ON PRICE CAPS UNDER UNCERTAINTY

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ABSTRACT. This paper shows how standard arguments supporting the imposition of price caps break down in the presence of demand uncertainty. In particular, though in the deterministic case the introduction or lowering of a price cap (above marginal cost) results in increased production, increased total welfare, decreased prices, and increased consumer welfare, we show that all of the above comparative statics predictions fail for generic uncertain demand functions. For example, for price caps sufficiently close to marginal cost, a decrease in the price cap always leads to a decrease in production and total welfare under certain mild conditions. Under stronger regularity assumptions, all of the monotone comparative statics predictions from the deterministic case also do not hold for a generic uncertain demand if we restrict attention to price caps in an arbitrary fixed interval (as long as the price caps are binding for some values in that interval).

KEYWORDS: Price caps, demand uncertainty, monotone comparative statics.

1. Introduction

There is a common economic perception that, in the context of a monopoly or oligopoly, price ceilings or caps[1] can be effective in combatting the exercise of market power. In words of a Federal Energy Regulatory Commissioner (Walsh (2001)): "If you cap those prices, you eliminate any incentive to withhold ... you may as well sell into the market at a capped price as long as you're covering your running cost and making a reasonable profit."

The classical rationale for the use of price caps is well known. Consider the case of a monopolist and a standard downward-sloping demand curve. If competition was perfect, the resulting price p_{pc} would be equal marginal cost, and the produced quantity q_{pc} would maximize efficiency and welfare. However, an unconstrained monopolist will maximize profits by equalizing marginal revenue and marginal cost. The resulting quantity q_m is less than the socially optimal quantity q_{pc} and leads to a price of $p_m > p_{pc}$. The imposition of a price cap at a level between p_{pc} and p_m results in an increased quantity produced by the monopolist because its marginal revenue is constant up to a production level where the price cap is no longer binding. As a result, the imposition of a price cap at a level between p_{pc} and p_m increases quantities produced, increases welfare, and decreases prices. Moreover, the "optimal" price cap is exactly p_{pc}, the perfectly competitive price. Thus, price caps and, in particular, price caps close to marginal costs, seem like an attractive tool to increase consumer welfare and total efficiency.

A typical criticism of the theory is on practical grounds. For example, it is difficult in practice to know what p_{pc} is (See Clarkson and Miller (1982), p. 461 ff). Nevertheless, price caps have been imposed in a wide variety of markets as mentioned above. In the electric power generation industry, for example, price caps have been used in the context of creating a deregulated market while at the same time providing a check on the exercise of horizontal market power. Borenstein (2002) discusses the use of price caps in the California electricity markets. That policy makers, and not just economists, use and apply price cap theory is shown by the quote above from a Federal Energy Regulatory Commissioner. Price caps also show up various forms in the regulation of monopolies. One such area that relies heavily on the typical theory of price caps is that of performance based rate making and "price cap regulation." The idea is that rather than using traditional

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[1]The focus here is not on macroeconomic situations in which price controls might be imposed to control inflation, but on situations in which price caps are or have been imposed on particular products such as apartment rents, electricity, gasoline, interest rates on credit card balances, or college tuition.

cost-of-service regulation with attendant problems such as lack of productivity improvements and disincentives to cost minimize, the regulator sets a price cap which it adjusts over time to, in theory, marginal costs incurred by a competitive firm. The Federal Communications Commission, for example, has indicated a preference for price cap regulation over cost-of-service methodologies (In the Matter of Policy and Rules Concerning Rates for Dominant Carriers, Order on Reconsideration, CC Docket No. $87-313$, FCC $89-91$, p. 19.). For an overview of considerations under price cap regulation see Parker (2002). For application to telecommunications, see Uri (2001), and to healthcare, see Mougeot and Naegelen (2005). The institutional details of price caps whether as applied to markets or regulated utilities are, however, beyond the scope of this paper. The purpose of this paper is to focus on the theoretical properties of price caps that underlie the justification of the use of price caps in a variety of contexts. We show that the predictions of the deterministic theory change drastically if demand is uncertain.

In order to make this point, the paper examines the impact of price caps in the context of a simple Cournot model.[2] We first consider a one-stage, deterministic, Cournot game in which $n \in \mathbb{N}$ firms choose production quantities.[3] Profit of each firm depends on the quantity produced by the firm and the price of the good which is a function of the total quantity produced by the industry. In addition, each firm has a constant marginal cost c. In the context of this model, we examine the introduction of a price cap $\overline{p} > c$. Theorem 1 shows that, if we respectively restrict attention to the equilibrium with the lowest or highest production quantity, the following monotone comparative static results hold in the deterministic case: (i) production is nonincreasing in the price cap; (ii) total welfare is nonincreasing in the price cap; (iii) as the price cap $\overline{p}$ approaches marginal cost, total welfare converges to the efficient level; (iv) consumer welfare is nonincreasing in the price cap; (v) average prices are well defined (in the sense that firms produce a positive quantity) and nondecreasing in the price cap. These results seem to confirm the intuition supporting the use of price caps, in particular price caps which are close to marginal cost.

We show that these results do not extend to environments with stochastic revenues. For this purpose, we consider an extension of the above model where demand is uncertain and risk neutral firms choose the production quantities before

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[2]Cournot models are widely used in applications, for example, in the analysis of electricity markets. See Daughety (1988) for an anthology of work on Cournot models and general applications. Also see Carlton and Perloff (1994).

[3]All our results thus apply to the monopoly case $n = 1$.

the demand realization (or, equivalently, revenue realization) is known. In this environment, none of the above five predictions is true in general. As a matter of fact, we show that, under weak assumptions, all of the conditions (i)–(v) fail for a generic uncertain demand schedule. Theorem 2 shows that if demand is stochastic and some additional regularity assumptions are satisfied, (i)–(ii) actually will be reversed for price caps close to marginal cost. In general, the quantitative effect of price caps close to marginal cost will depend on the exact form of the uncertainty. Theorem 3 provides a characterization and shows that if low realizations of demand are possible, although perhaps very improbable, the breakdown of production (and thus welfare) associated with a low price cap (above but close to marginal cost) will be severe. However, perverse effects are not limited to price caps close to marginal cost. Theorems 4 and 5 show that, under stronger regularity conditions, for a generic demand schedule, the above monotonicity properties (points (i)–(ii) and (iv)–(v)) fail in any interval in which the price cap is binding. Finally, Theorem 6 considers a modification of the model in which firms do not have to sell the entire quantity they produced, but rather they choose the optimal amount they want to sell after the uncertainty has been resolved. The statements of Theorems $1 - 5$ are robust to this modification.

The technical reason why the results of the deterministic case do not immediately generalize is that with stochastic demand, the single-crossing property that underpins the deterministic monotone comparative statics results, does not have to hold. (Of course, the single crossing condition is only sufficient for the monotonicity results, not necessary.) On an intuitive level, the underlying reason for the difference between the results of the stochastic and the deterministic model is the interplay of two effects that result from an increase in the price cap. To gain some basic intuition consider the case of a monopolist producing a quantity q at a current price cap $\bar{p}$. The payoff of the monopolist is then a weighted average of the payoff if the price cap is binding and the payoff if it is not. If the monopolist knew that the price cap would be binding he would like to increase his production. Only the possibility that the price cap is not binding prevents him from doing so. An increase in the price cap has now two resulting effects. The first effect is an increase in the incentive of the monopolist to choose a higher quantity as the benefits of producing a higher quantity increase when the price cap is binding. But the disadvantages of producing a higher quantity when the price cap is not binding do not change. But there is a second effect that decreases the monopolist's incentive to increase the quantity. The probability that the price cap is binding will decrease as the price cap increases. Our theorems establish that, under some regularity assumptions, the

first effect dominates the second and the monopolist produces a higher quantity at
the larger price cap. The effects of a low price cap can be particularly severe when
price caps are just above marginal costs. If there is a slight possibility that pro-
duction will be inefficient, then a price cap sufficiently close to marginal cost will
result in a complete stoppage of production.

Our results exhibit some duality to those of the papers by Deneckere et al. (1996,
1997) on minimum resale price maintenance. Deneckere et al. (1997) examines a
model of a monopoly manufacturer selling a product with little or no scrap value
through a competitive retail sector to consumers. The retailers must order inven-
tories before the demand uncertainty is resolved. In this environment it may be
optimal for the manufacturer to impose a minimum retail price instead of permit-
ting retail markets to clear. Consumers may also benefit from the minimum price
because prices are lower and sales are higher when demand is high. In essence,
with a binding minimum price the competitive retailers order more inventory and
so sales and welfare can be higher. Although our model is very different its equi-
libria have dual properties: A price cap may lead to lower production and reduced
welfare.

The paper is organized as follows. The next section introduces the model. Sec-
tion 3 contains the results. In Section 4 we present examples and discuss in detail
the significance of the underlying assumptions. Section 5 concludes. All proofs can
be found in the Appendix.

2. Model

We consider the case of n symmetric firms, where $n = 1, 2, \ldots$ is an arbitrary
natural number. Each firm produces the same homogeneous good at a constant
marginal cost c which is the same for all firms.[4]

Demand is given by a continuous price function $P(Q, \theta)$ which depends on $Q \in
\mathbb{R}_+$, the total quantity produced in the industry, and some random variable $\theta \in \mathbb{R}$
distributed according to a distribution F with bounded support.[5] We assume that
a high realization of θ leads to higher prices than a low realization, i.e. that $P(Q, \theta)$
is increasing in θ for any fixed value of Q. We also assume that for each fixed θ the
price function $P(Q, \theta)$ is decreasing in Q.

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[4]We consider constant marginal costs because in this case the comparative statics results are
very clear cut and simple to state. It is straightforward to generalize Theorems 2, 4 and 5 to the
case of a convex cost function.

[5]While F has bounded support, it will be convenient to assume that $P(Q, \theta)$ is defined for all
$\theta \in \mathbb{R}$ and $Q \in \mathbb{R}_+$.

Firms decide on the quantity they wish to produce before the realization of θ is known. Because firms are assumed to be risk neutral, they will maximize their expected profits. This means that in the case of no price caps, firm i chooses its production q to maximize its expected profits

$$\pi(q, Q_{-i}) = E(q \cdot (P(q + Q_{-i}, \theta) - c)),$$

where Q_{-i} is the quantity produced by all other firms. Similarly, in the more general case of a price cap $\overline{p}$, firm i chooses its production q to maximize its expected profits given by

$$\pi(q, Q_{-i}, \overline{p}) = E(\pi_\theta(q, Q_{-i}, \overline{p})),$$

where

$$\pi_\theta(q, Q_{-i}, \overline{p}) = q \cdot (\min(P(q + Q_{-i}, \theta), \overline{p}) - c)$$

are the profits for a given realization of θ. To guarantee that there exists a profit-maximizing quantity, we assume that $\lim_{Q \to \infty} P(Q, \theta) < c$ for all $\theta \in \mathbb{R}$.

In the following, we say that *production is gainful* if $E(P(0, \theta)) > c$. Clearly if $E(P(0, \theta)) < c$, no firm will ever produce a positive quantity no matter what the price cap is. As the analysis of the case $E(P(0, \theta)) = c$ is straightforward and without practical interest, we will in the following assume that production is gainful. While we do not require P to be differentiable, we assume there exists $L_1, L_2 > 0$ such that $|P(Q, \theta) - P(Q', \theta)| \leq L_1 \cdot |Q - Q'|$ and $|P(Q, \theta) - P(Q, \theta')| \geq L_2 \cdot |\theta - \theta'|$ for all $\theta, \theta' \in \mathbb{R}$ and $Q, Q' \in [0, Q^{Max}]$, where $Q^{Max} = \inf\{x : E(P(x, \theta)) = c\}$.[6]

We are interested in pure strategy symmetric Nash Equilibria, i.e. quantities q that solve $q \in \arg\max_{q'} \pi(q', (n-1) \cdot q, \overline{p})$. For any fixed price cap $\overline{p}$, denote the set of all such equilibria by $q^*(\overline{p})$. Since a price cap below c leads to no production, we restrict ourselves in the following to price caps $\overline{p}$ which are strictly larger than c.

3. Results

3.1. **Existence.** We start by establishing the existence of the studied equilibria.

Proposition 1. *For any price cap $\overline{p}$ the set $q^*(\overline{p})$ is nonempty.*

Existence of symmetric equilibria for similar models in which marginal costs are equal to zero was proven by Roberts and Sonnenschein (1976).[7] The argument of Roberts and Sonnenschein relies on the nonnegativity of the inverse demand

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[6]If P were continuously differentiable, we could replace the above assumption with the requirement that $\frac{d}{dQ} P < 0$ and $\frac{d}{d\theta} P > 0$ on the relevant region. Note that our assumptions are sufficiently flexible to allow both for nonnegative price functions or the possibility of negative prices.

[7]See also Example 1 in Milgrom and Roberts (1994).

function and therefore does not directly apply in the presence of a positive c if $E(\min(P(Q,\theta),\overline{p}) - c)) < 0$ for some $\overline{p}$ and Q. Instead, we use Lemma 1 below to establish a regularity property of the best response correspondence $b(\cdot, \overline{p})$.

Lemma 1. *Let* $b(x, \overline{p}) = \arg\max_q \pi(q, x, \overline{p})$ *be the best response of a player if his opponents together produce* $x \in \mathbb{R}_+$ *and the price cap is* $\overline{p}$.

Fix a price cap $\overline{p} > c$ *and quantity* $x_o \in \mathbb{R}_+$. *Then* $\lim_{x \uparrow x_o} b(x, \overline{p})$ *and* $\lim_{x \downarrow x_o} b(x, \overline{p})$ *exist[8] and*

$$\lim_{x \uparrow x_o} b(x, \overline{p}) = \min b(x_o, \overline{p}) \leq \max b(x_o, \overline{p}) = \lim_{x \downarrow x_o} b(x, \overline{p}),$$

where $\min b(x, \overline{p})$ *and* $\max b(x, \overline{p})$ *denote respectively the smallest and largest element in* $b(x, \overline{p})$.[9]

The existence of equilibria now follows immediately from Lemma 1 along the same lines as in Roberts and Sonnenschein (1976) or Milgrom and Roberts (1994).

By Proposition 1, the set $q^*(\overline{p})$ is nonempty. Note that it is also compact as the best response correspondence has a closed graph. We will denote the smallest and largest element in $q^*(\overline{p})$ by $q_L^*(\overline{p})$ and $q_H^*(\overline{p})$, respectively.

3.2. Deterministic Case.
If we refer to total and consumer welfare in the context of deterministic demand, we mean respectively $\int_0^Q (P(x,\theta) - c) \cdot dx$ and $\int_0^Q (P(x,\theta) - \min(P(Q,\theta), \overline{p})) \cdot dx$ respectively.[10] If demand is not deterministic by total and consumer welfare, we mean respectively the expected values of the above two expressions.

Theorem 1. *Assume deterministic demand and restrict attention to the equilibrium with the highest production quantity,* $q_H^*(\overline{p})$, *or the lowest production quantity,* $q_L^*(\overline{p})$. *Then in the corresponding equilibrium,*

- (i) *production is nonincreasing in the price cap;*
- (ii) *total welfare is nonincreasing in the price cap;*
- (iii) *as price cap approaches marginal cost, total welfare converges to the efficient level;*

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[8]When we say that $\lim_{x \uparrow x_o} b(x, \overline{p})$ exists, we mean that there exists a real number denoted by $\lim_{x \uparrow x_o} b(x, \overline{p})$ such that for any sequence $x_m \to x_o$ with $x_m \leq x_0$ and for any sequence $q^m \in b(x_m, \overline{p})$ it is the case that $q^m \to \lim_{x \uparrow x_o} b(x, \overline{p})$. The statement for $\lim_{x \downarrow x_o} b(x, \overline{p})$ has an analogous meaning.

[9]The existence of $\min b(Q_{-i}, \overline{p})$ and $\max b(Q_{-i}, \overline{p})$ is guaranteed; the best response correspondence has a closed graph with values that are uniformly bounded. (The existence of a uniform upper bound follows from $\lim_{Q \to \infty} P(Q, \theta) < c$ for all $\theta \in \mathbb{R}$.)

[10]Thus, in the case where the price cap is binding, consumer welfare is calculated under efficient rationing.

(iv) *consumer welfare is nonincreasing in the price cap;*

(v) *average prices are well-defined*[11] *(i.e. firms engage in production) and non-decreasing in the price cap.*

The proof of the theorem is based on Lemma 1 and the following observation.

Lemma 2. *In the case of deterministic demand (the support of θ consists of a single point), $\pi(q, Q_{-i}, \overline{p})$ satisfies the single crossing property as a function of $(q, -\overline{p})$ for any fixed Q_{-i}.*

It is noteworthy that the above result involves all Nash Equilibria. Milgrom and Roberts (1994, Example 1) illustrate their methods considering the Cournot model of Roberts and Sonnenschein (1976). They conclude that an increase in costs leads to a decrease in equilibrium quantities if they restrict themselves to the set of equilibria in which each firm chooses the highest quantity consistent with profit maximization. We are able to generate results involving the set of all equilibria because of the regularity conditions developed in Lemma 1.

3.3. **Uncertain Demand.** Note that in the case of stochastic demand, the function π will be still a convex combination of functions which (by Lemma 2) have the single crossing property. However, unlike the increasing differences property, the single crossing property is not always preserved under convex combinations. Therefore, in the case of stochastic demand, it is not immediately clear whether π will satisfy the single crossing property as in Lemma 2. Of course, even if π does not satisfy the single crossing property, this does not automatically imply that $q_L^*(\overline{p})$ and $q_H^*(\overline{p})$ cannot be monotonically decreasing.

We start our investigation by considering price caps close to marginal costs.[12]

Theorem 2. *Assume $F : \mathbb{R} \to [0, 1]$ is continuously differentiable.*[13]
Then there exists $\gamma > 0$ such that q_L^* and q_H^* are monotonically non-decreasing in $(c, c + \gamma)$.

Assume, in addition, P is continuously differentiable. Then there exists $\gamma > 0$ such that either both q_L^* and q_H^* are monotonically increasing in $(c, c + \gamma)$ or both are equal to zero on $(c, c + \gamma)$.

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[11]In the case of deterministic demand there actually will be just a unique price. We use the term average prices so that it also makes sense to talk about (v) in the context of uncertain demand.

[12]Price caps close to marginal costs have received special attention. (See, for example, the comment of the Federal Energy Regulatory Commissioner cited in the Introduction.)

[13]The continuous differentiability of F is not just a technical requirement. We discuss this assumption in detail in Section 4.

Thus, if price caps are sufficiently close to c, then a lowering of the price cap will actually reduce the production level. Note that for any θ - as long as θ is known - welfare arbitrary close to the maximal level can be achieved by choosing price caps sufficiently close to c. It might be surprising that while this holds for any fixed θ, it is not the case when the exact realization of θ is ex ante uncertain, in which case the optimal, welfare maximizing price cap is bounded away from c.

To see the most basic intuition behind this result, consider the case of a monopolist ($n = 1$). Imagine the current price cap is $\bar{p}_1$ and the monopolist charges some quantity q. The payoff of the monopolist is then a weighted average of the payoff $q \cdot \bar{p}_1$ if the price cap is binding, and $q \cdot P(q, \theta)$ if it is not. Note that if the monopolist knew that the price cap would be binding he would like to increase his production. Only the possibility that the price cap is not binding prevents him from doing so. Now, consider what happens if the price cap goes slightly up to a level $\bar{p}_2 > \bar{p}_1$. In this case, if the price cap is binding profits become $q \cdot (\bar{p}_2 - c)$ while profits remain $q \cdot (P(q, \theta) - c)$ if the price cap is not binding. There are two resulting effects. The first effect of an increase in the price cap is an increase in the incentive of the monopolist to choose a higher quantity as the benefits of producing a higher quantity increase when the price cap is binding but the disadvantages of producing a higher quantity when the price cap is not binding do not change. But there is a second effect that decreases the monopolist's incentive to increase the quantity. The probability that the price cap is binding will decrease as the price cap increases. Theorem 2 establishes that under the given assumptions, the first effect dominates the second.

As a direct corollary of Theorem 2 we can establish that the comparative statics results of Theorem 1 do not hold for a generic distribution. In the following, when we refer to the set of all distributions on the reals and its subsets we consider it equipped with the topology that corresponds to weak convergence (i.e. convergence in distribution).

Corollary 1. *Assume P is continuously differentiable. Denote the set of all distributions with bounded support for which production is gainful by $\mathcal{F}$. Restrict attention to the equilibrium with the highest or lowest production quantity.*

Then the set of $F \in \mathcal{F}$, for which at least one of the conditions (i), (ii) or (iii) of Theorem 1 holds, is nowhere dense in $\mathcal{F}$. Indeed, there is an open and dense set for which neither of those conditions is satisfied.

While the above results imply that, unlike in the deterministic case, price caps close to marginal costs are typically not welfare-improving, they leave open the question whether the effect of such price caps are "really bad" or whether, perhaps,

they still lead to welfare that, for example, is above what would be achieved without a price cap. The answer to this question, of course, depends on the distribution F.

We denote by $\underline{\theta}$ and $\overline{\theta}$ the minimal and maximal point in the support of F.

Theorem 3. *Let q^{**} be equal to the unique root[14] of the equation*

$$P(n \cdot q^{**}, \underline{\theta}) = c.$$

if there is a nonnegative root or equal to zero if the equation does not have a non-negative root. Then $q^*(\overline{p})$ converges to $\{q^{**}\}$ as $\overline{p}$ converges to c, $\overline{p} \searrow c$.

Theorem 3 states that as the price cap gets closer and closer to marginal cost, the firms produce the equilibrium quantity from a competitive market with the lowest demand. Building on the intuitive discussion after Theorem 2, we can provide a basic intuition for this result. A firm deciding about its production quantity must consider the implications of its choice on profits for the different demand realizations, in particular the profits for a binding price cap and also for a price below the price cap, $P(Q, \theta) < \overline{p}$. Then it chooses a quantity that constitutes a good compromise between the different quantities which would be produced if demand were known. As the price cap converges to c, the benefits of high production in anticipation of a high demand state will decrease. Thus, the marginal benefits of production in high demand states become relatively smaller than the marginal disadvantages of high production in low demand states. As a result, lower production quantities are chosen. In the limit the producer will focus only on the state in which demand is smallest and produce the corresponding quantity.

The theorem immediately allows us to conclude that in certain situations, the introduction of a price cap close to marginal cost will result in a decreased production quantity. In particular, if there is a slight possibility that production will be inefficient, the introduction of a price cap sufficiently close to marginal cost will decrease production to zero.

Corollary 2. *Assume that the ex post socially optimal production is equal to zero with some positive probability. Then a price cap sufficiently close to marginal cost c will result in a complete stoppage of production.*

3.4. **Price caps that are not close to marginal cost.** Corollary 1 was based on properties of the equilibrium set for price caps close to marginal costs. This raises the question whether the monotonicity properties from Theorem 1 remain true for uncertain demand schedules if we restrict attention to price caps which are

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[14]The equation cannot have multiple roots because P is decreasing in quantities.

still binding, but not necessarily close to c. To generalize Corollary 1, we make an extra assumption.[15] In this section we assume that *marginal profits are decreasing in other firms' output,* in the sense that $\frac{d}{dq}\pi_\theta(q, Q_{-i})$ is decreasing in Q_{-1} (the output of the other firms) for any $\theta \in \mathbb{R}$, $q \in \mathbb{R}_+$ and $Q_{-i} \in \mathbb{R}_+$.[16] Note that this implies that, for any given price cap $\bar{p}$, best responses are nonincreasing and thus there exists a unique equilibrium for each price cap. Slightly abusing notation, we denote this equilibrium by $q^*(\bar{p})$.

Let (p', p'') be an interval. We will say that the *price cap is binding* for some $\bar{p} \in (p', p'')$ if for some $\bar{p} \in [p', p'']$, $\Pr(P(n \cdot q^*(\bar{p}), \theta) > \bar{p}) > 0$. Clearly, if this is not the case, i.e., if $\Pr(P(n \cdot q^*(\bar{p}), \theta) < \bar{p}) = 1$ for all $\bar{p} \in [p', p'']$, it has to be that q^* is constant on $[p', p'']$.

In the following, we will say that *production is gainful for a price cap $\bar{p}$* if $E(\min(P(0, \theta), \bar{p})) > c$. Again, if $(\min(P(0, \theta), \bar{p})) < c$ then at a given price cap of $\bar{p}$, no firm would ever produce a positive quantity.

The following result shows that, for stochastic demand distributions, neither produced quantities nor total welfare will typically be nonincreasing functions of the price cap almost everywhere. That is, it does not matter what distribution of θ one chooses, one can always find a perturbation of that distribution in which the desirable properties of price caps (i) or (ii) of Theorem 1 fail.

Theorem 4. *Assume P is twice continuously differentiable and marginal profits are decreasing in other firms output. Let (p', p'') be an arbitrary nonempty interval. Let F be the set of all distributions with bounded support such that production is gainful for price caps in (p', p'') and the price cap is binding for some $\bar{p} \in (p', p'')$.*

Recall points (i)-(v) in Theorem 1. Then the set of distributions $F \in \mathsf{F}$ for which at least one of the conditions (i) or (ii) holds for price caps in (p', p'') is nowhere dense in F. Indeed, there is an open and dense set for which neither of those conditions is satisfied.

As a decrease in the price cap has a direct effect on average prices and consumer welfare, an additional assumption is needed to guarantee that (iii) and (iv) do not

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[15]The assumption that marginal profits are decreasing in other firms output is convenient in the proof of Theorem 4 as it stops alternative equilibria from appearing but it does not appear to be essential for the result.

[16]Actually, for any fixed $n \in \mathbb{N}$, the following results also hold under the slightly weaker alternative assumption that $q \cdot \frac{d^2}{dQ^2}P(n \cdot q, \theta) + (1 + \frac{1}{n}) \cdot \frac{d}{dQ}P(n \cdot q, \theta) \leq 0$ for all $q \in \mathbb{R}_+$ and $q \in \mathbb{R}_+$.

hold for a generic demand structure. Let

$$CS_\theta(Q) = \int_0^Q (P(x,\theta) - P(Q,\theta)) \cdot dx$$

be consumer surplus for a fixed θ if industry wide output is Q and if there is no price cap or if the price cap is so high it never binds. We will say that CS is twice continuously differentiable and $CS'' < 0$ if, for all $\theta \in \mathbb{R}$, $CS_\theta(\cdot)$ is twice differentiable, $CS_\theta''(Q)$ is continuous in both parameters, and $CS_\theta''(Q) < 0$ for all $Q \in \mathbb{R}_+$ and $\theta \in \mathbb{R}$.

Theorem 5. *Assume CS is twice continuously differentiable with $CS'' < 0$ and marginal profits are decreasing in other firms output. Let (p', p'') be an arbitrary nonempty interval. Let F be the set of all distributions with bounded support such that production is gainful for price caps in (p', p'') and the price cap is binding for some $\bar{p} \in (p', p'')$.*

Recall points (i)-(v) in Theorem 1. Then the set of distributions $F \in F$ for which at least one of the conditions (i), (ii), (iv), and (v) holds for price caps in (p', p'') is nowhere dense in F. Indeed, there is an open and dense set for which neither of those conditions is satisfied.

Our result that lowering a price cap can result in higher average prices is somewhat surprising and means that a regulator trying to lower prices with a price cap can have his effort backfire. There appear to be empirical examples where lower price caps coincided with higher average prices, see, for example, California Power Exchange (2000). (However, the question whether in this case the price increase was related to uncertainty about future demands or driven by other factors is beyond this paper.)

3.5. **Disposal.** The model we introduced in Section 2 corresponds to a situation where firms must sell the entire quantity they produced, that is, a situation where disposal is infinitely costly or where firms do not learn anything about the realization of demand before they have to make a decision on the sold quantity. Alternatively, one could consider situations where the uncertainty is resolved after the firms' production decisions but before the firms decide on the quantity they want to sell. Then, especially when demand turns out to be very low, firms might benefit from disposal as this allows them to affect prices. In the following we describe a variation of our model that allows for disposal.

Firms are now permitted to withhold some amount of their production quantity from the market and can dispose of the unsold units. We assume that there is a fixed per unit cost of disposal equal to a constant $\eta > -c$. The case $\eta > 0$

corresponds to a situation where disposal is costly, $\eta = 0$ corresponds to the case of free disposal, and a parameter $\eta \in (-c, 0)$ corresponds to a situation where firms can recover some, but not all of the production costs.[17] For example, one could imagine that firms can recover some of the inputs used in the product or that the product has some scrap value and can be sold elsewhere.

In the presence of disposal our model becomes a two-stage game. In the first stage each firm decides on a production quantity q. Subsequently the firms learn the true state of the world, θ. Once they know the state each firm decides on the quantity q' that it wants to sell. The second-stage optimization problem for firm i is to maximize

$$\max_{q' \in [0,q]} (q' \cdot \min(P(q' + Q'_{-i}, \theta), \bar{p}) - \eta \cdot (q - q') - c \cdot q)$$

given that the firm knows that Q'_{-i} is the total quantity sold by all other firms. In general, depending on the form of P, the quantity q, and the value θ the second stage game might have many symmetric equilibria. To avoid technicalities related to this fact we assume that for any fixed q and θ the above function is strictly concave and marginal revenue is decreasing in Q'_{-i}. Then define $q'(q, \bar{p})$ to be the unique optimal solution to the second-stage maximization problem in the symmetric second-stage equilibrium where all other firms happen to also sell q'. Let

$$\pi_\theta(q, Q_{-i}, \bar{p}) = q'(q, \bar{p}) \cdot \min(P(q'(q, \bar{p}) + Q'_{-i}, \theta), \bar{p}) - \eta \cdot (q - q'(q, \bar{p})) - c \cdot q.$$

Define the set of symmetric equilibria $q^*(\bar{p})$ again to be the set of quantities q such that q maximizes

$$\pi(q, Q_{-i}, \bar{p}) = E(\pi_\theta(q, Q_{-i}, \bar{p})).$$

Theorem 6. *Consider the modified model with disposal. The statements of Theorems 1-5 remain valid for the equilibria with the production quantities $q_L^*(\bar{p})$ and $q_H^*(\bar{p})$.*

4. Discussion

In this section we illustrate our results and the driving forces behind them using two numerical examples. Recall from our model description the assumption that there exists a constant $L_2 > 0$ such that $|P(Q, \theta) - P(Q, \theta')| \geq L_2 \cdot |\theta - \theta'|$ for all $\theta, \theta' \in \mathbb{R}$ and $Q \in [0, Q^{Max}]$. This assumption can be seen as a regularity requirement as it is automatically satisfied whenever P is continuously differentiable

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[17]Note that the case $\eta = -c$ would render this model equivalent to a deterministic model. Firms could produce sufficiently large quantities, then choose the optimal deterministic quantity after uncertainty is resolved, and finally dispose without a loss of any leftover production quantity.

and $\frac{d}{d\theta}P > 0$ on the relevant region. Together with the assumption that the distribution of θ is continuously differentiable it implies that for any quantity in the relevant range the distribution of market clearing prices is Lipschitz continuous. This Lipschitz continuity is central to our analysis since this property introduces a sufficient degree of uncertainty into our model.

We present two numerical examples. The first example satisfies the regularity assumption and exhibits the properties of Theorems 2 and 3. The second example does not satisfy the regularity assumption and as a result does not exhibit the properties. We explain how this failure can be traced back to the kind of uncertainty we require. We then conclude the discussion with an argument why the regularity assumption is economically sensible.

Example 1. Let demand be given by $P(Q, \theta) = \max\{0, \theta - Q\}$ where $\theta \in [10, 20]$ has the triangular distribution with support $[10, 20]$ and mode 15. There is a single firm with constant marginal cost of $c = 10$. For a sufficiently low price cap $\bar{p}$ the firm's objective function is as follows.

$$
\pi(Q, \bar{p}) = Q \cdot \left(\int_{10}^{\bar{p}+Q} (\theta - Q) \cdot \frac{1}{25}(\theta - 10)d\theta + \int_{\bar{p}+Q}^{15} \bar{p} \cdot \frac{1}{25}(\theta - 10)d\theta \right.
$$
$$
\left. + \int_{15}^{20} \bar{p} \cdot (\frac{1}{5} - \frac{1}{25}(\theta - 15))d\theta \right)
$$

Figure 1 shows the production quantity of the monopolist for price caps $\bar{p} \in (10, 10.4]$. Just as Theorems 2 and 3 predict, the production quantity converges monotonically to 0 as $\bar{p} \searrow c = 10$ since at $\underline{\theta} = 10$ it holds that $P(0, \underline{\theta}) = c$.
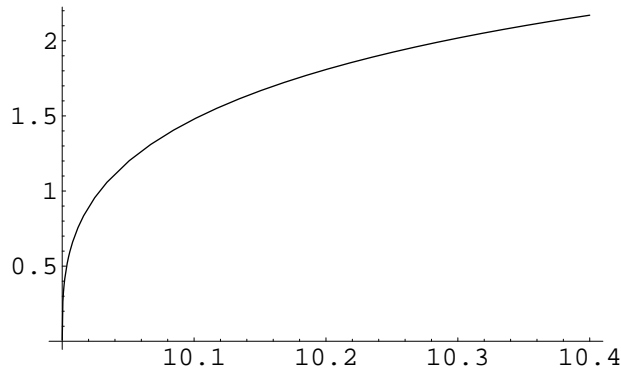

FIGURE 4.1. Production Quantity as Function of Price Cap

To gain some intuition why the production decreases as the price cap decreases we compare the firm's problem for two price caps, $\bar{p}_1 = 10.3$ and $\bar{p}_2 = 10.2$. At $\bar{p}_1$ the optimal production quantity is $Q_1 = 2.0183$, at $\bar{p}_2$ the optimal production quantity is $Q_2 = 1.8087$. Observe that Q_1 is the unique solution to $\frac{d}{dQ}\pi(\cdot, \bar{p}_1) = 0$. If the price cap is now lowered to $\bar{p}_2$ then the expected marginal profit becomes negative, $\frac{d}{dQ}\pi(Q_1, \bar{p}_2) = -0.0714$. Therefore, the firm must reduce its quantity in order to produce optimally. In order to see why expected marginal profit becomes negative we need to analyze the profit function in more detail.

The expected profit function is a weighted average of two components. For $\theta > 12.3183$ ($= \bar{p}_1 + Q_1$), the price cap is binding and profits are given by $R_{\theta, \bar{p}_1} = Q \cdot (\bar{p}_1 - c)$. Note that, for θ in this region, the marginal profit $MR_{\theta, \bar{p}_1} = \bar{p}_1 - c$ is always positive. In particular, if the monopolist knew in advance that the price cap is binding, then he would like to increase Q. For $\theta < 12.3183$, the price cap is not binding and profit is given by $R_\theta = Q \cdot ((\theta - Q) - c)$. For θ in this region, because of the monopolist's market power, marginal profit will be typically negative (as long as Q is larger then the quantity an unconstraint monopolist would choose if he knew θ). Here, at $\bar{p}_1$ and Q_1 the marginal revenues from the two components are respectively -0.2116 and 0.2116.

What happens if production is kept constant at Q_1 but the price cap is reduced to $\bar{p}_2 = 10.2$? Note that for realizations of θ for which the price cap was previously binding ($\theta > 12.3183$) the new lower price cap will still be binding. However, for θ in this region marginal profit decreases from $MR_{\theta, \bar{p}_1} = \bar{p}_1 - c$ to $MR_{\theta, \bar{p}_2} = \bar{p}_2 - c$. In other words, the monopolist's earlier incentive to produce a higher quantity is weakened. This explains why the monopolist might want to produce a lower quantity.

However, there is a second effect. As the price cap decreases slightly to $\bar{p}_2 = 10.2$ the new price cap will become binding for $\theta \in [12.2183, 12.3183]$ where it was not binding previously. While for $\theta < 12.2183$ ($= \bar{p}_2 + Q_1$) the price cap is still not binding and marginal profit remains unchanged, for $\theta \in [12.2183, 12.3183]$ marginal profit "jumps upwards" from $P(Q, \theta) - c - Q$ to $\bar{p}_2 - c$ as the term $-Q$ that comes from the monopolist's market power disappears. The change in marginal profit after the decrease in the price cap thus is the sum of two effects: a decrease coming from the case of $\theta > 12.3183$ for which the price cap was previously binding and an increase coming from the case of $\theta \in [12.2183, 12.3183]$ for which the price cap is binding at $\bar{p}_2$ but was not at $\bar{p}_1$.

In this example, the probability of a θ such that the price cap is binding at $\bar{p}_1$ is 89.2513%. The corresponding probability for $\bar{p}_2$ is 90.1586%. Thus, the probability

of a θ such that the price cap is binding at $\bar{p}_2$ but not binding at $\bar{p}_1$ is only 0.9073%. This last probability is very small and so it is not surprising that overall the first effect dominates and total marginal profit decreases. Indeed, numerically, the two effects correspond respectively to -0.0892 and 0.0178 giving the total expected marginal profit of -0.0714.

In summary, there are two driving forces needed for the first effect to dominate the second:

(1) For sufficiently small price caps there is always a positive probability that the price cap is binding. (Here, as before we use "binding" in the sense that the market clearing price without a cap would be strictly larger then the price cap).

(2) The probability that the price cap is binding changes only "very little" in a response to a small change in the price cap.

We contrast this example to a second example[18] with discrete uncertainty.

Example 2. Let demand be given by $P(Q, \theta) = \max\{0, \theta - Q\}$ where $\theta \in \{1, 2\}$, with each value having probability $\frac{1}{2}$. There is a single firm with constant marginal cost of $c = 0$. Then the firm's objective function for a given price cap $\bar{p}$ is

$$\pi(Q, \bar{p}) = Q \left(\frac{1}{2} \min\{\bar{p}, \max\{0, 1 - Q\}\} + \frac{1}{2} \min\{\bar{p}, \max\{0, 2 - Q\}\} \right).$$

For any price cap $\bar{p} \leq \frac{1}{3}$ the optimal solution to this optimization problem is $Q = 2 - \bar{p}$, that is, the optimal quantity is decreasing in the price cap. This fact contradicts the results of Theorems 2 and 3.

Observe first that neither of the two important properties (1) and (2) of the first example are satisfied. Consider again a sufficiently low price cap $\bar{p}_1 > c = 0$ and the corresponding optimal quantity Q_1. Then at $\bar{p}_1$ there is a zero probability that the price cap is binding in the sense that $P(Q, \theta)$ is strictly larger than $\bar{p}_1$. On the other hand, if the price cap is slightly decreased to a level $\bar{p}_2$ (and production held constant) the price cap will become binding (in the above sense) with probability 50%. As a result, in the example the second of the two effects highlighted in the discussion of Example 1 dominates.

The basic idea of Example 2 is that for sufficiently low price caps the quantities Q chosen by the monopolist are in a region such that $P(Q, \theta)$ is constant and equal to zero for $\theta < M$ where M is some constant which is larger then $\theta_{low} = 1$. Therefore, the maximization problem of the monopolist for those price caps looks essentially as if the monopolist would ignore low demand realizations and condition on $\theta > M$.

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[18]We are grateful to an anonymous referee for suggesting this example.

But the distribution of θ conditional on $\theta > M$ is deterministic. As a result the same monotone comparative statics results hold as if there was no uncertainty and the monopolist effectively knew that the state will be $\theta_{high} = 2$. It is exactly this "lack of uncertainty" that drives the discrepancy between the above example and Theorem 2. To challenge the predictions of the deterministic model we therefore need to impose a regularity condition, which ensures that the distribution of prices for any given production quantity cannot be deterministic even after conditioning on any event that has positive probability. Such a condition is the assumption that the distribution of θ is continuously differentiable and that there exists a constant $L_2 > 0$ such that $|P(Q, \theta) - P(Q, \theta')| \geq L_2 \cdot |\theta - \theta'|$ for all $\theta, \theta' \in \mathbb{R}$ and $Q \in [0, Q^{Max}]$. This assumption guarantees that, for production levels in the relevant range, the distribution of market clearing prices will be well behaved, in particular Lipschitz continuous.

A natural question to ask now is whether the assumption that the distribution of market clearing prices is well behaved is economically interesting. Or, whether models in which prices are uncertain but take only finitely many values might provide a better descriptive model. We think there is something unappealing and unnatural in a model with only finitely many possible prices and where all prices in the neighborhood of any given price have zero probability. It seems that most natural perturbations introducing some small randomness would invalidate this structure. In addition, recall that in the studied model, the demand side is captured by a classic demand function. We see this as a reduced model for a situation where the number of customers is large enough to induce essentially price taking behavior. However the regularity of the distribution of market clearing prices seems a natural requirement, if the number of customers is large and each customer has a valuation v that does not just take finitely many values but is distributed in some interval according to a continuous distribution that is allowed to depend on some fundamental uncertainty θ.

5. Conclusion

This paper has shown that the simple monotone comparative statics results which are usually used to justify price caps in the context of deterministic demand cease to hold if we consider firms facing an uncertain demand function. In the presence of such uncertainty, imposing a price cap or lowering its level can result in a reduced production and total welfare. In addition, it can lead to an increase

in average prices and a decrease in consumer welfare.[19] In the case of price caps close to marginal costs, we obtained a characterization of equilibrium production thus making it possible to quantitatively assess the effects of imposing such a cap. Unlike in the case where the future realization of demand is known, price caps close to marginal cost are never welfare maximizing. In the case of price caps "far away" from marginal costs, we showed that all of the standard comparative statics predictions fail for a generic uncertain demand schedule suggesting that also in this case, price caps should be used only with great caution.

Mathematical Appendix

In the following, we sometimes want to underline the dependence of functions or variables on the underlying distribution F. In this case, we write π_F, $q_F^*(\overline{p})$, etc. Slightly abusing notation, we continue to write $\pi_{\theta'}$ where $\theta' \in \mathbb{R}$ to denote profits if F is such that $\theta = \theta'$ with probability one, i.e.,

$$\pi_{\theta'}(q, Q_{-i}) = q \cdot (P(q + Q_{-i}, \theta') - c).$$

A.1. Proof of Proposition 1 and related results.

Proof of Lemma 1. The statement of the lemma consists of two parts, one involving $\lim_{x \downarrow x_o} b(x, \overline{p})$, the other involving $\lim_{x \uparrow x_o} b(x, \overline{p})$. We will prove the former; the proof of the latter is analogous.

Assume the first part of the lemma is not true, thus there exists a sequence $x_k \downarrow x_o$ and a sequence $q_k \in b(x_k, \overline{p})$ such that q_k converges to some point q^A which is not equal to the point $\max b(q, \overline{p})$ which we will denote by q^B.

As the best response correspondence has a closed graph, it must be the case that q^A is itself a best response. The definition of q^B therefore implies that $q^B \geq q^A$. As we assumed that q^A is not equal to q^B, it follows that $0 \leq q^A < q^B$.

For the sake of notation, it will be convenient to denote $x_k - x_o$ by h_k. Also, let $\phi(x) = E(\min(P(x, \theta), \overline{p}) - c))$ for all $x \in \mathbb{R}_+$. The remainder of the proof is organized in several steps.

(1) We already saw that both q^A and q^B must be best responses against x_o, in particular it must be that

$$q^A \cdot \phi(x_o + q^A) = q^B \cdot \phi(x_o + q^B).$$

---

[19]Of course, even if average prices increase in response to a decrease in price caps, such price caps could still be justified if the objective is to decrease price volatility rather than improve consumer welfare. (See Verleger (1993) for a discussion of this issue.)

and thus (as $0 \leq q^A < q^B$ implies $0 < q^B$)

$$\frac{q^A}{q^B} \cdot \phi(x_o + q^A) = \phi(x_o + q^B).$$

Note that $0 < q^B$ and $q^B \cdot \phi(x_o + q^B) \geq 0$ implies that $\phi(x_o + q^B) \geq 0$. But then $q^A < q^B$ implies that $\phi(x_o + q^A) > 0$. Hence,

$$\phi(x_o + q^A) > \phi(x_o + q^B).$$

(2) Since q^A is a best response against x_o, it must be that

$$
\begin{aligned}
q_k \cdot \phi(x_k + q_k) &= (q_k + h_k) \cdot \phi(x_o + q_k + h_k) - h_k \cdot \phi(x_o + q_k + h_k) \\
&\leq q^A \cdot \phi(x_o + q^A) - h_k \cdot \phi(x_o + q_k + h_k).
\end{aligned}
$$

(3) Since q_k is a best response against x_k, it must be that

$$(q^B - h_k) \cdot \phi(x_k + (q^B - h_k)) \leq q_k \cdot \phi(x_k + q_k).$$

(4) Combining the inequalities of the two previous steps, we get that

$$(q^B - h_k) \cdot \phi(x_k + (q^B - h_k)) \leq q^A \cdot \phi(x_o + q^A) - h_k \cdot \phi(x_o + q_k + h_k).$$

Note that $x_k + (q^B - h_k) = x_o + q^B$ and apply the first equation of Step 1 to conclude that

$$h_k \cdot \phi(x_o + q_k + h_k) \leq h_k \cdot \phi(x_o + q^B).$$

As $h_k > 0$ and $x_o + h_k = x_k$ this means that

$$\phi(x_k + q_k) \leq \phi(x_o + q^B).$$

(5) Our assumptions on P guarantee that Φ is continuous. Thus, as $x_k \to x_0$ and $q_k \to q^A$, the last step implies that

$$\phi(x_o + q^A) \leq \phi(x_o + q^B).$$

The last inequality in Step 5 contradicts the last inequality of Step 1. The contradiction shows that the statement of the lemma must be correct. $\square$

Proof of Proposition 1. Lemma 1 implies that the function $\max b(\cdot, \overline{p})$ is continuous but for upwards jumps. Thus, by Theorem 1 in Milgrom and Roberts (1994), the equation

$$q = \max b((n-1) \cdot q, \overline{p})$$

has a solution. The conclusion follows. $\square$

A.2. **Proof of Theorem 1 and related results.**

Proof of Lemma 2. Consider two quantities $q' > q''$ and two price caps $\overline{p}' < \overline{p}''$. Assume that, for a fixed Q_{-i}, $\pi_\theta(q', Q_{-i}, \overline{p}'') > \pi_\theta(q'', Q_{-i}, \overline{p}'')$. We will show that $\pi_\theta(q', Q_{-i}, \overline{p}') > \pi_\theta(q'', Q_{-i}, \overline{p}')$. We will compare the change in profits as the price cap changes from $\overline{p}''$ to $\overline{p}'$. It will be convenient to distinguish three cases:

(1) the price cap $\overline{p}'$ is binding for both quantities q' and q''. As $\overline{p}' > c$, this implies that producing the higher quantity is better after the price cap was decreased to $\overline{p}'$.

(2) the price cap $\overline{p}'$ is binding if production has the lower level q'' but is not binding if production is q'. As $\overline{p}' < \overline{p}''$, this means that after the decrease of the price cap to $\overline{p}'$ production at a level q'' became less attractive while production at a level of q' yields the same profits as before. As producing the higher quantity was better before, it also must be better now, after the decrease in the price cap.

(3) the price cap $\overline{p}'$ is not binding for both quantities q' and q''. In this case profits are the same both after and before a decrease in the price cap. The claim follows.

The second part of the proof which involves weak inequalities is analogous. $\square$

Lemma 3. *If π satisfies the single crossing property then $q_L^*(\overline{p})$ and $q_H^*(\overline{p})$ are both nonincreasing functions of $\overline{p}$.*[20]

Proof of Lemma 3. Define $\phi(Q) = E(\min(P(Q, \theta), \overline{p}) - c)$. Using Lemma 1, we already showed in the proof of Theorem 1 that $\max b(\cdot, \overline{p})$ is continuous but for upward jumps. Similarly, Lemma 1 implies that $\min b(\cdot, \overline{p})$ is also continuous but for upward jumps.

As π satisfies the single crossing property, the Monotonicity Theorem of Milgrom and Shannon (1994, p. 162, Theorem 4) implies that for any fixed Q_{-i} the functions $\min b(Q_{-i}, \cdot)$ and $\max b(Q_{-i}, \cdot)$ are nonincreasing. Note that $q_L^*(\overline{p})$ and $q_H^*(\overline{p})$ are respectively equal to the lowest and highest intersection of $\min b(Q_{-i}, \cdot)$ and $\max b(Q_{-i}, \cdot)$ with the diagonal. Theorem 1 in Milgrom and Roberts (1994) implies that $q_L^*(\overline{p})$ and $q_H^*(\overline{p})$ are nonincreasing. $\square$

Proof of Theorem 1. The theorem states that (i)-(v) hold if we restrict attention to the equilibrium with the highest or lowest production quantity. Lemmas 2 and 3 immediately imply that (i) holds. Note that total welfare

$$\int_0^Q (P(x, \theta) - c)) \cdot dx$$

---

[20]As always in this paper, we only consider price caps $\overline{p}$ which lie above marginal cost c.

is a monotonic function of total production Q as long as $P(Q, \theta) \geq c$. However, independently of the price cap, equilibrium total production will always satisfy $P(Q, \theta) \geq c$ as otherwise firms could decrease production. Point (ii) follows.

If demand is deterministic, then in equilibrium demand is equal to supply, i.e. for all $q \in q^*$, $P(n \cdot q, \theta) \leq \overline{p}$. Indeed, $P(n \cdot q, \theta) > \overline{p}$ in connection with the continuity of $P(\cdot, \theta)$ and $\overline{p} > c$ would mean that firms could increase their profits by slightly increasing production. Therefore (i) together with the observation made above and the monotonicity assumptions on the inverse demand function P implies (iii) and (iv). Property (v) now follows directly from (i) and (iv). $\square$

A.3. Proofs for Section 3.3.

Proof of Theorem 2. We will prove the statement of the theorem for q_L^*, the argument for q_H^* is analogous. Consider first the case where P is twice continuously differentiable in both of its parameters. Note that if $q_L^*(\overline{p}') = 0$ for some $\overline{p}' > c$ then $q_L^*(\overline{p}'') = 0$ for all $p'' \in (c, \overline{p}')$ and the theorem holds. Assume therefore, without loss of generality, that $q_L^*(\overline{p}') > 0$ for all $\overline{p}' > c$.

Define $q^{\max}$ to be the solution to

$$P(q^{\max}, \overline{\theta}) = c,$$

where $\overline{\theta}$ is the maximal point in the closure of the support of F. Let $b(x, \overline{p}) = \arg\max_q \pi(q, x, \overline{p})$ be the best response of a player if her opponents together produce x and the price cap is $\overline{p}$.

Note that

$$
\begin{aligned}
\frac{d^2}{dq \cdot d\overline{p}} \pi(q, Q_{-i}, \overline{p}) &= \frac{d}{dq} \int_{\widetilde{\theta}}^{\infty} q \cdot dF(\theta) \\
&= (1 - F(\widetilde{\theta})) + \frac{\frac{\delta}{\delta Q} P(q + Q_{-i}, \widetilde{\theta})}{\frac{\delta}{\delta \theta} P(q + Q_{-i}, \widetilde{\theta})} \cdot q \cdot f(\widetilde{\theta}) \\
&\geq (1 - F(\widetilde{\theta})) - \frac{L_1}{L_2} \cdot q^{\max} \cdot f(\widetilde{\theta})
\end{aligned}
$$

where $\widetilde{\theta}$ is equal to $\underline{\theta}$ if $P(q + Q_{-i}, \underline{\theta}) > \overline{p}$, equal to $\overline{\theta}$ if $P(q + Q_{-i}, \overline{\theta}) < \overline{p}$ and given by $P(q + Q_{-i}, \widetilde{\theta}) = \overline{p}$ otherwise. If, in addition, $q \in b(Q_{-i}, \overline{p})$ then,

$$
\begin{aligned}
0 &\leq (1 - F(\widetilde{\theta})) \cdot q \cdot (\overline{p} - c) + \int_{-\infty}^{\widetilde{\theta}} (P(q + Q_{-i}, \theta) - c) \cdot q \cdot dF(\theta) \\
&\leq (1 - F(\widetilde{\theta})) \cdot q \cdot (\overline{p} - c) + \int_{-\infty}^{\widetilde{\theta}} (\overline{p} - c - (\widetilde{\theta} - \theta) \cdot L_2) \cdot q \cdot dF(\theta) \\
&= q \cdot ((\overline{p} - c) + \int_{-\infty}^{\widetilde{\theta}} (\theta - \widetilde{\theta}) \cdot L_2 \cdot dF(\theta))
\end{aligned}
$$

and thus either $q = 0$ or $\int_{-\infty}^{\widetilde{\theta}} (\widetilde{\theta} - \theta) \cdot dF(\theta) \leq \frac{(\overline{p} - c)}{L_2}$.

Note that $\int_{-\infty}^{\widetilde{\theta}} (\widetilde{\theta} - \theta) \cdot dF(\theta)$ is monotonically increasing in $\widetilde{\theta}$. Choose $\gamma > 0$ such that if $\int_{-\infty}^{\widetilde{\theta}} (\widetilde{\theta} - \theta) \cdot dF(\theta) \leq \frac{\gamma}{L_2}$ then $\widetilde{\theta}$ is small enough so that

$$(1 - F(\widetilde{\theta})) - \frac{L_1}{L_2} \cdot q^{\max} \cdot f(\widetilde{\theta}) \geq \frac{1}{2}.$$

Putting the above observations together conclude that if $q, x \in \mathbb{R}_+$ and $\overline{p} > c$ satisfy $\overline{p} - c < \gamma$, $q \leq \max b(x, \overline{p})$, and $q > 0$ then $\frac{d^2}{dq \cdot d\overline{p}} \pi(q, x, \overline{p}) \geq \frac{1}{2}$.

Assume q_L^* is not monotonically nondecreasing in $(c, c + \gamma)$. Then there exist sequences $r^n, s^n \in (c, c + \gamma)$ such that $s^n - r^n \to 0$, $r^n < s^n$, and $q_L^*(r^n) > q_L^*(s^n)$. Without loss of generality, assume there exists $t \in (c, c + \gamma)$ such that $s^n, r^n \to t$ as $n \to \infty$. Note that γ was chosen so that $\frac{d^2}{dq \cdot d\overline{p}} \pi(q, x, \overline{p}) > \frac{1}{4}$ for all $(q, x, \overline{p})$ in some neighborhood of $\{(q', (n-1) \cdot q, t) : q \in q^*(t), q' \leq \max b((n-1) \cdot q, t)\}$. Then, however, Theorem 1 from Edlin and Shannon (1996) implies that $b(x, \overline{p})$ is monotonically increasing (as a function of $\overline{p}$) in some neighborhood of $\{(n-1) \cdot q, t) : q \in q^*(t)\}$ which means that q_L^* has to be monotonically increasing in some neighborhood of t. Contradiction!

Now consider the case where P does not satisfy the differentiability assumptions made above. Choose $\gamma > 0$ such that if $\int_{-\infty}^{\widetilde{\theta}} (\widetilde{\theta} - \theta) \cdot dF(\theta) \leq 2\frac{\gamma}{L_2}$ then $\widetilde{\theta}$ is small enough so that

$$(1 - F(\widetilde{\theta})) - \frac{L_1}{L_2} \cdot 8 \cdot q^{\max} \cdot f(\widetilde{\theta}) \geq \frac{1}{2}.$$

If q_L^* is not monotonic on $(c, c + \gamma)$, then there are points $r, s \in (c, c + \gamma)$ such that $r < s$ and $q_L^*(r) > q_L^*(s)$.

Choose a sequence of demand schedules P^k such that

- $q_k^{\max} : P^k(q^{\max}, \overline{\theta}) = c$ satisfy $q_k^{\max} < 2 \cdot q^{\max}$ where $q^{\max} : P^k(q^{\max}, \overline{\theta}) = c$;
- $|P^k(q, \theta) - P^k(q', \theta)| \leq 2 \cdot L_1 \cdot |q - q'|$ and $|P^k(q, \theta) - P^k(q, \theta')| \geq 2 \cdot L_2 \cdot |\theta - \theta'|$ for all $\theta, \theta' \in \mathbb{R}$, $q, q' \in \mathbb{R}\_+$ and $k \in \mathbb{N}$;
- $P^k \to P$ uniformly on $Support(F) \times [0, 2 \cdot q^{\max}]$
- P^k is twice continuously differentiable in both parameters
- the lowest equilibrium for a price cap s is the same for each demand schedule P^k as in P.

As in the limit the equilibrium set for price cap r and demand schedule P^k has to converge to a subset of $q^*(r)$, the established result for the differentiable demand schedules P^k implies the result for the nondifferentiable case. $\square$

Proof of Corollary 1. Let $F \in \mathcal{F}$ be an arbitrary distribution. Note that we can always find a continuously differentiable distribution $\widehat{F} \in \mathcal{F}$ arbitrary close to F. Theorem 2 together with standard continuity arguments implies that (i) does not

hold for $\widehat{F}$.[21] If (i) does not hold for $\widehat{F}$, there is an open neighborhood of $\widehat{F}$ for which (i) does not hold. We showed that for any $F \in \mathcal{F}$, there exists an open set on which (i) does not hold and which is arbitrary close to F. This fact implies that there is an open and dense set on which (i) does not hold. Analogous arguments imply that there are open and dense sets on which respectively (2) and (3) do not hold. Taking the intersection of those three sets we obtain an open and dense set on which none of the properties (i)-(iii) holds. This fact implies that the sets of $F \in \mathcal{F}$ for which at least one of the three statements holds is nowhere dense. $\square$

Proof of Theorem 3. Let $\overline{p}^m > c$ be a sequence of numbers such that $\lim_{m \to \infty} \overline{p}^m = c$ and q^m be a sequence of quantities such that $q^m \in q^*(\overline{p}^m)$. Our aim is to show that $q^m \to q^{**}$. For this purpose we examine the sequence $P(n \cdot q^m, \underline{\theta})$.

Assume that

$$\liminf_{m \to \infty} P(n \cdot q^m, \underline{\theta}) < c$$

Then there exists a subsequence q^{m_k} such that $\lim_{k \to \infty} P(n \cdot q^{m_k}, \underline{\theta}) < c$. As $\underline{\theta}$ lies in the support of θ, this means that with positive probability the firms will make a per unit loss which is bounded away from zero. Note that on the other hand, the potential per unit gains converge to zero as $\overline{p}$ approaches c. Thus it must be that q^{m_k} is equal to zero for large enough k. In this case, $\lim_{k \to \infty} P(n \cdot q^{m_k}, \underline{\theta}) < c$ implies $P(0, \underline{\theta}) < c$, i.e. the equation from the proposition does not have a solution.

Assume now that

$$\limsup_{m \to \infty} P(n \cdot q^m, \underline{\theta}) > c.$$

Then there exists a subsequence q^{m_k} such that $\lim_{k \to \infty} P(n \cdot q^{m_k}, \underline{\theta}) > c$. This means that $\lim_{k \to \infty} P(n \cdot q^{m_k}, \theta)$ is uniformly bounded away from c for all θ. Thus, for large enough k the price cap will be always binding. As $P(Q, \theta)$ is assumed to be continuous in Q, this means that firms would like to increase their production in equilibrium – contradiction. Hence $\limsup_{m \to \infty} P(n \cdot q^m, \underline{\theta}) \leq c$. $\square$

A.4. **Proofs of Theorems 4, 5, and 6.**

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[21]Indeed, either q_L^* and q_H^* are monotonically increasing on some interval $(c, c + \gamma)$ or equal to zero in that interval. In the former case (i) clearly does not hold. In the latter case, (i) would imply that q_L^* and q_H^* are equal to zero for all price caps. This, however, can not be, the fact that production is gainful implies that producing zero cannot be an equilibrium if the price cap is chosen high enough so that it is not binding if total production is zero. The contradiction shows that (i) also in this case cannot be true.

Proof of Theorem 4. Note that it is sufficient to show that for any continuously differentiable $F \in F$ with convex support, there is a $\widetilde{F} \in F$ arbitrary close to F such that q^* is nonmonotonic on (p', p''). Indeed, if this is the case, there will be a neighborhood of $\widetilde{F}$ for which q^* is nonmonotonic.

Assume the price cap is binding at some point in (p', p''). If q^* is not monotonically nonincreasing in (p', p'') then we can choose $\widetilde{F} = F$ and are done. Assume therefore q^* is monotonically nonincreasing in (p', p''). This fact combined with the assumption that the price cap is binding at some point in (p', p'') implies that the price cap is binding in some proper subinterval of (p', p''). Without loss of generality assume the price cap is binding on $(p,' p'')$.

Note that since marginal profits decrease in other firms output and prices are decreasing, $\pi_\theta(\cdot, Q_{-i})$ is concave for any $\theta \in \mathbb{R}$, $Q_{-i} \in \mathbb{R}_+$. As $\pi_\theta(\cdot, Q_{-i}, \overline{p})$ is equal to $\min(\pi_\theta(\cdot, Q_{-i}), (\overline{p} - c) \cdot q)$, it is also concave for any $\theta \in \mathbb{R}$, $Q_{-i} \in \mathbb{R}_+$, and $\overline{p} > c$. The equilibrium q^* is thus equal to the unique root of the first order condition[22]

$$\int_{-\infty}^{\infty} D_1\pi_\theta(q, (n-1) \cdot q, \overline{p}) \cdot dF = 0.$$

For a given distribution F, price cap $\overline{p}$, and quantity $q \in \mathbb{R}_+$, denote the left hand of the above expression by $\Gamma_F(q, \overline{p})$. It is straightforward to check that, as marginal profits decrease in other firms' output, $\Gamma_F(\cdot, \overline{p})$ is a decreasing function for any distribution F and price cap $\overline{p}$.

Choose a point $\overline{p} \in (p', p'')$. Define $\widetilde{\theta} : P(n \cdot q^*(\overline{p}), \widetilde{\theta}) = \overline{p}$. Fix a $\varepsilon > 0$ and consider the distribution $\widetilde{F}$ given by

$$\widetilde{F}(x) = \begin{cases} F(x) & x < \widetilde{\theta} - \varepsilon \\ F(\widetilde{\theta} - \varepsilon) + \alpha \cdot (F(\widetilde{\theta} + \varepsilon) - F(\widetilde{\theta} - \varepsilon)) & x\epsilon[\widetilde{\theta} - \varepsilon, \widetilde{\theta} + \varepsilon) \\ F(x) & \text{if } x \geq \widetilde{\theta} + \varepsilon \end{cases}$$

where α is chosen so that

$$\frac{\int_{\widetilde{\theta}-\varepsilon}^{\widetilde{\theta}+\varepsilon} D_1\pi_\theta(q^*(\overline{p}), (n-1) \cdot q^*(\overline{p}), \overline{p}) \cdot dF}{(F(\widetilde{\theta} + \varepsilon) - F(\widetilde{\theta} - \varepsilon))}$$

is equal to

$$\alpha \cdot D_1\pi_{\widetilde{\theta}-\varepsilon}(q^*(\overline{p}), (n-1) \cdot q^*(\overline{p})) + (1 - \alpha) \cdot \overline{p}.$$

Note that this definition implies that

$$\int_{-\infty}^{\infty} D_1\pi_\theta(q^*(\overline{p}), (n-1) \cdot q^*(\overline{p}), \overline{p}) \cdot d\widetilde{F} = 0$$

and thus $q^*(\overline{p})$ is also the unique equilibrium if θ is distributed according to $\widehat{F}$.

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[22]$D_1\pi_\theta$, the derivative with respect to the first parameter exists a.e. As F is assumed continuously differentiable and D_1P is bounded it is clear that the integral is well defined.

Finally observe that in some neighborhood of $\overline{p}$, the Implicit Function Theorem yields

$$\frac{dq^*}{d\overline{p}} = -\frac{\int_{\widetilde{\theta}}^{\infty} 1 \cdot d\widetilde{F}}{\int_{-\infty}^{\widetilde{\theta}} ((n+1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\widetilde{F}}$$

As $q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta) + \frac{d}{dQ} P(n \cdot q, \theta) \leq 0$ (by the decreasing marginal profit assumption) and $\frac{d}{dQ} P(n \cdot q, \theta) < 0$ it follows that $\frac{dq^*}{d\overline{p}} > 0$. $\square$

Proof of Theorem 5. The argument follows the same line as the proof of Theorem 4. After choosing $\widetilde{F}$, observe that in some sufficiently small neighborhood of $\overline{p}$, average prices are equal to

$$\int_{\widetilde{\theta}}^{\infty} \overline{p} \cdot d\widetilde{F} + \int_{-\infty}^{\widetilde{\theta}} P(n \cdot q^*(\overline{p}), \theta) \cdot d\widetilde{F}$$

where $\widetilde{\theta}$ is fixed and defined as in the proof of Theorem 4. Using the equality for $\frac{dq^*}{d\overline{p}}$ at the very end of the proof of Theorem 4 note that

$$\frac{d}{d\overline{p}}(\int_{\widetilde{\theta}}^{\infty} \overline{p} \cdot d\widetilde{F} + \int_{-\infty}^{\widetilde{\theta}} P(n \cdot q^*(\overline{p}), \theta) \cdot d\widetilde{F}) =$$

$$= \int_{\widetilde{\theta}}^{\infty} 1 \cdot d\widetilde{F} + \frac{dq^*}{d\overline{p}} \cdot \int_{-\infty}^{\widetilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q^*(\overline{p}), \theta) \cdot d\widetilde{F}$$

$$= \int_{\widetilde{\theta}}^{\infty} 1 \cdot d\widetilde{F} \cdot (1 - \frac{\int_{-\infty}^{\widetilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\widetilde{F}}{\int_{-\infty}^{\widetilde{\theta}} ((n+1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\widetilde{F}})$$

It is therefore enough to show that[23]

$$\frac{\int_{-\infty}^{\widetilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\widetilde{F}}{\int_{-\infty}^{\widetilde{\theta}} ((n+1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\widetilde{F}} > 1.$$

Note that $CS'' < 0$ implies that $\frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta) > 0$ for all θ. But then clearly

$$\int_{-\infty}^{\widetilde{\theta}} ((n+1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\widetilde{F} > \int_{-\infty}^{\widetilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\widetilde{F}.$$

But then (as the left hand side is negative)

$$\frac{\int_{-\infty}^{\widetilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\widetilde{F}}{\int_{-\infty}^{\widetilde{\theta}} ((n+1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\widetilde{F}} > 1$$

follows. $\square$

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[23] If average prices are decreasing in the price cap and quantity is increasing in the price cap the implication about consumer and total welfare follow immediately.

Sketch of proof of Theorem 6. We sketch how the arguments used in the proofs of Theorems 1-5 change in the modified model with disposal.

First note that the proof of the analog to Theorem 1 follows immediately from Theorem 1 since firms will never dispose of production in the deterministic case.

We sketch the argument to prove the analog of Theorem 3 before we discuss the proof of the analog of Theorem 2. To see that the analog of Theorem 3 is true assume - as in the proof of Theorem 3 - there exists a subsequence q^{m_k} such that $\lim_{k\to\infty} P(n \cdot q^{m_k}, \underline{\theta}) < c$. Consider first the case where $\lim_{k\to\infty} q^{m_k} > 0$. In this case, let $q_o > 0$ be a quantity such that $\lim_{k\to\infty} P(n \cdot q^{m_k}, \underline{\theta}) < P(n \cdot q_o, \underline{\theta}) < c$. Then if $\theta = \underline{\theta}$ and each firm sells less than q_o a firm's profit can be at most

$$-\frac{1}{2}(q^{m_k} - q_o) \cdot (c - \eta) + q^{max} \cdot (\overline{p}^{m_k} - c).$$

On the other hand, if $\theta = \underline{\theta}$ and each firm sells more than q_o a firm's profit can be at most

$$(P(n \cdot q_o, \underline{\theta}) - c) \cdot q_o.$$

Note that as $k \to \infty$ both expressions are smaller than and bounded away from zero. This is a contradiction as it would imply that firms make negative profits that are bounded away from zero for θ sufficiently close to $\underline{\theta}$ and sufficiently large k. As total revenue is bounded by $q^{max} \cdot (\overline{p}^{m_k} - c)$ which converges to zero this would mean that for sufficiently large k a firms expected equilibrium profit is negative - a contradiction!

It therefore must be the case that $\lim_{k\to\infty} n \cdot q^{m_k} = 0$. But then $\lim_{k\to\infty} P(n \cdot q^{m_k}, \underline{\theta}) < c$ together with the continuity of the price function implies $P(0, \underline{\theta}) < c$, which means the equation in the theorem does not have a solution. The remainder of the argument is analogous to the proof of Theorem 3.

To prove the analog of Theorem 2 follow the argument in the theorem's proof and note that it is still the case that

$$\begin{aligned}
\frac{d^2}{dq \cdot d\overline{p}} \pi(q, Q_{-i}, \overline{p}) &= \frac{d}{dq} \int_{\widetilde{\theta}}^{\infty} q \cdot dF(\theta) \\
&= (1 - F(\widetilde{\theta})) + \frac{\frac{\delta}{\delta Q} P(q + Q_{-i}, \widetilde{\theta})}{\frac{\delta}{\delta \theta} P(q + Q_{-i}, \widetilde{\theta})} \cdot q \cdot f(\widetilde{\theta}) \\
&\geq (1 - F(\widetilde{\theta})) - \frac{L_1}{L_2} \cdot q^{max} \cdot f(\widetilde{\theta})
\end{aligned}$$

where $\widetilde{\theta}$ is equal to $\underline{\theta}$ if $P(q + Q_{-i}, \underline{\theta}) > \overline{p}$, equal to $\overline{\theta}$ if $P(q + Q_{-i}, \overline{\theta}) < \overline{p}$ and given by $P(q + Q_{-i}, \widetilde{\theta}) = \overline{p}$ otherwise. Indeed, this fact follows directly from the observation that the optimal amount a firm will dispose off depends only on θ and the produced quantity q, but does not depend directly on the price cap $\overline{p}$. Now,

as in the proof of Theorem 2, consider the case where, in addition, $q \in b(Q_{-i}, \overline{p})$. Then the bound on profits we used in the proof of the analog of Theorem 3 yields that $\tilde{\theta}$ can be required to be arbitrary close to $\underline{\theta}$ if γ is sufficiently close to zero or the produced quantity is zero. The remainder of the argument is along the same lines as the proof of Theorem 2.

The analog of Theorems 4 and 5 are proven along the same lines as those theorems. The major difference is that the marginal revenue $D_1\pi_\theta(q^*(\overline{p}), (n-1) \cdot q^*(\overline{p}), \overline{p})$ appearing repeatedly in the proof has to be substituted with $\max(D_1\pi_\theta(q^*(\overline{p}), (n-1) \cdot q^*(\overline{p}), \overline{p}), -c - \eta)$.

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