A Micro-Foundation for Non-Deterministic Contests of the Logit Form

Matthias Dahm* and Nicolás Porteiro†

July 18, 2005

* Northwestern University.
† Universidad Pablo de Olavide.

Authors' Addresses:

Matthias Dahm

CMS-EMS (Center for Mathematical Studies in Economics and Management Science), Northwestern University, 2001 Sheridan Rd, Evanston, IL 60208, USA.

e-mail: m-dahm@kellogg.northwestern.edu.

Nicolás Porteiro

Dpto. Economía, Met. Cuantitativos e H Económica, Área Análisis Económico Universidad Pablo de Olavide, Carretera de Utrera, km 1. 41013 Sevilla, Spain.

e-mail: nporfre@upo.es.

Abstract

In models of non-deterministic contest, players exert irreversible effort in order to increase their probability of winning a prize. The most prominent functional form of the win probability in the literature is the so-called "logit" contest success function. We provide a simple micro-foundation of this function for the two contestant case. In this setting the contest administrator is a rational decision maker whose optimal choice is *deterministic*. However, from the point of view of the contestants the outcome of the contest is *probabilistic* because of an underlying uncertainty about the type of the administrator.

Keywords: Contests, Contest Success Function, Effort levels, Endogenous Contest.

Journal of Economic Literature Classification Numbers:

C72 (Noncooperative Games),

D72 (Economic Models of Political Processes: Rent-Seeking, Elections),

D74 (Conflict; Conflict Resolution; Alliances)

"...Just as there is a technology of production, there is a technology of conflict and struggle. The key to the latter is the Conflict Success Function (CSF). ..."

Hirshleifer (1989), p. 101

"...the analysis of equilibrium or endogenous contests has reached only a very preliminary and inadequate stage of development. Future progress in this direction will constitute a significant contribution to the theory of rent seeking, public choice and, more generally, to political economy. ..."

Nitzan (1994), p. 56

1. Introduction

The Nash demand game is a non-cooperative game that supports the Nash solution of two-person bargaining games (see Nash (1953), Trockel (2000)). In doing so it answers the basic question, how the *implicit* model of rational individual behavior supporting the Nash solution may look like.

The aim of this work is to carry out a similar exercise concerning the most prominent model of non-deterministic contests. More specifically we ask: Can we think about the underlying model of individual behavior in a way that is analogous to the way we think about the auction of an object? In the standard (incomplete information) model of an auction bidders face uncertainty about the competing bidders' type. Given this uncertainty they behave rationally and choose *deterministically* an optimal bid. The success of this bid is *non-deterministic*. For some types of competitors the bidder obtains the object while for others not.

In a contest game agents exert irreversible effort to increase their probability of winning a prize. Contests have been used to analyze a variety of situations including rent-seeking and rent-defending contests, lobbying, litigation, political campaigns, conflict, patent races, arms races, sports events, R&D competition or coalition formation.

Contests in which the player exerting the highest effort wins the prize with probability one, like in an all-pay auction, are called deterministic (or perfectly discriminating). In non-deterministic contests the probability of winning is given by a contest success function (henceforth CSF) which depends on the efforts of the players. In deciding on the mathematical formulation of the CSF the literature has largely used a CSF in logit form.[1] In the two contestant case this CSF is defined as

$$\pi_i(e) = \frac{g_i(e_i)}{g_i(e_i) + g_j(e_j)}, \ i \in \{1, 2\}, \ j \neq i. \tag{1.1}$$

In this formulation π_i is the probability that contestant C_i wins the contest given the vector of efforts $e = (e_1, e_2)$ of the two contestants and the effectivity functions $g_i(\cdot)$. The effectivity functions specify how effort enters the logit CSF and are assumed to be increasing. They are often the same for all players (symmetry or anonymity assump-

[1] Baik (1998) lists 23 papers that use a logit form CSF. We are aware of at least 7 other (recent) works.

tion). Concerning the specification of effectivity functions a large part of the literature has build on Tullock's (1980) suggestion of an exponential form.[2]

A better understanding of CSFs has been gained by the axiomatic characterizations of Skaderpas (1996) and Clark and Riis (1998). However, contests are intended to be a positive model. Therefore, its properties are necessarily part of the phenomena to be explained and not something to be assumed.[3]

The only attempt we are aware of to provide a micro-foundation for this model in which a CSF arises as the optimal choice of rational decision makers is our companion-paper Dahm and Porteiro (2004). There we offer a very similar approach that rests on an uncertainty about the state of the world.[4] However, there are recent attempts to endogenize various components of a contest game (for a brief survey see e. g. Nitzan (1994)).

We define a two-stage game played by a contest administrator and two contestants. In the first stage contestants exert irreversible effort which affects the payoff function

[2] Tullock has later admitted that "I used the exponential form when I wrote "Efficient Rent-Seeking", because I wanted a form which showed economics of scale, and that was the standard elementary textbook method of doing it." Tullock (1995), p. 190. We found this citation in Clark and Riis (1996). Amegashie (2003) proposes a variation of Tullocks CSF on tractability grounds. Hirshleifer (1989) proposes a function of the differences in effort because equilibrium behavior may capture better certain situations. Che and Gale (2000) motivate their difference form CSF by the fact that they are able – contrary to the literature using Tullocks formulation – to characterize the equilibrium for all levels of sensitivity of the outcome to contestants' efforts.

[3] This is different if we are concerned with the design of a contest. For example, it may be in the interest of a contest administrator to commit credibly to a CSF that is normatively appealing in order to induce participation in the contest.

[4] Clark and Riis (1996) adopt a random utility formulation in which it is assumed that the contestants view the contest administrator as maximizing a random utility function.

of the administrator. Given this effort the administrator chooses in the second stage to which contestant to give the prize. From the point of view of the lobbies, success in the contest is *non-deterministic* because of an underlying uncertainty about the type of the administrator. For some types of administrators the contestant obtains the prize while for others not. However, the administrator's choice is *deterministic*.

2. A Micro-Foundation for the Case of Two Contestants

Consider a contest administrator A who is pivotal in the decision to give a political prize, e.g. a procurement contract, to either of two contestants C_1 and C_2. The decision taken is denoted by D and we will write $D = C_i$ if contestant C_i is given the prize. Contestants' valuations for the prize are given by $V_i \geq 0$.[5]

In order to advance their aims contestants can exert effort $e_i \in \mathbb{R}_+$ at a cost $c_i(e_i)$. This effort is irreversible and affects the politician in form of an effectivity function $g_i(e_i)$. Suppose there is a parameter t which characterizes the type of the politician from the point of view of contestants. The parameter t is distributed on the line segment $[0, 1]$ according to some cumulative distribution function F. Define $t_1 = t$ and $t_2 = 1 - t$. We postulate the following functional form for the payoff of the contest administrator from

[5] In the two-contestants case it is unambiguous to interpret the political prize as a decision over a policy – in which case it is natural to assume contestants may obtain a non-zero utility from not obtaining the prize. We may define then the utility difference between both policies for contestant C_i as $V_i = V_i^i - V_i^j \geq 0$, $(i = 1, 2$ and $j \neq i)$ where V_i^j denotes the utility achieved by contestant i when the policy chosen is j. With more than two contestants a non-zero utility from not getting the prize may differ depending on the policy choice. This adds additional considerations to the optimization problem of contestants.

giving the prize to contestant C_i

$$U_A(D = C_i) = t_i g_i(e_i). \tag{2.1}$$

To provide an interpretation for this formulation consider the following example.

Example 2.1. *Contests are frequently used in the literature to model lobbying of legislators. Consider a political decision-maker who must decide among two policy alternatives with different consequences for the environment. There are two interest groups, one representing the industry and one environmental group. Both groups can dedicate effort, political pressure or propaganda campaigns, in order to lobby the electorate. An environmentally friendly voting record may prove important if the legislator decides later in his career to run for president. On the other hand, the politician may decide to become a lobbyist for the industry in the future. Both lobbies face uncertainty concerning the future plans of the politician.*

One way to capture such a situation may use the following effectivity function

$$g_i(e_i) = f(e_i) + \Gamma_i. \tag{2.2}$$

These functions differ only by an additive component Γ_i. The administrator's choice depends on three determinants:

1. *His type t representing the future plans of the legislator which are a random*

variable from the point of view of the lobbies.

2. *The effective efforts of the lobbies $f(e_i)$ reflecting the result of e. g. the propaganda campaigns.*

3. *The additive components Γ_i. One may think of Γ_1 as the relative advantage policy L_1 has over L_2 in terms of damage to the environment. Similarly, Γ_2 may capture benefits of a future employment as a lobbyist for industry L_2.*[6]

Coming back to the administrator's payoffs described in (2.1), note that although the lobbies know the effectivity functions $g_i(\cdot)$, they do not know whether $t \gtreqless \frac{1}{2}$. This implies that even if effort was known, lobbies would not know which policy the politician considers optimal. Moreover, they would not know how much better one policy is from the point of view of the politician.

The timing of the game is sequential. In the first stage lobbies exert effort simultaneously. Given this effort, the politician awards in the second stage the prize. The alternative chosen is the one that gives the highest payoffs to the politician.[7] We have the following result.

[6] An important determinant of the success of lobbying activities is access to politicians. Different access might be reflected in different Γs and in different functions $f(\cdot)$. For simplicity we abstract of the latter.

[7] This model has a structure similar to an all-pay auction as e. g. in Baye et al (1993 and 1996). The only difference is informational. Lobbies do not know the politicians type. Thus, the award of the prize is non-deterministic and the contest is governed by a CSF.

Theorem 2.2. *Assume t is distributed according to a symmetric density function and the politician's payoffs are given by (2.1). Then the problem of lobby L_i is*

$$\max_{e_i} \ \pi_i(e)V_i - c_i(e_i), \tag{2.3}$$

where $\pi_i(e)$ is a monotonically increasing transformation of the logit form specified in equation (1.1).

Proof: We have that

$$\begin{aligned} & U_{DM}(D = L_1) \geq U_{DM}(D = L_2) \\ \Leftrightarrow \quad & t \cdot g_1(e_1) \geq (1-t) \cdot g_2(e_2) \\ \Leftrightarrow \quad & t \geq \frac{g_2(e_2)}{g_1(e_1) + g_2(e_2)} \equiv \bar{t}. \end{aligned}$$

An effort vector e leads to a winning probability for lobby L_1 of

$$\pi_1(e) = 1 - F(\bar{t}). \tag{2.4}$$

Assuming $dF(t)$ is symmetric, we have that $1 - F(\bar{t}) = F(1 - \bar{t})$. This implies:

$$\begin{aligned} \pi_1(e) &= F\left(\frac{g_1(e_1)}{g_1(e_1) + g_2(e_2)}\right), \text{ and} \\ \pi_2(e) &= F\left(\frac{g_2(e_2)}{g_1(e_1) + g_2(e_2)}\right). \end{aligned} \tag{2.5}$$

This is a monotonically increasing transformation of the expression in equation (1.1).■

One interesting implication of this micro-foundation concerns symmetric contest. In the literature a contest is called symmetric if players are identical. The most common asymmetries considered refer to effectivity functions or valuations of the prize. Consider the case of symmetric effectivity functions, that is, $g_i(\cdot) = g(\cdot)$ for all i. In this case it is most visible that the decision-maker's type can be interpreted as being related to the relative ability of the lobbies to convert effort into utility for the politician. Their effectivity functions multiplied by t_i are the multiples of each other and this multiple is uncertain. *The symmetric contest is non-deterministic because it is an asymmetric contest with probability one and the degree of this asymmetry is uncertain.*

Not surprisingly, the logit form hinges on the different assumptions made. With a non-symmetric distribution of types or different support, probability mass would be shifted from one side of the threshold $\bar{t}$ to the other. In fact, one could interpret such an asymmetry as an ideological bias of the decision-maker. Consider, for instance that policy C_2 is the enviromentally friendly choice. Then, facing a politician of an uncertain type but characterized by a probability distribution that concentrates the probability mass on the left tail could be seen as a setting in which it is common knowledge for the contestants that the decision-maker sympathizes with pro-environmental positions (though his precise decision in a particular choice is uncertain). Even if in such a scenario still it would be true that efforts affect a threshold value for the random variable that determines the choice of the politician, this would give rise to an asymmetric contest which is qualitatively different from asymmetric contests in the literature. Similarly, it is important how effort enters equation (2.1). If effort differences are considered important, as e. g. in Baik (1998), then a formulation like $U_{DM}(D = L_i) = t_i\,(f(e_i) - f(e_j) + \Gamma_i)\,,\ i = 1, 2$, with $i \neq j$ may be natural but leads to a different CSF.[8]

3. Concluding Remarks

In this work we have shown that the two-contestant contest success function of the logit form can be derived from a rational choice environment. The merit of this approach is to visualize the implicit micro-level assumptions underlying this non-deterministic contest.[9]

There are at least two mayor shortcomings to our approach. Firstly, we do not offer any explication of what an effectivity function is. Rather we "shift the blackbox" from the CSF to the payoffs of the contest administrator.

Secondly, our model refers to the two contestant case. Mathematically, an extension of our approach to n-contestants is possible. Given that the logit CSF is characterized mainly by Luce's Choice Axiom we can define a sequential decision process of the contest administrator that works as follows.[10] At stage one the politician partitions the set of

---

[8] However, the family of logit form CSFs contains exponential effectivity functions in which case it depends on the effort difference.

[9] It is apparent that if we find the assumptions in our model questionable, then we cast doubt on the literature that uses logit form CSFs.

[10] For axiomatic characterizations see Skaperdas (1996) and Clark and Riis (1998).

lobbies into two coalitions. Each of these coalitions is treated as a single player whose effort is the sum of the efforts of its members and obtains an overall probability using the CSF described in (1.1). At the following stages we repeat this proceeding until all coalitions are singletons. The probability of contestant C_i is the product of all the probabilities associated to coalitions in which C_i was a member. This process leads to the desired probabilities and is independent of the particular partitioning process employed. However, pursuing this line of reasoning provides some problems, in particular: what is the economic interpretation of the parameter t at each stage?

Rather than a dead end we believe this to open avenues for future research. For example, if the contest administrator is characterized by a vector of priors $t = (t_1, \ldots, t_n)$ with $\sum_{i=1}^{n} t_i = 1$, one associated to each contestant, drawn from some distribution, a contest success function can in principle be defined. We conjecture that it is different from a contest success function of the logit form but we are confident that such an approach can lead to economically meaningful models of non-deterministic contests.

References

[1] Amegashie, J. Atsu (2003): "A Contest Success Function with Tractable Noise Parameter," unpublished manuscript, University of Guelph.

[2] Baik, Kyung Hwan (1998): "Difference-Form Contest Success Functions and Effort Levels in Contests," *European Journal of Political Economy* **14**, 685-701.

[3] Baye, Michael R.; Kovenock, Dan and Casper G. de Vries (1996): "The All-Pay Auction with Complete Information," *Economic Theory* **8**, 291-305.

[4] Baye, Michael R.; Kovenock, Dan and Casper G. de Vries (1993): "Rigging the Lobbying Process: An Application of the All-Pay Auction," *The American Economic Review* **83** (1), 289-294. 3.

[5] Che, Yeon-Koo and Ian Gale (2000): "Difference-Form Contests and the Robustness of All-Pay Auctions," *Games and Economic Behavior* **30**, 22-43.

[6] Clark, Derek and Christian Riis (1998): "Contest Success Functions: an Extension," *Economic Theory* **11**, 201–204.

[7] Clark, Derek and Christian Riis (1996): "On the win Probability in Rent-Seeking Games," Memorandum No. 5 from the Department of Economics, University of Oslo.

[8] Dahm, Matthias and Nicolás Porteiro (2004): "Tullock's Conjectures Concerning Biased Contest," unpublished manuscript, Northwestern University.

[9] Hirshleifer, Jack (1998): "Conflict and Rent-Seeking Success Functions: Ratio vs Difference Models of Relative Success," *Public Choice* **63**, 101–112.

[10] Nash, John F.: "Two-person cooperative games," *Econometrica* 21, 128–140.

[11] Nitzan, Shmuel (1994): "Modelling Rent-Seeking Contests," *European Journal of Political Economy* **10**, 41-60.

[12] Skaderpas, Stergios (1996): "Contest Success Functions," *Economic Theory* **7**, 283–290.

[13] Trockel, Walter (2000): "Implementations of the Nash solution based on its Walrasian characterization," *Economic Theory* **16**, 277–294.

[14] Tullock, Gordon (1995): "The reluctant gamesperson – A comment on Baye, Kovenock and De Vries," *Public Choice* **85**, 189–192.

[15] Tullock, Gordon (1980). Efficient rent seeking. In J. Buchanan, R. Tollison, and G. Tullock (Eds.), Toward a theory of the rentseeking society. College Station: Texas A&M University Press, 97–112.