Precautionary Bidding in Auctions*

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Abstract

We analyze bidding behavior in auctions when risk-averse bidders bid for an object whose value is risky. We show that, as risk increases, decreasingly risk-averse bidders will reduce their bids by more than the risk premium. Ceteris paribus, bidders will be better off bidding for a more risky object in first-price, second-price, English, and all-pay auctions with affiliated private values. We then extend the results to common value settings. This 'precautionary bidding' effect arises because the expected marginal utility of income increases with risk, so bidders are reluctant to bid so highly. We show that precautionary bidding also arises in response to common values risk. This precautionary bidding behavior can make decreasingly risk-averse bidders better off when they face a 'winner's curse' than when they do not.

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"It's kind of like Christmas. Except that you are afraid it won't meet your expectations. But most of it does." – Lynne Strauss, antique collector, on buying antiques in on-line auctions (quoted in The New York Times, November 12, 1998).

1 Introduction

In many real world auctions the value of the goods for sale is subject to ex post risk. At the time of the sale, bidders can only estimate the value of the object and they are well aware that the true value to them will be revealed only some time after the sale. Part of this risk is what can be termed as winner's curse risk: uncertainty about other buyers' (or the seller's) information which is not revealed in the course of the auction. However, there is also almost invariably *pure risk* in the valuations, arising from information that none of the buyers (nor the seller) can obtain, which will be resolved after the object has been allocated. The sale of oil tracts, art, antiques, wine, and procurement contracts provide obvious examples which exhibit both types of risk. In each case, there is something about the future resale price, authenticity, quality, etc., of these goods which cannot be perfectly foreseen, and which from the bidders' point of view is just pure white noise. Almost all of these risks are present in internet auctions also, which have recently received much attention.[1]

Despite the ubiquity of pure ex post risk, there has to date been no analysis of its effect on the bidding behavior of risk-averse agents.[2] The core of this paper is devoted to providing such an analysis, with somewhat surprising results. We then draw out the implications of the results on pure ex post risk for common value risk.[3]

[1]The *New York Times (*November 12, 1998) documents the risks involved in buying antiques in on-line auctions - the item purchased may be damaged, a fake, or may even never arrive. "The web is a suberb place to sell bad stuff... [internet] auctions are a great way to get rid of things you can't sell any other way – something that has damage, or has been repaired, or is a fake." Recent academic papers analyzing internet auctions include Bajari and Hortaçsu (2000), Lucking-Reiley et al. (1999) and Roth and Ockenfels (2000).

[2]Bidders display risk aversion in a variety of auction scenarios. For a survey of the experimental evidence see Kagel (1995). Econometric evidence based on data from timber auctions is provided by Paarsch (1992), Athey and Levin (2001) and Campo et al. (2000).

[3]The effect of the winner's curse has been studied in the literature with risk neutral bidders; we will adopt the framework of the seminal work of Milgrom and Weber (1982).

Our first result is to show that in all common auction forms (first price, second price, English and all-pay auctions), decreasingly absolute risk-averse (DARA) bidders will reduce their bids by more than the risk premium as pure risk increases. This implies that bidders will be better off bidding for a more risky object, ceteris paribus.

In the first price and all-pay auctions, bidders effectively engage in "precautionary bidding." As with precautionary saving, when agents face a risk their marginal utility of income rises.[4] This causes bidders to bid *less* aggressively because they value more highly each extra dollar of income, as compared to the increased probability of winning the object. This result is not obvious for the following reason. Under some conditions, decreasingly risk-averse individuals become more risk-averse in facing one risk (i.e., losing the object) when they are forced to face an independent risk (i.e., object value).[5] Since increasing the degree of risk aversion leads to *more* aggressive bidding in a first price auction,[6] we might therefore expect that increasing the riskiness of the object would make bidders more risk-averse and so raise bids and make them worse off. However, this latter effect turns out to be of second order compared to the precautionary bidding effect. Precisely when bidders have decreasing absolute risk aversion, the precautionary bidding effect is large enough to make bidders better off when they are bidding for a risky object than a safe one.

In the second-price and English auctions the bidders also reduce their bids by more than the value of the risk premium. But though the conditions for the effect to occur are the same (i.e., DARA), the intuition behind it is slightly different. Recall that in these auctions the bidders submit bids assuming that they will receive zero surplus from winning whenever they win.[7] This means that in the presence of noise, bidders with decreasing absolute risk aversion will reduce their bids by the large risk premium that would be required if their surplus were zero. But when they actually win, the bidders will have a positive surplus on average and their expected payment

[4] See Leland (1968), Sandmo (1970), Drèze and Modigliani (1972), and more recently, Kimball (1990).

[5] See Pratt and Zeckhauser (1987), Kimball (1993), Eeckhoudt et al (1996).

[6] This result has been established by Butters (1975), Matthews (1979), Holt (1980), Riley and Samuelson (1981), Maskin and Riley (1984).

[7] In second price and English auctions, one's bid does not directly affect how much one will pay, only under which circumstances one wins, so bidders should be increase their bids up to the point where they are indifferent to winning.

will have been reduced by a risk premium that was too large. So overall they will be better off. This decreasing risk premium effect is distinct from precautionary bidding. Notice, in particular, that it arises in the absence of any strategic effect on the part of the bidders.

Up until now, the auction literature has completely abstracted from the issue of pure ex post risk. We show that this abstraction is fully justified only in the case of CARA preferences. Bidders with CARA preferences reduce their bids by exactly the risk premium in all the above auction types and thus they are indifferent to the introduction of symmetric risk in their values. With CARA bidders, one could equally well analyze a 'certainty equivalent auction' where all bidders' initial values are adjusted downwards by the amount of the risk premium and then proceed as if there were no ex post risk on the object. By contrast, this is not true for DARA bidders. The latter are not indifferent to ex post risk, but instead positively benefit from the introduction of this type of uncertainty. Hence they have no incentive at all to collectively reduce the risks they face. We might even expect to see DARA bidders attempting to collectively commit not to acquire information about shocks which might affect their payoffs (e.g. not doing test drilling on a tract of oil for sale) if such actions are publicly observable.

The results also imply that a seller facing bidders with DARA preferences has an incentive to reduce the riskiness of the valuations. This is not simply because the risk reduction directly increases the bidders' willingness to pay – but also because it limits precautionary bidding and thus intensifies competition. Even a seller who is as risk-averse as the bidders will wish to provide insurance against the noise, if this is possible. This result may provide an explanation for the cooling-off rights found in some auctions.[8] By contrast, if the seller's objective function contains elements other than expected profit (e.g., popularity of the auction) then it could be a good idea to auction goods whose value is uncertain. Auctions with risky objects may become very popular with DARA bidders who think that there is a large surplus to be made. It is important to see that this effect arises with risk-averse buyers rather than risk-loving ones, and that the popularity of risky auctions is thus completely rational. This observation could help to explain the widespread popularity of on-line auctions: despite the evident risks involved in purchasing unseen goods from a complete stranger, risk-averse bidders do well out of such auctions, and participation in them need not be viewed as a form of

[8] See Asker (2000) for more discussion of auctions with cooling-off rights.

addictive gambling, as has sometimes been suggested in the popular press.

Our finding that the seller would like to reduce the white noise faced by buyers is distinct from the linkage principle (due to Milgrom and Weber [1982]). This principle implies that the seller should commit to reveal any information affiliated with the buyers' signals because the commitment reduces the winner's curse that the buyers face. Note, however, that the winner's curse arises because winning provides information about the value of the object, in which case even risk-neutral bidders should bid less aggressively. Conversely, it is completely possible for the private value of an object to an individual to be risky without any winner's curse implications for bidding; this will be the case when knowing that one is the winner provides no additional information about the value of the object. An obvious distinction between the linkage principle and the effects of white noise that we will focus on in this paper is that precautionary bidding will arise only when bidders are decreasingly risk-averse, whereas the linkage principle will hold even when bidders are risk-neutral, but have affiliated common values.

The behavior of risk-averse bidders in an environment with affiliated common values has to our knowledge been hardly studied at all. Towards the end of this paper we use the analysis of precautionary bidding to throw more light on this topic. We show that DARA bidders engage in precautionary bidding in response to the risk inherent in other bidders' signals not revealed in the course of the auction. Because of this, decreasingly risk-averse bidders may be better off in a common values than in a private values setting, something which will never arise with risk-neutral bidders. Thus what is a winner's *curse* to risk-neutral bidders can sometimes be a *blessing* to risk-averse bidders.

The comparison of common and private values auctions from the buyers' point of view is of practical importance because in some settings the bidders may be able to choose between entering auctions where they will face winner's curse risk and where they will not. In some cases this may be a straight choice about what type of object to buy. In other potential applications, whether an auction is largely a private or common values affair may be determined by prior moves taken by the bidders. For example, consider two firms which will later compete in procurement auctions. If – prior to the auctions – these two firms choose similar production technologies then the subsequent auctions will have a strong common-value component: one firm's estimate of the likely cost of fulfilling the contract is likely to be important information for the other firm. But if the two firms choose two very different technologies then they will have mostly private valuations for the contract. Our results suggest that if the firms are DARA then they will choose more similar technologies in order to benefit from the softened bidding that the winner's curse risk generates. Similar remarks apply to the choice of customer base by car, art, wine and antique retailers who buy their product in wholesale auctions. By choosing to serve customers with similar tastes they mitigate competition in the wholesale auction and the reduction in competition effect would counteract the bidders' desire to differentiate themselves. These and other applications and testable predictions are spelled out in more detail throughout the paper.

The paper is laid out as follows. In order to investigate the effect of pure risk on bidding behavior in isolation, we begin in section 2 by abstracting from the common-value framework and analyzing a model where bidders have affiliated private values. Then in section 3 we consider the consequences of adding white noise to the prize in common values auctions, and are able to prove the same result: that decreasingly risk-averse bidders will benefit from more risk in the good's value. In section 4 we show how the winner's curse can be a blessing for risk-averse buyers (they may be better off in a common rather than a private values auction). For the sake of fair comparison, we consider a natural (rent-preserving) mapping from common values to private values settings and show that whenever risk-neutral bidders are indifferent between the two settings, DARA bidders strictly prefer the common values auction. Finally in Section 5 we offer concluding remarks.

2 The affiliated private values model

Assume that there are n potential buyers for a given object. The seller's reservation value for the good is zero. Buyer i's best estimate for the private monetary value of the object is represented by his type, $\theta_i \in [\underline{\theta}, \overline{\theta}]$, $\underline{\theta} > 0$.

The joint ex ante distribution of (θ_i, θ_{-i}) has a positive density $f(\theta)$, and is symmetric and affiliated.[9] Denote the conditional density of $\theta_{-i}^{(1)} \equiv \max_{j \neq i}\{\theta_j\}$ given θ_i by $f(y \mid \theta_i)$, and the associated cumulative distribution by $F(y \mid \theta_i)$. Since θ_i and $\theta_{-i}^{(1)}$ are also affiliated, the conditional distribution of $\theta_{-i}^{(1)}$ given θ_i first-order stochastically dominates the distribution of $\theta_{-i}^{(1)}$ given θ_i' for all $\theta_i \geq \theta_i'$; that is, $\frac{\partial}{\partial \theta_i} F(y \mid \theta_i) \leq 0$. The intuition behind this

[9] For the definition and properties of affiliation see Milgrom and Weber (1982).

property is simple: if types are affiliated ("positively correlated") then for a larger θ_i the highest among $\{\theta_j\}_{j \neq i}$ is likely to be larger.[10]

The ex post monetary value of the object for buyer i is $\theta_i + z_i$, where z_i is the realization of a zero-mean random variable $\tilde{z}_i$. We assume that the $\tilde{z}_i$'s come from a symmetric joint distribution and that each $\tilde{z}_i$ is independent of (θ_i, θ_{-i}).[11] The noise is ex interim unobservable and uninsurable. When the $\tilde{z}_i$'s are degenerate, $\tilde{z}_i \equiv 0$, we say that the buyers have *deterministic valuations*, and when the $\tilde{z}_i$'s are non-degenerate, we say that they have *noisy valuations*. Note that we can interpret this "noise term" affecting bidders' values in either of two ways. First, it could be a result of common shocks (such as a change in the oil price or the amount of oil underground); or second, it could be buyer-specific symmetrically distributed shocks (such as unforeseen production costs).

The buyers evaluate their monetary surplus (consisting of their initial wealth minus the transfer paid to the seller, plus the object's value when they win) according to a strictly concave utility function, u, and they are expected utility maximizers. We normalize their initial wealth and $u(0)$ to zero, and assume that the object is valuable even for the lowest type, $\mathsf{E}u(\underline{\theta} + \tilde{z}_i) > 0$. We will use the notions of decreasing (increasing, and constant) absolute risk aversion (DARA, IARA, and CARA, respectively), defined the standard way as $-u''(x)/u'(x)$ being decreasing (increasing, and constant, respectively).

All our results carry through in a more general class of utility functions, $u(w, \theta)$, where the buyer's wealth (w = initial wealth minus possible payments) and the value of the object (θ when winning, 0 otherwise) enter as two separate arguments. In this case we require $u'_i > 0$, and $u''_{ij} < 0$ for $i, j \in \{1, 2\}$ for risk aversion. The Arrow-Pratt measure of *aversion to risk in the object's value* would be $r(w, \theta) \equiv -u''_{22}/u'_1 \equiv -\frac{\partial^2 u}{\partial \theta^2} / \frac{\partial u}{\partial w}$, and decreasing (increasing, or constant) absolute risk aversion should be interpreted as $r(w, \theta)$ decreasing (increasing, or constant) in θ. This said, to simplify notation we will not use the broader class of utility functions in this paper; instead we will follow the majority of the auction literature in expressing the object's value in monetary terms.

[10] Matthews (1987) notes that this property directly follows from Theorem 5 of Milgrom and Weber (1982), which states: If $X_1, ..., X_n$ are affiliated and $h(x_1, ..., x_n)$ is non-decreasing in its arguments then $\mathsf{E}[h(X_1, ..., X_n) \,|\, a_1 \leq X_1 \leq b_1, ..., a_n \leq X_n \leq b_n]$ is non-decreasing in $a_1, ..., a_n, b_1, ..., b_n$. For the property mentioned in the text, apply the theorem to $F(y \,|\, \theta_i) \equiv \mathsf{E}[1_{\{\max_{j \neq i}\{\theta_j\} \leq y\}} \,|\, \theta_i]$.

[11] Note that we allow the $\tilde{z}_i$'s to be correlated, or even $\tilde{z}_i \equiv \tilde{z}$ for all i.

2.1 Main results: precautionary bidding

We now analyze how DARA buyers' behavior and indirect utility changes as a result of more white noise being added to their affiliated private valuations. In particular, we will compare two situations, one where $\tilde{z}_i \equiv 0$ (deterministic valuations), and another where $\tilde{z}_i$ is an independent random variable with zero mean and finite variance (noisy valuations). We will show that DARA buyers end up being better off when noise is present in their valuations in each of the four most common auction forms: the English or button-, the first-price, the second-price, and the all-pay auction.[12]

Proposition 1 establishes that in the presence of noise the buyers reduce their bids by more than the appropriate risk premium in the second-price auction (SPA); and that consequently, DARA buyers are better off with risk in both the SPA and in the English auction. Unlike in the case of the first-price auction (FPA) or the all-pay auction (APA) there is no strategic effect stemming from decreasing risk aversion.

Proposition 1 *In the second-price auction with noisy valuations, decreasingly risk-averse buyers bid $\theta_i - \pi_0$, where π_0 satisfies $\mathsf{E}u(\tilde{z}_i + \pi_0) = 0$. Consequently, the (ex interim) expected utility of DARA buyers is higher with noisy rather than deterministic valuations in both the SPA and the English auction (button-auction).*

Proof. In the SPA it is a weakly dominant strategy to bid one's valuation when the valuations are deterministic.

Now assume that the valuations are noisy. Suppose that buyer i bids $b > \theta_i - \pi_0$ (where π_0 is as defined in the claim). His monetary surplus conditional on winning differs from the surplus he would receive by bidding $\theta_i - \pi_0$ only if $b^{(2)} > \theta_i - \pi_0$, where $b^{(2)}$ is the second highest bid. He wins the object with a bid of b, while he would have lost it with a bid of $\theta_i - \pi_0$. His utility from winning with bid b is $\mathsf{E}u(\theta_i + \tilde{z}_i - b^{(2)}) < \mathsf{E}u(\tilde{z}_i + \pi_0) = 0$, which is worse than not winning. A symmetric argument establishes that buyer i can only be worse off by bidding $b < \theta_i - \pi_0$ instead of $\theta_i - \pi_0$ by missing profitable opportunities.

[12] The results are confined to comparing noisy and deterministic valuations, but immediately extend to situations where another independent noise is added to make already noisy valuations still riskier. This is so because the DARA property is preserved under addition of independent mean-zero noise (see Kihlstrom, Romer, and Williams, 1981).

However, π_0 is greater than the risk premium necessary to compensate i's utility for $\tilde{z}_i$ conditional on winning because he pays less for the object than his bid (except in case of a tie), and his risk aversion is decreasing in his final wealth. This proves the claim that DARA bidders are better off in the SPA with noisy rather than deterministic valuations.

Finally, the same statement is true for the English (button-) auction because the SPA and the button-auction are outcome equivalent under private values. ■

In Lemma 2 below attention is restricted to *symmetric deterministic direct mechanisms* – direct mechanisms that have the form $\{G, b, a\}$. $G(\hat{\theta}_i, \theta_i)$ is the expected probability of buyer i winning the object with a type-announcement $\hat{\theta}_i$ and a true type θ_i; $b(\hat{\theta}_i)$ is the transfer this buyer pays conditional on winning, and $a(\hat{\theta}_i)$ is the transfer he pays conditional on losing. Note that both payments depend on i's own announcement only and are assumed to be deterministic, so these mechanisms do not represent all mechanisms. Our goal is merely to prove a claim that can be specialized to prove statements on the FPA and the APA (Propositions 3 and 4), mechanisms that do satisfy these conditions.[13]

Lemma 2 *Consider two symmetric deterministic direct mechanisms with the same allocation rule G and providing the same utility for the lowest type $\underline{\theta}$. Let these two mechanisms be $\{G, b, a\}$ and $\{G, \beta, \alpha\}$. Assume that $\{G, b, a\}$ is incentive compatible under deterministic valuations, and $\{G, \beta, \alpha\}$ is incentive compatible with a given specification of noisy valuations. Assume that the buyers have DARA preferences and that a, b, α, β are continuous.*

If $\alpha(\theta_i) \leq a(\theta_i)$ and $\frac{\partial}{\partial \theta_i} G(x, \theta_i) \leq 0$ for all θ_i, where the utilities that θ_i receives in the two mechanisms equal, then buyers are better off in mechanism $\{G, \beta, \alpha\}$ with noisy valuations than in mechanism $\{G, b, a\}$ with deterministic valuations.

Proof. In mechanism $\{G, b, a\}$ with deterministic values, buyer i's utility from announcing $\hat{\theta}_i$ when his true type is θ_i can be written as

$$U(\hat{\theta}_i, \theta_i) = G(\hat{\theta}_i, \theta_i)\, u(\theta_i - b(\hat{\theta}_i)) + [1 - G(\hat{\theta}_i, \theta_i)]\, u(-a(\hat{\theta}_i)).$$

[13] The method of proof is similar to the envelope-theorem type of arguments widely used in the literature starting with Milgrom and Weber (1982).

Let $V(\theta_i) \equiv U(\theta_i, \theta_i)$, that is,

$$V(\theta_i) = G(\theta_i, \theta_i)\, u(\theta_i - b(\theta_i)) + [1 - G(\theta_i, \theta_i)]\, u(-a(\theta_i)). \tag{1}$$

By the necessary condition of incentive compatibility, for all $\theta_i \in (\underline{\theta}, \overline{\theta})$,

$$V'(\theta_i) = G_2'(\theta_i, \theta_i)\; [u(\theta_i - b(\theta_i)) - u(-a(\theta_i))] \; + \; G(\theta_i, \theta_i)\, u'(\theta_i - b(\theta_i)), \tag{2}$$

where $G_2'(\theta_i, \theta_i) \equiv \frac{\partial}{\partial \theta_i} G(\hat{\theta}_i, \theta_i)|_{\hat{\theta}_i = \theta_i}$.

In mechanism $\{G, \beta, \alpha\}$ with noisy valuations, let buyer i's expected utility from announcing $\hat{\theta}_i$ when his true type is θ_i be $\tilde{U}(\hat{\theta}_i, \theta_i)$, and let $\tilde{V}(\theta_i) \equiv \tilde{U}(\theta_i, \theta_i)$. We have

$$\tilde{V}(\theta_i) = G(\theta_i, \theta_i)\, \mathsf{E} u(\theta_i + \tilde{z}_i - \beta(\theta_i)) + [1 - G(\theta_i, \theta_i)]\, u(-\alpha(\theta_i)), \tag{3}$$

and, by the envelope theorem, for all $\theta_i \in (\underline{\theta}, \overline{\theta})$,

$$\begin{aligned}\tilde{V}'(\theta_i) = G_2'(\theta_i, \theta_i)\; &[\mathsf{E} u(\theta_i + \tilde{z}_i - \beta(\theta_i)) - u(-\alpha(\theta_i))] \\ &+ \; G(\theta_i, \theta_i)\, \mathsf{E} u'(\theta_i + \tilde{z}_i - \beta(\theta_i)).\end{aligned} \tag{4}$$

Define $\pi(\theta_i)$, the risk premium compensating the winner's utility, as

$$u(\theta_i - b(\theta_i)) \equiv \mathsf{E} u(\theta_i + \tilde{z}_i - b(\theta_i) + \pi(\theta_i)).$$

By decreasing absolute risk aversion,

$$u'(\theta_i - b(\theta_i)) < \mathsf{E} u'(\theta_i + \tilde{z}_i - b(\theta_i) + \pi(\theta_i)). \tag{5}$$

Now compare (1) and (3). If $V(\theta_i) = \tilde{V}(\theta_i)$ and $\alpha(\theta_i) \leq a(\theta_i)$ for some θ_i then $\beta(\theta_i) \geq b(\theta_i) - \pi(\theta_i)$. Hence the bracketed difference in the first term of (2) is not less than the corresponding difference in (4),

$$u(\theta_i - b(\theta_i)) - u(-a(\theta_i)) \geq \mathsf{E} u(\theta_i + \tilde{z}_i - \beta(\theta_i)) - u(-\alpha(\theta_i)).$$

Since $G_2'(\theta_i, \theta_i) \leq 0$ by assumption, the first term in the expression for $V'(\theta_i)$ is less than or equal to the first term in the expression for $\tilde{V}'(\theta_i)$.

By (5) and $\beta(\theta_i) \geq b(\theta_i) - \pi(\theta_i)$,

$$u'(\theta_i - b(\theta_i)) < \mathsf{E} u'(\theta_i + \tilde{z}_i - \beta(\theta_i)).$$

Hence the second term in (2) is strictly less than the second term in (4).

We conclude that under the hypotheses of the Lemma, if $V(\theta_i) = \tilde{V}(\theta_i)$ for some $\theta_i \in (\underline{\theta}, \overline{\theta})$, then $V'(\theta_i) < \tilde{V}'(\theta_i)$.

By assumption, $V(\underline{\theta}) = \tilde{V}(\underline{\theta})$ and for all $\theta_i \in (\underline{\theta}, \overline{\theta})$, $V(\theta_i) = \tilde{V}(\theta_i)$ implies $V'(\theta_i) < \tilde{V}'(\theta_i)$. It is easy to show that $V(\theta_i) < \tilde{V}(\theta_i)$ follows for all $\theta_i \in (\underline{\theta}, \overline{\theta}]$. ■

Now we can prove,

Proposition 3 *Consider a first-price sealed bid auction with affiliated private values. Then decreasingly risk-averse buyers are better off with noisy rather than deterministic valuations.*

Proof. FPA belongs to the class of auctions represented by symmetric deterministic direct mechanisms. The bidder's actual bid function is $b(\theta_i)$ $[\beta(\theta_i)]$ under deterministic [noisy] values is and $a(\theta_i) = \alpha(\theta_i) \equiv 0$.

By the assumption that $\mathsf{E}u(\underline{\theta} + \tilde{z}) > 0$ all buyers participate in the FPA under deterministic or noisy valuations. The outcome will be efficient because the unique equilibrium of the FPA consists of symmetric increasing strategies (Maskin and Riley [1984], Theorem 2, applies under both deterministic and noisy values). Hence $G(\hat{\theta}_i, \theta_i) \equiv F(\hat{\theta}_i \,|\, \theta_i)$, where $F(\hat{\theta}_i \,|\, \theta_i)$ is the conditional distribution of $\max_{j \neq i}\{\theta_j\}_{j \neq i}$ given θ_i. By affiliation, $G_2'(., \theta_i) \leq 0$.

The hypothesis of Lemma 2 holds and therefore the claim follows. ■

The same result goes through for the all-pay auction provided that a symmetric equilibrium in increasing strategies exists in that game. The existence of an equilibrium of the APA is not guaranteed with affiliated signals; see Krishna and Morgan (1997).

Proposition 4 *Consider an all-pay auction with affiliated private values and suppose that a symmetric equilibrium in increasing strategies exists under both deterministic and noisy private values. Then decreasingly risk-averse buyers are better off with noisy rather than deterministic valuations.*

Proof. All types participate and use a symmetric increasing bid function; therefore $G(\hat{\theta}_i, \theta_i) \equiv F(\hat{\theta}_i \,|\, \theta_i)$ and $G_2'(., \theta_i) \leq 0$ by affiliation. The actual bid function corresponds to $b(\theta_i) \equiv a(\theta_i)$ and $\beta(\theta_i) \equiv \alpha(\theta_i)$ in the deterministic and noisy cases, respectively. Note that when type θ_i's utility under deterministic valuations equals that under noisy valuations in the APA, we have

$$
\begin{aligned}
&F(\theta_i \,|\, \theta_i)\, u(\theta_i - b(\theta_i)) + [1 - F(\theta_i \,|\, \theta_i)]\, u(-b(\theta_i)) \\
&\qquad = F(\theta_i \,|\, \theta_i)\, \mathsf{E} u(\theta_i + \tilde{z}_i - \beta(\theta_i)) + [1 - F(\theta_i \,|\, \theta_i)]\, u(-\beta(\theta_i)),
\end{aligned}
$$

and therefore $b(\theta_i) \geq \beta(\theta_i) \geq b(\theta_i) - \pi(\theta_i)$; so $\alpha(\theta_i) \leq a(\theta_i)$ holds as well.

The hypothesis of Lemma 2 holds and therefore the claim follows. ■

To summarize the results of this section: decreasingly risk-averse individuals bidding for an object in most common auction formats will be better off if the object's value becomes more risky. The intuition behind the result for the first-price and all-pay auctions is that when the object becomes risky, bidders' marginal utility of an extra dollar of income when they win will rise, and this effect will make them want to shade their bids in the presence of risk. The effect follows from the same property of the utility function which generates precautionary saving (i.e. $U(.)''' > 0$) in response to uncertain future income, and so by analogy we call the effect precautionary bidding. When bidders have decreasing risk aversion (strictly stronger than $U(.)''' > 0$), the increase in marginal utility due to risk persists even after bidders have been compensated by the amount of their risk premium (Kimball 1990), so that bids are reduced by more than the risk premium and the precautionary bidding effect is large enough to make bidders better off.[14] In the second-price and English auctions the result follows more directly from the fact that the risk premium is decreasing.

Interestingly, numerical calculations (available from the authors) show that the same result does *not* hold in the optimal auction.[15] Depending on the parameters, bidders may either gain or lose when the object for sale becomes riskier; and there appear to be no general results available. We conjecture that the reason for this is that the optimal auction can be thought of as a "two instrument" auction, as opposed to the common auction formats, which are "single instrument" mechanisms in the sense explained below.

[14] Note that the discussion in this paragraph suggests that *risk-averse* bidders with $U(.)''' < 0$ could bid strictly *more* for a risky object than for a safe one. We do not pursue this possibility because we do not regard such strongly increasing risk aversion as empirically plausible.

[15] For an analysis of the optimal auction with decreasingly risk-averse bidders, see Maskin and Riley (1984). Matthews (1983) analyses the case of constant absolute risk-aversion.

One way of looking at the results for the common auction formats is that bidders with low signals are less able to bear the risk on the object than bidders with higher signals. When risk is added, the low types reduce their bids (the single instrument) by more than the amount which the high types would require to keep them indifferent to imitating the low types. Therefore the high types get more utility from imitating low types and hence more utility than they did from the riskless auction. This must be so in single instrument auctions. But in the optimal auction the auctioneer has "two instruments". Although the auctioneer must reduce the expected payment made by low types, he can also redistribute this between payments when bidders lose and payments when they win. This is what makes the effect of risk on bidders' welfare ambiguous in the optimal auction. In choosing the appropriate combination of the two instruments, the seller faces the familiar trade-off between rent extraction and insurance. The intuition can be gained by considering a model with only two types of bidders: high value and low value. Raising the low type's payment when he wins and reducing it when he loses provides him with more insurance, so raising his total expected payment. But it also makes the low type's allocation more attractive to the high type, so reducing the expected payment which can be extracted from the latter. When risk is added to the object and the auction is then reoptimized, a given level of insurance for the low type allows less rent extraction from the high type than before, so the seller's optimal response to risk is to provide less insurance for the low-type. However, it is ambiguous whether the reduction in insurance is sufficient to actually reduce the welfare of the higher type.

2.2 Comparison of auctions in the presence of noise

In the first part of this section we saw that in all traditional auction forms risk-averse buyers reduce their bids by more than their risk premium when their valuations are noisy. The size of this effect is potentially substantial: in the SPA and English auction, bids are reduced by the amount of the risk premium if bidders had zero surplus.[16] But the magnitude of this effect varies across auction forms and we might wonder how the auction forms are ranked (either from the seller's or the buyers' point of view) when valuations

[16]Numerical computations were performed for the first-price auction using CRRA bidders (details available from the authors). The difference between the actual bid functions and hypothetical bid functions (where bids were reduced by the compensating risk premium) was shown to be as much as 10-15% of actual bids.

are noisy. In particular, does the differential reduction in the bids alter the preference ordering of the seller (or the buyers) over the different auction forms, relative to the situation when noise is not present? Let us now address this issue.

In fact, the answer is very simple: the results developed under deterministic valuations carry through. The presence of noise will not alter the ranking of different auction forms in the preferences of either the seller or DARA buyers. The seller's preferences are considered in Milgrom and Weber (1982), and Maskin and Riley (1984). It is shown that the seller prefers the first-price auction over the second-price auction under independent private values with risk-averse (not necessarily DARA) buyers. The buyers' preferences over auction forms are established by Matthews (1987). Under independent private values he shows that DARA buyers prefer the SPA over the FPA.[17] Note that both results are only valid for independent valuations; in contrast, affiliation of the buyers' signals will improve the expected revenue and decrease the DARA buyers' utility in the SPA relative to the FPA due to the linkage principle (Milgrom and Weber [1982]; Matthews [1987]).

The two results remain true with noisy valuations because both risk aversion and the DARA property are preserved after the introduction of noise (i.e., the two properties are preserved under expectations - see Pratt [1964]). This is a straightforward combination of existing results, but for the sake of clarity and completeness we state them formally in Proposition 5.

In Proposition 5 we also complete the preferences of (decreasingly risk averse) buyers over traditional auction forms by proving that they prefer the first-price auction to the all-pay auction. (This result is true more generally, under *affiliated* private values). To the best of our knowledge this result is not reported elsewhere in the literature. The expected-revenue comparison of the FPA and the APA with risk-averse buyers is not known.

Proposition 5 *Assume that the buyers are risk averse and have iid private values augmented by a symmetric additive ex post noise. The seller gets more revenue from every type of every buyer in the FPA than in the SPA.*

If the buyers' preferences exhibit decreasing absolute risk aversion then they prefer the SPA to the FPA, and the FPA to the APA. (The last relation

[17] Under independent private values the English and second-price auctions are outcome-equivalent. Therefore both the seller and the buyers are indifferent between these two formats.

remains true under affiliation, provided that an increasing equilibrium exists in the APA.)

Proof. The first two results follow from the cited literature and the fact that both risk aversion and the DARA property are preserved under expectation (Pratt [1964]).

Now we prove that under affiliated private values, buyers with DARA preferences prefer the first-price auction to the all-pay auction.

Denote the equilibrium bid function in the APA by $b(\theta_i)$ and the equilibrium bid function in the FPA by $\beta(\theta_i)$. The utility of type θ_i pretending $\hat{\theta}_i$ in the all-pay auction is,

$$U^{APA}(\hat{\theta}_i, \theta_i) = G(\hat{\theta}_i, \theta_i)\mathsf{E}u(\theta_i + \tilde{z} - b(\hat{\theta}_i)) + [1 - G(\hat{\theta}_i, \theta_i)]u(-b(\hat{\theta}_i)),$$

where $G(\hat{\theta}_i, \theta_i)$ is his expected probability of winning. Letting $V(\theta_i) \equiv U^{APA}(\theta_i, \theta_i)$, by the envelope theorem, for all $\theta_i \in (\underline{\theta}, \overline{\theta})$,

$$V'(\theta_i) = G_2'(\theta_i, \theta_i)[\mathsf{E}u(\theta_i + \tilde{z} - b(\theta_i)) - u(-b(\theta_i))] + G(\theta_i, \theta_i)\mathsf{E}u'(\theta_i + \tilde{z} - b(\theta_i)). \tag{6}$$

Similarly construct $U^{FPA}(\hat{\theta}_i, \theta_i) = G(\hat{\theta}_i, \theta_i)\mathsf{E}u(\theta_i + \tilde{z} - \beta(\hat{\theta}_i))$, and define $W(\theta_i) \equiv U^{FPA}(\theta_i, \theta_i)$. Again, by the envelope theorem, for all $\theta_i \in (\underline{\theta}, \overline{\theta})$,

$$W'(\theta_i) = G_2'(\theta_i, \theta_i)\mathsf{E}u(\theta_i + \tilde{z} - \beta(\theta_i)) + G(\theta_i, \theta_i)\mathsf{E}u'(\theta_i + \tilde{z} - \beta(\theta_i)). \tag{7}$$

Note that $V(\underline{\theta}) = W(\underline{\theta})$. Suppose $V(\theta_i) = W(\theta_i)$ at some $\theta_i \in (\underline{\theta}, \overline{\theta})$. Then, obviously, $0 \leq b(\theta_i) \leq \beta(\theta_i)$, and

$$\mathsf{E}u(\theta_i + \tilde{z} - \beta(\theta_i)) \leq \mathsf{E}u(\theta_i + \tilde{z} - b(\theta_i)) - u(-b(\theta_i)), \tag{8}$$

since $u(0) = 0$ and $u' > 0$. By (8) and $G_2' \leq 0$ (affiliation), the first term in (7) is not less than that in (6). Since $b(\theta_i) \leq \beta(\theta_i)$, the second term in (7) is strictly greater than its counterpart in (6) by the DARA-property. Hence $V'(\theta_i) < W'(\theta_i)$ whenever $V(\underline{\theta_i}) = W(\theta_i)$ at some $\theta_i > \underline{\theta}$. We conclude that $V(\theta_i) < W(\theta_i)$ for all $\theta_i \in (\underline{\theta}, \overline{\theta}]$. ■

2.3 Testable predictions

In this section we consider the empirical implications of our model and how it might be tested. The immediate prediction that DARA bidders are better off bidding for a risky object will certainly not be easy to verify. For a direct test one would need an accurate estimate of both the degree of bidders' risk aversion and the riskiness of the object, and estimating these parameters is a difficult econometric exercise. The joint identification of the degree of risk aversion and the distribution of signals is already problematic: see Campo et al (2000) for a recent attempt to tackle this problem.

An indirect but simpler test of our model would be to consider the popularity of an auction. If bidders are better off in risky auctions then one should expect to see more entry into such auctions. Suppose that bidders must choose between attending several auctions, for example because they have only a limited amount of time available. (This seems to be a reasonable description of entry into on-line auctions such as eBay.) Then in equilibrium we should see more bidders entering auctions which are riskier as bidder surplus must be equalized across auctions. This may explain the apparent puzzle as to why internet auctions are so popular despite the great risks associated with buying merchandise from a seller that one has never met. Our model suggests that it is precisely the riskiness of the auction which generates its popularity. A given number of bidders would gain from the presence of risk because the bids would be reduced by an amount which more than compensates them for the risk. Hence with endogenous entry, risky auctions must attract more bidders.[18]

Another situation which would yield the same result – that risky auctions attract more bidders – is where any entry cost has to be paid before bidders observe their types. For example, in forestry auctions a major component of the cost of bidding is the cost of paying a 'cruiser' to visit the tract and estimate its value before the auction, so that the cost of entry is sunk before the value of the object is known. Then tracts which are thought to be risky (e.g. because lack of roads makes it difficult to observe the different species of trees when cruising) should attract more interest from bidders for any given expected value. Similar remarks could apply to bidding for oil tracts (where the main cost is drilling to find out one's estimate of the value of

[18]Another formulation which would work in the same way is that bidders can only bid on a limited number of objects because of eligibility restrictions. For example, to bid in some auctions, bidders must put up a deposit on each item on which they are intending to bid to display their seriousness as bidders. Liquidity constraints would then effectively limit the number of items on which they can bid. In the Australian Spectrum auctions, bidders were asked to state in advance the maximum number of licences in which they were interested and paid an entry fee accordingly.

the tract). By contrast, consider the case when entry costs are paid after the bidders learn their type (as is commonly assumed in the literature,[19] but which actually seems to be less realistic in many applications). Then risk will still lead to precautionary bidding but the lowest types of bidders may not attend the risky auction - so that the net effect of risk on entry is ambiguous.

Note that although risk will reduce a seller's revenue with a fixed number of bidders, the overall effect with an endogenous number of bidders is less clear. This is because risk generates entry - especially by high types, in the case where bidders know their types before the entry decision - and this will tend to raise equilibrium bids. We leave this as a topic for future research.

3 Extension to common values

In this section we extend the analysis to a model where the valuations on which the ex post noise is imposed already have a common value component. We show that the earlier results go through in an affiliated common values model for the SPA and the FPA.

We modify the model of the previous section by allowing the buyers' deterministic valuations to depend on each others' signals. We assume that buyer i's deterministic valuation is $v(\theta_i, \{\theta_j\}_{j \neq i})$. Note that a buyer's valuation depends only on the collection of signals of the other bidders (besides his own), not on the identities of the other bidders. An alternative notation (used by Milgrom and Weber [1982] and others) would be to write the valuation function as $v(X_1, Y_1, ..., Y_{n-1})$, where X_1 stands for i's own signal (θ_i), and Y_k stands for the kth highest among the other bidders' signals ($\theta_{-i}^{(k)}$). In any case, we assume that v is strictly increasing in the buyer's own signal and weakly increasing in all other signals.[20]

The ex post stochastic part of the valuation, $\tilde{z}_i$, is the same as in the previous section: it is a zero-mean random variable drawn from a symmetric joint distribution that is independent of (θ_i, θ_{-i}). The buyer's ex post stochastic valuation for the object is then $v(\theta_i, \{\theta_j\}_{j \neq i}) + \tilde{z}_i$. We will say that the buyers have deterministic (or noisy) valuations when $\tilde{z}_i \equiv 0$ (or when $\tilde{z}_i$ is non-degenerate, respectively). We continue to assume that the

[19] A notable exception is Levin and Smith (1994) who analyze the type of model we describe above, where bidders must pay their entry costs before they receive their signal.

[20] The deterministic part of the valuations is equivalent to the (expected) valuations in the general symmetric affiliated model of Milgrom and Weber (1982).

object is valuable even for the lowest type, $\mathsf{E}u(v(\underline{\theta}, ..., \underline{\theta}) + \tilde{z}_i) > 0$.

As a preparation, define

$$\hat{u}(w\,;\,x, y) \equiv \mathsf{E}[u(v(\theta_i, \{\theta_j\}_{j\neq i}) + w) \,|\, \theta_i = x,\, \theta_{-i}^{(1)} = y]. \tag{9}$$

where $u(.)$ is strict DARA. We will use three key properties of this function:

1. $\hat{u}$ is strictly increasing in x, weakly in y, and it is a concave, strictly increasing utility function in variable w.

2. For $x' > x$ (but holding $y = \theta_{-i}^{(1)}$ fixed), $\hat{u}(w\,;\,x', y)$ is less risk-averse in w than is $\hat{u}(w\,;\,x, y)$ for all levels of w. Similarly, $\hat{u}(w\,;\,x, y')$ is less risk-averse than is $\hat{u}(w\,;\,x, y)$ iff $y > y'$. These properties follow because, by affiliation, the random variable $v(\theta_i, \{\theta_j\}_{j\neq i})$ given $\theta_i, \theta_{-i}^{(1)}$ increases both in θ_i and $\theta_{-i}^{(1)}$ in the monotone likelihood ratio sense and therefore the resulting expected utility function, $\hat{u}$, will exhibit a lower level of risk aversion in w for a higher θ_i or $\theta_{-i}^{(1)}$ (this is due to Jewitt [1987]; see also Eeckhoudt et al. [1996] and Athey [2000]).

3. The functions $-\frac{\partial}{\partial x}\hat{u}(w\,;\,x, y)$ and $-\frac{\partial}{\partial y}\hat{u}(w\,;\,x, y)$ are concave utility functions as well, and both exhibit a higher level of risk aversion in w than does $\hat{u}(w\,;\,x, y)$. This property directly follows from the previous one combined with the observation that decreasing absolute risk aversion of a utility function is equivalent to the negative of the marginal utility being more risk-averse than the utility function (Kimball [1990]).

We now prove the counterpart of Proposition 1 in the general symmetric affiliated model.

Proposition 6 *Consider the general symmetric affiliated model with decreasingly risk-averse buyers. The buyers are better off with noisy rather than deterministic valuations in the symmetric equilibria of the second-price auction and the English auction.*

Proof. First consider the SPA with deterministic common values. Let

$$\overline{v}(x, y) \equiv \mathsf{E}[v(\theta_i, \{\theta_j\}_{j\neq i}) \,|\, \theta_i = x,\, \theta_{-i}^{(1)} = y],$$

where $\theta_{-i}^{(1)} = \max_{j \neq i}\{\theta_j\}$ in accordance with our earlier notation. Define $\pi_0^{CV}(\theta_i)$ solving

$$\hat{u}(-\overline{v}(\theta_i, \theta_i) + \pi_0^{CV}(\theta_i)\,;\,\theta_i, \theta_i) = 0, \tag{10}$$

which means, by (9), that $\pi_0^{CV}(\theta_i)$ compensates bidder θ_i for the common value risk conditional on $\theta_{-i}^{(1)} = \theta_i$ at zero surplus:

$$\mathsf{E}[u(v(\theta_i, \{\theta_j\}_{j \neq i}) - \overline{v}(\theta_i, \theta_i) + \pi_0^{CV}(\theta_i)) \mid \theta_i,\, \theta_{-i}^{(1)} = \theta_i] = 0.$$

We claim that a symmetric increasing equilibrium exists, and it consists of bid functions

$$b(\theta_i) = \overline{v}(\theta_i, \theta_i) - \pi_0^{CV}(\theta_i). \tag{11}$$

It is easy to check that $b(\theta_i)$ is strictly increasing. To establish that it is best response to $\{b(\theta_j)\}_{j \neq i}$, suppose, towards contradiction, that i bids $b > \overline{v}(\theta_i, \theta_i) - \pi_0^{CV}(\theta_i)$ instead of (11). This makes a difference only if, for $\theta_{-i}^{(1)} = y$, $b > b(y) > \overline{v}(\theta_i, \theta_i) - \pi_0^{CV}(\theta_i)$. Then i will receive, instead of 0,

$$\begin{aligned}
\mathsf{E}[u(v(\theta_i, \{\theta_j\}_{j \neq i}) - \overline{v}(y, y) + \pi_0^{CV}(y)) \mid \theta_i,\, \theta_{-i}^{(1)} = y] &< \\
\mathsf{E}[u(v(y, \{\theta_j\}_{j \neq i}) - \overline{v}(y, y) + \pi_0^{CV}(y)) \mid \theta_i = \theta_{-i}^{(1)} = y] &= 0,
\end{aligned}$$

Hence bidding $b > \overline{v}(\theta_i, \theta_i) - \pi_0^{CV}(\theta_i)$ is not a good idea. An analogous argument works against bidding $b < \overline{v}(\theta_i, \theta_i) - \pi_0^{CV}(\theta_i)$. Therefore (11) is indeed an increasing equilibrium.[21]

If valuations are risky then in the symmetric equilibrium buyer θ_i bids

$$\beta(\theta_i) = \overline{v}(\theta_i, \theta_i) - \pi_0^{CV}(\theta_i) - \pi_0(\theta_i), \tag{12}$$

where $\pi_0(\theta_i)$ solves

$$\mathsf{E}_z \hat{u}(-\overline{v}(\theta_i, \theta_i) + \pi_0^{CV}(\theta_i) + \tilde{z}_i + \pi_0(\theta_i)\,;\,\theta_i, \theta_i) = 0. \tag{13}$$

That is, type θ_i further reduces his bid by the compensating risk premium for $\tilde{z}_i$ at the risky initial wealth (risky due to the common value risk) that

[21] With risk neutrality the equilibrium bid is $\overline{v}(\theta_i, \theta_i)$. We have shown that in the case of DARA bidders it is reduced by the risk premium $\pi_0^{CV}(\theta_i)$, which compensates the buyer for the risk of the others' signals at zero expected surplus.

gives him zero surplus. The derivation is identical to that of the equilibrium under deterministic valuations, (11), and therefore is omitted.

For all $y \leq x$, by property 2 of (9),

$$\hat{u}(-\overline{v}(y,y) + \pi_0^{CV}(y)\,;\,x,y) \leq \mathsf{E}_z\hat{u}(-\overline{v}(y,y) + \pi_0^{CV}(y) + \tilde{z}_i + \pi_0(y)\,;\,x,y) \tag{14}$$

because $\hat{u}(w\,;\,y,y) = \mathsf{E}_z\hat{u}(w + \tilde{z}_i + \pi_0(y)\,;y,y)$ at $w = -\overline{v}(y,y) + \pi_0^{CV}(y)$ by definition (compare (10) and (13)). But (14) is equivalent to

$$\begin{aligned} & \mathsf{E}[u(v(\theta_i,\{\theta_j\}_{j\neq i}) - \overline{v}(\theta_{-i}^{(1)},\theta_{-i}^{(1)}) + \pi_0^{CV}(\theta_{-i}^{(1)})) \mid \theta_i,\, \theta_{-i}^{(1)}] \\ \leq\; & \mathsf{E}[u(v(\theta_i,\{\theta_j\}_{j\neq i}) - \overline{v}(\theta_{-i}^{(1)},\theta_{-i}^{(1)}) + \pi_0^{CV}(\theta_{-i}^{(1)}) + \tilde{z}_i + \pi_0(\theta_{-i}^{(1)})) \mid \theta_i,\, \theta_{-i}^{(1)}], \end{aligned}$$

for all $\theta_{-i}^{(1)} \leq \theta_i$, and therefore

$$\begin{aligned} & \mathsf{E}[u(v(\theta_i,\{\theta_j\}_{j\neq i}) - \overline{v}(\theta_{-i}^{(1)},\theta_{-i}^{(1)}) + \pi_0^{CV}(\theta_{-i}^{(1)})) \mid \theta_i,\, \theta_{-i}^{(1)} \leq \theta_i] \\ \leq\; & \mathsf{E}[u(v(\theta_i,\{\theta_j\}_{j\neq i}) - \overline{v}(\theta_{-i}^{(1)},\theta_{-i}^{(1)}) + \pi_0^{CV}(\theta_{-i}^{(1)}) + \tilde{z}_i + \pi_0(\theta_{-i}^{(1)})) \mid \theta_i,\, \theta_{-i}^{(1)} \leq \theta_i]. \end{aligned}$$

By definition, this means that buyer θ_i is better off with noise $\tilde{z}_i$ and bidding $\beta(\theta_i)$, as compared to having no noise but bidding $b(\theta_i)$.

We now turn to the proof in the case of the English auction. In the efficient symmetric equilibrium with deterministic values, buyer i plans to quit at $v(\theta_i,\theta_{-i})$, such that for all active $j \neq i$, $\theta_j = \theta_i$, and for all inactive $j \neq i$, θ_j equals j's true type. By strict monotonicity of v in θ_i, lower types plan to quit earlier. When a buyer drops out, the other buyers infer his type and repeat the above calculation until only one buyer remains active. The winner will therefore pay $v(x,\theta_{-i})$ at $x = \theta_{-i}^{(1)}$, which is (weakly) less than his actual valuation because $\theta_i \geq \theta_{-i}^{(1)}$.

With noisy valuations, buyer i plans to quit at $v(\theta_i,\theta_{-i}) - \pi_0$ such that for all active $j \neq i$, $\theta_j = \theta_i$, for all inactive $j \neq i$, θ_j equals j's true type, and π_0 solves $u(0) = \mathsf{E}u(\tilde{z}_i + \pi_0)$. The winner pays $v(x,\theta_{-i}) - \pi_0$ at $x = \theta_{-i}^{(1)}$. Since the deterministic part of his surplus is non-negative, the compensating risk premium for noise $\tilde{z}_i$ is less than π_0, and he ends up being better off. ■

The following is the counterpart of Proposition 3.

Proposition 7 *Consider the general symmetric affiliated model with decreasingly risk-averse buyers. The buyers are better off with noisy rather than deterministic valuations in the symmetric equilibrium of the first-price auction.*

Proof. Let the equilibrium bid function under deterministic and noisy values be $b(\theta_i)$ and $\beta(\theta_i)$, respectively. Under deterministic values, in equilibrium, the utility of θ_i from pretending $\hat{\theta}_i$ is

$$U^{FPA}(\hat{\theta}_i,\theta_i) = \int_{\underline{\theta}}^{\hat{\theta}_i} \hat{u}(-b(\hat{\theta}_i)\,;\,\theta_i,\theta_{-i}^{(1)})\, f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)},$$

where $f(\theta_{-i}^{(1)}\,|\,\theta_i)$ is the probability density function of $\theta_{-i}^{(1)}$ conditional on θ_i. Let $V(\theta_i) \equiv U^{FPA}(\theta_i,\theta_i)$. By the envelope theorem, for all $\theta_i \in (\underline{\theta},\overline{\theta})$,

$$\begin{aligned} V'(\theta_i) &= \int_{\underline{\theta}}^{\theta_i} \tfrac{\partial}{\partial\theta_i}\hat{u}(-b(\theta_i);\theta_i,\theta_{-i}^{(1)})\, f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)} \\ &\quad + \int_{\underline{\theta}}^{\theta_i} \hat{u}(-b(\theta_i);\theta_i,\theta_{-i}^{(1)})\, \tfrac{\partial}{\partial\theta_i} f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)}. \end{aligned} \tag{15}$$

Similarly, if $\tilde{V}(\theta_i)$ denotes θ_i's indirect expected utility under noisy values in the equilibrium of the FPA, we have for all $\theta_i \in (\underline{\theta},\overline{\theta})$,

$$\begin{aligned} \tilde{V}'(\theta_i) &= \int_{\underline{\theta}}^{\theta_i} \tfrac{\partial}{\partial\theta_i} \mathsf{E}_z\hat{u}(-\beta(\theta_i)+\tilde{z}_i\,;\theta_i,\theta_{-i}^{(1)})\, f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)} \\ &\quad + \int_{\underline{\theta}}^{\theta_i} \mathsf{E}_z\hat{u}(-\beta(\theta_i)+\tilde{z}_i\,;\theta_i,\theta_{-i}^{(1)})\, \tfrac{\partial}{\partial\theta_i} f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)}. \end{aligned} \tag{16}$$

If $V(\theta_i) = \tilde{V}(\theta_i)$ for some $\theta_i \in (\underline{\theta},\overline{\theta})$ then

$$\int_{\underline{\theta}}^{\theta_i} \left[\mathsf{E}_z\hat{u}(-\beta(\theta_i)+\tilde{z}_i\,;\theta_i,\theta_{-i}^{(1)}) - \hat{u}(-b(\theta_i)\,;\theta_i,\theta_{-i}^{(1)}) \right] f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)} = 0. \tag{17}$$

The expression in brackets is weakly increasing in $\theta_{-i}^{(1)}$ whenever it is non-positive by property 3 of $\hat{u}$, while its expected value is 0; therefore the integrand switches sign only once, from negative to positive (it is quasi-monotonic). By affiliation, $\frac{\partial}{\partial\theta_i} f(\theta_{-i}^{(1)}\,|\,\theta_i)/f(\theta_{-i}^{(1)}|\,\theta_i)$ is increasing in $\theta_{-i}^{(1)}$. Therefore the product of the integrand in (17) and $\frac{\partial}{\partial\theta_i} f(\theta_{-i}^{(1)}\,|\,\theta_i)/f(\theta_{-i}^{(1)}|\,\theta_i)$ has a non-negative integral (an easy proof can be given along the lines of Lemma 1, Persico [2000]). That is,

$$\int_{\underline{\theta}}^{\theta_i} \left[\mathsf{E}_z\hat{u}(-\beta(\theta_i)+\tilde{z}_i\,;\theta_i,\theta_{-i}^{(1)}) - \hat{u}(-b(\theta_i)\,;\theta_i,\theta_{-i}^{(1)}) \right] \tfrac{\partial}{\partial\theta_i} f(\theta_{-i}^{(1)}\,|\,\theta_i)d\theta_{-i}^{(1)} \geq 0,$$

and so the second term in (16) is not less than that in (15).

By the DARA property, which is preserved under integration,

$$\begin{aligned} & \int_{\theta_{-i}\in[\underline{\theta},\theta_i]^{n-1}} \hat{u}'(-b(\theta_i)\,;\theta_i,\theta_{-i}^{(1)})\, f(\theta_{-i}^{(1)}\mid\theta_i)d\theta_{-i}^{(1)} \\ < & \int_{\theta_{-i}\in[\underline{\theta},\theta_i]^{n-1}} \mathsf{E}_z\hat{u}'(-\beta(\theta_i)+\tilde{z}_i\,;\theta_i,\theta_{-i}^{(1)})\, f(\theta_{-i}^{(1)}\mid\theta_i)d\theta_{-i}^{(1)}, \end{aligned} \tag{18}$$

and therefore $V'(\theta_i) < \tilde{V}'(\theta_i)$. Since $V(\underline{\theta}) = \tilde{V}(\underline{\theta}) = 0$, and, as we have just shown, $V(\theta_i) = \tilde{V}(\theta_i)$ implies $V'(\theta_i) < \tilde{V}'(\theta_i)$ for all $\theta_i \in (\underline{\theta},\overline{\theta})$, we must have $V(\theta_i) < \tilde{V}(\theta_i)$ for all $\theta_i \in (\underline{\theta},\overline{\theta}]$. This proves the claim. ■

We also note that a counterpart to Proposition 4 (precautionary bidding in the all-pay auction) can be proven under the assumptions of interdependent (common) values and *independent signals*.

4 When the winner's curse is a blessing

Up until now we have shown how an exogenous mean-zero risk causes decreasingly risk-averse bidders to engage in precautionary bidding. However, it is easy to see that the effect is more general than this, and that other forms of risk are likely to cause the same sort of behavior. In particular, in common-value contexts, the object is risky for the buyer because when he wins he does not necessarily know the signals of the other buyers, and these affect his valuation.

In this section we show that precautionary bidding effect arises in this set-up when valuations are stochastic **only** because of these common-value components. This is novel because the behavior of risk-averse bidders in common-value auctions is not well understood as such models are not readily tractable. We also show that it is possible for decreasingly risk-averse buyers to be better off when they face a winner's curse than when they are in an equivalent private values auction because they "underbid" due to precautionary bidding. This means that in equilibrium, the winner's curse is often not in fact a curse but a blessing for DARA bidders, something which cannot occur with risk-neutral bidders.

This is a relevant insight for theory because it can help us better understand the behavior of risk-averse bidders in common value settings. But it is also of practical importance, because in some settings bidders may be able to choose between entering auctions where they will face a significant winner's curse and those where they will not. In some cases this may be a straight choice about what type of object to buy: other things being equal, should I bid for a painting by a known artist for which the estimate of the resale value (common to all bidders) is likely to be important, or one by a lesser known artist which is valued merely for its aesthetic appeal? Conventional economic wisdom suggests that one should be wary of the former option because of the significant "winner's curse;" our results suggest that if bidders are DARA, this is not true.

In other cases, whether the auction of a given object is largely a private or common values affair may be determined by prior moves taken by the bidders. For example, consider two firms which will later compete in procurement auctions. If – prior to the auctions – these two firms choose similar production technologies, then the subsequent auctions will have a strong common-value component: one firm's estimate of the likely cost of fulfilling the contract is likely to be important information for the other firm. But if the two firms choose two very different technologies to one another, then the first firm's likely cost of production may not be relevant at all to the second firm's costs, and vice versa: then the auctions will take place in a private values setting. The results of this section suggest that if the firms are DARA,[22] they may be better off choosing technologies which are 'too correlated' (from the seller's and perhaps the social point of view) in order to benefit from the softened bidding which the winner's curse risk generates. Similar remarks could apply to the choice of customer base by car, art, wine and antique retailers who buy their product in wholesale auctions: ceteris paribus if they choose to serve customers with similar tastes, the common-value risk will be larger, because when bidding in auctions they will all be interested in estimating the **same** properties of the objects for sale. This reduction in competition effect would counteract the bidders' desire to differentiate themselves to avoid excessive competition in the retail market.

In the next subsection, we demonstrate using an example which permits

[22] For a model of why firms in imperfect capital markets will tend to display decreasing absolute risk aversion, see Froot and Stein (1998), and Froot, Scharfstein and Stein (1993). The basic insight is that the firm will appear risk-averse to variations in cash flow if each unit of external funds is increasingly costly to raise, because variance of internal cash flow raises expected external financing costs for given a investment plan. But the firm will be less averse to a given risk in cash flows if it begins with more internal capital ('wealth'), because it is unlikely to have to raise very large amounts of external finance: it can use its internal capital to fund its investments even if cash flows are low. This very naturally yields behaviour which is equivalent to decreasing absolute risk aversion.

an analytic solution that the precautionary effect (arising from the noise due to other buyers' signals) may overcompensate the winner's curse and so the buyers may end up better off in a common values auction compared to a private values situation. In subsection 4.2 we provide a more formal demonstration that it is decreasing absolute risk aversion which is driving the result.

4.1 A numerical example

Suppose that there are two potential buyers for a given object. The buyers' private information, θ_i for $i = 1, 2$, is independently and uniformly distributed on [0,1]. Assume that the buyers have CRRA(ρ) preferences, that is, they evaluate a stochastic monetary payoff, x, according to $\mathsf{E}\left[(x)^{1-\rho}/(1-\rho)\right]$, where $0 < \rho < 1$.

We will consider a first price auction (FPA) and compare the behavior of the buyers when their monetary valuations for the object are: (i) private, $v_i \equiv \theta_i$, and: (ii) pure common, $v_i = (\theta_i + \theta_{-i})/2$. We will explicitly calculate, for each case separately, the symmetric equilibrium bid functions and the buyers' expected utility from participating in the auction. We will show that for high values of ρ ($\rho > \overline{\rho} \approx 0.745$) the buyers are better off in the common values auction than in the private values auction.

There are two conflicting forces driving this result: the reduction in informational rent and precautionary bidding. Suppose that we move from the private values setup to the common values setup. On the one hand, the value of each bidder's private information becomes smaller. This increases competition and thus increases bids - and it is the only effect which arises for risk-neutral bidders. On the other hand, introducing common values also makes the value of the object riskier. Then DARA bidders will cut their bids by more than the appropriate risk premium – this is the additional precautionary bidding effect. We show that for high values of ρ (i.e. more steeply decreasing risk aversion) the precautionary effect dominates the informational rent effect.

(i) Private values. Denote the symmetric increasing equilibrium bid function by $b(\theta_i)$.[23]

[23] In the setups which we adopt for private and common values, it is well known that the symmetric equilibrium bid functions exist and are strictly increasing.

By bidding $\hat{b}_i = b(\hat{\theta}_i)$ against the other player's equilibrium strategy, buyer i with type θ_i gets an expected utility of

$$\Pr(\theta_{-i} < \hat{\theta}_i)\, u(\theta_i - b(\hat{\theta}_i)) = \hat{\theta}_i(\theta_i - b(\hat{\theta}_i))^{1-\rho}/(1-\rho).$$

In equilibrium the maximum is obtained at $\hat{\theta}_i = \theta_i$. The first order condition and $b(0) = 0$ yields the following differential equation for $b(\theta_i)$,

$$b'(\theta_i) = \frac{\theta_i - b(\theta_i)}{(1-\rho)\theta_i}, \quad b(0) = 0.$$

It is easy to check that the solution is $b(\theta_i) = \theta_i/(2-\rho)$. Note that as $\rho \to 0$, risk neutrality, the bid function becomes $B(\theta_i) = \frac{1}{2}\theta_i$.

Buyer i's utility can be calculated as

$$U^{PV}(\theta_i) \equiv \Pr(\theta_{-i} < \theta_i)\, u(\theta_i - b(\theta_i)) = \left(\frac{1-\rho}{2-\rho}\right)^{1-\rho} \frac{1}{1-\rho}\theta_i^{2-\rho}. \qquad (19)$$

(ii) Pure common values. Now assume that buyer i's valuation for the object is $(\theta_i + \theta_{-i})/2$. Let the symmetric increasing equilibrium bid function be $c(\theta_i)$.

The equilibrium condition is $\Pr(\theta_{-i} < \hat{\theta}_i)\, \mathrm{E}[u((\theta_i + \theta_{-i})/2 - c(\hat{\theta}_i)) \,|\, \theta_{-i} < \theta_i]$ maximized at $\hat{\theta}_i = \theta_i$. This, coupled with $c(0) = 0$, yields the following differential equation for $c(\theta_i)$:

$$c'(\theta_i) = \frac{(\theta_i - c(\theta_i))^{1-\rho}/(1-\rho)}{\int_0^{\theta_i} [(\theta_i + x)/2 - c(\theta_i)]^{-\rho}\, dx}, \quad c(0) = 0.$$

It is easy to check that the solution is $c(\theta_i) = \frac{1}{2}\theta_i$. Note that this result holds for all ρ between 0 and 1, and, in the limit, it holds for risk neutrality: $C(\theta_i) = \frac{1}{2}\theta_i$.

Buyer i's utility in equilibrium is

$$\begin{aligned} U^{CV}(\theta_i) &\equiv \Pr(\theta_{-i} < \theta_i)\ \mathrm{E}[u((\theta_i + \theta_{-i})/2 - c(\hat{\theta}_i)) \,|\, \theta_{-i} < \theta_i] \\ &= \frac{1}{2-\rho}\left(\frac{1}{2}\right)^{1-\rho} \frac{1}{1-\rho}\theta_i^{2-\rho} \end{aligned} \qquad (20)$$

Comparing equations (19) and (20) we find that $U^{PV} < U^{CV}$ if and only if

$$\left(\frac{1-\rho}{2-\rho}\right)^{1-\rho} < \frac{1}{2-\rho}\left(\frac{1}{2}\right)^{1-\rho}.$$

This is true for $\rho > \overline{\rho} \approx 0.745$.

Notice that in this particular example the total surplus available to be shared between the seller and the buyers is smaller in the common values case than in the private values case (i.e. the expected value of the object to the highest bidder is smaller). It is therefore surprising that the bidders can be better off with common values than with private values. Note also that in the limiting risk neutral case, $\rho \to 0$, the bid function is the same under private and common values, $B(\theta_i) = C(\theta_i)$. So the winner's curse is borne entirely by the buyers. It is generally true that risk-neutral buyers must bear at least part of the reduced surplus due to common values. This is because there is a reduction in the informational rent available to bidders when moving from private to common values whilst maintaining the same signal distribution. (In this example, each buyer's signal determines only half as much of his value as before and the other half is competed away by bidders.) Despite this reduction in informational rent, DARA bidders can be better off with common values if their precautionary effect is large enough, i.e. if risk aversion decreases quickly enough. This suggests that a natural next step is to find some way of renormalizing the way in which values are composed from signals in order to better compare like with like. This is the task which we undertake in the next subsection.

4.2 Comparison of common and private values

In this subsection we will demonstrate that the result of the previous numerical example was due to the DARA property displayed by the CRRA bidders. We show that decreasingly risk-averse buyers are better off in common values auctions than in "comparable" private values auctions.

In order to simplify the analysis, in this subsection we will assume that the signals are *independent*. That is, θ_i, $1 \leq i \leq n$, are drawn iid from $[\underline{\theta}, \overline{\theta}]$ according to some distribution F (with positive density f). The advantage of assuming independence is that the revenue equivalence theorem applies for risk-neutral bidders, which will greatly facilitate the task of finding comparable common and private value auctions.

Suppose that in a common values auction buyer i's valuation is $v(\theta_i, \theta_{-i})$. The first task is to decide which private values setup is comparable to this common values model. In order to isolate the precautionary effect from the reduction in informational rent discussed in section 4.1 above, the appropriate comparison seems to be between two settings that yield the *same* *informational rent* to the bidders. This means that the appropriate private-value normalization is an auction where buyer i's valuation is $t(\theta_i)$ such that a risk-neutral buyer with signal θ_i is indifferent between this private values auction and the original common values auction. We call such a private values model the *risk-neutral private-value equivalent* of the common values auction. We aim to show that whenever a risk-neutral buyer is interim indifferent between two auctions - one of them common values and the other private values - then DARA bidders will strictly prefer to be in the common values setting (and CARA bidders will be indifferent).

For a common value $v(\theta_i, \theta_{-i})$ we can find the corresponding private value $t(\theta_i)$ in the following way. Consider an efficient direct revelation mechanism that leaves 0 surplus with the lowest type under both common and private values.[24] Denote the utility of type θ_i in the private values auction by $V(\theta_i)$. By the envelope theorem,

$$V'(\theta_i) = F(\theta_i)^{n-1}t'(\theta_i), \quad V(\underline{\theta}) = 0.$$

Similarly, by letting $W(\theta_i)$ stand for the utility of type θ_i in this mechanism under common values, we have

$$W'(\theta_i) = F(\theta_i)^{n-1}\mathsf{E}[v'_{\theta_i}(\theta_i, \theta_{-i}) \mid \theta_{-i}^{(1)} \leq \theta_i], \quad W(\underline{\theta}) = 0,$$

where $v'_{\theta_i}(\theta_i, \theta_{-i}) \equiv \frac{\partial}{\partial \theta_i} v(\theta_i, \theta_{-i})$. In order to make a risk-neutral bidder indifferent between the two auctions (the one where his valuation is $v(\theta_i, \theta_{-i})$ and the one where it is $t(\theta_i)$) we must choose $t(\theta_i)$ so that $V(\theta_i) \equiv W(\theta_i)$, that is,

$$t'(\theta_i) \equiv \mathsf{E}[v'_{\theta_i}(\theta_i, \theta_{-i}) \mid \theta_{-i}^{(1)} \leq \theta_i], \quad t(\underline{\theta}) = \underline{\theta}. \tag{21}$$

A particularly tractable example for common values is where the effect of one's own signal and the effects of others' signals are additively separable

[24] Both the FPA and the SPA are examples of mechanisms with this property – therefore, we just consider their direct revelation counterpart. This is the advantage of the independence assumption in this context. Since the revenue equivalence theorem applies, the private value which makes a risk-neutral bidder indifferent between a common and private values first price auction will be the same as that which makes him indifferent with a second price auction. Without independence, the risk-neutral equivalents for the two auctions would have to be separately derived and may not be equal, further complicating the analysis.

(a special case of which is the linear model). Additive separability requires $v(\theta_i, \theta_{-i}) \equiv g(\theta_i) + h(\theta_{-i})$, or, after a suitable normalization of the signals,

$$v(\theta_i, \theta_{-i}) = \theta_i + w(\theta_{-i}). \tag{22}$$

It is easy to check that with separable signal effects under common values we get a risk-neutral equivalent private value $t(\theta_i) = \theta_i$. The additively separable formulation is particularly revealing because it highlights the idea (discussed above) that bidders obtain rents from their own signals only. Although the expected value of the object is larger in the common values case, the bidders compete away all the surplus associated with the additional value of the object which comes from positive realizations of the other bidders' signals.

It is interesting to note that if the selling mechanism is the English (button-) auction then buyers are indifferent between the additively separable common-value auction and its private-value equivalent, no matter what their risk preferences (maintaining the assumption of symmetry).[25] The intuition for this is simple: when a bidder wins a common value English (button-) auction, he knows the value of the object to him exactly because he has seen the points at which each of his opponents has dropped out. Thus bidders face no additional risk in the English common values auction relative to the private values case, and there is no precautionary bidding effect there. This fact further justifies the choice of the private-value equivalent.

In the following two propositions we establish that decreasingly risk-averse buyers are better off in a common values auction relative to the comparable private values auction when the mechanism is either a first or a second price auction. We prove this result under the additive separability assumption (22) in the case of the SPA, and under slightly more general circumstances in the case of the FPA.

Proposition 8 *Assume that in the iid signals common-value setup the valuation function satisfies (22), additive separability. Decreasingly risk-averse buyers are better off in the common values auction relative to the risk-neutral*

[25] When only two buyers remain in the English auction, say θ_i and θ_j, they both know all the other buyers' signals (the true $\hat{\theta}_k$s, $k \neq i, j$) from the drop-out prices. In the common values case they build this information into their valuations and bid $\theta_i + w(\theta_i, \hat{\theta}_{-ij})$ and $\theta_j + w(\theta_j, \hat{\theta}_{-ij})$, respectively. Suppose the winner is i, then his utility is $u(\theta_i + w(\theta_j, \hat{\theta}_{-ij}) - \theta_j - w(\theta_j, \hat{\theta}_{-ij})) = u(\theta_i - \theta_j)$. In the private values case if θ_i wins then he pays θ_j yielding a utility of $u(\theta_i - \theta_j)$. Therefore winner i's utility is the same in the common and private values setups for all realizations of θ_j.

equivalent private values auction when the auction mechanism is a second-price auction.

Proof. Define $\bar{v}(x,y)$ and $\bar{w}(y)$ in the following way:

$$\begin{aligned}
\bar{v}(x,y) &\equiv \mathsf{E}[v(\theta_i, \{\theta_j\}_{j\neq i}) \,|\, \theta_i = x,\, \theta_{-i}^{(1)} = y] \\
&= x + \mathsf{E}[w(\{\theta_j\}_{j\neq i}) \mid \theta_{-i}^{(1)} = y] \equiv x + \bar{w}(y).
\end{aligned}$$

Define $\pi^{CV}(x)$, solving

$$\begin{aligned}
\mathsf{E}[u(v(x, \{\theta_j\}_{j\neq i}) - \bar{v}(x,x) + \pi^{CV}(x)) \mid \theta_{-i}^{(1)} = x] &= \\
\mathsf{E}[u(w(\{\theta_j\}_{j\neq i}) - \bar{w}(x) + \pi^{CV}(x)) \mid \theta_{-i}^{(1)} = x] &= 0.
\end{aligned} \tag{23}$$

That is, $\pi^{CV}(x)$ compensates the expected utility for the common-value noise $w(\{\theta_j, \forall j \neq i \,|\, \theta_{-i}^{(1)} = x\}) - \bar{w}(x)$ at the initial wealth level.

We know from the proof of Proposition 6 that a symmetric increasing equilibrium exists, and it consists of bid functions

$$c(\theta_i) = \bar{v}(\theta_i, \theta_i) - \pi^{CV}(\theta_i).$$

Therefore type θ_i's expected utility is,

$$W(\theta_i) = \mathsf{E}[u(v(\theta_i, \{\theta_j\}_{j\neq i}) - \bar{v}(\theta_{-i}^{(1)}, \theta_{-i}^{(1)}) + \pi^{CV}(\theta_{-i}^{(1)})) \mid \theta_{-i}^{(1)} \leq \theta_i,\, \theta_i].$$

Rewrite this as

$$\begin{aligned}
W(\theta_i) &= \mathsf{E}[u(\theta_i - \theta_{-i}^{(1)} + w(\{\theta_j\}_{j\neq i}) - \bar{w}(\theta_{-i}^{(1)}) + \pi^{CV}(\theta_{-i}^{(1)})) \mid \theta_{-i}^{(1)} \leq \theta_i,\, \theta_i] \\
&> \mathsf{E}[u(\theta_i - \theta_{-i}^{(1)}) \mid \theta_{-i}^{(1)} \leq \theta_i,\, \theta_i],
\end{aligned} \tag{24}$$

where the inequality follows from (23), $\theta_i - \theta_{-i}^{(1)} > 0$ almost everywhere, and the DARA property.

On the other hand, in the risk-neutral equivalent private values auction, $t(\theta_i) = \theta_i$, and buyers bid their valuations. Therefore the expected utility of type θ_i from the auction is,

$$V(\theta_i) = \mathsf{E}[u(\theta_i - \theta_{-i}^{(1)}) \mid \theta_{-i}^{(1)} \leq \theta_i,\, \theta_i].$$

This, together with (24), implies $W(\theta_i) > V(\theta_i)$. ■

Proposition 9 *Assume that in the iid signals common-value setup the valuation function has the property that $v'_{\theta_i}(\theta_i, \theta_{-i})$ is non-increasing in all θ_j, $j \neq i$.*[26] *Decreasingly risk-averse buyers are better off in the common values auction relative to the risk-neutral equivalent private values auction when the auction mechanism is a first-price auction.*

Proof. The utility of type θ_i pretending $\hat{\theta}_i$ in the equilibrium of the FPA under common and (comparable) private values is

$$\begin{aligned} U^{CV}(\hat{\theta}_i, \theta_i) &= F(\hat{\theta}_i)^{n-1} \mathsf{E}[u(v(\theta_i, \theta_{-i}) - c(\hat{\theta}_i)) \mid \theta_{-i}^{(1)} \leq \hat{\theta}_i], \\ U^{PV}(\hat{\theta}_i, \theta_i) &= F(\hat{\theta}_i)^{n-1}\, u(t(\theta_i) - b(\hat{\theta}_i)), \end{aligned}$$

respectively, where $c(.)$ and $b(.)$ are the equilibrium bid functions in the two cases, respectively. Denoting $V(\theta_i) = U^{PV}(\theta_i, \theta_i)$ and $W(\theta_i) = U^{CV}(\theta_i, \theta_i)$, by the envelope theorem, for all $\theta_i \in (\underline{\theta}, \overline{\theta})$,

$$W'(\theta_i) = F(\theta_i)^{n-1} \mathsf{E}[\, v'_{\theta_i}(\theta_i, \theta_{-i})\, u'(v(\theta_i, \theta_{-i}) - c(\theta_i)) \mid \theta_{-i}^{(1)} \leq \theta_i], \quad (25)$$

$$V'(\theta_i) = F(\theta_i)^{n-1} t'(\theta_i) u'(t(\theta_i) - b(\theta_i)). \quad (26)$$

Whenever $V(\theta_i) = W(\theta_i)$ for some $\theta_i \in (\underline{\theta}, \overline{\theta})$, that is,

$$u(t(\theta_i) - b(\theta_i)) = \mathsf{E}[u(v(\theta_i, \theta_{-i}) - c(\theta_i)) \mid \theta_{-i}^{(1)} \leq \theta_i],$$

by the DARA property,

$$u'(t(\theta_i) - b(\theta_i)) < \mathsf{E}[\, u'(v(\theta_i, \theta_{-i}) - c(\theta_i)) \mid \theta_{-i}^{(1)} \leq \theta_i]. \quad (27)$$

Therefore, starting from (26) and then using (27), then (21), we get

$$\begin{aligned} F(\theta_i)^{-(n-1)}\, V'(\theta_i) &< t'(\theta_i)\, \mathsf{E}[\, u'(v(\theta_i, \theta_{-i}) - c(\theta_i)) \mid \theta_{-i}^{(1)} \leq \theta_i] \\ &= \mathsf{E}[v'_{\theta_i}(\theta_i, \theta_{-i}) \mid \theta_{-i}^{(1)} \leq \theta_i]\, \mathsf{E}[\, u'(v(\theta_i, \theta_{-i}) - c(\theta_i)) \mid \theta_{-i}^{(1)} \leq \theta_i] \\ &\leq \mathsf{E}[\, v'_{\theta_i}(\theta_i, \theta_{-i})\, u'(v(\theta_i, \theta_{-i}) - c(\theta_i)) \mid \theta_{-i}^{(1)} \leq \theta_i] \\ &= F(\theta_i)^{-(n-1)}\, W'(\theta_i), \end{aligned}$$

The second to last line follows because $v'_{\theta_i}(\theta_i, \theta_{-i})$ and $u'(v(\theta_i, \theta_{-i}) - c(\theta_i))$ are non-increasing in θ_{-i} (and therefore non-negatively covary in θ_{-i}); and the last line follows from (25). This establishes that for $\theta_i \in (\underline{\theta}, \overline{\theta}]$, $V(\theta_i) = W(\theta_i)$ implies $V'(\theta_i) < W'(\theta_i)$.

Since $V(\underline{\theta}) = W(\underline{\theta})$, we find that $V(\theta_i) < W(\theta_i)$ for all $\theta_i \in (\underline{\theta}, \overline{\theta}]$, which proves the claim. ■

[26] In words: the effect of i's signal on his own valuation does not increase as others' signals become higher. Additive separability is a special case of this model.

5 Conclusion

We have shown that decreasingly risk-averse bidders are better off when the value of the object auctioned becomes more risky. The bidders behave less aggressively, reducing their bids by more than the amount of the risk premium. This is true in first-price and all-pay auctions with stochastic affiliated private values because risk increases the marginal utility of income, as in the precautionary saving literature; so the trade-off between raising one's bid to win more often, and reducing it to win with a larger surplus, is shifted in favor of the latter. The same result holds in first-price auctions with affiliated common values and in all-pay auctions with common values and independent signals. We have shown that in second-price and English auctions also, decreasingly risk-averse bidders will be better off when the object becomes more risky. This is true for both private and common values. The bidders become better off because their optimal strategy is to bid their value, less the risk premium of the noise calculated at the initial wealth level. That is, bidders calculate the risk premium assuming that they will pay their bid - but most of the time when they win their payment is strictly less than their bid, so they are better off.

Our result – the "precautionary bidding" effect – is not immediately obvious. Consider, for example, the first-price auction. Risk (in the value of the object) typically increases the risk aversion of decreasingly risk-averse bidders, and we know that competition in the FPA intensifies when the buyers become more risk-averse. Therefore one might imagine that increasing the object's riskiness would leave bidders with less surplus than before. However, the effect of risk on risk aversion turns out to be of second-order importance compared to the effect on marginal utility.

It is well-known that, with risk-averse bidders, revenue is higher in a first-price auction than a second-price auction. This remains true under stochastic private values, independently of the utility function. However, the advantage of the first-price auction over the second-price auction can be diminished when risk is added; decreasingly risk-averse bidders behave less aggressively than previously in both auctions, and the effect can be larger in the first-price auction than the second-price auction. Moreover, the model presented here calls into question the result that the seller's revenue will be higher in a first-price auction when the bidders are more risk-averse. It remains true that if the object is risky then the fear of losing the object will motivate more risk-averse bidders to bid more highly. But against this, they will bid less highly because they dislike the riskiness of the object, and even less highly because of the precautionary effect. So it is an open question under exactly which circumstances the seller will earn higher expected revenue from more risk-averse bidders.[27]

The analysis of auctions with risk-averse bidders and common values is not in general very tractable and for this reason has been very little studied. However, we have shown that our model of how risk-averse bidders responding to mean-zero white noise risk can be used to understand the way in which bidders will react to negative expected value winner's curse risk. In particular, suppose that bidders move from a (certain) private values setting to a (risky) common values one. Then the precautionary bidding effect will typically cause bidders to reduce their bids by more than the risk premium associated with the negative expected value risk. This means that if risk aversion decreases fast enough then risk-averse bidders will be better off bidding in a common-values situation than a private-values situation. In other words, the "winner's curse" turns out to be a benefit for bidders! This can never happen with risk-neutral bidders; they will always be worse off bidding in the common values case because there are fewer informational rents available for bidders in that case.

We also provided a simple and natural way of renormalizing the social surplus in auctions that leaves risk-neutral buyers (or any buyers in an English auction) indifferent between common and private value settings. We then showed that decreasingly absolute risk-averse buyers will always prefer the common value setting in this case with first- or second-price auctions, precisely because of the precautionary bidding effect. This is clearly relevant to bidders' choices about which auctions to attend, and it is not paradoxical that we see lots of entry by risk-averse bidders into auctions which would seem to contain a large common value element. Moreover, in situations where bidders take actions prior to bidding (for example, in procurement auctions, choosing whether to operate using the same or different technologies) the bidders may even be able to choose the extent of the winner's curse (common value) element in the auction. This is because the prior action may determine for each bidder the relevance of the information available to other bidders. The further analysis of such two-stage games is a potentially complex but rewarding proposition which we leave to future research.

[27] Numerical computations were performed corroborating these claims. The calculations are available from the authors.

References

[1] Asker, J. (2000) Bidding up, buying out and cooling-off: an examination of auctions with withdrawal rights, Economic Theory 16:585-611

[2] Athey, S. (2000), Monotone Comparative Statics under Uncertainty, mimeo. (at mit.edu/athey/www), forthcoming in the Quarterly Journal of Economics.

[3] Athey, S., and J. Levin (2001), Information and Competition in US Forest Service Timber Auctions, Journal of Political Economy 109:375-417

[4] Bajari, P., and A. Hortaçsu (2000), Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions, mimeo., Stanford University, presented at the World Congress of the Econometric Society 2000.

[5] Butters, G. (1975), Equilibrium Price Distributions and the Economics of Information, Ph.D. thesis, University of Chicago.

[6] Campo, S., I. Perrigne, and Q. Vuong (2000), Semi-Parametric Estimation of First Price Auctions with Risk-Averse Bidders, mimeo., University of Southern California; presented at the World Congress of the Econometric Society 2000.

[7] Drèze, J. and F. Modigliani (1972), Consumption Decisions under Uncertainty, Journal of Economic Theory 85:163-90.

[8] Eeckhoudt, L., C. Gollier, and H. Schlesinger (1996), Changes in Background Risk and Risk-Taking Behavior, Econometrica 64:683-689.

[9] Froot, K., and J. Stein (1998), Risk Management, Capital Budgeting, and Capital Structure Policy for Financial Institutions: an Integrated Approach, Journal of Financial Economics 47:55-82

[10] Froot, K., D. Scharfstein, and J. Stein (1993), Risk Management: Coordinating Corporate Investment and Financing Policies, Journal of Finance 48:1629-1658.

[11] Holt, C. (1980), Competitive Bidding for Contracts under Alternative Auction Procedures, Journal of Political Economy, 88:433-445.

[12] Kagel, J., (1995), Auctions: A Survey of Experimental Research, Chapter 7, Handbook of Experimental Economics, J. Kagel and A. Roth eds., Princeton University Press

[13] Kihlstrom, R., D. Romer, and S. Williams (1981), Risk Aversion with Random Initial Wealth, Econometrica 49:911-920.

[14] Kimball, M. (1990), Precautionary Saving in the Small and in the Large, Econometrica 58:53-73.

[15] Kimball, M. (1993), Standard Risk Aversion, Econometrica 61:589-611.

[16] Krishna, V., and J. Morgan (1997), An Analysis of the War of Attrition and the All-Pay Auction, Journal of Economic Theory, 72:343-362.

[17] Leland, H. (1968), Saving and Uncertainty: The Precautionary Demand for Saving, Quarterly Journal of Economics 82:465-73

[18] Levin, D., and J. Smith (1994), Equilibrium in Auctions with Entry, American Economic Review 84:585-599.

[19] Lucking-Reiley, D., D. Bryan, N. Prasad, and D. Reeves (1999): Pennies from eBay: the Determinants of Price in Online Auctions, mimeo., Vanderbilt University.

[20] Maskin, E., and J. Riley (1984), Optimal Auctions with Risk Averse Buyers, Econometrica 52:1473-1518.

[21] Maskin, E., and J. Riley (1996a), Existence of the Equilibrium in Sealed High Bid Auctions, mimeo., Harvard University.

[22] Maskin, E., and J. Riley (1996b), Uniqueness of the Equilibrium in Sealed High Bid Auctions, mimeo., Harvard University.

[23] Matthews, S., (1979), Risk Aversion and the Efficiency of First and Second Price Auctions, Working Paper, University of Illinois.

[24] Matthews, S., (1987), Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View, Econometrica 55:633-646.

[25] Milgrom, P., and R. Weber (1982), A Theory of Auctions and Competitive Bidding, Econometrica 50:1089-1122

[26] Myerson, R., (1981), Optimal Auction Design, Mathematics of Operations Research, 6: 58-73.

[27] New York Times, 1998 November 12, "Riding a Mouse on the Quest for Antiques"

[28] Paarsch, H. (1992), An Econometric Analysis of British Columbia Timber Auctions, Working Paper, University of British Columbia

[29] Persico, N. (2000), Information Acquisition in Auctions, Econometrica, 68:135-48.

[30] Pratt, J. W., and R. J. Zeckhauser (1987), Proper Risk Aversion, Econometrica 55:143-154.

[31] Riley, J., and W. Samuelson (1981), Optimal Auctions, American Economic Review, 71: 381-392.

[32] Roth, A. and A. Ockenfels (2000), Last Minute Bidding and the Rules for Ending Second-Price Auctions: Theory and Evidence from a Natural Experiment on the Internet, mimeo., Harvard University.

[33] Ross, S. A. (1981), Some Stronger Measures of Risk Aversion in the Small and in the Large with Applications, Econometrica 49:621-638.

[34] Sandmo, A. (1970), The Effect of Uncertainty on Saving Decisions, Review of Economic Studies 37:353-60.

[35] Wilson, R. (1977), A Bidding Model of Perfect Competition, Review of Economic Studies 4:511-518.