted by x_i, and that the agent's equilibrium strategy is denoted by $a : [0,1] \to \{a_1, a_2\}$. In what follows we normalize the agent's productivity levels so that his expected productivity coincides with the estimated probability that $\omega = H$, that is, we set $H = 1$ and $L = 0$. This is without any loss of generality because the transformation is affine and the agent is risk neutral.

Let $W_i(\theta, x_i)$ denote the agent's expected first-period wage with type θ when he takes action a_i associated with market belief x_i. (The probability that the action generates a signal equal to the agent's true productivity, π_i, is a parameter that is suppressed by this notation.) The period-1 payoff, W_i, is determined as follows. First, given the agent's strategy and his chosen action, the market's updated (posterior) belief that the agent's productivity is high when the signal generated by his action is y can be calculated by Bayes' rule as

$$\Pr(\omega = H \,|\, y, a(\cdot), a_i) = \frac{\Pr(y \,|\, \omega = H, a(\cdot), a_i) \Pr(\omega = H \,|\, a(\cdot), a_i)}{\Pr(y \,|\, a(\cdot), a_i)}.$$

The market wage paid to the agent given his action choice, a_i, and the signal realization, y, is $w_i^y = \Pr(\omega = H \,|\, y, a(\cdot), a_i)$. Using the above equality,

$$w_i^H = \frac{\pi_i x_i}{\pi_i x_i + (1-\pi_i)(1-x_i)}, \tag{2}$$

$$w_i^L = \frac{(1-\pi_i) x_i}{(1-\pi_i) x_i + \pi_i (1-x_i)}. \tag{3}$$

Here w_i^H (respectively, w_i^L) is the wage that the agent receives when he chooses action a_i associated with market belief x_i and the publicly observed signal happens to be H (respectively, L). If $\pi_i = 1/2$ then $w_i^H = w_i^L = x_i$ because the signal does not provide any new information about ω. However, if $\pi_i > 1/2$ then the agent's wage is higher when the signal realization is higher, $w_i^H > w_i^L$.

$W_i(\theta, x_i)$ is the agent's expectation of his first-period wage given θ, that is,

$$W_i(\theta, x_i) = \theta \left[\pi_i w_i^H + (1-\pi_i) w_i^L\right] + (1-\theta)\left[(1-\pi_i) w_i^H + \pi_i w_i^L\right]. \qquad (4)$$

By substituting in w_i^H and w_i^L from (2) and (3) into this equation and rearranging terms we get

$$W_i(\theta, x_i) = \frac{\pi_i \theta + (1-\pi_i)(1-\theta)}{\pi_i x_i + (1-\pi_i)(1-x_i)} \pi_i x_i + \frac{(1-\pi_i)\theta + \pi_i(1-\theta)}{(1-\pi_i)x_i + \pi_i(1-x_i)}(1-\pi_i)x_i. \quad (5)$$

Notice that the agent's expected first-period wage is affine in θ, his initial belief regarding his productivity. We summarize other useful properties of $W_i(\theta, x_i)$ in the following lemma. Figure 1 illustrates W_i graphically.

Lemma 1 *If $\pi_i = 1/2$ then $W_i(\theta, x_i) \equiv x_i$. If $\pi_i > 1/2$ then the agent's expected wage in period 1 satisfies:*

(i) $W_i(\theta, 0) \equiv 0$ and $W_i(\theta, 1) \equiv 1$.

(ii) For all $x_i \in (0,1)$, $W_i(\theta, x_i)$ is strictly increasing in θ and x_i.

(iii) For all x_i, $W_i(x_i, x_i) = x_i$.

(iv) For all $x_i \in (0,1)$, $W_i(\theta, x_i)$ is strictly increasing in π_i if $\theta > x_i$. Conversely, $W_i(\theta, x_i)$ is strictly decreasing in π_i if $\theta < x_i$.

Proof. See the Appendix. ■

Notice that holding the precision of the signal-generating action (π_i) fixed, both the intercept and the slope of the short-run wage depend on the market's beliefs regarding the average productivity of the agent that takes that action. In particular, if the market's expectations are low (x_i is low), W_i starts out low and has a small slope. As we increase x_i, the expected talent associated with action a_i, the short-run expected wage increases and becomes more sensitive to the agent's private information. However, as the market's belief approaches certainty in the agent's high productivity, the wage becomes less and less sensitive to θ.

The comparison of first-period expected wage schedules resulting from different actions is difficult because the intercepts and slopes of the W_i functions depend on the relative precisions of the two signals (i.e., π_1 and π_2), and also the

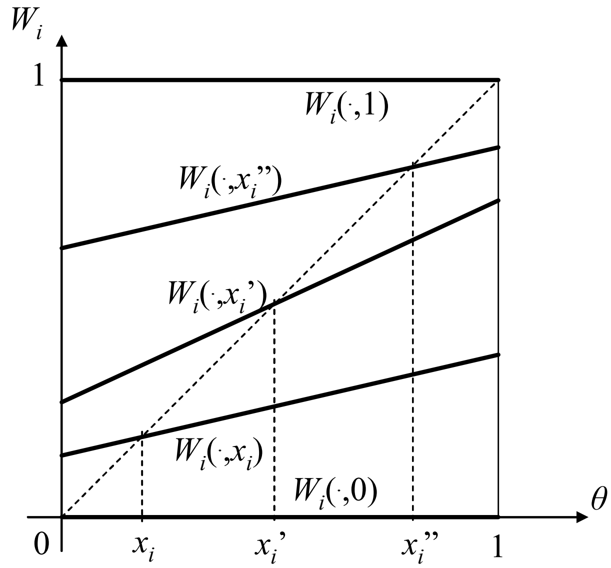

Figure 1: The first-period expected wage.

market's beliefs regarding the talent of the agent taking the different actions (i.e., x_1 and x_2). Since the market's beliefs are endogenous in the model, not much can be said in advance regarding the difference between W_1 and W_2 at a particular θ. By part (iii) of Lemma 1, if $x_1 = x_2 = x$ (i.e., the market's expectation of the agent's productivity is the same for both actions), then no matter how precise the actions are, the expected first period wage of type $\theta = x$ does not depend on the action choice, that is, $W_1(\theta, x) = W_2(\theta, x) = x$ for $\theta = x$. From part (iv) of Lemma 1 we also know that the expected first period wage (W_i) is increasing in the precision of the signal (π_i) if and only if the agent's type is greater than the market's expectation ($\theta > x_i$). A more precise signal is beneficial for the agent in the short run only if his type is better than the average type that chooses it.[8]

Now we turn to the characterization of the agent's second-period payoff.

Let $T_i(\theta, K)$ denote the agent's expectation at the beginning of period 1 of his benefit from the second-period option to choose between getting a constant payoff K and a payoff equal to his true productivity. (Again, the parameter π_i is implicit in our notation.) Recall that $\theta_i^y(\theta)$, defined in equation (1), denotes the agent's updated (posterior) belief at the beginning of period 2 that his productivity is

[8] From parts (ii) and (iv) of Lemma 1 it also follows that if $x_1 = x_2 = x$ but $\pi_1 > \pi_2$ then $W_1(\theta, x)$ crosses the 45 degree line at $\theta = x$ steeper than $W_2(\theta, x)$ does.

high given that action a_i generated signal y, and that his prior belief was θ. In particular, by Bayes' rule,

$$\theta_i^L(\theta) = \frac{(1-\pi_i)\theta}{(1-\pi_i)\theta + \pi_i(1-\theta)}, \tag{6}$$

$$\theta_i^H(\theta) = \frac{\pi_i\theta}{\pi_i\theta + (1-\pi_i)(1-\theta)}. \tag{7}$$

The property that observing y is informative for the agent regarding his productivity means that θ_i^y is a mean-preserving spread around θ, that is,

$$\Pr(y = L|\theta, a_i)\theta_i^L(\theta) + \Pr(y = H|\theta, a_i)\theta_i^H(\theta) \equiv \theta.$$

Since the agent chooses "in" over "out" in period 2 if and only if $\theta_i^y(\theta) \geq K$, the option value he gets from this choice is $\max\{\theta_i^y(\theta) - K, 0\}$. At the beginning of period 1, the agent does not know the realization of y yet, hence the expected value of his second-period option is

$$T_i(\theta, K) = E_y\left[\max\{\theta_i^y(\theta) - K, 0\} \mid \theta\right].$$

Figure 2 illustrates graphically the derivation and properties of $T_i(\theta, K)$. As it can be seen in the figure, $T_i(\theta, K) = 0$ for all θ such that $\theta_i^H(\theta) \leq K$, and $T_i(\theta, K) = \theta - K$ for all θ such that $K \leq \theta_i^L(\theta)$, and T_i is convex in θ.[9] Finally, for θ such that $\theta_i^L(\theta) < K < \theta_i^H(\theta)$, we have

$$\begin{aligned} T_i(\theta, K) &= \Pr\left(y = H \mid \theta, a_i\right)\left(\theta_i^H(\theta) - K\right) \\ &= \pi_i\theta - \left(\pi_i\theta - (1-\pi_i)(1-\theta)\right)K. \end{aligned} \tag{8}$$

There is a difference between the second-period benefit generated by action a_1 and a_2 that arises as follows. Action a_1 is more informative than a_2, hence the agent's posterior beliefs are more spread out under a_1 than they are under a_2:

$$\theta_1^L(\theta) < \theta_2^L(\theta) \leq \theta \leq \theta_2^H(\theta) < \theta_1^H(\theta).$$

Since the second-period option value, $\max\{\theta_i^y - K, 0\}$, is convex in θ_i^y, the period 1 expectation of it is greater under action a_1 when θ_i^y is more spread out. That is, a more informative action generates a greater payoff in the second period because

[9] It may be useful to note that the same qualitative properties would hold even if we had more than two possible realizations of y. The only difference would be that T_i would be more "smooth".

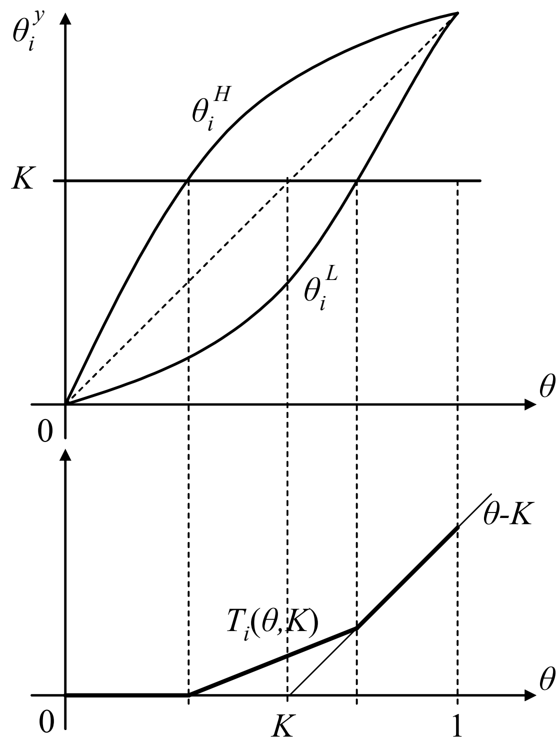

Figure 2: The second-period option value

the agent learns more about his own productivity and so the value of the option to stay in or get out is greater.[10] The property that a_1 generates a weakly greater period 2 benefit than a_2 does *for all types of the agent* implies that a_2 is a costlier action compared to a_1.[11] However, the "cost" of action a_2 is not monotonic in the agent's type (in fact, it is zero at $\theta = 0$ and $\theta = 1$).

The agent's total expected payoff from choosing action a_i in the first period (given that the market's belief associated with action a_i is x_i) is $W_i(\theta, x_i) + T_i(\theta, K)$. From the preceding analysis it is clear that our game is not, and cannot

[10]For this result to hold, what matters for the specification of the second-period payoff is that the agent's indirect profit in the continuation game be a *convex function* of his updated second-period belief regarding his productivity.

[11]In Figure 2 we can easily see the "cost" of a totally uninformative action relative to action a_i whose second-period payoff T_i is depicted in the lower panel. Notice that the agent's payoff in period 2 after choosing an uninformative signal-generating action in period 1 is $\max\{\theta - K, 0\}$. Therefore, the "cost" of choosing this action over a_i is $T_i(\theta, K) - \max\{\theta - K, 0\}$, which is zero near $\theta = 0$ and $\theta = 1$, and peaks at $\theta = K$.

be transfomed into, a Spencian (monotonic) signalling game.

3.2 Main results

A tuple $\langle a(\cdot), x_1, x_2\rangle$ is called an informative (or separating) rational-expectations equilibrium if $a(\theta) = a_1$ and $a(\theta') = a_2$ for some θ and θ' in $[0, 1]$, and

$$a(\theta) = \begin{cases} a_1 & \text{if } W_1(\theta, x_1) + T_1(\theta_1, K) > W_2(\theta, x_2) + T_2(\theta, K), \\ a_2 & \text{if } W_1(\theta, x_1) + T_1(\theta_1, K) < W_2(\theta, x_2) + T_2(\theta, K), \end{cases} \tag{9}$$

$$x_i = \Pr(\omega = H \mid a(\cdot), a_i). \tag{10}$$

The first condition requires that the agent choose his most preferred action with type θ given the market's beliefs and the payoff functions; the second condition states that the market's beliefs are rational given the agent's strategy.

In the following proposition we show that in any informative equilibrium, action a_2 is associated with on average higher types of the agent. That is, the agent choosing to generate a less precise signal indicates that his expected productivity is higher.

Proposition 1 *In any equilibrium where both actions are played with positive probability we have $x_1 \leq x_2$. That is, a relatively less informative signal is chosen, on average, by higher types of the agent.*

Proof. Suppose towards contradiction that $x_2 < x_1$.

If $x_2 = 0$ then type $\theta = 0$ must be choosing a_2 in the equilibrium. On the other hand, by Lemma 1, $W_2(\theta, x_2) = 0 < W_1(\theta, x_1)$ for all $\theta \in [0, 1]$. Together with $T_2(\theta, K) \leq T_1(\theta, K)$ (which follows from the fact that action a_1 is more informative than a_2) this implies $W_2(\theta, x_2) + T_2(\theta, K) < W_1(\theta, x_1) + T_1(\theta, K)$ for all $\theta \in [0, 1]$. Therefore, type $\theta = 0$ prefers to choose a_1, contradiction.

In the rest of the proof assume $x_2 > 0$.

By (iii) in Lemma 1, $W_2(x_2, x_2) = W_1(x_2, x_2) = x_2$, and by (ii) in Lemma 1, $W_1(x_2, x_2) < W_1(x_2, x_1)$ because $x_1 > x_2$. Similarly, $W_2(x_1, x_2) < W_2(x_1, x_1) = W_1(x_1, x_1) = x_1$. Therefore, for $\theta \in \{x_2, x_1\}$,

$$W_2(\theta, x_2) < W_1(\theta, x_1). \tag{11}$$

Recall that by equation (4) the expected first-period wage, $W_i(\theta, x_i)$, is affine in θ. Therefore, (11) must also hold for all $\theta \in [x_2, x_1]$. Since $T_2(\theta, K) \leq T_1(\theta, K)$, we conclude that for all $\theta \in [x_2, x_1]$,

$$W_2(\theta, x_2) + T_2(\theta, K) < W_1(\theta, x_1) + T_1(\theta, K). \tag{12}$$

Since $W_i(\theta, x_i)$ is affine in θ, either $W_1(\cdot, x_1)$ is steeper than $W_2(\cdot, x_2)$, or $W_2(\cdot, x_2)$ is steeper than $W_1(\cdot, x_1)$. We consider the two cases in turn.

Case 1. Suppose that $W_1(\cdot, x_1)$ is steeper than $W_2(\cdot, x_2)$ is. Then, inequality (12) holds for all $\theta > x_1$ as well. This implies that all types $\theta \geq x_2$ strictly prefer action a_1 over a_2. Hence,

$$\Pr\{\omega = H \,|\, a(\cdot), a_2\} < x_2,$$

which contradicts condition (10) in the definition of a separating equilibrium.

Case 2. Suppose that $W_2(\cdot, x_2)$ is steeper than $W_1(\cdot, x_1)$. Then, inequality (12) holds for all $\theta < x_2$ as well. This implies that all types $\theta \leq x_1$ strictly prefer action a_1 over a_2. Hence,

$$\Pr\{\omega = H \,|\, a(\cdot), a_2\} \geq x_1 > x_2,$$

which contradicts condition (10) in the definition of a separating equilibrium. This completes the proof. ■

Proposition 1 rules out the possibility of an equilibrium where a more precise signal-generating action is chosen by on average higher types. The reason why this result may not be obvious is that all else equal, higher types would benefit more from issuing a more informative signal. That is, an agent who is more confident in his productivity is less afraid of the market learning the truth. Moreover, the sensitivity of the agent's gross payoff to his own type depends on the market's beliefs about the productivity of the agent that takes the particular action, and the beliefs are endogenous.

The following proposition states that the types of the agent that choose a less precise signal do not dominate the types choosing a more informative one. In other words, the separating equilibrium is *not* a threshold equilibrium.

Proposition 2 *In any equilibrium where both actions are played with positive probability, there exist types $\theta < \theta' < \theta''$ such that both θ and θ'' choose action a_2 while θ' chooses a_1.*

Proof. By equations (2)-(3) and (4), the period 1 expected wage of the agent with type $\theta = 0$ choosing action a_i is

$$W_i(0, x_i) = \frac{(1-\pi_i)\pi_i x_i}{\pi_i x_i + (1-\pi_i)(1-x_i)} + \frac{\pi_i(1-\pi_i)x_i}{(1-\pi_i)x_i + \pi_i(1-x_i)}.$$

By part (ii) of Lemma 1, this expression is increasing in x_i, and by part (iv) of Lemma 1, it is strictly decreasing in π_i as long as $\pi_i > 1/2$. Therefore, in a separating equilibrium, by $\pi_1 > \pi_2$ and $x_1 \leq x_2$, we have $W_1(0, x_1) < W_2(0, x_2)$. The second-period option value for type $\theta = 0$ is zero, therefore that type's total expected payoff from playing action a_2 exceeds the payoff from playing a_1.

This establishes that some type θ sufficiently close to zero strictly prefers, and therefore chooses, action a_2. Since the average type choosing a_2 exceeds the average type choosing a_1, that is, $x_2 > x_1$, it must be the case that some type θ' below x_2 chooses a_1 and some type θ'' above x_2 chooses a_2. This completes the proof. ■

In the proof of Proposition 2 we showed that if the agent is nearly sure that he is not talented (θ is close to zero) then he prefers to send the least informative signal that is associated with on average the highest types. The reason for this is that the lowest type of the agent does not gain from learning about his true productivity—he knows it is low anyway—therefore he might as well choose the least informative action that is rewarded with the highest wage in the short run; moreover, type $\theta = 0$ also likes the fact that a less informative signal is less likely to generate a (correct) low signal about his productivity. The same calculation does not apply to the highest type, $\theta = 1$. Although the agent who is sure that his productivity is high does not gain from learning and likes to be perceived as a higher type, he may prefer a more informative signal that is more likely to generate a (correct) high signal about his talent. Formally, the expected first-period wage of type $\theta = 1$, $W_i(1, x_i)$, is increasing in x_i by Lemma 1, part (ii), but is also increasing in π_i by Lemma 1, part (iv), hence $W_1(1, x_1) < W_2(1, x_2)$ cannot be assured.

In the rest of the section we establish the existence of a separating equilibrium under various conditions. First, we consider a situation where the agent's type distribution approximates a discrete distribution on exactly three types: low, medium, and high. We establish sufficient conditions under which an equilibrium exists where the low and high types pool on a less informative signal and the medium type chooses a more informative one. Second, under the assumption that the type distribution is continuous, we show that for certain parameter values an equilibrium exists that partitions the types of the agent in three increasing subsets: low types, medium types, and high types. In this "tripartite equilibrium" the average of the low and high types is greater than the average of the medium types. The low and high types pool on a less informative action, while the medium types choose a more informative one.

Discrete type distribution. Suppose that the agent's type is distributed on three values, $\theta_L < \theta_M < \theta_H$, with probability weights (p_L, p_M, p_H) such that

$(p_L\theta_L + p_H\theta_H)/(p_L + p_H) > \theta_M$.[12] Assume that the type-independent payoff that the agent can opt for in the second period, K, is in the neighborhood of θ_M. This means that the medium type is the most likely to gain from learning about his true productivity. In fact, let us assume that as far as the extreme types (θ_L and θ_H) are concerned, the first-period signal, y, does not matter for their second-period choice between "in" and "out". If $K \approx \theta_M$ then a simple sufficient condition for this is

$$\frac{\pi_1\theta_L}{\pi_1\theta_L + (1-\pi_1)(1-\theta_L)} < \theta_M, \tag{13}$$

$$\frac{(1-\pi_1)\theta_H}{(1-\pi_1)\theta_H + \pi_1(1-\theta_H)} > \theta_M. \tag{14}$$

Note that by equations (6)-(7), the left-hand sides of the above inequalities equal $\theta_1^H(\theta_L)$ and $\theta_1^L(\theta_H)$, respectively. The two conditions mean that type θ_L chooses "out" in the second period even if the first-period signal is high (because $\theta_1^H(\theta_L) < \theta_M \approx K$), and type θ_H chooses "in" even if the realization of y is low (because $\theta_1^L(\theta_H) > \theta_M \approx K$). If conditions (13) and (14) hold for action a_1 then they also hold for a_2 because the latter action is less informative ($\pi_2 < \pi_1$). Therefore, neither signal generating action provides more "option value" for the extreme types, that is, action a_2 is "costless" for types θ_L and θ_H. As $\pi_1 \to 1/2$ the conditions (13)-(14) simplify to $\theta_L < \theta_M < \theta_H$; for $\pi_1 > 1/2$ the conditions essentially require that the three types be sufficiently "spread out".

In the separating equilibrium whose existence we want to establish the extreme types pool on action a_2 while the medium type chooses a_1. Therefore the market's equilibrium beliefs are $x_1 = \theta_M$ and $x_2 = (p_L\theta_L + p_H\theta_H)/(p_L + p_H)$; by assumption, $x_2 > x_1$. Assuming that inequalities (13) and (14) hold and $K \approx \theta_M$, the high and low types prefer a_2 over a_1 whenever

$$W_2(\theta, x_2) > W_1(\theta, \theta_M), \text{ for } \theta = \theta_L, \theta_H. \tag{15}$$

The medium type prefers a_1 over a_2 whenever

$$W_2(\theta_M, x_2) + T_2(\theta_M, K) < W_1(\theta_M, \theta_M) + T_1(\theta_M, K),$$

where $W_1(\theta_M, \theta_M) = \theta_M$ by part (iii) of Lemma 1, and $T_i(\theta_M, K) = \pi_i\theta_M - [\pi_i\theta_M K - (1-\pi_i)(1-\theta_M)]\, K$ by equation (8). Under the assumption $K \approx \theta_M$

[12] In Section 2 we assumed the distribution of θ has full support on $[0, 1]$. Therefore, the three-type "discrete" type distribution considered here is really an $(\varepsilon, 1-\varepsilon)$ mixture of (any) full-support type distribution and the distribution on $\{\theta_L, \theta_M, \theta_H\}$, where $\varepsilon > 0$ is arbitrarily small.

the expression for $T_i(\theta_M, K)$ becomes approximately $(2\pi_i - 1)\theta_M(1 - \theta_M)$, and the above inequality can be rewritten as

$$W_2(\theta_M, x_2) < \theta_M + 2(\pi_1 - \pi_2)\theta_M(1 - \theta_M). \tag{16}$$

In the following proposition we summarize the result that when K is sufficiently close to θ_M, the conditions (13)–(16) are sufficient for the existence of an equilibrium where θ_L and θ_H pool on action a_2 and θ_M plays a_1.

Proposition 3 *Suppose that the type distribution is discrete on $\theta_L < \theta_M < \theta_H$ with probability weights (p_L, p_M, p_H), such that $(p_L\theta_L + p_H\theta_H)/(p_L + p_H) > \theta_M$. If the inequalities (13)–(16) hold then, for K sufficiently close to θ_M, there exists an equilibrium where θ_L and θ_H choose a_2 and θ_M plays a_1 in the first period.*

The construction of the equilibrium is illustrated in Figure 3, which corresponds to the following numerical example.

Example 1 *Let $\pi_1 = 2/3$, $\pi_2 = 1/2$. Asssume that the support of the type distribution is $\theta_L = 1/3$, $\theta_M = K = 1/2$, $\theta_H = 2/3$, and the probability weights (p_L, p_M, p_H) are such that $x_2 \equiv (p_L\theta_L + p_H\theta_H)/(p_L + p_H) \in (5/9, 7/12)$, for example, $(p_L, p_M, p_H) = (7/48, 1/2, 17/48)$. There exists a separating equilibrium where θ_L and θ_H pool on action a_2 while θ_M chooses a_1.*

In the example the first-period wage of the agent choosing action a_2 is $W_2 \equiv x_2 \in (5/9, 7/12)$, because the action is uninformative.[13] This wage exceeds the first-period wage from action a_1 for any type of the agent because $W_1(1, 1/2) = 5/9$ by equation (5) and $\pi_1 = 2/3$. The parameters are chosen so that types θ_L and θ_H do not enjoy positive second-period option values from either first-period action, hence by $W_2 > W_1$, both θ_L and θ_H indeed strictly prefer action a_2 to a_1. Finally, type θ_M strictly prefers a_1 over a_2 because $W_1(\theta_M, \theta_M) + T_1(\theta_M, \theta_M) = \theta_M + (2\pi_1 - 1)\theta_M(1 - \theta_M) = 7/12 > W_2$.

Continuous type distribution. Any example with a three-type discrete distribution that satisfies the conditions of Proposition 3 (e.g., Example 1) can easily be transformed into an example with a continuous type distribution where, in a separating equilibrium, intervals of low and high types choose action a_2 and an interval of medium types play a_1. One way to do this is the following. Denote

[13] The assumption $\pi_2 = 1/2$ is a useful simplification for the purpose of calculating the example, but it is certainly not implied by the conditions of Proposition 3.

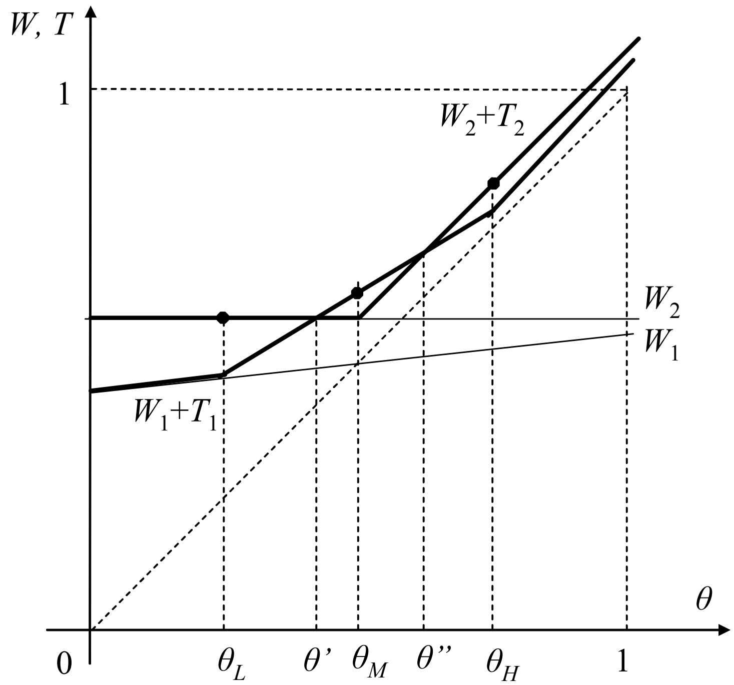

Figure 3: Existence of a separating equilibrium.

the types at which $W_1 + T_1$ and $W_2 + T_2$ intersect by θ' and θ'', as in Figure 3. Then, let the c.d.f. of the type distribution be

$$F(\theta) = \begin{cases} \left(\frac{\theta}{\theta'}\right)^{\frac{\theta_L}{\theta'-\theta_L}} p_L & \text{for } \theta \in [0, \theta'], \\ \left(\frac{\theta-\theta'}{\theta''-\theta'}\right)^{\frac{\theta_M-\theta'}{\theta''-\theta_M}} p_M + p_L & \text{for } \theta \in (\theta', \theta''), \\ \left(\frac{\theta-\theta''}{1-\theta''}\right)^{\frac{\theta_H-\theta''}{1-\theta_H}} p_H + p_M + p_L & \text{for } \theta \in [\theta'', 1]. \end{cases}$$

It is easy to see that this distribution is continuous with $F(\theta') = p_L$ and $F(\theta'') = p_L + p_M$, and that $E[\theta|\theta \leq \theta'] = \theta_L$, $E[\theta|\theta' \leq \theta \leq \theta''] = \theta_M$, and $E[\theta|\theta'' \leq \theta] = \theta_H$. Therefore, if the discrete model satisfies the conditions of Proposition 3 then there exists a separating equilibrium in the model where θ is drawn according to F such that types $\theta \leq \theta'$ and $\theta \geq \theta''$ choose a_2 while types $\theta \in (\theta', \theta'')$ choose a_1.

We now show that similar "tripartite" equilibria also exist for a nontrivial set of parameter values for *any* continuous type distribution. For example, a separating equilibrium exists for certain values of K (the second-period outside option) when a_2 is uninformative and a_1 is only a little more informative. The reason that we find this result interesting is that in the limit, when both actions are uninformative (or both are equally informative), separating equilibria do not exist.[14] However, by making a_1 slightly more informative than a_2 a tripartite equilibrium emerges.

Proposition 4 *For any continuous distribution of* θ *with full support on* $[0,1]$*, if* π_1 *is sufficiently close to* $\pi_2 = 1/2$ *then there exists* $K \in (0,1)$ *such that in a separating equilibrium, types* $\theta \in [0,A) \cup (B,1]$ *choose* a_1 *and types* $\theta \in [A,B]$ *choose* a_2*, where* $0 < A < K < B < 1$.

Proof. See the Appendix. ■

The intuition of how tripartite equilibria are sustained is the following. The low and high types are not keen on learning more about their true productivity, therefore they choose the action that is less informative, but is perceived better by the market in the first period. (We need to impose conditions on the parameters, e.g., that π_1 is sufficiently close to π_2, in order to ensure that high types are not better off by sending a more informative signal—see also the discussion after the proof of Proposition 2.) On the other hand, medium types are interested in updating their beliefs about their productivity, and are willing to be perceived as on average lower types by choosing a more informative action in the first period. For them, this action increases the value of the option to stay or quit in the second period so much so that it outweighs the "stigma" associated with its choice.

3.3 Discussion

In this section we shall discuss some variants of our model.

First, one may consider an alternative model where the signal generated by the agent's action in period 1 is observable only to the agent himself. The market still observes his action and pays him a credence wage in the first period; then, in the second period, the agent faces the same in/out decision as he does in the original model. Although this alternative model may not correspond to any real-life situations,[15] all of our results continue to hold there. Intuitively, in this variant, high types have even fewer reasons to choose a more informative action because such an action can no longer signal that the agent is "not afraid of the truth". In any separating equilibrium, choosing a less informative signal displays

[14] The reason is that in this case, both actions are "costless".

[15] For example, it is difficult to imagine that a young artist can create her first work, get rewarded based on whether it is conventional or experimental, and find out its quality without actually showing the piece to the outside world.

strength, and only intermediate (on average, lower) types take a more informative action.

Second, one may consider a variant of the model is where the agent does not face a second-period decision; instead, there is an exogenous cost of taking action a_2. (Recall that in our original model the "cost" of a_2 arises from a lower second-period payoff.) However, our Proposition 1 continues to hold: the high types' incentive to signal that they are not afraid of the truth is overwhelmed by their incentive to show strength by picking a costlier action. In fact, this result is so robust that it carries over to models where the two actions are not rankable according to their informativeness.[16]

Finally, let us discuss some of the simplifying assumptions imposed on the model. We limited the agent to choose between two actions, with each action generating a binary signal. These assumptions can be relaxed without compromising any of our results. Perhaps the only technical assumption that is important for our analysis is that the underlying talent of the agent (the state of nature, ω) is also binary. We made this assumption in order to ensure that the agent's private information (the posterior distribution over ω) is one-dimensional. The results of Section 3.2 remain unchanged as long as the agent's type, θ, is a one-dimensional variable indexing a convex set of probability vectors over the values of ω.

4 Conclusions

We have considered a model of signalling where the agent also learns about his talent from the realization of signal that his action generates. Our aim was to build the simplest, most tractable model of signalling with career concerns.

We found that in any equilibrium where at least two actions are played with positive probability, a less precise signal is always associated with a higher average productivity agent. However, the types of the agent that choose a less informative signal-generating action are not uniformly higher than the types that choose a more informative action. In fact, there always exist low, medium, and high agent-types such that the low and high types pool on the former action, while the medium type plays the latter one. We showed that this type of equilibrium indeed exists under fairly general conditions.

[16] Details of these arguments are available from the authors.

5 Appendix: Omitted Proofs

Proof of Lemma 1. (i) If $x_i = 0$ then $w_i^H = w_i^L = 0$ by (2)-(3), and so $W_i(\theta, 0) = 0$ by (4). If $x_i = 1$ then $w_i^H = w_i^L = 1$, hence $W_i(\theta, 1) = 1$ as well.

(ii) From (4),

$$\frac{\partial W_i(\theta, x_i)}{\partial \theta} = (2\pi_i - 1)(w_i^H - w_i^L),$$

which is positive because both terms in the product are positive for $\pi_i > 1/2$.

To see that W_i is strictly increasing in x_i, note that both w_i^H and w_i^L are strictly increasing in x_i provided $\pi_i > 1/2$, and that W_i is just a weighted average of w_i^H and w_i^L.

(iii) For $\theta = x_i$, (5) simplifies to $W_i(x_i, x_i) = \pi_i x_i + (1 - \pi_i)x_i = x_i$.

(iv) Differentiating (5) with respect to π_i yields

$$\frac{\partial}{\partial \pi_i} W_i(\theta, x_i) = \frac{(2\pi_i - 1)(\theta - x_i)(1 - x_i)x_i}{[\pi_i x_i + (1 - \pi_i)(1 - x_i)]^2 [(1 - \pi_i)x_i + \pi_i(1 - x_i)]^2}.$$

The sign of the right-hand side is the same as the sign of $(\theta - x_i)$ because the other terms are all positive. ■

Proof of Proposition 4. By assumption, a_2 is uninformative ($\pi_2 = 1/2$), hence $\theta_2^L(\theta) = \theta = \theta_2^H(\theta)$, and so $T_2(\theta, K) = (\theta - K)\,\mathbf{1}_{\theta \geq K}$. Since action a_1 is informative, the second-period benefit advantage of action a_1 over a_2, $T_1(\theta, K) - T_2(\theta, K)$, is positive whenever $\theta_1^L(\theta) < K < \theta_1^H(\theta)$, and zero otherwise. Recall that the agent's first-period wage is $W_i(\theta, x_i)$, as defined in (4). In the first period, the agent chooses a_i to maximize $W_i(\theta, x_i) + T_i(\theta, K)$.

Denote $\mu = E[\theta]$. By the continuity and full support of the distribution of θ, we have $\mu < 1$.

We claim that for π_1 sufficiently close to $\pi_2 = 1/2$, there exists $x_1^* \in [0, \mu)$ such that

$$W_1(1, x_1^*) = W_2(1, \mu) \equiv \mu, \text{ and} \tag{17}$$

$$\mu + T_2(x_1^*, x_1^*) < W_1(x_1^*, x_1^*) + T_1(x_1^*, x_1^*) < 1 + T_2(x_1^*, x_1^*). \tag{18}$$

To see this, first note that for any π_1 there exists $x_1^* \in (0, \mu)$ satisfying (17) because $W_1(\theta, x_1)$ is continuous in x_1, and by Lemma 1,

$$W_1(1, 0) = 0 < \mu = W_1(\mu, \mu) < W_1(1, \mu).$$

$W_1(\theta, x_1^*)$ is positive and increasing in θ, therefore

$$\begin{aligned}\mu - W_1(x_1^*, x_1^*) &< W_1(1, x_1^*) - W_1(0, x_1^*) \\ &= \frac{(2\pi_1 - 1)\,\pi_1 x_1^*}{\pi_1 x_1^* + (1-\pi_1)(1-x_1^*)} - \frac{(2\pi_1 - 1)(1-\pi_1) x_1^*}{(1-\pi_1) x_1^* + \pi_1(1-x_1^*)}.\end{aligned}$$

By equation (8), $T_1(\theta, K) - T_2(\theta, K)$ peaks at $\theta = K$, where

$$T_1(K, K) - T_2(K, K) = (2\pi_1 - 1)\, K\, (1-K)\,.$$

Therefore, a sufficient condition for $\mu + T_2(x_1^*, x_1^*) < W_1(x_1^*, x_1^*) + T_1(x_1^*, x_1^*)$ is

$$\frac{(2\pi_1 - 1)\,\pi_1 x_1^*}{\pi_1 x_1^* + (1-\pi_1)(1-x_1^*)} - \frac{(2\pi_1 - 1)(1-\pi_1) x_1^*}{(1-\pi_1) x_1^* + \pi_1(1-x_1^*)} < (2\pi_1 - 1)\, x_1^*\,(1-x_1^*)\,.$$

Since $\pi_1 > 1/2$, we may cross-divide by $(2\pi_1 - 1)x_1^* > 0$. However,

$$\frac{\pi_1}{\pi_1 x_1^* + (1-\pi_1)(1-x_1^*)} - \frac{1-\pi_1}{(1-\pi_1) x_1^* + \pi_1(1-x_1^*)} < 1 - x_1^*,$$

which holds for any $x_1^* \in (0,1)$ if π_1 is sufficiently close to $1/2$ because the left-hand side tends to zero as π_1 tends to $1/2$. Therefore for π_1 sufficiently close to $1/2$ and $x_1^*(\pi_1)$ satisfying (17), the first inequality in (18) holds. The second inequality in (18) also holds for π_1 close to $1/2$ because $x_1^*(\pi_1) < \mu < 1$ and $\lim_{\pi_1 \to 1/2} [T_1(\theta, K) - T_2(\theta, K)] = 0$ for all (θ, K).

In the rest of the proof fix π_1 and x_1^* such that (17) and (18) hold.

Define

$$\bar{x}_2 = x_1^* + T_1(x_1^*, x_1^*) - T_2(x_1^*, x_1^*).$$

That is, $\bar{x}_2$ is the highest x_2 such that $W_2(\theta, x_2) + T_2(\theta, x_1^*) \leq W_1(\theta, x_1^*) + T_1(\theta, x_1^*)$ at $\theta = x_1^*$. By inequality (18) and $W_2 \equiv x_2$, we have $\bar{x}_2 \in (\mu, 1)$.

Define, for all $x_2 \in [\mu, \bar{x}_2]$,

$$C(x_2) = \{K \mid \text{for } \theta = K,\, W_2(\theta, x_2) + T_2(\theta, K) \leq W_1(\theta, x_1^*) + T_1(\theta, K)\}\,.$$

This is the set of outside option levels (K's) such that type $\theta = K$ weakly prefers a_1 to a_2 given the market's beliefs x_1^* and x_2. It is easy to see that $C(x_2)$ is always an interval, $[\underline{c}(x_2), \bar{c}(x_2)]$, that contains x_1^*. Moreover, $C(\mu) = [\underline{K}, 1]$ with $\underline{K} \in (0, x_1^*)$, and $C(\bar{x}_2)$ is either $[x_1^*, \bar{c}(\bar{x}_2)]$ or $[\underline{c}(\bar{x}_2), x_1^*]$.

Define, for all $x_2 \in [\mu, \bar{x}_2]$ and $K \in C(x_2)$,

$$D(x_2, K) = \{\theta \mid W_2(\theta, x_2) + T_2(\theta, K) \leq W_1(\theta, x_1^*) + T_1(\theta, K)\}\,. \tag{19}$$

This is the set of types (θ's) that prefer action a_1 over action a_2 given the market's beliefs, $x_1 = x_1^*$ and x_2, and the outside option, K. Clearly, $D(x_2, K)$ is a non-empty inteval for all (x_2, K) in the domain, and both endpoints of this interval are continuous functions of x_2 and K.

It is easy to see that if K equals either $\underline{c}(x_2)$ or $\bar{c}(x_2)$ then $D(x_2, K) = \{K\}$. Therefore

$$E\left[\theta \mid \theta \in D(x_2, \underline{c}(x_2))\right] < x_1^* < E\left[\theta \mid \theta \in D(x_2, \bar{c}(x_2))\right].$$

Since the endpoints of $D(x_2, K)$ and the distribution of θ are continuous, the Intermediate Value Theorem implies that there exists $K = K(x_2)$ in the interior of $C(x_2)$ such that

$$E\left[\theta \mid \theta \in D(x_2, K(x_2))\right] = x_1^*.$$

For $x_2 = \mu$, $K(x_2) = K(\mu) \in (\underline{K}, 1)$, while for $x_2 = \bar{x}_2$, $K(x_2) = K(\bar{x}_2) = x_1^*$. $D(x_2, K(x_2))$, which always contains x_1^*, is a non-degenerate interval for all $x_2 \in [\mu, \bar{x}_2)$; however, $D(\bar{x}_2, K(\bar{x}_2)) = \{x_1^*\}$.

Define, for all $x_2 \in [\mu, \bar{x}_2]$,

$$\hat{x}_2(x_2) = \min\left\{\bar{x}_2, E\left[\theta \mid \theta \notin D(x_2, K(x_2))\right]\right\}.$$

This is a continuous function because D and the distribution of θ are both continuous. Notice that for $x_2 = \mu$, $\hat{x}_2(x_2) = \hat{x}_2(\mu) \in (\mu, \bar{x}_2]$ because $D(\mu, K(\mu))$ is a non-degenerate interval of θ with a conditional expectation $x_1^* < \mu$, while $E[\theta] = \mu$. For $x_2 = \bar{x}_2$, we have $\hat{x}_2(x_2) = \hat{x}_2(\bar{x}_2) = \mu$ because $D(\bar{x}_2, K(\bar{x}_2)) = \{x_1^*\}$ and the distribution of θ is continuous.

Since $\hat{x}_2(x_2)$ is continuous on $[\mu, \bar{x}_2]$ and $\hat{x}_2(\mu) > \mu = \hat{x}_2(\bar{x}_2)$, the Intermediate Value Theorem implies that there exists $x_2^* \in (\mu, \bar{x}_2)$ such that $\hat{x}_2(x_2^*) = x_2^*$.

Finally, we claim that for π_1 fixed above and $K = K(x_2^*)$, there exists a separating equilibrium where types $\theta \in [A, B] \equiv D(x_2^*, K(x_2^*))$ choose action a_1 and all other types choose a_2. This is easy to check. The market's rational-expectations beliefs must be that the average type choosing a_1 is $E\left[\theta \mid \theta \in D(x_2^*, K(x_2^*))\right] = x_1^*$, and the average type choosing a_2 is $E\left[\theta \mid \theta \notin D(x_2^*, K(x_2^*))\right] = x_2^*$. Given these beliefs, the set of types that prefer a_1 over a_2 is exactly $D(x_2^*, K(x_2^*))$ by equation (19). Since x_2^* is in the interior of $[\mu, \bar{x}_2]$ the interval $D(x_2^*, K(x_2^*)) \subset [0, 1]$ is non-degenerate and it contains both K and x_1^*. ■

References

[1] Avery, Christopher N., and Judith A. Chevalier, "Herding over the career," *Economics Letters*, 63 (1999), pp. 327-333.

[2] Brandenburger, A., and B. Polak, "When Managers Cover Their Posteriors: Making the Decisions the Market Wants to See" *RAND Journal of Economics*, 27:3 (1996), pp. 523-541.

[3] Fang, Hanming, "Social Culture and Economic Performance," *American Economic Review*, 91 (2001), pp. 924-937.

[4] Feltovich, Nick, Richmond Harbaugh, and Ted To, "Too Cool for School? Signalling and Countersignalling," *RAND Journal of Economics*, 33 (2002), 4:630-649.

[5] Holmström, Bengt, "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, 66 (1999), pp. 169-182.

[6] Hvide, H. K., "Education and the Allocation of Talent," *Journal of Labor Economics*, 21 (2003), 945-970.

[7] Morris, Stephen, "Political Correctness," *Journal of Political Economy*, 109 (2001), pp. 231-265.

[8] Nelson, P., "Advertising as Information," *Journal of Political Economy*, 82 (1974), pp. 729-754.

[9] Ottaviani, Marco, and Peter Norman Sørensen, "Reputational Cheap Talk," *RAND Journal of Economics*, vol.37:1 (2006) pp.155-175.

[10] Ottaviani, Marco, and Peter Norman Sørensen, "Professional Advice," *Journal of Economic Theory*, 126 (2006) pp.120-142.

[11] Prendergast, Canice, and Lars Stole, "Impetuous Youngsters and Jaded Oldtimers," *Journal of Political Economy*, 104 (1996), pp. 1105-1134.

[12] Ross, S. A., "The Determination of Financial Structure: The Incentive-Signalling Approach," *Bell Journal of Economics*, 8 (1977), pp. 23-40.

[13] Scharfstein, David, and Jeremy Stein, "Herd Behavior and Investment," *American Economic Review*, 80 (1990), pp.465-479.

[14] Spence, Michael, "Job Market Signaling," *Quarterly Journal of Economics*, 87:3 (1973), pp. 355-74.

[15] Teoh, S. H., and C. Y. Hwang, "Nondisclosure and Adverse Disclosure as Signals of Firm Value," *Review of Financial Studies*, 4 (1991), pp. 283-313.

[16] Veblen, T., *The Theory of the Leisure Class.* Macmillan, New York, 1899.

[17] Zahavi, A., "Mate Selection—A Selection for a Handicap," *Journal of Theoretical Biology*, 53 (1975), pp. 205-214.