Fat Products

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Abstract

The economics literature generally considers products as points in some characteristics space. Starting with Hotelling, this served as a convenient assumption, yet with more products being flexible or self-customizable to some degree it makes sense to think that products have positive measure. I develop a model where firms can offer interval long 'fat' products in the spatial model of differentiation. Contrary to the standard results profits of the firms can decrease with increased differentiation - there is a standard effect of lowering the incentive to cut prices, but there is also an incentive to provide more content sometimes resulting in lower profits. Consumer welfare increases unambiguously with respect to the standard model of Salop. I also find that it is profitable for firms to commit as an industry not to make fat products. If one firm is a leader and another is a follower, the leader accommodates the follower by settling for less profits if differentiation is small.

1 Introduction

Harold Hotelling was arguably the first to introduce product differentiation. In his model a product is a point in the linear space of characteristics. While that model is generally associated with differentiation in locations and distances, it is clear from the article that Hotelling had characteristics space in mind – he talks about how his model applies to things from sweetness of cider to

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political parties' positions on tariffs. Lancaster's work (1971) formally extended the definition of the product to a point in some characteristics space with many dimensions of differentiation, while giving credit to Hotelling for being the pioneer in the field:

> Hotelling had provided a hint as to a possible solution of the product variation problem by extending his model of pure spatial competition... Hotelling himself did not develop the idea further, Chamberlin ignored it, and no one else took it up.[1]

A product is generally defined as follows: "*a complete bundle of benefits or satisfactions that buyers perceive they will obtain if they purchase the product.*"[2] Why should we think of a product as a point in a characteristics space? Since Hotelling's article, the economic literature has represented products as points. This has proved to be a useful and convenient assumption that stood the test of time. The following question arises, however: "why should consumer's utility function be defined just over points?" A general definition of a product should have utility maps going from a set of characteristics to the real line. If we want to look at maps that are relatively better behaved, then we can look at maps of "contiguous" sets of characteristics – intervals in one dimensional Hotelling space. The cost function of the products can also be a map from the set of the product's characteristics to the real line. If results turn out to be broadly similar to those using utility and cost functions defined over points, then we can safely continue using the latter assumption. Otherwise, more general definitions are needed.

In this article, I examine a straightforward extension of point products to interval-long products in a one-dimensional spatial model. I refer to these as "fat products". A consumer's utility depends on whether or not her preferred point is inside the range of the product. If it is, then the consumer does not need to incur any travel or adjustment costs[3]. If it is not, then the consumer has to incur the costs of traveling to the border of the product. As a result, firms can position their product closer to some consumers without moving away from others. However, such flexibility is costly – a firm's cost of developing a product is a convex function of the length (measure) of the product.

I find that the firms would be willing to collude to make zero measure (point) products, but in the absence of collusion they develop products of positive measure. Moreover, firms might

[1] See Lancaster (1971), p. 16

[2] Wikipedia (English), search query "product".

[3] I will go on referring to travel or transportation costs throughout the article, although costs of adjustment to a different brand, or a different set of characteristics, provides an alternative interpretation.

incur losses as the degree of differentiation increases, because while in equilibrium prices rise, the equilibrium range of the product rises as well, in several instances resulting in an overall drop in profits. As a consequence if there is a free entry (or zero profit) condition, this would imply that as the market grows, or becomes more differentiated, the number of firms that can survive stays constant or even decreases, as each firm unilaterally finds it optimal to escalate its R&D spending and increase the measure of its product.[4]

I examine an extension, in the spirit of Stackelberg competition, where there are two firms, and one of them is a leader – it picks the price and the size of her product first. I find that with sufficiently small development costs the leader picks bigger price and measure than the follower, and ends up with higher market share and profits as one would expect. However, the result is reversed if the development costs are bigger (or the firms are more homogenous) – the leader accommodates the follower by picking a smaller measure than the follower will, and under some conditions even charging smaller price.

There are several branches of literature close in appearance to Fat Products. One of the most well-known is bundling. Bakos and Brynjolfsson (1999) examine bundling of many goods, with the application discussed being distribution of digital goods via internet. The model is built on the law of large numbers, and the assumption that consumers' valuations are i.i.d. The outcome is that bundling a very large number of products can be profitable because the firm can just charge the mean of the distribution. The bundling literature had focused on independently valued products and occasionally on complements. One of the very few articles on bundling of substitutes is by Venkatesh and Kamakura (2003) which finds that if the goods are highly substitutable, it is not a good idea to bundle to them. The interval-long products in the Hotelling space with each consumer interested in her ideal point can be viewed as a bundle, however it is a bundle of an uncountable number of goods (all the points in the Fat Product), where the value of the bundle is the value of the most valuable good in that bundle.

My model also has some similarity to articles describing the "crowding out" effect, in particular to Schmalensee (1978). While the intuition from that article is that incumbents fill up the whole

[4] This "escalation mechanism" is reminiscent of Sutton (1991). As firms' willingness to pay for broader products increases, Sutton's escalation mechanism kicks in. Related to this idea is recent work by Ellickson (2005) looking at supermarkets as natural oligopolies in terms of making their product offerings broader, the stores larger, and the aisles wider, therefore not letting in more firms as the market size grows.

arc of a circle, they do it for deterrence reasons. In my model, where there is no deterrence, firms fill up intervals to capture more consumers not worrying about potential entrants. The extension with a leader and a follower has some resemblance to Schmalensee results if the costs of development are not too high – the leader will choose to produce a product of big measure and force the follower into a market niche. Of course the products are still point products in Schmalensee (1978), which may make more sense as far as cereal is concerned.

Cheng and Nahm (2006) examine what happens in a vertical differentiation model when there is a system of a base product and an add-on which is valueless by itself. As the value of the base product increases, keeping the value of the system constant the pricing will go from complementary with the double marginalization problem to independent getting rid of the problem. My model examines the optimal boundary of products in the horizontal differentiation framework with competing symmetric firms.[5]

2 Applications

Here is how Gerard Debreu describes a product ("commodity") in Theory of Value (1959):

> a *commodity* is therefore defined by specification of all its physical characteristics, of its availability date, and of its availability location. As soon as one of these factors changes, a *different* commodity results.[6]

Contribution of this article is to think of locations, characteristics, and dates as ranges as opposed to being points. Location does not have to be a point. A consumer can request a delivery, and then a product can be at a different location without resulting in a different product. If delivery costs the same in some area then wheat in Chicago and wheat in Minneapolis can be thought of as the same product. A trip on Chicago's Elevated Line costs two dollars no matter if the customer goes one mile down the line to a store or some twenty miles from Evanston to Hyde Park[7]. If the consumer lives far from the end of a line, then she can walk or take a bus from the last stop, which will require extra expenses – just like the fat products model. In fact, any access good can be

[5] Discussion of other related literature is scattered throughout more relevant (for a specific article) sections.
[6] Debreu (1959), page 30. Italics are preserved from the original.
[7] $1.75 with a discount card.

thought of as a Fat Product. Consumers pay to access the good and pick what they want inside – the applications range from Disneyland to network access to all-you-can-eat buffets. The measure of the product is then the extent of the access provided.

Imagine a beach and two vendors selling ice-cream. Instead of being stationary they can walk around, and the consumers who are not in one of the route intervals (or who are dissatisfied with the price in the route they belong to) can come to the boundary of one of the routes and wait for the vendor to come by. I am interested in how long are the vendors' routes, and what price will they charge. Alternatively, two vendors can be at the opposite ends of the unit interval, deciding how much extra ground to cover (or maybe to just stay in one place if the optimum is zero)[8].

I do not intend the model to just be a spatial model – this is a model of product differentiation. Think of an office chair. One can adjust height up and down in a continuous interval – each firm does not have many lines of otherwise the same office chairs, each of a particular height. Think of how much sugar you put in your coffee for the same price. Sweetness of cider was one of Hotelling's applications, but if you come to the coffee shop and the sugar is free, then you can continuously adjust the sweetness to your own taste without paying more. Brightness and focus of a projector or a TV set, self-customizable products, and all of the above are examples of fat products. The consumer gets an interval of characteristics in one product, and can pick the one that she wants. Another example to look at is any software with options. The user can adjust how big is the window, the size of the font, the color of the letters – whatever that software specializes in, but still the available characteristics to the user are an interval of characteristics (i.e. how wide should the window of Acrobat Reader be – from zero to the width of the screen), for which the user does not have to pay more.

Lancaster in his book had provided another kind of application for fat products with respect to characteristics – combinable goods.

> If goods are combinable, so that two goods can be consumed simultaneously to give a characteristics collection that is a combination of the characteristics of the separate goods.... the problem must be approached anew..... In the combinable case, however, the individual could attain exactly the most-preferred collection of characteristics (that

[8] Thanks to Shane Greenstein for this example.

is the collection he or she would obtain from the most-preferred good, if it were available) by consuming goods Y and Z together.[9]

The fat products are combinable goods with the goods Y and Z being the endpoints of the interval, and bundle Y and Z sold by one firm. Imagine buying a cocktail set – every consumer can make their own favorite combination while paying the same price, while the combination can vary continuously. Alternatively one can mix hot and cold water in the tap to achieve the perfect temperature. There is already some literature on the combinable goods, started by Anderson and Neven (1989), who prove that with combinable goods firms end up playing the socially optimal strategies. The consumers in this literature can buy a bit of both products around them and mix them together, however these are point products being offered by different firms.[10]

An example with time for Fat Products is coupons that consumers can use in a given time period. The coupon does not change if it is used several days before or several days after, and the length of the product is just the expiration date of the coupon – then the value is zero. While the coupon loses the value after the expiration date, the companies issuing them still have to think over the length of the period when the coupon might be used.

Stores' operating hours is a close concept to a fat product – the consumer can go to the store at whatever time the store is opened without having to pay extra fees. Anyone checking in at a hotel can check in whatever time they want to in a given interval (say, from 4pm). Also operating hours are easily modeled as one-dimensional and intervals – it is hard to imagine a store or a hotel opening and closing for a few seconds each minute.

Shopping hours literature developed some models close to interval-long products in one dimensional space. The shops are picking the hours when are the stores open and prices. Consumers have optimal shopping times and incur disutility if they have to move their shopping hours if the store is closed at a particular time.[11] The literature is mainly interested in what happens if the

[9] Lancaster (1971), p. 56–58.

[10] The applications of this concept are TV viewing and advertisement, where viewers can see different channels in the same day, and pick the optimal mix for them. Two recent papers on the topic are Gal-Or and Dukes (2003) arriving at a conclusion that firms prefer to minimally diffirentiate their products and Gabszewicz et. al. (2004), where the authors find that the less viewers like advertising the closer will the TV stations come to eachother .

[11] Inderst and Irmen (2005) considers two shops choosing between being opened either at day, at night, never or always. The result is that there can be some asymmetric hours provision from two ex-ante similar stores. Also, if shopping hours are regulated, the retailers will charge higher prices and will be better off. Shy and Stenbacka (2006) considers continuous time intervals, yet keep prices as exogenous, again resulting in the possibility of asymmetric shopping hours, and in the fact that shops are not opened long enough from the social welfare point of view.

government regulates the shopping hours, and when is it possible to set asymmetric opening and closing hours, but while asymmetric distribution of consumers plays a big role in that literature, this model might still be useful in that context.

3 Monopoly

3.1 The Fat Products Monopolist Model

The model setup closely follows that of Salop, with the important differences in the parts describing the production possibilities and costs. The conceivable goods are located along the unit interval. The consumers are going to be the standard address consumers – located uniformly along the unit circumference circle, with some fixed reservation price for the product, say $R > 0$, and transportation costs $t \times d$ – with $d \geq 0$ being the distance between the consumer and the product[12], and $t \geq 0$ being the marginal cost of transportation. If the consumer has a choice, she is going to buy the product to maximize $U(d) - p$, where p is the price of the product, and $U(d) = R - td$. The outside option's utility is 0.

The monopolist can choose to make products in a form, and with the characteristics of an interval, say $[a, b]$, $0 \leq a \leq b \leq 1$, with the development/production cost of a product $C_{development}([a, b]) = c(b - a)$, where $c(\cdot)$ is a *function*. I will usually refer to the length of the interval offered as m[13]. The standard measure zero products will be developed/produced with some positive fixed cost, so $c(0) = F > 0$. Also let $c'(0) = 0$ to avoid the uninteresting case where the costs are so steep to expand from a point that no firm will be willing to take them. I will assume throughout the paper that $t > R$, i.e. in the simple monopoly case there are consumers who do not buy the good.

3.2 Non-Negative Measure Solution

To make sure that the second order conditions are satisfied I make the following assumption[14].

Sufficiently Convex Cost Assumption (M). *Let the cost function $c(m)$ of developing/producing a fat product of length m be such that $c''(m) \geq \dfrac{t}{4}$.*

[12] or the closest point on the interval in case of the product being an interval

[13] U for Utility, t for Transportation, R for Reservation, c for Cost and m for Measure

[14] For an example where this assumption is not satisfied (the costs are linear) and consumers are on a unit interval as opposed to a line see Appendix A. The qualitative results do not change substantially.

Does this assumption make sense? Since c is a cost function, it should be increasing. This assumption requires the cost function to be not just convex, but also have the second derivative bounded away from zero. Since adding an extra $\varepsilon > 0$ of measure to the product requires this ε to interact with the rest of the measure already in place, it seems natural to assume that as measure increases the addition of the same ε becomes more and more costly. Now we can move on to the proof.

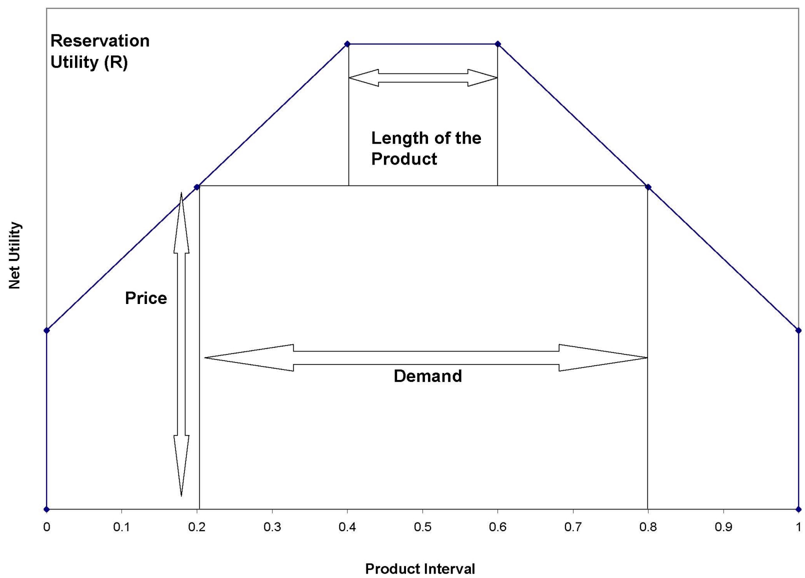

Figure 1. Consumers' utility with a Fat Monopolist.

Theorem 1 *(Fat Product Monopolist). A monopolist which has the ability to offer a Fat Product charges* $p^* = \dfrac{R}{2} + \dfrac{tm^*}{4}$ *and makes products with the measure of* m^**, such that* $c'(m^*) = p^*$.

Proof. The difference between the figure above and a standard spatial problem is that before the net utility (utility less the travel costs) would look like a triangle as opposed to a top of a trapezoid – there would be no flat part. The demand remains from the zero measure case, but there is also a portion added, equal to the length of the Fat Product. Therefore the demand for a given price for a non-negative length measure m will be $D(p) = m + \dfrac{2(R-p)}{t}$.

Therefore the profit of the monopolist will be $\Pi_{M+}(p, m) = p \times D(p) - c(m) = p(m + \dfrac{2R}{t}) -$

$\frac{2p^2}{t} - c(m)$.

I need to look at the Second Order Conditions to make sure that the function is concave in p and m. The Hessian is $H_M(m,p) = \begin{bmatrix} -\frac{4}{t} & 1 \\ 1 & -c''(m) \end{bmatrix}$. To ensure concavity the first leading principle major needs to be less than zero, and the determinant needs to be positive. The first leading major is $-4/t$, and since $t > 0$ it is negative. Looking at the determinant, we can see that the second order conditions are satisfied iff $c''(m) \geq \frac{t}{4}$.

Now look at the First Order Conditions:

$\frac{\partial \Pi}{\partial p} = m^* + \frac{2R}{t} - \frac{4p^*}{t} = 0 \Longrightarrow$

$$p^* = \frac{R}{2} + \frac{tm^*}{4}. \tag{1}$$

$\frac{\partial \Pi}{\partial m} = p^* - c'(m^*) = 0 \Longrightarrow$

$$c'(m^*) = p^*. \tag{2}$$

■

3.3 Comparing the Results

Notice that if we force $m = 0$, we get the results that we would have with the standard point products ($p_0^* = \frac{R}{2}$). The optimal measure is increasing in both R and t, which is intuitive as if the utility to be extracted from the consumers is high, the monopolist will offer a wider product, and if the transportation costs are higher, then the incentives to offer a wider product increase.

Note that the condition on the derivative of the cost function is exactly the marginal revenue equals the marginal cost condition. Increasing the measure by an epsilon increases the demand by epsilon, and therefore the revenue by epsilon times price. But increasing the measure by an epsilon costs the derivative of the cost function at the current measure times epsilon.

The prices are higher with a fat product since not only does the firm need to cover the development costs, but also now they can extract the full reservation price. The profits are of course higher as well, since measure equal to zero option was there for the firm to take. It is not obvious what happens with the number of the consumers served, and their welfare.

Corollary 1 *The number of consumers served is strictly higher under Fat Products than under* *standard assumptions.*

Proof. $D(p^*, m^*) = \dfrac{m^*}{2} + \dfrac{R}{t} > D(p_0^*)$. ∎

The effect of Fat Products on consumption is unambiguous - even though the price is going up, the more of the measure offered, the more product is going to be sold. It is not as straightforward for the consumer welfare.

Proposition 1 *Consumer welfare will increase under Fat Products iff* $R > \dfrac{3tm^*}{4}$.

Proof. Consumer welfare is the trapezoid above the price line and below the net utility curve on Figure 1. Therefore, its area is easy to calculate: $CW_+ = \dfrac{(D^* + m^*)}{2} \times (R - p) = \dfrac{R^2}{4t} - \dfrac{3t{m^*}^2}{16} + \dfrac{m^* R}{4}$. Compare this to the consumer welfare with $m = 0$, which is $CW_0 = \dfrac{R^2}{4t}$. $CW_+ > CW_0$ iff $-\dfrac{3t{m^*}^2}{16} + \dfrac{m^* R}{4} > 0$ or equivalently $R > \dfrac{3tm^*}{4}$. ∎

The result says that as R increases it is easier for the consumer welfare under Fat Products to be bigger than under the measure 0 case. This happens because all the consumers who are getting exactly what they want (i.e. located within the product interval) are getting much more utility than in the standard case even if the price is slightly higher, and increasing R increases all of their utilities more than it would under the standard case. As for the result in m^*, since the optimal measure is directly related to price, it is clear that as the measure increases, so does the price, and the welfare must decrease with respect to the measure 0 case. When the transportation costs increase, so will the price, as opposed to the standard case, and again the welfare will decrease comparatively.

Notice that we need the sufficiently convex cost assumption to make sure that the SOCs are satisfied, however if the assumption does not hold, the equilibrium might still exist. In the Appendix A I derive the equilibrium for linear costs of expanding the measure and the customers located at the unit interval. Both of these make the proof much harder and do not add much intuition to the result. The results are that if the cost of the expansion is low enough then the monopolist will cover the whole interval and charge $p = R$, leaving consumers with 0 welfare. Otherwise the monopolist goes back to the standard point product.

4 N Firms in Bertrand Competition

4.1 Introduction and Setup

I am going to solve the N firms problem, competing a′ la Bertrand. A given firm will take the prices and the measures by the other firms as fixed. The firms and the consumers will be in a circular city. For the local analysis I will use three firms on the unit interval – with the two fixed firms and the deviant firm, the prices and measures of the fixed firms fixed and equal. To provide intuition behind the math, the figure on the next page is provided. To make the matters even simpler, the reader can view this as a duopoly along the circumference competition – since the prices by the other firms are fixed, then it does not matter whether there are two fixed one side firms, or one fixed firm on both sides.

4.2 Fat Products versus Multi-Products

There is a big literature on Multi-Product firms – the ones which can make several products and price them accordingly[15]. The reasons not to make many points instead of an interval are fixed costs and economies of scale. Think of the two ice-cream vendors from the application section. Would it make sense to put dozens of stationary vendors instead? Probably not, because of the fixed cost of hiring each additional vendor. For the same reason, and economies of scale, it would not make much sense to develop dozens of chairs of different heights or several projectors of different brightness and focus which are otherwise the same. The literature on product lines and mass customization moves in similar direction as the Multi-Product firms literature. The benefit of an adjustable good versus a mass customized line of goods is still in fixed costs and economies of scale. Here is an example to illustrate this point from Zipkin's (2001) discussion on the limits of Mass Customization.

> ...there was talk of customizing car seats. Toyota even set up a prototype of a seat-measurement device at its visitor center in Toyota City. It never happened. Instead, adjustable seats developed rapidly. It is cheaper to construct adjustable seats than to customize.

[15] See Spence (1980) for example

If the fixed cost F of producing a point product is high enough, no firm would be willing to make two of them. The conditions on when does it cost less to produce one fat product or two endpoints of the product are easily derived[16]. What happens if the fixed costs go to zero, and the firms would prefer to make many point products? Then if there is more than one firm, we have the result due to Teitz (1968) that there does not exist a pure strategy Nash equilibrium where firms can choose to produce more than one product with linear travel costs for consumers. I will examine the oligopoly structure as if there is no option to develop more than one product – or that the fixed cost F is high enough.

Think of a belt – it has many adjustment positions (holes), but the adjustment is not continuous. This is not a multi-product offering. This is a fat product, where instead of an interval, the product is a set of multiple points. All the results hold the same, except for consumer welfare – the consumers whose favorite position is between two holes of the belt will get less welfare than with an interval. However, as long as there are a few points inside the interval, these consumers will not be marginal, therefore many points as one set as opposed to an interval only will matter for quantitative results on consumer welfare, while all the other results (including qualitative on consumer welfare) will go through as is now.

4.3 The Fat Products Oligopoly Model

The model setup is almost the same as that of Section 3, except that now there are N firms located symmetrically around the unit circle, and the products are arcs on the circle. The consumers are going to be the standard address consumers – located uniformly along the unit circumference circle, with some fixed reservation price for the product, say $R > 0$, and transportation costs $t \times d$ – with $d \geq 0$ being the distance between the consumer and the product, and $t \geq R$ being the marginal cost of transportation. If the consumer has a choice, she is going to buy the product such that to maximize $U(d) - p$, where p is the price of the product, and $U(d) = R - t \times d$.

The producers can choose to make products in a form, and with the characteristics of an interval (so an arc, since it is a circular city), say $[a, b]$, $0 \leq a \leq b \leq 1$, with the development/production cost of a product $C([a, b]) = c(b - a)$, where c is a function. I will still refer to the length of the interval

[16] $c(m) < 2F$, or equivalently the additional cost of developint a positive length product must be less than the fixed cost.

offered as m. The standard measure zero products will be developed/produced at some positive fixed cost, so $c(0) = F > 0$. Before proceeding with the proof, I need the following assumption to ensure that the second order conditions hold. The implications of the assumptions were discussed in the monopoly section. Notice that this bound is weaker than the one in the monopoly section, which makes sense since competition implicitly makes it harder for the firm to expand its' measure further.

Sufficiently Convex Cost Assumption (N). *Let the cost function $c(m)$ of developing/producing a fat product of length m be such that $c''(m) \geq \frac{t}{8}$.*

I only focus on pure strategy symmetric Nash Equilibria as the solution concept. Consider what happens if some consumers are left out of the market.

Lemma 1 *(Full Coverage Lemma). The only equilibrium such that there are consumers left out of the market is the trivial equilibrium where the global monopolist's problem is the same as the local monopolist's problem.*

Proof. See appendix B. ■

In this case the scaling up of the market is not valid, since as N goes up and everything else stays constant this equilibrium will eventually disappear, and it will be harder and harder to let every firm to optimize as if it is unconstrained by the neighbors. From now on I will assume that the parameters do not lead to the trivial case where a firm can act as a global monopolist, and move on to the case where there are no consumers are left out of the market.

4.4 No Consumer Left Out

Now that there are no consumers left out, we can use Figure 3 (see below). First, I need to examine the possibility of a price equals marginal cost equilibrium. The price equal to marginal cost situation can arise in the case where no consumer is left out if either all the firms produce measure 1/N products, or if all of them produce measure zero products. In either case, a deviant firm can unilaterally raise its price above zero. This will clearly bring in positive profits, and so we can not have a zero profit equilibrium in my model, unless one includes fixed costs of entry in the industry. To make sure that the figure below looks right, I need to show that products of competing firms do not intersect.

Lemma 2 *There is no symmetric Nash equilibrium where products of two firms intersect.*

Proof. See appendix B. ∎

In the main proof below I fix all the firms' measure and prices at the same level, and check whether a deviant firm has an incentive of moving away from the knife-edge equilibrium where the local markets just touch, and $p^* > c(m^*)$.

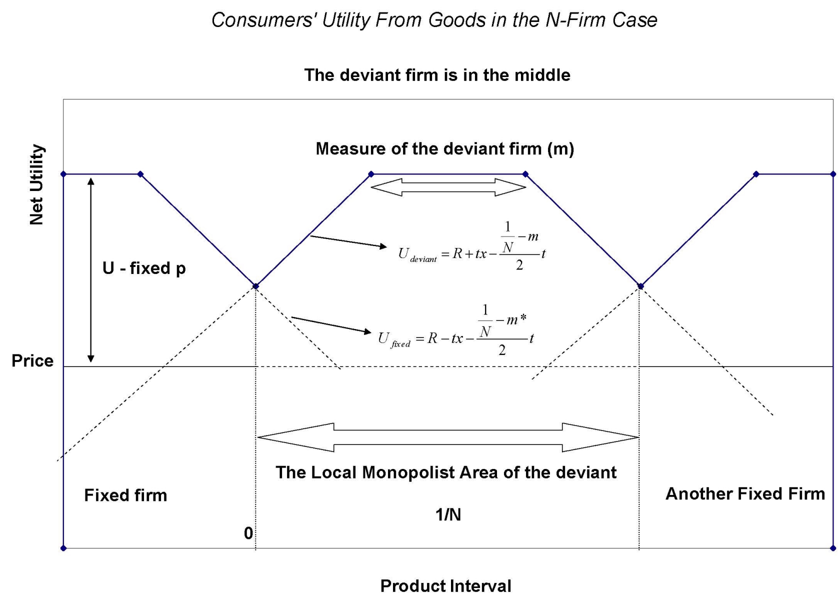

Figure 2. Oligopoly with Fat Products.

Theorem 2 *(The N-Firm Fat Product Competition). In the symmetric Nash equilibrium of N firms capable of offering fat products, each firm charges* $p^* = \dfrac{t}{N}$ *and makes a product with the measure* $m^* < \dfrac{1}{N}$*, such that* $c'(m^*) = \dfrac{t}{2N}$ *(The Optimal Measure Condition).*

Proof. (For details the reader is encouraged to look at Appendix B).

Fix the measure and the price of the fixed firms at m^* and p^*, and denote by m and p the measure and price of the deviant firm with respect to which it is going to maximize its' profit.

First, I find the demand function – which is determined by the marginal consumer – who is at the intersection of two $U(x)$ functions (on Figure 2 above), one of the consumers buying a deviant firm product, and the other one of the left Fixed firm. Just making the two equal to each other, the intersection point is $x(m,p) = \dfrac{m^* - m}{4} + \dfrac{p - p^*}{2t}$, so if the deviant firm lowers the measure of its' fat product, m, the intersect point will move to the right, cutting into deviant's market share. Raising price p has the same effect. Since the deviant firm competes with two fixed firms, one on each side, the demand for the deviant firm is $D_{deviant}(m,p) = \dfrac{1}{N} - 2x(m,p)$, and the profit is then $\Pi_{deviant}(m,p) = D_A(m,p) \times p - c(m)$.

To insure that the Second Order Conditions hold, use the Sufficiently Convex Cost assumption. Then using the First Order Conditions together with symmetry ($m = m^*$, $p = p^*$), get the optimal price $p = \dfrac{t}{N}$, and the optimal measure m^*, which, if an interior solution, has to satisfy the Optimal Measure Condition: $c'(m^*) = \dfrac{t}{2N}$, and cannot be outside of the interval $[0, \dfrac{1}{N})$.

Then to find the profit simply substitute the optimal values into the objective function, and the consumer welfare is N times the area of the trapezoid in the deviant firm region in Figure 2. ■

Corollary 2 *In the symmetric equilibrium each firm will get a profit of* $\Pi_+ = \dfrac{t}{N^2} - c(m^*)$, *and the consumer welfare is going to be* $CW_+ = \dfrac{1 + N + m^*}{2} \times (R - \dfrac{t}{N})$.

5 Comparison with Previous Literature

5.1 Summary of the Results

I considered two cases in the previous section. The trivial case equilibrium, with some consumers left out, happens when the local market optimization is the same as the global one, so the firms do not have an incentive to deviate even if the ex-post markets are expanded. We get N local monopolists, with strategies described in Section 3.

In the more interesting case, the market is covered, and the second order conditions are satisfied if the cost function of the measure of the product is sufficiently convex. I will focus on this scenario for the rest of the paper. Prices are the same as in Salop, profits are lower and consumer welfare is higher. The measure offered by firms in equilibrium is sometimes positive, and depends on the exact form of the cost function. This is intuitive and something that we would expect from the monopoly results.

5.2 Comparison with Salop

Corollary 3 *The equilibrium price is the same, the profits of the firms are lower, and the consumer welfare is higher in the Fat Products oligopoly than in the standard oligopoly equilibrium.*

To see the results of Salop one can just substitute $m = 0$ into the results of Theorem 2 – the fat products model is a generalization of Salop's model. The price is the same, which is the most unexpected result, especially after Theorem 1, where in the monopoly case the price charged was higher than the standard. The prices remain the same since what determines the prices is the slopes of the net utility curves at the marginal consumer (the intersection). Those remain the same as they were in Salop's model. Clearly this result is due to the assumption of linear travel costs. Profits are lower since there is the extra cost of providing fat products, while the revenue stays the same. The consumer welfare went up since in the Salop equilibrium the welfare would just be N triangles with bases of $\frac{1}{N}$ and heights of reservation price less the actual price. Now we have N trapezoids, and as long as the measure is positive consumer welfare goes up. In the limit case of measure being equal to one the consumer welfare will double.

The more firms there are in the industry, the overall profits can actually become higher. Whether it happens depends on the exact form of the cost function. This possibility arises because the diseconomies of scale issue in the cost of fat products is becoming less severe as N increases.

If we look at the fixed cost, like Salop did, the fixed cost in this industry to support a SPZE (sub-game perfect zero profit Nash equilibrium) would be less than the ones in the original model, since there is the cost of positive measure that the firms have to pay now that they did not have to worry about before.

Corollary 4 *It would be profitable for all the firms in the market to commit to making only $m = 0$ (standard) products.*

Recall the example of two ice-cream vendors. What this corollary means is that they would rather agree that both of them stand as opposed to walking around and try to appeal to more consumers. The intuition behind this result is as follows. Since the prices and the demand are the same as in Salop, the revenues are the same as well. However, the firms have to pay extra cost for making their products fat. There is an arms race which benefits in the end only the customers -

without the effects of the competitors the firms would happily make their products fatter, but with the competitors doing the same, fatter products only help the customers.

What happens to profits if t goes up, or as the products become more differentiated? Just looking at the equation $\Pi_+ = \frac{t}{N^2} - c(m^*)$ the immediate reaction is that they necessarily go up. However m^* depends on t, and as t goes up, so does m^*. The following numerical example illustrates an interesting point.

Example 1 *Let there be N firms in the city. Let $t \in (6, 8)$ and $c(m) = e^m$. Then the sufficiently enough cost condition is satisfied (since $e^m \geq 1 \geq \frac{t}{8}$ for $m \geq 0$) and $c(0) = 1 > 0$. Also let R be close to t so that all the firms being local monopolists is not an equilibrium.*

Then $c'(m^*) = \frac{t}{2N}$, and therefore $c(m^*) = \frac{t}{2N}$. Also notice that $m^* = \ln \frac{t}{6} \geq 0$. Thus the profit is $\Pi_+(t, N) = \frac{t}{N^2} - c(m^*) = \frac{t}{N^2} - \frac{t}{2N} = \left[\frac{2-N}{N^2}\right] t$. With $N > 2$ this decreases in t!

Therefore with the Fat Products we get the unexpected conclusion that *profits do not necessarily increase as the transportation costs go up*! This is a clear difference from the predictions of Salop (1979) since there we got a clear conclusion that the profits will go up as the transportation costs (differentiation between the products) go up, since there the profits per firm were simply $\frac{t}{N^2}$.

Proposition 2 *In equilibrium profits will go down as the transportation costs go up (firms become more differentiated) if and only if $c''(m^*) < \frac{t}{4}$.*

Proof. All is needed is the derivative of profits with respect to t, however first we need to implicitly differentiate the Optimal Measure Condition with respect to t:

$$c''(m^*)\frac{\partial m^*}{\partial t} = \frac{1}{2N} \Longrightarrow \frac{\partial m^*}{\partial t} = \frac{1}{2Nc''(m^*)}. \tag{3}$$

Then differentiate the profit function $\Pi = \frac{t}{N^2} - c(m^*)$:

$$\frac{\partial \Pi}{\partial t} = \frac{1}{N^2} - c'(m^*) \times \frac{\partial m^*}{\partial t} = \frac{1}{N^2} - \frac{t}{2N} \times \frac{1}{2Nc''(m^*)} = \frac{4c''(m^*) - t}{4N^2 c''(m^*)}. \tag{4}$$

To see when is profit going down with respect to t we now just need to check when is (4) less than zero. Since $c(\cdot)$ is convex by assumption, $c''(\cdot) > 0$. Therefore $\frac{\partial \Pi}{\partial t} < 0$ iff $c''(m^*) < \frac{t}{4}$. Notice that for the N firm part of the paper we just needed to assume that $c''(m^*) > \frac{t}{8}$, therefore the region where both inequalities are satisfied is not empty. ■

What is the intuition behind this surprising result? In Salop's model the reason that the profits go up with the increase in transportation costs is that now the rival firms do not have as much incentive to lower their prices to undercut, since after undercutting not as many consumers are going to switch because of the travel costs. This gives the firms the ability to price higher without fearing the competition.

Once again, recall the example with two ice-cream vendors. Suppose it becomes hotter, so the travel costs go up for the consumers. Then the vendors become more differentiated, but the value of walking closer to a given consumer increases. Therefore under certain conditions the vendors will do more additional walking than the additional profits that they will receive due to increased differentiation. In this model there are two different effects at play. First one is the one described above. The second one is that with increase in t expanding the measure of the product becomes more and more valuable way to attract consumers than it was before. And so instead of competition in prices, the firms engage in cut-throat competition in content. In equilibrium it is clear that even though the firms will offer more content, they will still attract the same number of consumers, therefore with increase in t we have a welfare transfer from firm profits to consumer welfare.

Why do the profits not go down with t if the costs are very convex? This happens because if the costs are too steep they act as a deterrent for firms to increase the measure too much, therefore this way the firms actually benefit from higher costs.

5.3 Comparison with Mass Customization

5.3.1 Setup

Mass customization in operations research is a model is of a base point product in the spatial model of differentiation, and a firm that can produce point products close by to the base product for a higher cost. The firm can charge consumers different prices for different products. Dewan et.al. (2003) examines a duopoly with quadratic costs of making products away from the base product. The main results are that the duopoly would offer less scope of products than a two-facility monopolist, the prices stay the same as in Salop's model, and that if the firms do not enter simultaneously, then the first entrant always achieves advantage. Mendelson and Parlakturk (2005) looks at a duopoly with a two stage game – in the first stage the firms decide how much (if at all) to invest in mass customization, and in the second stage they decide on the pricing structure. The findings are that a firm with either quality or cost disadvantage will not want to customize by itself, and that occasionally even free customization might hurt firm's profits.

A branch of economics closely related to mass customization is product line competition. This literature concerns firms which can offer several products for different prices. One of the first efforts is an insightful article by Klemperer (1992), which results are driven by customers having switching costs from one brand (or store) to another, and so the firms would try to position their products next to each other, as opposed to the standard Hotelling intuition. One of the latest efforts is Draganska and Jain (2005) empirically examining the effects of extending the product line (i.e. offering new flavors of yogurts) when the customers have preference for variety. They find that the product line length and price are substitutes from the firm's point of view - if the firm wants to increase the price and keep the market share constant it should increase the product line as well, which I find as well – if the firm wants to increase the price, it needs to extend the length of the interval as well.

I will assume an exact functional form for the cost function and compare the results of the equilibrium from the previous section to the results obtained by Dewan et.al. for Mass Customization. The Mass Customization model has exactly the same setup from the consumers' side as the standard Salop and the Fat Products model. The difference comes on the production side.

In Mass Customization, each firm has a focus point – the standard product, and can produce a tailored product x units away from the focus point for $c(x)$. In that article the authors assume a duopoly (so $N = 2$) with quadratic costs, with positive coefficients on x^2 and x (which makes it easy to satisfy the Convex Enough Cost Assumption) and the constant being zero. The price of the goods listed is the price of the focus good, with the optimal price of a tailored good is the "base price" of the focus good plus the transportation cost from the focus good to the tailored good (i.e. if the focus good is at 0 and has a price of 2, a tailored good at $\frac{1}{2}$ will be priced at $2 + \frac{t}{2}$). The offered scope is the length of the interval where a firm offers tailored products for any point on that interval.

For both models to work, I will assume the same cost structure here to make the comparison straightforward, and also a technical assumption on the relation of a, b and t for the Mass Customization model to produce answers. Firms earn more profits if they are in the Mass Customized industry versus a Fat Product industry, and produce a bigger scope of products than the measure of the Fat Product if the travel costs are sufficiently high.

Quadratic Cost Assumption. *Let the costs of customization (customizing a product x units away from the base product) and the costs of developing a product of measure x are the same: $c(x) = ax^2 + bx$ (Therefore $c'(x) = 2ax + b$, and $c''(x) = 2a$). Let $6b < t < 4a$.*

5.3.2 Comparison

Given the same values of parameters (R, t, a and b) I will compare the equilibrium price, measure and scope, and the profit in the Fat Products model and the Mass Customization model. The derivations for the Fat Products can be found in Section 4 and the appendix, the derivations for Mass Customization can be found in Dewan et.al.

The price of base product in the Mass Customization model is the same as the price in the Fat products model, both $t/2$. One would expect the prices to go down in both Mass Customization and the Fat Products models because the firms essentially get closer to each other and are now more competitive. On the other hand, the consumers have higher welfare, and the firms could potentially charge higher prices. Neither happens because while pricing for the marginal consumer, the firms have to remember about the pricing decisions inside the interval, being especially true for the Fat Products model where the firms can not price discriminate. Moreover, for the Mass Customization model the base price is $t/2$, however as the discussion in the previous sub-section mentioned, the tailored goods are going to be priced at $\frac{t}{2} + td$, where d is the distance from the last customized product, therefore the average price paid by a consumer in the Mass Customization model goes up. Therefore a mis-specification of the model can lead to overestimation of the actual price paid.

The scope offered by the Mass Customization firms is $x^* = \frac{t-6b}{12a-3t}$ (this is why the additional assumption was needed about a, b and t). From Theorem 2, and using the Quadratic Costs Assumption, $m^* = \frac{t-4b}{8a}$. Therefore we have the following corollary.

Corollary 5 *The measure in the Fat Products model is going to be bigger than the scope in the*

Mass Customization model if $t < \frac{4a}{3} + 4b$.

The cutoff is clearly increasing in both a and b, therefore with increasing costs there is a higher chance that the Fat Products equilibrium is going to produce a good which more consumers will have as their optimum. The firms with Mass Customized products can get something from increasing the scope (proportional to the travel costs), since the consumers who buy the goods inside the scope will have to pay higher prices. Therefore as travel costs increase the Mass Customizing firms will offer wider scope than the Fat firms.

The profit under the Fat Products model is $\Pi_+ = \frac{t}{N^2} - c(m^*)$. The profit in the Mass Customization model is $\Pi_{MC} = \frac{18(b^2 + at) - 5t^2}{18(4a - t)}$.

Corollary 6 *The per firm profit in the Fat Products model is going to be always smaller than the profit in the Mass Customization model. The difference in profits is* $\Pi_{MC} - \Pi_+ = \frac{t(9t^2 - 20at - 144b^2)}{576a(4a - t)}$.

The Fat Products scenario is the worst for the firms (and therefore the best for the consumers) because they have to charge the same price as in the standard case, yet pay up for more measure than the Mass Customization pay for scope. As the costs (either a or b) go up, the difference becomes smaller, yet with the assumptions made in the beginning of this section, the difference is going to be positive.

6 Comparison with Benchmarks

6.1 Free Entry

It is interesting to compare what happens with free entry - the limiting case being monopolistic competition. The firms will stop entering as soon as $\Pi_+(N) = 0$, which will happen when $\Pi_+(N) = 0$, or

$$N^* = \sqrt{\frac{t}{c(m^*)}} \tag{5}$$

Corollary 7 *The industry would support more firms if the firms could not develop fat products. The supported number of firms will go down as the firms become more differentiated under the same conditions as in Proposition 2.*

The expression above gives us a condition on the optimal N. As F (the fixed cost $c(0)$) goes up, the optimal N goes down. As t goes up, so does m^*, therefore the effect of t on N^* is hard to see. Similarly to the profit discussion in the section above, *higher differentiation does not necessarily lead to lower number of firms in monopolistic competition* as opposed to the result due to Salop. We can see immediately from the equations above that if in the absence of free entry firms can have profits decreasing in t, then so can the optimal number of firms. The intuition stays the same as before in the analysis of profits.

This result, while surprising, had been previously examined at lengths in the literature. The same mechanism works here as in Sutton's results on endogenous sunk costs. The firms sink more development costs as the market becomes bigger (or the transportation costs go up), resulting in lower profits, and therefore the market cannot accommodate more firms.[17]

6.2 Comparison with a Social Planner

Following the literature, it is interesting to see what happens when there is a social planner who is interested in maximizing the total welfare - the sum of consumer welfare and firms' profits. The case of N local monopolists in the market is not optimal since not everyone is getting served, even though everyone should be because the marginal cost of producing the good is 0 and all the consumers have positive valuations. In the competitive case as long as the market is covered, it does not matter for the purposes of total welfare what is the price. Therefore what the social planner needs to optimize is the total possible welfare (R) less the loss in transportation costs[18], less the loss due to the development of products.

$$TW = R - \frac{\frac{1}{N} - m}{2} t \times \frac{\frac{1}{N} - m}{2} \times N - c(m)N \tag{6}$$

Since the maximum welfare possible, R, is fixed, the social planner's problem reduces to minimizing the transportation cost loss and the loss due to development of products.

[17] What are the possible industries where the firms essentially offer fat products and Sutton's results apply? Ellickson (2005) examined supermarkets as application of endogenous sunk costs. As the market size grows, the supermarkets offer more product offering breadth, wider aisles, more check-outs, and so on – giving customers new options to use if they want for the same price.

[18] The term in the brackets - first term is height of the triangle above the utility lines and below R line in Figure 3; the second is half the base. There are N such triangles overall.

$$Loss = \frac{(1 - mN)^2}{4N} t + c(m)N \tag{7}$$

I will analyze only the social planner who can maximize with respect to either the measure that each firm produces (m) or the number of firms (N).

Proposition 3 *A social planner who optimizes total welfare with respect to the number of firms would choose less firms than would enter with free entry.*

Proof. Look at the first and second order conditions of the loss function with respect to N.

$\frac{\partial Loss}{\partial N} = \left[(\frac{1}{4N} - \frac{m}{2} + \frac{m^2 N}{4})t + c(m)N \right]' = -\frac{t}{4N^2} + \frac{m^2 t}{4} + c(m)$. Since the social planner needs to minimize loss, we have to make sure that the second derivative is bigger than zero. $\frac{\partial^2 Loss}{\partial N^2} = \frac{t}{2N^3} > 0$. Therefore the first order condition is enough to find the answer. Solving the FOC:

$$N_{sp} = \sqrt{\frac{t}{m^2 t + 4c(m)}} \tag{8}$$

However the equilibrium N from the free entry was $N^* = \sqrt{\frac{t}{c(m^*)}} > \sqrt{\frac{t}{m^2 t + 4c(m)}} = N_{sp}$.

■

This result supports the long running view in the literature that when there are significant business stealing effects in an industry, there will be too many firms. This is the case here, as since the market is covered, each new entrant does not create any business but just steals the neighbors' consumers. Coupled with the fixed cost, this creates the conclusion that there are too many firms with free entry. The optimal number of firms is not the only thing that the social planner can affect, therefore I need to also examine the measure effects.

Proposition 4 *A social planner who optimizes total welfare with respect to the measure of the product each firm uses, would choose products of lower measure than the firms would have chosen by themselves.*

Proof. Again I look at the first and second order conditions only with respect to m.

$\frac{\partial Loss}{\partial m} = \frac{mN - 1}{2} t + Nc'(m)$. The second derivative is: $\frac{\partial^2 Loss}{\partial m^2} = \frac{Nt}{2} + Nc''(m)$. This is always going to be bigger than zero since both terms are positive by definition of t and the sufficiently convex $c(\cdot)$ assumption. From the first order condition:

$$c'(m_{sp}) = \frac{t}{2N} - \frac{m_{sp}t}{2} \tag{9}$$

Since the optimal measure condition in Theorem 2 was $c'(m^*) = \dfrac{t}{2N}$ and $c''(m) > 0$, the result is that $m_{sp} < m^*$. ■

While one would expect the firms to produce not enough measure, since this greatly helps the consumer welfare, that does not happen. The same effect as we have seen in the proposition above works here as well - the firms in an arms race to see who can deliver the most measure, yet since everyone will deliver the same in equilibrium the firms end up hurting each other. The cost of the measure expansion hurts the firms more than it helps the consumers because the firms need not only to supply whatever the consumers might need, but much more than that to compete with its' neighbors.

6.3 Monopolist with N locations

Assume monopolist has N locations symmetrically distributed around the circle. The cost functions at each location are the same as in oligopoly. If the monopolist chooses not to cover the market, then this is equivalent to the N monopolists from the Fat Monopoly section. If the monopolist does cover the market, then it is clear that she will extract all the surplus from the consumers indifferent between locations ($\frac{1}{2N}$ away from a location). This price will therefore be $p_M^N = R - t(\frac{1}{2N} - \frac{m}{2})$.

Lemma 3 *Monopolist with N symmetric locations will charge a price of $p_M^N = R - t\frac{(1-mN)}{2N}$, and develop products with the same measure as N competing firms at same locations.*

Proof. The price part is presented above – if the monopolist covers the market, she has to extract all the surplus from marginal consumers. The market size is 1, therefore the monopolist has to optimize the price less development costs with respect to measure at each location. The profit function, and the derivative with respect to m is thus

$$\Pi^N(m) = R - t\frac{(1-mN)}{2N} - Nc(m) \tag{10a}$$

$$\frac{\partial \Pi}{\partial m} = \frac{t}{2} - Nc'(m) \tag{10b}$$

From the FOC we get $c'(m) = \frac{t}{2N}$ – exactly the same condition as before. ■

This comparison illustrates that the result of firms being worse off with fat products is not due to the fact that by creating fatter products the industry does not capture new consumers. The monopolist does not capture new consumers either, yet she is still willing to invest as much as the N firms who were losing money by developing fat products.

7 Relaxing Linear Transportation Costs Assumption[19]

Linear transportation costs for the customers is an assumption made throughout the literature on spatial models. However it is not clear why should that assumption be close to reality even for simple applications of spatial models, where the transportation costs actually represent the physical costs of going from one place to another, let alone applications where the costs represent how far away is the product from the customer's ideal product in some characteristics space. Even with physical transportation costs, there is an area where the customer can just walk, then there might be an area covered by the public transportation system, and so on, and there is no reason for the costs to vary linearly within each of the areas.

It would be a major drawback of the model if the reason why I get the interesting results is because of the linear transportation costs. Therefore it would be natural to assume some transportation costs function $t(d)$, where d is the distance from the customer to the product offered by a firm and $t(\cdot)$ is a strictly increasing differentiable function, with $t(0) = 0$. For simplicity, assume that the firms are in the equilibrium where all the consumers are served and that the profit function is concave in the price and measure of the product.

Proposition 5 *With transportation costs a function $t(\cdot)$, in the symmetric equilibrium firms' charge price $p = \dfrac{t'(\frac{1}{2N} - \frac{m^*}{2})}{N}$ and make products of measure m^*, such that $c'(m^*) = \dfrac{p}{2}$.*

Proof. The full proof is in the appendix. It follows the proof of the Theorem 2, except that now transportation costs are a function $t(\cdot)$. ■

I have derived the optimal price and measure, however the interesting results were what happens with the measure if the firms can restrict themselves, and what happens with the profits as the

[19] Thanks to Michael Whinston for raising this issue.

transportation costs go up.

Proposition 6 *It would be profitable for all the firms in the market to commit to making only* $m = 0$ *(standard) products if and only if* $\left[t'(\frac{1}{2N} - \frac{m^*}{2}) - t'(\frac{1}{2N})\right] < c(m^*)N^2$*. In particular, this is satisfied if the transportation cost function* $t(\cdot)$ *is convex.*

Proof. Similarly to the proof of the previous proposition, we can derive the optimal price if the firms are restricted to price at $m = 0$. This price turns out to be $\frac{t'(\frac{1}{2N})}{N}$. Therefore, it is profitable for the firms to sign a stand still agreement with respect to measure iff the profit with $m = 0$ is higher than the one from the previous proposition, or

$$\left[t'(\frac{1}{2N} - \frac{m^*}{2}) - t'(\frac{1}{2N})\right] < c(m^*)N^2. \tag{11}$$

Since the left hand side is bigger than zero, and $m^* > 0$, then if $t'(\cdot)$ is an increasing function, the right hand side is less than zero, and so the inequality is satisfied. ■

While this condition is not as clear as the one from corollary 4, where this was always satisfied, there is still a wide range of values where this condition holds. Clearly, as the transportation costs become more and more steep, the firms have to invest more and more into the cost of making fat products, giving us the result. However, if the transportation costs are sufficiently concave, then an increase in the optimal measure takes the marginal customer lower along the transportation cost function and allows the firms to charge a higher price.

As the transportation costs are now a function, to make comparative statics, I will look at transportation costs $\alpha \times t(\cdot)$, at $\alpha = 1$, and see how does increasing α effect the firms' profits.

Proposition 7 *In equilibrium profits will go down as the transportation costs go up (firms become more differentiated) if and only if* $c''(m^*) < \frac{t'(\frac{1}{2N} - \frac{m^*}{2})}{4}$.

Proof. The proof is in the appendix. It consists of differentiating the profit function with respect to α. ■

Again, we get the conclusion that it is possible to get profits going down as the firms become more differentiated. Moreover, the condition looks similar to the condition in proposition 2, which was $c''(m^*) < \frac{t}{4}$, and of course with linear costs t is the derivative. Overall, it is clear from this section that linear travel costs assumption was not necessary to achieve any of the results.

8 Fat Products with a Leader and a Follower

With any capacity or investment related problem an interesting question presents itself – what would happen in a Stackelberg setup. I examine what will be the optimal strategies of the firms if there are two firms in the market with the leader picking the price and the measure first, and the follower picking its' price and measure conditioning on the leader's choice. In the mass customization literature the result of a Stackelberg competition is that the leader will expand the scope of offerings more than in the standard duopoly to force the follower to produce less.[20] While Schmalensee (1978) did not explicitly look at a Stackelberg-type model in his seminal work in entry deterrence, one could view it as such since the incumbents had a chance to expand their product line. Judd (1985) later showed that the model sometimes does not lead to excessive entry by incumbents if the exit option is available to the incumbent. To make the predictions more clear I will disregard the possibility that a big enough measure of the leader might make the follower not enter at all, and look at the case where both the leader and the follower will produce the good. Therefore the two variables of interest are prices and measures of the firms.

Assume there are two firms a leader (L) and a follower (F). Firm L picks m_L and p_L first, and then firm F picks m_F and p_F. The cost of Fat Product development for each firm is $c(m) = F + am^2$, where $F > 0$ are fixed costs and $a > 0$.[21]

Proposition 8 *In a Stackelberg game with Fat Products, Leader charges* $p_L = \frac{t(16a-t)(12a-t)}{8a(32a-3t)}$ *and makes a product of* $m_L = \frac{t(12a-t)}{4a(32a-3t)}$. *Follower charges* $p_F = \frac{t(20a-2t)}{32a-3t}$, *and makes a product of* $m_F = \frac{t(20a-2t)}{4a(32a-3t)}$.

Proof. The proof is a standard Stackelberg procedure presented in the appendix. ■

The two main results are that the prices and the measures offered by the leader and the follower are generally different. Intuition from previous models leads to a belief that the measure of the

[20] See Dewan et. al. (2003)

[21] Similarily to the earlier sections, I need to make an assumption on the second derivative of the cost function to make sure that the second order conditions are satisfied. Now the assumption is $a > \frac{t}{10}$.

leader will be bigger than in the standard model, and the measure of the follower smaller, with the prices comparison being more ambiguous, but generally price of the leader being bigger than the price of the follower. The following two corollaries examine the price and measure comparisons between the leader and the follower.

Corollary 8 *The leader develops a product of a bigger measure if the development costs are low enough ($t > 8a$). Otherwise the follower develops a product of a bigger measure.*

Corollary 9 *The leader will charge a higher price unless $t \in [4a, 8a]$. If the travel costs are within this range, then the follower's price is higher.*

Both of the results are interesting in their own right. The result about measures is that if the differentiation is big enough then the leader will use the standard Stackelberg intuition and develop products of bigger measure and force the follower into a niche. However when the products are more homogeneous, then the leader is better off taking the niche strategy for herself. This is consistent with the basic Fat Products model where if the product differentiation is not big enough, the firms might be worse off developing a product of a bigger measure[22].

The corollary on prices shows that the prices exhibit an unusual behavior, where there is an interval over which the follower's price is higher. This can be viewed as accommodating the follower. As the products become more homogenous than in the accommodating interval, the leader will continue to underdevelop its' good, but charge higher prices to capture more profits, as eventhough the follower will undercut, there will still be enough profit for the leader. As the products become more differentiated than in the interval, the leader stops accommodating on measure and on price, as one would expect from the intuition of previous models.

Corollary 10 *The market share and the profit of the leader will be higher than that of the follower if and only if $t > 8a$.*

The corollary above is expected after the previous two, and the intuition stays the same – as the goods become more differentiated the leader will use standard Stackelberg in capacity intuition. She will overdevelop the product and force the follower into a market niche, trying to undercut the

[22] See corollary 4. Moreover, condition $t > 8a$ is exactly $c''(m^*) < \frac{t}{4}$ condition from the corollary, since here $c = am^2 + F$, and so $c'' = 2a$.

leader, but still ending up with less than a half of the market share. As the products become more homogenous the result of the basic model kicks in where the firms are better off not producing that much, and then the leader is forced into a niche developing a product of smaller measure and sometimes even charging less for it.

9 Conclusion

The discussion introduces the notion of fat products – products that are sets in the characteristic space as opposed to points. I have looked at an application of this idea to single dimension spatial model of Salop, and let products be intervals. I found, among other things, that contrary to the standard intuition, increasing travel costs can lead to lower profits in equilibrium. Also the equilibrium number of firms with free entry might go down as well as firms become more differentiated. Based on the results derived in the article, I find that the assumption of products as points has drawbacks and may lead to not representative results.

Making firms more differentiated might in fact lower profits, if the cost function is not too convex, because the optimal length of the interval goes up with the transportation costs. The result is due to two offsetting effects. First is the same as in Salop - as transportation costs increase the firms have less incentive to undercut competitors. On the other hand with positive measure products the firms will have more incentive to expand their measure. But since in equilibrium everyone will do that, this expansion just increases the costs without increasing the revenue. Also, if the firms could restrict themselves to producing only the standard, measure zero products, they would do that, since the revenues with fat products is the same as in the standard Salop model, yet they have to pay for the fat product development.

I also examine free entry and social planner's decisions. Since I already have the unexpected result of increase in transportation costs lowering the profits, I also derive that an increase in transportation costs might lead to a lower optimal number of firms in the industry with fixed costs, which mirrors the results of Sutton, but goes contrary to the point model of Salop. The social planner would have chosen fewer firms each producing products of smaller measure than under free entry. This confirms the standard intuition that when there is the customer stealing effect with fixed costs the number of firms is too big as compared to the social welfare optimum. Also while the longer products help consumers, they hurt the firms more.

I also check whether the findings above hold up when the transportation costs are not linear, but rather any strictly increasing function. The price will not stay the same, but there are still the offsetting effects on profits, and a range of parameters where the profits will go down with transportation costs going up. Overall, most of the results go through with a general transportation function.

If one firm is a leader in setting price and length of the product, the result is similar to the base model – with high enough differentiation the standard intuition of developing a product with more measure and forcing the follower to develop a smaller product applies. But high development costs force the leader into accommodating the follower and even receiving less market share and profits than the follower. This extension shows that the idea of Fat Products is applicable to a broader set of problems, for example, the firms can be asymmetric. Even without asymmetry there are many other avenues to consider – firms changing locations, consumers valuing Fat Product more than just the maximum of the points inside, possible mixed strategy equilibria where the intervals could intersect – the list goes on. However, in this paper I chose to focus on the most interesting extensions and leave others for future research.

Appendix

A Bounded Interval and Linear Costs

I prove the Monopolist Theorem under linear development cost assumption and a bounded interval. Let the measure cost function $C([a,b]) = C_M \times (b-a)$, where C_M is a positive constant. The proof is a simple two step procedure, where in the first step the optimal p is found for a fixed $m = M$, and then I maximize with respect to m. The complications arise since the SOCs are not satisfied, and the profit function is a piecewise function – a convex parabola until a cutoff after which the function is linear.

Theorem 3 *(Fat Product Monopolist). A monopolist who has the ability to offer a Fat Product will offer a product of measure* 1 *for the price of* R *if and only if* $C_m \leq R - \dfrac{R^2}{2t}$. *Otherwise the monopolist will offer a standard point product for* $p^* = \frac{R}{2}$.

Proof. First I Calculate the optimal price for a given measure. The demand is $D(p) = M + \dfrac{2(R-p)}{t}$, and therefore the problem is the following (since there are no production costs, and the development costs are fixed):

$$\max p(M + \frac{2R}{t}) - \frac{2p^2}{t}$$

$$s.t. D(p) \leq 1$$

$$p \leq R$$

transforming into the standard form:

$$\max -\frac{2}{t} \times p^2 + (M + \tfrac{2R}{t}) \times p$$

$$\frac{2}{t} \times p + 1 - M - \frac{2R}{t} \geq 0$$

$$-p + R \geq 0$$

If there are any Kuhn-Tucker points they must satisfy the following conditions:

$$-\frac{4}{t} \times p + M + \frac{2R}{t} + \lambda_1 \frac{2}{t} - \lambda_2 = 0 \tag{12a}$$

$$(\frac{2}{t} \times p + 1 - M - \frac{2R}{t}) \times \lambda_1 = 0 \tag{12b}$$

$$(-p + R) \times \lambda_2 = 0 \tag{12c}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{12d}$$

Consider what happens when $\lambda_1 = 0$ and $\lambda_2 = 0$. We get $p = \frac{mt+2R}{4}$, and $p \times D_+(p) = pm + \frac{2Rp-2p^2}{t} = \frac{(mt+2R)^2}{8t}$.

Consider $\lambda_1 > 0$ and $\lambda_2 = 0$. From the second condition on the Kuhn-Tucker points we get $p = -\frac{t}{2} + \frac{Mt}{2} + R$, then from the first condition we get: $\lambda_1 = -t + \frac{Mt}{2} + R$. The revenue in this case will be simply p, since the demand is 1. Since $\lambda_1 > 0$, for this case to happen $-t + \frac{Mt}{2} + R > 0 \Longrightarrow M > 2 - \frac{2R}{t}$.

Consider $\lambda_2 > 0$ and $\lambda_1 = 0$. From the third condition on the Kuhn-Tucker points we get $p = R$. Then from the first condition we get $\lambda_2 = M - \frac{2R}{t}$, so for this case to happen we need $M > \frac{2R}{t}$. In this case the demand will just be M, and so the revenue will be RM.

Consider $\lambda_1 > 0$ and $\lambda_2 > 0$. From the third condition on the Kuhn-Tucker points we get $p = R$. Plugging this into the second condition we get $M = 1$. In this case the revenue is R.

Now that we have bounds on M for both case 2 and case 3, and since we know that the cases are mutually exclusive, combine the bounds, and get for case 2: $2 - \frac{2R}{t} < M < \frac{2R}{t}$ and for case 3: $\frac{2R}{t} < M < 2 - \frac{2R}{t}$. Therefore, if $2R > t$ then case 2 will happen, and case 3 can not happen, and the opposite if $2R < t$, and if $2R = t$, neither of them can happen. Also notice that as M goes to 1, both case 2 and case 3 solutions go to case 4, and if $2R = t$, then case 1 solution goes to case 4 as well. Therefore we do not need to worry about case 4 being a special case. Overall, in the next step we will have to worry about three scenarios, here is the summary:

$$\begin{array}{cccc}
\text{Condition} & 2R = t & 2R > t & 2R < t \\
\text{Start with} & p = \dfrac{mt + 2R}{4} & p = \dfrac{mt + 2R}{4} & p = \dfrac{mt + 2R}{4} \\
\text{Switch when} & \text{Don't switch} & M = 2 - \dfrac{2R}{t} & M = \dfrac{2R}{t} \\
\text{Switch to} & ... & p = \dfrac{t}{2} + \dfrac{Mt}{2} + R & p = R
\end{array}.$$

Now we need to calculate the optimal measure given the price. I proceed by examining each of the cases above. Consider $2R = t$ first. Then, $\Pi_+(m) = \frac{(mt+2R)^2}{8t} - C_m \times m$, which is clearly a convex parabola in m. Therefore the minimum lies at one of the endpoints, therefore we just need to compare profits. $R - C_m \geq \frac{R^2}{2t}$ for $m = 1$ to be the maximum, so if $C_m \leq R - \frac{R^2}{2t} = \frac{3R}{4} = \frac{3t}{8}$ then it is optimal to have $m = 1$, and $p = R$. Profit is then $R - C_m$, and the consumer welfare is clearly zero. Otherwise the optimum is $m = 0$, and we get back to the Lemma 1 solution.

For $2R > t$ scenario the profit function is the same until the switch point, so let's assume that the optimal solution on that interval is the switch point (since the function is convex), and then compare the answers to $\frac{R^2}{2t}$. This gives us $\Pi_+(2 - \frac{2R}{t}) = \frac{(2t-2R+2R)^2}{8t} - C_m \times (2 - \frac{2R}{t})$. After the switching point the profit function becomes price – cost, so the switch point falls under this definition as well. The derivative of profit w.r.t. m becomes $\frac{t}{2} - C_m$. Therefore if $t \geq 2C_m$ then the optimal solution is $m = 1$, $p = R$. Let's compare it with $m = 0$ solution. Again, we get if $C_m \leq R - \frac{R^2}{2t}$, then $m = 1$ is optimal, and otherwise m = 0. If $t < 2C_m$ then $m = 2 - \frac{2R}{t}$, and $p = t/2$. Demand in this case will be 1, and so the profit will be $\frac{t}{2} - C_m \times (2 - \frac{2R}{t})$. Let's compare this with the profit from the $m = 0$ case. So for the measure to be positive, $\frac{t}{2} - C_m \times (2 - \frac{2R}{t}) \implies C_m \times (2 - \frac{2R}{t}) \leq \frac{t^2 - R^2}{2t} \implies C_m \leq \frac{t+R}{4}$. However, since $R < t < 2C_m$, we get $C_m \leq \frac{t+R}{4} < \frac{2t}{4} < C_m$, meaning that the first inequality never holds, so this sub case never occurs. Otherwise we go back to Lemma 1.

For the last, $2R < t$, scenario, the same thing happens as above before the switch point, and then after the switch point we get the profit function equal to $(RM - cost)$, with the switching point following this definition as well, so the derivative w.r.t. m becomes $R - C_m$. If $R \geq C_m$, then the optimal solution is $M = 1$, $p = R$, so the comparison with $m = 0$ is routine by now: we get if $C_m \leq R - \frac{R^2}{2t}$, then $m = 1$ is optimal, and otherwise $m = 0$. If $R < C_m$ then $M = \frac{2R}{t}$, $p = R$. Demand in this case will be M, and so the profit will be $\frac{2R(R-C_m)}{t}$, and the consumer welfare will be 0, since the price is the reservation price. Let's compare with the $m = 0$ profit. We get $\frac{2R(R-C_m)}{t} \geq \frac{R^2}{2t} \Longrightarrow 4R^2 - 4RC_m \geq R^2 \Longrightarrow C_m \leq \frac{3R}{4}$ for the measure to be positive. ■

B Proofs from the text

Proof of Lemma 1.

Proof. Suppose the equilibrium prices and measures are such that there are intervals of consumers who do not buy from either of the firms next to them. Then, effectively, the deviant firm's market share increases to include the 'left-out' consumers – the demand that it has now, plus the two intervals of the consumers on either side of that demand. This gives the local market of more than 1/N, since half of the left-out consumers came from the 1/N's of the fixed firms.

Whatever was optimal before in the local market for the deviant firm will not be optimal in the ex-post market for this firm, which now includes the consumers that the Left and the Right fixed firms had implicitly left out. The deviant firm will now optimize with respect to both price and the length of its' product on its' new local market, and the equilibrium will be violated, as a bigger measure of the interval and/or a higher price is going to be optimal now.

The only case where this does not make a difference to the optimization solution is when the local market monopoly solution is the same as the monopoly solution. This will happen if the development or the transportation costs are too high, so that the demand of the optimal monopoly price is less than 1/N. Then the deviant firm will not want to change its measure and the price, and so each firm will remain in a Bertrand - Nash Equilibrium, acting as a global monopolist in each of the local markets. ■

Proof of Lemma 2.

Proof. Suppose that the intervals did intersect. Then there would be a positive measure of consumers who can be captured by decreasing the price by $\varepsilon > 0$. Bertrand intuition applies and prices go to marginal cost of making one more product, which is 0, and the revenues go there as well. But then the firms would be better off deviating to making point products, since they still get as much revenue, but don't have to pay a big development cost of $c(m)$. ■

Proof of Theorem 2.

Proof. In this appendix I derive step-by-step the N-Firm Equilibrium with no consumers left out.

I will start with the derivation of the point x – the intersection of two utility functions, one for the deviant firm's product, the other for the fixed product. Deviant chooses m and p, and m^* and p^* stay fixed for the fixed firms.

$$U_{deviant} = R + tx - \frac{\frac{1}{N} - m}{2}t, \tag{13a}$$
$$U_{fixed} = R - tx - \frac{\frac{1}{N} - m^*}{2}t. \tag{13b}$$

Solving for the intersection, I find:

$$x = \frac{m^* - m}{4} + \frac{p - p^*}{2t}. \tag{14}$$

Since there is a fixed firm from both sides, the demand for the deviant firm is:

$$D_{deviant}(m,p) = \frac{1}{N} - 2x = \frac{1}{N} + \frac{m - m^*}{2} + \frac{p^* - p}{t}. \tag{15}$$

therefore the profit is, with the conditions that both m and p are non-negative.

$$\Pi_{deviant}(m,p) = D_d(s,p) \times p - c(m) = \frac{p}{N} + p\frac{m - m^*}{2} + p\frac{p^* - p}{t} - c(m). \tag{16}$$

Therefore,

$$\frac{\partial \Pi_d(m,p)}{\partial p} = \frac{1}{N} + \frac{m - m^*}{2} + \frac{p^*}{t} - \frac{2p}{t}, \tag{17a}$$
$$\frac{\partial \Pi_d(m,p)}{\partial m} = \frac{p}{2} - c'(m). \tag{17b}$$

It can be shown that the cost of m must satisfy the sufficiently convex cost assumption for the

SOCs to hold.[23]

$$\frac{\partial^2 \Pi_d(m,p)}{\partial m \partial p} = \frac{1}{2}, \tag{18a}$$

$$\frac{\partial^2 \Pi_d(m,p)}{\partial m^2} = -c''(m), \tag{18b}$$

$$\frac{\partial^2 \Pi_d(m,p)}{\partial p^2} = -\frac{2}{t}. \tag{18c}$$

and then the Hessian is

$$H_d(m,p) = \begin{bmatrix} -\frac{2}{t} & \frac{1}{2} \\ \frac{1}{2} & -c''(m) \end{bmatrix}. \tag{19}$$

To ensure concavity the Hessian needs to be negative semi-definite, and so the first leading principle major needs to be less than zero, and the second, the determinant, needs to be positive. The first leading major is $-2/t$, and since $t > 0$ it is negative. From examining the determinant:

$$c''(m) > \frac{t}{8}. \tag{20}$$

Now, I use the symmetry conditions and make $m = m^*$ and $p = p^*$, and use the FOCs. By setting $\frac{\partial \Pi_d(m,p)}{\partial p}$ and $\frac{\partial \Pi_d(m,p)}{\partial m}$ to zero, I find

$$p = \frac{t}{N},$$

$$c'(m) = \frac{t}{2N} \ (\mathit{Optimal\ Measure\ Condition}). \tag{21a}$$

Therefore, $D = \frac{1}{N}$, and $\Pi_+ = \frac{t}{N^2} - c(m^*)$. There is no development cost (except for the fixed costs) for the original Salop model firms, and their profit looks like this as well, therefore the profit went down for the firms. Notice that such m is unique, since the total cost function must be convex, and so the first derivative is strictly increasing. Consumer welfare is N times the area of the trapezoid in the center on Figure 2, which becomes

$$CW = N \times \frac{D(m^*,p^*) + m^*}{2} \times (R - p^*) = N \times \frac{\frac{1}{N} + m^*}{2} \times (R - \frac{t}{N}), \tag{22}$$

[23] Differentiating again we get

where m^* satisfies the Optimal Measure Condition. We can not see what happens with the total welfare without functional forms for the cost functions, but since the price is the same as it was in the original Salop model, and m^* is non-negative, we can say that the consumer welfare increased.

■

Proof of Proposition 5.

Proof. I examine two firms, one with base at 0, which will have a product of measure m and charge p for it, and the other one located at $\frac{1}{N}$, with the product of measure m^*, charging p^* for it - set up analogous to the one in the proof of Theorem 2. Consider a customer located at x between the two firms. Then the customer's utility from buying the two products are, respectively, $R - t(x - \frac{m}{2}) - p$ and $R - t(\frac{1}{N} - \frac{m^*}{2} - x) - p^*$. To find out the customer indifferent between the two products, just make the two utilities equal, and simplify to get

$$p - p^* = t(\frac{1}{N} - \frac{m^*}{2} - x^*) - t(x^* - \frac{m}{2}). \tag{23}$$

Assuming the other neighbor is also playing m^* and p^*, the demand for firm at 0 is $2x^*$. Therefore the profit of this firm will be $\Pi(p, m) = 2x^* \times p - c(m)$. Then, implicitly differentiating equation 23, with respect to p one can show that

$$\frac{\partial x^*}{\partial p} = \frac{-1}{t'(\frac{1}{N} - \frac{m^*}{2} - x^*) + t'(x^* - \frac{m}{2})}. \tag{24}$$

Then, implicitly differentiating equation 23, with respect to m, one can show that

$$\frac{\partial x^*}{\partial m} = \frac{t'(x^* - \frac{m}{2})}{2\left[t'(\frac{1}{N} - \frac{m^*}{2} - x^*) + t'(x^* - \frac{m}{2})\right]}. \tag{25}$$

Now we can take look at the first order conditions of the profit function:

$$\frac{\partial \Pi}{\partial p} = 2x^* + 2p \times \frac{\partial x^*}{\partial p} = 0, \tag{26a}$$

$$\frac{\partial \Pi}{\partial m} = 2p \times \frac{\partial x^*}{\partial m} - c'(m) = 0. \tag{26b}$$

Notice, that if we enforce the symmetry assumption to 23 ($p = p^*$ and $m = m^*$), we know that

$t(\frac{1}{N} - \frac{m^*}{2} - x^*) = t(x^* - \frac{m}{2})$, and therefore $x^* = \frac{1}{2N}$. Substituting from 24 and 25 into 26a:

$$p = \frac{t'(\frac{1}{2N} - \frac{m}{2})}{N}. \tag{27}$$

And from 26b I get:

$$c'(m) = \frac{p}{2}. \tag{28}$$

Again, if this m is bigger than $\frac{1}{N}$ than the firms end up playing the standard Bertrand, going down to marginal costs. However then the firms will have a profitable deviation to $m = 0$, and therefore there will be no symmetric equilibrium in this case. ■

Proof of Proposition 7.

Proof. With the new term α, the condition for the optimal measure stays the same ($c'(m^*) = \frac{p}{2}$) and the one for the optimal price becomes

$$p^* = \frac{\alpha t'(\frac{1}{2N} - \frac{m^*}{2})}{N}. \tag{29}$$

Therefore the profit for each firm is now $\Pi(p^*, m^*) = \frac{\alpha t'(\frac{1}{2N} - \frac{m^*}{2})}{N^2} - c(m^*)$. Before we differentiate it with respect to α, we have to derive $\frac{\partial m^*}{\partial \alpha}$ first. We will do it implicitly from the combination of new optimal price and the optimal measure conditions, and simplifying get

$$\frac{\partial m^*}{\partial \alpha} = \frac{2t'(\frac{1}{2N} - \frac{m^*}{2})}{\alpha t''(\frac{1}{2N} - \frac{m^*}{2}) + 4Nc''(m^*)}. \tag{30}$$

Now we can differentiate the profit[24], and check when is the derivative less than zero (getting rid

[24] $\frac{\partial \Pi}{\partial \alpha} = \frac{t'(\frac{1}{2N} - \frac{m^*}{2})}{N^2} - \frac{\alpha t''(\frac{1}{2N} - \frac{m^*}{2})}{2N^2} \frac{\partial m^*}{\partial \alpha} - c'(m^*) \frac{\partial m^*}{\partial \alpha} =$

$= \frac{t'(\frac{1}{2N} - \frac{m^*}{2})}{N^2} - \frac{\alpha t'(\frac{1}{2N} - \frac{m^*}{2}) t''(\frac{1}{2N} - \frac{m^*}{2})}{N^2 \times \left[\alpha t''(\frac{1}{2N} - \frac{m^*}{2}) + 4Nc''(m^*)\right]} - \frac{\alpha N \left[t'(\frac{1}{2N} - \frac{m^*}{2})\right]^2}{N^2 \times \left[\alpha t''(\frac{1}{2N} - \frac{m^*}{2}) + 4Nc''(m^*)\right]}.$

of common terms):

$$1 < \frac{\alpha t''(\frac{1}{2N} - \frac{m^*}{2}) + \alpha N t'(\frac{1}{2N} - \frac{m^*}{2})}{\alpha t''(\frac{1}{2N} - \frac{m^*}{2}) + 4Nc''(m^*)}. \tag{31}$$

A necessary condition for this inequality to hold is that the denominator has to be positive, since the numerator is always positive. Given that we have the denominator positive, and since $\alpha \to 1$:

$$c''(m^*) < \frac{t'(\frac{1}{2N} - \frac{m^*}{2})}{4}. \tag{32}$$

■

Proof of Proposition 8.

Proof. The game is equivalent to the one where the firms and the consumers are on a Hotelling half-unit interval, with L located at 0 and F located at $\frac{1}{2}$, with doubled payoffs and cost of double measure. Examining a consumer located at $x \in [\frac{m_L}{2}, \frac{1}{2} - \frac{m_F}{2}]$:

$$U_L(x) = R - (x - \frac{m_L}{2})t - p_L, \tag{33a}$$

$$U_F(x) = R - (\frac{1}{2} - \frac{m_F}{2} - x)t - p_F. \tag{33b}$$

Make the two equal to compute demand for both the leader and the follower. Notice that demand for the leader is $2x$ and the demand for the follower is $1 - 2x$. Then,

$$D_L = \frac{1}{2} + \frac{p_F - p_L}{t} + \frac{m_L - m_F}{2}, \tag{34a}$$

$$D_F = \frac{1}{2} + \frac{p_L - p_F}{t} + \frac{m_F - m_L}{2}. \tag{34b}$$

Fixing the measure and the price of the leader, the follower will maximize $\Pi_F = p_F D_F - a m_F^2$:

$$\frac{\partial \Pi_F}{\partial p_F} = \frac{1}{2} + \frac{p_L - 2p_F}{t} + \frac{m_F - m_L}{2}, \tag{35a}$$

$$\frac{\partial \Pi_F}{\partial m_F} = \frac{p_F}{2} - 2a m_F. \tag{35b}$$

The second derivatives and the cross-partial are then $\frac{\partial^2 \Pi_F}{\partial p_F^2} = -\frac{2}{t}$, $\frac{\partial^2 \Pi_F}{\partial m_F^2} = -2a$, and $\frac{\partial^2 \Pi_F}{\partial m_F \partial p_F} = \frac{1}{2}$.

Therefore for the Second Order Conditions to be satisfied we need

$$16a > t. \tag{36}$$

Assume that this condition on a holds. From 35b we get $m_F = \dfrac{p_F}{4a}$. Substitute this back into 35a, and make the expression equal to zero to get

$$p_F = \frac{4a}{16a - t}\left(t - tm_L + 2p_L\right). \tag{37}$$

Substitute 37 into 34b to get

$$D_L = \frac{1}{2} + \frac{4a}{16a-t} - \frac{4a}{16a-t}m_L + \frac{8a}{t(16a-t)}p_L - \frac{p_L}{t} + \frac{m_L}{2} - \frac{t - tm_L + 2p_L}{2(16a-t)}. \tag{38}$$

Then $\Pi_L = p_L D_L - am_L^2$, and

$$\frac{\partial \Pi_L}{\partial p_L} = \frac{12a - t}{16a - t} + \frac{4a}{16a-t}m_L - \frac{16a}{t(16a-t)}p_L, \tag{39a}$$

$$\frac{\partial \Pi_L}{\partial m_L} = \left(-\frac{4a}{16a-t} + \frac{1}{2} + \frac{t}{2(16a-t)}\right)p_L - 2am_L = \frac{4ap_L}{16a-t} - 2am_L. \tag{39b}$$

Notice that demands can be expressed as

$$D_L = 1 - \frac{4a(1-m_L)}{16a-t} - \frac{8a}{t(16a-t)}p_L, \tag{40a}$$

$$D_F = \frac{4a(1-m_L)}{16a-t} + \frac{8a}{t(16a-t)}p_L. \tag{40b}$$

The second derivatives and the cross-partial are: $\frac{\partial^2 \Pi_L}{\partial p_L^2} = -\frac{16a}{t(16a-t)}$, $\frac{\partial^2 \Pi_L}{\partial m_L^2} = -2a$, and $\frac{\partial^2 \Pi_L}{\partial m_L \partial p_L} = \frac{4a}{16a-t}$. Therefore for the Second Order Conditions to be satisfied we need

$$32a > 3t. \tag{41}$$

Notice that 41 implies the SOC for the follower (36). We can now examine the First Order Conditions. From 39b we have $m_L = \dfrac{2p_L}{16a-t}$. Substitute this into 39a, and set the expression equal to zero to get

$$p_L = \frac{t(16a-t)(12a-t)}{8a(32a-3t)}. \tag{42}$$

Therefore

$$\begin{aligned} m_L &= \frac{t(12a-t)}{4a(32a-3t)}, && (43a)\\ p_F &= \frac{20at-2t^2}{32a-3t}, && (43b)\\ m_F &= \frac{t(20a-2t)}{4a(32a-3t)}. && (43c) \end{aligned}$$

■

Proof of Corollary 8.

Proof. It can be shown that $m_L > m_F$ when $\frac{t(12a-t)}{4a(32a-3t)} > \frac{t(20a-2t)}{4a(32a-3t)}$ or:

$$t > 8a. \tag{44}$$

■

Proof of Corollary 9.

Proof. It can be shown that $p_L > p_F$ when $\frac{t(16a-t)(12a-t)}{8a(32a-3t)} > \frac{t(20a-2t)}{32a-3t}$, or

$$(t-8a)(t-4a) > 0. \tag{45}$$

■

Proof of Corollary 10.

Proof. It can be shown that $D_L > \frac{1}{2}$ when $1 - \frac{4a(1-m_L)}{16a-t} - \frac{8a}{t(16a-t)} p_L > \frac{1}{2}$, or

$$t > 8a. \tag{46}$$

From the previous derivations we know that

$$\begin{aligned} D_L &= 1 - \frac{20a-2t}{32a-3t} = \frac{32a-3t-20a+2t}{32a-3t} = \frac{12a-t}{32a-3t}, && (47a)\\ D_F &= \frac{20a-2t}{32a-3t}. && (47b) \end{aligned}$$

Then the profits are

$$\Pi_L = \frac{12a-t}{32a-3t}\frac{t(16a-t)(12a-t)}{8a(32a-3t)} - a\left(\frac{t(12a-t)}{4a(32a-3t)}\right)^2 - F, \tag{48a}$$

$$\Pi_F = \frac{20a-2t}{32a-3t}\frac{20at-2t^2}{32a-3t} - a\left(\frac{t(20a-2t)}{4a(32a-3t)}\right)^2 - F. \tag{48b}$$

Therefore for the leader to earn more the following condition must hold:

$$(t-8a)(t-(16+4\sqrt{2})a)(t-(16-4\sqrt{2})a) > 0. \tag{49}$$

Since $a > \frac{t}{10}$ by assumption, both second and third term are always negative, therefore the leader earns more iff $t > 8a$. ■

References

[1] Anderson, S. P. and D.J. Neven (1989): "Market Efficiency with Combinable Products, " *European Economic Review*, **33**, 707-719.

[2] Bakos, Y. and E. Brynjolfsson (1999): "Bundling Information Goods: Pricing, Profits, and Efficiency," *Management Science*, **45(12)**, 1613–1630.

[3] Cheng, L. and J. Nahm (2006): "Product Boundary, Vertical Competition, and the Double Mark-up Problem," *Rand Journal of Economics*, forthcoming.

[4] Debreu, G. (1959). **Theory of Value**. *Yale University Press, New Haven, CT.*

[5] Dewan, R., B. Jing, and A. Seidmann (2003): "Product Customization and Price Competition on the Internet,"*Management Science*, **49 (8)**, 1055–1070.

[6] Draganska, M. and D. Jain (2005): "Product-Line Length as a Competitive Tool," *Journal of Economics and Management Strategy*, **14(1)**, 1-28.

[7] Ellickson, P. (2005): "Does Sutton Apply to Supermarkets?," working paper.

[8] Gabszewicz, J.J., Laussel D., and N. Sonnac (2004): "Programming and Advertising Competition in the Broadcasting Industry," *Journal of Economics and Management Strategy*, **13(4)**, 657-669.

[9] Gal-Or, E. and A. Dukes (2003): "Minimum Differentiation in Commercial Media Markets," *Journal of Economics and Management Strategy*, **12(3)**, 291-325.

[10] Hotelling, H. (1929): "Stability in Competition," *Economic Journal*, **39**, 41–57.

[11] Inderst R. and A. Irmen (2005): "Shopping hours and price competition," *European Economic Review*, **49**, 1105-1124.

[12] Judd, K. (1985): "Credible Spatial Preemption," *Rand Journal of Economics*, **16(2)**, 153–166.

[13] Klemperer, P. (1992): "Equilibrium Product Lines: Competing Head-to-Head May Be Less Competitive," *American Economic Review*, **82(4)**, 740 - 755.

[14] Lancaster, K. (1971). **Consumer Demand: A New Approach**. *Columbia University Press, New York, NY.*

[15] Mendelson, H., and A. K. Parlakturk (2005): "Competitive Customization," working paper.

[16] Salop, S. C. (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, **10(1)**, 141–156.

[17] Schmalensee, R. (1978): "Entry deterrence in the ready-to-eat breakfast cereal industry," *Bell Journal of Economics*, **9**, 305 – 327.

[18] Shy, O. and R. Stenbacka (2006): "Service hours with asymmetric distributions of ideal service time," *International Journal of Industrial Organization*, **24**, 763–771.

[19] Spence, A. M. (1980): "Multi-Product Quantity-Dependent Prices and Profitability Constraints," *Review of Economic Studies*, **47(5)**, 821–841.

[20] Sutton, J. (1991). **Sunk Cost and Market Structure: Price Competition, Advertising, and the Evolution of Concentration**. *MIT Press, Cambridge, MA.*

[21] Teitz, M. B. (1968): "Locational Strategies for Competitive Systems," *Journal of Regional Science*, **8(2)**, 135–148.

[22] Venkatesh, R., and W. Kamakura (2003): "Optimal Bundling and Pricing under a Monopoly: Contrasting Complements and Substitutes from Independently Valued Products," *Journal of Business*, **76(2)**, 211–231.

[23] Zipkin, P. (2001): "The Limits of Mass Customization," *MIT Sloan Management Review*, **42(3)**, 81 – 87.