The Price of Advice*

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Abstract

We develop a model of consulting (advising) where the role of the consultant is that she can reveal signals to her client which refine the client's original private estimate of the profitability of a project. Importantly, only the client can observe or evaluate these signals, the consultant cannot. We characterize the optimal contract between the consultant and her client. It is a menu consisting of pairs of transfers specifying payments between the two parties (from the client to the consultant or vice versa) in case the project is undertaken by the client and in case it is not. The main result of the paper is that in the optimal mechanism, the consultant obtains the same profit as if she could evaluate the impact of the signals (whose release she controls) on the client's profit estimate.

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1 Introduction

An important question in economic theory is how information is transmitted between strategic agents with differing goals, and what type of mechanisms govern (or induce optimal) information transmission between such parties. In particular, one fascinating topic is the relationship between consultants (professional advisors) and their clients: How do consultants "create value," and what characterizes optimal contracts between them and their clients?

In this paper, we put forward a model where a consultant (she) is able to reveal signals to her client (him) that refine the client's original private estimate regarding the profitability of a project. The client's action as to whether he undertakes the project is contractible, but the information disclosed by the consultant is not. Indeed, we assume that only the client can observe (evaluate) the additional signals disclosed by the consultant, so the consultant does not know a priori whether her advice made the project look more or less profitable to the client. She may only make inferences from the client's action.

We characterize the optimal contract between the consultant and the client. The optimal contract can be represented by a menu that consists of pairs of transfers specifying payments between the two parties contingent on whether or not the project is undertaken by the client. If the client chooses an item from the menu, the consultant agrees to release to him whatever information she has, and the transfers take place according to the client's action.[1] In the optimal menu, there may be items where the client pays a fee to the consultant upon undertaking the project, as well as items where the consultant pays the client whenever the project is carried out. The client's choice among the different pairs of transfers depends on how optimistic or pessimistic he is regarding the profitability of the project prior to listening to the advice of the consultant. In interesting special cases, the client pays the consultant a positive fee exactly when her advice changes the client's mind as to whether or not to undertake the project. Intuitively, the consultant's advice is valuable because it may induce the client to take the action opposite to what he has planned.

The main result of the paper is that, while the consultant cannot observe the "new information" disclosed by her, she can design a contract in which she obtains

[1] A contract, in general, could be more complicated (for example, it could involve lotteries), however, we show that the optimal contract has this simple form.

the same profit as if she could. In other words, in the optimal contract, the client only enjoys information rents for the information he already has prior to meeting the consultant. The client does not get any rents from the information whose release is controlled by the consultant, even though that information becomes his (the client's) private information when released. We also show that the optimal contract does entail inefficiencies, that is, the first-best is not achieved.

Our way of modeling "professional advice"—where the advisor may disclose information that only the client can interpret—is new, perhaps unusual, but we believe is accurate in many instances. For a concrete example, think of the client as a potential buyer of a good (e.g., a car, or a firm's shares) who is uncertain of various characteristics of the good (e.g., the features of the car, the covariance of the stock's return with other assets' returns, etc.). Then, the consultant can be thought of as an expert who has additional information regarding the good's characteristics. Naturally, the consultant does not know the buyer's original value-estimate; neither does she know by how much her information increases or decreases the buyer's willingness to pay for the good, because that is also part of the buyer's private information (e.g., what features he values in a car, what his existing portfolio consists of, etc.). Our question is: What is the consultant's optimal contract, if the buyer's action (whether or not he buys the good) is contractible, but the effect of the consultant's information on the client's valuation is not?

More broadly, our model of consultancy is motivated by the widely held belief that the role of strategy and management advisors is to help uncover their clients' own ideas so that the clients can realize what they are capable of.[2] Consultants often only talk about the correct general criteria to be used in decision making (what types of trade-offs to consider, common fallacies, etc.), instead of the particularities of the client's decision problem. By discussing general ideas, industry trends, or similar cases, they provide useful information to the client: his private knowledge regarding his project becomes more nuanced. Nevertheless, the consultant may never learn exactly what effect her advice has had on the client's objective function. In many cases, it is conceivable that only the client's actions are observable and contractible. This is the type of potential information transmission that we attempt to model in

[2] For example, Accenture (a consultancy) advertises on its website the firm's "ability to act as a catalyst" to "bring [clients'] ideas to life".

the paper.[3]

The literature on information transmission between experts and client-customers (see Milgrom and Roberts (1986), Pitchik and Schotter (1987), Wolinsky (1993), Emons (1997) and the references therein) often treats the expert's information disclosure as *cheap talk* (à la Crawford and Sobel (1982)). The advisor has unverifiable information, and the question is how precisely she can reveal it if the interests of the parties are not perfectly aligned, and the client's actions are not contractible. Ours has not much in common with cheap-talk models as in our setup the expert does not know what effect her signal has on the client's action, but that action is contractible.

There are other models of professional advice (e.g., the attorney-client relationship) that our approach and results are more related to. This literature is motivated mostly by the observation that attorneys are paid contingent fees: payments that substantially differ depending on the success or failure of the client's case. The literature (see Dana and Spier (1993) and the references therein) offers several types of economic explanations for such contracts, among them risk sharing, liquidity constrained clients, and moral hazard problems associated with the attorney. Contingent fee contracts may also be optimal when there is asymmetric information between the attorney and the client. In the model of Scotchmer and Rubinfeld (1990), the attorney has private information about her own ability, while the client is better informed regarding the merits of the case. In Dana and Spier (1993), the attorney obtains superior information regarding the merits of the client's case after having offered the client a contract. In both models, contingent-fee contracts arise in equilibrium.

Our research contributes to this literature by considering an advisor (or attorney) that can make her client better informed about the client's project (or legal case), without becoming better informed herself. As we already said, our most interesting result is that it does not matter whether or not the consultant also becomes better informed as she discloses signals to the client, she gets the same expected payoff in both cases. The optimal contract in our model resembles those found in this

[3] It may be the case that the client intentionally restricts the consultant's ability to evaluate the client's options, and this is why only the client can interpret the consultant's advice. For example, the client may fear that the consultant would use her knowledge of the client's problem to advise his competitors.

literature; the main differences are that in our case, it is a *menu* of contingent fees, and that the transfers are conditional on the client's observable *action* (e.g., whether he decides to pursue the case).

Besides shedding light on issues concerning information disclosure and the appropriation of information rents, our model describes features of real-world contracts as well. As the literature cited above points out, attorneys often work under contingent-fee contracts (where the contingencies may represent the client's actions, such as settling or proceeding with the case), and the exact terms of the contracts vary across clients. This observation is consistent with our model, in which the optimal contract is a menu of contingent transfers that the client chooses from according to his assessment of the merits of the case prior to talking to the lawyer.

In other real-world examples of professional advice, such as management or IT consulting and real-estate advising, we also observe fees that are contingent on the client's action. In mergers, for example, it is customary for the consultant of the buyer to demand a "success fee" due upon the completion of the deal. Since the decision to acquire the target is ultimately the client's decision, we may interpret such success fees as action-contingent transfers. A buyer's agent (e.g., in a real estate transaction) may end up with a lower commission after the client has listened to her advice (e.g., the client buys the cheaper condo when he learns that the other unit, while more luxurious, has features that he finds appalling). Our model shows that this may be the outcome of an optimal contract between the advisor and her client: The buyer's agent should disclose as much information as she has even if she is unaware of its impact on the client's preferences.

In a related paper of ours (Eső and Szentes (2002)) we analyze the auction design problem where a monopolist can disclose, without observing, private signals to the buyers that refine their initial private valuation estimates for the object being sold. That paper characterizes the revenue-maximizing selling mechanism and show that in the optimal mechanism the seller discloses all available signals (which only the buyers can observe) and attains the same revenue as if she could directly observe the realizations of these signals. This result is similar to the one we obtain in the present paper, where the consultant obtains the same profit as if she could observe the effect of her signal on the client's valuation for the project. The problem is very different here, however, because it is not the seller, but a third party, that controls information relevant for the buyer (here, the client).

The analysis of the optimal contract in our model is similar to that of a Principal-Agent model where the value of the Agent's outside option depends on his type.[4] Principal-Agent models with type-dependent outside option have been studied in the literature by Lewis and Sappington (1989), Klibanoff and Morduch (1995), Maggi and Rodriguez (1995), and most generally by Jullien (2000). None of the above treatments applies directly in our framework, but the solutions exhibit certain common features. Also, naturally, in this literature the question whether the consultant could gain by directly observing the signals that she controls does not arise, because that is a question very specific to our actual model.

The paper is structured as follows. In the next section, we outline the model and introduce the necessary notation. In Section 3, we derive the optimal contract for the consultant. In Section 4, we compare the results with those obtained in a "benchmark" case, where the consultant can observe (upon release) the signal that she controls. We show that her payoff is the same in either optimal contract. Section 5 concludes.

2 The Model

2.1 The Environment

There are two risk neutral agents in the model: a consultant (she) and her client (he). The client can undertake a project at a cost r, where $r \in \mathbb{R}$ is commonly known. The project generates a stochastic ex-post monetary benefit, $V = v + s$, where v is the client's estimate of V, and s is an error term. Note that the additive structure is not an assumption, rather, $s \equiv V - v$ is the definition of the error term. While v is the client's private information, s is a signal that only the consultant can disclose him, without the consultant directly observing it. Notice that since s is not observed by the consultant (and neither is v), it does not matter whether she reveals without observing V or s, because in the latter case the client can compute the value of $V = v + s$. It is important to understand that this assumption—that

[4] The client can undertake the project without asking the consultant for advice. If his original value-estimate (type) is below the project's cost then the client's outside option is worth zero. If his estimate exceeds the project's cost then the client's outside option is the project's net profit, which is increasing in his type.

the sender of s does not observe s while the receiver does—is just the way we *model* the situation where the consultant is unaware of the effect of her information on the client's initial value-estimate.

We assume that v is drawn from a distribution F on the unit interval[5] with a positive density f that is twice differentiable and logconcave (i.e., $d^2 \ln f(v)/dv^2 \leq 0$). Logconcavity is an important, though standard, assumption in the literature on contracting with incomplete information. It implies, among other things, that the distribution satisfies certain monotone hazard rate conditions. In particular, for all $b \in [0,1]$, $(b-F)/f$ is weakly decreasing.[6] Many widely used density functions satisfy logconcavity (see Bagnoli and Bergstrom (1989)).

The other component of the client's ex-post valuation, s, is drawn from a distribution G with full support on $(-\infty, +\infty)$.[7] We assume that s and v are independently distributed, and that the expected value of s is zero. This means that the noise in the client's value-estimate is unrelated to the estimate itself. In other words, the client's original private signal, v, is an unbiased estimator of his expected valuation, and he has no other private information, for example, regarding the precision of this estimator. This assumption is made for the sake of conceptual clarity. In Appendix 2 we show under what conditions and how the analysis can be generalized when the error term ($s \equiv V - v$) is correlated with v.

What is the "value" of the consultant's services to the client? Intuitively, the closer v is to the cost of undertaking the project (r), the more valuable it is for the client to know V precisely. Formally: If the client observes s then he undertakes the project if and only if $v + s \geq r$, and his profit becomes

$$\int_{r-v}^{\infty} (v + s - r) dG(s). \tag{1}$$

When $v \leq r$, the client would not undertake the project without knowing s, which

[5] The normalization that the project's expected gross profit, v, falls between 0 and 1 is innocuous as r can be either positive, negative, or zero. All that this assumption implies is that the project's expected profit is bounded.

[6] This result is due to Prékopa (1971). For references, see also Fudenberg and Tirole (1991) and Jullien (2000).

[7] Intuitively, the full support assumption ensures that no realization of V (small or large) can be excluded given a particular estimate v. This assumption is made solely for ease of exposition. All our results go through (with more cumbersome notation) if the support of the distribution of s is not the whole real line.

would then yield zero profit; therefore, for him the value of knowing s is exactly (1). When $v \geq r$, the client would undertake the project without the consultant, and his expected profit would be $v - r$. By learning the value of s from the consultant, his payoff becomes (1), therefore his gain from knowing s is $\int_{r-v}^{\infty}(v+s-r)dG(s)-(v-r)$. Therefore, the client's willingness to pay for the consultant's information is

$$w(v) = \int_{r-v}^{\infty} (v + s - r)dG(s) - (v - r)\mathbf{1}_{v \geq r}, \tag{2}$$

where $\mathbf{1}$ is the indicator function. The function $w(v)$ is strictly increasing for $v < r$ and strictly decreasing for $v > r$. Intuitively, the closer is the client's estimate, v, to r, the more uncertain he is whether or not to undertake the project, and hence the more valuable it is for him to learn his actual valuation more precisely.

The release of information costs $K \geq 0$ to the consultant in monetary terms. We assume that K is less than the "value" of the consultant's information to any type of the client, that is, $w(0) \geq K$ and $w(1) \geq K$. This assumption means that it is always socially desirable for the consultant to release her information to the client. However, the consultant does not always do so in the optimal (second-best) contract, as we show in Section 3.

The simplest situation that corresponds to the model's formalism is where the client is the management of a firm contemplating the acquisition of another firm. The takeover price is r (commonly known); the client's initial estimate about the target's value is v (privately known). The consultant is an expert on mergers. She can help (at cost $K \geq 0$) the client learn the value of the target firm without her actually learning anything about the value of her advice. In the remaining part of this section, we turn to the description of contracts between the consultant and her client when the client's choice of undertaking the project is contractible but the consultant's information is not.

2.2 Feasible Contracts

In the interim stage (when the client already knows v) the consultant can offer a contract to the client. Naturally, the terms of the contract cannot depend on the realization of s; however, the client's decision whether or not to undertake the project is contractible. After a contract is offered by the consultant, the client may accept or reject it. The contract can specify whether or not the consultant will provide advice (disclose s) to the client, and in either case, transfers between the two parties that may be contingent on the client's action.[8]

The general revelation mechanisms that we consider in this section consist of four real-valued functions on the domain of types (the unit interval): a, c, p, and q. The consultant commits to a mechanism $\{a, c, p, q\}$, and the client, if he wants to participate in it, reports his type. For a reported type $v \in [0, 1]$, the client pays the consultant an up-front transfer of $c(v) \in \mathbb{R}$. (We normalize the direction of transfer payments from the client to the consultant; of course, transfers may be positive, negative, or zero.) The consultant discloses s to the client with probability $a(v) \in [0, 1]$. If s is not disclosed then the client has to undertake the project with probability $q(v) \in [0, 1]$. If s is disclosed then the client can decide whether or not to undertake the project, but if he does then he pays the consultant an additional premium, $p(v) \in \mathbb{R}$. Note that p determines whether or not the client undertakes the project when s is disclosed: he carries it out whenever $v + s \geq r + p(v)$.[9] Contracts with more complex transfer schemes can be rewritten in this simple form. For example, a mechanism in which the consultant requires a payment when s is not disclosed can be simplified without altering the client's incentives (or participation) by incorporating the expected value of that payment in the upfront fee.[10]

In our setup, the consultant acts as a monopolist when offering a contract to the client. This is an abstraction of the fact that advice is a differentiated product and consultants enjoy limited market power. In reality, contracts are negotiated between consultants and clients, and usually neither party has a complete advantage in the process. However, any situation where the bargaining power is shared between the consultant and the client can be modeled such that at time 0, a lottery determines who has the right to make the offer. If the client gets to offer a contract, the optimal contract is simple: he asks the consultant to give him advice in exchange for a transfer of K. In the rest of the paper, we focus on the case where the bargaining

[8] The consultant may offer to use a lottery to decide whether or not to disclose s to the client, but she cannot garble s (change its value if disclosed). Garbling the advice is not a practical possibility for real-world consultants, and it would trivialize the theoretical analysis.

[9] From the consultant's perspective (i.e., disclosing, but not knowing s), a client with type v undertakes the project with probability $1 - G(r + p(v) - v)$.

[10] One of our results, Theorem 2, directly implies that no contract can perform better for the consultant than the optimal revelation mechanism of the form $\{a, c, p, q\}$.

power is delegated to the consultant, which should be thought of as a subgame of the larger game describing the negotiation process.

The willingness-to-pay function in (2), together with the distribution of v, could be used to compute the monopoly price for the consultant's services, that is, an optimal flat fee that she could charge for disclosing (without observing) her information. However, the purpose of this paper is to investigate what (how much more) can be done when the client's action as to whether or not he undertakes the project is contractible. We will see that flat-fee contracts are not optimal.

A contract $\{a, c, p, q\}$ is incentive compatible if no type v of the client is strictly better off reporting $v' \neq v$, and it is individually rational if the payoff of type v from truthful reporting exceeds his payoff from not contracting at all with the consultant. The profit of the client with type v reporting v' is

$$\pi(v, v') = a(v') \int_{r+p(v')-v}^{\infty} [v + s - r - p(v')] \, dG(s) + (1 - a(v'))\, q(v')\, (v - r) - c(v'). \quad (3)$$

Denote the indirect profit function of type v in the mechanism by $\Pi(v) = \pi(v, v)$. Note that without a contract, the client undertakes the project if and only if $v \geq r$, hence his outside option is worth $\max\{0, v - r\}$. Therefore, a mechanism $\{a, c, p, q\}$ is incentive compatible and individually rational for type $v \in [0, 1]$, if and only if,

$$\Pi(v) \geq \max\{\pi(v, v'), v - r, 0\} \quad \text{for all } v' \in [0, 1]. \quad (4)$$

In what follows, we will not distinguish individual rationality from incentive compatibility, and say that the mechanism is incentive compatible if and only if (4) holds for all $v \in [0, 1]$.

3 The Consultant's Optimal Contract

We turn to the derivation of the optimal contract of the consultant. The consultant's problem is to find an incentive compatible mechanism $\{a, c, p, q\}$ that maximizes her ex-ante expected profit. We first characterize the client's profit in incentive compatible mechanisms. Then we derive the solution to the consultant's problem in special cases (Subsection 3.2). This provides the foundation for the general case, which we solve in Subsection 3.3. At the end of the section we illustrate our findings by numerical examples.

3.1 Incentive Compatible Mechanisms

Let $X(v)$ denote the probability that client type v, reporting his type truthfully, undertakes the project in mechanism $\{a, c, p, q\}$. That is,

$$X(v) = a(v)\,[1 - G(r + p(v) - v)] + (1 - a(v))\,q(v). \tag{5}$$

In the following lemma, first, we provide necessary conditions for the incentive compatibility of a mechanism. It turns out that in any incentive compatible contract, the client's indirect profit can be expressed in a familiar way as the profit of the lowest type plus the integral, over types lower than the client's actual type, of the probability of undertaking the project. We also provide (stronger) conditions that are sufficient for the incentive compatibility of a mechanism. We will use the results of the lemma by looking for the optimal mechanism among all mechanisms that satisfy the necessary conditions of incentive compatibility, and then we show that the optimal contract satisfies the sufficient conditions.[11]

The proofs of all lemmas are collected in Appendix 1.

Lemma 1 *If a mechanism $\{a, c, p, q\}$ is incentive compatible then X is weakly increasing, and for all $v \in [0, 1]$,*

$$\Pi(v) = \Pi(0) + \int_0^v X(z)dz, \tag{6}$$

and

$$\Pi(v) \geq \max\{0, v - r\}. \tag{7}$$

Conversely, if $a = \mathbf{1}_{v \in [\underline{v}, \bar{v}]}$ with $\underline{v} < r < \bar{v}$, p is weakly decreasing, $q = \mathbf{1}_{v \geq r}$, and (6)–(7) hold, then the mechanism is incentive compatible.

[11] It may be interesting to note that (6), (7) and the monotonicity of X are not sufficient for incentive compatibility. For example, one can show that if $a(v) = 1$ for all v in a ball around r then p has to be weakly decreasing at r.

Notice that the premium function, p, may introduce distortion in the client's decision. When s is disclosed to the client, he undertakes the project if and only if $v + s$ exceeds $r + p(v)$, which results in inefficiency whenever $p(v) \neq 0$. (When s is not disclosed, a similar distortion is introduced if $q(v) \neq \mathbf{1}_{v \geq r}$ for some v.) A decreasing premium function discriminates among different types of the client in a particular way: a higher type with the same ex-post valuation may undertake the project while a lower type may not. The lemma also states that the client's profit function must start at $v = 0$ from a non-negative level $\Pi(0)$, and must never go below his participation constraint: $\Pi(v) \geq \max\{0, v - r\}$ for all $v \in [0, 1]$. By incentive compatibility, the slope of Π at any point v must coincide with the probability that client type v undertakes the project. For example, if the client reporting v gets to learn s for sure ($a(v) = 1$) then the slope of Π at v is $X(v) = 1 - G(r + p(v) - v)$.

The significance of the lemma is that with its help (using the necessary conditions), we can characterize the expected payoff of the consultant in any incentive compatible mechanism. First, we can calculate the expected payoff of the client as

$$\int_0^1 \Pi(v)dF(v) = \Pi(0) + \int_0^1 \int_0^v X(z)dzdF(v).$$

The second term, using integration by parts, can be written as $\int_0^v X(z)dzF(v)\big|_{v=0}^1 - \int_0^1 F(v)X(v)dv$, hence

$$\int_0^1 \Pi(v)dF(v) = \Pi(0) + \int_0^1 (1 - F(v))\, X(v)dv.$$

The consultant's expected payoff is the difference between the social surplus and the client's profit. The interim social surplus is $E_s\left[(v + s - r)\mathbf{1}_{v+s\geq r+p(v)}\right] - K$ when s is disclosed, and $q(v)(v - r)$ when it is not. Therefore, in the mechanism the consultant's expected payoff is

$$W = \int_0^1 \left(a(v)\left[\int_{r+p(v)-v}^{\infty} (v + s - r)\, dG(s) - K\right] + (1 - a(v))q(v)\,(v - r)\right) dF(v) \\ - \int_0^1 (1 - F(v))\, X(v)dF(v) - \Pi(0).$$

Using (5) for X, this can be rewritten as

$$W = \int_0^1 \left(a(v) \left[\int_{r+p(v)-v}^{\infty} \left(v + s - r - \frac{1 - F(v)}{f(v)} \right) dG(s) - K \right] \right.$$
$$\left. + (1 - a(v))\, q(v) \left(v - r - \frac{1 - F(v)}{f(v)} \right) \right) dF(v) - \Pi(0). \quad (8)$$

In the proofs of Lemma 2 and Theorem 2 we will refer to this formula for the consultant's expected payoff, while in the proof of Lemma 3 we will derive a different one based on the results of Lemma 1.

In the rest of the section, using the results of Lemma 1, we replace c (the fee function) with Π (the client's indirect profit function). Converting $\{a, \Pi, p, q\}$ back into the form $\{a, c, p, q\}$ is straightforward, and we will do that after deriving the optimal mechanism.

3.2 Preliminary Analysis of the Consultant's Problem

We now characterize the consultant's optimal contract in certain special cases, which include cases when either $r \geq 1$ or $r \leq 0$ (see Lemmas 2 and 3 below). The results for these special cases form the basis of the derivation of the optimal contract for the general case ($r \in \mathbb{R}$) in Subsection 3.3.

In order to discuss the special cases, we introduce the following notation. Let $v^* \in [0, 1]$. For all $v \in [0, v^*]$, define

$$F_L(v) = \frac{F(v)}{F(v^*)}, \quad (9)$$

and for all $v \in [v^*, 1]$, define

$$F_H(v) = \frac{F(v) - F(v^*)}{1 - F(v^*)}. \quad (10)$$

That is, F_L and F_H are the cumulative distribution functions of the client's valuation conditional on v falling into the intervals $L = [0, v^*]$, and $H = [v^*, 1]$, respectively. Also, let $f_L = f/F(v^*)$ on L and $f_H = f/(1 - F(v^*))$ on H, that is, f_L and f_H are the conditional densities on the respective domains. These densities are logconcave because f is logconcave.

In the proof of Theorem 1 (Subsection 3.3), we will need general formulas for the consultant's expected payoff coming from client-types $v \in [0, v^*]$ and $v \in [v^*, 1]$ for an appropriate v^*, when the mechanism is incentive compatible for all $v \in [0, 1]$. To this end, we now characterize mechanisms that are incentive compatible for all $v \in [0, 1]$ and maximize the consultant's expected payoff conditional v belonging to L (and H, respectively) in certain cases. Since we require incentive compatibility on the whole unit interval, we can use the results of Lemma 1. However, we compute and maximize the consultant's profit as if she faced a buyer with $v \in L$ and $v \in H$, respectively. (For the purposes of this subsection, one may think of v^* as one of the endpoints of the unit interval. We exploit the fact that v^* can take intermediate values only in the proof of Theorem 1.)

Lemma 2 *Let $v^* \in [0, 1]$. Suppose that $r \geq 0$ and*

$$\int_{\underline{v}}^{v^*} \left[1 - G\left(r + \frac{1 - F_L(v)}{f_L(v)} - v\right)\right] dv \geq \max\{0, v^* - r\}. \tag{11}$$

Then, the mechanism that is incentive compatible for all $v \in [0, 1]$ and maximizes the consultant's payoff conditional on $v \in [0, v^*]$ is characterized by $a = \mathbf{1}_{v \in [\underline{v}, v^*]}$ with

$$\underline{v} = \min\left\{v \in [0, v^*] : \int_{r+p(v)-v}^{\infty} [v + s - r - p(v)]\, dG(s) \geq K\right\}, \tag{12}$$

$p = (1 - F_L)/f_L$, $q = \mathbf{1}_{v \geq r}$, and (6) with $\Pi(0) = 0$ and X defined by (5).

If condition (11) holds then the solution to the problem can be summarized in words as follows. The signal is disclosed with probability one to high types of the client ($v > \underline{v}$) and is never disclosed to low types ($v < \underline{v}$). If s is not disclosed then the client is instructed to carry out the project whenever $v \geq r$ (i.e., the consultant does not interfere with his choice). In contrast, if s is disclosed, the premium function introduces some distortion as it equals the inverse hazard rate of the distribution of v, which is positive and weakly decreasing in v.

When $v^* = 1$ and $r \geq 1$, inequality (11) automatically holds. Therefore, we have found the solution to the consultant's problem for the special case when $r \geq 1$.

The optimal mechanism when $r \geq 0$ and (11) hold, characterized by Lemma 2 with $v^* = 1$, is illustrated in Figure 1. The figure depicts the indirect profit function of the client together with his type-dependent participation constraint.

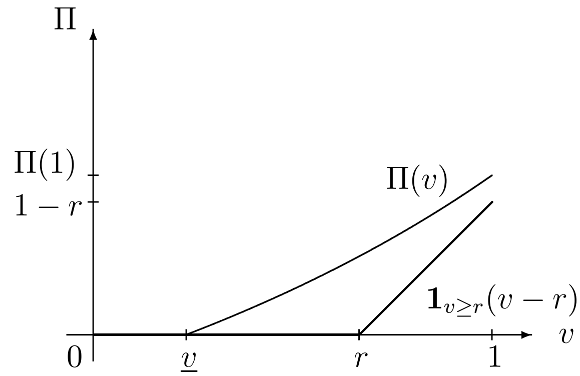

Figure 1: Client's profit in the optimal mechanism, $p = (1 - F)/f$

In the situation shown in Figure 1, those types of the client that fall below the threshold $\underline{v}$ do not get so observe s, never undertake the project, and get zero profit. If the client's original value-estimate is above the threshold, $v > \underline{v}$, then the consultant discloses s to him, and the probability that he undertakes the project becomes $1 - G(r + (1 - F(v))/f(v) - v)$. This is also the slope of the client's indirect profit function. Since the slope is between zero and one and (11) holds, the profit function never falls below the value of the outside option.

Now we derive the optimal contract in another special case.

Lemma 3 *Let $v^* \in [0, 1]$. Suppose that $r \leq 1$ and*

$$\bar{v} - r - \int_{v^*}^{\bar{v}} \left[1 - G\left(r - \frac{F_H(v)}{f_H(v)} - v\right)\right] dv \geq \max\{0, v^* - r\}. \tag{13}$$

Then, the mechanism that is incentive compatible for all $v \in [0, 1]$ and maximizes the consultant's payoff conditional on $v \in [v^*, 1]$ is characterized by $a = \mathbf{1}_{v \in [v^*, \bar{v}]}$ with

$$\bar{v} = \max\left\{v \in [v^*, 1] : \int_{-\infty}^{r+p(v)-v} (r + p(v) - v - s)\, dG(s) \geq K\right\}, \tag{14}$$

$p = -F_H/f_H$, $q = \mathbf{1}_{v \geq r}$, and (6) with $\Pi(1) = 1 - r$ and X defined by (5).

In words, if condition (13) holds then the solutions is the following. The signal is always disclosed to low types of the client ($v < \bar{v}$) and is never disclosed to high types ($v > \bar{v}$). If s is not disclosed then the client is instructed to carry out the project whenever $v \geq r$. If s is disclosed, the premium at which the client can undertake the project is negative and weakly decreasing in v as it equals $-F(v)/f(v)$.

When $r \leq 0$ and $v^* = 0$, (13) automatically holds. Therefore, we have found the solution to the consultant's problem for the special case when $r \leq 0$.

The optimal mechanism when $r \leq 1$ and (13) hold, characterized by Lemma 3 with $v^* = 0$, is illustrated in Figure 2. The figure depicts the indirect profit function of the client together with his type-dependent participation constraint.

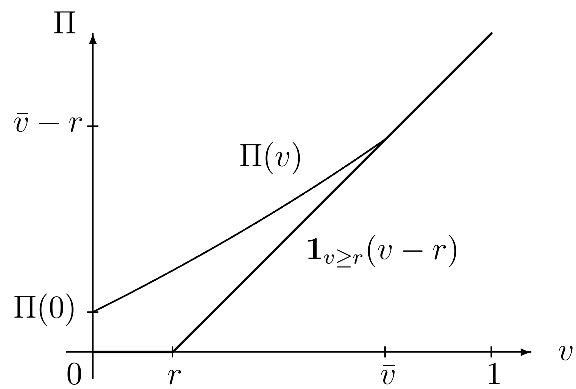

Figure 2: Client's profit in the optimal mechanism, $p = -F/f$

In the situation shown in Figure 2, those types of the client that are above the threshold $\bar{v}$ do not get so observe s, yet always undertake the project, and get an expected profit of $v-r$. If the client's original value-estimate is below the threshold, $v < \bar{v}$, then the consultant discloses s to him, and the probability that he undertakes the project becomes $1 - G(r - F(v))/f(v - v)$. This is also the slope of the client's indirect profit function. Since the slope is between zero and one, and (13) holds, the profit function never falls below the value of the outside option.

The solutions to the consultant's contract design problem when $r \geq 1$, and $r \leq 0$, respectively, are quite insightful and interesting on their own. When $r \geq 1$, the client is originally "pessimistic" in the sense that his estimate regarding the profitability of the project is always non-positive. He would never undertake the project without the consultant's advice (i.e., without learning that s is sufficiently large and positive). Note that in this case, the client has to pay the consultant if he decides to undertake the project: $p(v) > 0$ for all $v < 1$. On the other hand, when $r \leq 0$, the client's original profitability estimate is always "optimistic," $v \geq r$ for all v, and without the consultant's advice he would always undertake the project. In this case, it is the consultant who pays the client in case he undertakes the project:

$p(v) < 0$ for all $v > 0$. In other words, when $r \leq 0$, the client pays more to the consultant if he does not undertake the project than if he does.

In these special cases, the client has to make a net payment to the consultant *when the consultant's advice makes the client change his mind*: if he undertakes the project while $v \leq r$ for all v, or if he does not undertake the project while $v \geq r$ for all v. Of course, in general (when $r \in \mathbb{R}$) the consultant does not know *a priori* which of the two actions of the client signify that he has changed his mind. In the next subsection, we see how the contract is structured in this case.

3.3 The Optimal Contract in the General Case ($r \in \mathbb{R}$)

We now complete the analysis of the consultant's problem by deriving the terms of the optimal contract for any $r \in \mathbb{R}$. We will show that the consultant always discloses s to types between certain thresholds (denoted by $\underline{v}_b$ and $\bar{v}_b$), and never discloses it to any other types. If s is not disclosed then the consultant lets the client undertake the project whenever his original estimate exceeds the project's cost ($v \geq r$, no interference). If s is disclosed then the premium function is $p = (b - F)/f$ for some $b \in [0,1]$. Note that when $r \geq 1$, we have already established $b = 1$, while for $r \leq 0$, we have found $b = 0$ (see Lemmas 2 and 3). The optimal contract is structured so that the client is indifferent between his offer and his outside option at either endpoint of the range $[\underline{v}_b, \bar{v}_b]$, except possibly at one of the endpoints, when that point is on the boundary of $[0,1]$. Finally, the consultant is indifferent between excluding and including the boundary types in the optimal contract whenever these boundary points are inside the unit interval. Figure 3 depicts the client's indirect profit function in the optimal mechanism in a situation where $0 < \underline{v}_b < \bar{v}_b < 1$.

For all $b \in [0,1]$, let $p_b(v) = (b - F(v))/f(v)$, and define

$$\underline{v}_b = \min \left\{ v \in [0,1] : \int_{r+p_b(v)-v}^{\infty} (v + s - r - p_b(v))\, dG(s) \geq K \right\}, \quad (15)$$

$$\bar{v}_b = \max \left\{ v \in [0,1] : \int_{-\infty}^{r+p_b(v)-v} (r + p_b(v) - v - s)\, dG(s) \geq K \right\}. \quad (16)$$

If the premium function is set to $p_b = (b - F)/f$ then client type v that learns s from the consultant undertakes the project if and only if $v + s - r - p_b(v)$ is non-negative, and earns a profit that equals this value. Hence the interpretation of $\underline{v}_b$ is

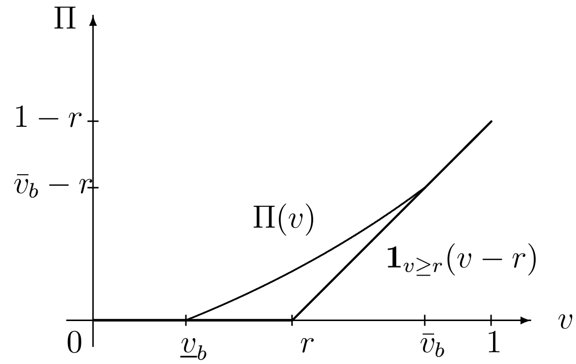

Figure 3: Client's profit in the optimal mechanism, $p = (b - F)/f$

that it is the lowest client type whose expected profit (before learning s, given that the premium is p_b) exceeds the cost of the consultant. Similarly, $\bar{v}_b$ is the highest client type whose expected profit (under the same conditions) exceeds the cost of the consultant.

Lemma 4 *For all $b \in [0,1]$, $\underline{v}_b < \bar{v}_b$ are well-defined by (15) and (16); moreover, $F(\underline{v}_b) \leq b \leq F(\bar{v}_b)$, and both $\underline{v}_b$ and $\bar{v}_b$ are continuous and weakly increasing in b with $\underline{v}_0 = 0$ and $\bar{v}_1 = 1$.*

The particular value of b that is used in the optimal mechanism is determined as follows:

$$\text{If } \int_{\underline{v}_1}^{1} \left[1 - G\left(r + \frac{1 - F(v)}{f(v)} - v\right)\right] dv \geq 1 - r \text{ then } b = 1; \text{ otherwise,} \tag{17}$$

$$\text{if } \int_{0}^{\bar{v}_0} \left[1 - G\left(r - \frac{F(v)}{f(v)} - v\right)\right] dv \leq \bar{v}_0 - r \text{ then } b = 0; \text{ otherwise} \tag{18}$$

$$\text{let } b \text{ any solution to } \bar{v}_b - r - \int_{\underline{v}_b}^{\bar{v}_b} \left[1 - G\left(r + \frac{b - F(v)}{f(v)} - v\right)\right] dv = 0. \tag{19}$$

The integrand in (19), $1 - G(r + p_b(v) - v)$, equals the probability that type v of the client that learns the value of s undertakes the project when the premium function is p_b. Intuitively, in (19), b is set so that among the client-types that the consultant contracts with, the "number" (Lebesgue-measure) of client-types that undertake the project under premium function p_b is the same as the "number" of client-types that would have undertook the project without learning s. The other two lines, (17)–(18), take care of corner solutions.

In order to see that there exists $b \in [0,1]$ satisfying (17)–(19), first note that the left-hand side of (19) is continuous in b. If (17) does not hold then this expression is positive at $b = 1$ (since $\bar{v}_1 = 1$ by Lemma 4). If (18) does not hold then the same expression is negative at $b = 0$ (since $\underline{v}_0 = 0$ by Lemma 4). Therefore, if neither (17) nor (18) holds then, by the Intermediate Value Theorem, there exists a $b \in (0,1)$, not necessarily unique, that satisfies (19).

Theorem 1 *Define b by (17)–(19), $\underline{v}_b$ and $\bar{v}_b$ by (15)–(16). In the consultant's optimal contract, for all $v \in [0,1]$, $a(v) = \mathbf{1}_{v \in [\underline{v}_b, \bar{v}_b]}$, $q(v) = \mathbf{1}_{v \geq r}$, and*

$$p(v) = \frac{b - F(v)}{f(v)}. \tag{20}$$

The client's profit function is, for all $v \in [0,1]$,

$$\Pi(v) = \Pi(0) + \int_0^v X(z)dz, \tag{21}$$

where X is defined by (5). Furthermore, if $\underline{v}_b > 0$ or $\bar{v}_b = 1$ then $\Pi(v) = 0$ for all $v \in [0, \underline{v}_b]$, while if $\underline{v}_b = 0$ or $\bar{v}_b < 1$ then $\Pi(v) = v - r$ for all $[\bar{v}_b, 1]$.

Proof. If $r \geq 1$ or $r \leq 0$ then the theorem is established by Lemmas 2 and 3. In the rest of the proof, suppose $r \in (0,1)$. We consider three cases: $b = 1$, $b = 0$, and $b \in (0,1)$.

If $b = 1$ then $\underline{v}_1 \leq r$ (because the left-hand side of the inequality in (17) does not exceed $1 - \underline{v}_1$), and $\bar{v}_1 = 1$ by Lemma 4. Let $v^* = 1$, hence $F_L \equiv F$. Note that (12) is equivalent to (15) at $b = 1$, therefore $\underline{v}_1 = \underline{v}$ as defined in (12). Condition (11) coincides with (17), therefore Lemma 2 applies: In the optimal mechanism, $a = \mathbf{1}_{v \in [\underline{v}_1, 1]}$, $q = \mathbf{1}_{v \geq r}$, and $p = (1 - F)/f$ as in (20). Moreover, Π satisfies (6), which is equivalent to (21), and $\Pi(0) = 0$, which implies $\Pi(v) = 0$ for all $v \in [0, \underline{v}_1]$. This completes the proof for the case $b = 1$.

If $b = 0$ then $\underline{v}_0 = 0$ by Lemma 4, and $\bar{v}_0 \geq r$ because the left-hand side of the inequality in (18) is non-negative. Let $v^* = 0$, hence $F_H \equiv F$. Note that (14) is equivalent to (16) at $b = 0$, therefore $\bar{v}_0 = \bar{v}$ as defined in (14). Condition (13) coincides with (18), therefore Lemma 3 applies: In the optimal mechanism, $a = \mathbf{1}_{v \in [0, \bar{v}_0]}$, $q = \mathbf{1}_{v \geq r}$, and $p = -F/f$ as in (20). Moreover, Π satisfies (6), which is equivalent to (21), and $\Pi(1) = 1 - r$, which implies $\Pi(v) = v - r$ for all $v \in [\bar{v}_0, 1]$. This completes the proof for the case $b = 0$.

Finally, suppose that $b \in (0,1)$. First we establish that the mechanism proposed in the theorem is incentive compatible. Note that $\underline{v}_b < r < \bar{v}_b$ because the integrand in (19) is always between zero and one, and hence the value of the integral is between zero and $\bar{v}_b - \underline{v}_b$. Also note that $p = (b - F)/f$ is weakly decreasing by the logconcavity of f. If we set a, q, and p according to the statement of the theorem, then

$$X(v) = \begin{cases} 0 & \text{for } v < \underline{v}_b, \\ 1 - G\left(r + \frac{b - F(b)}{f(b)} - v\right) & \text{for } v \in [\underline{v}_b, \bar{v}_b], \\ 1 & \text{for } v > \bar{v}_b. \end{cases}$$

By setting $\Pi(0) = 0$, (21) implies that $\Pi(v) = 0$ for all $v \in [0, \underline{v}_b]$. Combining (21) and (19), we get $\Pi(\bar{v}_b) = \bar{v}_b - r$, which implies $\Pi(v) = v - r$ for all $v \in [\bar{v}_b, 1]$, moreover,

$$\Pi(v) \geq \max\{0, v - r\} \quad \text{for all } v \in (\underline{v}_b, \bar{v}_b). \tag{22}$$

Therefore, the mechanism satisfies the sufficient conditions for incentive compatibility provided in Lemma 1. The only remaining question is whether it is optimal for the consultant.

Define $v^* = F^{-1}(b)$, that is, $F(v^*) = b$. Since $F(\underline{v}_b) \leq b < F(\bar{v}_b)$ by Lemma 4, we have $\underline{v}_b \leq v^* \leq \bar{v}_b$. Note that by (9)–(10) and $b = F(v^*)$,

$$\frac{b - F(v)}{f(v)} = \begin{cases} \dfrac{1 - F_L(v)}{f_L(v)} & \text{for all } v \in [0, v^*], \\ \dfrac{-F_H(v)}{f_H(v)} & \text{for all } v \in [v^*, 1]. \end{cases}$$

Therefore, (12) is equivalent to (15), and similarly, (14) is equivalent to (16), hence $\underline{v}_b = \underline{v}$ and $\bar{v}_b = \bar{v}$. By $\Pi(\underline{v}_b) = 0$, $\Pi(\bar{v}_b) = \bar{v}_b - r$, (21), and (19),

$$\begin{aligned} \Pi(v^*) = \int_{\underline{v}}^{v^*} \left[1 - G\left(r + \frac{1 - F_L(v)}{f_L(v)} - v\right)\right] dv \\ = \bar{v} - r - \int_{v^*}^{\bar{v}} \left[1 - G\left(r - \frac{F_H(v)}{f_H(v)} - v\right)\right] dv. \end{aligned} \tag{23}$$

By (22),

$$\Pi(v^*) \geq \max\{0, v^* - r\}. \tag{24}$$

But then, by (23) and (24), (11) holds, and Lemma 2 applies: by setting $p = (1 - F_L)/f_L = (b - F)/f$ we maximize the consultant's payoff conditional on $v \in [0, v^*]$. Similarly, by (23) and (24), (13) holds, and Lemma 3 applies: $p = -F_H/f_H = (b - F)/f$ is optimal conditional on $v \in [v^*, 1]$. Since the proposed mechanism maximizes the consultant's payoff conditional on $v \in [0, v^*]$ and conditional on $v \in [v^*, 1]$, it is unconditionally optimal. ■

Remark 1 *If $K = 0$ then from (15)–(16) it follows that the consultant discloses s to all types of the client.*

The up-front fee-schedule, c, can be determined from the definition of the client's profit, (3) with $v' = v$, combined with the equations characterizing the same in the optimal mechanism, $a = \mathbf{1}_{v \in [\underline{v}_b, \bar{v}_b]}$, $q = \mathbf{1}_{v \geq r}$, (20), and (21). Alternatively, we may proceed as follows. If $v \notin [\underline{v}_b, \bar{v}_b]$, then $c(v) = 0$ because the client does not get to observe s and $q(v) = \mathbf{1}_{v \geq r}$. If $v \in [\underline{v}_b, \bar{v}_b]$ then $a(v) = 1$, hence from a deviation to $v' \in [\underline{v}_b, \bar{v}_b]$ the client's profit is

$$\pi(v, v') = \int_{r+p(v')-v}^{\infty} [v + s - r - p(v')] \, dG(s) - c(v').$$

Local incentive compatibility, that is, the first-order condition of the maximization of $\pi(v, v')$ in v' (note that $p = (1-F)/f$ is differentiable) and $v' = v$ in the maximum yield, for all $v \in [\underline{v}_b, \bar{v}_b]$,

$$c'(v) = -p'(v) \left(1 - G(r + p(v) - v)\right). \tag{25}$$

This differential equation with a boundary condition for either $c(\underline{v}_b)$ or $c(\bar{v}_b)$ (whichever is more convenient) determines c. If $\Pi(\underline{v}_b) = 0$, which is the case if $\underline{v}_b > 0$ or $\bar{v}_b = 1$, then $c(\underline{v}_b) = w(\underline{v}_b)$, while if $\Pi(\bar{v}_b) = \bar{v}_b - r$, which is the case if $\bar{v}_b < 1$ or $\underline{v}_b = 0$, then $c(\bar{v}_b) = w(\bar{v}_b)$.

Since the resulting fee function, c, is non-decreasing, while p is non-increasing, a lower premium (paid in case the project is undertaken, chosen by better client types) requires the payment of a higher up-front fee, and vice versa. We quantify this relationship in numerical examples in the next subsection.

3.4 A Numerical Example

Assume that v is uniform on $[0,1]$, that is, $F(v) = v$ and $f(v) = 1$ on the domain. Let $r \in (0,1)$, and, for simplicity, set $K = 0$. This implies $a \equiv 1$ and that we can ignore q. Suppose that s is drawn from a uniform distribution on $[-\varepsilon, \varepsilon]$, with $\varepsilon > 0$, so

$$G(s) = \begin{cases} 0 & \text{if } s < -\varepsilon \\ (s+\varepsilon)/(2\varepsilon) & \text{if } -\varepsilon \le s \le \varepsilon \\ 1 & \text{if } s > \varepsilon \end{cases}. \tag{26}$$

The density, g, is zero outside $[-\varepsilon, \varepsilon]$, and equals $1/(2\varepsilon)$ on it.

According to Theorem 1, in the consultant's optimal contract,

$$p(v) = \frac{b - F(v)}{f(v)} = b - v,$$

where b can be calculated by solving

$$\int_0^1 G(r + b - 2v)dv = r, \tag{27}$$

and "snapping" the resulting value of b to 0, or 1, whenever it falls below 0, or above 1, respectively. In order to find the value of b, first define $\underline{\alpha} = (r + b - \varepsilon)/2$ and $\bar{\alpha} = (r + b + \varepsilon)/2$, the two thresholds of v where $r + b - 2v$ equals $-\varepsilon$ and ε, respectively. Since $G(r + b - 2v)$ is monotone decreasing in v, we have

$$G(r + b - 2v) = \begin{cases} 1 & \text{if } v \in [0, \underline{\alpha}) \\ (r + b + \varepsilon - 2v)/(2\varepsilon) & \text{if } v \in [\underline{\alpha}, \bar{\alpha}] \\ 0 & \text{if } v \in (\bar{\alpha}, 1] \end{cases}.$$

Case 1: $r < \varepsilon/2$. Suppose $r + b < \varepsilon$. Then $\underline{\alpha} < 0 < \bar{\alpha} < 1$, and

$$\int_0^1 G(r + b - 2v)dv = \frac{G(0)\bar{\alpha}}{2} = \frac{(r + b + \varepsilon)^2}{8\varepsilon}.$$

Equation (27) is satisfied by $b = \sqrt{8r\varepsilon} - \varepsilon - r$. (Note that if $r < \varepsilon/2$ then $b < \sqrt{4\varepsilon^2} - \varepsilon - r = \varepsilon - r$, so indeed, $r + b < \varepsilon$, as assumed.) It can be checked that $b \ge 0$ if and only if $r \ge (\sqrt{2} - 1)^2\varepsilon$.

Case 2: $r \in [\varepsilon/2, 1-\varepsilon/2]$. Suppose $r+b \in [\varepsilon, 2-\varepsilon]$. Then $0 \le \underline{\alpha} < \bar{\alpha} \le 1$, and

$$\int_0^1 G(r+b-2v)dv = \underline{\alpha} + \frac{\varepsilon}{2} = \frac{r+b}{2}.$$

Equation (27) is satisfied by $b = r$, and indeed, $r + b \in [\varepsilon, 2-\varepsilon]$ as assumed.

Case 3: $r > 1-\varepsilon/2$. Suppose $r+b > 2-\varepsilon$, $r > \varepsilon/2$. Then $0 < \underline{\alpha} < 1 < \bar{\alpha}$, and

$$\int_0^1 G(r+b-2v)dv = 1 - \frac{(1-\underline{\alpha})(1-G(1))}{2} = 1 - \frac{(2+\varepsilon-r-b)^2}{8\varepsilon}.$$

Equation (27) is satisfied by $b = 2+\varepsilon-r-\sqrt{8(1-r)\varepsilon}$. (Note that by $r > 1-\varepsilon/2$, i.e., $(1-r) < \varepsilon/2$, we have $b > 2+\varepsilon-r-\sqrt{4\varepsilon^2} = 2-\varepsilon-r$.) It can be checked that $b \le 1$ if and only if $r \le 1-(\sqrt{2}-1)^2\varepsilon$.

To summarize: if $r < (\sqrt{2}-1)^2\varepsilon$ then $b = 0$; if $r > 1-(\sqrt{2}-1)^2\varepsilon$ then $b = 1$; otherwise

$$b = \begin{cases} \sqrt{8r\varepsilon}-\varepsilon-r & \text{if } r \in \left[(\sqrt{2}-1)^2\varepsilon, \varepsilon/2\right] \\ r & \text{if } r \in [\varepsilon/2, 1-\varepsilon/2] \\ 2+\varepsilon-r-\sqrt{8(1-r)\varepsilon} & \text{if } r \in \left[1-\varepsilon/2, 1-(\sqrt{2}-1)^2\varepsilon\right] \end{cases}.$$

From now on, let us focus on the case where r is inside the unit interval and ε is small relative to r, specifically $r \in [\varepsilon, 1-\varepsilon]$, so in the optimal contract of our example $p(v) = r - v$. Then, we can easily determine the upfront fee in the optimal contract, $c(v)$. For $v = 0$, we need $\Pi(0) = 0 \Leftrightarrow c(0) = \int_r^\varepsilon (s-b-r)g(s)ds$. Since $r \ge \varepsilon$, the integral is empty, so $c(0) = 0$. For $v > 0$, we use (25), which can be rewritten as $c'(v) = 1 - G(2r-2v)$. Straightforward integration of this equation, with G defined in (26), yields,

$$c(v) = \begin{cases} 0 & \text{if } v < r-\varepsilon/2 \\ (v-r+\varepsilon/2)^2/(2\varepsilon) & \text{if } r-\varepsilon/2 \le v \le r+\varepsilon/2 \\ v-r & \text{if } r+\varepsilon/2 < v \end{cases}.$$

Note that client types $v < r-\varepsilon/2$ or $v > r+\varepsilon/2$ are essentially not served by the consultant, but all other types are.

Our example with uniformly distributed v and small, uniformly distributed ε (such that $r \in [\varepsilon, 1-\varepsilon]$) illustrates the interesting relationship between the upfront fee, c, and the premium, p, in the optimal menu offered by the consultant. The upfront fee that client type v is supposed to choose is strictly increasing in v. All client types that may benefit from knowing $s = V - v$ pay their fee and learn the realization of the shock. In exchange, the client agrees to paying a premium, which is decreasing in his type, in case he undertakes the project. Furthermore, this premium is positive for $v < r$ and negative for $v > r$. Therefore, in this particular example with $r \in [\varepsilon, 1 - \varepsilon]$, when the undecided client undertakes the project, he pays the consultant "extra" whenever learning the value of the additional signal changes his mind, and conversely, the consultant pays him back whenever it does not.[12] This happens in the optimal mechanism despite the fact that the consultant cannot observe directly whether her release of information actually changed the client's mind.

4 Main Result: Comparison with a Benchmark

In this section we derive the optimal contract when the consultant can in fact verify the realization of the signal $s \equiv V - v$ as she releases it (i.e., in case she decides to reveal it to the client).[13] We show that her payoff is the *same* as in the optimal contract of the previous section, where she could not observe s but could only contract on the client's action. This result, Theorem 2, is the main result of the paper.

When the consultant can verify the value of s as it is released by her, and can contract on the client's action as well, we can represent a contract by a truthful revelation mechanism consisting of four functions, $d^* : [0,1] \to \mathbb{R}$, $a^*, q^* : [0,1] \to [0,1]$, and $x^* : [0,1] \times \mathbb{R} \to [0,1]$. In this mechanism, first, the client reports his type, v, and gives an unconditional transfer $d^*(v)$ to the consultant. The consultant checks the value of s (incurring cost K) with probability $a^*(v)$, and instructs the client to undertake the project with probability either $q^*(v)$, in case she did not learn s, or $x^*(v, s)$, in case she did, respectively.[14] Incentive compatibility of a mechanism

[12] We made the same observation in the general model for $r > 1$, and $r < 0$, when we found that $p = -F/f < 0$, and $p = (1 - F)/f > 0$, in these two cases respectively.

[13] The benchmark case is somewhat similar (at least in spirit, if not in the details) to the model of Dana and Spier (1993), where the attorney becomes more informed than the client after the contract is signed.

[14] We will use "starred" symbols throughout this section to avoid confusion with notation used

$\{a^*, d^*, x^*, q^*\}$ means that no client type v has an incentive to report $v' \neq v$, for all $v, v' \in [0,1]$.

Define the client's overall probability of undertaking the project with truthfully reported type v as

$$X^*(v) = a^*(v) \int x^*(v,s)dG(s) + (1 - a^*(v))q^*(v). \tag{28}$$

Denote the client's deviation payoff when he has type v and reports v' by

$$\pi^*(v,v') = a^*(v') \int x^*(v',s)(v+s-r)dG(s)+(1 - a^*(v'))\, q^*(v')(v-r)-d^*(v'). \tag{29}$$

Finally, let $\Pi^*(v) = \pi^*(v,v)$ denote the client's indirect profit function.

Incentive compatibility with participation requires $\Pi^*(v) \geq \max\{\pi^*(v,v'), v - r, 0\}$ for all $v, v' \in [0,1]$. We have the following counterpart to Lemma 1 for the case when the consultant can verify the realization of s.

Lemma 5 *Assume the consultant can observe and contract on $s \equiv V - v$. A mechanism $\{a^*, d^*, x^*, q^*\}$ is incentive compatible if and only if X^* is weakly increasing and for all $v \in [0,1]$,*

$$\Pi^*(v) = \Pi^*(0) + \int_0^v X^*(z)dz, \tag{30}$$

and also

$$\Pi^*(v) \geq \max\{0, v - r\}. \tag{31}$$

This lemma allows us to characterize the consultant's expected payoff in any incentive compatible mechanism. First, the client's ex-ante expected profit in an incentive compatible mechanism can be written as

$$\begin{aligned}
\int_0^1 \Pi^*(v)dF(v) &= \Pi^*(0) + \int_0^1 \int_0^v X^*(z)dzdF(v) \\
&= \Pi^*(0) + \int_0^1 \int_z^1 X^*(z)dF(v)dz \\
&= \Pi^*(0) + \int_0^1 (1 - F(z))X^*(z)dz.
\end{aligned}$$

in the previous section.

The consultant's expected payoff, which is the difference between the social surplus and the client's profit, equals

$$W^* = \int_0^1 \left\{ a^*(v) \left[\int_{-\infty}^{\infty} (v+s-r) x^*(v,s) dG(s) - K \right] + (1-a^*(v))\, q(v)(v-r) \right\} dF(v)$$
$$- \int_0^1 (1-F(v))\, X^*(v) dF(v) - \Pi^*(0),$$

which can be rewritten, using (28), as

$$\int_0^1 \left\{ a^*(v) \left[\int_{-\infty}^{\infty} \left(v+s-r-\frac{1-F(v)}{f(v)} \right) x^*(v,s) dG(s) - K \right] + \right.$$
$$\left. (1-a^*(v)) \left(v-r-\frac{1-F(v)}{f(v)} \right) q^*(v) \right\} dF(v) - \Pi^*(0). \quad (32)$$

The final step before characterizing the optimal mechanism in the benchmark is to prove

Lemma 6 *Assume the consultant can observe and contract on s. In the optimal mechanism, for all $v \in [0,1]$ with $a(v) > 0$, there exists $s_v \in (-\infty, \infty)$ such that $x^*(v,s) = \mathbf{1}_{s \geq s_v}$ almost everywhere.*

In words, for a given type-announcement of the client, the consultant will ask him to carry out the project with a sufficiently high realization of s and not otherwise. The intuition behind this result is that once the client is induced to (truthfully) report v, the consultant has no reason to ban the client from undertaking a high-value (high-s) project while asking him to undertake a less valuable (low-s) one, because she can appropriate the efficiency gains due to directly knowing the realization of s.

The interesting consequence of the last lemma is the following. For all $v \in [0,1]$ with $a(v) > 0$, there exists $p^*(v)$ such that

$$x^*(v,s) = \begin{cases} 1 & \text{if } \; s \geq r + p^*(v) - v, \\ 0 & \text{otherwise.} \end{cases} \quad (33)$$

That is, in the optimal mechanism the rule specifying whether or not the client should undertake the project can be implemented via a premium function—a transfer that takes place whenever the client undertakes the project and is contingent only on the client's type (and not the realization of s).

Now we are ready to prove our main result. We show that the consultant cannot be made better off (relative to her optimal contract seen in Section 3) even if she finds out the value of s as she reveals it. In other words, compared to the optimal contract with unobservable s (see Theorem 1), the consultant cannot obtain a higher expected payoff even if the signal that she can release becomes observable.

Theorem 2 *Assume that the consultant can observe and contract on s. In the optimal mechanism, the consultant's expected payoff is the same as in the optimal mechanism where the realization of s is only observable to the client, but the consultant controls the disclosure of s and the client's action is contractible.*

Proof. Suppose that mechanism $\{a^*, d^*, x^*, q^*\}$ is incentive compatible and individually rational under observable and contractible s. The consultant's expected payoff, (32), can be rewritten using (33) as

$$W^* = \int_0^1 \left\{ a^*(v) \left[\int_{r+p^*(v)-v}^{\infty} \left(v + s - r - \frac{1-F(v)}{f(v)} \right) dG(s) - K \right] + \right. \\ \left. (1 - a^*(v)) \left(v - r - \frac{1-F(v)}{f(v)} \right) q^*(v) \right\} dF(v) - \Pi^*(0).$$

This expression is equivalent to W in (8), which is the consultant's payoff in any mechanism that satisfies the necessary conditions of incentive compatibility under *unobservable* s given in Lemma 1. From Theorem 1, we know that this is maximized in a^*, p^*, q^* and $\Pi^*(0)$ by setting $a^* = 1_{v \in [\underline{v}_b, \bar{v}_b]}$, $p^* = (b - F)/f$, $q^* = \mathbf{1}_{v \geq r}$, and $\Pi^*(0)$ where b satisfies (17)–(19), and $\underline{v}_b$, $\bar{v}_b$ are given by (15)–(16). Set $x^*(v, s)$ according to (33), which implies by (28)

$$X^*(v) = a^*(v) \left(1 - G(r + p(v) - v)\right) + (1 - a^*(v))q^*(v),$$

and let

$$d^*(v) = c(v) + (1 - G(r + p(v) - v))p(v). \tag{34}$$

The necessary conditions of incentive compatibility of any mechanism under *observable and contractible* s (see Lemma 5) are the same as those given in Lemma 1 under *unobservable* s. Therefore, the mechanism $\{a^*, d^*, p^*, q^*\}$ maximizes the consultant's objective among all incentive compatible mechanisms under observable and contractible s, and yields the same payoff for the consultant as the optimal mechanism of Theorem 1.

The only remaining question is whether the mechanism defined above is incentive compatible under observable and contractible s. This is clear, however, as the conditions given in Lemma 5, which this mechanism satisfies, are both necessary and *sufficient* for incentive compatibility. This completes the proof. ■

5 Conclusions

We analyzed a model of the advisor–client relationship where the role of the advisor is that she can disclose "clues" to the client that only he (the client) can understand. These clues, or signals, refine the client's original private estimate regarding the profitability of the client's project. We assumed that the client's action (whether or not he undertakes the project) is contractible, therefore the consultant can offer a deal where the client pays her differently depending on whether he undertakes the project upon evaluating her advice. We derived the consultant's optimal contract, which can be thought of as a menu of such transfer pairs. Some items on the menu may require the client to pay more if he undertakes the project, other items may require higher payments if he does not. The consultant discloses the additional signals only if the client agrees to one of the items.

In general, in the optimal contract, only clients with value estimates between certain thresholds take up the consultant's offer. Among those that do, clients with higher estimates choose transfer pairs where the signed difference between what they have to pay upon undertaking the project and upon not undertaking it are smaller. In interesting special cases of the model the optimal contract can be interpreted as one where the client pays the consultant more whenever her advice has made him change his mind whether to undertake the project.

The most interesting finding, we believe, is that the consultant's payoff in the optimal contract is the same as if she could in fact "decipher her own clues", that is, as if she knew how the client's value estimate changed by her advice. Even if the consultant is ignorant regarding how her advice affects her client, as long as she has the power to design their contract and can condition it on the decision of the client, she can do just as well as if she understood the precise effect of her advice.

Appendix 1: Omitted Proofs

Proof of Lemma 1. (Necessity.) Assume $\{a, c, p, q\}$ is incentive compatible. In the rest of the proof, consider arbitrary $v, v' \in [0,1]$ such that $v < v'$. Note that (7) is the participation constraint, which is necessary for all v.

Rewrite the definition of $\pi(v, v')$ as follows:

$$\begin{aligned}
\pi(v,v') &= a(v')\int_{r+p(v')-v'}^{\infty}[v'+s-r-p(v')]dG(s) \\
&\quad +(1-a(v'))q(v')\,(v'-r)-c(v') \\
&\quad +(v-v')\,[a(v')\,(1-G(r+p(v')-v'))+(1-a(v'))q(v')] \\
&\quad -a(v')\int_{r+p(v')-v'}^{r+p(v')-v}(v+s-r-p(v'))dG(s) \\
&\geq \Pi(v')-(v'-v)X(v').
\end{aligned}$$

The first two lines of the above expression for $\pi(v, v')$ give exactly $\Pi(v')$, the third line equals $(v - v')X(v')$, and the fourth line is nonnegative, hence the inequality on the last line holds.

Similarly,

$$\begin{aligned}
\pi(v',v) &= a(v)\int_{r+p(v)-v}^{\infty}(v+s-r-p(v))dG(s) \\
&\quad +(1-a(v))q(v)(v-r)-c(v) \\
&\quad +(v'-v)\,[a(v)(1-G(r+p(v)-v))+(1-a(v))\,q(v)] \\
&\quad +a(v)\int_{r+p(v)-v'}^{r+p(v)-v}(v'+s-r-p(v))dG(s) \\
&\geq \Pi(v)+(v'-v)X(v).
\end{aligned}$$

Incentive compatibility requires $\Pi(v) \geq \pi(v, v')$ and $\Pi(v') \geq \pi(v', v)$, therefore

$$(v'-v)X(v) \leq \Pi(v') - \Pi(v) < (v'-v)X(v').$$

Cross-dividing by $(v' - v) > 0$, we have for all $v, v' \in [0, 1]$ and $v < v'$,

$$X(v) \leq \frac{\Pi(v')-\Pi(v)}{v'-v} \leq X(v'). \tag{35}$$

From this, X is weakly increasing, and Π is differentiable with $d\Pi(v)/dv = X(v)$ almost everywhere. Since $X(v) \in [0,1]$ for all v, Π is Lipschitz-continuous by (35), therefore Π is integrable, and (6) follows.

(Sufficiency.) Assume that (6)–(7) hold, $a = \mathbf{1}_{v\in[\underline{v},\bar{v}]}$ for some $\underline{v} < r < \bar{v}$, $q = \mathbf{1}_{v\geq r}$, and p is weakly decreasing.

Note that $\pi(v,v') = \max\{0, v-r\}$ for all $v' \notin [\underline{v},\bar{v}]$, therefore, by (7), no type $v \in [0,1]$ has an incentive to deviate to any $v' \notin [\underline{v},\bar{v}]$.

In the rest of the proof, let v, $v' \in [0,1]$ such that $v < v'$. If $v' \in [\underline{v},\bar{v}]$ then by (3)

$$\pi(v,v') = \int_{r+p(v')-v}^{\infty} (v+s-r-p(v'))\,dG(s) - c(v').$$

Rewrite $\pi(v,v')$ as follows:

$$\begin{aligned}
\pi(v,v') &= \int_{r+p(v')-v'}^{\infty} (v'+s-r-p(v'))dG(s) - c(v') \\
&\quad - \int \mathbf{1}_{v'+s-r-p(v')\geq 0}(v'+s-r-p(v'))dG(s) \\
&\quad + \int \mathbf{1}_{v'+s+v-v'-r-p(v')\geq 0}(v'+s+v-v'-r-p(v'))dG(s) \\
&= \Pi(v') - \int\int_{s+v-v'}^{s} \mathbf{1}_{v'+\sigma-r-p(v')\geq 0}d\sigma dG(s),
\end{aligned}$$

where, on the last line, we used the identity (true for all v' and $s' \leq s$)

$$\mathbf{1}_{v'+s-r-p(v')\geq 0}(v'+s-r-p(v')) - \mathbf{1}_{v'+s'-r-p(v')\geq 0}(v'+s'-r-p(v')) = \int_{s'}^{s} \mathbf{1}_{v'+\sigma-r-p(v')\geq 0}d\sigma.$$

However,

$$\begin{aligned}
\int\int_{s+v-v'}^{s} \mathbf{1}_{v'+\sigma-r-p(v')\geq 0}d\sigma dG(s) &= \int\int_{v-v'}^{0} \mathbf{1}_{v'+s+x-r-p(v')\geq 0}dx dG(s) \\
&= \int_{v-v'}^{0}\int \mathbf{1}_{v'+s+x-r-p(v')\geq 0}dG(s)dx \\
&= \int_{v-v'}^{0} (1 - G(r+p(v')-v'-x))\,dx,
\end{aligned}$$

therefore

$$\pi(v,v') = \Pi(v') - \int_{v-v'}^{0} (1 - G(r + p(v') - v' - x))\, dx.$$

On the other hand, by (6),

$$\begin{aligned}
\Pi(v) &= \Pi(v') - \int_{v}^{v'} (1 - G(r + p(\nu) - \nu))\, d\nu \\
&= \Pi(v') - \int_{v-v'}^{0} (1 - G(r + p(v' + x) - v' - x))\, dx \\
&\geq \Pi(v') - \int_{v-v'}^{0} (1 - G(r + p(v') - v' - x))\, dx \\
&= \pi(v,v'),
\end{aligned}$$

where the inequality holds because $p(v'+x) \geq p(v')$ for all $x \leq 0$ by the monotonicity of p. Therefore, client type v has no incentive to imitate $v' > v$ such that $v' \in [\underline{v}, \bar{v}]$.

Similarly, for $v \in [\underline{v}, \bar{v}]$, $v \in [0,1]$, and $v < v'$ we can rewrite $\pi(v', v)$ as

$$\begin{aligned}
\pi(v',v) &= \int_{r+p(v)-v}^{\infty} (v + s - r - p(v)) dG(s) - c(v) \\
&\quad - \int \mathbf{1}_{v+s-r-p(v)\geq 0}(v + s - r - p(v)) dG(s) \\
&\quad + \int \mathbf{1}_{v+s+v'-v-r-p(v)\geq 0}(v + s + v' - v - r - p(v)) dG(s) \\
&= \Pi(v) + \int \int_{s}^{s+v'-v} \mathbf{1}_{v+\sigma-r-p(v)\geq 0} d\sigma dG(s).
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\int \int_{s}^{s+v'-v} \mathbf{1}_{v+\sigma-r-p(v)\geq 0} d\sigma dG(s) &= \int \int_{0}^{v'-v} \mathbf{1}_{v+s+x-r-p(v)\geq 0} dx dG(s) \\
&= \int_{0}^{v'-v} \int \mathbf{1}_{v+s+x-r-p(v)\geq 0} dG(s) dx \\
&= \int_{0}^{v'-v} (1 - G(r + p(v) - v - x))\, dx,
\end{aligned}$$

therefore

$$\pi(v',v) = \Pi(v) + \int_{0}^{v'-v} (1 - G(r + p(v) - v - x))\, dx.$$

On the other hand, by (6),

$$\begin{aligned}\Pi(v') &= \Pi(v) + \int_v^{v'} (1 - G(r + p(\nu) - \nu))\, d\nu \\ &= \Pi(v) + \int_0^{v'-v} (1 - G(r + p(v + x) - v - x))\, dx \\ &\geq \Pi(v) + \int_{v-v'}^{0} (1 - G(r + p(v) - v - x))\, dx \\ &= \pi(v', v),\end{aligned}$$

where the inequality holds because $p(v + x) \leq p(v)$ for all $x \geq 0$ by monotonicity of p. Therefore, a client type v' has no incentive to imitate $v < v'$ such that $v \in [\underline{v}, \bar{v}]$. We conclude that the mechanism is indeed incentive compatible. ■

Proof of Lemma 2. Steps identical to those that established formula (8) in the text yield the following characterization of the consultant's expected payoff, conditional on $v \in L$, in any incentive compatible mechanism:

$$W_L = \int_0^{v^*} \left(a(v) \left[\int_{r+p(v)-v}^{\infty} \left(v + s - r - \frac{1 - F_L(v)}{f_L(v)} \right) dG(s) - K \right] \right. \\ \left. + (1 - a(v))\, q(v) \left(v - r - \frac{1 - F_L(v)}{f_L(v)} \right) \right) dF_L(v) - \Pi(0). \quad (36)$$

We will now choose a, p, q and $\Pi(0)$ to maximize (36) pointwise (for all v), and then check whether the sufficient conditions of incentive compatibility in Lemma 1 hold. Set $\Pi(0) = 0$ and hence minimize the last term. For a given v and $a(v)$, the integrand in (36) is maximized by choosing $p(v) = (1 - F_L(v))/f_L(v)$ and $q(v) = \mathbf{1}_{v \geq r+p(v)}$. Since f_L is logconcave, the inverse hazard rate of the distribution, p, is weakly decreasing, hence $v - r - p(v)$ is strictly increasing in v. Call η the unique threshold where q changes from zero to one, that is, $\eta = r + p(\eta)$. In order to choose $a(v)$ optimally, set $a(v) = 1$ if the bracketed expression multiplying $a(v)$ exceeds the expression multiplying $(1 - a(v))$, and set $a(v) = 0$ otherwise. That is $a(v) = 1$ if and only if

$$\int_{r+p(v)-v}^{\infty} [v + s - r - p(v)]\, dG(s) - K \geq [v - r - p(v)]\, \mathbf{1}_{v \geq r+p(v)}, \quad (37)$$

where $p(v) = (1 - F_L(v)) / f_L(v)$. This inequality holds for $v \in [\underline{v}, v^*]$ with $\underline{v} < \eta$. To see this, note that at $v = v^*$, inequality (37) holds because $p(v^*) = 0$ and $w(v^*)$, defined by (2) as $\int_{r-v^*}^{\infty} (v^* + s - r)\, dG(s) - (v^* - r)\mathbf{1}_{v^* \geq r}$, exceeds K by assumption. If $r < v^*$ then the inequality continues to hold for all $v \in [\eta, v^*]$ as well because p, the inverse hazard rate of F_L, is weakly decreasing by the logconcavity of f_L, and as v decreases from v^* to η, the right-hand side of (37) falls faster than the left-hand side does.[15] As we decrease v further, the opposite is true, so we either find $\underline{v} > 0$ where (37) holds as equality with the right-hand side being zero, or $\underline{v} = 0$. Therefore, it is optimal to set $\bar{v} = v^*$ and $\underline{v}$ according to (12).

Note that (11) implies $\underline{v} < r$. Also note that the value of $q(v)$ only matters if the realization of s is not disclosed to client type v, that is, for $v < \underline{v}$. Since $\underline{v} < r$ and $q = \mathbf{1}_{v \geq r + p(v)}$, no type v below $\underline{v}$ is asked to undertake the project, and we may as well set $q = \mathbf{1}_{v \geq r}$.

The conditions on a, p, and q imposed by the sufficient conditions of Lemma 4 clearly hold. The only remaining sufficient condition of overall incentive compatibility is that the client's profit, defined by (6) and rewritten using $\Pi(\underline{v}) = 0$ as

$$\Pi(v) = \int_{\underline{v}}^{v} \left[1 - G\left(r + \frac{1 - F_L(z)}{f_L(z)} - z \right) \right] dz,$$

has to meet or exceed the value of his outside option, $\max\{0, v - r\}$. Since $X(z) \in [0, 1]$ for all $z \in [0, 1]$, it is sufficient to check whether $\Pi(v^*) \geq \max\{0, v^* - r\}$. But this condition is satisfied by (11) in the hypothesis of the lemma. ■

Proof of Lemma 3. The client's ex-ante expected profit conditional on $v \in H$ is

$$\begin{aligned}
\int_{v^*}^{1} \Pi(v) dF_H(v) &= \Pi(1) - \int_{v^*}^{1} \int_{v}^{1} X(z) dz dF_H(v) \\
&= \Pi(1) - \int_{v^*}^{1} \int_{v^*}^{z} X(z) dF_H(v) dz \\
&= \Pi(1) - \int_{v^*}^{1} \frac{F_H(z)}{f_H(z)} X(z) dF_H(z).
\end{aligned}$$

The consultant's expected payoff conditional on $v \in H$ is the difference between the

[15] When p (which is weakly decreasing) is differentiable, the derivative of the left-hand side of (37) with respect to v is $[1 - p'(v)]\,[1 - G(r + p(v) - v)]$ while that of the right-hand side is $[1 - p'(v)]$.

social surplus and the client's profit conditional on $v \in H$,

$$W_H = \int_{v^*}^{1} \left(a(v) \left[\int_{r+p(v)-v}^{\infty} (v + s - r)\, dG(s) - K \right] + (1 - a(v))\, q(v)(v - r) \right) dF_H(v) + \int_{v^*}^{1} \frac{F_H(v)}{f_H(v)} X(v) dF_H(v) - \Pi(1),$$

which can be rewritten, using (5) for $X(v)$, as

$$W_H = \int_{v^*}^{1} \left(a(v) \left[\int_{r+p(v)-v}^{\infty} \left(v + s - r + \frac{F_H(v)}{f_H(v)} \right) dG(s) - K \right] + (1 - a(v)) q(v) \left(v - r + \frac{F_H(v)}{f_H(v)} \right) \right) dF_H(v) - \Pi(1). \quad (38)$$

This expression can be used to study the optimal solution conditional on $v \in H$. We will choose a, p, q and $\Pi(1)$ to maximize (38) pointwise, and then check incentive compatibility of the resulting mechanism. First, set the profit of the highest client type equal to the value of this type's outside option, $\Pi(1) = 1 - r$ (recall that $r < 1$). For any given v and $a(v)$, the integrand in (38) is maximized by choosing $p(v) = -F_H(v)/f_H(v)$ and $q(v) = \mathbf{1}_{v \geq r+p(v)}$. Note that by the logconcavity of f_H, the ratio f_H/F_H is weakly decreasing, therefore p is weakly decreasing. Since $v - r - p(v)$ is strictly increasing in v, there exists a unique threshold, η', where q switches from zero to one, that is, $\eta' = r + p(\eta')$. In order to set $a(v)$ optimally, let $a(v) = 1$ if the bracketed term multiplying $a(v)$ exceeds the term multiplying $(1 - a(v))$, and let $a(v) = 0$ otherwise. That is, $a(v) = 1$ whenever (37) holds with $p = -F_H/f_H$, and $a(v) = 0$ otherwise. The inequality (37) holds at $v = v^*$ (by $w(v^*) - K \geq 0$), and for all $v \in [v^*, \eta']$ as well because for $v \leq \eta'$, the left-hand side of the inequality is increasing while the right-hand remains zero. For $v > \eta'$, the right-hand side of (37) may eventually rise faster than the left-hand side does, therefore there exists a $\bar{v} \leq 1$ such that (37) holds on the domain $v \in [v^*, 1]$ if and only if $v \in [v^*, \bar{v}]$. Therefore, set $a(v) = 1$ for $v \in [v^*, \bar{v}]$, where

$$\bar{v} = \max \left\{ v \in [v^*, 1] : \textstyle\int_{r+p(v)-v}^{\infty} [v + s - r - p(v)]\, dG(s) - K \geq v - r - p(v) \right\}.$$

This definition is equivalent to (14). By (13), $\bar{v} > r$. All types above $\bar{v}$ are asked to undertake the project in this mechanism, and for $v < \bar{v}$ we can set $q(v)$ freely, hence we may as well let $q = \mathbf{1}_{v \geq r}$.

The only condition of incentive compatibility that remains to be checked is that the client's profit in the mechanism, given by $\Pi(v) = \bar{v} - r - \int_v^{\bar{v}} X(z)dz$ or

$$\Pi(v) = \bar{v} - r - \int_v^{\bar{v}} \left[1 - G\left(r - \frac{F_H(z)}{f_H(z)} - z\right)\right] dz,$$

must not fall below the value of the client's outside option, $\max\{0, v - r\}$. Since $X(z) \in [0, 1]$ for all $z \in [0, 1]$, it is sufficient to check whether this condition holds at $v = v^*$. By assumption (13), it does. ■

Proof of Lemma 4. To see existence, let $v^* = F^{-1}(b)$, that is, $F(v^*) = b$. The left-hand side of the first inequality in (15) is continuous and strictly increasing in $\underline{v}_b$. At $\underline{v}_b = v^*$, the expression becomes $\int_{r-v^*}^{\infty} (v^* + s - r)\, dG(s)$, which equals $w(v^*) + \mathbf{1}_{v^*>r}(v^* - r)$ by (2). However, $w(v^*) > K$ by assumption, so indeed there exists $\underline{v}_b$ such that (15) holds. Similarly, the left-hand side of the inequality in (16) is continuous and strictly decreasing in $\bar{v}_b$. At $\bar{v}_b = v^*$, it becomes $\int_{-\infty}^{r-v^*} (r - v^* - s)\, dG(s) = \int_{r-v^*}^{\infty} (v^* + s - r)dG(s) + r - v^*$, which equals $w(v^*) + \mathbf{1}_{r>v^*}(r - v^*)$ by (2), and $w(v^*) > K$ by assumption. From this argument it is also clear that $F(\underline{v}_b) \leq b \leq F(\bar{v}_b)$, which then implies $\underline{v}_0 = 0$ and $\bar{v}_1 = 1$.

It is easy to see that $\underline{v}_b$ and $\bar{v}_b$ are continuous in b (no matter what the distribution of s is). Since the integral in (15) is strictly decreasing in b, and the integral in (16) is strictly increasing in b, both $\underline{v}_b$ and $\bar{v}_b$ are weakly increasing in b. ■

Proof of Lemma 5. Suppose first that the mechanism is incentive compatible, and $v, v' \in [0, 1]$, $v < v'$. Then, by the definition of π^*,

$$\pi^*(v, v') = \Pi^*(v') - (v' - v)X^*(v') \leq \Pi^*(v)$$

and

$$\pi^*(v', v) = \Pi^*(v) + (v' - v)X^*(v) \leq \Pi^*(v'),$$

where the inequalities follow from incentive compatibility. Therefore,

$$(v' - v)X^*(v) \leq \Pi^*(v') - \Pi^*(v) \leq (v' - v)X^*(v'),$$

and hence X^* is weakly increasing on $[0, 1]$, moreover, $d\Pi^*/dv = X^*$. Since Π^*

is continuous everywhere (which follows from the continuity of $\pi^*(v, v')$ in v and incentive compatibility), we get (30). Since the other conditions were established in the text, this concludes the proof of necessity.

Now assume that (30)–(31) hold and X^* is weakly increasing. If $v, v' \in [0, 1]$ and $v' < v$ then

$$
\begin{aligned}
\Pi^*(v) &= \Pi^*(v') + \int_{v'}^{v} X^*(z)dz \\
&\geq \Pi^*(v') + \int_{v'}^{v} X^*(v')dz \\
&= \Pi^*(v') + (v - v')X^*(v') = \pi^*(v, v'),
\end{aligned}
$$

where the inequality follows because X is weakly increasing. If $v' > v$ then similarly,

$$
\begin{aligned}
\Pi^*(v) &= \Pi^*(v') - \int_{v}^{v'} X^*(z)dz \\
&\geq \Pi^*(v') - \int_{v}^{v'} X^*(v')dz \\
&= \Pi^*(v') - (v' - v)X^*(v') = \pi^*(v, v').
\end{aligned}
$$

Therefore, the mechanism is indeed incentive compatible. ■

Proof of Lemma 6. The idea of the proof is that if $x^*(v, s)$ does not equal a step-function, $\mathbf{1}_{s \geq s_v}$ for some s_v given v, then the consultant can gain (weakly) by removing positive values of $x^*(v, s)$ at low realizations of s and reallocating them to higher realizations of s with $x^*(v, s) < 1$.

Let μ_G denote the measure generated by G on $\mathbb{R}$. Notice that if the claim of the lemma is not true then there is a type $v \in [0, 1]$ with $a^*(v) > 0$ and there exists a subset of $\mathbb{R}$, A, such that $\int_A x^*(v, s)d\mu_G \in (0, 1)$. Moreover, there exist subsets of A, call them B and C, such that $B \leq C$, and

$$
\int_B x^*(v, s)d\mu_G = \int_C x^*(v, s)d\mu_G \in (0, 1).
$$

We now show that the consultant can do weakly better by defining a new allocation rule $\hat{x}$ as follows.

$$\begin{aligned}\hat{x}^*(v,s) &= 0 \text{ if } s \in B,\\ \hat{x}^*(v,s) &= 1 \text{ if } s \in C, \text{ and}\\ \hat{x}^*(v',s) &= x^*(v',s) \text{ if } v' \neq v \text{ or } s' \notin B \cup C.\end{aligned}$$

Also, define $\hat{X}^*$ according to (28) and $\hat{\Pi}^*$ according to (30) so that $\hat{\Pi}^*(0) = \Pi^*(0)$. Note that since $\hat{X}^*(v) = X^*(v)$ for all v, the indirect profit function did not change either, $\hat{\Pi}^* = \Pi^*$. Hence the mechanism $\{a^*, \hat{x}^*, q^*, d^*\}$ is incentive compatible. However, in the consultant's objective function, the term

$$\int \left[\left(v + s - r - \frac{1-F(v)}{f(v)}\right) x^*(v,s) - K\right] dG(s) f(v) \\ \leq \int \left[\left(v + s - r - \frac{1-F(v)}{f(v)}\right) \hat{x}^*(v,s) - K\right] dG(s) f(v)$$

Hence x^* can be replaced by $\hat{x}^*$ without decreasing the consultant's objective function. ■

Appendix 2 (for online publication only): Extension to Correlated Signals

In this online appendix we show that our results extend to the case where the client's ex-post valuation for the project, V, cannot necessarily written as the sum of his interim valuation, v, and an independent error term, s. Recall that the error term, or equivalently V, is the signal that the consultant may reveal to the client without the consultant directly observing its value.

A2.1 Equivalent formalizations of the general model

Since the error term can always be defined as the difference between V and v, the most general model to consider is one where $V = v + z$, but z (the signal or error term that the consultant can disclose) may be correlated with v. Suppose that the conditional cdf of z given v is $G_v(z) \equiv G(z|v)$, and call this version the model with "*additive, correlated signals*."

Alternatively (and equivalently), the general model can be written so that $V = u(v, s)$ for some $u : R^2 \to R$, and s (the signal that the consultant can disclose) is independent of v. The model with additive, correlated signals can be transformed into this form by letting $s = G_v(z)$ and $u(v, s) = v + G_v^{-1}(s)$. Note that s is a random variable that is independent of v as it is distributed uniformly on $[0, 1]$ no matter what v is.[16] We call this formalization as the model with "*orthogonally normalized signals*." Obviously, the distibution of V conditional on v is the same in both versions of the model, as, by definition, $u(v, s) \equiv v + z$.

In what follows, we will first generalize the results of the paper using the formalization with orthogonally normalized signals (and general u). We derive the optimal contract for this case. Our most interesting result, namely, that the consultant can do just as well in the case where she cannot observe the signal controlled by her as if she could observe it remains true (under certain conditions) about the signal s, the orthogonally normalized part of the error term. This makes a lot of sense as the only "new" information in $V - v$ for the client is the part that is orthogonal to his private information, v. In order to obtain the results, we impose mild conditions on u. Essentially, we require that the client's original signal and the orthogonal part

[16] If X is a random variable with continuous cdf F then $F(X)$ is distributed uniformly on [0,1] because for all $y \in [0, 1]$, $\Pr(F(X) \le y) = \Pr(X \le F^{-1}(y)) = F(F^{-1}(y)) = y$.

of the consultant-controlled signal be substitutes. In the last part of this appendix we show what these conditions imply in the model of additive, correlated signals in terms of the conditional distribution of z given v.

A2.2 Generalizaton of the results

Suppose that the client's ex post valuation for the project is $V = u(v, s)$. Recall that v is distributed on $[0, 1]$ with log-concave pdf f, and s is distributed on $(-\infty, \infty)$ with pdf $g > 0$, and assume that v and s are statistically independent. (As we pointed out above, independence of v and s is not a restriction, just a normalization.)

We assume that all functions are at least twice differentiable, and partial derivatives of functions are denoted by subscripts. For example, $u_1 = \partial u(v, s)/\partial v$.

We impose the following conditions on the shape of the u function (whose arguments are the orthogonally normalized v and s).

1. $u_1 > 0$, $u_2 > 0$ (strictly increasing in both arguments),
2. $u_{12} \leq 0$ (v and s are substitutes)
3. $u_{11}/u_{11} \leq u_{12}/u_2$ (v and s are substitutes along iso-value curves).

Assumption 2 states that v and s are substitutes in the sense that as v increases, the marginal value of s ($\partial u/\partial s$) decreases. To see the meaning of Assumption 3 note that the total differential of u_1' (the change in the marginal value of the client's type) is $u_{11}''dv + u_{12}''ds$. Keeping u constant (moving along an iso-value curve) means $ds = -u_1'/u_2'dv$. Substituting this into the total differential of u_1' yields $(u_{11}'' - u_{12}''u_1'/u_2')dv$. This expression is non-positive for $dv > 0$ iff $u_{11}''/u_1' \leq u_{12}''/u_2'$. In words, Assumption 3 states that an increase in v, even if compensated by an decrease in s to keep u constant, weakly increases the marginal value of s.

In order to simplify the analysis, assume that the consultant's cost of disclosing s to the client is zero ($K = 0$), and that she is required to contract with the client, i.e., $a \equiv 1$. (It can be shown that $a \equiv 1$ actually follows from the assumption that information disclosure is costless.) As a result, we can ignore q. For notational convenience, normalize v so that it is an unbiased estimator of V, that is, the expectation of $u(v, s)$ with respect to s (given v) is v. We can represent the consultant's contract by the pair (p, c), where $c : [0, 1] \to R$ is the up-front fee the client pays to the consultant upon contracting with her; $p : [0, 1] \to R$ is the "premium" he pays if he undertakes the project after getting the consultant's advice.

The client that learns the consultant advice undertakes the project if and only if $u(v,s) \geq r$, while a client that doesn't contract with the consultant undertakes it whenever $v \geq r$. Define $\sigma(p,v)$ so that

$$u(v,\sigma(p,v)) \equiv p + r. \tag{39}$$

Note that σ is increasing in p and decreasing in v. Moreover, if v increases, $u_1'(v,\sigma(p,v))$ decreases by Assumption 3 on the u-function.

The client's expected payoff with type v facing an up-front fee c and a premium p can be written as

$$\tilde{U}(p,c,v) = \int_{\sigma(p,v)}^{\infty} [u(v,s) - r - p]\, dG(s) - c. \tag{40}$$

The consultant's payoff from the same can be written as

$$\tilde{W}(p,c,v) = [1 - G(\sigma(p,v))]\, p + c. \tag{41}$$

Note that the structure of the problem is very similar to that of an adverse selection (Principal-Agent) model with quasilinear utilities. Here c should be interpreted as the "transfer" and p as the "contractible action". The Agent's type has a positive impact on his utility,

$$\frac{\partial \tilde{U}}{\partial v} = \int_{\sigma(p,v)}^{\infty} u_1'(v,s) dG(s) > 0,$$

and the Spence-Mirrlees single-crossing condition holds because

$$\frac{\partial^2 \tilde{U}}{\partial p \partial v} = -\sigma_p'(p,v) u_1'(v,\sigma(p,v)) g(\sigma(p,v)) < 0. \tag{42}$$

In contract (p,c), define the deviation payoff of client-type v for a report $\hat{v}$ as

$$U(v,\hat{v}) = \int_{\sigma(p(\hat{v}),v)}^{\infty} [u(v,s) - r - p(\hat{v})]\, dG(s) - c(\hat{v}). \tag{43}$$

Incentive compatibility of the contract means $U(v,\hat{v}) \leq U(v,v)$ for all $v, \hat{v} \in [0,1]$. We now characterize all incentive compatible contracts in the model. Define the client's indirect payoff function as $U(v) \equiv U(v,v)$.

We claim that incentive compatibility of (p, c) is equivalent to

$$\frac{dU(v)}{dv} = \int_{\sigma(p(v),v)}^{\infty} u_1'(v,s)dG(s), \tag{44}$$

and $p(v)$ monotonically non-increasing.

We only sketch the proof, and assume differentiability throughout.

Necessity: By the envelope theorem,

$$\frac{dU(v)}{dv} = \frac{\partial U(v,\hat{v})}{\partial v}\bigg|_{\hat{v}=v} = \int_{\sigma(p(v),v)}^{\infty} u_1'(v,s)dG(s).$$

The first-order condition of maximizing (43) in $\hat{v}$ and obtaining the maximum at $\hat{v} = v$ is

$$FOC(v,\hat{v}) = -p'(\hat{v})\left[1 - G(\sigma(p(\hat{v}),v))\right] - c'(\hat{v}) = 0 \text{ for } \hat{v} = v.$$

The necessary second-order condition is that $\partial FOC(v,\hat{v})/\partial\hat{v} \leq 0$ at $\hat{v} = v$, equivalently, $\partial FOC(v,\hat{v})/\partial v \geq 0$ at $\hat{v} = v$, that is,

$$-p'(v)g(\sigma(p(v),v)) \geq 0,$$

so $p(v)$ is non-increasing.

Sufficiency: It follows from the first-order condition and the single-crossing condition.

Besides incentive compatibility, the indirect payoff function, $U(v)$, must also satisfy individual rationality (participation),

$$U(v) \geq \max\{v - r, 0\}. \tag{45}$$

It is easy to see that this constraint either binds at $v = 0$, or $v = 1$, or both. The reason is that by equation (44), dU/dv is between 0 and 1.

The consultant's expected payoff in an incentive compatible mechanism (p, c) is

$$W = \int_0^1 \left\{\left[1 - G(\sigma(p(v),v))\right] p(v) + c(v)\right\} dF(v).$$

Using the definition of the client's indirect payoff function, (43) with $\hat{v} = v$, this can be rewritten as

$$W = \int_0^1 \left\{ \int_{\sigma(p(v),v)}^{\infty} [u(v,s) - r] \, dG(s) - U(v) \right\} dF(v). \tag{46}$$

The consultant chooses $U : [0,1] \to R$ and $p : [0,1] \to R$ to maximize (46) subject to incentive compatibility and individual rationality. We solve this problem using optimal control, where the control variable is p, the state variable is U. We will ignore the constraint that p is non-increasing and verify it at the end. We will also ignore the two IR constraints (at $v = 0$ and $v = 1$) and use them as transversality conditions to pin down certain parameters at the end.

Assign multiplier $\mu(v)$ to the law of motion for U. The Hamiltonian becomes,

$$H = \left(\int_{\sigma(p(v),v)}^{\infty} [u(v,s) - r] \, dG(s) - U(v) \right) f(v) + \mu(v) \int_{\sigma(p(v),v)}^{\infty} u_1'(v,s) dG(s).$$

By Pontryagin's Maximum Principle, the derivative of the Hamiltonian with respect to the control variable must be zero, $\partial H / \partial p = 0$. Taking the derivative and simplifying terms yield

$$[u(v, \sigma(p(v),v)) - r] \, f(v) + \mu(v) u_1'(v, \sigma(p(v),v)) = 0. \tag{47}$$

Also, $-\partial H / \partial U = \dot{\mu}$, that is,

$$\mu'(v) = f(v). \tag{48}$$

Integrating (48) gives $\mu(v) = F(v) - B$ where B is a constant. Substituting this and $u(v, \sigma(p(v),v)) - r \equiv p(v)$ into (47), we get

$$\frac{p(v)}{u_1'(v, \sigma(p(v),v))} = \frac{B - F(v)}{f(v)}. \tag{49}$$

Note that for $u(v,s) = v + s$, (49) simplifies to $p(v) = (B - F(v))/f(v)$. In the case of a general u function, however, we only have an implicit solution for $p(v)$.

Transversality conditions (corresponding to the IR constraints) pin down the value of B. Indeed, if $U(0) > \max\{-r, 0\}$ then $\mu(0) = 0$ and hence $B = 0$. If $U(1) > \max\{1 - r, 0\}$ then $\mu(1) = 0$ and hence $B = 1$. Finally, if $U(v) = \max\{v - r, 0\}$ for both $v = 0$ and $v = 1$, then $B \in [0, 1]$ is determined by $U(0) = 0$ and $U(1) = 1 - r$, that is,

$$\int_0^1 \left[\int_{\sigma(p(v),v)}^{\infty} u_1'(v, s) dG(s) \right] dv = 1 - r.$$

By log-concavity of f, $(B - F(v))/f(v)$ is decreasing in v for all $B \in [0, 1]$. This, and the conditions imposed on u together imply that $p(v)$ as defined by (49), is weakly decreasing. This is so because if $p(v)$ were weakly increasing at v, then the ratio on the left-hand side of (49) would be weakly increasing as the denominator is weakly decreasing in v (as v goes up, if $p(v)$ goes up weakly then $u_1'(v, \sigma(p(v), v))$ goes down weakly by Assumptions 2 and 3 imposed on u).

Our main (and perhaps most interesting) result is that the consultant cannot do better than she does in this mechanism even if she is able to observe s as she reveals it to the client.

When s is observable by the consultant, we can represent any mechanism where all types of the client must be served as follows: the client reports his type, v, and pays the consultant $d^*(v)$; the consultant observes s, and instructs him to undertake the project with probability $x^*(v, s)$. If a client with type v reports type $\hat{v}$ then his payoff is

$$U^*(v, \hat{v}) = \int (u(v, s) - r) \; x^*(\hat{v}, s) dG(s) - d^*(\hat{v}).$$

Let $U^*(v) = U^*(v, v)$. From the first-order conditions of incentive compatibility we get

$$\frac{d}{dv} U^*(v) = \int u_1(v, s) \, x^*(v, s) dG(s).$$

The second-order condition is that $\int u_1(v, s) \, x^*(v, s) dG(s)$ is weakly increasing in v.

The consultant's payoff is the difference between the social surplus and the client's payoff. The client's expected payoff is

$$\begin{aligned} \int_0^1 U^*(v) dv &= U^*(0) + \int_0^1 \int_0^v \left(\int u_1(z, s) \, x^*(z, s) dG(s) \right) dz dF(v) \\ &= U^*(0) + \int_0^1 (1 - F(v)) \int u_1(v, s) \, x^*(v, s) dG(s) \, dv, \end{aligned}$$

where we used integration by parts to get the second line. Therefore, the consultant's expected payoff can be written as

$$\begin{aligned} W^* &= \int_0^1 \left[\int (u(v,s) - r)\, x^*(v,s) dG(s) - U^*(v) \right] dF(v) \\ &= \int_0^1 \int \left(u(v,s) - r - \frac{1-F(v)}{f(v)} u_1(v,s) \right) x^*(v,s) dG(s) dF(v) - U^*(0). \end{aligned}$$

We want to maximize W^* by choosing $x^*(v,s) \in [0,1]$ and $U^*(0) \in \mathbb{R}$ subject to the type-dependent participation constraint and the second-order conditions of incentive compatibility. Clearly, in the optimum, for all $v \in [0,1]$, $x^*(v,\cdot)$ must be a step-function that equals 0 for $s < s_v$ and 1 for $s > s_v$, almost everywhere. If it were not then we could reduce $x^*(v,s)$ on a positive measure of low s's and increase it on a positive measure of high s's (for a fixed v) without changing $dU^*(v)/dv = \int u_1(v,s) x^*(v,s) dG(s)$, yet increasing W^*. That is, we could increase the consultant's expected payoff while keeping the mechanism incentive compatible, which is not possible in the optimum.[17]

Since $x^*(v,s) = \mathbf{1}_{s \geq s_v}$ a.e., we can rewrite W^* as

$$W^* = \int_0^1 \int_{s_v}^{\infty} \left(u(v,s) - r - \frac{1-F(v)}{f(v)} u_1(v,s) \right) dG(s) dF(v) - U^*(0). \tag{50}$$

The consultant's expected payoff under *unobservable* s is given in equation (46). Using (44), the client's expected payoff under the same assumption can be written as

$$\begin{aligned} \int_0^1 U(v) dv &= U(0) + \int_0^1 \int_0^v \left(\int_{\sigma(p(z),z)}^{\infty} u_1'(z,s) dG(s) \right) dz\, dF(v) \\ &= U(0) + \int_0^1 (1 - F(v)) \int_{\sigma(p(v),v)}^{\infty} u_1'(v,s) dG(s) dv, \end{aligned}$$

where the second line is obtained after integration by parts. Substituting this into (46) yields,

$$W = \int_0^1 \int_{\sigma(p(v),v)}^{\infty} \left[u(v,s) - r - \frac{1-F(v)}{f(v)} u_1(v,s) \right] dG(s) dF(v) - U(0). \tag{51}$$

[17] This result corresponds to Lemma 6 in the model of the paper.

Note that (50) and (51) are equivalent. Therefore, W^* is maximized subject to the incentive and participation constraints by choosing $s_v = \sigma(p(v), v)$ where σ is defined by (39) and p by (49), and $U^*(0) = U(0)$. It is easy to check that the sufficient conditions of incentive compatibility hold in this mechanism, therefore we have found the maximum.

We conclude that the solution to the "benchmark" problem, where the consultant can observe the realization of s, is the same as that of our original problem. This result corresponds to Theorem 2 in the paper.

A2.3 The additive-correlated model

In this section of the appendix we translate the conditions imposed on the function $u(v,s) \equiv V$ in the model with orthogonally normalized signals (see Assumptions 1-3 in section A.2) into conditions on the joint distribution of v and the error term, $z \equiv V - v$. Recall that the assumptions made on u were easy to interpret, and meant, roughly speaking, that the client's original information, v, and the orthogonal part of the consultant-controlled signal, s, are substitutes in the client's ex-post valuation. The conditions imposed on the joint distribution of v and z express exactly the same assumptions. We provide them only for the sake of completeness, there really is no new insight to be gained from this transformation.

Recall that the conditional cdf of z conditional on v is denoted by G_v, and its positive density by g_v. Differentiating the identity $u(v,s) \equiv v + G_v^{-1}(s)$ yields

$$\begin{aligned} \frac{\partial u(v,s)}{\partial v} &= 1 + \frac{\partial G_v^{-1}(s)}{\partial v} \\ \frac{\partial u(v,s)}{\partial s} &= \frac{\partial G_v^{-1}(s)}{\partial s} = \frac{1}{g_v\left(G_v^{-1}(s)\right)}. \end{aligned} \tag{52}$$

Since $g_v > 0$, the second line is always positive. As far as the first line is concerned, we first show that for $s = G_v(z)$,

$$\frac{\partial G_v^{-1}(s)}{\partial v} = -\frac{\partial G_v(z)/\partial v}{g_v(z)}. \tag{53}$$

To see this, fix s and define $\widetilde{z}(v)$ implicitly by $s = G_v(\widetilde{z}(v))$. Differentiating this identity according to v and rearranging it we get

$$\frac{d\widetilde{z}(v)}{dv} = -\frac{\partial G_v(\widetilde{z}(v))/\partial v}{g_v(\widetilde{z}(v))}.$$

On the other hand, $G_v^{-1}(s) = \widetilde{z}(v)$, hence

$$\frac{d\widetilde{z}(v)}{dv} = \frac{\partial G_v^{-1}(s)}{\partial v}.$$

So, from (52) and (53) we can conclude that the assumption that u is increasing in v in the independent model translates to

$$1 - \frac{\partial G_v(z)/\partial v}{g_v(z)} > 0 \tag{54}$$

in the correlated linear model.

From the second line of (52), using the rule for derivatives of inverse functions and the chain rule we get,

$$\begin{aligned}\frac{\partial^2 u(v,s)}{\partial s \partial v} &= \frac{\partial^2 G_v^{-1}(s)}{\partial s \partial v} = \frac{\partial\left(1/g_v\left(G_v^{-1}(s)\right)\right)}{\partial v} \\ &= -\frac{1}{g_v^2\left(G_v^{-1}(s)\right)}\left(\left.\frac{\partial g_v(z)}{\partial v}\right|_{z=G_v^{-1}(s)}\right)\frac{\partial G_v^{-1}(s)}{\partial v}.\end{aligned} \tag{55}$$

Using (53) we can further rewrite it as

$$\frac{\partial^2 u(v,s)}{\partial s \partial v} = \left.\frac{1}{g_v^3(z)}\frac{\partial g_v(z)}{\partial v}\frac{\partial G_v(z)}{\partial v}\right|_{z=G_v^{-1}(s)}.$$

Since the density g_v is positive, the assumption $u_{12} \leq 0$ in the independent model translates into

$$\frac{\partial g_v(z)}{\partial v}\frac{\partial G_v(z)}{\partial v} \leq 0 \tag{56}$$

in the correlated linear model.

From (52) and (53)

$$\begin{aligned}\frac{\partial^2 u(v,s)}{\partial v^2} &= -\left.\frac{\partial\left[\partial G_v(z)/\partial v\right]/g_v(z)}{\partial v}\right|_{z=G_v^{-1}(s)} \\ &= -\left.\frac{g_v(z)\,\partial^2 G_v(z)/\partial v^2 - \left[\partial g_v(z)/\partial v\right]\left[\partial G_v(z)/\partial v\right]}{g_v^2(z)}\right|_{z=G_v^{-1}(s)}.\end{aligned}$$

Therefore, for $z = G_v^{-1}(s)$,

$$\begin{aligned}
&\frac{\partial^2 u(v,s)}{\partial v^2} \Big/ \frac{\partial u(v,s)}{\partial v} = \\
&-\frac{g_v(z)\,\partial^2 G_v(z)/\partial v^2 - [\partial g_v(z)/\partial v]\,[\partial G_v(z)/\partial v]}{g_v^2(z)} \Big/ \left[1 - \frac{\partial G_v(z)/\partial v}{g_v(z)}\right] \\
= \; &-\frac{g_v(z)\,\partial^2 G_v(z)/\partial v^2 - [\partial g_v(z)/\partial v]\,[\partial G_v(z)/\partial v]}{g_v^2(z)} \Big/ \left[\frac{g_v(z) - \partial G_v(z)/\partial v}{g_v(z)}\right] \\
= \; &-\frac{g_v(z)\,\partial^2 G_v(z)/\partial v^2 - [\partial g_v(z)/\partial v]\,[\partial G_v(z)/\partial v]}{g_v(z)\,(g_v(z) - \partial G_v(z)/\partial v)}.
\end{aligned}$$

And, by (55) and (52),

$$\frac{\partial^2 u(v,s)}{\partial v \partial s} \Big/ \frac{\partial u(v,s)}{\partial s} = \left.\frac{1}{g_v^2(z)} \frac{\partial g_v(z)}{\partial v} \frac{\partial G_v(z)}{\partial v}\right]_{z=G_v^{-1}(s)}.$$

Hence the assumption $u_{11}/u_1 \leq u_{12}/u_2$ in the independent model translates to

$$-\frac{g_v(z)\,\partial^2 G_v(z)/\partial v^2 - [\partial g_v(z)/\partial v]\,[\partial G_v(z)/\partial v]}{g_v(z)\,(g_v(z) - \partial G_v(z)/\partial v)} \leq \frac{1}{g_v^2(z)} \frac{\partial g_v(z)}{\partial v} \frac{\partial G_v(z)}{\partial v}.$$

After multiplying both sides by $g_v(z)\,(g_v(z) - \partial G_v(z)/\partial v)$, which is non-negative by (54),

$$\begin{aligned}
\frac{\partial g_v(z)}{\partial v} \frac{\partial G_v(z)}{\partial v} - g_v(z) \frac{\partial^2 G_v(z)}{\partial v^2} \quad &\leq \quad \frac{(g_v(z) - \partial G_v(z)/\partial v)}{g_v(z)} \frac{\partial g_v(z)}{\partial v} \frac{\partial G_v(z)}{\partial v} \\
&= \quad \frac{\partial g_v(z)}{\partial v} \frac{\partial G_v(z)}{\partial v} - \frac{(\partial G_v(z)/\partial v)^2}{g_v(z)} \frac{\partial g_v(z)}{\partial v}.
\end{aligned}$$

Notice that the first term is the same on both sides of the inequality. Hence, Assumption 3 is equivalent to

$$\frac{\partial^2 G_v(z)/\partial v^2}{(\partial G_v(z)/\partial v)^2} \geq \frac{\partial g_v(z)/\partial v}{g_v^2(z)}. \tag{57}$$

Summary:

$$\text{Assumption 1: } u_1, u_2 > 0 \Leftrightarrow 1 - \frac{\partial G_v(z)/\partial v}{g_v(z)} > 0$$

$$\text{Assumption 2: } u_{12} \leq 0 \Leftrightarrow \frac{\partial g_v(z)}{\partial v} \frac{\partial G_v(z)}{\partial v} \leq 0,$$

$$\text{Assumption 3: } \frac{u_{11}}{u_1} \leq \frac{u_{12}}{u_2} \Leftrightarrow \frac{\partial^2 G_v(z)/\partial v^2}{(\partial G_v(z)/\partial v)^2} \geq \frac{\partial g_v(z)/\partial v}{g_v^2(z)}.$$

The meaning of the latter two conditions, as we discussed above, is the same as that of Assumptions 2 and 3: The client's original signal, v, and the component of the error term that is orthogonal to v, are substitutes in V. It seems to us that the equivalent formulation where the signals are orthogonally normalized (as described in section A.2.2) show this interpretation much more clearly.

References

[1] Bagnoli and Bergstrom (1989), "Log–concave probability and its applications," Working Paper, University of Michigan.

[2] Crawford, P. V., and and J. Sobel (1982), "Strategic Information Transmission," *Econometrica*, Vol. 50, No. 6: 1431-1451.

[3] Dana, J. D., and K. E. Spier (1993), "Expertise and Contingent Fees: The Role of Asymmetric Information in Attorney Compensation," *Journal of Law, Economics, and Organization*, Vol. 9, 349-367.

[4] Emons, W., "Credence Goods and Fraudulent Experts," *RAND Journal of Economics*, Vol. 28, 107-119.

[5] Eső P. and B. Szentes (2002), "Optimal Information Disclosure in Auctions: The Handicap Auction," Northwestern University, CMS-EMS DP#1352.

[6] Fudenberg, D., and J. Tirole (1991), *Game Theory*, MIT Press, Cambridge, USA

[7] Jullien, Bruno (2000), "Participation Constraints in Adverse Selection Models," *Journal of Economic Theory*, 93, 1-47.

[8] Klibanoff, P., and J. Morduch (1995), "Decentralization, Externalities, and Efficiency," *Review of Economic Studies,* 62, 223-247.

[9] Lewis, T., and D. Sappington (1989), "Countervailing Incentives in Agency Problems," *Journal of Economic Theory*, 49, 294-313.

[10] Maggi, G., and A. Rodriguez (1995), "On Countervailing Incentives," *Journal of Economic Theory*, 66, 238-263.

[11] Milgrom, P., and J. Roberts (1986), "Relying on the Information of Interested Parties," *RAND Journal of Economics*, Vol 17, pp. 18-32.

[12] Pitchik, C., and A. Schotter (1987), "Honesty in a Model of Strategic Information Transmission," *American Economic Review*, Vol. 77, No. 5, pp. 1032-1036; Erratum in Vol. 78 (1988), p. 1164.

[13] Prékopa, A. (1971), "Logarithmic concave measures with applications to stochastic programming," *Acta Sci. Math.,* Szeged, 32, 301-316.

[14] Scotchmer, S., and D. L. Rubinfeld (1990), "Contingent Fees for Attorneys: An Economic Analysis," *RAND Journal of Economics*, Vol. 24, 343-356.

[15] Wolinsky, A. (1993), "Competition in a Market for Informed Experts' Services," *RAND Journal of Economics*, Vol. 24, pp. 380-398.