A Market Based Approach to Property Tax

Asher Wolinsky

Department of Economics

Northwestern University

Evanston, IL 60208

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Abstract

This paper discusses the possibility of basing the the administration of property taxes on market forces or even privatizing it. The basic idea is that owners will be free to determine the assessed valuations of their own properties, but these assessments will constitute an obligation to sell the property to whomever offers that price. With the aid of a simple formal model we discuss the manner in which such a method would induce owners to choose assessments that reflect the value. The obvious objection is that many owners would resent the risk of unintended sale of their house so much that it is impractical or even silly to consider this idea. In response we outline a softened version that would still offer some of the benefits but would reduce the pain considerably.

1   Introduction

This paper discusses the possibility of basing the administration of property taxes on market forces and perhaps even privatizing it. The basic idea is that property owners will be free to determine the assessed valuations of their own properties, but such an assessment will constitute an obligation to sell the property to whomever offers that price. The obvious objection to this idea is that it is impractical or even silly owing to the deep resentment with which many owners might view the risk of unintended sale of their house. While

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this objection is probably sufficiently substantial for dismissing this idea as a practical alternative, at least in its crudest form, it would still be helpful to explore it somewhat more systematically and to think of softened versions that might be less objectionable.

Property taxes usually take the form of a flat rate tax applied to the "assessed valuation" of the property. The assessed valuation is determined by a government bureaucracy that makes a direct effort to evaluate the properties by collecting information on their physical attributes and on current sales prices. I do not know whether governments define precisely the value notion that they try to estimate, but the general sense that comes across from looking at government publications is that they try to estimate the price that each property would yield if it were offered for sale in the current market. Owing to the heterogeneity of real estate properties and to the continuous changes in the market, the assessment process requires substantial resources and often results in a very noisy outcome. Casual examination of recent sales data even within a narrowly defined neighborhood (not to speak about the entire county or school districts which are the relevant areas) often reveals very substantial variation of the ratio of the actual price to the assessed valuation across houses.

From a purely economic perspective, the variation in tax burdens across similar properties may not matter much. One may think of this as a form of lump sum taxation. According to this view, properties are assigned different tax obligations somewhat randomly. These obligations are capitalized into the value of the house and the owners at the time of the assignment gain or lose some fraction of their wealth accordingly. However, this view is not entirely accurate. Since the assignment of the tax obligations is not purely random, it involves a non-trivial administrative effort and costly (both for the government and individuals) lobbying efforts[1]. In any case, whether or not this variation matters much from a pure economic perspective, it is clearly an acceptable principle that property taxes should reflect the value of properties in an equitable manner. Our discussion takes this goal as given and does not inquire about its economic rationale.

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[1]The rules of thumb used in reaching assessments may also affect the manner in which houses are constructed by creating incentives to substitute square footage from areas that are counted as more valuable in the computation of assessments into areas that are not. Such practice clearly exists but I think that it is mainly motivated by building regulations that place restrictions on the square footage of prime living area, rather than by tax considerations.

As mentioned above, the market based mechanism in its crudest form will let owners determine their own assessments, but these assessments will constitute an obligation to sell the property at that price. The implementation of such an arrangement will have to be accompanied by a detailed set of rules that will regulate the different aspects of the assessment process and the transactions that might result from it. For example, such rules will have to specify the frequency with which assessments can be updated, the time period within which houses can be bought after a new assessment was posted, the manner in which purchases and ownership transfers will take effect, the manner in which multiple purchase offers will be handled, etc. The advantage of this mechanism is that it might result in closer correspondence between the "true value" and the tax burden, might limit the bureaucratic arbitrary discretion and might reduce the public administration cost (although it might create some new private costs instead).

The obvious difficulty that cannot be eliminated even with judicious choice of rules is that homeowners might resent the uncertainty of being bought out by surprise. Owners may of course reduce the likelihood of such an event by posting a sufficiently high price that will deter prospective buyers. But it is conceivable that either by committing an error or by taking a calculated risk, owners who do not desire to sell might be forced to do so. The rules might help them by giving them sufficient time to vacate their home and by allowing them to negotiate with the prospective buyer, but they cannot entirely eliminate this potential hardship.

In response to this difficulty we may consider hybrid alternatives that would substantially reduce the hardship of surprise purchase. Such alternatives would still have a centralized assessment process for those who want it, but owners would be allowed instead to determine their own assessment which would then constitute an obligation to sell. Thus, while the hardship of an unwanted purchase would not be eliminated, it would be confined to those owners who choose to take this risk. One advantage of such methods is that the actual administration of the assessment process could be contracted out to profit seeking firms in a manner that maintains proper incentives (which seems much less feasible for the administration of the prevailing method). In fact, we may also think of a variation in which the firm that administers the assessment process is the one that has the right to buy the property a contesting owner. Of course, the implementation of such methods would have

to take care of some new incentive problems[2], but we will argue that they do not seem insurmountable.

The ideas discussed in this paper are extremely simple and perhaps do not require elaborate theoretical arguments. Nevertheless, we use a simple formal model to examine the relations that might emerge between self assessments and the underlying value parameters. the and explores some of its extensions to establish that, at least on some abstract level, this idea might work. An extension of this model also explores how assessments might be related to values in presence of asymmetric information about the condition of the house.

Finally, it is useful to reiterate that the ideas discussed here are very far from being a truly practical proposal. Even if these ideas have worthwhile practical implications, their implementation will require a great deal of care and much better understanding of the benefits and costs involved.

2   The Model

The values that are taxed by the property tax are determined in the context of the market for real estate. This is a rather complex market in which heterogenous and indivisible units are traded in processes that combine auctioning with bilateral negotiations. The task of providing anything that is close to a complete equilibrium model is daunting and will be much too ambitious for the purpose of expressing the simple idea that we are about to discuss. We therefore adopt a rather partial reduced form model that focuses on the decisions of individual owners, without an attempt to model the manner in which values are determined endogenously in equilibrium. Towards the end of the paper we discuss in some detail the manner in which this reduced form model might be embedded into a more fundamental equilibrium model.

The benefit of owning a home consists of a common value component v and a private value component u, both measured in monetary terms. The gross benefit accruing to an owner of a home with common and private components v and u respectively is $v + u$. Under the usual property tax systems the house is assigned an assessed valuation p which is then taxed at a flat rate t. The net benefit accruing to the homeowner is then $v + u - tp$. We

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[2]For example, how to keep the authority that adminesters the assessments from understatintg the value of all houses in a manner that empties the owners' right to determine their own assesment.

may think of the component v as representing those attributes over which there is broad agreement such as size, location and the state of repair as well as attributes of the community such as the quality of services. The component u reflects elements that are specific to an individual owner such as emotional attachment, idiosyncratic individual tastes or costs associated with moving to another house.

Consider now the scheme whereby the owner is free to select the assessed valuation p, but is obligated to sell the house for that price. Such a mechanism could take many different forms with respect to such details as the frequency with which owners can update their assessment, the window of time within which prospective purchase offers can be made and the manner in which multiple offers might be dealt with. Since our purpose here is just to consider the general idea, it will be useful to abstract from these details by focusing on a very simple model.

There is an owner with private valuation component u_O. The owner announces a price/assessed-valuation p. Then a potential buyer with private valuation component u_B appears, observes p and decides whether to purchase the house. If no purchase is made, the owner pays the tax tp and gets the payoff $v + u_O - tp$, and the prospective buyer's payoff is zero; in the event of a purchase, the owner's payoff is p while the buyer's payoff is $v + u_B - p - tp$. The owner decides on p without knowing u_B, but with knowledge of the probability distribution G from which u_B is drawn. It is assumed that u_B is independent of u_O, and that the function G is strictly increasing on $[\underline{u}, \overline{u}]$ and differentiable (with density g). It may be useful to think that $\underline{u} < 0 < \overline{u}$ and that $E(u_B) = 0$ so that v is the expected value of the house. However, these features are not required for the analysis. In addition, it is assumed that the function

$$x - \frac{1 - (1+t)G(x)}{(1+t)g(x)} \tag{1}$$

is increasing in x. The latter assumption is technical. As we will see, it is helpful in streamlining some of the later arguments, but it is not crucial for the analysis. This assumption is a variation on a standard assumption of increasing hazard rate that is frequently employed in the literature on mechanism design.

We will consider two regimes with respect to the information. In one regime, the common component v is known to both, while in the other it is private information of the owner. In both regimes the private components are private information.

Before proceeding to the analysis, let us make one further comment on the model. Notice that the buyer's tax payment is calculated on the basis of the assessed valuation selected by the owner. One may think of a more complicated recursive formulation in which the buyer's tax payment is based on a possibly different assessed valuation selected by the buyer upon assuming ownership. While such formulation might be more complete, its recursive nature will complicate matters considerably.

3 The complete information regime

The owner's *strategy* is a price $p = p(v, u_O)$ and the buyer's *strategy* is a probability $q = q(v, u_B, p)$ of purchasing the house, given the price chosen by the owner. The owner's expected utility of announcing the price p is

$$w(v, u_O, p) \equiv E\{q(v, u_B, p)[v + u_0 - tp] + [1 - q(v, u_B, p)]p\}$$

where the expectation is with respect to u_B. The buyer's expected utility is $q[v + u_B - tp - p]$. An *equilibrium* is a pair of strategies $p^*(v, u_O)$ and $q^*(v, u_B, p)$ such that $p = p^*(v, u_O)$ maximizes the owner's expected utility and $q^*(v, u_B, p)$ maximizes the buyer's utility for any pT. Obviously, $q^*(v, u_B, p) = 1$ if $v + u_B - tp > p$, i.e., if the benefit of purchase exceeds the price, and $q^*(v, u_B, p) = 0$ if $v + u_B - tp < p$. Therefore, $p^*(v, u_O)$ prescribes price p that maximizes

$$w(v, u_O, p) \equiv G[(1+t)p - v][v + u_0 - tp] + [1 - G[(1+t)p - v]]p \qquad (2)$$

Proposition 1 :*The equilibrium prices are unique and have the form*

$$p^*(v, u_O) = r(u_O) + \frac{v}{1+t} \qquad (3)$$

where $r(u_O)$ is increasing in u_O.

Proof. Condition (1) implies that $Arg \max_p w(0, u_O, p)$ is a singleton. Let $r(u_O) = Arg \max_p w(0, u_O, p)$. Observe from (2) that, for any v, p and p', $w(v, u_O, p) \leq w(v, u_O, p')$ iff $w(0, u_O, p - \frac{v}{1+t}) \leq w(0, u_O, p' - \frac{v}{1+t})$. Thus,

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since for all p, $w(v, u_O, p) \le w(v, u_O, p^*(v, u_O))$, it follows that $p^*(v, u_O) = r(u_O) + \frac{v}{1+t}$.

Suppose now that $u'_O > u_O$ but $r(u'_O) \le r(u_O)$. Observe that

$$
\begin{aligned}
w(0, u'_O, r(u_O)) &= w(o, u_O, r(u_O)) + (u'_O - u_O)G((1+t)r(u_O)) \\
&> w(0, u_O, r(u'_O)) + (u'_O - u_O)G((1+t)r(u'_O)) = w(0, u'_O, r(u'_O))
\end{aligned}
$$

where the inequality follows from the optimality of $r(\bullet)$ which in turn implies that, for any u, $r(u) \le \inf\{r : G[(1+t)r] = 1\}$. But $w(0, u'_O, r(u_O)) > w(0, u'_O, r(u'_O))$ contradicts the optimality of $r(u'_O)$. Thus, $r()$ is increasing. ∎

Remark: Notice that the technical assumption (1) was only used to establish that $Arg\max_p w(0, u_O, p)$ is a singleton. If that condition were not imposed, it would still follow from the proof that any selection $r(u_O)$ from $Arg\max_p w(0, u_O, p)$ would be monotonically increasing and hence $Arg\max_p w(0, u_O, p)$ would be a singleton except perhaps for a countable number of values of u_O. So, the equilibrium prices would still be unique except perhaps for a measure 0 of points.

The first order condition for $r(u_O)$ (obtained by differentiating the RHS of (2) w.r.t. p at $v = 0$) yields

$$
r(u_O) - \frac{1 - (1+t)G[(1+t)r(u_O)]}{(1+t)^2 g[(1+t)r(u_O)]} = \frac{u_O}{1+t}
$$

Substituting this into (3) gives

$$
p(v, u_O) = \frac{v + u_O}{1+t} + \frac{1 - (1+t)G[(1+t)r(u_O)]}{(1+t)^2 g[(1+t)r(u_O)]} \tag{4}
$$

Formula (4) gives the precise sense in which p reflects the value of the house. $\frac{v+u_O}{1+t}$ is the full value of the house for the owner. At this price the owner is exactly indifferent between keeping the house and selling it. The owner's chosen price $p(v, u_O)$ might be below or above this level. For $u_O = G^{-1}(\frac{1}{1+t})$ the term $\frac{1-(1+t)G[(1+t)r(u_O)]}{(1+t)^2 g[(1+t)r(u_O)]}$ vanishes and $p(v, u_O) = \frac{u_O+v}{1+t}$. For $u_O < G^{-1}(\frac{1}{1+t})$ that term is positive and $p(v, u_O) > \frac{u_O+v}{1+t}$, while for $u_O > G^{-1}(\frac{1}{1+t})$ that term is negative and $p(v, u_O) < \frac{u_O+v}{1+t}$. Thus, the price incorporates fully the value of the common component v. It depends positively on u_O as well, but the effect of u_O is moderated by strategic considerations. The relationship

between the price and $\frac{u_O+v}{1+t}$ reflects the trade-off between the profit in case that the house is indeed bought and the tax burden in case that it is retained. If owners with high private value u_O chose the price $\frac{u_O+v}{1+t}$, they might be bought out only by a prospective buyer who has a sufficiently high private value u_B. Since this occurs with low probability, such owners are better off keeping the price below $\frac{u_O+v}{1+t}$ in order to save on the tax. They will lose in the low probability event that they will be bought out, but the gain in tax compensates for that. In contrast, if owners with particularly low u_O chose the price $\frac{u_O+v}{1+t}$, they would be likely to sell. By choosing a higher price they increase their profit in the event of sale at the expense of a higher tax in case they keep the house.

4   Discussion

This section contains an informal discussion of some of the issues associated with the adoption of a self assessment method of the sort described above.

How the outcome of self-assessment compares to that of centralized assessment

To compare the equilibrium self assessment $p^*(v, u_O)$ to the government determined assessment, we have to identify the latter in our model. The problem is that it is not obvious what notion of value the government tries to capture and it is even harder to determine what it ends up with. For concreteness, suppose that, using its direct evaluation methods, the government observes correctly v and the expectation of u_O, $E(u_O)$, over the owners population. And suppose that $\frac{v}{1+t}$ or $\frac{v+E(u_O)}{1+t}$ is designated as the assessed valuation[3]. Clearly, $p^*(v, u_O)$ differs from either of these magnitudes in that it varies across houses with the same v value due to variation in the u_O values of the actual owners. The extent of this variation depends on the distribution of u_O among owners. Recall that u_O reflects personal tastes including the costs associated with moving. These factors, in particular the cost of moving, may vary over the life cycle of the owners. Thus, even for a given house and owner $p^*(v, u_O)$ might be expected to vary across time as well. Now, since in practice the government does not observe v but instead comes

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[3]Obviously, even if the expectation of the private value component for a given house over the entire population is 0, it does not imply that $E(u_O) = 0$ owing to self selection reflected in the ownership.

up with rather noisy estimates, the centrally determined assessments do not coincide with $\frac{v}{1+t}$ or $\frac{v+E(u_O)}{1+t}$ either. So, the comparison is between a method that generates arbitrary random deviations from $\frac{v}{1+t}$ and a method that generates deviations that reflect individual preferences, with higher assessments imposed on those who attach greater values to their homes.

The costs associated with the two alternative methods.

The existing centralized assessment process involves direct administration costs. There are also individual costs associated with the appeal processes that inevitably accompany the assessment process. The self assessment alternative outlined here would also involve costs: Owners or consultants in their employ will have to obtain current information on their local real estate market in order to arrive at the appropriate assessment. Thus, the self assessment method would in fact privatize rather than totally eliminate some of the administration costs. We have no basis to determine how the costs under the two systems might compare. But, even ignoring popular views regarding the relative efficiency of private vs. public administrations, the special advantage that the private effort would have in this case is that it would have access to the information that homeowners obtain in the natural course of owning their properties.

Both alternatives also involve psychic costs. The centralized assessment by the government aggravates owners by the arbitrariness with which it assigns quite different assessments to houses that sell for similar prices. The main psychic cost under the alternative self assessment method would be associated with the possibility of unwanted purchase that will be discussed further below.

Problems in implementation

While the simplicity and transparency of the market based assessment method seem appealing, there are some obvious problems in its implementation that might render it impractical. Implementation will have to be accompanied by rules regulating the many important details of the process such as the posting and updating of the assessments and the procedures governing the transactions that might arise. Among other things, such rules will have to determine the time period within which purchase offers can be made, how multiple offers are dealt with and how and when the ownership of a house that was purchased in this process might be transferred. A wise choice of such rules might avoid many potential problems. But however wisely such rules

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would be chosen, there are some inherent problems that would accompany this method in any case.

First, there is the above mentioned problem of facing an undesired purchase. Imagine an unprepared homeowner who suddenly learns that his home has been bought and he has to move out. This might be quite distressing even if the owner knowingly posted a somewhat low price in order to save on taxes, but there is also the possibility that a less than fully informed owner posted a too low price by error, or that market conditions changed rapidly and turned what had been a reasonably priced house into a bargain. These problems can be somewhat mitigated by careful choice of the rules. For example, an owner whose house was bought might be given sufficient time to move out. Protection from changing market conditions can be achieved by limiting the period within which purchases can be made to, say, the first 3-6 months in an assessment cycle. Undesirable purchase might sometimes be averted by free bargaining between the owner and the prospective purchaser who might agree to withdraw the offer in exchange for suitable compensation. But whatever steps are taken to minimize these complications, the situation of an unwanted purchase cannot be entirely avoided[4]. Although some of the responsibility for such problems would rest with the owners who could prevent them by posting sufficiently high assessments, the punishment for errors or calculated risks might sometimes seem too severe and hence politically unacceptable.

A second and related problem is that, either out of anger at an undesirable purchase or as a means of blackmailing the buyer, an owner might damage the house in the period that elapses between the purchase and the change of ownership. Again, suitable regulations regarding the transaction might reduce this problem to some extent, but it probably may not be entirely eliminated.

There might be other potential complications associated with the feature that all houses are essentially offered for sale. For example, a developer might take advantage of this system to purchase a block of properties at once thus avoiding the free riding and other difficulties that make it hard to consolidate a large block of properties. It is not clear that everybody would view this as a problem. But in case it is viewed as such, it can be avoided by spreading

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[4]One may also not count on the emergence of special insurance arrangements that would promise owners a large compensation in case they will be forced to sell at an assessed price approved by the insurer. This is because such insurance would be susceptible to collusion between owners and prospective buyers.

the assessment cycles of neighboring homes or by enacting regulations that prohibit such transactions.

A hybrid method that avoids the undesirable purchase problem

In response to the difficulty associated with an unwanted purchase, we may consider hybrid alternatives that would substantially reduce this hardship. Such alternatives would still have a centralized assessment process for those who want it, but owners would be allowed instead to determine their own assessment which would then constitute an obligation to sell. Thus, while the hardship of an unwanted purchase would not be eliminated, it would be confined to those owners who choose to take this risk. The government might try circumvent this method by understating the value of all properties, which would not be affect the tax revenues owing to adjustments of the tax rates (governments do it often even under the current system to avoid appeals based on sales value). But this problem can be solved by devising an index that will capture the actual dollar value of an assessment dollar in the said county and the purchase would have to be at the price calculated on this basis.

Privatization

An important feature of both the pure self assessment method and the hybrid method just introduced is that the administration of the assessment process could be contracted out to profit seeking firms in a manner that maintains proper incentives. Such private firms might be entrusted with the assessment process by tying their compensation to the total value they generate or by simply auctioning to them the tax revenues from blocks of property. These firms would be prevented from abusing their powers by the individuals' right to determine their own assessments according to either of the above methods. In fact, one possible variation would give the right of purchase only to the firm that administers the process rather than to anybody. One might think that under such variation, an owner who contests the firm's assessment and sets his own assessment would have to allow the firm to inspect his property and then might face a purchase.

In contrast, if the prevailing centralized assessment process were privatized, profit seeking assessors will either have no incentive to maximize revenue or else will have incentives to overtax the owners.

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5 Discussion of the modeling

This section discusses possible extensions of the basic model of Section 3. First, it shows that the general features of the equilibrium self assessments survive the introduction of asymmetric information. Second, it discusses the foundations of the reduced form model we use.

Asymmetric information about values

The formal model of Section 3 assumes symmetric information with regard to the common component v. In practice, both owners and prospective buyers might have private information about v. A prospective buyer's private information about v might arise in the less common situations in which the prospective buyer has some sort of inside information about future development plans in the area. It might be possible to reduce the use of such inside information to some extent by restricting the participation of individuals who might have access to such information (in much the same way it is done in financial markets) and by restricting the number of properties that a single buyer can purchase in a certain area at a certain time. The more common source of asymmetric information is the owner's private information about the condition of the house. The following discussion extends the basic model to accommodate the latter form of information asymmetry.

Assume that the common component v is private information of the owner. The prospective buyer holds a prior probability belief over v with support on $[\underline{v}, \overline{v}]$. The idea is that certain objective characteristics of the house are known to the owner but not to prospective buyers. The interaction is as before. The owner's strategy is a price $p = p(v, u_O)$ and the buyer's strategy is a purchase probability $q = q(u_B, p)$. The additional element is the prospective buyer's beliefs regarding the distribution of v given p. Let $v_B(p)$ denote the expected value of v according to the buyer's beliefs given p.

The owner's expected utility of announcing the price p is

$$w(v, u_O, p) \equiv E\{q(u_B, p)[v + u_0 - tp] + [1 - q(u_B, p)]p\} \qquad (5)$$

where the expectation is with respect to u_B. The buyer's expected utility of purchasing at price p with probability q given the belief $v_B(p)$ is $q[v_B(p) + u_B - tp - p]$.

An *equilibrium* is a pair of strategies $p^*(v, u_O)$ and $q^*(u_B, p)$ and beliefs summarized by $v_B(p)$ such that: (i) $q^*(u_B, p)$ maximizes the buyer's utility

given the beliefs $v_B(p)$; (ii) $p^*(v, u_O)$ is the price p that maximizes the owner's expected utility given $q^*(\bullet, p)$; (iii) the beliefs are confirmed in equilibrium in the sense that, for any p such that $p = p^*(v, u_O)$ for some v and u_O, we have $v_B(p) = E(v \mid p^*(v, u_O) = p)$.

Notice that the owner's payoff depends on v and u_O only through their sum. A *separating* equilibrium is an equilibrium in which p is increasing in the sum $v + u_O$. The following claim characterizes the separating equilibria.

Proposition 2 : *There exists a separating equilibrium. The equilibrium price schedule $p(v, u_O)$ is of the form*

$$
\begin{array}{lll}
p(v, u_O) > \frac{v+u_O}{1+t} & for & v + u_O < y^* \\
p(v, u_O) = \frac{v+u_O}{1+t} & for & v + u_O = y^* \\
p(v, u_O) < \frac{v+u_O}{1+t} & for & v + u_O > y^*
\end{array}
$$

where y^* is the unique solution to $y - E(v \mid v + u_O = y) = G^{-1}(\frac{1}{1+t})$.

Proof. : Equilibrium condition (i) implies that $q^*(u_B, p) = 1$ if $v_B(p) + u_B - tp \geq p$ and $q^*(u_B, p) = 0$ otherwise. Therefore, given $q^*(\bullet, \bullet)$ and $v_B(p)$, the owner's expected utility (5) can be written as

$$
w(v, u_O, p) \equiv \tag{6}
$$
$$
G[(1 + t)p - v_B(p)][v + u_0 - tp] + (1 - G[(1 + t)p - v_B(p)]) \, p
$$

Thus, $p^*(\bullet, \bullet)$ is an equilibrium price schedule iff, for all v and u_O, $p^*(v, u_O)$ is the price p that maximizes (6) where $v_B(p)$ satisfies condition (iii).

We construct a separating equilibrium price schedule, $p(v, u_O)$, as follows. Let $\underline{p}$ be the p solution for

$$
(1 + t)p - \underline{v} - \frac{1 - (1 + t)G[(1 + t)p - \underline{v}]}{(1 + t)g[(1 + t)p - \underline{v}]} = \underline{u}
$$

and let $\overline{p}$ be the p solution for

$$
(1 + t)p - \overline{v} - \frac{1 - (1 + t)G[(1 + t)p - \overline{v}]}{(1 + t)g[(1 + t)p - \overline{v}]} = \overline{u}
$$

By (1) there are unique such $\underline{p}$ and $\overline{p}$. Let $E(v \mid z)$ be a shorthand for $E(v \mid v + u_O = z)$ and define the function $\rho(z)$ as follows (this function

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will give later the equilibrium price by defining $p(v, u_O) = \rho(v + u_O))$. For $z \in [\underline{v} + \underline{u}, y^*)$, $\rho(z)$ is the solution to the differential equation

$$\rho'(z) = \frac{\frac{\partial E(v|z)}{\partial z}[(1+t)\rho(z) - z]}{(1+t)[(1+t)\rho(z) - z - \frac{1-(1+t)G[(1+t)\rho(z)-E(v|z)]}{(1+t)g[(1+t)\rho(z)-E(v|z)]}]} \tag{7}$$

with initial conditions $\rho(\underline{v} + \underline{u}) = \underline{p}$. Observe that such solution exists and satisfies $\rho(z) > \frac{z}{1+t}$, $(1+t)G[(1+t)\rho(z) - E(v \mid z)] \leq 1$ and $(1+t)\rho(z) - z - \frac{1-(1+t)G[(1+t)\rho(z)-E(v|z)]}{(1+t)g[(1+t)\rho(z)-E(v|z)]} > 0$. Hence, $\rho(z)$ is increasing over this range and $\lim_{z \to y^*} \rho(z) = \frac{y^*}{1+t}$. Analogously, for $z \in (y^*, \overline{v} + \overline{u}]$, $\rho(z)$ is also the solution to (7) with terminal condition $\rho(\overline{v} + \overline{u}_O) = \overline{p}$. Observe that here the solution satisfies $\rho(z) < \frac{z}{1+t}$, $(1+t)G[(1+t)\rho(z) - E(v \mid z)] \geq 1$ and $(1+t)\rho(z) - z - \frac{1-(1+t)G[(1+t)\rho(z)-E(v|z)]}{(1+t)g[(1+t)\rho(z)-E(v|z)]} < 0$. Hence, $\rho(z)$ is increasing over this range too and $\lim_{z \to y^*} \rho(z) = \frac{y^*}{1+t}$. Finally, for $z = y^*$, define $\rho(y^*) = \frac{y^*}{1+t}$.

Next, let us construct the buyer's beliefs $v_B(p)$ that will support $p(v, u_O) = \rho(v + u_0)$ as an equilibrium price schedule

$$v_B(p) = \begin{cases} 0 & for \quad p < \underline{p} \\ E(v \mid \rho^{-1}(p)) & for \quad p \in [\underline{p}, \overline{p}] \\ \overline{v} & for \quad p > \overline{p} \end{cases} \tag{8}$$

Thus, for $p \in [\underline{p}, \overline{p}]$, the beliefs simply invert ρ; for $p < \underline{p}$ and $p > \overline{p}$ which will be off equilibrium prices, the beliefs assign the minimum and maximum values of v respectively.

Let us now verify that $\rho(v + u_0)$ is an equilibrium price schedule. By construction, $p = \rho(v + u_0)$ satisfies the first order condition for maximization of (6)

$$[(1+t) - \frac{\partial E(v \mid \rho^{-1}(p))}{\partial p}]g[(1+t)p - E(v \mid \rho^{-1}(p))][v + u_0 - (1+t)p] \tag{9}$$
$$-(1+t)G[(1+t)p - E(v \mid \rho^{-1}(p))] + 1 = 0$$

where $v_B(p)$ was substituted out using (8). Since for $p > \rho(v + u_0)$ the LHS of (9) is negative and for $p < \rho(v + u_0)$ it is positive, $p = \rho(v + u_O)$ maximizes $w(v, u_O, p)$ over all $p \in [\underline{p}, \overline{p}]$. By the choice of $\underline{p}$ and $\overline{p}$ and by (1), for any v, u_O, $w(v, u_O, p) \leq w(v, u_O, \underline{p})$ for $p \leq \underline{p}$, and $\overline{w}(v, u_O, p) \leq w(v, u_O, \overline{p})$ for $p \geq \overline{p}$.

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Next, observe that, by construction, the beliefs are confirmed on the path, i.e., $v_B(p) = E(v \mid p(v, u_O) = p)$. Therefore, $p(v, u_O) = \rho(v + u_O)$ is a separating equilibrium price schedule.

Finally, observe that, by construction, this equilibrium has the form specified in the statement of the proposition. ∎

Thus, a separating equilibrium resembles the complete information equilibrium in the following ways. First, the equilibrium price assessment reflects both the common component v and the private component u_O. It is increasing in both. Second, the price overstates the owner's value of the house for relatively low values of $v + u_O$ and understates the owner's value for high values of $v + u_O$. As in the complete information case, the relationship between the price and $\frac{u_O + v}{1+t}$ reflects the trade-off between profit in case the house is indeed purchased and the tax burden in case it is retained. Owners with high $v + u_O$ values are better off keeping the price below $\frac{u_O + v}{1+t}$ in order to save on the tax, since the probability that they will be forced to sell and hence make a loss is relatively low (it takes a buyer with particularly high u_B). In contrast, owners with particularly low values of $v + u_O$ are better off with a price above $\frac{u_O + v}{1+t}$. In this manner they increase their profit in the relatively likely event of sale at the expense of a higher tax in case they keep the house.

The main conclusion here is that the analysis of the self assessment equilibrium is robust to the introduction of owners' private information in a sense that the equilibrium outcome remains qualitatively similar to what it was under complete information. The implication is that the complete information model offers a reasonable framework for the discussion[5].

The reduced form nature of the model

The analysis so far has been conducted in context of a rather rudimentary reduced form model of the market setting. The demand side is captured by the function G that describes the probability that a prospective buyer will purchase a given house as a function of only the price p and the common

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[5]Symmetric information might be a reasonable working assumption for another reason as well. While owners often have meaningful information and this might have important consequences for the trading in ordinary real estate markets, this information is often not totally private. It might be known to a previous owner or to a service provider (contractor, realtor) who dealt with this house. This might deter owners from posting an assessment that underestimates the value of their house, as there might be professional bargain hunters who will collect such information. From analytical point of view this situation is essentially like symmetric information.

value component v of that house. In particular, this description ignores the effect of the other prices and the values that are available in that market. The purpose of the following discussion is to relate this reduced form to a more fundamental model.

Suppose that the relevant market consists of n houses whose owners post assessments/prices simultaneously. Let v^i and p^i denote the common value component and price of house i. As before, a prospective buyer arrives on the scene and observes the v^i's and p^i's of the houses (we are back at the complete information scenario). Let u_B^i denote the buyer's private value component for house i and assume that the u_B^i's are independent draws from a distribution H. Now, the buyer would choose to buy house j if

$$v^j + u_B^j - p^j \geq \max\{\max_i[v^i + u_B^i - p^i], 0\}$$

The probability of this event is

$$\Psi^j[(p^i, v^i)_{i=1}^n] = \int_{(1+t)p^j - v^j} \prod_{i \neq j} H[x - (v^i - v^j) + (1+t)(p^i - p^j)]h(x)dx \quad (10)$$

Notice that Ψ^j depends on p^j and v^j only through $(1+t)p^j - v^j$. Define $G^j((1+t)p^j - v^j \mid (p^i, v^i)_{i \neq j}) = 1 - \Psi^j[(p^i, v^i)_{i=1}^n]$ and observe that G^j is increasing in $(1+t)p^j - v^j$. Now, G^j is the probability that the prospective buyer will not purchase house j and thus it is the counterpart of the function G that was used throughout to describe that probability.

The qualitative differences between G^j and G are that G^j depends on the entire distribution of prices and values $(p^i, v^i)_{i=1}^n$ and specifically on j (since $(p^i, v^i)_{i \neq j}$ varies with j). The dependence on j (as distinct from p^j and v^j) would be negligible, if we think of a large population of homes and imagine that a prospective buyer gets to consider only a randomly drawn sample of m houses, where m is small relative to n. In this case, $\Psi^j[(p^i, v^i)_{i=1}^n]$ will be an expectation of the RHS of (10) over all samples of size m that include j, and it would be a reasonable approximation to omit the index j and think of $G(\bullet \mid (p^i, v^i)_{i=1}^n)$ as being the same function for all homes. The dependence on the entire distribution rather than on just the p and v of a given house is of greater importance. If the demand side is modeled in this way, there should be another equilibrium condition that requires that the price distribution arising from the optimal pricing decisions of the individual owners with respect to $G(\bullet \mid (p^i, v^i)_{i=1}^n)$ indeed coincides with the actual distribution $(p^i)_{i=1}^n$.

Observe now that the main model of this paper is a reduced form of the model outlined here. It implicitly assumes that an equilibrium price distribution exists and hence omits the additional equilibrium condition. Instead, the analysis focuses on characterizing the relationship between the price and the value parameters, given the equilibrium distribution.