# On the Optimality of Privacy in Sequential Contracting*

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**Abstract**

This paper considers an environment where two principals sequentially contract with a common agent and studies the exchange of information between the two bilateral relationships. We show that when (a) the upstream principal is not personally interested in the decisions taken by the downstream principal, (b) the agent's exogenous private information has a "vertical" structure in the sense that the sign of the single crossing condition is the same for upstream and downstream decisions, and (c) preferences in the downstream relationship are separable, then the upstream principal optimally commits to full privacy, whatever price the downstream principal is willing to pay to receive information. On the contrary, when any of the above conditions is violated, the upstream principal may find it strictly optimal to disclose a (noisy) signal of the agent's exogenous type and/or the result of his upstream contractual activity, even if she can not make the downstream principal pay for the information she receives. We also show that disclosure does not necessarily reduce the equilibrium payoff of the agent and may lead to a Pareto improvement for the three players.

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# 1 Introduction

In many contracting environments, multiple principals sequentially interact with the same agent.[1] In organizations, for example, a division manager (in the role of an upstream principal) who hires a worker (agent) typically expects the latter to change employer after a while and pass under the control of other managers (downstream principals). In politics, the ruling administration (upstream principal) that signs a procurement contract with a contractor (agent), or that offers a trade policy to a lobby, expects its counterpart to contract also with the next appointed administration (downstream principal). Similarly, in commerce, a seller (principal) who sets up a menu of contract offers, expects her buyers (agents) to procure products and services also from other vendors. In financial contracting, a venture capitalist or an investment bank (upstream principal) who offers a contract to a start-up (agent) anticipates that the firm will also contract with other investors, as well as with suppliers, retailers and, perhaps, regulatory agencies (downstream principals).

In environments characterized by sequential bilateral contracting, a (downstream) principal who offers a contract to an agent makes the best possible use of any information that derives from the agent's upstream contractual experiences. First, the surplus in a downstream relationship may well depend on upstream decisions. For example, the ability, or the cost, for a worker to perform a task is likely to depend on the activities the agent performed on behalf of his previous employers. Similarly, in a trade relationship, the willingness to pay for a certain product or service usually depends on the complementarity or substitutability with products and services procured from upstream suppliers. When this is the case, a downstream principal is likely to be interested in the *endogenous information* the agent may possess about upstream decisions. Second, the observation of the result of upstream contractual activities, whenever possible, is also a useful signal of the agent's *exogenous private information* (i.e. information the agent may have acquired prior to any contractual relationship). For example, the terms of a financial contract between a venture capitalist and an entrepreneur may convey information to downstream investors about the probability of success of the project as well as the personal characteristics of the entrepreneur, information that is likely to affect the result of downstream financial contracting. Similarly, in a trade relationship, the history of past purchases of a consumer may reveal information about the consumer's preferences and may thus influence the personalized offers a consumer receives from downstream vendors.

In a sequential bilateral contracting environment, an upstream principal is thus likely to take advantage of her Stackelberg position by designing the upstream relationship in such a way that optimally controls for the influence it has on downstream contracting. There are two ways a contract can affect another one: directly, through the decisions that are stipulated (*contractual externalities*) and indirectly, through the information that the contract discloses (*informational externalities*). In this paper we investigate how a principal should control for both informational and contractual externalities by designing a mechanism that screens the agent's exogenous type and strategically

[1] The distinction between a principal and an agent is often just conventional: in what follows, we will refer to a principal as the party who offers the contract. We also adopt the convention of using masculine pronouns for the agent and female pronouns for the principals.

discloses exogenous and endogenous information to a downstream principal.

Our first result shows that when (a) the upstream principal is not personally interested in the decisions taken by the downstream principal, (b) the agent's exogenous private information has a "vertical" structure in the sense that the sign of the single crossing condition is the same for upstream and downstream decisions, and (c) preferences in the downstream relationship are (additive) separable in the two contractual decisions, then the optimal disclosure policy consists in keeping all information secret. This is true even if the upstream principal can sell information to the downstream principal.

Note that when the agent and the downstream principal's preferences are separable, the optimal contract in the downstream relationship does not depend on upstream decisions. The only benefits from influencing downstream contracting by disclosing information about the agent's exogenous type then come from either an *information trade* effect, i.e. the possibility to make the downstream principal pay for the information she receives, and/or a *rent shifting* effect, namely the possibility for the upstream principal to increase the agent's rent in the downstream relationship by disclosing a noisy signal of his type. Both effects may well be positive. To illustrate the rent shifting effect, consider the following example. Suppose there are two sellers who sequentially contract with a common buyer. Suppose the buyer has either a low or a high valuation for the downstream product. If the downstream seller's prior beliefs assign high probability to the agent's high valuation (for example because the proportion of high-valuation buyers is significantly high), then the optimal price in the downstream relationship will be equal to the agent's high valuation and leave no surplus to either type. The upstream principal can then adopt a policy which discloses two signals, say $s_1$ and $s_2$, as function of the agent's type. Suppose she discloses signal $s_2$ with certainty when the agent reports he is a low type and with probability $\delta$ when he reports he is a high type. If $\delta$ is sufficiently low, $s_2$ becomes informative of the low type and induces the downstream seller to reduce her price. Furthermore, when $s_2$ is disclosed with positive probability also when the agent is a high type – that is when $\delta \in (0, 1)$ – it gives the latter a strictly positive rent in the downstream relationship. The *rent shifting* effect then consists in making the agent pay a higher price for the increase in his expected utility with the second principal.

However, due to the asymmetry of information between the upstream principal and the agent, the latter must be provided with incentives to truthfully reveal his type. These incentives, in the form of an informational rent, are a function of the disclosure policy adopted by the principal. We show that when the sign of the single crossing condition in the agent's preferences is the same for upstream and downstream decisions, the increase in the rent the upstream principal must leave to the agent when she discloses information always offsets both the rent shifting and the information trade effects. As a consequence, when the upstream principal is not personally interested in downstream decisions, there is no advantage in disclosing information and the optimal policy consists in committing to full privacy.

We next investigate in which environments disclosure can be profitable for the upstream principal. As a converse to the previous result, we prove that when any of the above three conditions is relaxed, there exist preferences for which disclosure is strictly optimal for the upstream principal, even in the least favorable case where she is not able to make the downstream principal pay for the information she receives (that is, in the absence of any information trade effect). We examine separately the determinants for the disclosure of exogenous information and the determinants for the disclosure of endogenous information. To this aim, we first consider environments where the agent and the downstream principal's preferences are separable, so that disclosure is uniquely about the agent's exogenous type. We show that when the upstream principal is personally interested in the decisions of the downstream principal, she may well accept to pay the incentive costs of disclosure if this leads to more favorable outcomes in downstream contracting. We then relax the assumption of constant sign of the single crossing condition and show that when the decisions of the two principals are horizontally differentiated in the agent's preferences so that a higher valuation for the upstream decision signals a low valuation for the downstream, disclosing information does not necessarily increase the rent the upstream principal must leave to the agent; it may actually help reducing it. Indeed, by increasing the rent in the downstream relationship for those types that value the upstream decision the least, disclosure creates useful countervailing incentives which help reducing the payoff differential across types which in turn allows the upstream principal to extract more surplus from the agent.

Finally, in the last part of the paper, we relax the separability assumption and consider environments where the downstream principal is interested in receiving information about upstream decisions. To isolate disclosure of endogenous information from disclosure of exogenous information, we assume the agent's type does not influence the marginal surplus in the downstream relationship, so that disclosure is uniquely about upstream decisions and is motivated by the nonseparability of the agent's preferences. We show that by introducing sufficient uncertainty in upstream contracting (for example through lotteries, mixed strategies, or simply by taking different decisions with different types), the upstream principal may create a rent for the agent vis a vis the downstream principal. However, since lotteries on upstream decisions are costly for they induce inefficient trade, the optimal mechanism may also require the adoption of a policy that discloses information about upstream decisions to the downstream principal. We show that the strategy of combining endogenous uncertainty with disclosure may pay both when the decisions of the two principals are *complements* as well as *substitutes* in the agent's preferences.

For each environment discussed above, we also examine the effects of disclosure on individual payoffs and on total welfare, defined as the sum of expected utilities. We compare the equilibrium contracts when the upstream principal is not allowed, or able, to disclose information with the contracts that are offered in equilibrium when disclosure is possible. We show that, perhaps contrary to what one might have expected, disclosure need not harm the agent, it may actually increase his surplus in the two relationships. The effect of disclosure on welfare, is however in general ambiguous. By reducing the distortions in the downstream relationship that are due to the initial asymmetry of information, disclosure of exogenous information tends to increase efficiency in downstream contracting. At the same time, it may introduce novel distortions on upstream decisions required by incentive compatibility, that reduce efficiency in the upstream relationship. A similar result holds for disclosure of endogenous information: in this case, changes in upstream decisions introduced by the possibility to sustain downstream rents affect not only the surplus in the upstream relationship but also the value the agent and the downstream principal attach to downstream contracting.

**Outline.** The rest of the paper is organized as follows: Section 1.1 relates the paper to the literature; Section 2 describes the sequential contracting game and illustrates how the optimal policies can be obtained through a mechanism design approach; Section 3 contains the main theorem and analyses the possible determinants for information disclosure; Section 4 concludes. Technical proofs are confined to the Appendix.

## 1.1 Related Literature

This paper is related to a few lines of research in contract theory and industrial organization with asymmetric information.

**Information sharing and certification.** Strategic information sharing among firms has been examined in the literature of oligopolistic competition (see, for example Raith, 1998, for a survey) and in the financial intermediation literature (Padilla and Pagano, 1998, and Pagano and Jappelli, 1993, among others). In these papers, the informed firms can decide to share information with rivals before competing. Conversely, in our model upstream principals are initially uninformed, learn information by contracting with the agent, and also create new information by undertaking decisions that are relevant to downstream principals. Optimal disclosure policies have been analyzed also by Lizzeri (1999) in a model where certification intermediaries possess a technology to test the quality of the product of a seller and commit on what to disclose to competitive buyers. In this paper, we assume the only way a principal may learn the agent's private information is through a screening mechanism.

**Consumers' privacy.** A recent literature on consumers' privacy considers environments where sellers have the possibility to use information about individual purchase histories for product customization and price discrimination (Acquisti and Varian, 2002, Dodds, 2002, and Taylor, 2003a,b). Contrary to our paper, this literature however does not endogenize the choice of the disclosure policy.

**Auctions followed by downstream strategic interactions.** Informational linkages across markets have been studied also in the literature on auctions followed by resale or product market competition. Haile (1999) examines bidders' incentives to signal information to the secondary market in auctions followed by resale. Katzman and Rhodes-Kropf (2002) and Zhong (2002) study the effect of different bid announcement policies on the seller's expected revenue in auctions followed by Bertrand and Cournot competition. Calzolari and Pavan (2003) and Zheng (2002) study optimal auctions with resale and derive the monopolist's revenue-maximizing selling procedure and disclosure policy.

**Sequential common agency.** Sequential common agency models have been analyzed by Baron (1985), Bergman and Välimäki (2004), Martimort (1999), and Prat and Rustichini (1998). In these works, principals sequentially offer their contracts but decisions are taken only after the agent has received all proposals. On the contrary, in this paper the agent first contracts with an upstream principal, reveals exogenous information, takes a secret payoff relevant decision, and then enters into a new bilateral relationship with a second principal. This timing is more suitable to examine the design of optimal disclosure policies from the upstream principal's viewpoint.

**Contracting with externalities.** Segal (1999), and Segal and Whinston (2003) provide a general and unifying framework for contracting with direct multilateral externalities and show how they can result in inefficiencies in the equilibrium contracts. Martimort and Stole (2003) also consider the role of direct externalities in a simultaneous common agency game. In this paper, we combine direct externalities with *informational externalities* and show how they are optimally fashioned through the design of disclosure policies.

# 2 The Contracting Environment

## 2.1 Model Description

**Players.** A single agent, $A$, sequentially contracts with two principals, $P_1$ and $P_2$.[2] In what follows we will often find it convenient to think of $P_1$ and $P_2$ as two differentiated sellers.

**Allocations and Preferences.** Each principal contracts with $A$ over a *decision* $x_i \in X_i$ and a *transfer* $t_i \in T_i \equiv \mathbb{R}$ from $A$ to $P_i$. The vector $\mathbf{x} \equiv (x_1, x_2) \in \mathbf{X} \equiv X_1 \times X_2$ will denote a profile of decisions for the two principals. The agent's preferences are represented by the payoff function $U_A = v_A(\mathbf{x}, \theta) - t_1 - t_2$, whereas the two principals' preferences by $U_i = v_i(\mathbf{x}, \theta) + t_i$, for $i = 1, 2$. The variable $\theta \in \Theta$ constitutes the agent's exogenous private information. We will assume that $X_i$ and $\Theta$ are finite sets with $X_i = \{0, 1\}$ and $\Theta \equiv \{\underline{\theta}, \overline{\theta}\}$. $x_i = 0$ will denote the "status quo" i.e. $x_i = 0$ in the absence of any contract between $A$ and $P_i$, whereas $x_i = 1$ the decision to trade. The two principals are assumed to share a common prior $\Pr(\overline{\theta}) = p = 1 - \Pr(\underline{\theta})$. To save on notation, we will let $\Delta\theta \equiv \overline{\theta} - \underline{\theta} > 0$, $\Delta_\theta v_i(\mathbf{x}, \theta) \equiv v_i(\mathbf{x}, \overline{\theta}) - v_i(\mathbf{x}, \underline{\theta})$, $\Delta_{x_1} v_i(\mathbf{x}, \theta) \equiv v_i(1, x_2, \theta) - v_i(0, x_2, \theta)$, and similarly for $\Delta_{x_2} v_i(\mathbf{x}, \theta)$ and $\Delta_\theta[\Delta_{x_1} v_i(\mathbf{x}, \theta)]$. Finally, we assume $v_A(\mathbf{x}, \theta) = v_1(\mathbf{x}, \theta) = v_2(\mathbf{x}, \theta) = 0$ for any $\theta \in \Theta$ if $\mathbf{x} = (0, 0)$.

That $\Theta$ and $X_i$ are finite sets simplifies the description of the stochastic mechanisms offered by the two principals. That they contain two elements is used only when we solve for the optimal lotteries. Note that our main result does not depend on the finiteness of $\Theta$ and $X_i$: as we show in the Appendix, Theorem 1 extends to environments where $\theta$ is continuously distributed over an interval $\Theta = [\underline{\theta}, \overline{\theta}]$ as well as to $X_1 = X_2 = \mathbb{R}_+$.

**Contracts and Privacy Policies.** Each principal offers the agent a mechanism (hereafter also referred to as a contract). A mechanism $\phi_2 \in \Phi_2$ for $P_2$ consists of a message space $\mathcal{M}_2$ along with a mapping from $\mathcal{M}_2$ onto $X_2$ and $T_2$: formally, $\phi_2 : \mathcal{M}_2 \mapsto X_2 \times T_2$. Let $x_2(m_2) \in X_2$, and $t_2(m_2) \in T_2$ denote respectively the decision and the transfer associated with message $m_2$.[3] On her part, $P_1$ offers $A$ a mechanism $\phi_1 \in \Phi_1$ that is characterized by a message space $\mathcal{M}_1$, a set of signals

[2] The model can also be read as one in which there is a continuum of agents with independent types, provided there are no direct externalities among the agents and the principals' payoffs are additive in the trades with the different agents. See, for example, Taylor (2003a).

[3] Note that in this environment, $P_2$ never gains by offering a stochastic mechanism.

$S$ that $P_1$ will disclose to $P_2$, and a mapping $\phi_1 : \mathcal{M}_1 \mapsto \Delta(X_1 \times S) \times T_1$. $\delta_1(m_1) \in \Delta(X_1 \times S)$ and $t_1(m_1) \in T_1$ stand for the joint lottery over $X_1 \times S$ and the expected transfer associated with message $m_1 \in \mathcal{M}_1$. With the standard abuse of notation in mechanism design, $\delta_1(x_1, s|m_1)$ will denote the conditional probability of decision $x_1$ and signal $s$, given $m_1$. The mechanism $\phi_1$ embeds a *disclosure policy* $d : \mathcal{M}_1 \to \triangle(S)$. When the agent chooses message $m_1$, $P_1$ discloses signal $s$ to $P_2$ with probability $d(s|m_1) = \sum_{x_1 \in X_1} \delta_1(x_1, s|m_1)$. We do not assign any specific meaning to the set $S$, but we assume it is sufficiently rich to generate any desired system of posterior beliefs of $P_2$: as we show below, since $\Theta$ and $X_1$ are finite, it will suffice to treat $S$ also as a finite set. This abstract representation of information transmission between the two contractual relationships allows to replicate fairly general disclosure policies without imposing a priori restrictions. Note that the mechanism $\phi_1$ is (possibly) stochastic for two reasons: First, $P_1$ may want to create uncertainty about the decision $x_1$ in order to fashion the contract offered by $P_2$; second, it may be in the interest of $P_1$ not to reveal to $P_2$ all the information disclosed in the upstream relationship. In other words, $P_1$ may find it optimal to disclose to $P_2$ only a noisy signal of $(\theta, x_1)$.[4] $P_1$ is not exogenously compelled to release any particular information, so that she can select the disclosure policy she wants.[5]

We assume each principal can perfectly commit to her mechanism, which also implies that $P_1$ can credibly commit to the disclosure policy of her choosing. With this assumption we rule out two possible scenarios. In the first, $P_1$ discloses to $P_2$ more information than allowed by the contract $\phi_1$. In the second, $P_1$ announces to $P_2$ a disclosure policy $d$, but then secretly offers $A$ a side contract characterized by a different policy. As standard in common agency games, we also assume each principal cannot contract over the decisions of the other principal.

Finally, we denote with $\tau(\phi_1)$ the price $P_2$ pays to observe the signals disclosed by $\phi_1$. We want $\tau(\phi_1)$ to be the price for information and not for the distribution over $X_1$. To this aim, we assume $\tau(\phi_1)$ is contracted after $\phi_1$ has been executed, so that $P_1$ can not threat $P_2$ to take different decisions in case she does not pay $\tau$. Instead of modelling explicitly a bargaining game between $P_1$ and $P_2$, for the scope of our analysis, it will suffice to define a set of reasonable rational prices that can be the result of possible bargaining games. We do it in the following way. Let $\mathbb{E}U_2(\phi_1)$ be the expected payoff for $P_2$ in the continuation game where she observes the signals $s$ disclosed by $\phi_1$ and $\mathbb{E}U_2^{ND}(\phi_1)$ in the continuation game in which she does not. We define the set of rational prices as $\tau(\phi_1) = \gamma[\mathbb{E}U_2(\phi_1) - \mathbb{E}U_2^{ND}(\phi_1)]$ for $\gamma \in [0, 1]$. The parameter $\gamma$ captures the fraction of the value $P_2$ attaches to the information disclosed by $\phi_1$ that can be appropriated by $P_1$ through the price $\tau(\phi_1)$. Clearly, $\tau(\phi_1) = 0$ for any $\gamma$ if $\phi_1$ does not disclose any information.

**Timing: a sequential contracting game**

- At $\mathbf{t = 0}$, $A$ privately learns $\theta$.
- At $\mathbf{t = 1}$, $P_1$ commits to a public mechanism $\phi_1 \in \Phi_1$. If $A$ rejects $\phi_1$, the game ends and all players are left with their reservation payoffs that are normalized to zero. If, on the contrary, $A$

[4] Because of quasi-linearity, $P_2$ is never interested in learning $t_1$.

[5] If $P_1$ were obliged to disclose $m_1$, she might find it optimal to induce $A$ to randomize over $\mathcal{M}_1$ (see Bester and Strausz, 2001, and Laffont and Tirole 1990, for dynamic models where a principal lacking of commitment for future decisions induces the agent to randomize in order to reduce the information revealed in the early contracting stages).

accepts $\phi_1$, he chooses a message $m_1$, pays an expected transfer $t_1(m_1)$, a decision $x_1 \in X_1$ is taken with probability $\delta_1(x_1|m_1) = \sum_{s\in S} \delta_1(x_1, s|m_1)$, and a signal $s \in S$ is selected with probability $d(s|m_1) = \sum_{x_1\in X_1} \delta_1(x_1, s|m_1)$. The realization of the joint lottery $\delta_1(m_1)$ is commonly observed by $A$ and $P_1$.
- At $\mathbf{t} = \mathbf{2}$, $P_2$ pays $\tau(\phi_1)$, receives information $s$ from $P_1$ and offers $A$ her own mechanism $\phi_2 \in \Phi_2$. If $A$ rejects $\phi_2$, the game is over. Otherwise, $A$ reports a message $m_2$ which induces a decision $x_2(m_2)$ and a transfer $t_2(m_2)$.

That it is optimal for $P_1$ to make her mechanism public is proved in Pavan and Calzolari (2003): they show that any outcome of a mixed strategy equilibrium where $P_1$ randomizes over different mechanisms and discloses the realizations of the mixed strategy only to the agent can be replicated through a *stochastic* public mechanism which is disclosed also to $P_2$, but whose realization (the extended type discussed in the next section) remains the agent's private information. It is important to note that the fact that $\phi_1$ is public implies that $P_2$ can observe the mapping $\phi_1 : \mathcal{M}_1 \mapsto \Delta(X_1 \times S) \times T_1$, *but not* $m_1$ and $x_1$.

That the game ends after $A$ rejects $\phi_1$ is clearly not without loss of generality. However, note that in a game where $A$ can contract with $P_2$ after rejecting $\phi_1$, there exist equilibria where $P_1$ informs $P_2$ about the agent's decision to reject $\phi_1$, such that all types obtain zero surplus with $P_2$ out-of-equilibrium. These equilibria also satisfy forward induction refinements such as the *intuitive criterion* of Cho and Kreps (1987). Instead of relying on equilibrium selection arguments to determine $A$'s outside option with $P_1$, given the focus of the analysis, we prefer to assume it is exogenously fixed to zero.

**Strategies and equilibrium**

The game described above is a sequential version of the simultaneous common agency games with adverse selection examined in Martimort (1992), Martimort and Stole (2002, 2003), and Stole (1991). A strategy for $P_1$ is simply the choice of a mechanism $\phi_1 \in \Phi_1$. For $P_2$, a strategy $\phi_2(\phi_1, s)$ is a mapping from $\phi_1$ and $s$ into the set of feasible contracts $\Phi_2$.[6] The agent's strategy, $\boldsymbol{\phi}_A = (\phi_A^1, \phi_A^2)$, specifies the reports to each principal as a function of the agent's information set, i.e. $m_1 = \phi_A^1(\theta, \phi_1)$, and $m_2 = \phi_A^2(\theta, \phi_1, m_1, x_1, t_1, s, \phi_2)$. Because of their appeal in most applications, we will limit attention to Markov strategies in which the agent's behavior with $P_2$ depends only on the mechanism $\phi_2$ and the payoff-relevant component of the history $h_2 \equiv (\theta, \phi_1, m_1, x_1, t_1, s, \phi_2)$; i.e. we restrict attention to strategies such that $m_2 = \phi_A^2(\theta, x_1, t_1, \phi_2)$.

A strategy profile $\phi = (\phi_1, \phi_2, \phi_A)$ is a *perfect Bayesian equilibrium* if and only if: each principal selects a mechanism that is sequentially optimal given the agent's and the other principal's strategies; for each signal $s$ on the equilibrium path, $P_2$ updates her beliefs using Bayes' rule; $A$ announces only payoff-maximizing messages.

[6] Although $\phi_2$ depends on $\phi_1$, the feasibility of the decisions contemplated in $\phi_2$ does not depend on the particular decision $x_1$. This is a restriction. Calzolari and Pavan (2003), for example, consider the design of optimal disclosure policies for an auctioneer that expects her buyers to resell in a secondary market. As resale can take place only if a buyer received the good in the primary market, the feasibility of an allocation in the secondary market depends on the decisions taken in the primary market, so that the above assumption is clearly violated in auctions followed by resale.

## 2.2 Contracts Design

In games where agents contract with multiple principals, using the (standard version of) the Revelation Principle to construct equilibria is not without loss of generality [Epstein and Peters (1999), Martimort and Stole (2002), and Peters (2001)]. Among the possible solutions that have been recently proposed in the literature, here we follow Pavan and Calzolari (2003). They show that the entire equilibrium set of any common agency game can be characterized by restricting attention to Markovian direct mechanisms in which the message space $\mathcal{M}_i = \Theta_i^E$ coincides with the extended type space and it includes only payoff-relevant variables. In the case competition among the principals is sequential, preferences are quasi-linear, and the agent follows Markov strategies, the extended type space reduces to $\Theta_1^E \equiv \Theta$ and $\Theta_2^E \equiv \Theta \times X_1$.

Consider the extended type $\theta_2^E = (\theta, x_1)$. Let $v_A\left(x_2, \theta_2^E\right)$ be the value $A$ attaches to $x_2$ when he has type $\theta$ and decision $x_1$ has been taken in the upstream relationship, i.e. $v_A\left(x_2, \theta_2^E\right) \equiv v_A(x_1, x_2, \theta)$. Similarly, $v_2\left(x_2, \theta_2^E\right) \equiv v_2(x_1, x_2, \theta)$. Then for any $s \in S$, we let[7]

$$U_A^2(\theta_2^E; s) \equiv v_A\left(x_2(\theta_2^E; s), \theta_2^E\right) - v_A\left(0, \theta_2^E\right) - t_2(\theta_2^E; s)$$

denote the surplus $A$ expects from $P_2$ when he truthfully reports his extended type $\theta_2^E = (\theta, x_1)$, and

$$U_A^2(\theta_2^E, \widehat{\theta}_2^E; s) \equiv v_A(x_2(\widehat{\theta}_2^E; s), \theta_2^E) - v_A\left(0, \theta_2^E\right) - t_2(\widehat{\theta}_2^E; s)$$

the corresponding payoff when he announces $\widehat{\theta}_2^E \neq \theta_2^E$. Furthermore, let

$$S\left(d; \phi_1\right) \equiv \{s : d(s|\theta) > 0 \text{ for some } \theta \in \Theta\}$$

denote the set of signals in the range of the disclosure policy $d$ induced by the mechanism $\phi_1$. For any truthful mechanism $\phi_1$ and signal $s \in S\left(d; \phi_1\right)$, $P_2$'s posterior beliefs about $\theta_2^E = (\theta, x_1)$ will be denoted by

$$\mu(\theta_2^E; s) \equiv \Pr((\theta, x_1)|s) = \frac{\delta_1(x_1, s|\theta)\Pr(\theta)}{\sum_{\theta \in \Theta}\sum_{x_1 \in X_1}\left[\delta_1(x_1, s|\theta)\right]\Pr(\theta)}.$$

An optimal mechanism for $P_2$, $\phi_2(s) = (x_2(\theta_2^E; s), t_2(\theta_2^E; s))$, then solves the following program

$$\mathcal{P}_2(s) : \begin{cases} \max\limits_{\phi_2 \in \Phi_2} \sum\limits_{\theta_2^E \in \Theta_2^E} \left[v_2\left(x_2(\theta_2^E; s), \theta_2^E\right) + t_2(\theta_2^E; s)\right]\mu(\theta_2^E; s) \\ \text{such that for } \textit{any } \theta_2^E \text{ and } \widehat{\theta}_2^E \in \Theta_2^E \\ U_A^2(\theta_2^E; s) \geq 0, & (IR_2) \\ U_A^2(\theta_2^E; s) \geq U_A^2(\theta_2^E, \widehat{\theta}_2^E; s). & (IC_2) \end{cases}$$

The individual rationality constraints $(IR_2)$ guarantee that $A$ accepts $\phi_2$, while the incentive compatibility constraints $(IC_2)$ that he has the correct incentives to truthfully announce his extended

[7] To simplify the notation, in what follows we will omit the dependence of $\phi_2(\phi_1, s) = (x_2(\theta_2^E; s, \phi_1), t_2(\theta_2^E; s, \phi_1))$ on $\phi_1$.

type. Note that $U_A^2(\theta_2^E; s)$ is defined as the *additional* surplus $A$ obtains from contracting with $P_2$; that is, his total surplus, net of the payoff from the interaction with $P_1$. Also, note that we assume there is no way $A$ can credibly disclose $(x_1, t_1)$ to $P_2$ so that the latter has to give him incentives for truthful information revelation.

At $t = 1$, $P_1$ designs a mechanism $\phi_1$ and a reaction $\phi_2(s)$ that solve

$$\mathcal{P}_1 : \begin{cases} \max\limits_{\substack{\phi_1 \in \Phi_1 \\ \phi_2(s) \in \Phi_2}} \sum\limits_{\theta \in \Theta} \left\{ \sum\limits_{x_1 \in X_1} \sum\limits_{s \in S} \left[ v_1(x_1, x_2(\theta_2^E; s), \theta) \right] \delta_1(x_1, s|\theta) + t_1(\theta) \right\} \Pr(\theta) + \tau(\phi_1) \\ \text{subject to} \\ U_A(\theta; \phi_1) \equiv \sum\limits_{x_1 \in X_1} \sum\limits_{s \in S} \left[ v_A(x_1, 0, \theta) + U_A^2(\theta_2^E; s) \right] \delta_1(x_1, s|\theta) - t_1(\theta) \geq 0, \text{ for any } \theta \in \Theta, & (IR_1) \\ U_A(\theta; \phi_1) \geq \sum\limits_{x_1 \in X_1} \sum\limits_{s \in S} \left[ v_A(x_1, 0, \theta) + U_A^2(\theta_2^E; s) \right] \delta_1(x_1, s|\hat{\theta}) - t_1(\hat{\theta}), \text{ for any } \theta \text{ and } \widehat{\theta} \in \Theta, & (IC_1) \\ \phi_2(s) \text{ solves } \mathcal{P}_2(s) & (SR) \end{cases}$$

In addition to standard individual rationality and incentive compatibility constraints for the agent, the $(SR)$ constraint in $\mathcal{P}_1$ guarantees the sequential rationality of $P_2$'s reaction $\phi_2(s)$. Note that treating $\phi_2(s)$ as a choice variable in $\mathcal{P}_1$ amounts to selecting among all possible equilibria the one which is most favorable to $P_1$. This selection is clearly arbitrary, but consistent with standard mechanism design analysis: we want to examine the properties of upstream and downstream decisions which maximize $P_1$'s expected payoff under minimal sequential rationality constraints for the agent $[(IR_1)$ and $(IC_1)]$ and for the downstream principal $[(SR)]$ — For similar selection arguments in dynamic contracting with a single principal, see Laffont and Tirole (1990).

## 3 Optimal Disclosure Policies

In this section we characterize the solution to $\mathcal{P}_1$ and discuss its implications in terms of disclosure policies. Before illustrating the properties of the optimal mechanisms, we find it useful to introduce some formal definitions. We start with the notion of disclosure and of optimal mechanisms which induce it.

**Definition 1** .

*$P_1$'s mechanism discloses information if and only if it assigns positive measure to signals that lead to different posterior beliefs over $\Theta_2^E$ : Formally, there exist signals $s_l \in S(d; \phi_1)$ and $s_m \in S(d; \phi_1)$, with $s_l \neq s_m$, such that $\mu(\theta_2^E; s_l) \neq \mu(\theta_2^E; s_m)$ for some $\theta_2^E \in \Theta_2^E$.*

*Information disclosure is considered optimal for $P_1$ if and only if there exists a mechanism $\phi_1$ that discloses information and solves $\mathcal{P}_1$, and there are no other solutions to $\mathcal{P}_1$ that induce no disclosure.*

Next, we define the properties of individual preferences that will play a role for disclosure.

**Definition 2** .

***Independence.*** *Player i's preferences are independent of* $x_j$ *if* $v_i(x_i, x_j, \theta) = v_i(x_i, \theta)$.

***(Additive) Separability.*** *Player i's preferences are separable in* $x_1$ *and* $x_2$ *if* $v_i(x_1, x_2, \theta) = v_i^1(x_1, \theta) + v_i^2(x_2, \theta)$.

***Sign of Single Crossing Condition.*** *The single crossing condition in the agent's preferences has the same sign for* $x_1$ *and* $x_2$ *if a higher (lower)* $\theta$ *indicates a higher (lower) valuation for both* $x_1$ *and* $x_2$*, i.e.* $sign\{\Delta_\theta[\Delta_{x_1} v_A(\mathbf{x}, \theta)]\} = sign\{\Delta_\theta[\Delta_{x_2} v_A(\mathbf{x}, \theta)]\}$ *for any* $x_1$ *and* $x_2$.

We are now ready to state the main result.

**Theorem 1** .

*Part (i). Assume the following hold: (a)* $P_1$*'s preferences are independent of* $x_2$*, (b) the sign of the single crossing condition in the agent's preferences is the same for upstream and downstream decisions, (c)* $P_2$ *and A's preferences are separable. Then, no disclosure is optimal for* $P_1$ *for any rational price* $\tau(\phi_1)$ *that* $P_2$ *is willing to pay to receive information from* $P_1$.

*Part (ii). When any of the conditions in (i) is violated, there exist preferences for which disclosure is optimal for* $P_1$*, even if she does not sell information to* $P_2$*, that is even if* $\tau(\phi_1) = 0$ *for any* $\phi_1$.

Condition (a) implies that $P_1$ is not personally affected by $x_2$ and condition (b) that the agent's private information is of a "vertical" form; without loss of generality, we assume the sign of the single crossing condition is positive in either relationship. When preferences are separable and $X_i = \{0, 1\}$, this is equivalent of saying that the values $v_A^1(1, \theta)$ and $v_A^2(1, \theta)$ the agent attaches to the products, or services, of the two principals are both increasing in $\theta$.[8] Condition (c) implies the reaction $\phi_2(s) = \left(x_2(\theta; s), U_A^2(\theta; s)\right)$ does not depend on $x_1$. It follows that under (a)-(c), the only benefits from influencing downstream decisions by disclosing information about $\theta$ come from (i) a *rent shifting* opportunity, namely the possibility to induce $P_2$ to leave $A$ a larger rent and then appropriate (part of) it through the transfer $\bar{t}_1$, and/or (ii) an *information trade* effect, i.e. the possibility to make $P_2$ pay a price $\tau(\phi_1)$ for the information she receives about $\theta$.

Let $W_2(x_2, \theta) \equiv v_2^2(x_2, \theta) + v_A^2(x_2, \theta)$ denote the downstream surplus, and $\phi_2^{ND} \equiv (x_2^{ND}(\theta), U_A^{2ND}(\theta))$ the mechanism $P_2$ offers in case she does not receive information from $P_1$. When preferences in the downstream relationship are separable, $\phi_2^{ND}$ does not depend on $\phi_1$. As illustrated in the Appendix, the proof for this result consists in showing that for any individually rational and incentive compatible mechanism $\phi_1$ – with reaction $\phi_2(s)$ – there exists another individually rational and incentive compatible mechanism $\phi_1^{ND}$ – with reaction $\phi_2^{ND}$ – which does not disclose information, it induces the same distribution over $X_1$, and such that[9]

$$\begin{aligned}
&\mathbb{E}U_1(\phi_1) - \mathbb{E}U_1(\phi_1^{ND}) = (1-\gamma) \sum_{\theta\in\Theta} \left[ \sum_{s\in S} U_A^2(\theta; s) d(s|\theta) - U_A^{2ND}(\theta) \right] \Pr(\theta) + \\
&+\gamma \sum_{\theta\in\Theta} \left[ \sum_{s\in S} W_2(x_2(\theta; s), \theta) d(s|\theta) - W_2(x_2^{ND}(\theta), \theta) \right] \Pr(\theta) + \qquad (1) \\
&- \sum_{\theta\in\Theta} \left[ U_A(\theta; \phi_1) - U_A\left(\theta; \phi_1^{ND}\right) \right] \Pr(\theta) \le 0.
\end{aligned}$$

[8] This follows from the fact that $v_i^1(0, \theta) = v_i^2(0, \theta) = 0$.

[9] To compact notation, we omit the dependence of $\mathbb{E}U_1(\phi_1)$ and $U_A(\theta; \phi_1)$ on $\phi_2$.

Clearly, when $\gamma = 0$, the information trade effect is absent, and the only benefit from disclosure must come from the rent shifting effect, which corresponds to the first term in (1). Conversely, when $\gamma = 1$, the rent shifting motivation vanishes since any additional dollar $P_1$ can extract from $A$ for the rent he expects from downstream contracting must be deducted from the price $\tau(\phi_1)$ that can be charged to $P_2$. In this case, the benefit from disclosure must come from the possibility to increase efficiency in the downstream relationship, as indicated in the second term in (1). Both the rent shifting and the information trade effects might well be positive. However, disclosure also affects the rents $P_1$ must leave to the agent in order to induce him to truthfully reveal his type, as indicated in the last term in (1). Under (b) and (c), if $\phi_1$ is optimal, then necessarily $U_A\left(\underline{\theta};\phi_1\right) = 0$ and

$$U_A\left(\overline{\theta};\phi_1\right) = \Delta_\theta v_A^1(1,\theta)\delta_1(1|\underline{\theta}) + \sum_{s\in S} U_A^2(\overline{\theta},s)d(s|\underline{\theta}) \geq 0,$$

where $\Delta_\theta v_A^1(1,\theta) = v_A^1(1,\overline{\theta}) - v_A^1(1,\underline{\theta})$. Among all mechanisms which induce the same distribution over $X_1$ as $\phi_1$ without disclosing information, consider a mechanism such that $U_A\left(\underline{\theta};\phi_1^{ND}\right) = 0$ and

$$U_A\left(\overline{\theta};\phi_1^{ND}\right) = \Delta_\theta v_A^1(1,\theta)\delta_1^{ND}(1|\underline{\theta}) + U_A^{2ND}(\overline{\theta}).$$

It is possible to show that if $\phi_1$ is individually rational and incentive compatible, so is $\phi_1^{ND}$. Furthermore,

$$\begin{aligned}\sum_{\theta\in\Theta}\left[U_A\left(\theta;\phi_1\right) - U_A\left(\theta;\phi_1^{ND}\right)\right]\Pr(\theta) &= p\left[U_A\left(\overline{\theta};\phi_1\right) - U_A\left(\overline{\theta};\phi_1^{ND}\right)\right] \\ &= p\left[\sum_{s\in S} U_A^2(\overline{\theta};s)d(s|\underline{\theta}) - U_A^{2ND}(\overline{\theta})\right].\end{aligned} \tag{2}$$

When (b) and (c) hold, (2) is positive and thus disclosure increases the expected rent $P_1$ must leave to $A$. What is more, this third effect always dominates the first two, making information disclosure undesirable from $P_1$'s viewpoint. To see this, consider first $\gamma = 0$, so that there is no information trade effect, and suppose $\phi_1$ embeds a privacy policy $d(s|\theta)$ which discloses only two signals, $s_1$ and $s_2$, where signal $s_1$ is more informative of type $\bar{\theta}$ and $s_2$ of type $\underline{\theta}$; formally, let $d(s_1|\bar{\theta}) = d(s_1|\underline{\theta}) + \varepsilon$ and $d(s_2|\bar{\theta}) = d(s_2|\underline{\theta}) - \varepsilon$. Then, under (b), $P_2$ leaves no surplus to $\underline{\theta}$ and a rent to $\bar{\theta}$ equal to

$$U_A^2(\bar{\theta};s) = \Delta_\theta v_A^2(x_2(\underline{\theta};s),\theta) \tag{3}$$

which is increasing in the posterior odds $\mu(\underline{\theta};s)/\mu(\overline{\theta};s)$ and hence in $d(s|\underline{\theta})/d(s|\overline{\theta})$, so that[10]

$$\sum_{s\in S} U_A^2(\bar{\theta};s)d(s|\bar{\theta}) - \sum_{s\in S} U_A^2(\bar{\theta};s)d(s|\underline{\theta}) = \left[U_A^2(\bar{\theta};s_1) - U_A^2(\bar{\theta};s_2)\right]\varepsilon \leq 0.$$

This result clearly generalizes to disclosure policies $d(s|\theta)$ with more than two signals: The most profitable signals for $A$, i.e. the signals that lead to downstream rents for $\overline{\theta}$, are always disclosed

[10] That $U_A^2(\bar{\theta};s)$ increases in $\mu(\underline{\theta};s)/\mu(\overline{\theta};s)$ follows from the fact that $x_2(\underline{\theta},s)$ solves (4), that is $x_2 = 1$ if $\mu(\underline{\theta};s)W_2(1,\underline{\theta}) - \mu(\overline{\theta};s)\Delta_\theta v_A^2(1,\theta) \geq 0$ and $x_2 = 0$ otherwise.

with a higher probability when $A$ announces $\underline{\theta}$ than $\bar{\theta}$. It follows that when $\gamma = 0$, the net effect of disclosure on $P_1$'s payoff is

$$\mathbb{E}U_1(\phi_1) - \mathbb{E}U_1(\phi_1^{ND}) = p \sum_{s \in S} U_A^2(\bar{\theta}; s)[d(s|\overline{\theta}) - d(s|\underline{\theta})] \leq 0.$$

Next, consider the information trade effect ($\gamma = 1$), i.e. the possibility to increase $\tau(\phi_1)$ by reducing the distortions in downstream contracting which are due to the asymmetry of information. In any optimal downstream mechanism, the decisions $x_2(\overline{\theta}; s)$ for the high type are never distorted and are independent of $s$, whereas the decisions for the low type solve the efficiency versus rent extraction trade off

$$x_2(\underline{\theta}; s) = \arg \max_{x_2 \in X_2} \left\{ \mu(\underline{\theta}; s) W_2(x_2, \underline{\theta}) - \mu(\overline{\theta}; s) \Delta_\theta v_A^2(x_2, \theta) \right\}. \tag{4}$$

It follows that when $\gamma = 1$,

$$\begin{aligned} &\mathbb{E}U_1(\phi_1) - \mathbb{E}U_1(\phi_1^{ND}) = \\ &(1-p) \left[ \sum_{s \in S} W_2(x_2(\underline{\theta}; s), \underline{\theta}) d(s|\underline{\theta}) - W_2(x_2^{ND}(\underline{\theta}), \underline{\theta}) \right] - p \left[ \sum_{s \in S} U_A^2(\overline{\theta}; s) d(s|\underline{\theta}) - U_A^{2ND}(\overline{\theta}) \right]. \end{aligned}$$

Using $U_A^2(\bar{\theta}; s) = \Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta)$ and $U_A^{2ND}(\overline{\theta}) = \Delta_\theta v_A^2(x_2^{ND}(\underline{\theta}), \theta)$, the above reduces to

$$\begin{aligned} &\sum_{s \in S} [(1-p) W_2(x_2(\underline{\theta}; s), \underline{\theta}) - p \Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta)] d(s|\underline{\theta}) + \\ &- \left[ (1-p) W_2(x_2^{ND}(\underline{\theta}), \underline{\theta}) - p \Delta_\theta v_A^2(x_2^{ND}(\underline{\theta}), \theta) \right] \end{aligned}$$

which is negative since in the absence of disclosure $P_2$ chooses a decision

$$x_2^{ND}(\underline{\theta}) = \arg \max_{x_2 \in X_2} \left\{ (1-p) W_2(x_2, \underline{\theta}) - p \Delta_\theta v_A^2(x_2, \theta) \right\}.$$

We thus conclude that under (a)-(c), disclosure is never optimal for $P_1$: for any mechanism $\phi_1$ that discloses information, there exists another mechanism $\phi_1^{ND}$ that induces the same distribution over $X_1$ without disclosure, such that $\mathbb{E}U_1(\phi_1^{ND}) \geq \mathbb{E}U_1(\phi_1)$. This result does not depend on the discreteness of $\Theta$, $X_1$ and $X_2$. As we show in the Appendix, Theorem 1 extends to environments where $\theta$ is continuously distributed over $[\underline{\theta}, \overline{\theta}]$ and $X_i = \mathbb{R}_+$ for $i = 1, 2$, under the usual additional assumptions for the continuous case which guarantee that in the canonical single mechanism designer problem, the optimal policies $x_i(\theta)$ are deterministic with no bunching.

It is interesting to compare the result in Theorem 1-part (i) with Baron and Besanko (1984). They consider a dynamic single-principal single-agent relationship and show that when preferences are additive separable and type is constant over time, the optimal long term contract under full commitment consists in a sequence of static optimal contracts. Although at a superficial look, the two results may appear similar, they are substantially different. First, in Baron and Besanko, the principal maximizes the intertemporal payoff $v_1(\cdot) + v_2(\cdot)$, whereas in our setting $P_1$ maximizes only $v_1(\cdot)$. Second, even if $P_1$ were to maximize the joint surplus, she would not offer the static optimal contracts. Indeed, this would be the case if the payoff of the downstream principal were not only separable but also independent of $x_1$, as it is in Baron and Besanko. When instead it is only separable, as in Theorem 1, the contract that maximizes the two principals' joint surplus $v_1(x_1,\theta)+v_2^1(x_1,\theta)+v_2^1(x_2,\theta)$ is different than the sequence of independent contracts that are offered in equilibrium when $P_1$ does not disclose information to $P_2$.[11]

Finally, note that part (ii) in Theorem 1 provides a converse to part (i): it shows that when any of the three conditions in (i) is violated, disclosure may be optimal for $P_1$ even in the least favorable case where $\tau(\phi_1)=0$. This in turn explains the choice on the above conditions as possible determinants for information disclosure in multiple principals models. The proof follows from the results in the rest of this section where we relax each condition separately.

### 3.1 Disclosure of Exogenous Information

In this section we consider environments where the decisions in the downstream relationship are not sensitive to the allocation determined in the upstream relationship. As a consequence, $P_2$ is interested in receiving information about $x_1$ only if this is indirectly informative about $\theta$.

**Condition 1** *The agent's preferences are separable:* $v_A(x_1,x_2,\theta)=a(\theta)\,x_1+b(\theta)\,x_2$, *with* $\Delta b\equiv b(\overline{\theta})-b(\underline{\theta})>0$; $P_2$*'s preferences are independent of* $x_1$*:* $v_2(x_1,x_2,\theta)=m_2x_2$.

That the surplus in the downstream relationship depends on $\theta$ only through its effect on $A$'s preferences and that $P_2$'s payoff is independent of $x_1$ shortens the exposition without any serious effect on the results.[12] To make $P_2$ interested in receiving information about $\theta$, we also assume that $m_2+b(\underline{\theta})\geq 0$. That is, under complete information, contracting in the downstream relationship generates positive surplus for any $\theta$.

To save on notation, in what follows we let $\overline{a}\equiv a(\overline{\theta})$, $\underline{a}\equiv a(\underline{\theta})$, $\bar{b}\equiv b(\overline{\theta})$, and $\underline{b}\equiv b(\underline{\theta})$. With preferences as in Condition 1, the optimal contract $\phi_2(s)$ assigns the same allocation to the two extended types $\theta_2^E=(\theta,1)$ and $\theta_2^E=(\theta,0)$. Furthermore, $(IR_2)$ binds for $\theta=\underline{\theta}$ and $(IC_2)$ for $\theta=\overline{\theta}$. The optimal contract $\phi_2(s)$ thus consists in a simple take-it-or-leave-it offer at a price

$$t_2(s)=\begin{cases}\bar{b} & \text{if } \left(\bar{b}+m_2\right)\mu(\bar{\theta};s)>\underline{b}+m_2,\\ \underline{b} & \text{if } \left(\bar{b}+m_2\right)\mu(\bar{\theta};s)\leq\underline{b}+m_2,\end{cases}$$

where $\mu(\bar{\theta};s)=\Pr(\bar{\theta}|s)=\mu((\bar{\theta},1);s)+\mu((\bar{\theta},0);s)$. As a result, $P_1$ needs to disclose only two signals, $s_1$, and $s_2$, such that $t_2(s_1)=\overline{b}$, and $t_2(s_2)=\underline{b}$.[13] Hence, signal $s_1$ stands for any information that

[11] Under assumptions (a)-(c) these contracts correspond to the long run contract that a single principal with payoff $v_1(x_1,\theta)+v_2^1(x_2,\theta)$ would offer in Baron and Besanko. Indeed, $P_1$ does not internalize the externality of $x_1$ on $x_2$.

[12] First note that adding a term $q_2(\theta)\,x_1$ in $P_2$'s preferences does not affect the downstream decisions. Second, as in standard screening models, letting $m_2$ depend on $\theta$ does not add much to the analysis since the virtual surplus for the $P_2-A$ relationship already depends on $\theta$ through its effect on $A$'s payoff.

[13] It is immediate to prove that for any mechanism $\phi_1$ that discloses more than two signals, there exists another mechanism $\phi'$ that discloses at most two signals which is payoff equivalent for all players.

induces $P_2$ to offer a high price $\bar{b}$, whereas $s_2$ a low price, $\underline{b}$. From Bayes rule, this is compatible with $P_2$'s sequential rationality, if and only if $\phi_1$ satisfies the following constraints

$$\begin{array}{ll} d(s_1|\overline{\theta}) \geq Hd(s_1|\underline{\theta}), & (SR_1) \\ d(s_2|\overline{\theta}) \leq Hd(s_2|\underline{\theta}), & (SR_2) \end{array} \tag{5}$$

where $H \equiv \left(\frac{1-p}{p}\right)\left(\frac{m_2+\underline{b}}{\Delta b}\right)$. Given $s_{1,}$ trade in the downstream relationship occurs only if $\theta = \overline{\theta}$ and $A$ receives zero surplus, whereas, given $s_2$, trade occurs with both types, and $\overline{\theta}$ enjoys a downstream informational rent equal to $\Delta b$. When $H < 1$ – equivalently $p\left(\bar{b} + m_2\right) > \underline{b} + m_2$ – the severity of the adverse selection problem is such that $P_2$ asks a high price that leaves no surplus to the agent, if she receives no information from $P_1$. When this is the case, we will say that $P_2$'s prior beliefs are *unfavorable* to $A$. On the contrary, $P_2$'s beliefs are *favorable* when $H \geq 1$. Note that when beliefs are unfavorable, $(SR_1)$ is implied by $(SR_2)$ and no disclosure is formally equivalent to releasing only signal $s_1$, whereas the opposite is true with favorable beliefs in which case no disclosure corresponds to releasing only signal $s_2$.

### 3.1.1 Direct Externalities

Suppose now $P_1$ is personally affected by $x_2$; we want to show that when this is the case she may find it optimal to disclose information in order to fashion the decisions that will be taken in the downstream relationship, even if this comes at the cost of a higher rent for the agent. To illustrate, we assume the sign of the single crossing condition in the agent's preferences is the same for upstream and downstream decisions in which case $A$'s valuations for $x_1$ and $x_2$ are positively correlated. This guarantees that disclosure is costly for $P_1$ and hence, when optimal, it is necessarily motivated by the direct externality of $x_2$ on $U_1$.

**Condition 2** *$P_1$ is personally interested in $x_2$ : $v_1(x_1, x_2, \theta) = m_1 x_1 + e x_2$. The sign of the single crossing condition in the agent's preferences is the same for upstream and downstream decisions: $sign(\Delta a) = sign(\Delta b)$.*

The externality $e$ can be either positive or negative.[14] We also assume that there is always value from contracting in the upstream relationship, that is $m_1 + a(\theta) \geq 0$ for any $\theta$.

Under Conditions (1) and (2), the surplus $A$ expects from the two contractual relationships given $\phi_1$ is thus

$$\begin{array}{l} U_A(\overline{\theta}) = \delta_1(1|\overline{\theta})\bar{a} + d(s_2|\overline{\theta})\Delta b - t_1(\overline{\theta}), \\ U_A(\underline{\theta}) = \delta_1(1|\underline{\theta})\underline{a} - t_1(\underline{\theta}), \end{array}$$

As standard, at the optimum constraints $(\underline{IR}_1)$ and $(\overline{IC}_1)$ bind, and thus $P_1$'s optimal contract maximizes

$$\begin{array}{c} \mathbb{E}U_1 = p\delta_1(1|\overline{\theta})\left(m_1 + \bar{a}\right) + (1-p)\,\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + \\ + pe + (1-p)\,d(s_2|\underline{\theta})e - p\left[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})\right]\Delta b \end{array}$$

[14] Assuming marginal effects of $x_2$ on $x_1$ is not needed to illustrate the role of externalities on disclosure. A full fledged analysis with marginal effects is available upon request.

subject to the constraints

$$\begin{array}{ll} \left[\delta_1(1|\overline{\theta}) - \delta_1(1|\underline{\theta})\right]\Delta a \geq \left[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})\right]\Delta b & (\underline{IC}_1) \\ d(s_1|\overline{\theta}) \geq H d(s_1|\underline{\theta}), & (SR_1) \\ d(s_2|\overline{\theta}) \leq H d(s_2|\underline{\theta}). & (SR_2) \end{array}$$

Note that since preferences in the the downstream relationship are separable and there are no marginal externalities of $x_2$ on $v_1(x_1, x_2, \theta) + v_A^1(x_1, \theta)$, the program for the optimal upstream mechanism can be written by decomposing the joint lottery $\delta_1(x_1, s|\theta)$ into a disclosure policy $d(s|\theta)$ and a trade policy $\delta_1(1|\theta)$, where $d(s|\theta)$ and $\delta_1(1|\theta)$ can be treated as independent distributions. This also implies that $\delta_1(1|\theta)$ can either be read as the probability of trade, or as the level of trade, with $\delta_1(1|\theta) \in [0, 1]$. As we will see in the next section, things are different with non separable preferences, for then the joint distribution over $X_1$ and $S$ clearly matters in determining the surplus $A$ and $P_1$ expect from downstream contracting. Also note that $\mathbb{E}U_1$ is the total joint surplus of $P_1$ and $A$, net of the rent $U_A(\overline{\theta}) = \delta_1(1|\underline{\theta})\Delta a + d(s_2|\underline{\theta})\Delta b$ that $P_1$ must leave to $\overline{\theta}$ to induce truthful information revelation. As for the externality, $\overline{\theta}$ always trades with $P_2$, whereas $\underline{\theta}$ trades if and only if signal $s_2$ is disclosed: it follows that the expected externality of $x_2$ on $P_1$ is equal to $pe + (1-p)\, d(s_2|\underline{\theta})e$. Since $\mathbb{E}U_1$ is increasing in $d(s_2|\overline{\theta})$ and since a higher $d(s_2|\overline{\theta})$ relaxes $(\underline{IC}_1)$, it is always optimal for $P_1$ to maximize $d(s_2|\overline{\theta})$, whose upper bound is given by $(SR_2)$ when beliefs are unfavorable and by $(SR_1)$ when they are favorable. Finally, note that constraint $(\underline{IC}_1)$ is an "adjusted" monotonicity condition which reduces to the standard weak monotonicity condition $\delta_1(1|\overline{\theta}) \geq \delta_1(1|\underline{\theta})$ when no information is disclosed. On the contrary, when $P_1$ discloses information, monotonicity becomes strict for it requires $\delta_1(1|\underline{\theta}) < \delta_1(1|\overline{\theta})$. It follows that there are two possible costs associated with disclosure. The first is the extra rent $\left[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})\right]\Delta b$ that $P_1$ must leave to $\overline{\theta}$, which comes from the fact that the most favorable signal for $\overline{\theta}$, $s_2$, is sent with a higher probability when $A$ reports $\widehat{\theta} = \underline{\theta}$ than $\widehat{\theta} = \overline{\theta}$. The second is the reduction in the upstream level of trade with $\underline{\theta}$ required by $(\underline{IC}_1)$. However, note that while it is always optimal for $P_1$ to trade with the high type, trading with the low type is desirable only if $m_1 + \underline{a} - p\Delta a/(1-p) \geq 0$, that is only if the "virtual surplus" for $\underline{\theta}$ is positive.

We shall now derive the conditions that lead to the optimality of information disclosure. Consider first the case where $P_2$'s prior beliefs are unfavorable to $A$ so that $SR_2$ binds and hence the extra rent that $P_1$ must leave to $\overline{\theta}$ when she discloses information becomes $(1-H)d(s_2|\underline{\theta})\Delta b$. If $m_1 + \underline{a} - p\Delta a/(1-p) \leq 0$, $P_1$ never wants to trade with $\underline{\theta}$ and thus a necessary and sufficient condition for disclosure is that

$$(1-p)e \geq p(1-H)\Delta b, \tag{6}$$

where the left hand side is the marginal externality generated by an increase in $d(s_2|\underline{\theta})$, whereas the right hand side is the marginal increase in the rent for $\overline{\theta}$. When instead $m_1 + \underline{a} - p\Delta a/(1-p) > 0$, the marginal cost of $d(s_2|\underline{\theta})$ also takes into account the reduction in the level of trade with $\underline{\theta}$ imposed by the $(\underline{IC}_1)$ constraint, which, using $SR_2$ and $\delta_1^*(1|\overline{\theta}) = 1$, reduces to $\delta_1(1|\underline{\theta}) \leq 1 - (1-H)\frac{\Delta b}{\Delta a} d(s_2|\underline{\theta})$. It follows that in this case disclosure is optimal for $P_1$ if and only if

$$(1-p)e \geq p(1-H)\Delta b + (1-p)(1-H)\frac{\Delta b}{\Delta a}[m_1 + \underline{a} - \frac{p}{1-p}\Delta a]. \tag{7}$$

Combining (6) with (7), we conclude that a necessary and sufficient condition for the optimality of disclosure when $P_2$'s prior beliefs are unfavorable is that

$$e > E \equiv \max\left\{m_1 + \underline{a};\ \frac{p}{1-p}\Delta a\right\}(1-H)\frac{\Delta b}{\Delta a} > 0.$$

Things are symmetrically opposite with favorable beliefs, i.e. when $P_2$ is expected to trade with either type if she learns nothing about $\theta$. In this case, disclosure is optimal only when $P_1$ has strong incentives to reduce the level of trade in the downstream relationship, which occurs for large negative externalities.

We summarize these results in the following Proposition [the formal proof in the Appendix also contains the complete characterization of the optimal contracts for all parameters configurations].

**Proposition 1** *Assume preferences are defined by conditions (1) and (2).*

*When $P_2$'s prior beliefs are* ***unfavorable*** *to A, information disclosure is optimal for $P_1$ for sufficiently large positive externalities; that is, if and only if* $e > E \equiv \max\left\{m_1 + \underline{a};\ \frac{p}{1-p}\Delta a\right\}\frac{\Delta b}{\Delta a}(1-H) > 0$.

*When $P_2$'s prior beliefs are* ***favorable****, disclosure is optimal for sufficiently small negative externalities, i.e. if and only if* $e < E \leq 0$.

As for the structure of the optimal contract with disclosure, consider first the case of unfavorable beliefs. When $\Delta b(1-H)/\Delta a \leq 1$, $P_1$ discloses signal $s_2$ with probability one when $\theta = \underline{\theta}$, thus maximizing the positive effect of the externality. Hence, the optimal disclosure policy is $d^*(s_2|\underline{\theta}) = 1$ and $d^*(s_2|\overline{\theta}) = H < 1$, whereas the optimal level of trade with $\underline{\theta}$ is $\delta_1^*(1|\underline{\theta}) = 0$ if $m_1 + \underline{a} - p\Delta a/(1-p) < 0$, whilst $\delta_1^*(1|\underline{\theta}) = 1 - \Delta b(1-H)/\Delta a$ otherwise, where the upper bound on $\delta_1(1|\underline{\theta})$ comes from $(\underline{IC}_1)$. When instead $\Delta b(1-H)/\Delta a > 1$, the rent that $P_1$ must leave to $\overline{\theta}$ when $d(s_2|\underline{\theta}) = 1$ is so high that it is impossible to prevent $\underline{\theta}$ from mimicking the high type, as indicated by the $(\underline{IC}_1)$ constraint: $\delta_1(1|\underline{\theta}) \leq 1 - (1-H)\frac{\Delta b}{\Delta a}d(s_2|\underline{\theta})$. In this case, to maximize $d(s_2|\underline{\theta})$, $P_1$ never trades with $\underline{\theta}$ and the optimal disclosure policy is thus $d^*(s_2|\underline{\theta}) = \frac{\Delta a}{\Delta b(1-H)} < 1$ and $d^*(s_2|\overline{\theta}) = \frac{\Delta a H}{\Delta b(1-H)} < 1$. Note that in either case, $P_1$ never fully informs $P_2$ about $\theta$. Indeed, full disclosure is always costly (in terms of rent for $\overline{\theta}$ and forgone trade with $\underline{\theta}$) and is either unnecessary to induce the desired level of trade in the downstream relationship (when $d^*(s_2|\underline{\theta}) = 1$), or incentive incompatible (when $\Delta b(1-H)/\Delta a > 1$).

Next, consider favorable beliefs (i.e. $H > 1$) and assume large negative externalities. Recall that in this case disclosure is formally equivalent to releasing signal $s_1$. At the optimum, $(SR_1)$ binds – that is $d(s_1|\overline{\theta}) = Hd(s_1|\underline{\theta})$ – $(SR_2)$ is implied by $(SR_1)$, and $(\underline{IC}_1)$ reduces to $\delta_1(1|\underline{\theta}) \leq 1-(H-1)\frac{\Delta b}{\Delta a}d(s_1|\underline{\theta})$, with $d(s_1|\underline{\theta}) \leq 1/H$. When $(H-1)\frac{\Delta b}{\Delta a}\left(\frac{1}{H}\right) \geq 1$, to minimize the negative effect of the externality without violating $(\underline{IC}_1)$, $P_1$ must trade only with $\overline{\theta}$ and the optimal disclosure policy is $d^*(s_1|\underline{\theta}) = \frac{\Delta a}{(H-1)\Delta b} < 1$ and $d^*(s_1|\overline{\theta}) = \frac{\Delta a H}{(H-1)\Delta b} \leq 1$. When instead $(H-1)\frac{\Delta b}{\Delta a}\left(\frac{1}{H}\right) < 1$, the optimal disclosure policy is $d^*(s_1|\overline{\theta}) = 1$ and $d^*(s_1|\underline{\theta}) = 1/H$, whereas the optimal level of trade depends on the sign of the "virtual surplus" $m_1 + \underline{a} - p\Delta a/(1-p)$. If this is negative, $P_1$ never trades with $\underline{\theta}$, whereas if it is positive, then $\delta_1^*(1|\underline{\theta}) = 1 - [\Delta b\,(H-1)]\,/\,(\Delta a H)$ where the upper bound on $\delta_1(1|\underline{\theta})$ is determined by $(\underline{IC}_1)$.

Using the properties of the optimal contracts, we now turn attention to the effects of disclosure on individual payoffs. We compare the three players' payoffs under the optimal contracts described above with those under the contracts that would be offered in case $P_1$ were not allowed, or able, to disclose information. Because preferences are separable in the downstream relationship, these contracts simply consist in a take-it-or-leave-it offer at price $t_1 = \overline{a}$ if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geq 0$ and at price $t_1 = \underline{a}$ otherwise.

**Corollary 1** *Assume preferences are defined by Conditions (1) and (2).*

*When $P_2$'s prior beliefs are **unfavorable** to $A$, disclosure leads to a Pareto-improvement: $P_1$ and $A$ are strictly better off, whereas $P_2$ is indifferent.*

*When $P_2$'s prior beliefs are **favorable**, disclosure makes $A$ worse off, $P_1$ better off, and leaves $P_2$ indifferent. The effect of disclosure on total welfare is positive for large negative externalities and negative otherwise.*

To see why $P_2$ is not affected by disclosure, suppose beliefs are unfavorable. Under any of the optimal contracts offered by $P_1$, constraint $(SR_2)$ always binds, whereas constraint $(SR_1)$ is slack. This means that for $s = s_1$, $P_2$ strictly prefers to offer the same contract she would offer if she did not receive any information ($t_2 = \overline{b}$), whereas for $s = s_2$, she is just indifferent between setting $t_2 = \overline{b}$ and $t_2 = \underline{b}$. Furthermore, since $P_2$'s preferences are independent of $x_1$, $P_2$ is not affected by changes in the distribution over $X_1$ that may be introduced when $P_1$ discloses information. As a consequence, $P_2$ is just as well off as in the absence of disclosure. A symmetric argument holds for favorable beliefs.

Next, consider the effect of disclosure on $A$ and recall that the low type never gets any surplus, whereas the expected payoff for the high type is $U_A^*(\overline{\theta}) = \delta_1^*(1|\underline{\theta})\Delta a + d^*(s_2|\underline{\theta})\Delta b$. Assume first unfavorable beliefs. If $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$, $A$ clearly benefits from disclosure for he expects no surplus in the absence of information transmission. If instead $m_1 + \underline{a} - \frac{p}{1-p}\Delta a > 0$, then $U_A^*(\overline{\theta})$ depends on whether at the optimum $(\underline{IC}_1)$ binds or not, as indicated above. When it binds, $\delta_1^*(1|\underline{\theta}) = 0$, $d^*(s_2|\underline{\theta}) = \frac{\Delta a}{\Delta b(1-H)}$ and thus $U_A^*(\overline{\theta}) = \frac{\Delta a}{(1-H)}$; when it does not, $\delta_1^*(1|\underline{\theta}) = 1 - \Delta b(1-H)/\Delta a$, $d^*(s_2|\underline{\theta}) = 1$, and $U_A^*(\overline{\theta}) = \Delta a + \Delta b H$. In either case, $U_A^*(\overline{\theta}) > \Delta a$ and hence $A$ strictly benefits from disclosure.

Things are different with favorable beliefs. In this case, $P_1$ induces $P_2$ to set a higher price ($t_2 = \overline{b}$ instead of $t_2 = \underline{b}$) with positive probability and reduces the level of trade with the low type to satisfy $(\underline{IC}_1)$. As a consequence, $A$ always suffers from disclosure. The effect on total welfare then depends on how strong the externality is. For moderate values, the negative effect on $A$ prevails, and hence welfare decreases with disclosure, whereas the opposite is true for large (negative) externalities.

### 3.1.2 Horizontal Differentiation and Countervailing Incentives

We now turn attention to environments where the agent has horizontally differentiated preferences for the decisions of the two principals, that is where his valuations for $x_1$ and $x_2$ are (perfectly) negatively correlated. We continue to assume that preferences in the downstream relationship are as in Condition 1, but now let $\Delta a < 0$. In this case, $A$ faces *countervailing incentives* when reporting his type to $P_1$ : By announcing $\bar{\theta}$, $A$ reveals he has a higher valuation for $x_2$, but a lower valuation for $x_1$.[15] When this is the case, $P_1$ may benefit from the possibility to disclose information to $P_2$, even in the absence of direct externalities. To illustrate, we assume

**Condition 3** *$P_1$'s preferences are independent of $x_2$: $v_1(x_1, x_2, \theta) = m_1 x_1$; the single crossing condition in the agent's preferences has opposite sign for $x_1$ and $x_2$: $sign(\Delta a) = -sign(\Delta b)$.*

Under Conditions 1 and 3, $P_1$'s optimal mechanism maximizes

$$\mathbb{E}U_1 = p\left\{\delta_1(1|\overline{\theta})(m_1 + \overline{a}) + d(s_2|\overline{\theta})\Delta b - U_A(\overline{\theta})\right\} + (1-p)\left\{\delta_1(1|\underline{\theta})(m_1 + \underline{a}) - U_A(\underline{\theta})\right\}$$

subject to the participation constraints $U_A(\overline{\theta}) \geq 0$, $U_A(\underline{\theta}) \geq 0$, the incentive compatibility constraints

$$\begin{array}{ll} U_A(\overline{\theta}) \geq U_A(\underline{\theta}) + d(s_2|\underline{\theta})\Delta b - \delta_1(1|\underline{\theta})\left|\Delta a\right|, & (\overline{IC}_1) \\ U_A(\underline{\theta}) \geq U_A(\overline{\theta}) - d(s_2|\overline{\theta})\Delta b + \delta_1(1|\overline{\theta})\left|\Delta a\right|, & (\underline{IC}_1) \end{array}$$

and $P_2$'s sequential rationality constraints

$$\begin{array}{ll} d(s_1|\overline{\theta}) \geq H d(s_1|\underline{\theta}), & (SR_1) \\ d(s_2|\overline{\theta}) \leq H d(s_2|\underline{\theta}), & (SR_2) \end{array}$$

with $H \equiv \left(\frac{1-p}{p}\right)\left(\frac{m_2+b}{\Delta b}\right)$. Note that $\overline{\theta}$ continues to obtain $\Delta b$ more than $\underline{\theta}$ when $P_2$ receives signal $s_2$, which, conditional on announcing $\widehat{\theta} = \theta$, occurs with probability $d(s_2|\theta)$. On the other hand, $\overline{\theta}$ has now a lower valuation than $\underline{\theta}$ for $x_1$ and thus obtains $|\Delta a|$ less than the latter from trading with $P_1$, which occurs with probability $\delta_1(1|\theta)$. It follows that when $A$'s private information has opposite effects on his valuations for $x_1$ and $x_2$, it is not possible to determine *a priori* which $(IR_1)$ and $(IC_1)$ constraints bind since this depends on which of the two countervailing incentives dominates. Nevertheless, at least one $(IR_1)$ and one $(IC_1)$ constraint must bind in any optimal mechanism and trade with $\underline{\theta}$ always occurs with probability one, i.e. $\delta_1^*(1|\underline{\theta}) = 1$.

As for the optimal disclosure policy, when $P_2$'s prior beliefs are favorable to $A$ (i.e. $H > 1$) so that $P_2$ is expected to set $t_2 = \underline{b}$ in the event she receives no information from $P_1$, no disclosure is always optimal, for having $P_2$ making an offer at a low price increases the value $\overline{\theta}$ attaches to upstream contracting and may even help reducing the rent for $\underline{\theta}$. To see this, consider first $|\Delta a| \geq \Delta b$, in which case the binding constraints are $(\overline{IR}_1)$ and $(\underline{IC}_1)$ (the formal proof is in the Appendix). Since $\mathbb{E}U_1$ is increasing in $d(s_2|\overline{\theta})$, and since $U_A(\underline{\theta})$ is decreasing in $d(s_2|\overline{\theta})$, at the

[15] For example, $v_A(x_1, x_2, \theta) = (1-\theta)x_1 + \theta x_2$. See Mezzetti (1997) for an analysis of countervailing incentives in (simultaneous) common agency games with similar preferences.

optimum necessarily $d^*(s_2|\overline{\theta}) = d^*(s_2|\underline{\theta}) = 1$, that is $P_1$ does not disclose any information to $P_2$. As for the optimal level of trade with $\overline{\theta}$, $\delta^*(1|\overline{\theta}) = 1$ if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geq 0$ and $\delta^*(1|\overline{\theta}) = \frac{\Delta b}{|\Delta a|}$ otherwise. In the first case, the low type enjoys a rent equal to $U_A^*(\underline{\theta}) = |\Delta a| - \Delta b$, whereas in the second $P_1$ appropriates all surplus from both types. When instead $|\Delta a| < \Delta b$, and hence $(\underline{IR}_1)$ and $(\overline{IC}_1)$ bind, reducing $d(s_2|\underline{\theta})$ may help containing $U_A(\overline{\theta})$, but comes at the cost of reducing the probability $P_2$ offers a low price to $\overline{\theta}$. Since $P_2$'s sequential rationality requires $d(s_2|\overline{\theta}) \leq d(s_2|\underline{\theta})$, the net effect of a reduction in $d(s_2|\underline{\theta})$ on $\mathbb{E}U_1$ is negative so that $P_1$ is again better off setting $d^*(s_2|\overline{\theta}) = d^*(s_2|\underline{\theta}) = 1$, and leaving a rent to $\overline{\theta}$ equal to $U_A^*(\overline{\theta}) = \Delta b - |\Delta a|$. Moreover, since in this case $\delta(1|\overline{\theta})$ has no bite on the rent for $U_A(\underline{\theta})$, at the optimum, $\delta^*(1|\overline{\theta}) = 1$, that is, trade occurs with both types with probability one.

Consider next the case where $P_2$'s prior beliefs are unfavorable (i.e. $H > 1$). In the absence of information disclosure, the optimal mechanism simply consists in trading with either type at a low price $t_1 = \overline{a}$ if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geq 0$ and only with the low type at a high price $t_1 = \underline{a}$ otherwise. In this second case, disclosure is always optimal for $P_1$: By sending signal $s_2$ with probability $d^*(s_2|\underline{\theta}) = \min\left\{1, \frac{|\Delta a|}{\Delta b}\right\}$ when $A$ reports a low type and with probability $d^*(s_2|\overline{\theta}) = Hd(s_2|\underline{\theta})$ when he reports a high type, $P_1$ can fully appropriate the surplus $d^*(s_2|\overline{\theta})\Delta b$ that $\overline{\theta}$ expects from downstream contracting without leaving $A$ any rent. Also note that when $\Delta b > |\Delta a|$, it never pays to increase $d(s_2|\underline{\theta})$ above $\frac{|\Delta a|}{\Delta b}$ since above this threshold there are no countervailing incentives and hence at the margin $P_1$ would also have to increase $U_A(\overline{\theta})$ by $\Delta b$ with a negative net effect of $(H-1)\Delta b$ on $\mathbb{E}U_1$. On the other hand, disclosure allows $P_1$ to increase the level of trade with $\overline{\theta}$ to $\delta_1^*(1|\overline{\theta}) = \frac{\Delta b}{|\Delta a|}d^*(s_2|\overline{\theta}) > 0$, without increasing the rent for $\underline{\theta}$. Increasing $\delta_1(1|\overline{\theta})$ above this threshold would require to increase $U_A(\underline{\theta})$ by $|\Delta a|$ with a total negative effect of $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a|$ on $\mathbb{E}U_1$.

Things are more complicated when $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geq 0$, for disclosure may then come at the expenses of a reduction in the level of trade with $\overline{\theta}$, which is now costly for $P_1$. Indeed, using $(SR_2)$ and $\delta_1^*(1|\underline{\theta}) = 1$, note that $(\overline{IC}_1)$ and $(\underline{IC}_1)$ require that

$$\delta_1(1|\overline{\theta}) \leq 1 - (1-H)\frac{\Delta b}{|\Delta a|}d(s_2|\underline{\theta}). \tag{8}$$

Because $\mathbb{E}U_1$ is now increasing in $\delta_1(1|\overline{\theta})$, for any $d(s_2|\underline{\theta}) \leq \frac{|\Delta a|}{(1-H)\Delta b}$, it is always optimal to maximize $\delta_1(1|\overline{\theta})$ whose upper bound is given by (8). Using (8) and $(\underline{IC}_1)$, we also have that for any $d(s_2|\underline{\theta}) \leq \frac{|\Delta a|}{\Delta b}$, $U_A^*(\overline{\theta}) = 0$ and $U_A^*(\underline{\theta}) = |\Delta a| - d(s_2|\underline{\theta})\Delta b \geq 0$. It follows that for any $d(s_2|\underline{\theta}) \leq \frac{|\Delta a|}{\Delta b}$, the marginal effect of $d(s_2|\underline{\theta})$ on $\mathbb{E}U_1$ is now given by

$$p\left\{H\Delta b - (1-H)\frac{\Delta b}{|\Delta a|}(m_1 + \overline{a})\right\} + (1-p)\Delta b$$

where the first term combines the marginal increase in the downstream surplus for $\overline{\theta}$, $H\Delta b$, with the marginal reduction in the upstream surplus due to the contraction in the level of trade with

$\overline{\theta}$, whereas the second term is the marginal reduction of the rent for the low type. It follows that disclosure is optimal for $P_1$ if and only if

$$m_1 + \overline{a} - \frac{1-p}{p} |\Delta a| \leq \frac{H |\Delta a|}{p(1-H)}. \tag{9}$$

Note that raising $d(s_2|\underline{\theta})$ above $\frac{|\Delta a|}{\Delta b}$ never pays since in this case $P_1$ should also increase $U_A(\overline{\theta})$ by $\Delta b$, as indicated in $(\overline{IC}_1)$, without being able to further reduce $U_A(\underline{\theta})$: the net effect of disclosing signal $s_2$ with probability $d(s_2|\underline{\theta}) > \frac{|\Delta a|}{\Delta b}$ is thus equal to $p\left\{H\Delta b - (1-H)\frac{\Delta b}{|\Delta a|}(m_1 + \overline{a}) - \Delta b\right\}$ which is clearly negative. Hence, when (9) holds, the optimal disclosure policy is again $d^*(s_2|\underline{\theta}) = \min\left\{1, \frac{|\Delta a|}{\Delta b}\right\}$, and $d^*(s_2|\overline{\theta}) = Hd^*(s_2|\underline{\theta})$, whereas the optimal level of trade between $P_1$ and $\overline{\theta}$ is determined by (8).

We summarize the above results in the following

**Proposition 2** *Assume preferences are defined by Conditions (1) and (3). Information disclosure is optimal for $P_1$ if and only if $P_2$'s prior beliefs are unfavorable to $A$ and $m_1 + \overline{a} - \frac{1-p}{p} |\Delta a| \leq \frac{H|\Delta a|}{p(1-H)}$.*

Contrary to the case of externalities, the motivation for information disclosure examined in this section is merely the possibility to benefit from the *rent shifting* effect. That is, $P_1$ discloses information to induce $P_2$ to lower her price so that $\overline{\theta}$ can enjoy a positive downstream informational rent. This policy never pays when $A$'s valuations for $x_1$ and $x_2$ are positively correlated, for any increase in downstream surplus is more than compensated by the increase in $U_A(\overline{\theta})$ required to induce $A$ to truthfully reveal his type. In contrast, when the two decisions are horizontally differentiated in the agent's preferences so that $A$ faces countervailing incentives when reporting his type to $P_1$, it is possible for the upstream principal to fully appropriate the surplus $\overline{\theta}$ expects from downstream contracting without leaving $\overline{\theta}$ any informational rent. The only cost associated with disclosure is then the reduction in the level of trade with $\overline{\theta}$ required by incentive compatibility. It follows that, when the "virtual" surplus from trading with the high type $m_1 + \overline{a} - \frac{1-p}{p} |\Delta a|$ is small, disclosure is always optimal for $P_1$. Note that disclosure is however only partial, since perfectly informing $P_2$ about the agent's type would allow the latter to extract all surplus from $A$ and is therefore never optimal for $P_1$.

Consider next the effect of disclosure on individual payoffs and on welfare. Under the optimal disclosure policy, $P_2$ is indifferent between offering the same price she would offer without information from $P_1$ and lowering the price in case she observes $s_2$. Furthermore, $P_2$ is not personally interested in the upstream decision so that she is not affected by possible changes in the distribution over $X_1$. As a consequence, under Conditions (1) and (3), $P_2$ is not affected by disclosure. As for the agent, when $m_1 + \overline{a} - \frac{1-p}{p} |\Delta a| < 0$, $A$ obtains the same payoff as when $P_1$ is not allowed/able to disclose information, that is $U_A^*(\theta) = 0$ for any $\theta$. When instead $m_1 + \overline{a} - \frac{1-p}{p} |\Delta a| \geq 0$, disclosure reduces the rent of the low type from $|\Delta a|$ to

$$U_A^*(\underline{\theta}) = -d^*(s_2|\overline{\theta})\Delta b + \delta_1^*(1|\overline{\theta}) |\Delta a| = |\Delta a| - d^*(s_2|\underline{\theta})\Delta b < |\Delta a|$$

without affecting that of the high type and therefore makes $A$ strictly worse off. In terms of total welfare, when $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|<0$, disclosure boosts efficiency in either relationships by increasing the level of trade and thus increases welfare. In contrast, when $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|\geq 0$, disclosure increases the level of trade in the downstream relationship, but reduces that in the upstream, with a net effect on welfare

$$\Delta W=-p[1-\delta_1^*(1|\overline{\theta})](m_1+\overline{a})+(1-p)(m_2+\underline{b})d^*(s_2|\underline{\theta})$$

which is positive if and only if $m_1+\overline{a}\leq\frac{H|\Delta a|}{1-H}$, that is if and only if the value of trading between $\overline{\theta}$ and $P_1$ is sufficiently small.[16] We conclude that

**Corollary 2** *When preferences are defined by Conditions (1) and (3), the possibility for $P_1$ to disclose information increases welfare if and only if* $m_1+\overline{a}\leq\max\left\{\frac{1-p}{p}\left|\Delta a\right|;\ \frac{H|\Delta a|}{1-H}\right\}$. *$P_1$ strictly benefits from disclosure, $P_2$ is indifferent, and $A$ is indifferent if $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|<0$ and worse off otherwise.*

### 3.2 Disclosure of Endogenous Information

In the rest of the section we consider situations where the agent's marginal utility in the downstream relationship depends on upstream decisions, such as in the case of a buyer whose valuation for a downstream product or service depends on the products and services purchased from upstream sellers, or a worker whose ability to perform a task in the downstream relationship depends on the activities done in the past while working for an upstream principal.

To isolate the effects associated with the disclosure of endogenous information – i.e. information about upstream decisions – from those associated with the disclosure of exogenous information, we assume the surplus in the downstream relationship depends on $x_1$, but not on $\theta$. Furthermore, we rule out direct externalities between the two principals, so that disclosure, when optimal, is motivated uniquely by the non separability of the agent's preferences for the two decisions.

**Condition 4** *The agent's preferences are not separable in $x_1$ and $x_2$: $v_A(x_1,x_2,\theta)=a(\theta)\,x_1+bx_2+gx_1x_2$, with $\Delta a\equiv a\left(\overline{\theta}\right)-a\left(\underline{\theta}\right)\geq 0$. The two principals have preferences $v_i(x_1,x_2,\theta)=m_ix_i$ for $i=1,2$.*

The decisions $x_1$ and $x_2$ are *complements* if $g>0$ and *substitutes* if $g<0$. To make things interesting, we assume trade always generates positive surplus in either relationship, that is $m_1+a(\theta)\geq 0$ for any $\theta$, $m_2+b\geq 0$, and $m_2+b+g>0$.[17] With preferences as in Condition 4, the optimal mechanism $\phi_2(s)$ assigns the same allocation to the two extended types $\theta_2^E=\left(x_1,\overline{\theta}\right)$ and $\theta_2^E=\left(x_1,\underline{\theta}\right)$, for any $x_1$. Furthermore, with complements, $(IR_2)$ binds for $x_1=0$ and $(IC_2)$ for

[16] The threshold on $m_1+\overline{a}$ is obtained by substituting $\delta_1^*(1|\overline{\theta})=1-(1-H)\frac{\Delta b}{|\Delta a|}d^*(s_2|\underline{\theta})$ and $(1-p)(m_2+\underline{b})=pH\Delta b$.

[17] This also guarantees that $P_2$ is indeed interested in receiving information about $x_1$.

$x_1 = 1$ and thus the optimal contract $\phi_2(s)$ consists in a take-it-or-leave-it offer at a price

$$t_2^{com}(s) = \begin{cases} b+g & \text{if } (m_2+b+g)\,\mu(1;s) > m_2+b, \\ b & \text{if } (m_2+b+g)\,\mu(1;s) \leq m_2+b, \end{cases}$$

where $\mu(1;s) = \mu((1,\underline{\theta});s) + \mu\big((1,\bar{\theta});s\big)$ denotes $P_2$'s posterior beliefs that trade occurred in the upstream relationship.

Conversely, when $x_1$ and $x_2$ are substitutes, $(IR_2)$ binds for $x_1 = 1$, and $(IC_2)$ binds for $x_1 = 0$, so that the optimal price is

$$t_2^{sub}(s) = \begin{cases} b & \text{if } (m_2+b)\mu(0;s) > m_2+b+g, \\ b+g & \text{if } (m_2+b)\mu(0;s) \leq m_2+b+g, \end{cases}$$

with $\mu(0;s) = 1 - \mu(1;s)$. That is, in the case of complements, $P_2$ asks a low price if she believes it is unlikely that $A$ traded with $P_1$, whereas with substitutes if she believes trade occurred with high probability. It follows that to create the desired informational linkage with the downstream relationship, $P_1$ needs to disclose only two signals, $s_1$, and $s_2$, such that

$$\begin{array}{ll} t_2(s_1) = b + g\mathbb{I}(g>0), & (SR_1) \\ t_2(s_2) = b + g\mathbb{I}(g<0), & (SR_2) \end{array} \tag{10}$$

where $\mathbb{I}(g>0)$ is the indicator function assuming value one if $g > 0$ and zero otherwise. As in the previous section, signal $s_1$ stands for information that induces $P_2$ to set a high price, whereas signal $s_2$ a low price. Note that, contrary to the case where disclosure is about exogenous preferences, the mechanism $\phi_1$ is now itself informative about $x_1$. However, to be consistent with Definition 1, in what follows we will say that $P_1$ discloses information to $P_2$ only when the optimal mechanism $\phi_1$ requires a privacy policy $d(s|\theta)$ that assigns positive measure to both signals $s_1$ and $s_2$. Finally, note that in this simple model, if the two goods are neither complements nor substitutes, i.e. $g = 0$, the two contractual relationships are completely unrelated and thus information disclosure is irrelevant.

### 3.2.1 The complements case

Under Condition 4, $A$ obtains a rent $g$ with $P_2$ only when he trades with $P_1$ in the upstream relationship and $P_2$ receives signal $s_2$. It follows that the surplus $A$ expects from the two contractual relationships given $\phi_1$ is

$$\begin{array}{l} U_A(\bar{\theta}) = \delta_1(1,s_1|\bar{\theta})\bar{a} + \delta_1(1,s_2|\bar{\theta})(\bar{a}+g) - t_1(\bar{\theta}), \\ U_A(\underline{\theta}) = \delta_1(1,s_1|\underline{\theta})\underline{a} + \delta_1(1,s_2|\underline{\theta})(\underline{a}+g) - t_1(\underline{\theta}). \end{array}$$

At the optimum, constraints $(\underline{IR}_1)$ and $(\overline{IC}_1)$ bind, so that $P_1$'s optimal contract maximizes

$$\begin{aligned} \mathbb{E}U_1 \;\; = \;\; & p\delta_1(1,s_1|\bar{\theta})\,(m_1+\bar{a}) + (1-p)\delta_1(1,s_1|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right) + \\ & + p\delta_1(1,s_2|\bar{\theta})\,(m_1+\bar{a}+g) + (1-p)\delta_1(1,s_2|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a+g\right) \end{aligned}$$

subject to the following constraints

$$
\begin{aligned}
&\delta_1(1,s_1|\overline{\theta})+\delta_1(1,s_2|\overline{\theta})\geq\delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta}), && (\underline{IC}_1)\\
&g[p\delta_1(1,s_1|\overline{\theta})+(1-p)\,\delta_1(1,s_1|\underline{\theta})]\geq(m_2+b)\left[p\delta_1(0,s_1|\overline{\theta})+(1-p)\,\delta_1(0,s_1|\underline{\theta})\right], && (SR_1)\\
&g[p\delta_1(1,s_2|\overline{\theta})+(1-p)\,\delta_1(1,s_2|\underline{\theta})]\leq(m_2+b)\left[p\delta_1(0,s_2|\overline{\theta})+(1-p)\,\delta_1(0,s_2|\underline{\theta})\right]. && (SR_2)
\end{aligned}
$$

As with exogenous information, $\mathbb{E}U_1$ is the joint surplus that $P_1$ and $A$ expect from the two contractual relationships, net of the rent that $P_1$ must leave to $A$ to induce truthtelling. However, in contrast with the previous section, since $A$ and $P_1$ share the same information about $x_1$ and the value $A$ attaches to $x_2$, $P_1$ can appropriate all surplus $A$ expects from downstream contracting. It follows that with endogenous information, disclosure does not induce the same negative incentives discussed in Theorem 1. Indeed, the rent $A$ obtains with $P_1$ is uniquely determined by the upstream level of trade, is independent of the disclosure policy selected by $P_1$, and is the same as in the absence of downstream contracting, i.e. $U_A(\overline{\theta})=\delta_1(1|\underline{\theta})\Delta a$ and $U_A(\underline{\theta})=0$. This also implies that constraint $(\underline{IC}_1)$ is the standard monotonicity condition on the level of trade, and never binds at the optimum. The remaining constraints, $(SR_1)$ and $(SR_2)$, are simple rewriting of the sequential rationality constraints as in (10) using Bayes rule. Note that to maximize the probability $P_2$ offers a low price when $x_1=1$, that is to maximize $p\delta_1(1,s_2|\overline{\theta})+(1-p)\,\delta_1(1,s_2|\underline{\theta})$, $P_1$ sends signal $s_2$ with probability one when $x_1=0$. Signal $s_1$ is then perfectly informative of the decision to trade and hence constraint $(SR_1)$ never binds and can be neglected. The optimal disclosure policy with the corresponding downstream price can then be qualitatively represented by the following diagram:

$$
\begin{array}{ccccc}
x_1=1 & \longrightarrow & s_1 & \rightarrow & t_2=b+g\\
 & \searrow & & & \\
x_1=0 & \longrightarrow & s_2 & \rightarrow & t_2=b
\end{array}
$$

Also note that contrary to the case of exogenous information, the disclosure policy $d(s|\theta)$ and the trade policy $\delta_1(1|\theta)$ can not be treated as independent distributions, for the correlation between $x_1$ and $s$ is exactly what determines the surplus $A$ and $P_1$ expect from downstream contracting.

We now discuss the optimal mechanism $\phi_1^*$ and derive conditions under which $P_1$ strictly benefits from disclosure. Consider first $m_1+\underline{a}-\frac{p}{1-p}\Delta a\geq m_2+b$. In this case, $P_1$ finds it optimal to trade with both types with probability one, even if this implies $A$ will not enjoy any rent with $P_2$. Indeed, from $(SR_2)$, the maximal surplus that $P_1$ can appropriate from downstream contracting is always bounded from above by $(m_2+b)\Pr(0,s_2)$, where $\Pr(0,s_2)$ is the total probability that trade does not occur in the upstream relationship.[18] It follows that when $m_1+\underline{a}-\frac{p}{1-p}\Delta a\geq m_2+b$, the "virtual" surplus $P_1$ can generate by trading with either type is higher than the downstream surplus she can appropriate by not trading and inducing an informational rent in the downstream relationship and hence the optimal mechanism is $\delta_1^*(1,s_1|\overline{\theta})=\delta_1^*(1,s_1|\underline{\theta})=1$, which involves no disclosure.

When instead $m_2+b>m_1+\underline{a}-\frac{p}{1-p}\Delta a$, $P_1$ finds it profitable to sacrifice trade with the low type to give $A$ a positive expected rent in the downstream relationship. The properties of the

[18] Recall that no trade is always associated with signal $s_2$.

optimal mechanism then depend on $P_2$'s willingness to ask a low price in the event $P_1$ trades only with the high type. From $(SR_2)$, if $\delta_1^*(1, s_2|\overline{\theta}) = \delta_1^*(0, s_2|\underline{\theta}) = 1$, $P_2$ finds it optimal to ask $t_2 = b$ if and only if $gp \leq (m_2 + b)(1-p)$, that is if and only if the complementarity is not too large. Assume this is the case. Then, if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + g \leq 0$, $P_1$ never trades with $\underline{\theta}$, for even if $P_2$ is expected to ask a low price, the extra rent $P_1$ must leave to $\overline{\theta}$ when she trades with $\underline{\theta}$ more than compensates for the surplus $P_1$ can extract from the low type. The optimal mechanism is then simply $\delta_1^*(1, s_2|\overline{\theta}) = \delta_1^*(0, s_2|\underline{\theta}) = 1$. When, instead, $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + \dot{g} > 0$, $P_1$ finds it optimal to trade also with the low type in case $P_2$ offers him a low price. However, this occurs only if trade in the upstream relationship is uncertain. Furthermore, since it is always more profitable to trade with $\overline{\theta}$ than with $\underline{\theta}$, at the optimum, $P_1$ trades with probability one with $\overline{\theta}$ and with probability $\delta_1^*(1, s_2|\underline{\theta}) = [(m_2 + b)(1-p) - gp] / [(1-p)(m_2 + b + 1)]$ with $\underline{\theta}$, where $\delta_1^*(1, s_2|\underline{\theta})$ guarantees that $P_2$ is indeed willing to ask a low price. Trade is thus stochastic, but the optimal contract does not require information disclosure.[19]

Things are more difficult for $P_1$ when $gp > (m_2 + b)(1-p)$, i.e. when $P_2$ is expected to ask a high price in the event $P_1$ trades with certainty with $\overline{\theta}$ and with probability zero with $\underline{\theta}$. In this case, $P_1$ has two options. The first is to sacrifice trade also with $\overline{\theta}$ and guarantee that $P_2$ will ask a low price with certainty. The second is to trade with probability one with the high type, and use the disclosure policy to induce a low price in the downstream relationship with probability positive but less than one. When $m_1 + \bar{a} \leq m_2 + b$, the cost of scarifying trade with $\overline{\theta}$ is low and hence $P_1$ maximizes the downstream informational rent by setting $\delta_1^*(0, s_2|\underline{\theta}) = 1$ and $\delta_1^*(1, s_2|\overline{\theta}) = (m_2 + b) / [p(m_2 + b + g)] = 1 - \delta_1^*(0, s_2|\overline{\theta})$.[20] The optimal mechanism is again stochastic and it involves no disclosure. When, instead, $m_1 + \bar{a} > m_2 + b$, the cost of scarifying trade with $\overline{\theta}$ is high and hence $P_1$ prefers to induce a downstream rent only by releasing a noisy signal of the upstream decision to trade. The optimal mechanism is then $\delta_1^*(0, s_2|\underline{\theta}) = 1$ and $\delta_1^*(1, s_2|\overline{\theta}) = (1-p)(m_2 + b)/(gp) = 1 - \delta_1^*(1, s_1|\overline{\theta})$. Note that trade is deterministic, but uncertain to $P_2$, for it is a function of the agent's exogenous type.

We summarize the above results in the following

**Proposition 3** *Assume preferences are defined by Condition (4) and $x_1$ and $x_2$ are complements. Information disclosure is optimal for $P_1$ if and only if (i) $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b < m_1 + \bar{a}$ and (ii) $g > (m_2 + b)(1-p)/p$.*

When preferences are as in Condition (4), $P_1$ trades off the surplus she can appropriate by trading with $A$ with the surplus she can appropriate by forgoing trade to induce an informational rent in the downstream relationship. Under Condition (i) in Proposition (3), the value of trading with the high type is higher than the value of creating a downstream rent, whereas the opposite is true for the low type. Hence, at the optimum, $P_1$ trades with certainty with $\overline{\theta}$ and with probability

[19] The value for $\delta_1^*(1, s_2|\underline{\theta})$ comes from $(SR_2)$ substituting $\delta_1^*(1, s_2|\overline{\theta}) = 1$, and $\delta_1^*(1, s_2|\underline{\theta}) = 1 - \delta_1^*(0, s_2|\underline{\theta})$.

[20] Recall that we are considering the case $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b$ so that at the optimum $P_1$ never trades with $\underline{\theta}$, that is $\delta_1^*(0, s_2|\underline{\theta}) = 1$. The value of $\delta_1^*(1, s_2|\overline{\theta})$ is then determiend by $(SR_2)$ using the feasibility constraint $\delta_1^*(1, s_2|\overline{\theta}) = 1 - \delta_1^*(1, s_1|\overline{\theta}) - \delta_1^*(0, s_2|\overline{\theta})$.

zero with $\underline{\theta}$. On the other hand, under Condition (ii), this trade pattern is not sufficient to induce $P_2$ to ask a low price. It thus becomes optimal for $P_1$ to adopt a noisy disclosure policy which discloses signal $s_2$ with certainty when $A$ does not trade (that is, when $\theta = \underline{\theta}$) and with probability $d^*(s_2|\overline{\theta}) = \delta_1^*(1, s_2|\overline{\theta}) < 1$ when he trades (that is, when $\theta = \overline{\theta}$). By choosing $\delta_1^*(1, s_2|\overline{\theta})$ sufficiently low, $P_1$ then induces $P_2$ to leave $\overline{\theta}$ a rent with positive probability which is in turn fully appropriated by $P_1$ through the price she charges for upstream contracting.

The reason why disclosure can be optimal when preferences are not separable is that it allows $P_1$ to exert influence on downstream contracting without relying exclusively on upstream decisions. In this simple model, to induce a low downstream price without disclosing information $P_1$ would need to trade only with the high type with probability less than one. Disclosure allows $P_1$ to increase the level of trade in the upstream relationship still inducing a low downstream price with positive probability.

We now turn attention to the effects of disclosure on individual payoffs and total welfare. To this aim, consider the contracts that $P_1$ would offer if disclosure were not possible. Under the conditions in Proposition (3), among all contracts that induce $P_2$ to ask a high price, the optimal one consists in trading with both types at price $t_1 = \underline{a}$ if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geq 0$, and with the high type only at price $t_1 = \overline{a}$ otherwise. On the other hand, as we formally prove in the Appendix, among all contracts that induce $P_2$ to ask a low price, the optimal one is such that $P_1$ trades only with $\overline{\theta}$ with probability $(m_2 + b) / [p(m_2 + b + g)]$. Comparing the payoff for $P_1$ under these two contracts, we obtain that when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a > 0$ and $g < \Delta a(m_2 + b)/(m_1 + \underline{a} - m_2 - b)$, $P_1$ would find it optimal to trade with either type with certainty if disclosure were not possible. Clearly, in this case, disclosure benefits $P_1$ but negatively affects $A$ and $P_2$: by reducing the level of trade with the low type, $P_1$ decreases the rent for $\overline{\theta}$ and also the surplus $P_2$ can extract from $\underline{\theta}$. Furthermore, since it is always efficient to trade in either relationship, disclosure also reduces total welfare.

In all other cases, disclosure leads to a Pareto improvement since it does not affect the level of trade with $\underline{\theta}$ – and thus the surplus $A$ obtains from the two relationships – and it either increases or leaves unchanged the level of trade with $\overline{\theta}$. As for $P_2$, she clearly benefits from disclosure in case it boosts trade in the upstream relationship, whereas she is indifferent otherwise. To see this, note that under the optimal contract with disclosure, $P_2$ is indifferent between asking a low and a high price when she receives signal $s_2$ and strictly prefers a high price when she observes $s_1$. On the other hand, if $P_1$ were to trade only with $\overline{\theta}$ without disclosing information, $P_2$ would offer a high price with certainty. It follows that when disclosure does not affect the marginal distribution over $X_1$, $P_2$'s expected payoff is the same as without disclosure. Finally, since $P_1$ discloses signal $s_2$ with probability one when she does not trade with $A$ (that is when $A$ is a low type), she makes trade occur with certainty in the downstream relationship and hence also increases downstream efficiency.

We conclude that

**Corollary 3** *When preferences are defined by Condition (4) and $x_1$ and $x_2$ are complements, the possibility for $P_1$ to disclose information makes $P_2$ and $A$ worse off and reduces welfare if*

*$m_1+\underline{a}-\frac{p}{1-p}\Delta a \geq 0$ and $g \geq \Delta a(m_2+b)/(m_1+\underline{a}-m_2-b)$. It leads to a Pareto improvement in all other cases.*

### 3.2.2 The substitutes case

Consider finally an environment where $x_1$ and $x_2$ are substitutes in the agent's preferences, in which case $A$ obtains a positive surplus with $P_2$ only if he does *not* trade with $P_1$, and $P_2$, believing that trade occurred in the upstream relationship with sufficiently high probability, asks a low price $t_2 = b+g$. Substituting $U_A(\overline{\theta}) = [\delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta})]\Delta a$ and $U_A(\underline{\theta}) = 0$ into $P_1$'s payoff, we have that the optimal contract maximizes

$$\begin{aligned}\mathbb{E}U_1 &= p[\delta_1(1,s_1|\overline{\theta})+\delta_1(1,s_2|\overline{\theta})]\,(m_1+\bar{a})+p\delta_1(0,s_2|\overline{\theta})\,|g|+\\ &\quad +(1-p)\,[\delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta})]\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right)+(1-p)\delta_1(0,s_2|\underline{\theta})\,|g|\end{aligned}$$

subject to the following constraints

$$\begin{array}{lr}\delta_1(1,s_1|\overline{\theta})+\delta_1(1,s_2|\overline{\theta}) \geq \delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta}), & (\underline{IC}_1)\\ |g|\left[p\delta_1(0,s_1|\overline{\theta})+(1-p)\,\delta_1(0,s_1|\underline{\theta})\right] \geq (m_2+b+g)\left[p\delta_1(1,s_1|\overline{\theta})+(1-p)\,\delta_1(1,s_1|\underline{\theta})\right], & (SR_1)\\ |g|\left[p\delta_1(0,s_2|\overline{\theta})+(1-p)\,\delta_1(0,s_2|\underline{\theta})\right] \leq (m_2+b+g)\left[p\delta_1(1,s_2|\overline{\theta})+(1-p)\,\delta_1(1,s_2|\underline{\theta})\right]. & (SR_2)\end{array}$$

Note that, conditional on trading, the information $P_1$ discloses to $P_2$ does not have any direct effect on the surplus $A$ expects from downstream contracting. However, as indicated in $(SR_2)$, disclosing signal $s_2$ with probability one when trade occurs maximizes the possibility of sending signal $s_2$ also when trade occurs and hence maximizes the informational rent $A$ obtains with $P_2$. It follows that at the optimum $\delta_1^*(1,s_1|\theta) = 0$ for any $\theta$, which also implies that constraint $(SR_1)$ never binds. The optimal disclosure policy can then be qualitatively represented by the following diagram:

$$\begin{array}{ccccc} x_1 = 1 & \longrightarrow & s_2 & \longrightarrow & t_2 = b+g < b \\ & \nearrow & & & \\ x_1 = 0 & \longrightarrow & s_1 & \longrightarrow & t_2 = b \end{array}$$

Also note that since the "virtual" surplus $m_1+\underline{a}-p\Delta a/(1-p)$ that can be generated by trading with the low type is lower than that with the high type, $m_1+\bar{a}$, it is always more profitable for $P_1$ to sacrifice trade with $\underline{\theta}$ before reducing the level of trade with $\overline{\theta}$. This also suggests that constraint $(\underline{IC}_1)$ will not bind and hence it will be neglected.

The optimal contract is then obtained by comparing the "virtual" surplus $P_1$ can appropriate by trading with either type with that she can obtain by not trading and making $P_2$ offer a low price. Clearly, when $|g| \leq m_1+\underline{a}-\frac{p}{1-p}\Delta a$, the rent $A$ obtains with $P_2$ is so small that it never pays to sacrifice trade in the upstream relationship and hence the optimal contract is $\delta_1^*(1,s_2|\underline{\theta}) = \delta_1^*(1,s_2|\overline{\theta}) = 1$. On the contrary, when $m_1+\underline{a}-\frac{p}{1-p}\Delta a < |g|$, $P_1$ finds it optimal to reduce the level of trade with $\underline{\theta}$ so as to induce $P_2$ to leave him an informational rent. As with complements, the optimal mechanism then depends on the price $P_2$ is expected to ask in case trade in the upstream relationship occurs if and only if $A$ is a high type. Given this trade policy, when $|g|(1-p) \leq (m_2+b+g)p$, or equivalently $|g| \leq (m_2+b)p$, $P_2$ asks a low price. In this case, the optimal contract for $P_1$ is $\delta_1^*(0,s_2|\underline{\theta}) = 1$ and $\delta_1^*(1,s_2|\overline{\theta}) = 1$ if $|g| \leq m_1 + \bar{a}$, that is when the value of the downstream rent is lower than the surplus of trading with $\overline{\theta}$. If instead $|g| > m_1 + \bar{a}$, then it becomes attractive for $P_1$ to let also the high type enjoy a downstream rent with positive probability and the optimal mechanism is thus

$$\begin{aligned} \delta_1^*(0,s_2|\underline{\theta}) &= 1, \\ \delta_1^*(1,s_2|\overline{\theta}) &= |g|/[(m_2+b)p] = 1 - \delta_1^*(0,s_2|\overline{\theta}). \end{aligned}$$

Next consider $(m_2+b)p < |g| \leq m_2+b$, in which case $P_1$ needs to trade with positive probability also with $\underline{\theta}$ if she wants to induce a low price in the downstream relationship.[21] Constraint $(SR_2)$ then necessarily binds and hence for $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g > 0$ the optimal mechanism is

$$\begin{aligned} \delta_1^*(1,s_2|\overline{\theta}) &= 1, \\ \delta_1^*(1,s_2|\underline{\theta}) &= [|g| - p(m_2+b)] / [(1-p)(m_2+b)] = 1 - \delta_1^*(0,s_2|\underline{\theta}). \end{aligned}$$

When, instead, $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g < 0$, which is possible only if the "virtual" surplus $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$, the rent $P_1$ must leave to $\overline{\theta}$ in case she trades with $\underline{\theta}$ is so high that it never pays to trade with the low type, even accounting for the fact that trading increases the surplus $P_1$ can appropriate from $P_2$. When this is the case, $P_1$ trades only with $\overline{\theta}$ and adopts a disclosure policy which sends signal $s_2$ with certainty when $x_1 = 1$ (that is, when $\theta = \overline{\theta}$) and with probability $d^*(s_2|\underline{\theta}) = p(m_2+b+g)/[(1-p)|g|]$ when $x_1 = 0$ (that is, when $\theta = \underline{\theta}$). As with complements, information disclosure allows $P_1$ to choose a more profitable trade pattern in the upstream relationship and at the same time fashion (albeit imperfectly) the result of downstream contracting.

We summarize the conditions under which disclosure is optimal in this simple model in the following proposition.

**Proposition 4** *Assume preferences are defined by Condition (4) and $x_1$ and $x_2$ are substitutes. Information disclosure is optimal for $P_1$ if and only if (i) $p(m_2+b) < |g| \leq m_2+b$ and (ii) $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g < 0$.*

That $P_1$ finds it optimal to disclose information only when the substitutability between the two goods $|g| \leq m_2 + b$ follows directly from the fact that if $|g| > m_2 + b$, then $P_2$ never accepts to trade at a price $t_2 = b + g$ and thus disclosure is irrelevant. To understand why $P_1$ then strictly benefits from the possibility to release information when the other conditions in Proposition 4 hold, consider the optimal contracts $P_1$ would offer in case information disclosure were not possible. Among all contracts that induce $P_2$ to set a high price, the one that maximizes $P_1$'s payoff consists in trading only with $\overline{\theta}$ at price $t_1 = \overline{a}$ and gives an expected payoff equal to $p(m_1 + \bar{a})$.[22] In

[21] When $|g| > m_2 + b$, $P_2$ always asks a high price $t_2 = b$, whatever her beliefs about $x_1$.

[22] Recall that *(ii)* implies $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$.

contrast, with information disclosure, $P_1$ can sustain the same level of trade in the upstream relationship, and at the same time induce $P_2$ to leave $\underline{\theta}$ a downstream rent with positive probability. Disclosure thus trivially improves upon this contract. If instead $P_1$ were to induce a low price in the downstream relationship without disclosing information, she would have to trade with sufficiently high probability also with $\underline{\theta}$ for otherwise, under Condition *(i)* in Proposition 4, $P_2$ would ask a high price. Since it is always more profitable to trade with the high type than with the low type, among all contracts that induce a low price in the downstream relationship, the optimal one is such that $P_1$ trades with certainty with $\overline{\theta}$ and with probability $\delta(1|\underline{\theta}) = [|g| - p(m_2 + b)] / [(1 - p)(m_2 + b)]$ with $\underline{\theta}$, where $\delta(1|\underline{\theta})$ is the minimal level of trade necessary to induce $t_2 = b + g$, as indicated in $(SR_2)$. With respect to this contract, the optimal one with disclosure gives $\underline{\theta}$ a lower rent with $P_2$, but allows $P_1$ to extract more surplus from $\overline{\theta}$ by limiting the informational rent she must leave to the latter to induce truthful information revelation. Under Condition *(ii)*, this second effect dominates and hence at the optimum $P_1$ trades only with the high type and uses the disclosure policy to induce $P_2$ to leave $\underline{\theta}$ a rent with positive probability.

We now turn to the effects of disclosure on individual payoffs.

**Corollary 4** *Assume preferences are defined by Condition (4) and $x_1$ and $x_2$ are substitutes. The possibility for $P_1$ to disclose information leads to a Pareto improvement if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \leq \frac{|g|(m_2+b+g)}{p(m_2+b)-|g|}$. Otherwise, $A$ is worse off, $P_1$ and $P_2$ better off, and disclosure is welfare increasing (decreasing) if and only if $|g| \geq (<)\ m_1 + \underline{a}$.*

When $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \leq \frac{|g|(m_2+b+g)}{p(m_2+b)-|g|}$, without disclosure, $P_1$ would trade only with the high type at a price $t_1 = \overline{a}$ which in turn would induce $P_2$ to ask a high price $t_2 = b$. In this case, disclosure is clearly welfare enhancing, for it does not affect the marginal distribution over $X_1$, but it increases the level of trade in the downstream relationship by reducing the distortions that are due to the endogenous asymmetry of information between $A$ and $P_2$. What is more, disclosure leads to a Pareto improvement: $A$ and $P_2$ obtain exactly the same payoff as without disclosure, whereas $P_1$ is strictly better off. That $A$ is indifferent follows from the fact that with either contract, $P_1$ trades only with the high type and hence $A$ does not obtain any informational rent. That $P_2$ is not affected by disclosure follows from the fact that $P_1$ does not change the level of trade in the upstream relationship and from the fact that the optimal disclosure policy makes $P_2$ just indifferent between asking a high price with probability one – as in the absence of disclosure – or reducing the price conditional on $s_2$.

On the contrary, when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a > \frac{|g|(m_2+b+g)}{p(m_2+b)-|g|}$, the optimal mechanism without disclosure is such that $P_1$ trades also with $\underline{\theta}$ with probability $[|g| - p(m_2 + b)] / [(1 - p)(m_2 + b)]$ and induces $P_2$ to ask a low price with certainty. In this case, $A$ strictly suffers from disclosure since by reducing the level of trade with $\underline{\theta}$, $P_1$ also reduces the rent for $\overline{\theta}$. On the other hand, $P_2$ benefits from the reduction in the level of trade in the upstream relationship – which in the case of substitutes reduces the agent's valuation for $x_2$ – and thus is strictly better off under disclosure. Finally, for the effect of disclosure on total welfare, note that disclosure reduces the level of trade in the upstream relationship, without affecting the level of trade in the downstream relationship. This in turn is welfare increasing if and only if $|g| \geq m_1 + \underline{a}$, i.e. if and only if it is efficient not to trade with $\underline{\theta}$ in the upstream relationship.

## 4 Concluding Remarks

This paper has considered the dynamic interaction between two principals who sequentially contract with the same agent. The focus of the analysis has been the study of disclosure policies that optimally control for the exchange of information between the two bilateral relationships. We have shown that the optimal policy from the viewpoint of an upstream principal who can perfectly commit to any mechanism of her choosing consists in keeping all information secret when (a) the upstream principal is not personally interested in the decisions of the downstream principal, (b) the sign of the single crossing condition in the agent's preferences is the same for upstream and downstream decisions and (c) the marginal surplus in the downstream relationship is independent of upstream decisions. This result is robust to the possibility for the upstream principal to sell information to the downstream principal. On the contrary, when any of these conditions is violated, there exist preferences for which the upstream principal finds it strictly optimal to disclose a noisy signal of the agent's exogenous type and/or upstream contractual decisions, even if she can not make the downstream principal pay for the information she receives. We have also shown that the possibility to disclose information need not necessarily harm the agent and may boost efficiency and lead to a Pareto improvement when it reduces the asymmetry of information in the downstream relationship and increases trade in the upstream.

In order to highlight the various effects at play, we have examined the determinants for the disclosure of exogenous and endogenous information separately. Furthermore, the results have been derived assuming the upstream principal can perfectly commit to whatever policy she chooses. The design of optimal privacy policies in specific environments where disclosure may be driven by a combination of the different determinants discussed above represents an interesting line for future research. Similarly, relaxing the assumption of full commitment may deliver more insights on the welfare effects of disclosure and on the desirability of regulatory intervention in the adoption of privacy-protecting policies. Despite the limits of the model, we expect the strategic effects highlighted in the analysis, as well as the motivations for disclosure discussed in the paper, to play an important role also in the study of more complex environments.

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# 5 Appendix

**Proof of Theorem 1 – Part (i).** Under conditions (a) and (c), $P_1$, $P_2$ and $A$ 's preferences can be written as

$$\begin{aligned} v_1(x_1,x_2,\theta) &= v_1(x_1,\theta), \\ v_2(x_1,x_2,\theta) &= v_2^1(x_1,\theta)+v_2^2(x_2,\theta), \\ v_A(x_1,x_2,\theta) &= v_A^1(x_1,\theta)+v_A^2(x_2,\theta), \end{aligned} \tag{11}$$

with $v_1(0,\theta)=v_i^j(0,\theta)=0$ for $j=1,2$, and $i=2,A$. Define $W_1(x_1,\theta)\equiv v_1(x_1,\theta)+v_A^1(x_1,\theta)$ and $W_2(x_2,\theta)\equiv v_2^2(x_2,\theta)+v_A^2(x_2,\theta)$.

The proof proceeds in four steps.

**Step 1**. If $P_2$ and $A$'s preferences are separable, then without loss of generality the reaction $\phi_2(s)$ is independent of $x_1$ so that $x_2(\theta_2^E;s)=x_2(\widetilde{\theta}_2^E;s)$ and $U_A^2(\theta_2^E;s)=U_A^2(\widetilde{\theta}_2^E;s)$ for any two extended types $\theta_2^E=(\theta,x_1)$ and $\widetilde{\theta}_2^E=(\widetilde{\theta},\widetilde{x}_1)$ such that $\theta=\widetilde{\theta}$. Indeed, when this is not true, there always exists another reaction $\phi_2'(s)$ that is payoff-equivalent for all players and which is independent of $x_1$. It follows that to characterize the optimal mechanism for $P_1$, one can restrict attention to downstream mechanisms $\phi_2(s)=(x_2(\theta;s),U_A^2(\theta;s))$. This also implies that when $P_2$ does not receive information from $P_1$, her optimal mechanism is independent of $\phi_1$ and will be denoted by $\phi_2^{ND}=(x_2^{ND}(\theta),U_A^{2ND}(\theta))$. Also note that when $W_2(1,\theta)=0$ for one of the two types, information disclosure is irrelevant since $P_2$ always contracts only with one type, whatever her posterior beliefs about $\theta$. In this case the result trivially holds. In what follows, we thus assume $W_2(1,\theta)>0$ for any $\theta$. When the sign of the single crossing condition in the agent's preferences is positive, $(\overline{IC}_2)$ and $(\underline{IR}_2)$ constraints bind so that

$$U_A^2(\underline{\theta};s)=U_A^{2ND}(\underline{\theta})=0,\quad U_A^2(\overline{\theta};s)=\Delta_\theta v_A^2(x_2(\underline{\theta};s),\theta),\quad U_A^{2ND}(\overline{\theta})=\Delta_\theta v_A^2(x_2^{ND}(\theta),\theta). \tag{12}$$

Furthermore, $(\overline{IR}_2)$ are always satisfied, whereas $(\underline{IC}_2)$ reduce to

$$x_2(\overline{\theta};s)\geq x_2(\underline{\theta};s),\qquad x_2^{ND}(\overline{\theta})\geq x_2^{ND}(\underline{\theta}).$$

It follows that at the optimum

$$\begin{aligned} x_2(\overline{\theta};s) &= x_2^{ND}(\overline{\theta})=\arg\max\nolimits_{x_2\in X_2}\left\{W_2(x_2,\overline{\theta})\right\}=1, \\ x_2(\underline{\theta};s) &= \arg\max\nolimits_{x_2\in X_2}\left\{\mu(\underline{\theta};s)W_2(x_2,\underline{\theta})-\mu(\overline{\theta};s)\Delta_\theta v_A^2(x_2,\theta)\right\}, \\ x_2^{ND}(\underline{\theta}) &= \arg\max\nolimits_{x_2\in X_2}\left\{(1-p)W_2(x_2,\underline{\theta})-p\Delta_\theta v_A^2(x_2,\theta)\right\}, \end{aligned} \tag{13}$$

which also implies that $(\underline{IC}_2)$ never bind.

**Step 2**. Using $\tau(\phi_1)=\gamma\left[\mathbb{E}U_2(\phi_1)-\mathbb{E}U_2^{ND}(\phi_1)\right]$, we have that for any individually rational and incentive compatible mechanism $\phi_1$ – with downstream reaction $\phi_2(s)$

$$\begin{aligned} \mathbb{E}U_1(\phi_1) = {} & \sum_{\theta\in\Theta}\left\{W_1(1,\theta)\delta_1(1|\theta)+\sum_{s\in S}U_A^2(\theta;s)d(s|\theta)-U_A(\theta;\phi_1)\right\}\Pr(\theta)+ \\ & +\gamma\left[\mathbb{E}U_2(\phi_1)-\mathbb{E}U_2^{ND}(\phi_1)\right] \end{aligned}$$

with

$$\begin{aligned}
\mathbb{E}U_2(\phi_1) &= \sum_{\theta\in\Theta}\left\{\sum_{s\in S}\left[W_2(x_2(\theta;s),\theta)-U_A^2(\theta;s)\right]d(s|\theta)+v_2^1\left(1,\theta\right)\delta_1(1|\theta)\right\}\Pr(\theta),\\
\mathbb{E}U_2^{ND}(\phi_1) &= \sum_{\theta\in\Theta}\left\{W_2(x_2^{ND}(\theta),\theta)-U_A^{2ND}(\theta)+v_2^1\left(1,\theta\right)\delta_1(1|\theta)\right\}\Pr(\theta).
\end{aligned}$$

Suppose $\phi_1$ is optimal so that it solves $\mathcal{P}_1$; then $(\overline{IC}_1)$ and $(\underline{IR}_1)$ necessarily bind, $\left(\overline{IR}_1\right)$ is slack and

$$\begin{array}{l}
U_A\left(\underline{\theta};\phi_1\right)=0,\\
U_A\left(\overline{\theta};\phi_1\right)=\Delta_\theta v_A^1(1,\theta)\delta_1(1|\underline{\theta})+\sum_{s\in S}U_A^2(\overline{\theta};s)d(s|\underline{\theta}),\\
\Delta_\theta v_A^1(1,\theta)[\delta_1(1|\overline{\theta})-\delta_1(1|\underline{\theta})]+\sum_{s\in S}U_A^2(\overline{\theta};s)[d(s|\overline{\theta})-d(s|\underline{\theta})]\geq 0. \qquad (\underline{IC}_1)
\end{array} \tag{14}$$

**Step 3**. Now, let $\phi_1^{ND}$ be another mechanism that does not disclose information, it induces the same distribution over $X_1$ as $\phi_1$ – i.e. such that $\delta_1^{ND}(1|\theta)=\delta_1(1|\theta)=\sum_{s\in S}\delta_1(1,s|\theta)$ for any $\theta$ – and such that

$$U_A\left(\underline{\theta};\phi_1^{ND}\right)=0,\quad U_A\left(\overline{\theta};\phi_1^{ND}\right)=\Delta_\theta v_A^1(1,\theta)\delta_1^{ND}(1|\underline{\theta})+U_A^{2ND}(\overline{\theta}). \tag{15}$$

Under (b), the mechanism $\phi_1^{ND}$ – with associated reaction $\phi_2^{ND}$ – is also individually rational and incentive compatible provided that $\delta_1^{ND}(1|\overline{\theta})\geq\delta_1^{ND}(1|\underline{\theta})$. We then have that

$$\mathbb{E}U_1(\phi_1^{ND})=\sum_{\theta\in\Theta}\left\{W_1(1,\theta)\delta_1^{ND}(1|\theta)+U_A^{2ND}(\theta)-U_A\left(\theta;\phi_1^{ND}\right)\right\}\Pr(\theta)$$

and thus

$$\begin{array}{l}
\mathbb{E}U_1(\phi_1)-\mathbb{E}U_1(\phi_1^{ND})=(1-\gamma)\sum_{\theta\in\Theta}\left[\sum_{s\in S}U_A^2(\theta;s)d(s|\theta)-U_A^{2ND}(\theta)\right]\Pr(\theta)+\\
+\gamma\sum_{\theta\in\Theta}\left[\sum_{s\in S}W_2(x_2(\theta;s),\theta)d(s|\theta)-W_2(x_2^{ND}(\theta),\theta)\right]\Pr(\theta)+\\
-\sum_{\theta\in\Theta}\left\{U_A\left(\theta;\phi_1\right)-U_A\left(\theta;\phi_1^{ND}\right)\right\}\Pr(\theta).
\end{array} \tag{16}$$

Using (12), (14) and (15), (16) reduces to

$$\begin{array}{l}
\mathbb{E}U_1(\phi_1)-\mathbb{E}U_1(\phi_1^{ND})=(1-\gamma)\,p\left[\sum_{s\in S}U_A^2(\overline{\theta};s)d(s|\overline{\theta})-\sum_{s\in S}U_A^2(\overline{\theta};s)d(s|\underline{\theta})\right]+\\
+\gamma\sum_{s\in S}\left[(1-p)W_2(x_2(\underline{\theta};s),\underline{\theta})-p\Delta_\theta v_A^2(x_2(\underline{\theta};s),\theta)\right]d(s|\underline{\theta})+\\
-\gamma\left[(1-p)W_2(x_2^{ND}(\underline{\theta}),\underline{\theta})-p\Delta_\theta v_A^2(x_2^{ND}(\underline{\theta}),\theta)\right].
\end{array} \tag{17}$$

**Step 4**. To prove $\mathbb{E}U_1(\phi_1) \leq \mathbb{E}U_1(\phi_1^{ND})$, consider first the last two terms in (17). From (13), the difference between these two terms is never positive. Next, consider the first term in (17). From (13), we have that $U_A^2(\overline{\theta}; s)$ is increasing in the posterior odds $\frac{\mu(\underline{\theta};s)}{\mu(\overline{\theta};s)}$ and hence in $\frac{d(s|\underline{\theta})}{d(s|\overline{\theta})}$. That

$$\sum_{s\in S} U_A^2(\overline{\theta}; s) d(s|\underline{\theta}) \geq \sum_{s\in S} U_A^2(\overline{\theta}; s) d(s|\overline{\theta}) \tag{18}$$

then follows from standard Representation Theorems (see, for example, Milgrom, (1981) – Proposition 1). To see this, it suffices to relabel the signals so that all $s \in \mathbb{R}$, with $s_1 < s_2 < ... < s_N$, and $\frac{d(s_1|\underline{\theta})}{d(s_1|\overline{\theta})} < \frac{d(s_2|\underline{\theta})}{d(s_2|\overline{\theta})} < ... < \frac{d(s_N|\underline{\theta})}{d(s_N|\overline{\theta})}$. It follows that $U_A^2(\overline{\theta}; s)$ is increasing in $s$. The inequality in (18) is then satisfied if and only if $d(s|\underline{\theta})/d(s|\overline{\theta})$ is increasing in $s$, which holds by construction. This also implies that if $(\underline{IC}_1)$ is satisfied in $\phi_1$, then $\delta_1^{ND}(1|\overline{\theta}) \geq \delta_1^{ND}(1|\underline{\theta})$ and hence $(\underline{IC}_1)$ is satisfied also in $\phi_1^{ND}$. We can conclude that for any mechanism $\phi_1$ that solves $\mathcal{P}_1$, there always exists another mechanism $\phi_1^{ND}$ that also solves $\mathcal{P}_1$ and which does not disclose information. This completes the proof of part (i) in the theorem. The proof for part (ii) follows from Propositions 1, 2, 3, and 4. ■

**Proof of Theorem 1 – Part (i): Continuum of types and decisions.**

Assume now $\theta \in \Theta \equiv [\underline{\theta}, \overline{\theta}]$, with absolutely continuous log-concave cumulative distribution function $F(\theta)$ with density $f(\theta)$ strictly positive over $\Theta$. Furthermore, assume $X_1 = X_2 = \mathbb{R}_+$ and let $v_A^i(x_i, \theta)$, $v_1(x_1, \theta)$ and $v_2^2(x_2, \theta)$ be thrice continuously differentiable with $\frac{\partial^2 v_1(x_1,\theta)}{\partial x_1^2} < 0$, $\frac{\partial^2 v_1(x_1,\theta)}{\partial x_1 \partial \theta} \geq 0$, $\frac{\partial^2 v_2^2(x_2,\theta)}{\partial x_2^2} < 0$, $\frac{\partial^2 v_2^2(x_2,\theta)}{\partial \theta \partial x_2} \geq 0$, $\frac{\partial v_A^i(x_i,\theta)}{\partial \theta} > 0$, $\frac{\partial^2 v_A^i(x_i,\theta)}{\partial x_i^2} < 0$, $\frac{\partial^2 v_A^i(x_i,\theta)}{\partial x_i \partial \theta} \geq 0$, $\frac{\partial^3 v_A^i(x_i,\theta)}{\partial \theta \partial x_i^2} \geq 0$, and $\frac{\partial^3 v_A^i(x_i,\theta)}{\partial \theta^2 \partial x_i} \leq 0$, for $i = 1, 2$. The above conditions are standard in the continuous case (see Fudenberg and Tirole, 1991, Chapter 7) and guarantee that a single mechanism designer would optimally choose two deterministic policies $x_i(\theta)$ with no bunching.

Let $d(s|\theta)$ denote a probability measure over $S$ and $\delta_1(x_1|\theta)$ a probability measure over $X_1$, with $S \subseteq \mathbb{R}$.

In what follows, we prove the result for the case $\gamma = 1$: note that if disclosure is not optimal when $\gamma = 1$, it is also not optimal for any $\gamma < 1$.

As with discrete types, since $P_2$ and $A$'s preferences are separable, the reaction $\phi_2(s)$ is independent of $x_1$, and will be denoted by $\phi_2(s) = (x_2(\theta; s), U_A^2(\theta; s))$. Similarly, $\phi_2^{ND} = (x_2^{ND}(\theta), U_A^{2ND}(\theta))$ will denote the mechanism $P_2$ offers in case she does not receive any information from $P_1$. Given $\phi_1$, $P_2$'s expected pay-off, respectively when she observes and when she does not observe the signals $s$ is thus

$$\mathbb{E}U_2(\phi_1) = \int_{\Theta} \left\{ \int_{S} \left[W_2(x_2(\theta; s), \theta) - U_A^2(\theta; s)\right] \mathrm{d}d(s|\theta) + \int_{X_1} v_2^1(x_1, \theta)\, \mathrm{d}\delta_1(x_1|\theta) \right\} \mathrm{d}F(\theta),$$

$$\mathbb{E}U_2^{ND}(\phi_1) = \int_{\Theta} \left\{ W_2(x_2^{ND}(\theta), \theta)) - U_A^{2ND}(\theta) + \int_{X_1} v_2^1(x_1, \theta)\, \mathrm{d}\delta_1(x_1|\theta) \right\} \mathrm{d}F(\theta).$$

It follows that, given any individually rational and incentive compatible upstream mechanism $\phi_1$ – with associated downstream reaction $\phi_2(s)$ – $P_1$'s expected payoff is

$$\begin{aligned}\mathbb{E}U_1(\phi_1) &= \int_\Theta \left\{ \int_{X_1} W_1(x_1,\theta)\mathrm{d}\delta_1(x_1|\theta) + \int_S U_A^2(\theta;s)\mathrm{d}d(s|\theta) - U_A(\theta;\phi_1) \right\} \mathrm{d}F(\theta) + \\ &\quad + \mathbb{E}U_2(\phi_1) - \mathbb{E}U_2^{ND}(\phi_1) \\ &= \int_\Theta \left\{ \int_{X_1} W_1(x_1,\theta)\mathrm{d}\delta_1(x_1|\theta) + \int_S W_2(x_2(\theta;s),\theta)\mathrm{d}d(s|\theta) - U_A(\theta;\phi_1) \right\} \mathrm{d}F(\theta) + \\ &\quad - \int_\Theta \left\{ W_2(x_2^{ND}(\theta),\theta)) - U_A^{2ND}(\theta) \right\} \mathrm{d}F(\theta)\end{aligned} \tag{19}$$

where

$$\begin{aligned}&U_A(\theta;\phi_1) = \int_{X_1} v_A^1(x_1,\theta)\,\mathrm{d}\delta_1(x_1|\theta) + \int_S U_A^2(\theta,s)\mathrm{d}d(s|\theta) - t_1(\theta), \\ &U_A^2(\theta;s) = v_A^2(x_2(\theta;s),\theta) - t_2(\theta;s).\end{aligned}$$

The optimal mechanism for $P_1$ maximizes (19) subject to individual rationality and incentive compatibility constraints for $A$, as well as sequential rationality constraints for $P_2$, as in $\mathcal{P}_1$ in the main text. Let the value of this program be denoted by $\mathbb{E}U_1(\phi_1^*)$.

Now suppose $P_1$ could control directly $x_2$ and $t_2$ and could commit to them at $t=1$. That is, suppose $P_1$ could offer $A$ a fictitious mechanism $\widetilde{\phi}_1 = (\widetilde{\delta}_1(x_1|\theta), \widetilde{d}(s|\theta), \widetilde{x}_2(\theta;s), U_A(\theta;\widetilde{\phi}_1))$ such that $A$ reports his type only at $t=1$, and on the basis of the report $\theta$, $P_1$ selects a lottery $\widetilde{\delta}_1(x_1|\theta)$, a disclosure policy $\widetilde{d}(s|\theta)$, a downstream decision $\widetilde{x}_2(\theta;s)$ and a total rent $U_A(\theta;\widetilde{\phi}_1)$. Note that the combination of the disclosure policy $\widetilde{d}(s|\theta)$ with the policy $\widetilde{x}_2(\theta;s)$ induces a lottery over $X_2$. Hence, think of $\widetilde{\phi}_1$ as a stochastic mechanism that for each $\theta$ specifies a lottery over $X_1$, a lottery over $X_2$ and a rent $U_A(\theta;\widetilde{\phi}_1)$. Suppose also that $P_1$ wishes to maximize (19) subject to individual rationality and incentive compatibility constraints. Under the assumptions discussed above, it is well known that the mechanism $\widetilde{\phi}_1$ which maximizes (19) is deterministic and is characterized by two monotonic (increasing) policies $\widetilde{x}_1(\theta)$ and $\widetilde{x}_2(\theta)$. Let the value of this fictitious program be

$$\begin{aligned}\mathbb{E}U_1(\widetilde{\phi}_1) &= \int_\Theta \left\{ W_1(\widetilde{x}_1(\theta),\theta) + W_2(\widetilde{x}_2(\theta),\theta) - U_A(\theta;\widetilde{\phi}_1) \right\} \mathrm{d}F(\theta) + \\ &\quad - \int_\Theta \left\{ W_2(x_2^{ND}(\theta),\theta)) - U_A^{2ND}(\theta) \right\} \mathrm{d}F(\theta).\end{aligned}$$

Then clearly $\mathbb{E}U_1(\widetilde{\phi}_1) \geq \mathbb{E}U_1(\phi_1^*)$. In other words, $\mathbb{E}U_1(\widetilde{\phi}_1)$ is an upper bound on the payoff for $P_1$. We want to show that when $P_1$ can control only upstream decisions, she can still guarantee herself $\mathbb{E}U_1(\widetilde{\phi}_1)$ by choosing not to disclose any information to $P_2$ and delegating to her the choice over

$x_2(\theta)$ and $t_2(\theta)$. If this is true, then no disclosure is necessarily optimal. To prove this claim, note that $\widetilde{\phi}_1 = (\widetilde{x}_1(\theta), \widetilde{x}_2(\theta), U_A(\theta; \widetilde{\phi}_1))$ is incentive compatible if and only if

$$\frac{\partial U_A(\theta;\widetilde{\phi}_1)}{\partial\theta} = \frac{\partial v_A^1\left(\widetilde{x}_1(\theta),\theta\right)}{\partial\theta} + \frac{\partial v_A^2\left(\widetilde{x}_2(\theta),\theta\right)}{\partial\theta}$$

and

$$\frac{\partial v_A^1\left(\widetilde{x}_1(\theta),\theta\right)}{\partial\theta\partial x_1}\frac{\partial\widetilde{x}_1(\theta)}{\partial\theta} + \frac{\partial v_A^1\left(\widetilde{x}_2(\theta),\theta\right)}{\partial\theta\partial x_2}\frac{\partial\widetilde{x}_2(\theta)}{\partial\theta} \geq 0 \tag{20}$$

almost everywhere[23]. This implies that

$$U_A(\theta;\widetilde{\phi}_1) = U_A(\underline{\theta};\widetilde{\phi}_1) + \int_{\underline{\theta}}^{\theta}\frac{\partial v_A^1\left(\widetilde{x}_1(z),z\right)}{\partial z}\mathrm{d}z + \int_{\underline{\theta}}^{\theta}\frac{\partial v_A^2\left(\widetilde{x}_2(z),z\right)}{\partial z}\mathrm{d}z.$$

At the optimum, $U_A(\underline{\theta};\widetilde{\phi}_1) = 0$, in which case individual rationality constraints are also satisfied. Integrating by parts we then have that $\widetilde{x}_1(\theta)$ and $\widetilde{x}_2(\theta)$ maximize

$$\int_{\Theta}\left\{W_1(\widetilde{x}_1(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)}\frac{\partial v_A^1\left(\widetilde{x}_1(\theta),\theta\right)}{\partial\theta}\right\}\mathrm{d}F(\theta) + \int_{\Theta}\left\{W_2(\widetilde{x}_2(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)}\frac{\partial v_A^2\left(\widetilde{x}_2(\theta),\theta\right)}{\partial\theta}\right\}\mathrm{d}F(\theta) \tag{21}$$

subject to (20). Log-concavity of $F(\theta)$, along with the assumptions on preferences discussed above, guarantees that the schedules $\widetilde{x}_1(\theta)$ and $\widetilde{x}_2(\theta)$ that maximize (21) point-wise are increasing in $\theta$ in which case (20) is satisfied and does not bind.

Note that in the absence of information disclosure $P_2$ would offer $A$ a mechanism $\phi_2^{ND} = (x_2^{ND}(\theta), U_A^{2ND}(\theta))$ such that $x_2^{ND}(\theta)$ maximizes the "virtual" surplus $W_2(x_2,\theta) - \frac{1-F(\theta)}{f(\theta)}\frac{\partial v_A^2(x_2,\theta)}{\partial\theta}$ pointwise and $U_A^{2ND}(\theta) = \int_{\underline{\theta}}^{\theta}\frac{\partial v_A^2\left(x_2^{ND}(z),z\right)}{\partial z}\mathrm{d}z$. That is, $\widetilde{x}_2(\theta) = x_2^{ND}(\theta)$. It follows that even if $P_1$ controls only $x_1(\theta)$, she can always guarantee herself $\mathbb{E}U_1(\widetilde{\phi}_1)$ by offering $A$ a deterministic mechanism such that $x_1(\theta) = \widetilde{x}_1(\theta)$ and $t_1(\theta) = v_A^1\left(x_1(\theta),\theta\right) - \int_{\underline{\theta}}^{\theta}\frac{\partial v_A^1(x_1(z),z)}{\partial z}\mathrm{d}z$ and committing not to disclose any information to $P_2$. This proves the result. ■

**Proof of Proposition 1.** In the following, we prove that disclosure is optimal when $H < 1$ (respectively, $H \geq 1$) if and only if $e > E$ (respectively, $e \leq E$). We do so by deriving the optimal disclosure policy $d(s|\theta)$ and the optimal level of trade $\delta_1(1|\theta)$ for all possible cases.

Recall from the main text that $\phi_1^*$ maximizes

$$\begin{aligned}\mathbb{E}U_1 = p\delta_1(1|\overline{\theta})\left(m_1+\bar{a}\right) + (1-p)\,\delta_1(1|\underline{\theta})\left(m_1+\underline{a}-\tfrac{p}{1-p}\Delta a\right) + \\ + pe + (1-p)\,d(s_2|\underline{\theta})e - p\left[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})\right]\Delta b\end{aligned}$$

[23] We omit the qualification almost everywhere henceforth.

subject to the constraints

$$\begin{array}{ll} \left[\delta_1(1|\overline{\theta}) - \delta_1(1|\underline{\theta})\right] \Delta a \geq \left[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})\right] \Delta b, & (\underline{IC}_1) \\ d(s_1|\overline{\theta}) \geq H d(s_1|\underline{\theta}), & (SR_1) \\ d(s_2|\overline{\theta}) \leq H d(s_2|\underline{\theta}). & (SR_2) \end{array}$$

First, note that $(SR_1)$ and $(SR_2)$ cannot be both slack. If this were the case, $P_1$ could reduce $d(s_1|\overline{\theta})$ and increase $d(s_2|\overline{\theta})$, enhancing her payoff and relaxing $(\underline{IC}_1)$. Second, using $d(s_1|\theta) = 1 - d(s_2|\theta)$, constraint $(SR_1)$ can be rewritten as

$$d(s_2|\overline{\theta}) \leq H d(s_2|\underline{\theta}) + 1 - H. \quad (SR_1)$$

When $H < 1$, if $(SR_2)$ is satisfied, so is $(SR_1)$, whereas when $H \geq 1$, $(SR_1)$ implies $(SR_2)$. Since at least one of these two constraints must bind, it follows that for $H < 1$, $(SR_2)$ binds and $(SR_1)$ is slack, whereas the opposite is true for $H \geq 1$.

Also note that by maximizing $\delta_1(1|\overline{\theta})$, $P_1$ maximizes the objective function without violating any of the constraints. Hence, at the optimum, trade occurs with probability one when $\theta = \overline{\theta}$, i.e. $\delta_1^*(1|\overline{\theta}) = 1$.

**Unfavorable beliefs**: $H < 1$. Substituting $(SR_2)$, that is $d(s_2|\overline{\theta}) = H d(s_2|\underline{\theta})$, the program reduces to

$$\mathcal{P}_1^{Unf}: \begin{cases} \quad \max \; p\,(m_1 + \bar{a}) + (1-p)\,\delta_1(1|\underline{\theta}) \left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + \\ \quad\quad + pe + d(s_2|\underline{\theta})\left[(1-p)\,e - p\,(1-H)\,\Delta b\right] \\ \text{subject to} \\ \left[1 - \delta_1(1|\underline{\theta})\right] \Delta a \geq d(s_2|\underline{\theta})\,(1-H)\,\Delta b. \qquad\qquad (\underline{IC}_1) \end{cases}$$

First, assume $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$ so that the optimal level of trade with $\underline{\theta}$ is $\delta_1^*(1|\underline{\theta}) = 0$. If $(1-p)\,e < p\,(1-H)\,\Delta b$, the optimal policy is no disclosure, that is $d^*(s_1|\theta) = 1$ for any $\theta$. If instead $(1-p)\,e \geq p\,(1-H)\,\Delta b$, then for $\frac{\Delta b}{\Delta a}(1-H) \leq 1$, it is optimal to set $d^*(s_2|\underline{\theta}) = 1$ and $d^*(s_2|\overline{\theta}) = H$, whereas for $\frac{\Delta b}{\Delta a}(1-H) > 1$, $(\underline{IC}_1)$ is binding and the optimal disclosure policy is $d^*(s_2|\underline{\theta}) = \frac{\Delta a}{\Delta b(1-H)}$ and $d^*(s_2|\overline{\theta}) = \frac{\Delta a H}{\Delta b(1-H)}$.

Next, assume $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geq 0$. If $(1-p)\,e < p\,(1-H)\,\Delta b$, then the optimal level of trade with $\underline{\theta}$ is $\delta_1^*(1|\underline{\theta}) = 1$ and no disclosure is again optimal. If on the contrary $(1-p)\,e \geq p\,(1-H)\,\Delta b$, then $(\underline{IC}_1)$ binds, for otherwise $P_1$ could increase her expected payoff by increasing $d(s_2|\underline{\theta})$ and/or $\delta_1(1|\underline{\theta})$. Substituting $\delta_1(1|\underline{\theta}) = 1 - d(s_2|\underline{\theta})\frac{\Delta b}{\Delta a}(1-H)$ into the objective function in $\mathcal{P}_1^{Unf}$ gives

$$\mathbb{E}U_1 = p\,(m_1 + \bar{a} + e) + (1-p)\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + (1-p) d(s_2|\underline{\theta})\,(e - E)$$

where

$$E = \frac{p}{1-p}(1-H)\,\Delta b + \frac{\Delta b}{\Delta a}\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right)(1-H) = \frac{\Delta b}{\Delta a}(1-H)\,(m_1 + \underline{a}).$$

Note that $E \geq \frac{p}{1-p}(1-H)\Delta b$ when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geq 0$. Hence, if $e \leq E$, then $\delta_1^*(1|\underline{\theta}) = 1$ and $d^*(s_1|\theta) = 1$ for any $\theta$. On the contrary, when $e > E$, $P_1$ maximizes $d(s_2|\underline{\theta})$ under the constraint $\delta_1(1|\underline{\theta}) \geq 0$, i.e. $d(s_2|\underline{\theta}) \leq \frac{\Delta a}{\Delta b(1-H)}$. For $\frac{\Delta a}{\Delta b(1-H)} \geq 1$, then $d^*(s_2|\underline{\theta}) = 1$, $d^*(s_2|\overline{\theta}) = H$, and $\delta_1^*(1|\underline{\theta}) = 1 - \frac{\Delta b}{\Delta a}(1-H)$. On the contrary, for $\frac{\Delta a}{\Delta b(1-H)} < 1$, it is optimal to set $d^*(s_2|\underline{\theta}) = \frac{\Delta a}{\Delta b(1-H)}$ $d^*(s_2|\overline{\theta}) = \frac{\Delta a H}{\Delta b(1-H)}$ and $\delta_1^*(1|\underline{\theta}) = 0$.

We conclude that when $P_2$'s prior beliefs are unfavorable, disclosure is optimal if and only if

$$e > E \equiv \max\left\{m_1 + \underline{a};\ \frac{p}{1-p}\Delta a\right\} \frac{\Delta b}{\Delta a}(1-H).$$

**Favorable beliefs:** $H \geq 1$. Using $d(s_1|\overline{\theta}) = Hd(s_1|\underline{\theta})$ and $d(s_2|\theta) = 1 - d(s_1|\theta)$ gives

$$\mathcal{P}_1^{Fav}: \begin{cases} \max \quad p(m_1+\bar{a}) + (1-p)\,\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + \\ \quad + e - d(s_1|\underline{\theta})\left[(1-p)\,e - p(1-H)\,\Delta b\right] \\ \text{subject to} \\ \left[1 - \delta_1(1|\underline{\theta})\right]\Delta a \geq (H-1)\,\Delta b d(s_1|\underline{\theta}) \qquad (\underline{IC}_1) \end{cases}$$

The proof follows the same steps as for unfavorable beliefs.

Assume first $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$ so that $\delta_1^*(1|\underline{\theta}) = 0$. When $(1-p)\,e \geq p(1-H)\,\Delta b$, the optimal policy is no disclosure, that is $d^*(s_1|\theta) = 0$ for any $\theta$. On the contrary, if $(1-p)\,e < p(1-H)\,\Delta b$, then $\mathbb{E}U_1(\phi_1)$ is increasing in $d(s_1|\underline{\theta})$ and therefore for $\frac{\Delta a H}{\Delta b(H-1)} \geq 1$, $d^*(s_1|\underline{\theta}) = 1/H$ and $d^*(s_1|\overline{\theta}) = 1$ (where the upper bound on $d^*(s_1|\underline{\theta})$ comes from $SR_1$) whereas for $\frac{\Delta a H}{\Delta b(H-1)} < 1$, $(\underline{IC}_1)$ binds and thus $d^*(s_1|\underline{\theta}) = \frac{\Delta a}{\Delta b(H-1)}$ and $d^*(s_1|\overline{\theta}) = \frac{\Delta a H}{\Delta b(H-1)}$.

Next, assume $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geq 0$. If $(1-p)\,e \geq p(1-H)\,\Delta b$, then $\delta_1^*(1|\underline{\theta}) = 1$ and $d^*(s_1|\theta) = 0$ for any $\theta$. If on the contrary $(1-p)\,e < p(1-H)\,\Delta b$, then $(\underline{IC}_1)$ binds, for otherwise $P_1$ could increase her expected payoff by increasing $d(s_1|\underline{\theta})$ and / or $\delta_1(1|\underline{\theta})$. Substituting for $\delta_1(1|\underline{\theta})$ into the objective function in $\mathcal{P}_1^{Fav}$ gives

$$\mathbb{E}U_1 = p(m_1+\bar{a}) + (1-p)\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + e - (1-p)d(s_1|\underline{\theta})\,(e-E)$$

Hence, if $e > E$, then again $\delta_1^*(1|\underline{\theta}) = 1$ and $d^*(s_1|\theta) = 0$ for any $\theta$. On the contrary, when $e \leq E$ and $\frac{\Delta a H}{\Delta b(H-1)} \geq 1$, then $d^*(s_1|\underline{\theta}) = 1/H$, $d^*(s_1|\overline{\theta}) = 1$ and $\delta_1^*(1|\underline{\theta}) = 1 - \frac{\Delta b(H-1)}{\Delta a H}$. On the contrary, for $\frac{\Delta a H}{\Delta b(H-1)} < 1$, $d^*(s_1|\underline{\theta}) = \frac{\Delta a}{\Delta b(H-1)}$, $d^*(s_1|\overline{\theta}) = \frac{\Delta a H}{\Delta b(1-H)}$ and $\delta_1^*(1|\underline{\theta}) = 0$.

Summarizing, disclosure is optimal when $H \geq 1$ if and only if $e < E$. ■

**Proof of Proposition 2.**

The proof identifies conditions for the optimality of disclosure and constructs the optimal mechanism for all possible parameters' configurations. From the main text, $\phi_1^*$ maximizes

$$\mathbb{E}U_1 = p\left\{\delta_1(1|\overline{\theta})(m_1+\overline{a}) + d(s_2|\overline{\theta})\Delta b - U_A(\overline{\theta})\right\} + (1-p)\left\{\delta_1(1|\underline{\theta})(m_1+\underline{a}) - U_A(\underline{\theta})\right\}$$

subject to the constraints

$$
\begin{array}{ll}
U_A(\overline{\theta}) \geq 0, & (\overline{IR}_1) \\
U_A(\underline{\theta}) \geq 0, & (\underline{IR}_1) \\
U_A(\overline{\theta}) \geq U_A(\underline{\theta}) + d(s_2|\underline{\theta})\Delta b - \delta_1(1|\underline{\theta}) \left|\Delta a\right|, & (\overline{IC}_1) \\
U_A(\underline{\theta}) \geq U_A(\overline{\theta}) - d(s_2|\overline{\theta})\Delta b + \delta_1(1|\overline{\theta}) \left|\Delta a\right|, & (\underline{IC}_1) \\
d(s_1|\overline{\theta}) \geq Hd(s_1|\underline{\theta}), & (SR_1) \\
d(s_2|\overline{\theta}) \leq Hd(s_2|\underline{\theta}). & (SR_2)
\end{array}
$$

First, note that in any optimal contract, $\delta_1^*(1|\underline{\theta}) = 1$, for otherwise $P_1$ could increase $\delta_1(1|\underline{\theta})$ increasing the objective function without violating any of the constraints. Second, note that $(SR_1)$ always binds and $(SR_2)$ is slack when $H \geq 1$, whereas the opposite is true when $H < 1$ (the argument is identical to that in the proof of Proposition 1).

**Favorable beliefs.** From $(SR_1)$, $d(s_2|\overline{\theta}) = 1 - H + Hd(s_2|\underline{\theta})$. Suppose that $d(s_2|\underline{\theta}) < 1$. Then by reducing $d(s_1|\underline{\theta})$ to zero and increasing $U_A(\overline{\theta})$ by $\Delta b d(s_1|\underline{\theta})$, $P_1$ increases her payoff, without violating any of the constraints. Hence, $d^*(s_2|\underline{\theta}) = d^*(s_2|\overline{\theta}) = 1$, that is no disclosure is always optimal. When $\Delta b > |\Delta a|$, the optimal contract is such that $U_A^*(\overline{\theta}) = \Delta b - |\Delta a|$, $U_A^*(\underline{\theta}) = 0$, and $\delta_1^*(1|\overline{\theta}) = 1$. When instead $|\Delta a| \geq \Delta b$, then $\delta_1^*(1|\overline{\theta}) = \frac{\Delta b}{|\Delta a|}$ and $U_A^*(\overline{\theta}) = U_A^*(\underline{\theta}) = 0$ if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$; if on the contrary $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geq 0$, then $U_A^*(\overline{\theta}) = 0$, $U_A^*(\underline{\theta}) = |\Delta a| - \Delta b$ and $\delta_1^*(1|\overline{\theta}) = 1$.

**Unfavorable beliefs.** First, observe that at the optimum $(\underline{IC}_1)$ must be saturated. Indeed, if this were not true, then necessarily $U_A(\underline{\theta}) = 0$ and $\delta_1(1|\overline{\theta}) = 1$, for otherwise $P_1$ could reduce $U_A(\underline{\theta})$ and/or increase $\delta_1(1|\overline{\theta})$ enhancing her payoff. But then from $(\underline{IC}_1)$ and $(\overline{IC}_1)$, $0 \geq U_A(\overline{\theta}) - d(s_2|\overline{\theta})\Delta b + |\Delta a| \geq [d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b$, which is consistent with $d(s_2|\underline{\theta}) \geq d(s_2|\overline{\theta})$ only if no information is disclosed – that is $d(s_2|\underline{\theta}) = d(s_2|\overline{\theta})$ – and $U_A(\overline{\theta}) - d(s_2|\overline{\theta})\Delta b + |\Delta a| = 0$, in which case $(\underline{IC}_1)$ is saturated. Next, we establish that $U_A^*(\overline{\theta}) = 0$. Again, suppose this is not true. Then, necessarily $U_A(\underline{\theta}) = 0$, for otherwise $P_1$ could reduce both rents by the same amount. Using the result that $(\underline{IC}_1)$ necessarily binds, we have that $U_A(\overline{\theta}) = d(s_2|\overline{\theta})\Delta b - \delta_1(1|\overline{\theta})|\Delta a|$. Replacing $U_A(\overline{\theta})$ and $U_A(\underline{\theta})$ into $\mathbb{E}[U_1]$, gives

$$
\mathbb{E}U_1 = p\left\{\delta_1(1|\overline{\theta})(m_1 + \overline{a}) + \delta_1(1|\overline{\theta})\left|\Delta a\right|\right\} + (1-p)\left\{m_1 + \underline{a}\right\}
$$

which is increasing in $\delta_1(1|\overline{\theta})$. But then $\delta_1(1|\overline{\theta}) = \min\left\{d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|};\ 1 - (1-H)d(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}\right\}$, where the upper bound comes from $(\overline{IR}_1)$ and $(\overline{IC}_1)$, substituting $U_A(\overline{\theta})$ and $U_A(\underline{\theta})$ and using $(SR_2)$, that is $d(s_2|\overline{\theta}) = Hd(s_2|\underline{\theta})$. If $\Delta b \leq |\Delta a|$, $\min\left\{d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|};\ 1 - (1-H)d(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}\right\} = d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}$ for all $d(s_2|\underline{\theta})$ and thus $\delta_1(1|\overline{\theta}) = d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}$, which implies that $U_A(\overline{\theta}) = 0$. If instead $\Delta b > |\Delta a|$, then $\delta_1(1|\overline{\theta})$ is maximized at $d(s_2|\underline{\theta}) = \frac{|\Delta a|}{\Delta b}$, and again $U_A(\overline{\theta}) = 0$. Substituting

$U_A(\overline{\theta})$ and $U_A(\underline{\theta})$ from $(\overline{IR}_1)$ and $(\underline{IC}_1)$ and using $(SR_2)$, the program for $\phi_1^*$ reduces to

$$\widehat{\mathcal{P}}_1^{HD}:\begin{cases}\max\limits_{\phi_1\in\Phi_1} \quad p\delta_1(1|\overline{\theta})(m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|)+(1-p)\left(m_1+\underline{a}\right)+d(s_2|\underline{\theta})H\Delta b\\ \text{subject to}\\ \delta_1(1|\overline{\theta})\geq d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}, & (\underline{IR}_1)\\ \delta_1(1|\overline{\theta})\leq 1-d(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}(1-H). & (\overline{IC}_1)\end{cases}$$

If $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|<0$, $(\underline{IR}_1)$ binds. Replacing $\delta_1^*(1|\overline{\theta})=d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}$ into the objective function in $\widehat{\mathcal{P}}_1^{HD}$ gives

$$\mathbb{E}U_1=d(s_2|\underline{\theta})H\Delta b\left[1+\frac{p}{|\Delta a|}(m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|)\right]+(1-p)\left(m_1+\underline{a}\right),$$

which is increasing in $d(s_2|\underline{\theta})$ and maximized by setting $d^*(s_2|\underline{\theta})=\min\left\{1,\frac{|\Delta a|}{\Delta b}\right\}$, where the upper bound comes from $(\overline{IC}_1)$. Hence, any mechanism such that $d^*(s_2|\underline{\theta})=\min\left\{1,\ \frac{|\Delta a|}{\Delta b}\right\}$, $d^*(s_2|\overline{\theta})=Hd^*(s_2|\underline{\theta})$, $\delta_1^*(1|\underline{\theta})=1$, and $\delta_1^*(1|\overline{\theta})=H\frac{\Delta b}{|\Delta a|}d^*(s_2|\underline{\theta})$ is optimal.

If on the contrary, $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|\geq 0$, then $(\overline{IC}_1)$ binds, in which case $P_1$'s payoff reduces to

$$\begin{aligned}\mathbb{E}U_1=&\left\{-p\left(m_1+\overline{a}-\tfrac{1-p}{p}\left|\Delta a\right|\right)\tfrac{\Delta b}{|\Delta a|}\left(1-H\right)+H\Delta b\right\}d(s_2|\underline{\theta})+\\ &+p\left(m_1+\overline{a}-\tfrac{1-p}{p}\left|\Delta a\right|\right)+(1-p)(m_1+\underline{a}).\end{aligned}$$

For $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|\leq\frac{H|\Delta a|}{p(1-H)}$, $\mathbb{E}U_1$ is again increasing in $d(s_2|\underline{\theta})$ and thus $d^*(s_2|\underline{\theta})=\min\left\{1,\frac{|\Delta a|}{\Delta b}\right\}$. In this case, the optimal mechanism is: $d^*(s_2|\underline{\theta})=\min\left\{1,\ \frac{|\Delta a|}{\Delta b}\right\}$, $d^*(s_2|\overline{\theta})=Hd^*(s_2|\underline{\theta})$, $\delta_1^*(1|\underline{\theta})=1$, and $\delta_1^*(1|\overline{\theta})=1-(1-H)\frac{\Delta b}{|\Delta a|}d^*(s_2|\underline{\theta})$.

Finally, if $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|>\frac{H|\Delta a|}{p(1-H)}$, then $\mathbb{E}U_1$ is decreasing in $d(s_2|\underline{\theta})$ and thus $d^*(s_2|\underline{\theta})=d^*(s_2|\overline{\theta})=0$ and $\delta_1^*(1|\underline{\theta})=\delta_1^*(1|\overline{\theta})=1$.

We conclude that disclosure occurs if and only if beliefs are unfavorable and $m_1+\overline{a}-\frac{1-p}{p}\left|\Delta a\right|\leq\frac{H|\Delta a|}{p(1-H)}$. ■

**Proof of Proposition 3.**

The optimal mechanism $\phi_1^*$ maximizes

$$\begin{aligned}\mathbb{E}U_1 \quad = \quad & p\delta_1(1,s_1|\overline{\theta})\left(m_1+\bar{a}\right)+(1-p)\delta_1(1,s_1|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right)+\\ &+p\delta_1(1,s_2|\overline{\theta})\left(m_1+\bar{a}+g\right)+(1-p)\delta_1(1,s_2|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a+g\right)\end{aligned}$$

subject to

$$\begin{aligned}&\delta_1(1,s_1|\overline{\theta})+\delta_1(1,s_2|\overline{\theta})\geq\delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta}), && (\underline{IC}_1)\\ &g[p\delta_1(1,s_1|\overline{\theta})+(1-p)\,\delta_1(1,s_1|\underline{\theta})]\geq(m_2+b)\left[p\delta_1(0,s_1|\overline{\theta})+(1-p)\,\delta_1(0,s_1|\underline{\theta})\right], && (SR_1)\\ &g[p\delta_1(1,s_2|\overline{\theta})+(1-p)\,\delta_1(1,s_2|\underline{\theta})]\leq(m_2+b)\left[p\delta_1(0,s_2|\overline{\theta})+(1-p)\,\delta_1(0,s_2|\underline{\theta})\right]. && (SR_2)\end{aligned}$$

At the optimum, $(SR_1)$ never binds and thus can be neglected. Indeed, $\delta_1^*(0, s_1|\theta) = 0$ for any $\theta$ is always optimal, for reducing $\delta_1(0, s_1|\theta)$ and increasing $\delta_1(0, s_2|\theta)$ relaxes $(SR_1)$ and $(SR_2)$ without affecting $\mathbb{E}U_1$. Constraint $(\underline{IC}_1)$ can also be ignored as it is always satisfied at the optimum.

Next, observe that the maximal expected surplus that $P_1$ can appropriate from $P_2$ by reducing the level of trade in the upstream relationship and disclosing signal $s_2$ instead of $s_1$ is bounded from above by the right hand side in $(SR_2)$. On the other hand, the cost of creating a downstream rent is the surplus that $P_1$ must forgo in the upstream relationship when she does not trade. It follows that when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geq m_2 + b$,

$$\begin{aligned} & p\delta_1(0, s_2|\overline{\theta})\,(m_1 + \bar{a}) + (1-p)\,\delta_1(0, s_2|\underline{\theta}) \left( m_1 + \underline{a} - \frac{p}{1-p}\Delta a \right) \\ > \; & \left[ p\delta_1(0, s_2|\overline{\theta}) + (1-p)\,\delta_1(0, s_2|\underline{\theta}) \right] (m_2 + b) \\ > \; & g[p\delta_1(1, s_2|\overline{\theta}) + (1-p)\,\delta_1(1, s_2|\underline{\theta})] \end{aligned}$$

and hence $\delta_1^*(1, s_1|\theta) = 1$ for any $\theta$ is always optimal. On the contrary, when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b$, then necessarily $\delta_1^*(1, s_1|\underline{\theta}) = 0$, for otherwise $P_1$ could transfer an $\varepsilon$ probability from $\delta_1(1, s_1|\underline{\theta})$ to $\delta_1(0, s_2|\underline{\theta})$ and then increase $\delta_1(1, s_2|\underline{\theta})$ by $\frac{\varepsilon(m_2+b)}{g}$ and reduce $\delta_1(1, s_1|\underline{\theta})$ by the same amount, enhancing her payoff, without violating $(SR_2)$.

Suppose first $-g \leq m_1 + \underline{a} - \frac{p}{1-p}\Delta a \leq m_2 + b$, in which case $(SR_2)$ always binds, for the unconstrained solution is $\delta_1^*(1, s_2|\overline{\theta}) = \delta_1^*(1, s_2|\underline{\theta}) = 1$. Note that, if $\delta_1^*(0, s_2|\underline{\theta}) < 1$, then necessarily $\delta_1^*(1, s_2|\overline{\theta}) = 1$, since otherwise $P_1$ could transfer an $\varepsilon$ probability from $\delta_1(0, s_2|\overline{\theta})$ to $\delta_1(1, s_2|\overline{\theta})$ and a $\frac{p}{1-p}\varepsilon$ probability from $\delta_1(1, s_2|\underline{\theta})$ to $\delta_1(0, s_2|\underline{\theta})$, increasing $\mathbb{E}U_1$ and preserving $(SR_2)$. This also implies that for $pg \leq (1-p)\,(m_2 + b)$, $P_1$ trades with both types and hence $\delta_1^*(1, s_2|\overline{\theta}) = 1$ and

$$\delta_1^*(1, s_2|\underline{\theta}) = \frac{(1-p)\,(m_2 + b) - pg}{(1-p)\,[m_2 + b + g]} = 1 - \delta_1^*(0, s_2|\underline{\theta}),$$

whereas for $pg > (1-p)\,(m_2 + b)$, necessarily $\delta_1^*(0, s_2|\underline{\theta}) = 1$ and $\delta_1^*(1, s_2|\overline{\theta}) < 1$. The optimal mechanism in this case depends on the comparison between $m_1 + \bar{a}$ and $m_2 + b$. If $m_1 + \bar{a} > m_2 + b$, then $\delta_1^*(0, s_2|\overline{\theta}) = 0$. To see this, note that by reducing $\delta_1(0, s_2|\overline{\theta})$ and $\delta_1(1, s_2|\overline{\theta})$, respectively by $\varepsilon$ and $\frac{\varepsilon(m_2+b)}{g}$, and by increasing $\delta_1(1, s_1|\overline{\theta})$ by $\varepsilon \left[ \frac{(m_2+b)}{g} + 1 \right]$, $P_1$ enhances $\mathbb{E}U_1$, without violating $(SR_2)$. It follows that for $m_1 + \bar{a} > m_2 + b$,

$$\delta_1^*(1, s_2|\overline{\theta}) = \frac{(1-p)\,(m_2 + b)}{pg} = 1 - \delta_1^*(1, s_1|\overline{\theta}),$$

where the upper bound on $\delta_1^*(1, s_2|\overline{\theta})$ comes from $(SR_2)$. By the same argument, if $m_1 + \bar{a} \leq m_2 + b$, then $\delta_1^*(1, s_1|\overline{\theta}) = 0$ and

$$\delta_1^*(1, s_2|\overline{\theta}) = \frac{m_2 + b}{p[m_2 + b + g]} = 1 - \delta_1^*(0, s_2|\overline{\theta}).$$

Finally, consider $m_1+\underline{a}-\frac{p}{1-p}\Delta a \leq -g$. In this case, $\delta_1^*(0,s_2|\underline{\theta})=1$ is always optimal. Following the same steps as in the previous case, when $pg \leq (1-p)(m_2+b)$, then $\delta_1^*(1,s_2|\overline{\theta})=1$, whereas for $pg > (1-p)(m_2+b)$,

$$\delta_1^*(1,s_2|\overline{\theta}) = \begin{cases} \frac{(1-p)(m_2+b)}{pg} = 1-\delta_1^*(1,s_1|\overline{\theta}) & \text{if } m_1+\bar{a} > m_2+b \\ \frac{m_2+b}{p[m_2+b+g]} = 1-\delta_1^*(0,s_2|\overline{\theta}) & \text{otherwise.} \end{cases}$$

From the above results, we conclude that disclosure is optimal if and only if (i) $m_1+\underline{a}-\frac{p}{1-p}\Delta a < m_2+b < m_1+\bar{a}$, and (ii) $pg > (1-p)(m_2+b)$. ■

**Proof of Corollary 3.** The proof derives the optimal contract $P_1$ would offer under Conditions (i) and (ii) of Proposition 3, were she unable or prohibited to disclose information to $P_2$. The analysis of the effects of disclosure on welfare is in the main text.

Among all contracts that induce $P_2$ to set a high price $t_2=b+g$, the maximal payoff for $P_1$ is clearly achieved by trading with both types at price $t_1=\underline{a}$ if $m_1+\underline{a}-\frac{p}{1-p}\Delta a \geq 0$, and with the high type only at price $t_1=\overline{a}$ otherwise. In contrast, the optimal contract that induces $P_2$ to ask a low price $t_2=b$ solves

$$\mathcal{P}_1^{ND}: \begin{cases} \max \quad p\delta_1(1|\overline{\theta})(m_1+\bar{a}+g)+(1-p)\,\delta_1(1|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a+g\right) & \\ \text{subject to} & \\ \delta_1(1|\overline{\theta}) \geq \delta_1(1|\underline{\theta}), & (\underline{IC}_1) \\ g\left[p\delta_1(1|\overline{\theta})+(1-p)\,\delta_1(1|\underline{\theta})\right] \leq (m_2+b)\left[p\left(1-\delta_1(1|\overline{\theta})\right)+(1-p)\left(1-\delta_1(1|\underline{\theta})\right)\right], & (SR_2) \end{cases}$$

where constraint $(SR_2)$ guarantees that $t_2=b$ is indeed sequentially rational for $P_2$. Under Condition (ii), that is for $pg > (1-p)(m_2+b)$, constraint $(SR_2)$ always binds, whatever the sign of $m_1+\underline{a}-\frac{p}{1-p}\Delta a+g$. Using $(SR_2)$, we have that $\mathbb{E}[U_1]$ is maximized at $\delta_1(1|\overline{\theta})=\frac{m_2+b}{p(m_2+b+g)}$ and $\delta_1(1|\underline{\theta})=0$ and yields $\mathbb{E}U_1=\frac{p(m_2+b)(m_1+\bar{a}+g)}{p(m_2+b+g)}$. The optimal contract is obtained by comparing this payoff with that associated with the contract that induces a high downstream price. When $m_1+\underline{a}-\frac{p}{1-p}\Delta a \geq 0$, $\frac{(m_2+b)(m_1+\bar{a}+g)}{m_2+b+g} \geq p(m_1+\bar{a})$ if and only if $g \leq \Delta a(m_2+b)/\left[m_1+\underline{a}-m_2-b\right]$, whereas for $m_1+\underline{a}-\frac{p}{1-p}\Delta a < 0$, $\frac{(m_2+b)(m_1+\bar{a}+g)}{m_2+b+g} \geq m_1+\underline{a}$ if and only if $g \leq \frac{(1-p)(m_2+b)(m_1+\overline{a})}{p(m_1+\overline{a})-m_2-b}$. ■

**Proof of Proposition 4.**

The optimal contract maximizes

$$\begin{aligned} \mathbb{E}U_1 \quad = \quad & p[\delta_1(1,s_1|\overline{\theta})+\delta_1(1,s_2|\overline{\theta})](m_1+\bar{a})+p\delta_1(0,s_2|\overline{\theta})\,|g| \\ & (1-p)\left[\delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta})\right]\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right)+ \\ & +(1-p)\delta_1(0,s_2|\underline{\theta})\,|g| \end{aligned}$$

subject to

$$\begin{aligned} & \delta_1(1,s_1|\overline{\theta})+\delta_1(1,s_2|\overline{\theta}) \geq \delta_1(1,s_1|\underline{\theta})+\delta_1(1,s_2|\underline{\theta}), && (\underline{IC}_1) \\ & |g|\left[p\delta_1(0,s_1|\overline{\theta})+(1-p)\,\delta_1(0,s_1|\underline{\theta})\right] \geq (m_2+b+g)\left[p\delta_1(1,s_1|\overline{\theta})+(1-p)\,\delta_1(1,s_1|\underline{\theta})\right], && (SR_1) \\ & |g|\left[p\delta_1(0,s_2|\overline{\theta})+(1-p)\,\delta_1(0,s_2|\underline{\theta})\right] \leq (m_2+b+g)\left[p\delta_1(1,s_2|\overline{\theta})+(1-p)\,\delta_1(1,s_2|\underline{\theta})\right]. && (SR_2) \end{aligned}$$

At the optimum, $\delta_1^*(1,s_1|\theta)=0$ for any $\theta$. Indeed, by reducing $\delta_1(1,s_1|\theta)$ and increasing $\delta_1(1,s_2|\theta)$, $P_1$ relaxes $(SR_1)$ and $(SR_2)$ with no effect on $(\underline{IC}_1)$ and the objective function. It follows that constraint $(SR_1)$ can be neglected. Constraint $(\underline{IC}_1)$ will also be ignored as it never binds at the optimum. Also, in all cases, $\delta_1^*(0,s_1|\overline{\theta})=0$, for otherwise $P_1$ could reduce $\delta_1(0,s_1|\overline{\theta})$ and increase $\delta_1(1,s_2|\overline{\theta})$ relaxing $(SR_2)$ and $(\underline{IC}_1)$ and enhancing $\mathbb{E}U_1$.

If $|g|\leq m_1+\underline{a}-\frac{p}{1-p}\Delta a$, the solution is simply $\delta_1^*(1,s_2|\overline{\theta})=\delta_1^*(1,s_2|\underline{\theta})=1$.

If, instead, $m_1+\underline{a}-\frac{p}{1-p}\Delta a\leq|g|\leq m_1+\bar{a}$, then the unconstrained solution is $\delta_1^*(1,s_2|\overline{\theta})=\delta_1^*(0,s_2|\underline{\theta})=1$, which satisfies $(SR_2)$ if and only if $|g|(1-p)\leq(m_2+b+g)p$, or equivalently $|g|\leq p(m_2+b)$. If on the contrary $p(m_2+b)<|g|\leq(m_2+b)$, then constraint $(SR_2)$ binds and hence $\delta_1(0,s_2|\underline{\theta})<1$. The optimal mechanism then depends on the sign of $m_1+\underline{a}-\frac{p}{1-p}\Delta a+m_2+b+g$. Suppose it is positive; then $\delta_1^*(0,s_1|\underline{\theta})=0$, in which case $\delta_1^*(1,s_2|\underline{\theta})=\frac{|g|-p(m_2+b)}{(1-p)(m_2+b)}$ and $\delta_1^*(0,s_2|\underline{\theta})=1-\delta_1^*(1,s_2|\underline{\theta})$ are determined directly from $(SR_2)$. Indeed, by reducing $\delta_1^*(0,s_1|\underline{\theta})$ by $(1+\frac{m_2+b+g}{|g|})\varepsilon$ and increasing $\delta_1(1,s_2|\underline{\theta})$ and $\delta_1(0,s_2|\underline{\theta})$, respectively by $\varepsilon$ and $\frac{m_2+b+g}{|g|}\varepsilon$, $P_1$ increases $\mathbb{E}U_1$ preserving $(SR_2)$. By a similar argument, if $m_1+\underline{a}-\frac{p}{1-p}\Delta a+m_2+b+g<0$, then necessarily $\delta_1^*(1,s_2|\underline{\theta})=0$, in which case $\delta_1^*(0,s_2|\underline{\theta})=\frac{p(m_2+b+g)}{(1-p)|g|}$ is determined from $(SR_2)$ and $\delta_1^*(0,s_1|\underline{\theta})=1-\delta_1^*(0,s_2|\underline{\theta})$.

Finally, if $|g|>m_1+\bar{a}$, then $(SR_2)$ always binds, for the unconstrained solution is $\delta_1^*(0,s_2|\overline{\theta})=\delta_1^*(0,s_2|\underline{\theta})=1$. Note that if $\delta_1(0,s_2|\overline{\theta})>0$, then necessarily $\delta_1(0,s_2|\underline{\theta})=1$. Otherwise, $P_1$ could transfer an $\varepsilon$ probability from $\delta_1(0,s_2|\overline{\theta})$ to $\delta_1(1,s_2|\overline{\theta})$ and $\frac{p}{1-p}\varepsilon$ probability from either $\delta_1(1,s_2|\underline{\theta})$ or $\delta_1(0,s_1|\underline{\theta})$ to $\delta_1(0,s_2|\underline{\theta})$ increasing $\mathbb{E}U_1$ without violating $(SR_2)$. It follows that for $|g|\leq p(m_2+b)$, $P_1$ maximizes her payoff by setting

$$\begin{aligned}\delta_1^*(0,s_2|\underline{\theta}) &= 1,\\ \delta_1^*(1,s_2|\overline{\theta}) &= |g|/[(m_2+b)p]=1-\delta_1^*(0,s_2|\overline{\theta}),\end{aligned}$$

whereas for $|g|>p(m_2+b)$, necessarily $\delta_1^*(1,s_2|\overline{\theta})=1$, in which case the solution coincides with that for the case $m_1+\underline{a}-\frac{p}{1-p}\Delta a\leq|g|\leq m_1+\bar{a}$.

We conclude that disclosure is optimal if and only if (i) $p(m_2+b)<|g|\leq m_2+b$ and (ii) $m_1+\underline{a}-\frac{p}{1-p}\Delta a+m_2+b+g<0$. ■

**Proof of Corollary 4.** In what follows, we construct the optimal contract for $P_1$ under Conditions (i) and (ii) of Proposition 4, in case $P_1$ is unable or prohibited to disclose information to $P_2$. The analysis of the effects of disclosure on welfare and individual payoffs is in the main text.

Note that under Condition (ii), $m_1+\underline{a}-\frac{p}{1-p}\Delta a<0$, which in turn implies that among all contracts that induces a high downstream price the optimal one is $\delta_1(1|\overline{\theta})=1$ and $\delta_1(0|\underline{\theta})=1$. In contrast, among all contract that induces a low price, the one that maximizes $P_1$'s payoff solves

$$\mathcal{P}_1^{ND}:\left\{\begin{array}{l}\max\quad p\left\{\delta_1(1|\overline{\theta})\,(m_1+\bar{a})+\delta_1(0|\overline{\theta})\,|g|\right\}+(1-p)\left\{\delta_1(1|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right)+\delta_1(0|\underline{\theta})\,|g|\right\}\\ \text{subject to}\\ \delta_1(1|\overline{\theta})\geq\delta_1(1|\underline{\theta}),\\ |g|\left[p\delta_1(0|\overline{\theta})+(1-p)\,\delta_1(0|\underline{\theta})\right]\leq(m_2+b+g)\left[p\delta_1(1|\overline{\theta})+(1-p)\,\delta_1(1|\underline{\theta})\right],\end{array}\right.$$

where $(SR_2)$ necessarily binds at the solution. Using the two feasibility constraints $\delta_1(0|\overline{\theta}) = 1 - \delta_1(1|\overline{\theta})$ and $\delta_1(0|\underline{\theta}) = 1 - \delta_1(1|\underline{\theta})$, constraint $(SR_2)$ reduces to

$$\delta_1(1|\underline{\theta}) = \frac{|g|}{(1-p)(m_2+b)} - \frac{p}{1-p}\delta_1(1|\overline{\theta}).$$

Constraint $(\underline{IC}_1)$ then requires that $\delta_1(1|\overline{\theta}) \geq \frac{|g|}{m_2+b}$ and the objective function is increasing in $\delta_1(1|\overline{\theta})$. Hence, the solution to $\mathcal{P}_1^{ND}$ is $\delta_1(1|\overline{\theta}) = 1$ and $\delta_1(1|\underline{\theta}) = \frac{|g|-p(m_2+b)}{(1-p)(m_2+b)}$.

Comparing the payoff for $P_1$ under the above two contracts, we have that the optimal mechanism in case $P_1$ does not disclose information induces a low price if and only if

$$\frac{|g|-p(m_2+b)}{(1-p)(m_2+b)}\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right)+|g|\left[1-\frac{|g|-p(m_2+b)}{(1-p)(m_2+b)}\right]\leq 0$$

or equivalently $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \leq \frac{|g|(m_2+b+g)}{p(m_2+b)-|g|}$. ■