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Optimal Multiproduct Nonlinear Pricing with Correlated Consumer Types*

By

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Abstract: In this paper we examine the design of nonlinear prices by a multiproduct monopolist faces customers with multidimensional but correlated types. We show that the monopoly can exploit the correlations between consumers' types to design pricing mechanisms that fully extract the surplus from each consumer. Our main insight is that regardless of the dimensionality of the consumers types and the number of good produced by the monopoly, the surplus that each consumer gets after buying is a scalar. Hence, it is possible to design a two step mechanism where in the first step the monopoly induces the consumers to make efficient purchasing decisions (given their private information), and in the second step he extracts the surplus from each consumer via a (random) fixed fee.

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1. Introduction

The mechanism design literature has traditionally focused on cases where the private information of agents can be captured by a single parameter. Yet, in many applications of mechanism design (e.g., nonlinear pricing, the design of product lines, optimal regulation, optimal taxation, etc.), agents have multiple characteristics and cannot be sorted out in a satisfactory manner according to only one of these characteristics. Indeed, in many applications, the data on agents is only imperfectly correlated with their types so the ability to use as many characteristics as possible is highly advantageous. The fact that agents may have multidimensional types is troubling because, as Sibley and Srinagesh (1997), Armstrong (1996), Rochet and Chone (1998), and Rochet and Stole (1999) demonstrate, some of the insights obtained in the unidimensional case do not generalize to the multidimensional case. Moreover, apart from highly specific cases, the standard methods that have been developed in the literature to characterize optimal mechanisms in the unidimensional case are not directly applicable in the multidimensional case.

Recently though, some advance was made in relaxing the restrictive assumption that agents are characterized by a single parameter. Wilson (1993) develops an innovative and powerful technique for characterizing optimal nonlinear prices in the unidimensional case and shows that this technique can also be used to solve specific examples in the multidimensional case. Sibley and Srinagesh (1997) analyze optimal multiproduct nonlinear pricing and show that if the demand curves are uniformly ordered (i.e., the ordering of the demand functions in any market gives rise to a an ordering of the utility functions that is independent of prices), the monopoly's problem amounts to designing optimal prices separately for each market. Armstrong (1996) examines a similar problem and shows that generically, the monopoly will exclude some low value consumers from all markets. He then goes on to show that in a special class of cases, it is possible to extend the standard approach, initiated by Mirrlees (1971), to obtain closed form solutions for the monopoly's problem. Armstrong (1999) shows that if the number of products offered by the monopolist is large, then it is possible to design almost optimal cost-based two part tariffs. Rochet and Chone (1998) design a new technique, called the "sweeping" procedure, to deal with the bunching problem that was pointed out by Armstrong (1996). Using their technique, they extend the Mussa and Rosen (1978) model of quality discrimination by a monopolist to the case where each consumer is parametrized by a multidimensional vector of characteristics. Rochet and Stole (1999) extend the Mussa and Rosen model in yet another direction: they consider the case in which the type of each agent specifies not only the agent's marginal utility from quality but also the reservation utility of the agent (i.e., the utility if the agent does not buy). Hence, consumers in their model have two-dimensional types. They develop a methodology for addressing this class of problems and show that although many properties of the optimal solution in the single dimensional case re-emerge, at the bottom of the distribution it may no longer be optimal to distort the level of quality or to ensure complete separation of types.

While these papers make an important progress in analyzing a very difficult and empirically relevant problem, they retain the traditional assumption (e.g., of Mussa and Rosen 1978, Myerson 1981, and Maskin and Riley 1984) that consumers' types are independently drawn some commonly known distribution. In practice though, the independence assumption appears to be quite strong since it suggests that information about one agent is completely uninformative about the types of other agents. For instance, in the context of nonlinear prices, it suggests that the data about the demands of existing customers cannot be used to predict the pattern of demands of new customers. In this paper we relax the independence assumption and examine the optimal design of nonlinear prices by a multiproduct monopolist who faces multidimensional consumers whose types are correlated. We show that the correlation between types has a dramatic impact on the results. In particular, the monopolist can use the correlation between types and design a pricing mechanism that fully extracts the (expected) surplus from each agent. That is, the monopolist can completely overcome the informational asymmetry problem and obtain, in expectation, the same payoff as in the full information case.

The main insight in this paper is that although the monopoly produces multiple products and although each consumer has a multidimensional type, it is possible to construct simple mechanisms (e.g., incremental cost pricing or a Clarke-Groves mechanism) to induce consumers to make the optimal buying decisions. While these mechanisms induce efficient outcomes, they do a poor job in extracting consumers' surplus. But since at this point the surplus of each consumers is a scalar, we are back in the unidimensional case.[1] Assuming that consumers' types are correlated, we can augment the mechanism with (correlated) fixed fees that will extract, in expectation, the surplus from each consumer, using the techniques of Crémer and McLean (1985, 1988) when the set of consumers' types is finite and McAfee and Reny (1992) when it is infinite. The fixed fee that each consumer pays is random from the consumer's point of view because it depends on the reports of other consumers, whose exact types are not known to the consumer in advance (although they are correlated with the consumer's own type).

Our approach can be viewed as a two step mechanism. In the first step the monopoly ensures efficiency by using a pricing mechanism that induces consumers to maximize their surplus. Then in the

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[1] Sharkey and Sibley (1993) and Armstrong and Vickers (1999) also exploit the fact that consumer surplus is a scalar and study competition with nonlinear prices in terms of the surplus induced by potentially complex and multidimensional pricing schemes.

second step, the monopolist extracts the surplus from each consumer through appropriate fixed fee.[2] When the set of consumers' type is finite, the fixed fees fully extract the expected surplus from each consumer while in the infinite case the surplus extraction is arbitrarily close to being full. This two step approach generalizes the Crémer and McLean (1985) model of monopoly price discrimination to the case where the monopoly produces multiple goods and faces a multidimensional consumers. Essentially, the role of the first step in our approach is to reduce the monopoly's problem to the problem in their paper which is how to extract (a unidimensional) surplus from consumers who have correlated types. Shinotsuka and Wilkie (1999) have used a similar two step approach in the context of optimal multi-object auctions.

Our results are in stark contrast to the independent types case: first the monopoly can get, in expectation, the entire social surplus. This suggests that at least theoretically, it may be possible to overcome the informational asymmetry problem provided that buyers are risk neutral, are not liquidity constrained, and have correlated types.[3] Second, the characterization of the optimal mechanism is fairly straightforward and it only requires to solve a system of linear inequalities (albeit quite large). Third, our approach does not require us to impose many of restrictions on the preferences of consumers, the distribution of types, and the monopoly's cost functions, that Wilson (1993), Armstrong (1996, 1999), Armstrong and Rochet (1999), Rochet and Chone (1998), Rochet and Stole (1999), and Sibley and Srinagesh (1997) had to make in order to make a progress in analyzing the much harder independent types case. In particular, we do not need to assume that consumer preferences satisfy any type of a single crossing property.

Although the fixed fee of each consumer depends in our model on the reports of all other consumers, the monopoly can also condition the fixed fees on a public signal that is correlated with the

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[2] This approach is reminiscent of Matthews (1997) who shows that it is optimal for a principal who needs to induce an agent to exert effort while insuring the agent against risk to break the contracting process with the agent into two steps. First, before the agent exerts effort, the principal should offer a menu of output levels that the agents needs to deliver and associated wages. Then, after the agents exerted effort but before uncertainty was realized, the principle should sell the agent insurance. Unlike in Matthews where the problem is one of moral hazard, we do not need to worry about the timing of the two steps as the problem that we consider is one of adverse selection.

[3] Since in reality buyers are likely to be risk averse and subject to limited liability constraints, our results should be interpreted as a benchmark. Robert (1991) shows that in the context of a single good auction problem, the seller cannot extract all the surplus from buyers who are risk averse or subject to limited liability constraints. The characterization of optimal nonlinear prices when buyers are risk averse is of course a much harder problem. Indeed, so far the literature on the multidimensional screening (e.g., Armstrong 1996 and 1999, Armstrong and Rochet 1999, Rochet and Chone 1998, Rochet and Stole 1999, and Sibley and Srinagesh, 1997) has only considered the case where buyers are risk neutral.

buyers' types (e.g., the state of the economy, or even the aggregate demand of all other consumers), provided that this signal is realized after consumers make their purchasing decisions. If the public signal is "sufficiently" correlated with the buyer's types, then it can play the same role that the reports of other consumers play in our model. In fact, a public signal could be simpler to use in the design of the fixed fees. The idea that public signals could be a substitute for reports by other agents has been explored by Riordan and Sappington (1988) in the context of procurement contracts. They derive necessary and sufficient conditions under which a single buyer who faces a single seller who is privately informed about his cost, can nonetheless get the full surplus from the relationship.

The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3 we characterize the optimal pricing mechanism when the monopoly's cost function is decomposable in the sense that the cost of serving one consumer is independent of the quantity produced for other consumers. In Section 4 we illustrate our approach by means of an example. Then we extend the results in Section 5 to the case where the monopoly's cost is not decomposable, and in Section 6 to the case where consumers have continuous type spaces. We conclude in Section 7.

2. The model

A monopoly provides k products or services to a set of consumers $N = \{1,...,n\}$. The utility function of consumer $i \in N$ when he buys a bundle of products, $q_i \in \Re^k_+$, at a price p_i, is quasi-linear and given by $V(q_i,t_i) - p_i$, where t_i is an m_i dimensional vector of characteristics that describes the consumer's (multidimensional) type and is drawn from a compact non-empty metric space $T_i \subset \Re^{m_i}$. We assume that the gross utility function, $V(q_i,t_i)$, is continuous and increasing in q_i, and if a consumer does not buy anything, his/her gross utility is $V(0,t_i) = 0$. Let $T = T_1 \times ... \times T_n$ be the consumers type space and let $T_{-i} = T \backslash T_{-i}$ be the type space of all consumers but consumer i. We will call elements of T **consumer profiles**. There is a continuous density function, f, defined over T. We denote by $f_i(t_i,t_{-i})$ the marginal density of f, and use $f(t_{-i} \mid t_i) = f(t_i,t_{-i})/f_i(t_i,t_{-i})$ to denote the probability distribution on T_{-i}, conditional on consumer i's type being t_i. The monopoly has a continuous and increasing cost function $C:\Re^k_+ \rightarrow \Re_+$. We assume that all consumers and the monopolist are risk neutral.

In most of the paper we shall assume that T_i is a finite set for all $i \in N$ and use ℓ_i to denote the cardinality of the set T_i (i.e., the number of possible types that consumer i can assume). Using analogous notation, we denote the cardinality of the set T_{-i} by $\ell_{-i} = \underset{j \neq i}{X} \ell_j$. In Section 6 below, we shall relax the assumption that the consumers type space is finite and allow T_i to be infinite for all $i \in N$.

As usual we invoke the revelation principle and restrict attention to direct revelation mechanisms. Under a (direct revelation) pricing mechanism, each consumer reports his/her type to the monopoly, and given a vector of reported types, $\hat{t} \in T$, the monopoly chooses an allocation of bundles $q(\hat{t}) \equiv (q_1(\hat{t}),...,q_n(\hat{t}))$ and a corresponding vector of prices $p(\hat{t}) \equiv (p_1(\hat{t}),...,p_n(\hat{t}))$. We will often refer to pairs of bundles and prices as offers. Using this terminology, a pricing mechanism simply specifies a list of offers, one for each reported type of consumer. We now introduce three useful definitions:

Definition 1: *A pricing mechanism is ex post efficient at consumer profile t if*

$$q^*(t) \in \underset{q}{Argmax} \sum_{i=1}^{n} V(q,t_i) - C(q). \tag{1}$$

That is, a pricing mechanism is ex post efficient if it selects an allocation q(t) that maximizes the social surplus.

Definition 2: *A pricing mechanism is incentive compatible if for all $i \in N$, all t_i, $\hat{t}_i \in T_i$, and all $t_{-i} \in T_{-i}$*

$$V(q_i(t),t_i) - p_i(t) \geq V(q_i(t_{-i},\hat{t}_i),t_i) - p_i(t_{-i},\hat{t}_i). \tag{2}$$

When a pricing mechanism is incentive compatible, each consumer prefers to make a truthful report so $\hat{t}_i = t_i$ for all $i \in N$.

Definition 3: *Given a pricing mechanism and given his/her report, consumer i's expected surplus is*

$$S_i(\hat{t}_i|t_i) \equiv \sum_{t_{-i} \in T_{-i}} \left[V(q_i(t_{-i},\hat{t}_i),t_i) - p_i(t_{-i},\hat{t}_i)\right] f(t_{-i}|t_i). \tag{3}$$

A pricing mechanism is Bayesian incentive compatible if, for all $i \in N$ and all t_i, $\hat{t}_i \in T_i$,

$$S_i(t_i) \equiv S_i(t_i|t_i) \geq S_i(\hat{t}_i|t_i), \tag{4}$$

and it is interim individually rational if for all $i \in N$ and all t_i, $\hat{t}_i \in T_i$,

$$S_i(t_i) \geq 0. \tag{5}$$

Bayesian incentive compatibility is a weaker concept than incentive compatibility because it only requires that each consumer will find it optimal to make a truthful report, given the consumer's beliefs (rather than the consumer's actual knowledge) about other agents' types.

Given an incentive compatible and interim individually rational pricing mechanism, M, the monopoly's expected profit is given by

$$\pi(M) = \sum_{t \in T} \left[\sum_{i=1}^{n} p_i(t) - C(q(t)) \right] f(t). \tag{6}$$

Note that in writing $\pi(M)$ we use the fact that M is incentive compatible to evaluate the allocation q at the true consumer profile t (rather than the vector of reports, $\hat{t}$) and use the fact that M is interim individually rational to integrate profits over all possible realizations of consumer profiles (i.e., no type of consumer is ever excluded).

If the monopoly knew the true consumer profile, it could have engaged in first degree price discrimination and earn a profit equal to the maximal social surplus in every possible state. Thus the maximal social surplus places an upper bound on the expected profit that the monopoly can attain. We will say that a pricing mechanism fully extracts the surplus if it enables the monopoly to reach this upper bound in expectation.[4] Formally, we introduce the following definition:

Definition 4: *A pricing mechanism M is said to fully extract the surplus if*

$$\pi(M) = \sum_{t \in T} \left[\sum_{i=1}^{n} p_i^*(t) - C(q^*(t)) \right] f(t), \tag{7}$$

where $(p_1^*(t),\dots,p_n^*(t))$ and $q^*(t) \equiv (q_1^*(t),\dots,q_n^*(t))$ are the prices and bundles offered under first degree price discrimination.

[4] Note that the monopoly's profit may, in some states, exceed the social surplus (in which case at least some consumers will end up with a negative utility), while in other states, it may fall short of the social surplus (in which case at least some consumers will have a positive utility); yet in expectation, each consumer breaks even so the expected profit of the monopoly is maximal.

When a pricing mechanism fully extracts the surplus, the monopoly completely overcomes the problem of informational asymmetry in the sense that, in expectation, it does as well as under first degree price discrimination. The following lemma provides a straightforward but useful characterization of the conditions under which full extraction of the surplus is possible. The lemma is closely related to Lemma 1 in Crémer and McLean (1988).

Lemma 1: *A pricing mechanism M fully extracts the surplus if and only if it is ex-post efficient at the true consumer profile t and if $S_i(t_i) = 0$ for all $i \in N$.*

Proof: First, we prove that ex-post efficiency and $S_i(t_i) = 0$ for all $i \in N$ imply that M fully extracts the surplus. To this end, note that by Definition 1, ex post efficiency implies that the surplus is maximal at every consumer profile. Since $S_i(t_i) = 0$ for all $i \in N$, the monopoly fully captures the expected surplus. Next we prove that if M fully extracts the surplus then it must be that M is ex post efficient and $S_i(t_i) = 0$ for all $i \in N$. By definition 4, M fully extracts the surplus if, on average, it generates a surplus that equals the maximal level of social surplus. But since by definition, M cannot achieve more than the maximal level of social surplus at any consumer profile, it can generate on average the maximal level of social surplus only if it does so at every consumer profile (i.e., it is ex post efficient). Clearly then, if M fully extracts the social surplus, each consumer must end up, on average, with a 0 level of utility. ■

3. Decomposable cost functions

In this section we consider the case where the cost of serving one customer is independent of the number of units produced for other customers. To this end we introduce the following definition:

Definition 5: *The cost function is said to be decomposable if $C(q) = \sum_{i \in N} C_i(q_i)$, where $C_i(q_i)$ is the cost of producing for consumer i.*

Note that decomposability does not place any restrictions on the cost function of serving any individual consumer. In particular it need not exhibit constant returns to scale or to scope. The case of decomposable cost functions has also been considered by Armstrong (1996). It includes as a special case constant marginal costs which are typically assumed in the literature (e.g., Mussa and Rosen 1978, Maskin and Riley 1984, Sibley and Srinagesh, 1997, and Rochet and Chone, 1998), but is much more general because it allows the customer specific costs to be nonlinear. In the next lemma we prove an important implication of decomposability that will be useful in what follows.

Lemma 2: *If the cost function is decomposable, then the efficient allocation at a consumer profile t is such that each consumer gets an offer that depends only on the consumer's own report.*

Proof: When the cost function is decomposable, the social surplus can be written as

$$\sum_i \left[V(q_i, t_i) - C_i(q_i) \right]. \tag{8}$$

Since this expression is separable across consumers, the efficient allocation at consumer profile t is such that $(q_1^*(t_i),...,q_n^*(t_i))$, i.e., the bundle of each consumer depends only on the consumer's own report. ■

Next, we introduce the augmented incremental cost (AIC) mechanism.

Definition 6: *Given a vector of reported types $\hat{t}$, the augmented incremental cost (AIC) mechanism specifies for each consumer $i \in N$,*

(i) *a bundle $q_i^*(\hat{t})$ that corresponds to the efficient allocation at $\hat{t}$,*

(ii) *a usage fee, r_i, such that $r_i(q(\hat{t})) = C(q(\hat{t})) - C(q_{-i}(\hat{t}),0)$.*

(iii) *a (random) fixed fee, z_i, that depends on $\hat{t}_{-i}$.*

Under the AIC mechanism, each consumer reports his/her type to the monopoly. Given a vector of reports, $\hat{t}$, each consumer gets a bundle of products dictated by the efficient allocation at $\hat{t}$, and pays a usage fee equal to the incremental cost of producing his/her bundle, assuming that all other consumers are already served, and a (random) fixed fee that depends on the vector of reports. When the cost function is decomposable, the incremental cost of producing the bundle q_i is simply $C_i(q_i) - C_i(0)$. Hence, the AIC mechanism selects in this case the allocation that maximizes the social surplus at the reported consumer profile $\hat{t}$, $(q_1^*(\hat{t}_1),...,q_n^*(\hat{t}_n))$, and sets a usage fees such that $r_i(q_i^*(\hat{t}_i)) = C_i(q_i^*(\hat{t}_i)) - C_i(0)$. The fixed fees dictated by the AIC mechanism will be characterized in Theorem 1 below.

By construction, the AIC mechanism is ex post efficient. In the next lemma we prove that the mechanism is also incentive compatible and individually rational.

Lemma 3: *If the cost function is decomposable, then an AIC mechanism with zero fixed fees is incentive compatible and gives each consumer a nonnegative expected surplus.*

Proof: Following a standard argument, let $q^*(\hat{t}_{-i},t_i)$ denote the allocation that the AIC mechanism selects at a vector of reports $(\hat{t}_{-i},t_i)$, in which consumer i makes a truthful report and all other consumers send arbitrary reports (which could, but need not, be truthful). Since the AIC mechanism is ex post efficient, it maximizes the social surplus at any vector of reports, including $(\hat{t}_{-i},t_i)$. Hence, the social surplus at $q^*(\hat{t}_{-i},t_i)$ must be at least as high as the social surplus at any other allocation, and in particular at $q^*(\hat{t}_{-i},\hat{t}_i)$ which is the allocation that the mechanism selects when consumer i's report is $\hat{t}_i$ instead of t_i. Therefore,

$$\sum_i V(q_i^*(\hat{t}_{-i},t_i),t_i) - C(q^*(\hat{t}_{-i},t_i)) \geq \sum_i V(q_i^*(\hat{t}_{-i},\hat{t}_i),t_i) - C(q^*(\hat{t}_{-i},\hat{t}_i)). \tag{9}$$

Using Lemma 2 and the assumption that the cost function is decomposable, the inequality becomes:

$$V(q_i^*(t_i),t_i) - C_i(q_i^*(t_i)) \geq V(q_i^*(\hat{t}_i),t_i) - C_i(q_i^*(\hat{t}_i)). \tag{10}$$

Adding $C_i(0)$ to both sides of the inequality and recalling that the usage fees are such that for all $i \in N$, $r_i(q_i^*(\hat{t}_i)) = C_i(q_i^*(\hat{t}_i)) - C_i(0)$, we obtain:

$$V(q_i^*(t_i),t_i) - r_i(q_i^*(t_i)) \geq V(q_i^*(\hat{t}_i),t_i) - r_i(q_i^*(\hat{t}_i)). \tag{11}$$

Hence, absent fixed fees, the mechanism is incentive compatible.

Since the AIC mechanism is ex post efficient and incentive compatible, it selects the efficient allocation at the truthful vector of reports t. Hence, the social surplus at $q^*(t)$ must be at least as high as the social surplus at any other allocation, and in particular, at the allocation $(q_{-i}^*,0)$ in which every consumer but i gets the same bundle as in $q^*(t)$ while consumer i gets 0. Hence,

$$\sum_i V(q_i^*(t),t_i) - C(q^*(t)) \geq \sum_{j \neq i} V(q_j^*(t),t_j) + V(0,t_i) - C(q_{-i}^*(t),0). \tag{12}$$

Using Lemma 2 and the assumption that the cost function is decomposable, recalling that $V(0,t_i) = 0$, and rearranging terms, the inequality becomes:

$$V(q_i^*(t_i),t_i) - \left(C_i(q_i^*(t_i)) - C_i(0)\right) \geq 0. \tag{13}$$

Noting that the expression inside the brackets is just the usage fee of consumer i, equation (13) implies that absent a fixed fee, each consumer gets a nonnegative expected surplus. ■

Lemma 3 shows that when the cost function is decomposable, then under an AIC mechanism with no fixed fee, it is a dominant strategy for each consumer to make a truthful report. The reason for this is that when the cost function is decomposable, the social surplus is separable across consumers, so the bundle that maximizes the utility of each consumer is also the one that maximizes social surplus. Therefore, it is optimal for each consumer to report truthfully and thereby enable the monopoly to select the bundle that maximizes the consumer's utility.

Since the usage fees only cover the monopoly's variable costs, consumers are left with the entire surplus. The monopoly's problem then is how to extract this surplus without violating incentive compatibility and interim individual rationality. We now show that if consumer types are correlated, the monopoly can design a random fixed fee that will fully extract the surplus from each consumer. The key observation here is that although consumers' types are multidimensional, the surplus that each consumer gets before paying the fixed fee is a scalar. Therefore, the monopoly's problem becomes similar to the unidimensional problem considered by Crémer and McLean (1988). To adapt their approach to the current problem, we introduce the following condition:

Condition CM1 (Crémer and McLean 1988): *For all $i \in N$, there does not exist a type $t_i \in T_i$ and a list of ℓ-1 numbers, $(\rho_1,\ldots,\rho_{\ell-1})$, such that:*

$$f(t_{-i}\,|\,t_i) = \sum_{j \neq i} \rho_j f(t_{-i}\,|\,t_j), \qquad \textit{for all } t_{-i} \in T_{-i}. \tag{14}$$

When Condition CM1 holds, the $\ell_i \times \prod_{j \neq i} \ell_j$ matrix of conditional probabilities given consumer i's type, is of rank ℓ_i. Roughly speaking, this means that every realization of consumer i's type gives the consumer some information about the types of other consumers in the sense that it induces a different belief about the likelihood of different realizations of other consumers' types. The implication of condition CM1 is that since the matrix of conditional probabilities given consumer i's type, is of rank ℓ_i, there exists for any vector $x_i \in \Re^{\ell_i}$, another vector $z_i \in \Re^{\ell_{-i}}$, such that for all $i \in N$,

$$x_i(t_i) = \sum_{t_{-i} \in T_{-i}} z_i(t_{-i}) f(t_{-i}\,|\,t_i), \qquad \textit{for all } t_i \in T_i. \tag{15}$$

In particularly, if we set x_i equal to the expected surplus that each consumer gets at an efficient allocation with truthful reports when he pays only a usage fee, i.e.,

$$x_i(t_i) = \sum_{t_i \in T_i} \left[v_i(q_i^*(t),t_i) - r_i(q_i^*(t)) \right] f(t_{-i} \mid t_i), \tag{16}$$

then a fixed fee $z_i(t_{-i})$ that satisfies equation (15), guarantees full extraction of the surplus from consumer i. We can now state the following theorem:

Theorem 1: *Suppose that the cost function is decomposable and Condition CM1 holds and let the fixed fees be implicitly defined by equation (15), with $x_i(t_i)$ being defined by (16). Then the AIC mechanism is incentive compatible, interim individually rational, and fully extracts the surplus.*

Proof: Lemma 3 shows that when the cost function is decomposable, the AIC mechanism without fixed fees is incentive compatible. Since $z_i(t_{-i})$ does not depend on consumer i's report, the AIC mechanism with fixed fees $(z_1(t_{-1}), \ldots, z_n(t_{-n}))$ is still incentive compatible. Moreover, by construction, $z_i(t_{-i})$ extracts the full surplus from each consumer, so $S_i(t_i) = 0$ for all $i \in N$ and all $t_i \in T_i$. Hence, the mechanism is interim individually rational. Finally, by Lemma 1, since the mechanism selects the efficient allocation at the true consumer profile and since $S_i(t_i) = 0$ for all $i \in N$ and all $t_i \in T_i$, the mechanism fully extracts the surplus. ■

Theorem 1 shows that when the cost function is decomposable and Condition CM1 holds, the AIC mechanism allows the monopoly to obtain, in expectation, the same profit it would get under first degree price discrimination. In other words, the monopoly completely overcomes the informational asymmetry problem. Moreover, following Theorem 1 in Crémer and McLean (1988), one can construct an example that shows that if condition CM1 fails, then it is impossible to find an incentive compatible and individually rational pricing mechanism that will guarantee full extraction of the surplus for any arbitrary choice of the gross utility functions.

We now turn to Bayesian incentive compatible pricing mechanisms. Such mechanisms will allow the monopolist to extract the full expected surplus in a broader class of problems than those considered so far. In particular they will allow the monopolist to extract the full expected surplus in some cases where condition CM1 fails. The difference though is that now, it is no longer a dominant strategy for each consumer to make a truthful report; rather, assuming that all other consumers report truthfully, each consumer will find it worthwhile to report truthfully given the consumer's beliefs about the types of other consumers. We now introduce the following condition:

Condition CM2 (Crémer and McLean 1988): *For all $i \in N$, there does not exist a type $t_i \in T_i$ and a list of ℓ_i-1 nonnegative numbers, $(\rho_1,\ldots,\rho_{\ell-1})$, such that:*

$$f(t_{-i}\,|\,t_i) = \sum_{j\neq i} \rho_j f(t_{-i}\,|\,t_j), \qquad \textit{for all } t_{-i} \in T_{-i}. \tag{17}$$

Condition CM2 is weaker than condition CM1 because it requires the non existence of a list of nonnegative numbers $(\rho_1,\ldots,\rho_{\ell-1})$ such that equation (17) holds, whereas condition CM1 requires the non existence of any list of numbers $(\rho_1,\ldots,\rho_{\ell-1})$ (negative or positive), such that the equation holds.

Given Condition CM2, we can construct an AIC mechanism that will fully extract the surplus in a larger class of cases than those permitted by Condition CM1. As before, the usage fee will be as in Definition 6. To construct the fixed fee, note that when Condition CM2 holds, Farkas' Lemma implies that, for all $i \in N$ and all $t_i \in T_i$, there exists a vector $g_{t_i} \in \Re^{\ell_{-i}}$, that attaches a number to each vector of reports by consumers other than i, such that

$$a_i(t_i) \equiv \sum_{t_{-i}\in T_{-i}} g_{t_i}(t_{-i}) f(t_{-i}\,|\,t_i) > 0, \tag{18}$$

and

$$\sum_{t_{-i}\in T_{-i}} g_{t_i'}(t_{-i}) f(t_{-i}\,|\,t_i) \leq 0, \qquad \textit{for all } t_i' \in T_i \setminus t_i. \tag{19}$$

Now, given a vector of reported types $\hat{t}$, let the fixed fee paid by consumer $i \in N$ be

$$z_i^{AIC}(\hat{t}) \equiv S_i^{AIC}(\hat{t}_i\,|\,t_i) + \lambda_i\left(a_i(\hat{t}_i) - g_{\hat{t}_i}(t_{-i})\right). \tag{20}$$

where $S_i^{AIC}(\hat{t}_i\,|\,t_i)$ is defined by equation (3) when there is no fixed fee and when $q_i(.)$ and $p_i(.)$ are dictated by the AIC mechanism, λ_i is a sufficiently large positive scalar, and $g_{\hat{t}_i}(t_{-i})$ satisfies inequalities (18) and (19). Notice that by construction, if the reports are truthful, then

$$\underset{t_{-i}\in T_{-i}}{E}\left[z_i^{AIC}(t)\,|\,t_i\right] = S_i^{AIC}(t_i) + \lambda_i \sum_{t_{-i}\in T_{-i}} \left(a_i(t_i) - g_{t_i}(t_{-i})\right) f(t_{-i}\,|\,t_i)$$
$$= S_i^{AIC}(t_i) + \lambda_i\left(a_i(t_i) - \sum_{t_{-i}\in T_{-i}} g_{t_i}(t_{-i}) f(t_{-i}\,|\,t_i)\right) = S_i^{AIC}(t_i). \tag{21}$$

This implies that if all consumers report their types truthfully (i.e., $\hat{t} = t$), the fixed fees fully extract the surplus from each consumer in expectation.

Theorem 2: *Suppose that the cost function is decomposable and Condition CM2 holds and let the fixed fees be defined by equation (20), where $S_i^{AIC}(\hat{t}_i\,|\,t_i)$ is evaluated at $q^*(t)$ which is the efficient allocation given truthful reports. Then the AIC mechanism is Bayesian incentive compatible, interim individually rational, and fully extracts the surplus.*

Proof: By Lemma 3 and the assumption that the cost function is decomposable, the AIC mechanism without fixed fees is incentive compatible. Hence it is also Bayesian incentive compatible. We now need to show that adding the fixed fees does not change this property. To this end, note that if consumer i believes that others are going to report their types truthfully, the expected value of the consumer's fixed fee when reporting $\hat{t}_i$, conditional on the consumer's type t_i, is

$$\underset{t_{-i}\in T_{-i}}{E}\left[z_i^{AIC}(t_{-i},\hat{t}_i)\,|\,t_i\right] = S_i^{AIC}(\hat{t}_i\,|\,t_i) + \lambda_i \sum_{t_{-i}\in T_{-i}} \left(a_i(\hat{t}_i) - g_{\hat{t}_i}(t_{-i})\right) f(t_{-i}\,|\,t_i)$$
$$= S_i^{AIC}(\hat{t}_i\,|\,t_i) + \lambda_i\left(a_i(\hat{t}_i) - \sum_{t_{-i}\in T_{-i}} g_{\hat{t}_i}(t_{-i}) f(t_{-i}\,|\,t_i)\right). \tag{22}$$

Since inequalities (18) and (19) imply that the expression in brackets vanishes if $\hat{t}_i = t_i$ (i.e., i makes a truthful report) and is positive otherwise, we can ensure that for all $t_i \in T_i$, the conditional expectation of the fixed fee exceeds $S_i^{AIC}(t_i)$ by making λ_i sufficiently large. Recalling that under truthful report, consumer i's expected fixed fee is $S_i^{AIC}(t_i)$, truthful reporting is Bayesian incentive compatible. Moreover, since by construction, each consumer is left with a 0 expected surplus after paying a fixed fee, the mechanism is also interim individually rational. Finally, by Lemma 1, since the mechanism selects the efficient allocation at the true consumer profile and since each consumer is left with a 0 expected surplus, the mechanism fully extracts the surplus. ■

4. An example

In this section we illustrate our two-step approach with an example which is based on the example in Section 3 in Armstrong and Rochet (1999). A monopoly produces goods A and B, at constant marginal costs, c^A and c^B, and serves two consumers, 1 and 2. The preferences of the two consumers are additively separable in the two goods and quasi-linear in income. Using P_i to denote the total payment of consumer i, the net utility of consumer i is:

$$U(q^A, q^B, t_i^A, t_i^B) = t_i^A \sqrt{q^A} + t_i^B \sqrt{q^B} - P_i, \qquad t_i^A, t_i^B \in \{H, L\}. \tag{23}$$

Since t_i^A and t_i^B can be either H or L, each consumer has four possible types: {(L,L),(L,H),(H,L),(H,H)}. To simplify the notations, we shall call these types 1,2,3, and 4, so that (L,L) is type 1, (L,H) is type 2, (H,L) is type 3, and (H,H) is type 4.

Unlike in Armstrong and Rochet's original example where consumers' types are independent, here we assume that consumers types are correlated. Let the matrix of conditional probabilities be given by

$$H = \begin{vmatrix} r & 0 & 2/3 - r & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/4 & 1/4 \end{vmatrix}, \tag{24}$$

where the entry in the l'th row and the m'th column represents the probability that consumer i assigns to the event that consumer j's type is m when consumer i's type is l; for instance, 1/2 is the probability that consumer i assigns to consumer j being of type 1, when consumer i's type is 4. To ensure that all entries in the first row are nonnegative, we assume that $0 \le r \le 2/3$. Note that the matrix H ensures that consumers' types are correlated because the rows of H are not identical so knowing one's type conveys information about the other consumer's type.

Now, suppose that the monopoly uses the AIC mechanism. Since the monopoly has a linear cost function, the incremental cost of each unit of A is c^A and the incremental cost of each unit of B is c^B. Therefore, if consumer i's type is t_i = 1,2,3,4, and the consumer buys the bundle $(q^A(t_i), q^B(t_i))$, then the total payment of the consumer is

$$P_i = c^A q^A(t_i) + c^B q^B(t_i) + z_i, \tag{25}$$

where z_i is a fixed fee. Given P_i, the optimal bundle for consumer i, given t_i, is

$$q^A(t_i) = \left(\frac{t_i}{2c^A}\right)^2, \quad q^B(t_i) = \left(\frac{t_i}{2c^B}\right)^2. \tag{26}$$

Substituting these quantities into equation (23) and rearranging, the surplus of each type of consumer is

$$S^{AIC}(1) = \frac{L^2}{4c^A} + \frac{L^2}{4c^B}, \quad S^{AIC}(2) = \frac{L^2}{4c^A} + \frac{H^2}{4c^B},$$
$$S^{AIC}(3) = \frac{H^2}{4c^A} + \frac{L^2}{4c^B}, \quad S^{AIC}(4) = \frac{H^2}{4c^A} + \frac{H^2}{4c^B}. \tag{27}$$

The remaining task is to construct the fixed fees, z_1 and z_2, to extract the full surplus from each consumer. We will consider two cases: one in which condition CM1 holds, and one in which it fails.

Case 1: condition CM1 holds

Condition CM1 holds whenever M has full rank. Since the determinant of H equal (5-12r)/108, H has a full rank iff $r \neq 5/12$. As we saw in Section 3, when H has a full rank, the fixed fee of each consumer depends only on the reports of other consumers but not on the consumer's own report. Given that the other consumer can make four possible reports (corresponding to each of the four possible types), we need to construct a vector (z_1,z_2,z_3,z_4) such that the following four individual rationality constraints hold:

$$S^{AIC}(l) = q_{l1}z_1 + q_{l2}z_2 + q_{l3}z_3 + q_{l4}z_4, \quad l = 1,2,3,4. \tag{28}$$

Solving the system yields

$$z_1 = \frac{-3S^{AIC}(1) + 3(1-3r)(S^{AIC}(2) - S^{AIC}(3)) + 4(2-3r)S^{AIC}(4)}{5-12r},$$
$$z_2 = \frac{-6S^{AIC}(1) + 6(1-3r)S^{AIC}(2) + 9(1-2r)S^{AIC}(3) - 4(1-6r)S^{AIC}(4)}{5-12r},$$
$$z_3 = \frac{9S^{AIC}(1) + 3(2-3r)(S^{AIC}(2) - S^{AIC}(3)) - 4(1+3r)S^{AIC}(4)}{5-12r},$$
$$z_4 = \frac{-3S^{AIC}(1) - 4(3-7r)(S^{AIC}(2) - S^{AIC}(3)) + 4(2-3r)S^{AIC}(4)}{5-12r}. \tag{29}$$

Note that as $r \to 5/12$, the correlation between the consumers' types vanishes, and as a result, the fixed fees blow up.

Case 2: condition CM1 fails but condition CM2 holds

Now suppose that r = 5/12. Then the determinant of H vanishes, so condition CM1 fails and hence it is impossible to construct fixed fees that will fully extract the surplus from each consumer and will induce truthful revelation as a dominant strategy. Condition CM2 however may still hold provided that we cannot find 3 nonnegative numbers, (ρ_1,ρ_2,ρ_3), such that

$$q_{l1} = \rho_1 q_{l2} + \rho_2 q_{l3} + \rho_3 q_{l4}, \qquad l = 1,2,3,4, \tag{30}$$

where q_{lm} is the entry in the l'th raw and the m'th column in H. To prove the nonexistence of such 3 numbers, consider the following expression:

$$W \equiv \underset{\rho_1 \geq 0, \rho_2 \geq 0, \rho_3 \geq 0}{Min} \left\{ \sum_{l=1}^{4} \left(q_{l1} - \rho_{l2} q_{l2} - \rho_{l3} q_{l3} - \rho_{l4} q_{l4} \right)^2 \right\}. \tag{31}$$

Condition CM2 holds if $W \neq 0$, because then there do not exist 3 nonnegative numbers, (ρ_1,ρ_2,ρ_3), such that condition (30) is satisfied. In our example, given that r = 5/12, we have W = 1/25, so condition CM2 holds. Now, we can construct one fixed fee for each combination of reports such that the monopoly will extract the full expected surplus from each consumer. That is, we can construct the vector $(z_{11},\ldots,z_{14},z_{21},\ldots,z_{24},z_{31},\ldots,z_{34},z_{41},\ldots,z_{44})$ such that each consumer will get a 0 expected payoff by making a truthful report, and a nonpositive payoff by making a false report. Hence the following 16 constraints need to hold:

$$S^{AIC}(l) - q_{l1}z_{l1} + q_{l2}z_{l2} + q_{l3}z_{l3} + q_{l4}z_{l4} = 0, \qquad l = 1,2,3,4, \tag{32}$$

$$S^{AIC}(l) - \left(q_{11}z_{l1} + q_{12}z_{l2} + q_{13}z_{l3} + q_{14}z_{l4}\right) \leq 0, \qquad l = 2,3,4, \tag{33}$$

$$S^{AIC}(l) - \left(q_{21}z_{l1} + q_{22}z_{l2} + q_{23}z_{l3} + q_{24}z_{l4}\right) \leq 0, \qquad l = 1,3,4, \tag{34}$$

$$S^{AIC}(l) - \left(q_{31}z_{l1} + q_{32}z_{l2} + q_{33}z_{l3} + q_{34}z_{l4}\right) \leq 0, \qquad l = 1,2,4, \tag{35}$$

and

$$S^{AIC}(l) - \left(q_{41}z_{l1} + q_{42}z_{l2} + q_{43}z_{l3} + q_{44}z_{l4}\right) \leq 0, \qquad l = 1,2,3. \tag{36}$$

The problem of finding a vector of fixed fees that satisfies the 4 equalities and 12 inequalities is a straightforward linear programming problem and can be solved using standard techniques. It turns out that the problem has multiple solutions. For instance, it can be verified that one family of solutions to this problem is

$$z_{l3} \in \Re, \qquad z_{l1} = z_{l4} = \frac{4\,S^{AIC}(l) - z_{l3}}{3}, \qquad z_{l2} = \frac{5\,S^{AIC}(l) - 2z_{l3}}{3}, \qquad l = 1,2,3,4. \tag{37}$$

For this family of solutions, all the constraints are holding with equality.

5. General cost functions

Having considered the case of decomposable cost functions, we next turn to the harder case where the cost of serving one customer depends on the quantity produced for other customers. In other words, the cost function is now given by C(q), where q = $(q_1,...,q_n)$ is an allocation of bundles. In order to characterize the solution to the monopoly's problem in this case we introduce the augmented Clarke-Groves (ACG) mechanism.

Definition 7: *Given a vector of reported types $\hat{t}$, the augmented Clarke-Groves (ACG) mechanism specifies for each consumer $i \in N$,*

(i) *a bundle $q_i^*(\hat{t})$ that corresponds to the efficient allocation at $\hat{t}$,*

(ii) *a usage fee, $w_i(\hat{t})$, such that*

$$w_i(\hat{t}) = \left[C(q^*(\hat{t})) - \sum_{j \neq i} V_j(q_j^*(\hat{t}),t_j)\right] - \left[C(q^*(\hat{t}_{-i})) - \sum_{j \neq i} V_j(q_j^*(\hat{t}_{-i}),t_j)\right],$$

where $q^*(\hat{t}_{-i})$ is an n-1 dimensional vector that specifies the optimal allocation of bundles when consumer i is not served.

(iii) *a fixed fee, z_i, that depends on $\hat{t}$.*

The ACG mechanism differs from the AIC mechanism in that the usage fees are no longer independent of the reports of other consumers. Specifically, the usage fee of each consumer is equal to the difference between (i) the difference between the monopoly's cost and the utilities of all consumers but i, when all consumers are served, and (ii) the difference between the monopoly's cost and the utilities of all consumers but i when the consumer i is excluded. In other words, the usage fee corresponds to the payment that will result from applying the Clarke-Groves mechanism to the monopoly's problem. Since the ACG mechanism is by construction ex post efficient, the ACG mechanism maximizes the social surplus. The monopoly's problem then is to construct the fixed fees in a way that will allow it to capture the entire surplus. But before we turn to the characterization of the fixed fees, we first prove that in the absence of fixed fees, the ACG mechanism is incentive compatible and individually rational.

Lemma 4: *Absent fixed fees, the ACG mechanism is ex post efficient, incentive compatible, and gives each consumer a nonnegative expected surplus.*

Proof: Note that the second bracketed term in $w_i(\hat{t})$ does not depend on consumer i's report. Hence, when the consumer decides what to report, his/her maximization problem could be written as follows:

$$\underset{\hat{t}_i \in T_i}{Max}\ V_i(q_i^*(\hat{t}_{-i},\hat{t}_i),t_i) - C(q^*(\hat{t}_{-i},\hat{t}_i)) - \sum_{j \neq i} V_j(q_j^*(\hat{t}_{-i},\hat{t}_i),t_j). \tag{38}$$

Since $q^*(\hat{t})$ is efficient, it is by definition optimal for consumer i to make a truthful report. Therefore the mechanism is incentive compatible. Moreover, given truthful reports, the efficiency of $q^*(t)$ implies that

$$\sum_{i \in N} V_i(q_i^*(t),t_i) - C(q^*(t)) \geq \sum_{j \neq i} V_j(q_j^*(t_{-i}),t_j) - C(q^*(t_{-i})). \tag{39}$$

Using the definition of $w_i(t)$, this implies in turn that for all $i \in N$:

$$V_i(q_i^*(t),t_i) - w_i(t) \geq 0, \quad \textit{for all } t_i \in T_i. \tag{40}$$

Hence absent fixed fees, the ACG mechanism is individually rational. Since all consumers participate and make truthful reports, the monopoly is able to chose the ex post efficient allocation. ■

Next we characterize the fixed fees. After paying a usage fee, the expected surplus of consumer i, conditional on the consumer's type, is

$$y_i(t_i) = \sum_{t_i \in T_i} \left[V_i(q_i^*(t),t_i) - w_i(t)\right] f(t_i \mid t_{-i}). \tag{41}$$

Given $y_i(t_i)$, we can now use exactly the same proofs as in Theorems 1 and 2 (the only difference is that now $y_i(t_i)$ replaces $x_i(t_i)$), to establish the following result:

Theorem 3: *(i) Suppose that Condition CM1 holds and let the fixed fees $(z_1^{ACG}(t_{-1}),\ldots,z_n^{ACG}(t_{-n}))$ be implicitly defined by*

$$y_i(t_i) = \sum_{t_{-i} \in T_{-i}} z_i(t_{-i}) f(t_{-i} | t_i), \qquad \textit{for all } t_i \in T_i. \tag{42}$$

Then the ACG mechanism is incentive compatible, interim individually rational, and it fully extracts the surplus.

(ii) Suppose that condition CM1 fails but condition CM2 holds, and let the fixed fees be defined by

$$z_i^{ACG}(t) = S_i^{ACG}(\hat{t}_i | t_i) + \lambda_i^{ACG}\left(a_i(\hat{t}_i) - g_{\hat{t}_i}(t_{-i})\right), \tag{43}$$

where $a_i(t_i)$ and $g_i(t_{-i})$ are as in equation (20). Then, for sufficiently large λ_i^{ACG}, the ACG mechanism with fixed fees $(z_1^{ACG}(t),\dots, z_n^{ACG}(t))$ is Bayesian incentive compatible, interim individually rational, and fully extracts the surplus.

6. The case of infinite types space

In this section we extend our results to the case where the consumers' type space is infinite. Specifically, we consider the case where for each $i \in N$, the type space, T_i, is a non-empty compact convex subset of $\Re^n$. In addition we make the following three assumptions: (i) there are no degenerate types in the sense that $f_i(t_i,t_{-i}) > 0$ for all $i \in N$, all $t_i \in T_i$, and all $t_{-i} \in T_{-i}$; (ii) the gross utility function $V(q_i,t_i)$ is continuous in q_i and t_i for all $i \in N$, and the monopoly's cost function is decomposable and continuous in each q_i; and (iii) for each t_i, the surplus maximization problem of each consumer has a bounded solution. The latter assumption implies that we restrict attention to cases where the optimal bundles lie in some compact subset of $K \subset \Re^k_+$. To simplify matter we retain the assumption that the monopoly's cost are decomposable. We now show the following result:

Lemma 5: *Under the AIC mechanism, the expected surplus of each customer before paying the fixed fee, is continuous in the consumer's type.*

Proof:[5] First note that since the monopoly's cost is decomposable, the surplus of each consumer, $S(t_i) = V_i(q_i^*) - c(q_i^*)$, depends only on t_i but not on t_{-i}. Second, since the gross utility function is continuous in q_i and t_i and the cost function is continuous in q_i on K, then by the Theorem of the Maximum, $S(t_i)$ is

[5] We thank Aviad Heifetz for his help with the proof.

continuous in t_i. As by assumption $f(t_i,t_{-i})$ is continuous on T, it is also uniformly continuous on T since T is compact. Uniform continuity means, in particular, that for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $|t_i,t_i'| < \delta$, then $|f(t_i,t_{-i}) - f(t'_i,t_{-i})'| < \varepsilon$. Hence

$$\begin{aligned} \left|f_i(t_i,t_{-i}) - f_i(t_i',t_{-i})\right| &= \left|\int_{T_{-i}} f(t_i,t_{-i})dt_{-i} - \int_{T_{-i}} f(t_i',t_{-i})dt_{-i}\right| \\ &< \int_{T_{-i}} \left|f(t_i,t_{-i}) - f(t_i,t_{-i})\right| dt_{-i} < \varepsilon\, \phi(T), \end{aligned} \tag{44}$$

where $\phi(T)$ is the finite volume of compact set T. Therefore $f_i(t_i,t_{-i})$ is continuous. By the assumption that $f_i(t_i,t_{-i}) > 0$, we have that $f(t_{-i}\,|\,t_i)$ is continuous in t_i. Noting that T and K are compact and using the Dominated Convergence Theorem, this implies that if the sequence $\{t_i^k\}$ converges to t_i then

$$\int_{T_{-i}} S_i(t_i^k) f(t_{-i}\,|\,t_i^k)dt_{-i} \to \int_{T_{-i}} S_i(t_i) f(t_{-i}\,|\,t_i)dt_{-i}, \tag{45}$$

as required. ■

Having established that the expected surplus of each consumer is continuous in the consumer's type and recalling that the AIC Mechanism is Bayesian Incentive Compatible for any finite subset of type profiles, we can invoke Theorem 2 in McAfee and Reny (1992). This theorem implies that under the continuous analogue of condition CM2 (which we define below), we can define for any $\varepsilon > 0$, a fine enough grid over the type space T, such that after applying the AIC mechanism with fixed fees as in Theorem 2 above, each consumer will be left with an expected net surplus ε. Given that the AIC mechanism leaves each consumer with a unidimensional surplus, the application of McAfee and Reny (1992) is straightforward.

Condition MR2 (McAfee and Reny 1992): *For all $i \in N$, there does not exist a type $t_i \in T_i$ and a positive measure, $\mu \in \Delta(T_i)$, such that:*

$$\mu(t_i) \neq 1 \;\; and \;\; f(t_{-i}\,|\,t_i) = \int_{T_i} f(t_{-i}\,|\,\tilde{t}_i)\,\mu(d\tilde{t}_i). \tag{46}$$

For any finite grid T' over the type space T, condition MR2 implies condition CM2. Thus, we can invoke Theorem 2 above that states that the AIC mechanism is Bayesian Incentive Compatible and fully extracts the surplus for the type space T'. As the surplus of each consumer is continuous in the consumer's type, we can apply Theorem 2 in McAfee and Reny (1992) directly to establish the following result:

Theorem 4: *Suppose that Condition MR2 holds. Then for any $\varepsilon > 0$, we can define a fine enough grid over the type space T, such that the AIC mechanism with fixed fees as in Theorem 2 above, is incentive compatible, interim individually rational, and extracts all but ε of each consumer's surplus.*

Theorem 4 indicates that when the consumers' type space is infinite, the monopoly can come arbitrarily close to full surplus extraction from every consumer. Hence, one again, the correlation between consumers' types can be used to overcome the informational asymmetry problem.

7. Conclusion

In this paper we examined the design of nonlinear prices by a multiproduct monopolist who faces customers with multidimensional types. In general, this problem is very hard because one cannot use standard techniques to solve it. Recently, some important progress was made by Wilson (1993), Armstrong (1996, 1999), Sibley and Srinagesh (1997), Armstrong and Rochet (1999), Rochet and Chone (1998), and Rochet and Stole (1999). These papers maintain the traditional assumption in the literature that consumers types are independently drawn from some known distribution. In reality however, consumer types are often correlated (e.g., affected by the same macroeconomic or technological factors). Our paper shows that in that case, the monopoly can exploit these correlations and design pricing mechanisms that (fully) extract the surplus from each consumer.

The main insight is that the monopolist does not need to achieve efficiency and rent extraction all at one step. Instead, it is possible to design a two step approach where in the first step the monopoly offers a simple usage fee to induce consumers to make efficient purchasing decisions given their private information, and then in the second step, it extracts the surplus from each consumer via a (random) fixed fee. The advantage of this two step approach is that regardless of the dimensionality of the consumers' types and the number of products that each consumer buys from the monopoly, the surplus that each consumer gets after buying is a scalar. Hence, we can design the fixed fees by adopting the techniques developed by Crémer and McLean (1985, 1988) for the finite type space and McAfee and Reny (1992) for the infinite case. This implies that at least theoretically, the monopoly can completely overcome the informational asymmetry problem even though it produces many products and even if it faces multidimensional consumers. Our approach also has the advantage that it can be easily extended to the case where the monopoly is regulated and is allowed to capture only a fraction of the consumer surplus. In that case the first step remains unchanged while in the second step, the fixed fees have to be simply multiplied by the required fraction.

Although the pricing mechanisms that we propose can completely solve the informational asymmetry problem, they have at least two practical limitations that are common in the literature. First, the mechanisms require common knowledge of the exact distribution of types. Second, the mechanisms rely heavily on the assumption that consumers are risk neutral and are not liquidity constrained (i.e., can always pay the fixed fees).

These limitations suggest that our results should be mainly viewed as a benchmark. This benchmark shows that with correlations among types, the multidimensionality of the problem in of itself should not prevent the monopoly from extracting the full surplus. Moreover, the optimal pricing mechanism takes a surprisingly simple form that consists of either incremental or Clarke-Groves prices plus fixed fees. This reinforces the view of McAfee and Reny (1992) that factors like knowledge of the distribution of types and risk aversion are important for the design of optimal nonlinear prices. In future research it would be interesting to examine how well simple but robust mechanisms that combine incremental cost pricing with fixed fees perform when these factors are present.

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