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COMPARATIVE ANALYSIS OF INCOME CONTINGENT PLANS

by

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ABSTRACT

Income contingent loans are a relatively new means of credit for financing students' investment in higher education. In this study a comparative analysis of three different types of income contingent loan plans is presented. The three types of plans chosen incorporate a variety of features, and a comparison of these three plans, therefore, provides an insight into the characteristics of these features. The comparison is made from an individual participant's and an institution's viewpoint. Sensitivity analysis of the environmental parameters such as, interest rates, inflation, repayment period, and expected future incomes of borrowers, on the economic measures of optimally designed plans are also discussed.

1. INTRODUCTION

In recent years, the cost of education at American colleges and universities has been increasing faster than revenues from grants and gifts. In order to reduce the consequent budgetary deficit, most institutions of higher education have raised tuition substantially, thereby transferring an increased portion of the costs to students. As a result, students are relying more heavily on loans to finance their education. Loans for financing higher education are not as easily available as loans for investing in physical facilities. Further, even when such loans are available, they are usually conventional fixed term loans. From a student's viewpoint, such loans have two major drawbacks--the burden of repayment is greatest in the earlier years of the student's lifetime, and the fixed repayment is due even when the student's investment in education has not paid off in terms of income. In order to overcome these drawbacks, Friedman and Kuznets [4] proposed equity funding for higher education. Since their proposal appeared, a number of alternate proposals have been considered [see Hartman [5], and Johnstone [8] for details]. Most of these proposals have the following features:

- A formula relating the amount to be repaid by a borrower in a given year to the borrower's annual income.
- A maximum repayment period beyond which the liability of the borrower terminates, regardless of previous repayments.
- An opt-out, or exit-out, provision, whereby the liability of the borrower terminates if the repayments made by the borrower satisfy this provision.

These proposals often are referred to as income contingent loans because the repayments depend on income. The particular specifications of the above features differentiate various income contingent loan proposals. Currently, such plans are available to students at a few universities. The provisions of the plans instituted at the different universities vary. However, the underlying structure of the plans instituted can be classified into three categories, named after universities at which such plans have been instituted, namely, Duke, Yale, and Harvard. These three plans incorporate a variety of features, and a comparison of these three plans, therefore, provides an insight into the characteristics of these features, thus allowing one to choose the 'best' features of the different plans to come up with another one. In this paper we compare the above three plans from the borrower's and institution's viewpoints.

Duke Plan

Under this family of loan plans, an individual receives a loan at a stated rate of interest and agrees to repay in installments until the repayments equal the amount borrowed, together with the interest compounded annually, or until a stated termination year is reached. The annual repayments are proportional to the amount borrowed. For each unit borrowed, the individual repays a fixed percentage (called the repayment tax rate) of the person's adjusted gross income, but, under no circumstances, less than a specified minimum annual repayment. At the termination year, any further liability of the individual is forgiven.

Thus, the parameters in a Duke plan are

$\rho \equiv$ minimum annual repayment per unit borrowed

$\tau \equiv$ repayment tax rate, or fraction of annual income to be repaid each year, per unit borrowed

$N \equiv$ maximum repayment period beyond which the borrower need no longer pay, regardless of past payments

$R \equiv$ stated exit-out rate of interest charged to the borrower.

The borrower's liability terminates earlier than year N if the individual's accumulated repayments, compounded at this rate, equal the amount borrowed, compounded at this rate.

Let Y_j represent the income of a borrower in year j and suppose that repayments start in year 1. Then the amount Z_j to be repaid in year $j \leq N$ is given by

(1) $$Z_j = \begin{cases} 0 & \text{if the borrower has exited out before year } j, \\ \text{minimum}\left[\left\{(1+R)^{j+k} - \sum_{t=1}^{j-1} Z_t(1+R)^{j-t}\right\}, \text{maximum } (\rho, \tau Y_j)\right] & \text{otherwise,} \end{cases}$$

assuming that the individual borrows one unit (dollar) and k years are allowed until the first repayment is due. Thus for a student who borrows \$3500 in toto, the repayment is $3500Z_j$.

Yale Plan

Under this family of loan plans, an individual receives a loan and agrees to repay in annual installments until the repayments equal a multiple of the amount borrowed, together with the interest compounded annually, or until a stated termination year is reached. As in Duke plans, the individual repays, for each unit borrowed, a fixed percentage of the individual's income, but, under no circumstances, less than a specified minimum annual repayment. The difference between Duke plans and Yale plans are in the exit-out option. Whereas in Duke plans the exit-out option is stated in terms of an interest rate, in Yale plans the exit-out option is stated in terms of the multiple of the amount borrowed that the individual has to repay together with the interest thereon compounded at the institution's cost of borrowing. Yale plans are characterized by the following parameters

$\rho \equiv$ minimum annual repayment per unit borrowed

$\tau \equiv$ tax rate, or fraction of annual income, to be repaid each year, per unit borrowed

$N \equiv$ maximum repayment period beyond which the borrower need no longer pay, regardless of past repayments

$\mu \equiv$ multiple of the amount borrowed. The borrower's liability terminates earlier than year N if the individual's accumulated repayments, discounted at the institution's cost of borrowing, equal μ times the amount borrowed.

As in Duke plans, we assume that an individual borrows one unit (dollar) in year 1 and that a k year time lag is allowed until the first repayment. The repayment Z_j in year $j \leq N$ under the Yale plan is given by

$$(2) \qquad Z_j = \begin{cases} 0 & \text{if the borrower has exited out before year } j \\ \text{minimum}\left\{ \mu(1+r)^{j+k} - \sum_{t=1}^{j-1} Z_t(1+r)^{j-t},\ \text{maximum}\ (\rho, \tau Y_j) \right\} & \text{otherwise,} \end{cases}$$

where r is the institution's cost of borrowing.

Harvard Plan

Under this family of loan plans, an individual repays, for each unit borrowed, according to a graduated repayment schedule, but under no circumstances more than a stated percentage of annual income. If a borrower pays less than a scheduled repayment, then the difference, together with the interest at the stated interest rate, is deferred and added to the following year's scheduled repayment. Any deferred amount at the end of the stated repayment period becomes due in subsequent years but not beyond a stated maximum repayment period, and also subject to the condition that the borrower does not pay more than a specified fraction of income for each unit borrowed. Any deferred amount remaining after the maximum repayment period is forgiven.

Harvard plans that we have considered incorporate most of the salient features of the plan instituted at Harvard University. There are some differences, however, and these are discussed in Jain [6]. Our version of Harvard plans is characterized by the following parameters

ρ ≡ scheduled repayment in the first year, per unit borrowed

α ≡ annual increment to the schedule of repayments per unit borrowed

τ ≡ maximum fraction of annual income a borrower repays, regardless of the scheduled annual repayment, per unit borrowed

R ≡ stated interest rate charged to borrowers

N_E ≡ repayment period for an individual who makes all the scheduled repayments

N ≡ maximum repayment period, beyond which no repayments are due, regardless of past repayments. $(N \geq N_E)$

With the above parameters, the scheduled repayment in year j equals $\rho + (j-1)\alpha$ per unit borrowed. The graduated repayment schedule until N_E is designed to ensure that the amount loaned is repaid, together with the interest thereon at the rate R, and so the parameters ρ, α, R and N_E jointly satisfy

$$\sum_{j=1}^{N_E} \left[\rho + (j-1)\alpha\right]\left(\frac{1}{1+R}\right)^{j+k} = 1, \qquad (3)$$

where k refers to the time lag until the first repayment. The repayment Z_j made in year j by an individual who borrows one unit(dollar), assuming that the first repayment is in year 1, is given by

$$Z_1 = \text{minimum } \{\rho, \tau Y_1\} \qquad (4)$$

$$Z_j = \text{minimum } \{\rho + (j-1)\alpha + (1+R)D_{j-1}, \tau Y_j\} \quad \text{for } j = 2,3,\ldots,N_E \qquad (5)$$

$$Z_j = \text{minimum } \{(1+R)D_{j-1}, \tau Y_j\} \quad \text{for } j = N_E+1, N_E+2, \ldots, N \qquad (6)$$

$$Z_j = 0 \quad \text{for} \quad j > N, \qquad (7)$$

where

$$D_1 = (\rho - Z_1) \qquad (8)$$

$$D_j = \{\rho + (j-1)\alpha + (1+R)D_{j-1} - Z_j\} \quad \text{for } j = 2,3,\ldots,N_E \qquad (9)$$

$$D_j = \{(1+R)D_{j-1} - Z_j)\} \quad \text{for } j = N_E+1, N_E+2, \ldots, N. \qquad (10)$$

Approach to Comparisons

To compare the three plans, we must select criteria for evaluation, which can also be viewed as the lending institution's criteria for determining specific plan parameter values for its potential population or cohort of borrowers. Income contingent plans currently are financed through universities to provide a manageable form of credit for their students, and typically, the universities have not tried to generate profit through these plans, but have sought to make zero-profit, that is, to break even. Since many zero-profit plans can be designed for a postulated cohort of borrowers, we consider two additional criteria in determining zero-profit plans.

A feature of income contingent loan plans is the mutualization of risk, that is, individuals whose investment in education does not yield higher incomes pay back less, whereas individuals with higher incomes pay back more. One criterion, then, for determining a plan is to maximize the degree of protection for low income earners in a plan, subject to the plan's being zero-profit. In a zero-profit plan, the difference in the present value of repayments made by low income earners and the amount borrowed is equal to the excess payments made by high income earners. Therefore maximizing the degree of protection to low income earners in zero-profit plans is equivalent to maximizing the positive subsidy contributed. Plans designed with this criterion are henceforth referred to as maximum or optimal Subsidy plans, or simply as Subsidy plans.

An alternative criterion for determining zero-profit plans is to maximize the total welfare of the individuals in the potential cohort of borrowers. In order to obtain the welfare of the individuals we postulate that the utility function for individuals is present discounted value, and consequently the total welfare equals the sum of the present discounted value of all the loan repayments by the individuals in the cohort. We refer to such loan plans as maximum or optimal Welfare plans, or simply as Welfare plans.

In the future, income contingent plans might be financed by commercial establishments seeking to generate profit. Therefore, the third criterion for determining plan parameter values that we consider is to maximize the present value of the profit to the institution financing the plan. We refer to such loan plans as maximum or optimal Profit plans, or simply as Profit plans.

An important consideration in the design of specific plans is the level of participation in that plan by individuals in the postulated cohort of borrowers. If, for example, the tax rate and exit rate are large, then some individuals, particularly those that expect higher incomes in the future, may not participate. Thus a plan that was designed to be zero-profit for a postulated cohort may generate a loss due to the non participation by some high income earners. Further, if the institution adjusts some of the parameters (increases the tax rate, for example) to ensure zero-profit, more potential borrowers may elect not to participate, thus frustrating the institution's desire to have a zero-profit plan. This phenomena whereby only individuals with low income expectations participate is referred to as "adverse selection" in the literature, but has not been considered explicitly in determining plan parameter values in most of the previous studies. Berner et al [1] consider this problem by assuming different participation rates for individuals from different income expectations. The participation rates are, however, obtained arbitrarily.

A factor affecting adverse selection is the acceptability of a plan. A loan plan is termed acceptable to an individual if the present discounted value of repayments, discounted at the individual's discount factor, is less than or equal to the loan amount. With our postulates that the utility of individuals is present discounted value, and that all individuals who find a plan acceptable will elect to participate in it, adverse selection will not occur in a loan plan if the loan plan is acceptable to each individual in the cohort. In this paper we restrict plan parameter choices to values that are acceptable to all individuals in the assumed cohort. For Duke and Harvard plans and all three criteria discussed earlier, it has been shown by Jain [6] that if the subjective discount rate of individuals is the same for each individual (as we assume in this paper), then there exists an optimal set of plan parameter values that would be acceptable to each individual in the cohort. Thus, for Duke and Harvard plans, no changes occur in optimal plans by restricting plan parameter choices to values that are acceptable to all individuals in the cohort. Optimal Yale plans, however, are not always acceptable to all individuals in the cohort. But for the cohorts considered in this paper, the repayments made by individuals are identical under the Yale and Duke optimal Subsidy and Welfare plans, and Jain [6] has proved that under certain assumptions (that are postulated in this paper), when there is such an equivalence between Yale and Duke plans, then restricting plan parameter choices to values that are acceptable to all individuals in the cohort does not affect optimality of Yale plans. Optimal Profit Yale plans, however, are not identical with regard to repayments, to the corresponding optimal Profit Duke plans, and thus restricting plan parameter choices to values that are acceptable to all individuals in the cohort determines plans with lower profits. To compare all the three plans and the repayments made by individuals under the three plans, we only consider maximum Profit Yale plans that are acceptable to all individuals in the cohort. Thus the plans that we refer to as maximum Profit Yale plans in this paper are really maximum Profit Yale plans acceptable to all individuals in the cohort.

In Section 2 we state the assumptions made in determining optimal plans, and define the various economic measures of merit. In Section 3 we examine the selected economic measures of different optimal plans. For maximum Subsidy and Welfare plans, we discuss how subsidy, cohort welfare, and long term capital requirements differ among the Duke, Yale, and Harvard plans. For maximum Profit plans, we look at profit, cohort welfare, and pay back period. We compare the economic measures of the income contingent plans with those of conventional loan plans. We also discuss the effect of changes in the environmental parameters on these measures and on the relative difference among Duke, Yale, and Harvard plans. In Section 4 we examine how individuals with different incomes fare in the different optimal plans. In Section 5 we study the effect on repayments when the institution's cost of financing changes after the plan has been offered. In Section 6 we determine the effect on plan parameters and economic measures from changes in the composition of the cohort, that is, when the percentage of borrowers with different incomes is altered. We also examine the effect of offering to a cohort optimal plans designed for another cohort. Finally, in Section 7 we summarize the conclusions, and comment on the specific Duke University, Yale University, and Harvard University plans that have been implemented at these institutions.

2. ASSUMPTIONS AND DEFINITIONS

Environmental Parameters

In the computation of optimal plans, information is required on the following environmental parameters:

- Lag between the time at which the loan is executed and the time at which the first repayment is made.
- Cohort composition, namely, the number of different borrowers as differentiated by their income profiles
- Cost to the institution of financing a loan plan
- Each individual's subjective cost of borrowing
- Rate of inflation
- Maximum repayment period for which a plan is designed

We next discuss our assumptions concerning the above parameters.

Time Lag

We assume throughout that all individuals in a cohort borrow in a single year only, and that for all individuals there is a time lag of 5 years.

Cohort Composition

We postulate that individuals who borrow from the loan plan will have incomes represented by one of the five income profiles given in Table 1. This broad spectrum of individual incomes, ranging from high to low, is representative of graduates from a leading private university offering undergraduate, graduate, and professional degrees, in subjects as diverse as music, liberal arts, sciences, law, and medicine. The amounts in Table 1 represent the annual adjusted gross income in base year dollars (1971 in

TABLE 1

INDIVIDUAL INCOMES

Year	Profile 1	2	3	4	5
1	9000	7600	8100	7720	7500
2	11125	8250	8533	8200	7716
3	13250	8900	8966	8680	7932
4	15375	9550	9399	9160	8148
5	17500	10200	9832	9640	8364
6	19625	10850	10265	10120	8580
7	21750	11500	10698	10600	8796
8	23875	12150	11131	11080	9012
9	26000	12800	11564	11560	9228
10	28125	13450	11997	12040	9445
11	30250	14100	12430	12520	9662
12	32375	14750	12863	13000	9879
13	34500	15400	13296	13480	10096
14	36625	16050	13729	13960	10313
15	38750	16700	14162	14440	10530
16	40875	17350	14595	14920	10747
17	43000	18000	15028	15400	10964
18	45125	18650	15462	15880	11181
19	47250	19300	15896	16360	11398
20	49375	19950	16330	16840	11615
21	51500	20600	16764	17320	11832
22	53625	21250	17198	17800	12049
23	55750	21900	17632	18280	12266
24	57875	22550	18066	18760	12483
25	60000	23200	18500	19240	12700
26	60000	23200	18500	19240	12700
27	60000	23200	18500	19240	12700
28	60000	23200	18500	19240	12700
29	60000	23200	18500	19240	12700
30	60000	23200	18500	19240	12700
31	60000	23200	18500	19240	12700
32	60000	23200	18500	19240	12700
33	60000	23200	18500	19240	12700
34	60000	23200	18500	19240	12700
35	60000	23200	18500	19240	12700

our study). The incomes of individuals are projected for 35 years. Since all the income contingent plans considered in this analysis have a maximum repayment repayment period not exceeding 35 years, the incomes of individuals beyond the 35th year are not required.

Information about individual incomes alone is not enough. We also need to specify the percentage of individuals in the different profiles and the amounts borrowed by individuals. This information can be combined in a composite figure, namely, the percentage of the total amount borrowed by individuals in the different profiles. This percentage, together with the individual incomes, provides complete information on the composition of the cohort. In the analyses discussed in Sections 3 through 6, we assume that the percentages are

Profile	Percentage of Borrowing
1	25.0%
2	4.5%
3	12.5%
4	50.0%
5	8.0%

When inflation is incorporated then the annual incomes must be adjusted. For a rate of inflation equal to d percent, the inflated income of an individual in year j equals the uninflated income given in Table 1, multiplied by the factor $(1+d)^{j+k}$, where k is the time lag discussed earlier.

Institution's Cost of Financing a Plan

The institution's cost (rate of interest) for financing an income contingent plan is assumed to be constant over time. This cost, denoted by r, is measured in terms of cost per dollar per year. In our analysis, we allow r to take the values .02, .025, and .03 per dollar. This value of r refers to the real interest cost. When inflation is introduced, the nominal cost of financing the plan increases. With a rate of inflation equal to d, the nominal cost equals $(1+r)(1+d)-1$.

Individual's Subjective Cost of Financing

The individual's subjective cost of financing education refers to the cost that the individual is prepared to pay per dollar borrowed per year. This rate is the real (uninflated) cost per unit borrowed per year that the individual is prepared to pay, or alternately, is the cost that the individual would incur if the loan were from outside sources. This cost, denoted by $\bar{R}$, is assumed to be constant over time and the same for all individuals in the cohort.

Since the institution's cost of financing a plan is close to the prime rate, and the cost of borrowing to individuals is usually greater than the prime rate, $\bar{R}$ is generally greater than r. But this need not be true in all cases, because individuals who borrow from National Defense Student Loans or under the Federal Guaranteed Loan Program often pay less than the prime rate. In these federal loans, however, there is an element of subsidy that the government provides, whereas in our analysis we consider only non-subsidized schemes of financing education. When subsidy is available to the institution, then the institution's cost r decreases by a corresponding amount.

In the following analysis we have allowed $\bar{R}$ to take the values .04 and .05 per dollar exclusive of inflation. When inflation is considered, then this cost becomes $(1+\bar{R})(1+d)-1$, where d is the rate of inflation. For example, if d equals 5 percent and $\bar{R}$ equals 4 percent then the nominal interest rate that individuals are prepared to pay equals 9.20 percent.

In determining optimal plans we have restricted the values of r and $\bar{R}$ to those in Table 2. The differences between $\bar{R}$ and r is henceforth referred to as the spread, s, between the interest rates, and varies between 1.5 to 3 percent in our study. The spread, s, has a significant effect on the economic measures of optimal plans.

TABLE 2

Values for $(r,\bar{R})$

r \ $\bar{R}$	.04	.05	.06
.02	√	x	x
.025	√	√	x
.03	x	√	√

√ - considered in study

Rate of Inflation

We have determined optimal plans with and without inflation. The rates of inflation that we treat are 4 and 5 percent. When inflation is considered, appropriate adjustments have to be made to the incomes of individuals, the cost of financing to the institution, and the subjective cost of financing to the individual.

Maximum Repayment Period

The maximum repayment period, denoted by N, is a plan parameter. Under mild assumptions on the incomes of borrowers, Jain [6] has proved that the longer the repayment horizon, the greater the optimal value of Subsidy, Welfare, and Profit in the corresponding Duke, Yale, and Harvard plans. Since the criteria we consider are monotone in N, rather than postulate an arbitrary upperbound for N, we have selected values of N equal to 10, 25, 30, and 35 years so as to provide sensitivity analysis with respect to this parameter.

Economic Measures

Economic measures of interest for zero-profit plans differ from those for maximum profit plans. For maximum Subsidy and Welfare plans the economic measures we consider are subsidy, cohort welfare, and long term capital requirements. For maximum Profit plans we consider profit, cohort welfare, and pay back period. In comparing the economic measures of optimal Duke, Yale, and Harvard plans, we restrict the environmental parameters (such as N, r, $\bar{R}$, d) to the same values for determining the optimal plans. We next discuss each of the economic measures.

In zero-profit plans, some individuals with high incomes contribute subsidy whereas individuals with low incomes receive subsidy. The subsidy, when contributed by an individual, equals the present value of repayments, discounted at the institution's discount factor $\frac{1}{1+r}$, in excess of the amount loaned. The total subsidy in a plan, which we henceforth refer to simply as <u>subsidy</u>, equals the sum of the positive subsidies contributed by all individuals. In the following analysis, subsidy is measured in terms of \$1000 loaned to the cohort.

By an individual's welfare we refer to the difference between the amount loaned to the individual and the present value of repayments made by the individual, discounted at the individual's discount to the $\frac{1}{1+\bar{R}}$.

Recall, from our discussion on adverse selection, that an individual participates in a plan only if the individual's welfare is non-negative under that plan. By cohort welfare we mean the sum of all the individual's welfare. In our analysis, cohort welfare and individual welfare are measured in terms of a $1000 loaned to the cohort, or the individual, respectively.

If identically constituted cohorts borrow the same amount year after year, then the capital requirements over time of zero-profit plans stabilize at a constant amount, provided that there is no inflation. When an inflation rate d is introduced, then, after the initial years, the capital requirements increase at a rate of (1+d) every year. Hence, even under inflation the capital requirements eventually stabilize if we consider the uninflated value. By long term capital requirements we refer to the capital requirements in terms of real dollars (uninflated) when identical cohorts borrow equal amounts year after year. The long term capital requirements are measured for $1,000 loaned to identical cohorts over time.

By institution's repayments, or simply repayments, we mean the present value of repayments discounted at the institution's discount factor for every $1000 loaned. The institution's repayments for a cohort in zero-profit plans equals $1000.

By profit we mean the repayments made by a cohort, discounted at the institution's discount rate, in excess of the amount loaned. The profit for maximum Profit loan plans is per $1000 loaned to the cohort.

For maximum Profit plans, by pay back period we mean the number of years after which long term capital requirements equals zero, when the same plan is offered to identical cohorts year after year.

Computational Details

The computation of optimal plans, given the values of the environmental parameters, is a non-linear programming problem in the plan parameters. Qualitative results regarding the optimal values of the plan parameters in Duke, Yale, and Harvard Subsidy, Welfare, and Profit plans have been established by Jain [6]. For example, Jain has shown that in an optimal Subsidy Duke plan, the stated interest rate R equals $\bar{R}$, under the assumptions that we have made. Using the qualitative results, where available, and solving for the remaining values of the plan parameters by means of a search grid, we determined optimal plans from a computer optimization program.

Tables 3, 4, and 5 give the plan parameter values and economic measures for some of the computed maximum Subsidy, maximum Welfare, and maximum Profit plans. In these tables the values of the plan parameters α and N_E for Harvard plans are not shown. The reason is that α equals its maximum allowable value (in our study the maximum value is 20% of ρ, the repayment due in year 1 under Harvard plans). Jain [6] has proved that under certain assumptions α equals its maximum allowable value. Further, Jain has also shown that under the same set of assumptions N_E equals N for optimal Harvard Subsidy, Welfare, and Profit plans, and therefore in this study we assumed that N_E equals N.

Table 3

PLAN PARAMETER VALUES AND ECONOMIC MEASURES FOR MAXIMUM SUBSIDY PLANS

Plan Parameters/Economic Measures	d%	r%	$\bar{R}$%	N = 10 D	N = 10 Y	N = 10 H	N = 30 D	N = 30 Y	N = 30 H
Minimum Repayment ($\rho \times 10^3$) for D and Y; and Repayment in 1st year for H.	0	2	4	110	110	78.6	40	40	20.3
	0	3	5	120	120	86.5	48	48	24.8
	0	3	6	110	110	95.1	35	35	29.9
	4	3	6	120	120	139.4	48	48	61.2
	5	3	6	0	0	152.8	48	48	71.8
Tax Rate ($\tau \times 10^6$)	0	2	4	103.6	103.6	116.4	24.2	24.2	29.7
	0	3	5	115.1	115.1	128.2	29.4	29.4	36.5
	0	3	6	116.4	116.4	121.1	29.8	29.8	32.0
	4	3	6	122.0	122.0	121.6	30.3	30.3	32.7
	5	3	6	122.5	122.5	121.7	31.2	31.2	32.8
Exit Rate/Multiple R% for D and H, and μ for Y	0	2	4	4.00	1.19	4.00	4.00	1.43	4.00
	0	3	5	5.00	1.19	5.00	5.00	1.40	5.00
	0	3	6	6.00	1.31	6.00	6.00	1.80	6.00
	4	3	6	10.24	1.30	10.24	10.24	1.78	10.24
	5	3	6	11.30	1.30	11.30	11.30	1.76	11.30
Subsidy	0	2	4	48.4	48.4	54.4	107.6	107.6	129.4
	0	3	5	47.2	47.2	53.5	101.2	101.2	123.2
	0	3	6	76.9	76.9	83.5	199.3	199.3	199.3
	4	3	6	75.5	75.5	80.5	194.3	194.3	186.6
	5	3	6	75.4	75.4	79.9	190.1	190.1	185.4
Cohort Welfare	0	2	4	164.8	164.8	173.7	294.4	294.4	326.4
	0	3	5	162.2	162.2	171.2	282.3	282.3	315.0
	0	3	6	235.5	235.5	242.9	419.4	419.4	427.7
	4	3	6	236.6	236.6	240.7	416.1	416.1	419.7
	5	3	6	236.8	236.8	240.7	416.0	416.0	419.0
Long Term Capital Requirements	0	2	4	11466	11466	12159	24652	24652	28374
	0	3	5	11975	11975	12746	26563	26563	31227
	0	3	6	12231	12231	12711	30587	30587	31386
	4	3	6	12287	12287	12571	30165	30165	30255
	5	3	6	12296	12296	12543	29995	29995	30150

D $\triangleq$ Duke Plans

Y $\triangleq$ Yale Plans

H $\triangleq$ Harvard Plans

Table 4

PLAN PARAMETER VALUES AND ECONOMIC MEASURES FOR MAXIMUM WELFARE PLANS

Plan Parameters/Economic Measures				N = 10			N = 30		
	d%	r%	$\bar{R}$%	D	Y	H	D	Y	H
Minimum Repayment ($\rho \times 10^3$)	0	2	4	0	0	64.5	0	0	13.4
for D and Y, and	0	3	5	0	0	71.3	0	0	16.6
Repayment in 1st year	0	3	6	0	0	71.3	35	35	16.6
for H	4	3	6	0	0	105.5	35	35	61.2
	5	3	6	0	0	115.8	35	35	71.8
Tax Rate ($\tau \times 10^6$)	0	2	4	116.2	116.2	191.3	31.3	31.3	71.8
	0	3	5	127.9	127.9	211.3	38.4	38.4	88.8
	0	3	6	122.5	122.5	211.3	29.8	29.8	88.8
	4	3	6	122.5	122.5	180.6	32.7	32.7	32.7
	5	3	6	122.5	122.5	173.4	32.8	32.8	32.8
Exit Rate/Multiple	0	2	4	4.00	1.19	2.00	4.00	1.38	2.00
R% for D and H, and	0	3	5	5.00	1.18	3.00	5.00	1.35	3.00
μ for Y	0	3	6	6.00	1.30	3.00	6.00	1.80	3.00
	4	3	6	10.24	1.30	7.12	10.24	1.73	10.24
	5	3	6	11.30	1.30	8.15	11.30	1.73	11.30
Subsidy	0	2	4	46.9	46.9	0	95.2	95.2	0
	0	3	5	46.0	46.0	0	89.2	89.2	0
	0	3	6	75.3	75.3	0	199.3	199.3	0
	4	3	6	75.4	75.4	0	183.5	183.5	186.6
	5	3	6	75.4	75.4	0	183.1	183.1	185.4
Cohort Welfare	0	2	4	166.8	166.8	178.8	302.0	302.0	341.0
	0	3	5	164.0	164.0	176.0	289.9	289.9	330.2
	0	3	6	236.8	236.8	250.3	419.4	419.4	445.2
	4	3	6	236.8	236.8	243.6	417.2	417.2	419.7
	5	3	6	236.8	236.8	243.6	417.2	417.2	419.0
Long Term Capital	0	2	4	11603	11603	12531	25310	25310	29996
Requirements	0	3	5	12122	12122	13159	27351	27351	33279
	0	3	6	12296	12296	13159	30587	30587	33279
	4	3	6	12296	12296	12745	29859	29859	30255
	5	3	6	12296	12296	12644	29849	29849	39150

Table 5

PLAN PARAMETER VALUES AND ECONOMIC MEASURES FOR MAXIMUM PROFIT PLANS

Plan Parameters/Economic Measures	d%	r%	$\bar{R}$%	N = 10 D	N = 10 Y	N = 10 H	N = 30 D	N = 30 Y	N = 30 H
Minimum Repayment ($\rho \times 10^3$) for D and Y; and Repayment in 1st year for H	0	2	4	120	100	78.6	0	40	20.3
	0	3	5	0	120	86.5	0	48	24.8
	0	3	6	0	120	95.1	0	48	30.0
	4	3	6	0	0	139.4	0	48	61.2
	5	3	6	0	0	152.8	0	35	71.8
Tax Rate ($\tau \times 10^6$)	0	2	4	147.6	166.0	232.9	51.4	44.9	108.9
	0	3	5	187.7	181.7	256.4	61.4	55.1	132.6
	0	3	6	177.7	170.3	281.8	72.9	62.2	160.0
	4	3	6	177.7	170.3	238.7	72.9	62.2	109.9
	5	3	6	177.7	170.3	228.8	72.9	62.2	74.3
Exit Rate/Multiple R% for D and H, and μ for Y	0	2	4	4.00	1.16	4.00	4.00	1.30	4.00
	0	3	5	5.00	1.16	5.00	5.00	1.28	5.00
	0	3	6	6.00	1.26	6.00	6.00	1.44	6.00
	4	3	6	10.24	1.26	10.24	10.24	1.44	10.24
	5	3	6	11.30	1.26	11.30	11.30	1.44	11.30
Profit	0	2	4	183.9	162.4	217.7	384.0	278.0	518.0
	0	3	5	179.8	159.3	213.7	364.0	256.0	493.0
	0	3	6	284.9	246.5	333.8	592.0	413.0	803.0
	4	3	6	284.9	246.5	322.1	591.5	412.8	672.3
	5	3	6	284.9	246.5	319.2	591.5	412.8	648.2
Cohort Welfare	0	2	4	10.1	18.6	0	21.0	89.0	0
	0	3	5	0	18.2	0	20.0	95.0	0
	0	3	6	10.6	40.9	0	19.0	129.0	0
	4	3	6	10.6	40.8	0	19.3	128.9	0
	5	3	6	10.6	40.8	0	19.4	128.3	17.2
Pay Back Period	0	2	4	46	48	45	52	59	52
	0	3	5	40	42	40	47	54	48
	0	3	6	33	35	33	39	45	40
	4	3	6	33	35	32	39	45	40
	5	3	6	33	35	32	39	45	40

3. ECONOMIC ANALYSIS

In this section we examine the economic and financial measures of the optimal Duke, Yale, and Harvard plans.

Subsidy

Optimal Subsidy Plans

The subsidies generated in the different plans vary considerably. The minimum amount of subsidy in the optimal plans that we determined is $36 per $1000 loaned to the cohort, whereas the maximum is $222. The amount of subsidy contributed in optimal Subsidy plans is very sensitive to the parameters N and s (the spread or R-r), because as N or s increases, the repayments made by individuals with income in profile 1 increase considerably, thereby increasing the subsidy that they contribute. Subsidy increases by 200 to 300 percent when N increases from 10 years to 35 years. The subsidies generated in the optimal plans decrease, though not significantly as r increases, and s is constant.

The subsidy generated in Harvard optimal Subsidy plans is almost always greater than that generated in Duke and Yale optimal Subsidy plans. Since conventional loans at r have zero subsidy, each of the three types of plans are preferred to conventional loans when the objective is to maximize subsidy.

When inflation is considered, the subsidy in Duke and Yale optimal Subsidy plans generally decreases. However, this decrease is not very significant. In fact the subsidy remains unchanged when ρ, the minimum annual repayment, equals 0 in optimal plans. For Harvard optimal Subsidy plans, the subsidy is affected significantly when inflation is introduced, because the annual increment to the scheduled repayments is constant. Consequently, the uninflated value of the maximum annual amount due increases in the initial periods and decreases in the later periods when inflation is introduced, thereby reducing the present value of repayments (and hence of subsidy too) for individuals who exit out. Nevertheless, the subsidy in Harvard optimal Subsidy plans remains greater than that of Duke and Yale optimal Subsidy plans under inflation.

Optimal Welfare Plans

For the Duke and Yale optimal Welfare plans the subsidy generated of course is less than that of optimal Subsidy plans, but the difference is not large. Further, the effect of an increase in N, s, r, or d on subsidy in Duke and Yale optimal Welfare plans is similar to the effect of such changes for optimal Subsidy plans.

Without inflation, the subsidy in Harvard optimal Welfare plans is zero, owing to the fact that R, the stated interest rate for such plans, equals r, and therefore no individual receives or contributes subsidy. Under inflation, when d is large some subsidy is generated in Harvard optimal Welfare plans, provided that N is large. In fact, when the rate of inflation d equals 5 percent and the maximum repayment period N is greater than 25 years, the Harvard optimal Subsidy and optimal Welfare plans are identical, and therefore so are the subsidies.

Cohort Welfare

Optimal Subsidy Plans

The cohort's welfare varies significantly with the maximum repayment period N, the difference s between $\overline{R}$ and r, and to a lesser extent on the institution's cost r and the rate of inflation d. The range of values for the cohort's welfare is between \$128 and \$458 per \$1000 loaned to the cohort.

By design the cohort's welfare is less for optimal Subsidy plans as compared to optimal Welfare plans.

The cohort's welfare under conventional loans at r is less than those under any of the three types of plans. Further, this difference increases as N or s increases. The cohort's welfare for the Harvard optimal Subsidy plans is greater than for the corresponding Duke and Yale plans.

The cohort's welfare increases as the maximum repayment period increases for each of the three types of plans and for conventional loans too. This increase is from 90 to 100 percent for Duke, Yale, and Harvard plans and is about 50 to 90 percent for conventional loans when the maximum repayment period increases from 10 years to 35 years. The increase is generally greater for large s.

The cohort's welfare increases as s increases. For income contingent loan plans, the increase in cohort welfare is between 30 to 50 percent when s increases from 2 percent to 3 percent. Under conventional loans this increase is between 30 to 45 percent. The increase is generally smaller for large N. The increase is generally greater for Duke and Yale plans.

The cohort's welfare in optimal Subsidy plans generally increases for Duke and Yale plans when inflation is introduced, but this increase is insignificant. In fact, the cohort's welfare for Duke and Yale plans is unchanged when the minimum annual repayment ρ equals 0.

For Harvard optimal Subsidy plans and conventional loans, the cohort's welfare decreases as d increases. This decrease is very small--less than 0.5 percent for an increase in d of one percent in Harvard plans. For conventional loans this increase is greater, especially for large N, where it is as much as 3 percent.

Optimal Welfare Plans

As discussed earlier, optimal Welfare Duke and Yale plans are not significantly different from the corresponding optimal Subsidy plans. The effect of an increase in N, s, r, or d on cohort welfare for Duke and Yale optimal Welfare plans is similar to the effect of such changes for optimal Subsidy plans.

Harvard optimal Welfare plans are different, however, from the corresponding optimal Subsidy plans. The cohort's welfare under Harvard optimal Welfare plans is greater than that under optimal Subsidy plans by as much as 5 percent. The effect of changes in N, s, and r on cohort welfare in Harvard optimal Welfare plans is similar to the effect of such changes in optimal Subsidy plans.

The introduction of a high rate of inflation in Harvard plans often causes a change in the structure of Harvard optimal Welfare plans because the exit rate (stated interest rate, R) changes. For large N and d, Harvard optimal Welfare plans are identical to Harvard optimal Subsidy plans, and therefore so is the cohort welfare.

Further, as d increases, the difference in the cohort's welfare between any one of the three income contingent plans and conventional loans increases. Therefore, as the rate of inflation increases, Duke, Yale, and Harvard optimal Welfare plans are increasingly preferred to conventional loans if the objective is to maximize the cohort's welfare.

Long Term Capital Requirements

Optimal Subsidy Plans

The long term capital requirements are generally greater for Harvard optimal Subsidy plans as compared to the corresponding Duke and Yale optimal Subsidy plans. The difference between them is as much as 22 percent. For small s, the percentage difference in long term capital requirements increases as N increases. For large s, the percentage difference decreases as N increases, provided that N is large. In fact, when N equals 35 years and s equals 3 percent, long term capital requirements for Duke, Yale, and Harvard optimal Subsidy plans are equal.

Long term capital requirements of conventional loans are less than those of Duke, Yale, and Harvard optimal Subsidy plans. The difference is small for small N, but is often as large as 20 percent when N equals 35 years. When inflation is introduced, then the difference in long term capital requirements between income contingent loans and conventional loans increases, and the difference is often as much as 60 percent.

Long term capital requirements (in terms of real dollars) are not affected significantly for Duke and Yale optimal Subsidy plans when inflation is introduced. In fact, when the minimum annual repayment ρ equals zero, they are unchanged by an increase in d. For Harvard optimal Subsidy plans and for conventional loans the long term capital requirements decrease as d increases.

Optimal Welfare Plans

Long term capital requirements of optimal Welfare plans are generally greater than those of optimal Subsidy plans for Duke, Yale, and Harvard plans. The difference in long term capital requirements between the two different criteria plans is negligible for Duke and Yale plans. For Harvard plans, the difference is often as much as 9 to 10 percent. The percentage difference is generally larger for large N, except for large d, when the two Harvard plans are identical.

Profit in Optimal Profit Plans

The profit generated in optimal Profit plans varies considerably with respect to N and s. Profit in Harvard optimal plans is greatest, followed by Duke optimal plans, conventional loans at $\bar{R}$, and Yale optimal plans. The difference in profits between Harvard and Yale plans ranges from 25 to 110 percent. Generally the difference in profits between the different types of plans increases as N or s increases.

For Duke plans the increase varies between 120 to 130 percent as N increases from 10 to 35 years. For Yale plans the corresponding increase is from 70 to 80 percent, and for Harvard plans it is from 160 to 175 percent. For conventional loan plans at $\bar{R}$, this increase is about 120 percent.

Duke and Yale optimal Profit plans are not significantly affected by introducing inflation. Typically the profits for such plans decrease when the rate of inflation increases; this decrease is usually less than 1 percent when d changes from 0 percent (no inflation) to 5 percent.

The profits generated in Harvard optimal Profit plans and in conventional loans decrease significantly as d increases. As d changes from 0 to 5 percent, the decrease ranges from 4 to 20 percent for Harvard optimal plans, and from 5 to 25 percent for conventional loans. The decrease is generally greater for large N.

Cohort Welfare in Optimal Profit Plans

Cohort's welfare is not a very meaningful economic measure for maximum Profit plans. For conventional loans and for optimal Harvard plans, cohort welfare is zero. For Duke and Yale optimal Profit plans, cohort welfare is positive. Cohort's welfare is greatest in Yale optimal plans.

For Duke and Yale optimal plans, cohort's welfare increases as N or s increases, but decreases as r or d increases. Except for the increase due to an increase in N, the other changes are insignificant.

Pay Back Period in Optimal Profit Plans

Recall that pay back period is meaningful only for optimal profit plans and refers to the number of years after which capital requirements of the plan equal 0. Pay back period is generally longest for Yale optimal plans, followed by Harvard and Duke optimal plans, and conventional loans at $\bar{R}$. The difference between the latter three is generally less than 2 years. The difference between Yale and Harvard optimal plans is often as much as 6 to 8 years.

Pay back period for all the plans

- increases as N increases. For an increase in N of 5 years, pay back period increases by about 2 years.
- decreases as s increases for a fixed r. This decrease is between 7 to 9 years for a 1 percent increase in s.
- decreases as r increases for a fixed s. This decrease is between 4 to 6 years for a 1 percent increase in r.

When d increases, pay back periods in Duke and Yale optimal plans are generally unchanged. Pay back periods in Harvard optimal plans and conventional loans decrease, though not significantly.

4. ECONOMIC ANALYSIS FOR INDIVIDUALS

In this section we examine how individuals with different income profiles fare with respect to

- present value of repayments discounted at the institution's interest rate, also referred to as institution's repayments, or repayments, and
- individual's welfare

for the three different models and the three different criteria. This analysis is helpful in understanding how individuals with different income expectations perceive income contingent loan plans.

For the cohort considered (see Table 1), individuals whose income is represented by profile 1 have the highest income in each year, followed by individuals whose income is represented by profile 2. Individuals whose income is represented by profile 3 have a higher initial income but a smaller annual increment than individuals whose income is represented by profile 4, resulting in a lower income beyond year 10. Individuals whose income is represented by profile 5 clearly have the lowest income in the assumed cohort. Because of the similarity in the sensitivity analysis of individuals whose incomes are represented by profiles 2, 3, and 4, it is convenient to group them together. With this grouping, individuals can be classified into the following three categories:

- high income earners, represented by profile 1,
- average income earners, represented by any of the three profiles, 2, 3, or 4,
- low income earners, represented by profile 5.

Present Value of Repayments Discounted at the Institution's Interest Rate

Optimal Subsidy Plans

In Duke, Yale, and Harvard optimal Subsidy plans, repayments discounted at the institution's interest rate, are proportional to the incomes of the borrowers. High income earners repay the most and contribute subsidy, whereas all other borrowers receive subsidy. For individuals receiving subsidy, the amount of subsidy that they receive generally increases as N increases and as s (difference between $\bar{R}$ and r) increases. Further, low income earners receive more subsidy than average income earners.

Repayments by high income earners are greater for optimal Harvard plans as compared to corresponding Duke and Yale optimal plans. For average income earners, repayments are greater for N large and smaller for N small under optimal Harvard plans, as compared to corresponding optimal Duke and Yale plans. For low income earners, repayments under optimal Harvard plan are less than those under corresponding optimal Duke and Yale plans.

Under conventional loans, the present value of repayments of all individuals equals the amount loaned. For this reason, high income earners repay more under Duke, Yale, and Harvard plans than they would under conventional loans, whereas average and low income earners repay less.

Optimal Welfare Plans

In Duke and Yale optimal Welfare plans, the repayments, discounted at the institution's interest rate, again are proportional to the income of the borrowers. In optimal Welfare plans, some average income earners (profile 2) do contribute subsidy when s is small. The repayments by high income earners is less under optimal Welfare plans than under corresponding optimal Subsidy plans; this also is true of low income earners (profile 5). Average income earners repay more under optimal Subsidy plans than under optimal Welfare plans. One interesting point to observe here is that for Duke and Yale plans, low income earners are subsidized more in optimal Welfare plans than in optimal Subsidy plans because, under optimal Subsidy plans, average income earners receive greater subsidy than in optimal Welfare plans.

Harvard optimal Welfare plans are significantly different from corresponding optimal Subsidy plans when inflation is not considered. In Harvard optimal Welfare plans without inflation, all individuals, regardless of their income, neither contribute nor receive subsidy. As a consequence, high income earners repay less in terms of present value of repayments discounted at the institution's cost in optimal Welfare plans than they do under the corresponding optimal Subsidy plans. Average and low income earners, however, repay more under optimal Welfare plans than under the corresponding optimal Subsidy plans. When inflation is introduced, then, as mentioned earlier, Harvard optimal Subsidy and Welfare plans become similar for large d, N, and s.

In optimal Welfare plans without inflation, the repayments made by high income earners in Harvard plans are less than those in the corresponding Duke and Yale optimal Welfare plans. For average income earners, the repayments under Harvard plans are generally greater (except for individuals with profile 2) than those under the corresponding Duke and Yale optimal Welfare plans. Low income earners repay more under Harvard plans. When inflation is introduced, the Harvard optimal Welfare and Subsidy plans tend to become identical for large N and d, and, in such cases, the remarks made for optimal Subsidy plans hold.

Repayments made by high income earners in Duke and Yale optimal Welfare plans are greater than under conventional loans when r is the same. Repayments of average and low income earners (except profile 2) are lower under optimal Duke and Yale plans.

Since repayments under Harvard optimal Welfare plans for all individuals equal the amount borrowed, repayments under a Harvard optimal Welfare plan and a conventional loan at r are the same. When inflation is introduced, then for large N and s the conclusions for optimal Subsidy plans hold, since Harvard optimal Subsidy and Welfare plans become identical.

Optimal Profit Plans

Optimal Profit plans are significantly different from optimal Subsidy and Welfare plans. Duke and Yale optimal plans differ from each other for this criterion. In Duke optimal Profit plans, repayments (institution's repayments) are greatest for average income earners, followed by high and low income earners, because average income earners exit out later than high income earners and, therefore, make greater repayments.

For Yale optimal Profit plans, repayments made by high and average income earners are equal. In Yale plans, like Duke plans, average income earners exit out later. But in Yale plans individuals exit out when their repayments (discounted at the institutions' interest rate) equal the exit multiple μ. Hence, repayments are generally the same for high and average income earners. Low income earners, however, do not exit out from Yale optimal Profit plan, and their repayments are therefore less than those made by high and average income earners. In fact, in some cases, low income earners are even subsidized.

Under Harvard optimal Profit plans without inflation, all individuals, regardless of their income, exit out, and repayments made by high, low, and average income earners are approximately equal. When the rate of inflation d, the repayment period N, and the spread s are large, then repayments made by low income earners is less than that made by average and high income earners.

Generally repayments made by all individuals are greatest under optimal Profit Harvard plans, followed by Duke and Yale optimal Profit plans. There is one exception--high income earners repay more under optimal Profit Yale plans than under the corresponding optimal Profit Duke plans. Also for large N, s, and d, repayments of low income earners are the same under Duke, Yale, and Harvard optimal Profit plans.

The institution's repayments are the same for all individuals under conventional loans. Thus, repayments made under conventional loans are less than repayments under Harvard optimal Profit plans for all individuals. They are greater than those under Duke and Yale optimal Profit plans for average and low income earners. When the rate of inflation is large, the difference in repayments decreases between conventional loans and Duke and Yale optimal Profit plans.

Individual Welfare

Optimal Subsidy Plans

In Duke, Yale, and Harvard optimal Subsidy plans individual welfare is the greatest for low income earners, followed by average and high income earners. For high income earners, individual welfare is generally zero, but is positive for average and low income earners. When N and s are large, then under Duke, Yale, and Harvard optimal Subsidy plans, individual welfare is positive for all individuals. This means that the high income earners are generally indifferent between a conventional loan at $\bar{R}$ and optimal Subsidy plans, except when N and s are large. Average and low income earners prefer any one of the Duke, Yale, and Harvard optimal Subsidy plans to a conventional loan at $\bar{R}$.

Individual welfare under optimal Subsidy plans for average (except profile 2) and low income earners is generally greater for Harvard plans. This means that average income earners (except for profile 2) and low income earners generally prefer Harvard optimal Subsidy plans to Duke and Yale optimal Subsidy plans. As a consequence, if all individuals in the cohort voted to select one plan from all the zero-profit plans, then Harvard optimal Subsidy plans would be the democratic choice. The difference in individual welfare between Harvard and Duke or Yale optimal Subsidy plans is not too significant for the average income earner, especially for small N.

Under conventional loans at r, individual welfare for each borrower equals a constant positive amount since $\bar{R} > r$ for the cases that we consider. Therefore, high income earners prefer a conventional loan at r to any of the three income contingent loan plans. Average and low income earners prefer income contingent loan plans. A comparison of individual welfare under optimal Subsidy income contingent loan plans and conventional loans at r is not really appropriate as it presumes that an individual's discount factor is $\frac{1}{1+r}$ instead of $\frac{1}{1+\bar{R}}$. A comparison at $\bar{R}$ is more appropriate, and then all the three optimal Subsidy income contingent plans are at least as preferable as conventional loans by all individuals.

Optimal Welfare Plans

For Duke and Yale plans, the difference in individual welfare under optimal Welfare plans and optimal Subsidy plans is not very significant. Individual welfare under Duke and Yale optimal Welfare plans for high income earners is generally the same as under optimal Subsidy plans. In some instances (when ρ is greater than zero for optimal Subsidy plans), individual welfare for high income earners is less under optimal Welfare plans. Individual welfare of average income earners is generally less in Duke and Yale optimal Welfare plans than the corresponding optimal Subsidy plans. For profiles 3 and 4 this difference is not very significant. Individual welfare for low income earners under Duke and Yale optimal Welfare plans is generally greater than under the corresponding optimal Subsidy plans.

When inflation is not considered, Harvard optimal Welfare plans are significantly different from optimal Subsidy plans. Individual welfare for all borrowers is generally constant when inflation is not considered. For this reason individual welfare under Harvard optimal Welfare plans is generally greater for the high income earners, and less for average and low income earners (except profile 2), as compared to the corresponding optimal Subsidy plans. Further, individual welfare under Harvard optimal Welfare plans (without inflation) is greater for high income earners, and lower for average (except for profile 2 when N and s are small) and low income earners, as compared to the corresponding Duke and Yale optimal Welfare plans. When inflation is introduced, then for small d the above remarks holds, and for large d and N, since Harvard optimal Subsidy and Welfare plans become identical, the remarks for optimal Subsidy plans hold.

Individual welfare under Duke and Yale optimal Welfare plans is less for high income earners and greater for average and low income earners than individual welfare under a conventional loan at r. The difference in individual welfare between the Duke and Yale optimal Welfare plans and conventional loans at r is greater for low income earners as compared to average income earners. For Harvard optimal Welfare plans (without inflation), individual welfare is greater than under a conventional loan at r, for all individuals. Further, this difference is generally not dependent on the income of the individual. But recall that a comparison of individual welfare under Duke, Yale, and Harvard plans and under a conventional loan at r is not very appropriate. When we compare individual welfare of Duke, Yale, and Harvard optimal Welfare plans with that of a conventional loan at $\overline{R}$, then individual welfare is

. generally the same under Duke and Yale plans, and conventional loans at $\overline{R}$, but greater under Harvard plans for high income earners

- greater under each of the three income contingent plans than under a conventional loan at $\bar{R}$ for average and low income earners.

<u>Optimal Profit Plans</u>

For Duke optimal Profit plans, individual welfare generally equals zero for high and average income earners, but is positive for low income earners.

For Yale optimal Profit plans, individual welfare equals zero for high income earners, and is positive for average and low income earners.

For Harvard optimal Profit plans (without inflation), individual welfare for all individuals equals zero. When inflation is introduced, then, for large N and d, individual welfare for low income earners is sometimes positive.

Thus, high income earners generally are indifferent between any of the three income contingent Profit plans and a conventional loan at $\bar{R}$. Average income earners generally prefer Yale Profit plans, and are usually indifferent between Duke, and Harvard Profit plans and a conventional loan at $\bar{R}$. Some average income earners (profile 3) prefer Duke optimal Profit plans when N and s are large. Low income earners generally prefer Yale Profit plans, followed by Duke Profit plans, and are generally indifferent between Harvard Profit plans and conventional loans at $\bar{R}$.

5. SENSITIVITY ANALYSIS ON INSTITUTION'S COST OF FINANCING PLANS

Throughout the analyses so far we have assumed that the institution's cost r is known, and already have discussed the effect of an increase in r. The institution's cost of financing plans, however, is subject to change over time as economic conditions change. An important question from the institution's viewpoint is the effect of a change in its cost after a plan has been implemented. We examine next the effect of a 1 percent change in the institution's cost of financing for optimal Subsidy, Welfare, and Profit plans.

When the institution's cost of financing varies, repayments made by the cohort, discounted at the institution's rate, also vary. Thus, plans initially designed to be zero-profit do not remain zero-profit. A possible strategy that the institution can use to reestablish the zero-profit condition is to increase or decrease the maximum repayment period. In this section, we examine how to adjust the maximum repayment period for Subsidy and Welfare plans to ensure that the cohort discharges its debt (cohort termination).

Prior to the analysis, we must point out that when the institution's cost of financing increases, some Yale optimal Subsidy and Welfare plans are no longer acceptable to all individuals in our postulated cohort, because the exit-out option in the Yale plan is a multiple of the amount of the loan discounted at the institution's interest rate. Thus, when the institution's cost increases, then individuals in profile 1 who exit out under the original plan do not do so any longer. But in the original plan, repayments of such individuals discounted at $\frac{1}{1+\bar{R}}$ equal the amount loaned. Thus, in the new plan (with the institution's cost increased) repayments discounted at $\frac{1}{1+\bar{R}}$ are greater than the amount loaned and therefore are unacceptable. We have not considered this factor of unacceptability, or of individuals not participating in such plans, arguing that if the institution does not know the future cost of financing at the time of offering the plan, then neither do the individuals borrowing.

For Profit plans we do not change the maximum repayment period, but discuss the effects of the change in the institution's cost on profit, individual repayments, and pay back period.

Optimal Subsidy Plans

As mentioned earlier, when the cost of financing the plan increases (decreases), the present value of repayments discounted at the institution's factor decreases (increases) for all the plans. The increase (decrease) is generally the greatest for Harvard Subsidy plans, followed by Duke and Yale Subsidy plans. The change (increase or decrease) is usually from 3 to 5 percent for a 1 percent change in the interest rate when N equals 10 years, and from 5 to 10 percent when N equals 25 years.

In order to ensure zero-profit plans, when the maximum repayment period N equals 10 years, an increase of only 1 year is usually sufficient for each of the three Subsidy plans when the interest rate of financing the plan increases by 1 percent. When the interest rate decreases by 1 percent, cohort termination, however, does not take place before the 10th year of repayments.

When the maximum repayment period is 25 years, a 1 percent increase in the institution's cost of financing delays cohort termination significantly, between 5 to 6 years for Duke Subsidy plans, 3 to 4 years for Yale Subsidy plans, and 4 to 5 years for Harvard Subsidy plans.

Thus, in this sense Yale plans may be considered to be the least sensitive from the institution's viewpoint to changes in the interest rate. This result is to be expected, because, as we discussed earlier, the exit-out option of Yale plans depends on the institution's cost of financing the plan.

When N equals 25 years, a 1 percent decrease in the institution's cost of financing hastens cohort termination by 1 to 2 years for Duke and Yale Subsidy plans, and by 1 year for Harvard Subsidy plans. Note that the resulting increase in the repayment period is generally much greater than the resulting decrease when the institution's cost of financing varies by 1 percent.

<u>Optimal Welfare Plans</u>

The effect of a change in the institution's cost for Duke and Yale Welfare plans is similar to that of Subsidy plans discussed above, both with respect to the direction of the change and its magnitude.

When the institution's cost of financing increases, Harvard Welfare plans cannot be made zero-profit by increasing the maximum repayment period because the exit rate R equals r, the institution's original cost of financing the plan. Thus, for a given N, the repayment ρ in the first year and the annual increment α in repayments are determined such that

$$\sum_{j=1}^{N} [\rho + (j-1)\,\alpha] \left(\frac{1}{1+r}\right)^{j+k} = 1 \ .$$

Consequently, with an increase in r, the left hand side of the above expression decreases, and therefore individuals who make all the scheduled repayments do not repay the amount borrowed plus interest.

Optimal Profit Plans

The present value of profit increases (decreases) as the institution's cost of financing the plan decreases (increases). With respect to the magnitude of the change in profit for a 1 percent change in the institution's cost, Harvard Profit plans are most sensitive (about 5 percent when N equals 10 years, and 9 percent when N equals 25 years), followed by Duke Profit plans (about 5 percent when N equals 10 years, and 8 percent when N equals 25 years), and Yale plans (less than 1 percent when N equals 10 years, and from 1 to 5 percent when N equals 25 years). For Harvard and Duke Profit plans, the increase in profit due to a decrease in the institution's cost is of the same order of magnitude as the decrease in profit due to an increase in the institution's cost. For Yale Profit plans, however, the decrease is of considerably greater magnitude than the increase.

Individual welfare in Duke and Harvard Profit plans is unaffected by changes in the institution's cost, because repayments are not dependent on the institution's cost. Individual welfare under Yale Profit plans is affected by about 5 percent from a 1 percent change in the institution's cost. Individual welfare under Yale Profit plans decreases (increases) when the institution's cost increases (decreases).

Pay back period for Duke and Harvard Profit plans increases (decreases) as the institution's cost increases (decreases), and the change in pay back period is between 2 and 4 years for a 1 percent change in the institution's cost. Generally the difference in pay back period decreases as N increases, and is greatest for Duke plans. For Yale plans, however, the effect on pay back period of a change in the institution's cost is reversed. Pay back period decreases (increases) as the institution's cost increases (decreases) because in Yale plans, total repayments increase (decrease) when the institution's cost increases (decreases). The change in pay back period for Yale Profit plans is usually between 1 and 3 years for a 1 percent change in the institution's cost, and generally is less when N is large.

6. SENSITIVITY ANALYSIS ON COHORT COMPOSITION

In all the foregoing analyses, we have considered a single cohort of borrowers. The composition of borrowing by individuals in this cohort, which we henceforth refer to as cohort A, is shown again in Table 6. Since the cohort composition, that is, the mixture of incomes of individuals in the borrowing population may differ we

- compare optimal plans designed for different cohorts
- examine the effect on repayments when optimal plans designed for one cohort are subscribed by another.

The first comparison provides insights about the sensitivity of economic measures of optimal plans to changes in the income mixture of the cohort. This analysis is critical if an institution is considering whether to offer different plans to different groups (such as students in Medicine, Law, Graduate School, Undergraduate programs, etc.). The second comparison is important because the institution inevitably designs its plan without full knowledge of the mixture of incomes of the participating individuals. For Subsidy and Welfare plans, we will allow the maximum repayment period N to vary in order to ensure that a cohort discharges its debt. (This is analagous to the cohort termination provision of Yale University's plan.)

For the purposes of these analyses, two other cohorts are considered, referred to as cohorts B and C. Specifically, the composition of borrowing by individuals in different profiles in the cohorts considered is given in Table 6. These two cohort compositions are the cases where the amount borrowed by profile 4 individuals increases or decreases by 10 percent from the original cohort A. The reason for changing the profile 4 representation is that this profile is the largest single group in the original cohort. Since profile 4 individuals have a lower than average income, cohort B has a smaller percentage of high income earners, and cohort C has a higher percentage of high income earners, as compared to cohort A. Note that for comparability we assume that the total amount loaned to the cohorts remains the same. (This assumption is important when we make the second comparison above.)

Table 6

Profile	Cohort Composition A	B	C
1	25.0%	20.0%	30.0%
2	4.5%	3.6%	5.4%
3	12.5%	10.0%	15.0%
4	50.0%	60.0%	40.0%
5	8.0%	6.4%	9.6%

Economic Comparison of Optimal Plans Designed for Different Cohorts

Optimal Subsidy Plans

Subsidy is the only measure that is significantly affected. Subsidy decreases as the percentage of individuals in profile 4 increases, because the percentage of individuals contributing subsidy (generally, subsidy is contributed by profile 1 individuals) decreases. Thus, optimal Subsidy plans designed for cohort C have the largest amount of subsidy, followed by cohorts A and B. The difference in subsidy between cohort C and cohort B is often as much as 60 to 70 percent for Duke, Yale, and Harvard optimal Subsidy plans.

The other measures, namely, cohort welfare and long-term capital requirements are not significantly affected. Usually cohort C has higher cohort welfare and long-term capital requirements as compared to cohorts A and C for Duke, Yale, and Harvard optimal Subsidy plans, the reason being that the optimal tax rate for cohort C is less than those for cohorts A and B.

Optimal Welfare Plans

For Duke and Yale optimal Welfare plans, the effects of changes in cohort composition on the economic measures are similar to those of optimal Subsidy plans already discussed. Harvard optimal Welfare plans are unaffected by a change in the cohort composition.

Optimal Profit Plans

For Duke optimal Profit plans, profits increase by 5 to 10 percent as Profile 4 increases from 40 percent (cohort C) to 60 percent (cohort B). This impact seems unusual as individuals within profile 4 have a lower than average income. But it is easily explained by the fact that individuals who exit out later in Duke plans contribute greater profit than individuals who exit out early.

The cohort welfare of Duke optimal Profit plans is not significantly affected by changes in the composition of the cohort. Typically, the cohort's welfare is greatest for cohort B followed by cohorts A and C; the difference is usually less than 1 percent.

Pay back period for Duke optimal Profit plans is not affected significantly, especially when N is large.

For Yale optimal Profit plans, the effect of cohort changes on economic measures is similar to that for Duke Profit plans. In Yale plans, the change in profit is generally less than that of the corresponding Duke plans.

The economic measures (and the plan parameters) for Harvard optimal Profit plans are unaffected by changes in the cohort composition because all individuals exit out. Hence, with no change in the incomes of individuals, the plans and the economic measures do not vary.

<u>Economic Comparisons of Plans Designed for One Cohort Subscribed by a Different Cohort</u>

We next discuss the effect on institution repayments when optimal plans for one cohort are subscribed by another. When a zero-profit plan designed for one cohort is subscribed by another, the plan is generally not zero-profit for the second cohort. In order to make the plan zero-profit we increase or decrease the maximum repayment period N. Since the income profile data is for 35 years, the following analyses are done for optimal plans in which N equals 10 and 25 years, because some of the 30 and 35 year plans would not be zero-profit in 35 years, which is the maximum period that we have considered.

Optimal Subsidy Plans

Because the average income of individuals is the lowest for cohort B and highest for cohort C, the present value of repayments discounted at the institution's rate

- decreases for Duke, Yale, and Harvard plans when cohort C optimal plans are offered to cohorts A and B
- decreases for Duke, Yale, and Harvard plans when cohort A optimal plans are offered to cohort B, but increase when cohort A optimal plans are offered to cohort C
- increases when cohort B optimal plans are offered to cohort A and C.

The change in institution repayments for Duke and Yale plans is of the same order of magnitude as that of Harvard plans. However, the difference in repayments for Duke, Yale, and Harvard plans increases as N or s increases, because the differences in institution repayments between high and low income earners increase.

For all three types of plans, when cohort A plans are offered to cohort B, the maximum repayment year has to be increased in order to ensure cohort termination. When the maximum repayment period is 10 years, then a repayment period of 11 years is sufficient to ensure cohort termination when a cohort A or C plan offered to cohort B. When a cohort B plan is offered to cohort A or C, the cohort termination period is unchanged when N is 10 years. But when N is 25 years, the repayment period increases. Cohort A plans need an extra 2 to 3 years in order to ensure cohort termination when offered to cohort B, and about 1 year less when offered to cohort C. Similarly, when cohort C plans are offered to cohorts A and B, the cohort termination requires an extra 1 to 3 years for cohort A and 2 to 5 years for cohort B. When cohort B plans are offered to cohorts A and C, the cohort termination period decreases from 0 to 1 years for cohort A and by 1 to 3 years for cohort C. Thus the decrease in the repayment period is generally less than the increase in the repayment period when the composition of the cohort changes.

Optimal Welfare Plans

For Duke and Yale optimal Welfare plans, the effect of offering plans designed for one cohort to another is similar to the effect on Subsidy plans discussed above. Harvard optimal Welfare plans are not affected by a change in the cohort composition.

Optimal Profit Plans

The plan parameters of optimal plans for the three different cohorts A, B, and C are generally the same for each type of plan. Consequently, the effect on repayments when optimal plans designed for one cohort are subscribed by another is insignificant.

7. SUMMARY OF CONCLUSIONS

In this section we summarize the results of the empirical comparison of the three types of plans and then comment on the specific plans which have been instituted at Duke, Yale, and Harvard Universities.

Comparison of Duke and Yale Plans

Optimal Subsidy and Welfare Plans

For the cohorts considered, Duke and Yale optimal plans are identical with respect to the repayments made by individuals, and with respect to the financial characteristics such as subsidy, cohort welfare, and long term capital requirements. Thus, it may seem that for optimal Subsidy and Welfare plans there is no difference between Duke and Yale plans. But this conclusion is not warranted, as Yale plans are less sensitive than corresponding Duke plans to changes in the institution's cost of financing. Further, if the cohorts were different, then as shown by Jain [6] we could expect that Duke optimal plans have greater subsidy, cohort welfare, and long-term capital requirements than corresponding Yale optimal plans.

Optimal Profit Plans

From the institution's viewpoint, profits obtained from Duke plans are significantly greater than that from corresponding Yale plans. The differences in the profit increase as the repayment period N or the spread s increases. Further, the pay back period is also smaller under Duke plans.

Cohort welfare and individual welfare for average and low income earners are significantly greater under Yale optimal plans.

Comparison of Duke and Harvard Plans

Optimal Subsidy Plans

Subsidy, cohort welfare, and long-term capital requirements under Harvard plans are greater than under the corresponding Duke plans. When inflation is considered, the magnitude of the differences in the above characteristics decreases as d increases. Also, for large N and s, Duke and Harvard optimal Subsidy plans are identical with respect to the repayments made by individuals and therefore all the economic measures that we consider.

Optimal Welfare Plans

Cohort welfare and long term capital requirements are greater under Harvard optimal Welfare plans as compared to the corresponding Duke plans. Subsidy is smaller, however, under Harvard Welfare Plans. The magnitude of the differences in these characteristics increases as N increases. When inflation is introduced, then, for small N and d, the preceding remarks still hold. For large N and d, Harvard optimal Welfare plans become identical to optimal Subsidy plans; then the differences between Duke and Harvard Welfare plans are similar to the differences of optimal Subsidy plans discussed earlier. When inflation is not considered, financial viability of Harvard Welfare plans cannot be achieved if the institution's cost of financing increases after the plan has been instituted.

Optimal Profit Plans

Profit under Harvard plans is significantly greater than than of corresponding Duke plans, but cohort welfare is often smaller. The difference in profits increases as N or s increases but decreases as d increases. Pay back periods for the Duke and Harvard Profit plans are virtually the same.

<u>Comparison of the Yale and Harvard Plans</u>

Optimal Subsidy and Welfare Plans

Since Yale and Duke optimal Subsidy and Welfare plans are identical for the cohorts considered, the comparison of Yale and Harvard plans is similar to the comparison of Duke and Harvard plans discussed earlier.

Optimal Profit Plans

Profits under Harvard optimal plans are significantly greater than under the corresponding Yale plans. The difference in profits increases as N or s increase, but decreases as d increases. The pay back period is smaller for the Harvard plans.

Cohort welfare is greater under Yale Profit plans, and the differences increase as N or s increases.

Comparison of the Specific Plans Instituted at Duke, Yale and Harvard Universities

We comment here on the specific plans instituted at Duke, Yale, and Harvard Universities. We determine Duke, Yale, and Harvard plans that are good approximations to the plans instituted at these universities and then compare the plans so determined. Hence, although our comments on the three plans are fairly general and are expected to hold under most circumstances, there can be exceptions because of the following factors

- Duke, Yale, and Harvard plans that we have considered represent only the major features of the actual plans offered at Duke, Yale, and Harvard Universities.
- we made several simplifying assumptions as set forth in Section 2.

Duke Plan Approximation to Duke University's Plan

Under the plan instituted at Duke University, the minimum repayment is \$36 x 10^{-3} and the tax rate \$36 x 10^{-7} for every dollar borrowed, the exit rate is 8 percent, and the maximum repayment period is 30 years. Since Duke University is financing this plan, the cost of financing is taken to be such that a Duke plan with the above parameters would be zero-profit. This internal cost of financing equals 6.6 percent approximately. Since the exit rate 8% is presumably considerably less than $\bar{R}$, the Duke University plan is not close to an optimally designed Duke Subsidy or Welfare plan.

Yale Plan Approximation to Yale University's Plan

Under the plan instituted at Yale University, the minimum repayment is $\$29 \times 10^{-3}$ and the tax rate is $\$40 \times 10^{-7}$ for every dollar borrowed, the exit multiple is 1.5, and the maximum repayment period is 35 years. Cohort termination actually is expected around 25 years because of the high tax rate and exit-out option. In order to determine the cohort termination year, that is, the implicit maximum repayment period, we have to know the institution's cost of financing the plan. Since Yale University's plan is being funded through loans from commercial institutions, we safely can assume that the cost of financing is greater than the 6.6 percent (implicit) cost of financing Duke University's plan. Further the cost is also likely to be greater than the cost of financing Harvard University's plan, since that is funded under the federal guaranteed loan program. Assuming the cost of financing the Yale plan to be a little over seven percent, which was a reasonable rate at the time of institution of the plan, we have that cohort termination will occur close to the 25th year. Because of the large exit multiple, Yale University's plan can be considered close to optimally designed Yale Subsidy or Welfare plan, and will be acceptable to all individuals only if $\bar{R}$ is close to 10 percent.

Harvard Plan Approximation to Harvard University's Plan

In the plan instituted at Harvard University, repayments discounted at the institution's cost equal at most the amount borrowed. Because of the high tax rate (6 percent of income) regardless of borrowing and the extended repayment period for those who defer, we can expect the subsidy in such a plan to be small. Thus Harvard University's plan can be considered as a Harvard plan with N = 10, R = 7 percent, a high tax rate, and α = 15 percent of ρ. Further the exit or stated interest rate R equals the institution's cost r, and therefore Harvard University's plan may be considered close to an optimally designed Harvard Welfare plan.

Analysis of the Differences

We now summarize the differences in the economic measures of the plans instituted at Duke, Yale, and Harvard Universities:

(1) Maximum subsidy is generated by Yale's plan followed by Duke's and Harvard's.

(2) Cohort welfare is greatest under Duke's plan, followed by Yale's and Harvard's.

(3) The long term capital requirements are greatest under Duke's plan, followed by Yale's and Harvard's.

(4) Yale's plan is acceptable to all individuals in the postulated cohort only if the subjective interest rate that individuals are prepared to pay is close to 10 percent (including inflation), whereas for Duke's and Harvard's plans, the rates are 8 and 7 percent, respectively.

The above differences are due to the specific features of the plans and the selection of the environmental parameters N and r. For example, Duke University's plan has the longest maximum repayment period and also the least implicit cost of financing. For these two reasons, cohort welfare is greatest under Duke University's plan.

The differences in the implicit cost of financing the three different universitites' plans give a relative idea of the extent to which the respective universities have externally subsidized such plans.

Yale's plan is not subsidized externally, whereas, Harvard's is to a small extent by the federal government, and Duke's is to an even greater extent by Duke University.

One other major difference in the three plans is that Yale's and Harvard's plans have a variable maximum repayment period whereas Duke's plan has a fixed repayment period of 30 years. In fact, even though the cohort termination is expected around the 25th year for Yale's plan, if the cost of financing were to increase, or if the income of individuals were far below those in Table 1, then students borrowing under Yale's plan would pay for a longer term (up to as many as 10 more years). Thus, Yale University's plan has the most flexibility to adapt to uncertainties in the cost of financing and income realizations of students. Harvard University's plan also has some flexibility to adapt to income realizations, but cannot adapt to changes in the cost of financing. Duke University's plan has no flexibility.

We have not looked at administrative costs in our analysis. Because of the high tax rate in Harvard University's plan, most borrowers can be expected to pay the scheduled repayment, and therefore administrative costs can be expected to be small as incomes of most individuals need not be verified.

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