Discussion Paper No. 129

An Impossibility Theorem for Deterministic Organizations*

by

Jean-Marie Blin
and
Mark A. Satterthwaite

January 24, 1975

Department of Managerial Economics and Decision Sciences
Graduate School of Management
Northwestern University
Evanston, Illinois 60201

*Conversations with Theodore Groves, Leonid Hurwicz, John Ledyard, Elisha Pazner, and Hugo Sonnenschein were of great help in this paper's genesis and development. Any errors, however, are solely our invention.

ABSTRACT

An Impossibility Theorem for Deterministic Organizations

by

Jean-Marie Blin and Mark A. Satterthwaite, January 1975

An organization is a group of rational agents each of whom picks a strategy. If their profile of chosen strategies is given, then a singlevalued function from strategy profiles to possible outcomes determines the organization's outcome. Agents have preferences which are not _a priori_ restricted to some subset of the possible orderings of the outcome set. Agents, in their attempts to maximize, choose a strategy profile which is an equilibrium according to whatever equilibrium concept (core, Nash equilibrium, etc.) is appropriate.

The primary theorem shows that if such an organization can achieve at least three distinct outcomes, then a singlevalued function cannot exist between agents' preference profiles and equilibrium outcomes. A preference profile must exist for which either no equilibrium outcome exists or at least two equilibrium outcomes exist. Thus such organizations cannot be deterministic and must leave agents room for strategic maneuvering. Proof follows from the Gibbard and Satterthwaite theorem on strategy-proof voting procedures.

An Impossibility Theorem for Deterministic Organizations

By: Jean-Marie Blin
and
Mark A. Satterthwaite

I. Introduction

Does there exist a singlevalued function from the preferences of the rational agents who comprise an organization to the organization's realized equilibrium outcomes? This paper proves that, in general, one does not exist. The preferences of rational agents, of course, do influence the outcome of an organization's decision process, but except in very special cases they do not invariably determine a unique equilibrium outcome. In general the strategic choices of each agent concerning how to pursue his preferences also influence the outcome. Thus if an observer has knowledge of the agents' preferences and the organization's structure but no knowledge concerning the agents' volitional choices of particular strategies, then the observer is generally unable to predict with certainty the decision process's equilibrium outcome. The best he can always do is to eliminate some of the feasible outcomes as realistic possibilities and then say that one of the remaining feasible outcomes will be the actual outcome. This inability to do better stems not from an incomplete model on the observer's part; rather it stems from the intrinsic indeterminancy which arises whenever two or more conflicting, rational agents try to outwit each other.

The paper's formal results may be summarized as follows. An organization is a group of agents for whom everything including their initial endowments and technology is fixed except for their preferences among the possible outcomes and their choices of strategies. Agents' choices of strategies determine the organization's outcome and each agent, if he is rational, chooses that strategy which he believes will secure him the best attainable outcome. Our question is: if all agents are rational, then do their preferences uniquely determine their choices of strategies or does an element of volition also enter into their choices? We call organizations where preferences do uniquely determine their strategic choices strategy-free because it is only in such organizations that the organization's outcome is uninfluenced by the agents' strategic manueverings. Therefore an organization is strategy-free if and only if a singlevalued function exists which maps agents' preferences into the set of possible outcomes. Theorem 2, the paper's primary result, states that if an organization is strategy-free and has (a) rational agents who are not consistently ignorant of the consequences of their actions, (b) a range of attainable outcomes containing at least three elements, and (c) the preferences of its agents are not a priori restricted to some subset of the collection of all possible orderings of the set of attainable outcomes, then the organization is dictatorial. An organization is dictatorial if one agent -- the dictator -- has the power against the opposition of all other agents to choose that outcome which he most prefers.

Conditions (a) and (b) are not strict. Only condition (c) is violated by a large variety of the organizations familiar to economic theory. For example, an exchange economy is an organization where the agents' preferences over the attainable outcomes are a priori restricted to be convex and selfish. Consequently, Theorem 3 does not exclude the existence of an allocation mechanism for an exchange economy that is both non-dictatorial and strategy-free. We conjecture, however, that no such mechanism exists. Proof of this conjecture would definitively establish that most organizations modeled by economic theory are not strategy-free.

Therefore this paper's primary contribution is to prove that, for some preference profile of the agents, every non-dictatorial organization with rational agents whose preferences are free to vary among a range of feasible outcomes is a game. This conclusion should not be too surprising because its truth appears to have been taken as self-evident by game theorists. For instance, Shapley [20] has written:

> An explanation for this multiplicity may be found in the essential ambiguity of decision-making in the presence of several independent free wills. Simple rationality (utility maximization) is not a sufficient determinant of behavior when one ventures beyond simple cases like the one-person decision problem or the two person game with directly opposed interests. Determinateness (i.e. uniqueness of outcome) may be desirable in a solution, but it can generally be obtained only at the price of oversimplifying or ignoring the observed tendencies toward organized cooperation (e.g. markets, political parties, cartels, etc.) when real people cope with the indeterminancy of real-life multilateral competition.

Nevertheless within the context of two strands of recent economic thought this paper's question is an important one that needs proof.

First, within the social choice literature as originated by Black [2] and most importantly Arrow [1] the question of strategic misrepresentation of preferences on the part of agents is of great interest. Early investigators of this question within the social choice context were Vickery [21], Farquharson [4], and Dummett and Farquharson [3]. Recently Gibbard [5] and Satterthwaite [17][18] independently proved that if the number of attainable alternatives is at least three, then no non-dictatorial voting mechanism exists which always gives every agent an incentive to reveal their true preferences. Zeckhauser [23], also independently, has proven a similar theorem. Subsequently Pattanaik [16], Kalai, Pazner, and Schmeidler [12], and Schmeidler and Sonnenschein [17] have all made significant additional to this literature.

The second strand of economic thought which is integral to this paper is the idea of incentive compatable organizations. In essence the question of incentive compatability as first asked by Hurwicz [10] [11] and then by, for example, Ledyard and Roberts [15], Groves [7] [8], Groves and Ledyard [9], and Green and Laffont [6] is the same question as asked by Gibbard [5] and Satterthwaite [18]. The only difference in the questions is that in the incentive compatability case the environment of the question has been the economist's usual model of an economy with consumers, firms, private goods, and public goods. This contrasts with the social choice literature which has generally confined itself to a less structured environment where there are only agents whose collective task is to rank or choose among a set of simple alternatives. But, as this paper seeks to show, this difference is more superficial than fundamental. Theorem 2 applies to organizations which differ from the organizations usually modeled by economic theory only with respect to the domain of admissible preferences. The theorem's proof, however, is based on the Gibbard [5] and Satterthwaite [18] strategy-proofness theorem which is a product of the social choice literature.

The organization of this paper is straightforward. Section two establishes the basic model and notation which we use throughout. Sections three, four, and five present material which is necessary for section six's statement proof and discussion of Theorem 2. Finally in sections seven and eight we apply our results. First, in section seven, we show using Theorem 2 that the Gibbard-Satterthwaite theorem on strategy-proof voting procedures continues to hold when less stringent equilibrium concepts are substituted for the Nash equilibrium which both Gibbard [5] and Satterthwaite [18] used. Second, in section eight, we show how our results relate to the problem of designing incentive compatable organizations.

2. Formulation

An <u>organization</u> Γ consists of a set of agents $\mathcal{I}$, a set of admissible strategies $\mathcal{S}$, a set of outcomes $\mathcal{X}$, a set of admissible preference profiles $\mathcal{R}$, an outcome function g, a solution correspondence ω, and an effective range $\mathcal{Y}$. We represent this structure concisely by the seven-tuple $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$.[1] An organization Γ functions as follows. Each of the organization's n <u>agents</u> $i \in \mathcal{I}$ picks a <u>strategy</u> s_i from his countable set of admissible strategies $\mathcal{S}_i$. A strategy s_i is a perfectly general and abstract representation of communication and action by participant i. For example, s_i might represent offers to buy and sell particular commodities at specific prices and in specific quantities. A strategy may involve the physical production of goods from raw materials. It may also include contingency plans of great variety and complexity, e.g. if agent j does this, then agent i will do the following, but if he does that, then agent i will do the opposite, etc. Let the n-tuple $s = (s_1, \ldots, s_n)$ be a <u>strategy profile</u> and let $\mathcal{S} = \prod_{i \in \mathcal{I}} \mathcal{S}_i$ be the set of <u>admissible strategy profiles</u>.

Given the agents' choice of a strategy profile $s \in \mathcal{S}$, then the <u>outcome function</u> g determines the consequences of their actions. This function g is singlevalued, has a domain of $\mathcal{S}$, a range of $\mathcal{X}$ and associates a single outcome with each admissible strategy profile. The <u>alternative set</u> $\mathcal{X}$ is the collection of all possible outcomes which can result from the participants' choice of a strategy profile, i.e. for every $x \in \mathcal{X}$ there exists a $s \in \mathcal{S}$ such that $g(s) = x$.

Each agent $i \in \mathcal{I}$ has preferences concerning the outcome. Let the preferences of agent i be represented by the weak order R_i defined over $\mathcal{X}$, i.e. R_i is reflexive, complete, and transitive. Thus if $x, y \in \mathcal{X}$ and $i \in I_n$, then $x \; R_i \; y$ means that agent i prefers outcome x over outcome y or is indifferent bewteen outcomes x and y. Let $x \; \overline{R}_i \; y$ denote strict preference of x over y. Therefore $x \; \overline{R}_i \; y$ is equivalent to the conjunction of $x \; R_i \; y$ and $\sim y \; R_i \; x$ where $\sim$ represents negation. Similarly let indifference, which can be denoted by $x \; R_i \; y$ and $y \; R_i \; x$, be more simply denoted by $x \; \tilde{R}_i \; y$. Clearly an agent's preferences R_i are purely internal to his mind. An observer can never know with certainty what an agent's preferences are; he can only, infer from the agent's actions and statements what they are likely to be. Nevertheless, despite their unobservable nature, an agent's preferences are the controlling motives for his actions.

Individual preferences are not necessarily free to vary unrestrictedly. Let $\mathcal{R}_i$ represent the collection of R_i that are admissible for participant i and let $\mathcal{R} = \prod_{i \in \mathcal{I}} \mathcal{R}_i$ represent the collection of admissible preference profiles. For example, if $\mathcal{X}$ is commodity space, then a sensible restriction on each participant's admissible preferences is that they be convex, i.e., $\mathcal{R}_i$ is the collection of all possible R_i that are convex π for agent i over $\mathcal{X}$. Restriction of preferences, however, should not be forced. If an agent's preferences could conceivably be a particular weak order R_i, then $\mathcal{R}_i$ must be defined to include R_i as an element. As a baseline, let π represent the collection of all logically possible preferences R_i that are defined over $\mathcal{X}$ and let π^n, the n-fold cartesian product of π, represent the collection of all logically possible preference profiles $R = (R_1, \ldots, R_n)$ that are defined over $\mathcal{X}$. Similarly let ρ represent the collection of all logically possible preferences R_i that are defined over $\mathcal{X}$ and that exclude indifference among outcomes. Finally let ρ^n represent the n-fold cartesian product of ρ.

Given $\mathcal{J}$, $\mathcal{X}$, $\mathcal{S}$, g, his own preference profile $R_i \in \mathcal{R}$, and any knowledge he may have concerning other agents' preferences, every agent $i \in \mathcal{J}$ tries to pick that strategy which maximizes the outcome with respect to his own preferences R_i. In general his choice of strategy will not be straightforward in the sense that his optimal strategy s_i will depend on the strategies which the other agents pick. This means, in the formal sense first defined by von Neumann and Morgenstern [22], that the agents are engaged in a game. Consequently what equilibrium strategy profile the participants will settle on is not immediately evident. For example, in some contexts, a reasonable assumption is that the equilibrium strategy profile will be a Nash equilibrium. In other contexts, a more reasonable assumption is that the equilibrium will be an element of the core.

Therefore, given an organization Γ, the crucial problem in analyzing its behavior is to define the correspondence between the agents preferences and their strategies. Let this correspondence be called the <u>solution correspondence</u> and let it be represented by the symbol ω. This correspondence $\omega : \mathcal{R} \rightarrow \mathcal{S}$ associates with each admissible preference profile a set of admissible strategy profiles. The set $\omega(R)$ may have one element, several elements, or be empty.

If a strategy profile s is an element of $\omega(R)$, then it is an admissible strategy profile and, given that the agents have preferences $R \in \mathcal{R}$, it is both an equilibrium profile and an attainable profile.

An <u>equilibrium strategy profile</u> $s \in \mathcal{S}$ for the preference profile $R \in \mathcal{R}$ means that if the agents should ever attain the profile s, then each agent i is satisfied with the outcome in the sense that he, either individually or in coalition, will not change his strategy from s_i to some $s_i' \neq s_i$. Thus an equilibrium strategy profile is a stable profile. An <u>attainable strategy profile</u> is a profile which the agents might conceivably reach. In other words, if $s = (s_1,\ldots,s_i,\ldots,s_n) \in \mathcal{S}$ is attainable, then a sequence of plausible communications among the agents must exist which would contemporaneously lead each agent i to consider making s_i his strategy. Of course, if the profile s is attainable but not an equilibrium, then when it is attained some agent will decide to change his strategy from s_i to s_i'. Similarly, a profile s may be an equilibrium profile for preference R but not be attainable. Therefore, if some observer knows only that the agents' preference profile is $R \in \mathcal{R}$, then he should place a strictly positive probability of occurrence on each strategy profile $s \in \omega(R)$. If he places zero probability of occurrence on a strategy profile $s \in \mathcal{S}$, then it should either be an unattainable profile or not an equilibrium profile. Once the agents do attain a particular equilibrium strategy profile $s \in \omega(R)$, then the outcome is determined: $x = g(s)$. Let the <u>effective range</u> of an organization

$\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ be the set $\mathcal{Y}$ of outcomes $x \in \mathcal{X}$ for which a preference profile $R \in \mathcal{R}$ exists such that, for some $s \in \omega(R)$, $x = g(s)$.

Four types of organizations play an important role in this paper: dictatorial, singlevalued, essentially singlevalued, and strategy-free organizations. The role each plays becomes clear as the paper progresses. An organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ is <u>dictatorial</u> if an agent $i \in \mathcal{I}$ exists such that, for all $R \in \mathcal{R}$, all $z \in \mathcal{Y}$, and all $s \in \omega(R)$, $g(s)\ R_i\ z$. Agent i is called the dictator because he always obtains an outcome which he ranks as high as any other outcome contained within $\mathcal{Y}$. An organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{R}, \mathcal{X}, g, \omega, \mathcal{Y} \rangle$ is <u>singlevalued</u> if and only if ω is singlevalued for all $R \in \mathcal{R}$. Thus, if an organization is singlevalued, then with each preference profile $R \in \mathcal{R}$ is associated a unique strategy profile $s = \omega(R)$ and, consequently, a unique outcome $x = g(s)$. An organization Γ is <u>essentially singlevalued</u> if and only if (a) it is not singlevalued, (b) for all $i \in \mathcal{I}$, for all $R \in \mathcal{R}$, and for all $s', s'' \in \omega(R)$, $g(s')\ \tilde{R}_i\ g(s)$, and (c), for all $R \in \mathcal{R}$, $\omega(R)$ is not empty. In other words, if Γ is essentially singlevalued, then for some $R \in \mathcal{R}$ the correspondence ω is not singlevalued in a trivial manner: no agent prefers any one potential outcome $g(s')$ to any other potential outcome $g(s'')$. Finally, an organization Γ is <u>strategy-free</u> if and only if it is either singlevalued or essentially singlevalued.

Our motivation for calling strategy-free any organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ that is either singlevalued or essentially singlevalued is simple. Consider an organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ that is singlevalued. Each preference profile $R \in \mathcal{R}$ determines a unique outcome: $x = g[\omega(R)]$. No room for successful strategic manuevering on each agent's part exists because no matter how much manuevering is done, the equilibrium outcome is still $x - g[\omega(R)]$.[2] Hence the label strategy-free organization. Contrast this with an organization that is not strategy-free. Two cases exist. First, suppose that, for some $R \in \mathcal{R}$, $\omega(R)$ is empty, i.e. no equilibrium strategy profile exists. Nevertheless some strategy profile $s \in \mathcal{S}$ is settled upon by the agents and the identity of this profile is determined by the agents' strategic behavior. Those agents who most skillfully, forcefully, and luckily pursue their own goals tend to do better than other agents. Second, suppose that, for some $R \in \mathcal{R}$, some $i \in \mathcal{I}$, and some s', $s'' \in \omega(R)$, $g(s')\ \overline{R}_i\ g(s'')$. By definition both s' and s'' are possible equilibria. Again the determining factor as to which profile is actually realized will be the agents' strategic behavior.

3. Voting Procedures

Gibbard [5] and Satterthwaite [17][18] have studied a specific class of strategy free organizations called voting procedures. A voting procedure is any organization V for which the set of admissible strategy profiles is identical to the set of admissible preference profiles, i.e. $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, v, \omega, \mathcal{Y} \rangle$. Consequently, instead of the argument of a voting procedure's outcome function v being a strategy profile s, the argument is a profile $R' = (R_1', \ldots, R_n') \in \mathcal{R}$ of stated preferences. This stated preference profile is not necessarily identical to the profile of "true" preferences $R = (R_1, \ldots, R_n)$ which guides each agent's behavior. If the stated preferences $R_i' \in \mathcal{R}$ of an agent $i \in \mathcal{I}$ are identical to his preferences $R_i \in \mathcal{R}$, then R_i' is his sincere strategy. If, however, his stated preferences $R_i' \in \mathcal{R}$ are not identical to his preferences R_i, then R_i' is a sophisticated strategy. If within a particular profile of stated preferences every agent uses his sincere strategy, then that profile is a sincere strategy profile. Let the notation R/R_i and R/R_i' denote respectively $(R_1, \ldots, R_{i-1}, R_i, R_{i+1}, \ldots, R_n) = R$ and $(R_1, \ldots, R_{i-1}, R_i', R_{i+1}, \ldots, R_n)$.

The solution concept which Gibbard and Satterthwaite used in their studies is Nash equilibrium. Consider a voting procedure $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, v, \omega, \mathcal{Y} \rangle$. V is manipulable at sincere strategy profile $R = (R_1, \ldots, R_i, \ldots, R_n) \in \mathcal{R}$ by agent i if and only if a $R_i' \in \mathcal{R}_i$ exists such that

(1) $$v(R/R_i')\overline{R}_i\ v(R/R_i).$$

Thus, if V is manipulable at R by agent i, then he can improve the outcome relative to his own preferences R_i by employing the sophisticated strategy R_i' instead of his sincere strategy R_i. The voting procedure V is strategy-proof if and only if for all agents $i \in \mathcal{J}$ no sincere strategy profile $R \in \mathcal{R}$ exists at which agent i can manipulate the outcome. Therefore, if V is strategy-proof, then every admissible sincere strategy profile is a Nash equilibrium.

If a voting procedure $V = \langle \mathcal{J}, \mathcal{R}, \mathcal{X}, \mathcal{R}, v, \omega, \mathcal{Y} \rangle$ is strategy-proof, then V is singlevalued and therefore strategy-free. This may be seen by supposing that an agent's preferences are $R_i \in \mathcal{R}_i$. His optimal strategy is then to play his sincere strategy R_i because, no matter what the other agents' strategies are, the definition of strategy-proofness implies that no sophisticated strategy exists which results in a more preferred outcome. Therefore his best strategy is his sincere strategy. Consequently, if V is strategy-proof, then $\omega(R) = R$ because every agent's equilibrium strategy is his sincere strategy.

The conclusion which Gibbard [5] and Satterthwaite [18] proved is an impossibility theorem for strategy-proof voting procedures.

Theorem 1. (Gibbard-Satterthwaite). Consider a strategy-proof voting procedure $V = \langle \mathcal{J}, \mathcal{R}, \mathcal{X}, \mathcal{R}, v, \omega, \mathcal{Y} \rangle$. One of the following three conditions must be violated:

1.1 $\mathcal{R} = \pi^n$ or $\mathcal{R} = \rho^n$;

1.2 $\mathcal{Y}$ has at least three elements;

1.3 V is non-dictatorial.

If none of these conditions are violated, then V is not strategy-proof.

Proofs of this proposition are found in both Gibbard [5] and Satterthwaite [17] [18]. In addition Schmeidler and Sonnenschein [19] have recently constructed a more concise proof of the theorem. An alternative statement of the theorem is that if (1.1) and (1.2) hold, then every strategy-proof voting procedure is dictatorial.

This negative result concerning the impossibility of non-dictatorial strategy-proof voting procedures can be circumvented in two ways. First, if $\mathcal{Y}$ has two elements, then majority rule and its variants are strategy-proof. Second, if $\mathcal{Y}$ has at least three elements, then sufficient restriction on the set of admissable preferences allows construction of a strategy-proof voting procedure V. The truth of this assertion can be seen by considering the case where all agents have identical preferences. In such a case the question of strategy-proofness is meaningless. The agents are unanimous and therefore no agent has any need to misrepresent his preferences.

The question remaining concerns the intermediate cases: how far must the set of admissible preferences $\mathcal{R}$ be restricted so that a strategy proof voting procedure satisfying conditions (1.2) and (1.3) exists? We do not know the answer, but we conjecture that restricting $\mathcal{R}$ to the extent which economic models usually restrict $\mathcal{R}$ is not sufficient to allow construction of a strategy-proof voting procedure.

Conjecture 1. Consider a strategy-proof voting procedure $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, v, \omega, \mathcal{Y} \rangle$ where $\mathcal{I}$ contains n agents, $n \geq 2$. Let there be m private goods and q public goods, $m + q \geq 2$. Let $\mathcal{X}$ be a vector space in the positive orthant of $\mathbb{R}^{nm+q}$ such that each $x \in \mathcal{X}$ is an nm+q dimensional vector whose first nm elements specify a quantity of each private good for each agent and whose last q elements specify a quantity of each public good. One of the following three conditions must be violated:

1.1′ For all $i \in \mathcal{I}$, $\mathcal{R}_i$ is the collection of all possible preference orderings R_i over $\mathcal{X}$ that are both convex and selfish;

1.2′ For each $i \in \mathcal{I}$ there exist distinct outcomes $x, y, z, \in \mathcal{Y}$ such that for each of the six possible strict orderings of x, y, and z an admissible preference ordering $R_i \in \mathcal{R}_i$ exists which orders x, y, and z in that order;

1.3′ V is non-dictatorial.

If none of these conditions are violated, then V is not strategy-proof.

The reason why we think this conjecture is true stems from the fact that over admissible domains $\mathcal{R}$ such as defined by (1.1′) it has been impossible to construct a social welfare function satisfying Arrow's conditions [1] of non-negative response, citizen's sovereignty, and independence of irrelevant alternatives.[3]

The duality which Satterthwaite [18] has shown to exist between strategy-proof voting procedures and social welfare functions satisfying Arrow's conditions when $\mathcal{R} = \rho^n$ then suggests that it is also impossible to construct a strategy-proof voting procedure over a domain $\mathcal{R}$ restricted as described by (1.1′).

4. Equivalent Organizations and Implicit Voting Procedures

Consider two organizations $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y}\rangle$ and $\Gamma' = \langle \mathcal{I}, \mathcal{S}', \mathcal{X}, \mathcal{R}, g', \omega', \mathcal{Y}'\rangle$. Define F, the <u>preference-outcome correspondence</u> for Γ, as the composition of g and ω: given a $R \in \mathcal{R}$, $x \in F(R)$ if and only if an $s \in \omega(R)$ exists such that $x = g(s)$. If $\omega(R)$ is empty, then F(R) is also empty. Define F', the preference-outcome correspondence for Γ', analogously. The organizations Γ and Γ' are <u>equivalent at profile $R \in \mathcal{R}$</u> if and only if (a) F(R) and F'(R) are not empty, (b) for all $x \in F(R)$ an $x' \in F'(R)$ exists such that $x \tilde{R}_i x'$ for all $i \in \mathcal{I}$, and (c) for all $x' \in F'(R)$ an $x \in F(R)$ exists such that $x' \tilde{R}_i x$ for all $i \in \mathcal{I}$. Organizations Γ and Γ' are <u>equivalent</u> if and only if they are equivalent at all $R \in \mathcal{R}$. Clearly, this definition implies that a sufficient, but not necessary, condition for Γ and Γ' to be equivalent is that, for all $R \in \mathcal{R}$, $F(R) \equiv F'(R)$. Moreover the definition implies that if Γ and Γ' are equivalent organizations, then no agent has a rational reason to prefer Γ or Γ' over the other.

It is easily shown that our equivalence definition for voting procedures fulfills the requirements of reflexivity, symmetry, and transitivity which mathematical convention usually requires of equivalence relations. Our definition immediately implies that (a) any Γ is equivalent to itself and (b) if Γ is equivalent to Γ', then Γ is equivalent to Γ. Therefore reflexivity and symmetry are satisfied. Transitivity does not follow immediately from the definition, but is easily proven as follows. Suppose that, while Γ and Γ' are equivalent and Γ' and Γ'' are equivalent, Γ and Γ'' are not equivalent. This implies, without loss of generality, that for some $R \in \mathcal{R}$, some $i \in \mathcal{I}$, and some $x \in F(R)$ no $x'' \in F''(R)$ exists such that $x'' \tilde{R}_i x$. But, because Γ and Γ' are equivalent, a $x' \in F'(R)$ exists such that $x' \tilde{R}_i x$. Similarly, because Γ' and Γ'' are equivalent, a $x'' \in F''(R)$ exists such that $x'' \tilde{R}_i x'$. Transitivity of the indifference relation $\tilde{R}_i$ therefore implies that $x'' \tilde{R}_i x$ where $x'' \in F''(R)$ and $x \in F(R)$. This contradicts the assumption that no such x'' exists.

For any essentially singlevalued organization $\Gamma = \langle \mathcal{I}, \mathcal{A}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ we can easily construct an equivalent singlevalued organization $\Gamma' = \langle \mathcal{I}, \mathcal{A}', \mathcal{X}, \mathcal{R}, g', \omega', \mathcal{Y}' \rangle$. Set $g' \equiv g$. Define for each $R \in \mathcal{R}$, $\omega'(R) = s$ where s is an arbitrarily chosen element of $\omega(R)$. Therefore ω' is singlevalued which means that Γ' is a singlevalued organization. Pick an arbitrary $i \in \mathcal{I}$ and arbitrary $R \in \mathcal{R}$. The essential singlevaluedness of Γ implies that, for all $s, s' \in \omega(R)$,

(2) $$g(s) \tilde{R}_i g(s').$$

By the construction of ω', a $s'' \in \omega(R)$ exists such that $s'' = \omega'(R)$. Therefore (2) implies that, for all $s \in \omega(R)$

(3) $$g(s'') \tilde{R}_i g(s).$$

By our definition that $g' \equiv g$, $g'(s'') \equiv g(s'')$. Therefore substitution gives, for all $s \in \omega(R)$,

(4) $$g'(s'') \tilde{R}_i g(s).$$

Because the choice of $i \in \mathcal{I}$ and $R \in \mathcal{R}$ were arbitrary, the conclusion is general: for all $i \in \mathcal{I}$ and $R \in \mathcal{R}$, (4) holds whenever $s'' = \omega'(R)$ and $s \in \omega(R)$. Thus Γ and Γ' are equivalent. Finally, to complete the construction of Γ', let $\mathcal{S}' = \{s \mid s = \omega'(R)$ for some $R \in \mathcal{R}\}$ and let $\mathcal{Y}' = \{x \mid x = g'(s)$ for some $s \in \mathcal{S}'\}$. This result is summarized by Lemma 1.

Lemma 1. If an organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y}\rangle$ is essentially singlevalued, then a singlevalued organization $\Gamma' = \langle \mathcal{I}, \mathcal{S}', \mathcal{X}, \mathcal{R}, g, \omega', \mathcal{Y}'\rangle$ that is equivalent to Γ can be constructed.

This lemma is the justification for calling an essentially singlevalued organization essentially singlevalued.

If an organization Γ is singlevalued, then a voting procedure V can be constructed which is equivalent to Γ. This construction from a singlevalued organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y}\rangle$ of an equivalent organization $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, f, \omega', \mathcal{Y}\rangle$ that satisfies the requirements for being a voting procedure proceeds as follows. Since the outcome function g and equilibrium correspondence ω of the organization Γ are both singlevalued, the preference-outcome mappying F of Γ is also singlevalued: $F(R) = g[\omega(R)]$ for all $R \in \mathcal{R}$. This function F maps the unobservable preference profiles $R \in \mathcal{R}$ of the agents onto the effective range $\mathcal{Y}$ of Γ. Define the outcome function f for the organization V to be identical to F : $f(R) \equiv F(R)$ for all $R \in \mathcal{R}$. Therefore its set of admissible strategies is $\mathcal{R}$, the set of admissible preferences. Define the equilibrium correspondence ω' such that, for all $R \in \mathcal{R}$, $\omega'(R) = R$. These definitions for f and ω' together imply that the effective range of V is unchanged from that of Γ. Call the resulting organization $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, f, \omega', \mathcal{Y} \rangle$ the implicit voting procedure of Γ. This organization V is in fact a voting procedure because its set of admissible strategy profiles is identical to the set of admissible preference profiles. Moreover V is equivalent to Γ because the construction of V guarantees that both it and Γ have identical preference-outcome correspondences. Lemma 2 summarized this result.

Lemma 2. Any singlevalued organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ is equivalent to its implied voting procedure. $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, f, \omega', \mathcal{Y} \rangle$ where, for all $R \in \mathcal{R}$, $f(R) = g[\omega(R)]$ and $\omega'(R) = R$.

This technique of constructing for every singlevalued organization in equivalent voting procedure is a generalization of Gibbard's method [5] of rewriting game forms equivalently as voting schemes.

5. Ignorant and Knowledgeable Agents

The agents of organizations may be characterized in particular situations to be either ignorant or knowledgeable. Consider an organization $\Gamma = \langle \mathcal{I}, \mathcal{A}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ with preference-outcome correspondence F. An agent $i \in \mathcal{I}$ is <u>ignorant at preference profile $R \in \mathcal{R}$</u> if and only if an $R_i' \in \mathcal{R}$ exists such that (a) $F(R/R_i)$ and $F(R/R_i')$ are not empty, (b) for all $x \in F(R/R_i)$ and all $x' \in F(R/R_i')$,

$$(5) \qquad x' \, R_i \, x$$

and (c) for some $y \in F(R/R_i)$ and some $y' \in F(R/R_i')$,

$$(6) \qquad y' \, \overline{R}_i \, y.$$

In other words, an agent is ignorant at preference profile $R \in \mathcal{R}$ if the outcomes which may result if he acts as if his preferences are R_i' instead of R_i, as they truly are, dominate the outcomes which may result if he continues to act as if his preferences are R_i.

An organization's agents are <u>ignorant</u> if some preference profile $R \in \mathcal{R}$ exists at which some agent $i \in \mathcal{I}$ is ignorant. An organization's agents are <u>knowledgeable</u> if they are not ignorant. A general property that follows from these definitions is that if two organization Γ and Γ' are equivalent and Γ has knowledgeable agents, then Γ' also has knowledgeable agents.

Lemma 3. Consider two equivalent organizations $\Gamma = \langle \mathcal{I}, \mathcal{A}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ and $\Gamma' = \langle \mathcal{I}, \mathcal{A}', \mathcal{X}, \mathcal{R}, g', \omega', \mathcal{Y}' \rangle$. Organization Γ has knowledgeable agents if and only if Γ' has knowledgeable agents. Similarly, Γ has ignorant agents if and only if Γ' has ignorant agents.

Proof. Suppose $\Gamma = \langle \mathcal{I}, \mathcal{A}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ has knowledgeable agents and $\Gamma' = \langle \mathcal{I}, \mathcal{A}', \mathcal{X}, \mathcal{R}, g', \omega', \mathcal{Y}' \rangle$ has ignorant agents. Also assume equivalence between Γ and Γ'. Let F and F′ be the respective preference-outcome correspondences of Γ and Γ'. Since Γ' has ignorant agents a $R \in \mathcal{R}$ and $R_i' \in \mathcal{R}_i$ exist for some $i \in \mathcal{I}$ such that (a), for all $w \in F'(R/R_i)$ and all $x \in F'(R/R_i')$.

(7) $\qquad x\ R_i\ w$

and (b), for some $w' \in F'(R/R_i)$ and some $x' \in F'(R/R_i')$,

(8) $\quad x'\ \overline{R}_i\ w'$.

Because Γ is equivalent to Γ' a $w \in F'(R/R_i)$ exists if and only if a $y \in F(R/R_i)$ exists such that $y\ \tilde{R}_i\ w$. Similarly a $x \in F'(R/R_i')$ exists if and only if a $z \in F(R/R_i')$ exists such that $x\ \tilde{R}_i\ z$. The transitivity of the indifference relation $\tilde{R}_i$ therefore allows substitution of y for w, y′ and w′, z for x, z′ for x′, and F for F′ in relationships (7) and (8): (a) for all $y \in F(R/R_i)$ and all $z \in F(R/R_i')$

(9) $\qquad z\ R_i\ y$

and (b), for some $y' \in F(R/R_i)$ and some $z' \in F(R/R_i')$,

(10) $\qquad z'\ \overline{R}_i\ y'$.

But this means, contrary to our assumption, that agent i in organization Γ is ignorant at preference profile R. Therefore Γ has ignorant agents if Γ' has ignorant agents. ||

Consideration of an organization $\Gamma = \langle \mathcal{I}, \mathcal{A}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ with ignorant agents illustrates our reasons for defining ignorant and knowledgeable agents in the manner which we do. Let F be the preference-outcome correspondence for Γ. Because Γ has ignorant agents a preference profile $R \in \mathcal{R}$ exists at which some agent $i \in \mathcal{I}$ is ignorant: for some $R_i' \in \mathcal{R}_i$, all $w \in F(R/R_i)$, and all $x \in F(R/R_i')$,

(11) $\qquad x \; R_i \; w$

and for some $w' \in F(R/R_i)$ and some $x' \in F(R/R_i')$,

(12) $\qquad x' \; \overline{R}_i \; w'$.

Thus if agent i would act as if his preferences were R_i' instead of R_i, then the outcome associated with R_i' would be at least as good as among outcome associated with R_i and, with positive probability, might be better. But because agent i is ignorant of the opportunity, he acts in his normal manner as described by $F(R/R_i)$.

Suppose for whatever reason the situation changes and agent i, no longer ignorant, recognizes that R is the preference profile and realizes that he can secure at least as good outcome and possibly a better outcome by pretending to himself that his preferences are R_i' and acting accordingly. If he is rational he will take the opportunity; he will act as if his preferences are R_i' instead of R_i. The other agents will never know of this internal misrepresentation by agent i because they can not read his mind.

It is consequently a very strong assumption to assume that an agent is ignorant. To assume that agent i is ignorant at preference profile R implies either that (a) he will never identify the preference profile R when it occurs or (b) that if on some occasion he does realize that the preference profile is R, then he will not understand that he can improve the outcome by pretending that his preferences are R_i'. But, with respect to (a), agents sometimes do know the preferences of other agents and, with respect to (b), agents do sometimes understand what the consequences of their actions may be. Thus a weaker and more realistic assumption is that agents are not ignorant, but that they recognize such situations at least part of the time. In other words, instead of defining F such that, for all $w \in F(R/R_i)$ and all $x \in F(R/R_i')$, $x\ R_i\ w$ and, for some $w' \in F(R/R_i)$ and some $x' \in F(R/R_i')$, $x\ \overline{R}_i\ w$, F should be redefined such that if $x \in F(R/R_i')$, then also $x \in F(R/R_i)$. Such a redefinition of F means that agent i is no longer ignorant at preference profile R because he is allowed to realize that it is in his best interest to act as if his preferences are R_i'.

This discussion of the motivation for our definitions of ignorant and knowlegeable agents also illustrates our motivation for defining $\omega(R)$ such that every $s \in \omega(R)$ is necessarily an attainable strategy profile, as well as being an equilibrium strategy profile. For example, suppose for some organization $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$, some preference profile $R \in \mathcal{R}$, and some agent $i \in \mathcal{I}$ the solution correspondence is defined such that

$$(13) \qquad s = F(R/R_i),$$

(14) $s', s'' = F(R/R_i')$,

(15) $g(s'') \bar{R}_i g(s)$, and

(16) $g(s) \bar{R}_i g(s')$

where $R_i' \in \mathcal{R}_i$. Since the outcomes associated with the preference profile R/R_i' do not dominate the outcomes associated with preference profile R/R_i agent i is not ignorant at profile R/R_i. But suppose for preference profile R/R_i' strategy profile s' is only an equilibrium strategy profile, but not also an attainable strategy profile. This means that if agent i acts as if his preferences are R_i', then he can be certain that s'' will be the equilibrium strategy profile on which the agents will settle. That is, he can be certain of obtaining the preferred outcome of $g(s'')$ if he acts as if his preferences are R_i'. Therefore in reality the outcome associated with profile R/R_i' does dominate the outcome associated with R/R_i and agent i is ignorant if he does not act accordingly. Consequently, the force of our definitions of ignorant and knowledgeable agents depends critically on our definition that $\omega(R)$ contains only attainable equilibria.

6. The Possibility of a Strategy-Free Organization

This section presents and proves the main result of this paper: a non-dictatorial strategy-free organization can only exist if admissible preferences are restricted, the effective range contains no more than two elements, or the agents are ignorant. The surprising aspect of this result is that it is completely independent of the choice of solution concept. We start with Theorem 1: a theorem concerning the stability of one specific class of organizations, voting procedures, when the solution concept is specifically Nash equilibrium. Now we show that the particularistic results of Theorem 1 imply the very general results of Theorem 2 which are valid for all organizations where admissible preferences are unrestricted no matter what the solution concept is. This jump depends on two ideas.

The first idea is Gibbard's [5] technique, which section four described, for reducing any singlevalued organization to its implicit voting procedure. The second idea, which forms the basis for our definitions and interpretations of knowledgeable and ignorant agents, is the concept of privacy as developed by Hurwicz [10]. The concept of privacy states that outcomes must depend solely on observable actions and characteristics of agents. An agent's preferences are unobservable so according to the privacy principle, outcomes must not depend directly on them. This is why, within our model of organizations, the outcome function g has as its argument agents' strategies or stated preferences, not their true preferences.

Theorem 2. Let $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ be a strategy-free organization. Let $\Gamma' = \langle \mathcal{I}, \mathcal{S}', \mathcal{X}, \mathcal{R}, g, \omega', \mathcal{Y}' \rangle$ be a single-valued organization that is equivalent to Γ. One of the following four conditions must be violated:

2.1 $\mathcal{R} = \pi^n$ or $\mathcal{R} = \rho^n$;

2.2 $\mathcal{Y}'$ has at least three elements;

2.3 Γ is non-dictatorial;

2.4 the agents of Γ are knowledgeable.

If none of these conditions are violated, then Γ is not a strategy-free organization.

Proof. Let $\Gamma = \langle \mathcal{I}, \mathcal{S}, \mathcal{X}, \mathcal{R}, g\ \omega, \mathcal{Y} \rangle$ be a strategy-free organization with knowledgeable agents. If Γ is essentially singlevalued, then on the authority of Lemma 1, construct an equivalent and singlevalued $\Gamma' = \langle \mathcal{I}, \mathcal{S}', \mathcal{X}, \mathcal{R}, g, \omega', \mathcal{Y}' \rangle$. If Γ is singlevalued, then let $\Gamma' = \Gamma$. This is permissible because every organization is equivalent to itself. Now construct $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, f, \omega'', \mathcal{Y}' \rangle$, the implied voting procedure of Γ', where, for all $R \in \mathcal{R}$, $f(R) = g'[\omega'(R)]$ and $\omega''(R) = R$. Lemma 2 states that V and Γ' are equivalent which imples that, because equivalence is transitive among organizations, V and Γ are equivalent. Therefore, by Lemma 3, V has knowledgeable agents because Γ has knowledgeable agents. Let F, F′ and F″ respectively be the preference-outcome correspondences of Γ, Γ', and V. F′ and F″ are singlevalued functions by definition. The construction of V from Γ' implies that $F'' \equiv F'$. Moreover, because Γ and Γ' are equivalent at every $R \in \mathcal{R}$, (a) if $x \in F(R)$, then $x \tilde{R}_i x'$ where $x' = F'(R)$ and (b) if $y' = F'(R)$, then a $y \in F(R)$ exists such that $y' \tilde{R}_i y$.

Our first substantive step in the proofis to show that V is a strategy-proof voting procedure because it has knowledgeable agents. Suppose the contrary: the agents of V are knowledgeable and V is not strategy-proof. This implies that an $i \in \mathcal{I}$, $R \in \mathcal{R}$, and $R_i' \in \mathcal{R}_i$ exist such that agent $i \in \mathcal{I}$ can manipulate V at sincere strategy R, i.e.

$$(17) \qquad f(R/R_i') \ \overline{R}_i \ f(R/R_i)$$

Substitution of $F''(R) = f[\omega''(R)] = f(R)$ into (13) gives

$$(18) \qquad F''(R/R_i') \ \overline{R}_i \ F''(R/R_i)$$

which implies, contrary to our hypothesis, that agent i is ignorant at R/R_i. Therefore the implied voting procedure V must be strategy-proof.

Suppose that $\mathcal{Y}'$, the effective range of both Γ' and V, has at least three elements and the set of admissible preference profiles is either $\mathcal{R} \equiv \pi^n$ or $\mathcal{R} \equiv \rho^n$. These two assumptions correspond to conditions (1.1) and (1.2) in Theorem 1. Moreover V is strategy-proof because Γ has knowledgeable agents. Therefore Theorem 1 implies that V is a dictatorial voting procedure. Suppose agent $i \in \mathcal{I}$ is the dictator of V. This means that, for all $R \in \mathcal{R}$ and all $x \in \mathcal{Y}'$, $F''(R) \ R_i \ x$. Therefore, because $F'' \equiv F'$ and $\mathcal{Y}'$ is the effective range of Γ', for all $R \in \mathcal{R}$ and all $x \in \mathcal{Y}'$, $F'(R) \ R_i \ x$, i.e. Γ' is dictatorial because V is dictatorial.

Suppose that even though V and Γ' are dictatorial Γ is not dictatorial. In particular, suppose that agent $i \in \mathcal{R}$ is not the dictator at preference profile $R \in \mathcal{R}$. Therefore a $x \in \mathcal{Y}$ and $y \in F(R)$ exist such that $x \bar{R}_i y$. Let $y' = F'(R)$. The equivalence of Γ and Γ' implies that $y' \tilde{R}_i y$. Consequently, by transitivity, $x \bar{R}_i y'$. There are two cases to consider: $x \in \mathcal{Y}'$ and $x \notin \mathcal{Y}'$. Suppose first that $x \in \mathcal{Y}'$. This contradicts the earlier result that Γ' is dictatorial: if $x \in \mathcal{Y}'$ and $y' = F'(R)$, then $x \bar{R}_i y'$ implies that Γ' is not dictatorial.

Now suppose that $x \notin \mathcal{Y}'$. Recall that $x \in \mathcal{Y}$. Therefore a $R' = (R_1', \ldots, R_i', \ldots, R_n') \in \mathcal{R}$ and $x' \in \mathcal{Y}'$ exist such that $x \in F(R')$ and $x' = F'(R')$. The equivalence of Γ and Γ' therefore implies $x' \tilde{R}_i x$. As a substitute for Γ' we can construct a new singlevalued $\Gamma^* = \langle \mathcal{I}, \mathcal{S}^*, \mathcal{X}, \mathcal{R}, g, \omega^*, \mathcal{Y}^* \rangle$ such that its preference-outcome correspondence F^* is identical to F' at all $R \in \mathcal{R}$ except at R': define $F^*(R) \equiv F'(R)$ for all $R \in \mathcal{R}$ except that $F^*(R') = x$ instead of x'. Note that Γ^* is equivalent to Γ and has the property that $x \in \mathcal{Y}^*$. In exactly the same manner that we constructed V we can construct the implied voting procedure V^* from Γ^*. Similarly V^* and Γ^* are dictatorial for the same reasons that V and Γ' are dictatorial.

Now return to the hypothesized counter-example where agent i is not the dictator for Γ at preference profile R, i.e. $x \in \mathcal{Y}$, $y \in F(R)$, and $x \bar{R}_i y$. The construction of Γ^* from Γ and Γ' implies that $y' \tilde{R}_i y$ where $y' = F^*(R)$. Consequently, by transitivity, $x \bar{R}_i y'$. By construction, $x \in \mathcal{Y}^*$. But this means that contrary to our earlier result, Γ^* is not dictatorial: if $x \in \mathcal{Y}^*$ and $y' = F^*(R)$, then $x \bar{R}_i y'$ implies that Γ^* does not have agent i as its dictator at preference profile R. Therefore agent i must be the dictator of Γ at preference profile $R \in \mathcal{R}$. Moreoever the same argument shows that agent i is the dictator of Γ at every profile $R \in \mathcal{R}$. ||

Conditions (2.2), (2.3), and (2.4) are not very strong and hence most organizations would appear to satisfy them. Only condition (2.1) appears strong enough for it to be likely that many organizations would violate it. Specifically, condition (2.2) that $\mathcal{Y}'$ contain at least three elements is very weak and is unlikely to be violated. Every organization but the most trivial can generate at least three outcomes. Condition (2.3) is even weaker because most organizations of any size are demonstrably not dictatorial. Condition (2.4) that agents are knowledgeable is also weak because agents are either knowledgeable or ignorant and, as we argued in section five, knowledgeable agents are the weaker of the two possible assumptions.

This leaves condition (2.1) that $\mathcal{R} = \pi^n$ or $\mathcal{R} = \rho^n$. It is strong because agents' preferences can often be a priori restricted to a subset of π^n or ρ^n. Nevertheless the strength of this condition may be a reflection of the state of our knowledge rather than a statement about the minimal conditions under which no strategy-free organization exists. If Conjecture 1 is true as we believe it is, then the same proof that we used to prove Theorem 2 remains valid and proves that the following conjecture must also be true.

Conjecture 2. Let $\Gamma = \langle \mathcal{I}, \mathcal{A}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ be a strategy-free organization where $\mathcal{I}$ contains n agents, $n \geq 2$. Let there be m private goods and q public goods, $m + q \geq 2$. Let $\mathcal{X}$ be a vector space in the positive orthant of $\mathbb{R}^{nm}$ such that each $x \in \mathcal{X}$ is an nm+q dimensional vector whose first nm elements specify a quantity of each private good for each agent and whose last q elements specify a quantity of each public good. Let $\Gamma' = \langle \mathcal{I}, \mathcal{A}', \mathcal{X}, \mathcal{R}, g, \omega', \mathcal{Y}' \rangle$ be a singlevalued organization that is equivalent to Γ. One of the following four conditions must be violated:

2.1′ For each $i \in \mathcal{I}$, $\mathcal{R}_i$ is the collection of all possible preference orderings R_i over $\mathcal{X}$ that are both convex and selfish;

2.2′ For each $i \in \mathcal{I}$ there exist distinct outcomes $x, y, z, \in, \mathcal{Y}'$ such that for each of the six possible strict orderings of x, y, and z an admissible preference ordering $R_i \in \mathcal{R}_i$ exists which orders x, y, and z in that order;

2.3′ Γ is non-dictatorial;

2.4′ The agents of Γ are knowledgeable.

If none of these conditions are violated, then Γ is not a strategy-free organization.

7. Application: Counter Threats and Stability of Sincere Strategy Profiles

As section three reported, Gibbard [5] and Satterthwaite [18] have studied the stability of sincere strategy profiles for voting procedures V = $\langle \mathcal{J}, \mathcal{R}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y}\rangle$ using Nash equilibrium as their solution concept. Pattanaik [16] has pointed out that this is a very stringent solution concept, because it assumes that an agent will substitute a sophisticated strategy for his sincere strategy without considering the possibility that other agents will change their strategies in order to block his gain or, conceivably, to punish him for employing a sophisticated strategy. Clearly the more an agent takes such considerations into account the less willing he will be to substitute a sophisticated stragegy for his sincere strategy.

Hence, Pattanaik asked, if a solution correspondence does take into account the existence of counter threats by other agents, might it be possible to construct a voting procedure V such that every agent always finds it optimal to employ his sincere strategy. In such a voting procedure each threat of an agent to employ a sophisticated strategy would be blocked by the threats of other agents' countering sophisticated strategies. To answer his question, Pattanaik defined two specific solution concepts, each of which takes into account that counterthreats may prevent an agent from using a sophisticated strategy which is optimal in the Nash equilibrium sense. He then showed that an analogue of Theorem 1 continues to hold: even when these particular, less stringent solution concepts are used: a sincere strategy profile R $\in \mathcal{R}$ exists such that some agent has an incentive to employ a sophisticated strategy R_i'.

Our intent here is to show that these interesting results can be derived as corrolaries of this paper's Theorem 2. In essence the question that Pattanaik asked was: for a non-dictatorial voting procedure $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$ where $\mathcal{R} = \pi^n$ and $\mathcal{Y}$ has at least three elements can the outcome function g and solution concept be defined such that (a) the agents are knowledgeable and (b) the solution correspondence ω as implied by the chosen solution concept has the property that, for all $R \in \mathcal{R}$, $\omega(R) = R$. Condition (a) is implicit throughout Pattanaik's paper as the assumption that each agent picks that strategy which maximizes the outcome with reference to his own preferences. Condition (b) is a simple restatement that the goal is to construct a voting procedure where every agent always finds it optimal to use his sincere strategy.

Recall that every voting procedure V is merely a special type of organization. This means that condition (b), since it requires that ω be singlevalued, is a requirement that V be a strategy-free organization. Consequently Theorem 2 applies: the requirements that $\mathcal{R} = \pi^n$, $\mathcal{Y}'$ has at least three elements, the agents be knowledgeable, V be strategy-free, and V be non-dictatorial are mutually inconsistent. Therefore we have proved the following theorem.

Theorem 3. Consider a voting procedure $V = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, g, \omega, \mathcal{Y} \rangle$. If V is essentially singlevalued, let $V' = \langle \mathcal{I}, \mathcal{R}, \mathcal{X}, \mathcal{R}, g, \omega', \mathcal{Y}' \rangle$ be an equivalent singlevalued voting procedure. If V is not essentially singlevalued, then let $V' \equiv V$. One of the following five statements, no matter what solution concept is used, must be violated:

(3.1) $\mathcal{R} = \pi^n$ or $\mathcal{R} = \rho^n$;

(3.2) $\mathcal{V}'$ has at least three elements;

(3.3) V is non-dictatorial;

(3.4) the agents of V are knowledgeable;

(3.5) $\omega(R) = R$ for all $R \in \mathcal{R}$.

Thus the answer to Pattanaik's question is negative for all possible solution concepts, not just for the solution concepts which he tested.

8. Application: Incentive Compatible Organization

Hurwicz [10] [11] has asked if organization can be designed so that individuals do not have an incentive to misrepresent their preferences in order to secure a more favorable outcome for themselves. He calls such an organization, when it exists, inventive compatible. The motivation for searching for incentive compatible organizations stems from the observation that misrepresentation of preferences often leads to non-optimal outcomes. For example, the free-rider problem,which prevents the achievement of Pareto optimality within an exchange economy that has public goods, is caused by the incentive that each agent has to misrepresent how much an additional unit of public good is worth to him.

A specific type of organization whcih Hurwicz [10] has analyzed in some detail is the competitive allocation mechanism. The mechanism works as follows. Each agent begins with an initial allocation of goods and is then asked to reveal his entire preference ordering R_i subject to the restriction that the revealed R_i is both convex and selfish over the feasible set of outcomes $\mathcal{X}$. Let each agent's true preferences be R_i and his revealed preferences be R_i'. After the agents have revealed R_i', a set of prices are calculated that would be the competitive market clearing prices if the profile of revealed preferences $R' = (R_1',\ldots,R_i',\ldots,R_n')$ were true profiles. Given these calculated competitive prices, a budget constraint is calculated for each agent: the final allocation must be worth no more than the initial allocation when both the initial and final allocations are valued at the calculated prices. Subject to this budget constraint, a final allocation is calculated for each agent by picking that possible bundle of goods which agent i most prefers according to his revealed preferences R_i'. This set of final allocations is market clearing because the trades among the agents are based on a set of prices that are perfectly competitive for the revealed preference profile R'.

Hurwicz [10] showed that when the number of agents is finite this mechanism gives each agent an incentive not to reveal his true preferences R_i, but rather to reveal a biased preference ordering $R_i' \neq R_i$. In other words, the profile of true preferences $R = (R_1,\ldots,R_i,\ldots,R_n)$ is not a Nash equilibrium. The conclusion which follows from this result is that this competitive allocation mechanism can not be relied on to produce Pareto optimal outcomes. If all agents did report their true preferences, then the final outcome would be Pareto optimal because it would be a perfectly competitive outcome. But in general, unless the set of agents is atomless and agents have zero influence on prices, each agent does have an incentive to misrepresent which means that it is pure chance if all misrepresentations do cancel out and the final outcome is Pareto optimal.

These results are in agreement with what Conjecture 1 leads us to expect. The competitive mechanism described above is a voting procedure where the solution concept is Nash equilibrium. If the competitive mechanism were incentive compatible, then the solution correspondence would be, for all $R \in \mathcal{R}$, $\omega(R) = R$. In other words, if the competitive mechanism were incentive compatible, then it would be a strategy-proof procedure. But, if

Conjecture 1 is true, the competitive mechanism can only be strategy-proof if it violates at least one of conditions (1.1′) through (1.3′). Inspection shows that none of the conditions are violated. The competitive mechanism is non-dictatorial and Hurwicz defines the collection of admissible revealed preferences to be those preferences that are convex and selfish. Finally, condition (1.2′) is satisfied because the assumption of convex, selfish preferences is not very restrictive and the competitive mechanism gives a wide range of outcomes as preferences are varied over this admissible domain.

This argument is very general. Conjecture 1 applies to any type of non-dictatorial voting procedure, not just to the competitive mechanism that Hurwicz analyzed. Hence the conjecture leads us to expect that no non-dictatorial, incentive compatable mechanism exists for allocating goods within an exchange economy. Conjecture 2 goes even further: it states that even if incentive compatability is redefined using a less stringent notion of stability than Nash equilibrium, then still no non-dictatorial, incentive compatable organization with knowledgeable agents exists within the exchange economy environment.

Footnotes

1. The definition of organization used here is adapted from Ledyard [14].

2. This assumes that the agents will eventually settle on an equilibrium strategy profile.

3. For a good discussion of this, see Kramer [13].

References

[1] K. J. Arrow, _Social Choice and Individual Values_, 2nd ed., Wiley, New York, 1963.

[2] D. Black, _The Theory of Committees and Elections_, Cambridge University Press, Cambridge, England, 1958.

[3] A. Dummett and R. Farquharson, "Stability in Voting," _Econometrica_, _29_ (1960), 33-44.

[4] R. Farquharson, _Theory of Voting_, Yale University Press, New Haven, 1969.

[5] A. Gibbard, "Manipulation of Voting Schemes: A General Result," _Econometrica_, _41_ (1973), 587-602.

[6] J. Green and J. Laffont, "On the Revelation of Preferences for Public Goods," Technical Report No. 140, Economic Series, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1974.

[7] T. Groves, "Incentives in Teams," _Econometrica_, _41_ (1973), 617-63.

[8] T. Groves, "Information, Incentives, and the Internalization of Production Externalities," Discussion Paper #87, Center for Mathematical Studies in Economics and Management Science, Northwestern University, May 1974.

[9] T. Groves and J. Ledyard, "An Incentive Mechanism for Efficient Resource Allocation in General Equilibrium with Public Goods," Discussion Paper #119, Center for Mathematical Studies in Economics and Management Science, Northwestern University, Dec. 1. 1974.

[10] L. Hurwicz, "On Informationally Decentralized Systems," in C. B. McGuire and R. Radner, eds., Decision and Organization, North-Holland, Amsterdam, 1972, pp. 297-336.

[11] L. Hurwicz, "The Design of Mechanisms for Resource Allocation," American Economic Review, 63 (May 1973), 1-30.

[12] E. Kalai, E. Pazner, and D. Schmeidler, "Collective Choice Correspondences as Admissible Outcomes of Social Bargaining Processes," Mimeo, Tel-Aviv University, June 1974.

[13] G. Kramer, On a Class of Equilibrium Conditions for Majority Rule," Econometrica, 41 (March 1973), 285-97.

[14] John Ledyard, "A Characterization of Organizations and Environments which are Consistent with Preference Revelation," Discussion Paper No. 5, Center for Mathematical Studies in Economics and Management Science, Northwestern University, September 1972.

[15] L. Ledyard and D. J. Roberts, "On the Incentive Problem with Public Goods," Discussion Paper No. 116, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1974.

[16] Prasanta K. Pattaniak, "Threats, Counter-Threats, and Strategic Voting," forthcoming in Econometrica.

[17] M. A. Satterthwaite, The Existence of a Strategy-Proof Voting Procedure: A Topic in Social Choice Theory, Ph.D. Dissertation, University of Wisconsin, Madison, 1973.

[18] M. A. Satterthwaite, "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions," forthcoming in Journal of Economic Theory.

[19] D. Schmeidler and H. Sonnenschein, "The Possibility of a Cheat-Proof Social Choice Function: A Theorem of A. Gibbard and M. Satterthwaite," Discussion Paper No. 89, Center for Mathematical Studies inEconomics and Management Science, Northwestern University, May 1974.

[20] L. S. Shapley, Utility Comparison and the Theory of Games," in La Decision, Centre National la Recherche Scientifique, Paris, 1969.

[21] W. Vickery, "Utility,Strategy, and Social Decision Rules," Quarterly Journal of Economics, 74 (1960), 507-535.

[22] J. Von Neumann and O. Morgenstern, Theory of Games and Economic Behavior, 2nd ed., Princeton University Press, Princeton, N. J. 1947.

[23] R. Zeckhauser, "Voting Systems, Honest Preferences, and Pareto Optimality," American Political Science Review, 67 (1973), 934-946.