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ON THE OPTIMAL NUMBER OF REPRESENTATIVES*

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Abstract.
We study a model of public decision-making in simple public goods economies with moral hazard and adverse selection. Economic agents must invest resources (or provide effort) to discover their own preferences. We consider direct revelation mechanisms based on sampling. A sample of agents is drawn in the population, and each member of the sample reports a preference type to a Principal. The determinants of the "representative sample" size are studied. The structure and magnitude of effort and sampling costs affects the optimal number of representatives. If the net social value of effort is high, first and second best optimality require a maximal sample (or "direct democracy"). If, on the contrary, effort is too costly, the recourse to samples ("representative democracy") is justified as a second best. To obtain the results, we not only take effort and revelation incentives into account, but also restrict decision rules to satisfy an additional property of robustness to opportunistic manipulation by the Principal, which forbids the use of a priori knowledge in public decision procedures.
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1. Introduction

In contemporary democracies, the production of public goods affects the well-being of a large number of citizens, whereas a typically much smaller number of individuals, technocrats or representatives, is in charge of the decision. This is true at almost all levels of society: there are parliaments at the national level, councils at the local level, and even committees within public and private organizations, as soon as some form of club good is involved. Direct democracy and direct expression are rarely used. The fact that, in spite of the extension of democratic values, institutions remain everywhere relatively narrow representations should, at some point, be explained as an equilibrium phenomenon.

The problem of the optimal number of seats in Parliament dates back, at least, to the 1787 debates surrounding the Constitution of the United States. Madison[1] briefly addresses the question in Federalist 10:

> In the first place, it is to be remarked that however small the Republic may be, the Representatives must be raised to a certain number, in order to guard against the cabals of a few; and however large it may be, they must be limited to a certain number, in order to guard against the confusion of a multitude.
>
> — Madison, *Federalist 10* (in Pole (1987), p. 155.)

But the problem has not attracted much attention in the recent formal literature on Political Economy and Political Science. The present contribution proposes a normative theory of the number of representatives, in a stylized economy with differential information. We hope that this theory can constitute a step towards a better understanding of the recourse to restricted representation, and of its welfare implications.

The costs associated with the acquisition of information and with the preparation of decisions play a major role in the formation of representative institutions. Only a naive view of democracy or organization can neglect the personal resources devoted by individuals to collective decision-making. Indeed, the most important input of democracy is time (leisure for the citizen), and time has a non-negligible opportunity cost. It is thus important to discuss the effect of variable, and fixed individual effort costs on the optimal size of representative samples. The forces driving the division of labor in almost every kind of human organization help understanding the emergence of experts and — in the realm of politics — of professional politicians, as well as the tendency of most committees to give birth to various kinds of sub-committees, etc. For instance, Gilligan and Krehbiel (1990) have analyzed the informational role of Congress committees; among other things, they provide a nice description of the costs of forming these committees, which is directly relevant for the interpretation of the "costs or representation" introduced in the following.

Protection against the opportunistic behavior of technocrats then becomes a major justification for the existence of representative institutions and of collective decision rules. Individuals being self-interested, in practice, benevolent planners do not exist. It follows that among the constraints bearing on the search for an optimal organization of collective decision-making, it is important to pay attention to rules that are robust to opportunism.

[1] The Anti-Federalists have also discussed the topic (see Section 6 below).

The Principal, defined as the agent or group of agents in charge of executing public decisions, should not have any unforeseen manipulation opportunity.

In the following, the analysis sheds light on the existing tradeoff between the exploitation of scale economies, suggesting that the smallest possible number of individuals should specialize in collective decision-making, and the democratic requirement that public decisions should reflect the collective will. Intuitively, optimal collective decision mechanisms should belong to the class of partial representation mechanisms.

To be more specific, the paper focuses on a public good production problem, in a society characterized by decentralized information on preferences. To capture the idea that collective decision-making is costly (*i.e.*, that one must invest resources to figure out the issues at hand and to make one's mind), we assume that agents do not have a complete knowledge of their willingness to pay for the public good, or *type*. However, they can improve this knowledge through some unobservable and costly individual choice of *effort*. We thereby extend the traditional Mechanism Design problem under asymmetric information with a dimension of moral hazard, since in our economy, the accuracy of each individual piece of information depends on some unobservable personal investment. The approach takes three types of constraints into consideration; *(i)*, the moral hazard problem, and its associated incentive constraints; *(ii)*, the adverse selection problem, and its associated preference revelation constraints; and in addition, *(iii)*, we constrain ourselves to public decision procedures satisfying a property of robustness to opportunistic manipulation by the Principal.

The emphasis put on political non-manipulability of public decision rules echoes a long standing tradition of economics, well represented by the Public Choice school (*e.g.*, Mueller (1989)), as well as more recent developments in Political Economy. Persson and Tabellini (1999), and Dixit (1996) provide excellent discussions of the main ideas in this field. Osborne and Slivinski (1996), and Besley and Coate (1998) recently proposed and developed the "citizen-candidates" approach, assuming that candidates in an election are always members of the set of economic agents, and can only commit to be "themselves:" once elected, citizen-candidates always choose their preferred policies. We apply these important ideas here. The opportunistic manipulation problem can be summarized by saying that the execution of public decisions does not rely on the existence of a benevolent planner: the Principal is always chosen in the set of agents, and rationally pursues his (her) private interest. As a consequence, in our setting, public decision rules should depend neither on *a priori* parameters nor on prior probability distributions on the space of possible preferences, simply because these parameters could be manipulated by the bureaucratic Principal. In other words, society is constrained to use *non-parametric* mechanisms, in the sense of Hurwicz (1972). We find below that this requirement immediately excludes the first best decision rule, for it happens to involve a Bayesian-like combination of observations and *a priori* information.

Let us define the first best optimum as the allocation of effort, joint with the public decision rule, which would be implemented by a benevolent, Bayesian and utilitarian planner, if this planner were able to monitor all effort variables, and to observe all private information. Because of effort costs, and because of the ex ante symmetry of agents (*i.e.*, all agents have the same ability to produce information), we find that the first best optimum has two possible structures: Either *all* agents are required to exert some positive effort and to transmit information, which is a form of *Direct Democracy*, or, all agents are required to choose effort zero and to transmit nothing. In this latter case, the planner uses the decentralized *a priori* information on the distribution of preferences to compute the public good production: We call this the *Reign of Tradition.*

We then consider the second best situation in which efforts cannot be monitored, information is private, and benevolent planners do not exist. The results can be summarized as follows: When the first best requires Direct Democracy, then, Direct Democracy is approximately implementable under conditions of asymmetric information. In contrast, when the Reign of Tradition defined above is first best optimal, it is not implementable as a second best. The problem is then to find a non-manipulable mechanism to collect decentralized information on preferences for the public good. If individuals are *ex ante* symmetric, we find that it is optimal to sample among agents: n individuals should be drawn at random out of the population of size $N \geq n$. The role of the sampled agents is, by simply representing themselves, to create a reduced mirror image of the population. In a sense, we do not expect more from representatives, in democratic societies, provided that we accept the principle of representation.

The second best analysis permits one to determine the optimal number of representatives. This number depends on a tradeoff between two opposite effects. On the one hand, sampled agents incur fixed costs, and variable costs of effort. These costs will tend to limit the sample size. On the other hand, the statistical precision of the information produced will be poor if the number of representatives is too low. It follows that a large assembly is required if fixed costs are low and the dispersion of preferences is high, whereas a smaller oligarchic assembly emerges for higher fixed costs. Finally, it is optimal to use a single representative, or technocrat, when the dispersion of preferences is small and costs are very high.

The cost-benefit analysis underpinning these second best results, depends in an important way on the property that the effort level of each representative varies inversely with the number of representatives. Intuitively, when the number of participants to the decision-making process is large, the influence of each individual agent on the final outcome becomes negligible, and as a consequence, effort incentives — the rewards of investment in information acquisition — also tend to vanish.[2] Now, when second best optimality requires a sample, the effort of representatives happens to be a social waste of resources. In this case, each individual effort must be interpreted as an attempt at influencing the collective decision in favor of particular interests, as a form of distortive "lobbying." It follows that, apart from its direct cost, the social benefit of an additional representative can be decomposed into a purely statistical effect, due to the reduction of sampling errors,

[2] The first students of pivotal mechanisms had recognized the potential importance of a tradeoff between the quantity and the quality of information in possible applications of revelation procedures. In the last chapters of their pionneering book, Green and Laffont (1979) explicitly considered sampling procedures and sketched the analysis of effort incentives (see also Green and Laffont (1977)). The results presented here are different, since in our model, sampling leads to a form of effort over-investment. The problem of incentives to produce information is tightly connected with the notion of individual *influence* in a public good mechanism. On this notion, see Al Najjar and Smorodinsky (1998), see also Mailath and Postlewaite (1990).

and a reduction of distortions, due to the discouragement of effort.

In the last section of this paper, we propose a preliminary empirical analysis of the size of representative institutions, using political data to run regressions. We try to explain the number of representatives in 111 countries as a function of the total population, of population density (that we take as a crude indicator of its heterogeneity), and of a dummy variable indicating developing countries (which can be viewed as a rough measure of the social cost of maintaining representatives). The regression of the number of representatives on total population provides a benchmark with which different political systems can be compared. For instance, among the advanced economies, the United States and Israel do not seem to have enough representatives. In contrast, France and Italy seem to have too many representatives. As a matter of fact, both France and Italy have more representatives than the United States in absolute terms.

Our normative theory of the number of representatives completely abstracts from voting procedures; yet, in the realm of modern politics, representation relies on voting mechanisms, not on random sampling. The reader could expect us to discuss this point.[3] People being different in their ability to produce and process information, an optimistic explanation for the recourse to voters is that elections are a way to screen the most able agents. Therefore, if the distributions of abilities and preferences are independent in the citizen's population, the result of elections will not be different from the outcome of random sampling, *in terms of represented preference types.* On the contrary, if there is a statistical correlation between abilities and preferences, voting procedures would systematically yield a biased sample of representatives. In the following, we study the simple case in which all citizens have the same ability to become representatives; more work remains to be done to study asymmetric cases rigorously. In any case, a fully integrated view of elections and legislative bargaining as a multi-stage game, paying attention to strategic behavior at each stage, as recently advocated by Myerson (1999), is beyond the reach of this paper. Austen-Smith and Banks (1988), Alesina and Rosenthal (1996)), among other contributions, have shown that an integrated game-theoretic approach leads to subtle difficulties. In the domain of organization design, random sampling of representatives is a shortcut, allowing us to concentrate on the optimal number of representatives.

Recent studies of the internal organization of the U.S. Congress show that representatives work to produce relevant information, to bargain on policy issues, and find compromises, while economizing on (transaction) costs.[4] Representatives are compensated to discuss and bargain at length whith each other, until they reach, in one way or another,

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[3] Taken literally, the proposed model describes a form of "radical democracy," in the sense of Aristotle: sampling is a radical way to ensure that representatives will resemble the represented people. The ancient Greeks, in Athens, used random drawings to choose their legislators and the members of trial juries. Socrates was sentenced to death by a jury of 501 randomly drawn citizens. The Athenian People's Assembly itself, with its 6000 members, was in fact a random sample of the citizen population (For details on these points, see Hansen (1991); see also Manin (1995), who proposes an history of the recourse to chance draws in political institutions). Closer to us, in the domain of justice, trial juries are randomly drawn in the population.

[4] Weingast and Marshall (1988) have applied the tools of Industrial Organization and Transaction Costs theories to Congress, while the most recent contributions in this field emphasize the informational aspects of decision-making (*e.g.*, the survey article by Shepsle and Weingast (1994)).

a production decision. In our framework, once drawn, representatives reach a decision by means of an information revelation process financed by taxes. More precisely, we assume that they use a pivotal or Clarke-Groves revelation mechanism (see Clarke (1971) and Groves (1973)).

It is a difficult task to model legislative bargaining with the help of game theoretic methods; recent work shows that the adopted public policies depend on the constitutional details of the legislature's organization, insofar as they define rules of a game played by representatives. See Baron and Ferejohn (1989), and, among other recent contributions, Diermeier and Feddersen (1998). On the other hand, the classic work on probabilistic voting indicates that under simple symmetry assumptions, competing office-motivated, plurality-maximizing candidates in an election will converge on a political platform which maximizes a social welfare function (an utilitarian sum of the citizen's utilities); see Hinich *et al.* (1972), Coughlin and Nitzan (1981), Lindbeck and Weibull (1987), (1993). A kind of reduced-form or black-box modelling of the legislative procedure is needed to obtain a tractable (and reasonably not too particular) formulation of the social value of an additional representative. To this end, we have constructed a stylized model of the representative institution in which the "legislative bargaining" process yields an approximate Pareto optimum. The Groves transfer schedule is a shortcut for all the incentives that a good public decision system should provide to its representatives.[5]

The class of Groves mechanisms is particularly attractive when applied to a random subset of agents; it is revealing in dominant strategies, and at the same time, approximately optimal when the sample is large enough (see Gary-Bobo and Jaaïdane (1997)). These results rely on the built-in division of labour between representatives (the sampled) and politically passive citizens (the non-sampled). In addition, in a quasi-linear environment, the Groves class is essentially the only subset of mechanisms satisfying these desirable properties (see the characterization results of Green and Laffont (1979), and Holmström (1979), Moulin (1986)). Finally, these mechanisms meet our additional requirements of robustness to opportunistic manipulation by the Principal: their production decision rule, as well as their taxation schedules are independent of prior probabilistic knowledge about citizens' preferences. But the most interesting aspect of this property is probably that, once submitted to a Clarke-Groves tax, the randomly chosen representatives are unanimously willing to pursue the common interest, insofar as the representation is a correct mirror image of the population's preferences. Under a suitably defined Groves mechanism, each representative will internalize the social welfare of the *other representatives.*

In the following, Section 2 presents the model and discusses the assumptions; Section 3 provides an analysis of the first best optimum; Section 4 describes a class of public decision mechanisms and the related revelation and incentive constraints; Section 5 develops the analysis of second best optimal sample sizes; Section 6 presents some empirical results; and finally, concluding remarks are gathered in Section 7.

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[5] Note that Clarke (1977) and Tullock (1977) went as far as to discuss the practical possibility of using pivotal mechanisms in the U.S. Congress to decide on public good production.

2. The Model

We consider an economy comprising N agents, indexed by $i = 1, \dots, N$. The agents are both consumers and contributors (taxpayers). There are two goods in the economy; one public good, which is financed through contributions (taxes) and the production level of which is a collective decision problem, and one private good, called money, in terms of which taxes and subsidies are denominated. Let q denote the public good production level and t_i agent i's tax. The agents are assumed to have linear preferences for the public good. Let us assume that if agent i participates in the collective decision mechanism and exerts a positive effort $e_i \geq 0$, his or her utility function, denoted u_i, writes

$$u_i = \theta_i q - t_i - \phi(e_i) - F, \tag{1}$$

with F the fixed cost of participation, ϕ the variable cost of effort, and θ_i is agent i's type. It is understood that if the agent does not participate in the decision mechanism, and if she does not exert any effort, then, her utility is simply $u_i = \theta_i q - t_i$.

2.1. A model of decentralized knowledge

Agent i's marginal willingness to pay for the public good θ_i is a hidden characteristic. Agent i does not know her own type *ex ante*, but is endowed with some prior information about herself: some prior probability distribution with mean μ_i. To make the analysis tractable, we assume that all individuals, conditional on their *a priori* mean type μ_i, are drawn from the same family of probability distributions. Each member of the family is obtained from the other by a translation. More precisely, let x_i denote agent i's centred type, *i.e.*,

$$x_i = \theta_i - \mu_i. \tag{2}$$

We assume the following.

Assumption 1
For all $i = 1, ..., N$,
(a), x_i is independently and identically distributed according to the probability distribution K, with density k, defined on the real line;
(b), the probability K has a zero mean and a finite variance denoted σ^2.

The interpretation is that each agent i belongs to a given "region", defined by political, ethnic, religious or social groups. It must be understood that an agent's "region" is not observable by other agents. The average preference type in agent i's region, μ_i, is the commonly held view on the public good in this "region". We are not ruling out common value problems (*i.e.*, the limiting case of a public good about which everybody agrees), but generically, we assume the existence of some *a priori* heterogeneity of preferences. Formally, this prior mean μ_i is as a random variable, and the μ_i, $i = 1, \dots, N$, are assumed to be independent drawings in some underlying probability distribution.

Assumption 2
For all $i = 1, ..., N$, μ_i is an independent drawing in the probability distribution P, with mean $\bar{\mu}$ and variance z^2.

Let E and V denote the expectation and variance operators, respectively. Going back to our interpretation in terms of regions, $E(\mu_i) = \bar{\mu}$ is the prior *interregional* (or national) mean, while $V(\mu_i) = z^2$ is the *interregional* variance of preference types. In contrast, $V[\theta_i|\mu_i] = \sigma^2$ is the *intraregional* variance of types. The total dispersion of preferences is obtained as the sum of the intraregional and interregional variances, that is, $V(\theta_i) = \sigma^2 + z^2$.

We do not assume that the probability distribution P or its moments are common knowledge among the agents. This is meant to capture the intuitive idea that in a decentralized economy, a given agent has some "local" prior information about her preferences (*i.e.*, μ_i), but has neither a global view of society (in the form of the prior overall average type $\bar{\mu}$), nor an exact knowledge of the other agents' type distribution (in the form of the probability distribution P). Finally, a description of decentralized knowledge in probabilistic form for this economy would be complete if we endowed each agent with some (subjective) probability distribution on the types of others. Since these probabilistic beliefs play no role in the sequel, we will avoid introducing formal notation to describe them.

2.2. Information Acquisition

Agents do not know their own type with precision *ex ante.* In the absence of better information, they rally to the view commonly held in their reference group, and see themselves as μ_i. Now, if they are in a position to acquire information, for instance while working through technical files or expert reports on the public project and its consequences, they will learn many things on the issue at hand, and will certainly revise their initial position. This does not mean that all informed agents will agree about the appropriate decision, unless we are in the particular case of a common value problem. They keep personal stakes in the public project and possibly ideological biases, but their perception of the project's impact on their utility will be much more accurate.

To capture these ideas, we assume that, at the cost of some effort, denoted e_i, agent i receives a signal denoted s_i, the precision of which depends on the effort level, and which conveys information on the unknown type θ_i. The total cost of effort is $\phi(e_i) + F$. To fix ideas, F can be understood as the fixed cost of participation in the collective decision mechanism, and e_i as the individual's input into the process. We make the following assumption.

Assumption 3
The cost function ϕ is non-decreasing, convex, continuously differentiable and $\phi(0) = 0$.

Assumption A3 implies that all individuals are identical in their ability to process and digest information. This is obviously a simplifying assumption.

We assume that the informative signals produced by means of effort, are drawn from a conditional probability distribution, described in the next assumption.

Assumption 4

(a) The s_i, $i = 1, \ldots, N$ are independent from each other;
(b), for all i, s_i is distributed according to a conditional probability density denoted $h(s_i|x_i; e_i)$, and parameterized by the effort level e_i.

If the reader recalls that $x_i = \theta_i - \mu_i$, Assumption 4 means that, apart from the effort level e_i, the signal's distribution depends only on the discrepancy between θ_i and its prior mean μ_i. The posterior density of x_i knowing $(s_i; e_i)$, denoted f, is given by

$$f(x_i|s_i; e_i) = \frac{h(s_i|x_i; e_i)k(x_i)}{\int h(s_i|x_i; e_i)k(x_i)dx_i}. \tag{3}$$

Let $\hat{x}$ be defined as follows.

$$\hat{x}(s_i; e_i) = E[x_i|s_i; e_i] = \int x_i f(x_i|s_i; e_i)dx_i. \tag{4}$$

Since all agents are assumed to make a rational use of their information, agent i's prediction of her own type, denoted $\hat{\theta}_i$, can therefore be defined as the sum of $\hat{x}$ and the prior mean μ_i,

$$\hat{\theta}_i(s_i; e_i) = E[\theta_i|s_i, \mu_i; e_i] = \hat{x}(s_i; e_i) + \mu_i. \tag{5}$$

We wish to capture the intuitive idea that agent i's prediction $\hat{\theta}_i$ will be better, the higher the effort. More precisely, we assume that the precision of the signal increases with effort, in the sense that it reduces the posterior variance of type θ_i knowing the signal s_i. Thanks to a well known identity of probability theory, *i.e.*, $V(Y) = E[V(Y|Z)] + V[E(Y|Z)]$, and using (5) above, the variance of θ_i, conditional on μ_i, can be decomposed as follows, $V(\,\theta_i\,|\,\mu_i\,) = E[V(\,\theta_i\,|\,s_i, \mu_i\,)\,|\,\mu_i\,] + V[\hat{\theta}_i\,|\,\mu_i\,]$, which, with our notations and assumptions, yields the equivalent identity,

$$\sigma^2 = E[V(\,\theta_i\,|\,s_i, \mu_i\,)\,|\,\mu_i\,] + V[\hat{x}], \tag{6}$$

given that $V[\hat{\theta}_i|\mu_i] = V[\hat{x}|\mu_i] = V[\hat{x}]$. A glance at (3) and (4) above shows that $V[\hat{x}]$ is a function of the effort variable e_i, but does not depend on μ_i. Hence, we pose

$$v(e_i) = V[\hat{x}]. \tag{7}$$

The following assumption formally describes the way in which the effort variable affects the variance of the agent's type prediction.

Assumption 5

(a), $v(e_i)$ is increasing and continuously differentiable with respect to e_i;
(b), $0 \leq v(e_i) \leq \sigma^2$, $v(0) = 0$ and $v(e_i) \to \sigma^2$ as $e_i \to +\infty$;
(c), the derivative $v'(e_i)$ is strictly quasi-concave.

By identity (6) above, Assumption 5a is tantamount to assuming that agent i's effort decreases the posterior variance of her type $\hat{\theta}_i$ conditional on the signal s_i. Intuitively, this is due to the fact that when e_i increases, $\hat{\theta}_i$ "tracks" the real unobserved type θ_i more closely, therefore increasing the variability of the former. And for the same reason, the predictor's variance is bounded above by σ^2, the *a priori* variance of the underlying type, conditional on μ_i. The additional assumption $v(0) = 0$ carries the intuitive idea that effort zero yields an uninformative signal. To obtain this property, it is sufficient to require that, when effort is zero, the agent's posterior prediction is equal to its prior mean, *i.e.*, $\hat{\theta}_i(s_i; 0) \equiv \mu_i$, for then, $v(0) = V[\mu_i|\mu_i] = 0$. Under these assumptions, it is easy to check that $v'(0) = 0$: The assumption $v(0) = 0$ is therefore incompatible with a concave v. Function v will typically be S-shaped, implying that the derivative v' is bell-shaped (which corresponds to Assumption 5c).

2.3. Quadratic cost function

The public good is produced in quantity q by means of a technology described by a total *per capita* cost function denoted C. Since we do not conduct an asymptotic analysis in the sequel, we can ignore the possible dependence of C on N. It is thus assumed, without loss of generality, that the *total* cost of production can be written $NC(q)$. Finally, in order to avoid unnecessary technicalities, we make the following simplifying assumption.

Assumption 6
$C(q) = (1/2)q^2$.

The model being now completely specified, we turn to the study of first best optimal production of the public good.

3. The First Best Optimum

3.1. First best optimal public decision function

Assume for the moment that an abstract, ideal planner checks and commands effort levels. More precisely, assume that a sample of n individuals is chosen at random, with $n \leq N$. Assume also that all the interim information in the economy can be used in computations, that is, let the planner know the parameters $\mu = (\mu_1, \ldots, \mu_N)$ and the probability densities k and h. Index sample members from 1 to n, non-sample agents being conventionally indexed from $n+1$ to N. The planner then demands a vector of efforts $e = (e_1, \ldots, e_n) \geq 0$ from sample members, while non-sample members provide effort zero and do not pay the fixed cost F. A vector of signals $s = (s_1, \ldots, s_n)$ is produced and observed. The planner finally chooses a public decision $q(s)$ as a function of s.

Assume finally that the planner is utilitarian, and wishes to maximize the sum of the agents' expected utilities. The first best objective is then to choose $q(.)$, e and n so as to maximize,

$$E_s\Big\{E_\theta\Big\{\sum_{i=1}^{N}\theta_i q(s) - NC[q(s)]\,\Big|\, s, \mu\,; e\Big\}\,\Big|\, \mu, e\Big\} - \sum_{i=1}^{n}\phi(e_i) - nF. \tag{8}$$

This form of the objective is obtained when the budget constraint is taken into account. Let $t_i(s)$ be agent i's tax, contingent on signal s. Then, the ex post budget balance condition writes, for all s,

$$\sum_{i=1}^{N} t_i(s) = NC[q(s)]. \tag{9}$$

The first order conditions for welfare maximization say that for each $(s;e)$, $q(s)$ should equate the marginal *per capita* cost with the average estimated willingness to pay,

$$C'[q(s)] = \frac{1}{N}\sum_{i=1}^{n} \hat{\theta}_i(s_i;e_i) + \frac{1}{N}\sum_{i=n+1}^{N} \mu_i, \tag{10}$$

for all $(s;e)$. Note that in expression (10), the right hand side is nothing but the average of the conditional type forecasts. Since it is assumed that non-sample members exert a zero effort, there are $N-n$ individuals for which the type forecast is simply the *a priori* parameter μ_i.

Note also that the first best production decision is in fact a function of the predictors $\hat{\theta}_i$ and μ_i; it depends on signals s only through the vector $(\hat{\theta}_i)_{i=1,\ldots,n}$. This suggests a possibility of decentralization if agent i can be made responsible for the production of $\hat{\theta}_i$. Define the n-vector $\hat{\theta} = (\hat{\theta}_i)_{i=1,\ldots,n}$. Then, optimal production can be conveniently redefined as a function of $\hat{\theta}$. Given that, under Assumption 6, $C'(q) = q$, define

$$q^*(\hat{\theta}) = (1/N)\Big[\sum_{1}^{n} \hat{\theta}_i + \sum_{n+1}^{N} \mu_i.\Big] \tag{11}$$

Then, clearly, $q(s) = q^*[\hat{\theta}(s)]$.

3.2. First best optimal effort levels

Using the independence assumption (Assumption 1), the Planner's objective (8) can be rewritten,

$$W = E_s\Big\{\sum_{i=1}^{n} \hat{\theta}_i(s_i;e_i)q(s) + \sum_{i=n+1}^{N} \mu_i q(s) - NC[q(s)]\,\Big|\,\mu, e\Big\} - \sum_{i=1}^{n} \phi(e_i) - nF. \tag{12}$$

The first best effort vector e^* maximizes (12) with respect to e. In general, the conditions for optimality do not have a simple expression. However, the quadratic cost case, which can be viewed as a first approximation for more general convex cost functions, is analytically tractable, and has a great illustrative power. In this case, q^* is simply a linear function of the predictors, and straightforward computations lead to the following result.

Lemma 1. *The first best welfare, denoted $W^*(e,n)$, is a function of the effort levels and of the sample size, and can be expressed as,*

$$W^*(e,n) = (N/2)\Big(\frac{1}{N}\sum_{i=1}^{N}\mu_i\Big)^2 + \sum_{i=1}^{n}\Big(\frac{v(e_i)}{2N} - \phi(e_i) - F\Big). \tag{13}$$

For proof, see the appendix

In expression (13), the first term on the right hand side is the expected value of social surplus obtained when all efforts are set equal to zero and q is simply equal to the average of the μ_is. The second term represents the additional welfare attributable to the effort of sampled agents. The necessary conditions for the maximization of (13) with respect to e can be written as follows:

$[(\frac{1}{2N})v'(e_i) - \phi'(e_i)]e_i = 0,$
$e_i \geq 0,$
$(\frac{1}{2N})v'(e_i) - \phi'(e_i) \leq 0$, for all $i = 1,\ldots,n$.

A solution with respect to ε of the equation

$$\Big(\frac{1}{2N}\Big)v'(\varepsilon) = \phi'(\varepsilon) \tag{14}$$

exists if $2N\phi'(0)$ is not too large with respect to the maximum of v'.

[Insert FIGURE 1 about here]

Let $e(N)$ be the solution of the problem $\max_{e\geq 0}\{(1/2N)v(e) - \phi(e)\}$. This solution necessarily satisfies (14), if it is strictly positive. We deduce that all sampled individuals will provide the same effort level e^* at the optimum, as a result of our symmetry assumptions. This level depends on N, the size of the total population.

3.3. First best optimal sample size

If e_i is everywhere replaced with $e(N)$ in the first best welfare function (13), a function depending on n only is obtained, that is,

$$W^*[e(N),n] = (N/2)\hat{\mu}^2 + n\Big[\frac{v(e(N))}{2N} - \phi(e(N)) - F\Big], \tag{15}$$

where

$$\hat{\mu} = \frac{1}{N}\sum_{1}^{N}\mu_i. \tag{16}$$

Hence, if

$$(1/2N)v(e(N)) - \phi(e(N)) \geq F, \tag{17}$$

the planner will demand $e^* = e(N)$ from all individuals, and the optimal sample size is $n^* = N$: Everybody participates in information production. If the costs are higher than the benefits, that is, if $(1/2N)v(e(N)) - \phi(e(N)) < F$, the planner prefers to give up any form of effort requirement, *i.e.*, sets $e^* = 0$ and the optimal sample size is $n^* = 0$: The planner then makes use of the *a priori* parameters μ and simply chooses to produce $C'(q) = \hat{\mu}$.

To sum up, if (17) holds, the expected social welfare is given by $(1/2)[N\hat{\mu}^2 + v(e(N))] - N(\phi(e(N)) + F)$. If (17) does not hold, then the expression for expected welfare reduces to $(1/2)N\hat{\mu}^2$, since nobody is sampled, so that no effort cost is incurred and $v(0) = 0$.

We conclude that in our simple quadratic model, sampling is never optimal in the first best world.

Proposition 1. *The first best optimum will either be a form of Direct Democracy, i.e., if (17) holds, $n^* = N$ and $e^* = e(N)$, or the exact opposite, an ideal Reign of Tradition, i.e., if (17) does not hold, $n^* = 0$ and $e^* = 0$.*

In the *Reign of Tradition* case, the benevolent planner relies on the *a priori* knowledge μ to choose the optimal public project's scale, *i.e.*, $q^* = \hat{\mu}$.

4. A Public Decision Mechanism under Asymmetric Information

We now turn to the more realistic conditions of asymmetric information: neither effort levels e, nor signals s are observable. To these assumptions, we add the constraint that society must organize itself without the help of a disembodied benevolent planner.

To avoid confusion, in the second best world, we will conventionally call *Principal*, the agent who effectively carries out the mechanism. Whichever the social process by which this Principal is nominated or elected, he or she can be viewed as a drawing in the probability distribution describing preferences. In other words, the Principal is an economic agent. It would then be inconsistent to suppose that he or she will not adopt some form of opportunistic behavior. In fact, if the public decision rule depends on *a priori* parameters such as μ, the Principal will exert some effort to discover her own preferences, and will choose the value of the parameters that best suits what she learned about herself. Consider, for instance, the first best solution which relies on the a priori information μ. Implementation of this rule by an opportunist Principal, with posterior type $\hat{\theta}_p$, will always lead to $q^* = \hat{\theta}_p$. In practice, the Reign of Tradition becomes dictatorship.

More generally, since μ is not known, and is not common knowledge among the agents, the robustness to manipulation constraint leads to rejection of the first best Bayesian production rule (10). Indeed, the difficulty with (10) is that to implement it, a way of eliciting the parameters μ_i should be designed, by means of a well-defined mechanism, relying on verifiable declarations. But this is not always possible. At the very moment at which agent i is nominated as sample member, she will exert an effort which *destroys* μ_i, so that μ_i cannot be revealed any more. In other words, what agent i believed about her tastes behind the "veil of ignorance" cannot be retrieved once she becomes informed. To collect the needed prior parameters μ_i, the Principal should make sure that sample members choose effort zero, for then, it would be possible to have them reporting $\hat{\theta}_i(s; 0) = \mu_i$, but this cannot simply be assumed.

4.1. The sampling mechanism

The robustness to manipulation constraint leads to rejection of all public production rules which rely on *a priori* parameters. This corresponds to the notion of a non-parametric mechanism, in the sense of Hurwicz (1972). The problem is then to find a production rule which depends on verifiable agent declarations $\hat{\theta}$ only, that is, a function $q = \xi(\hat{\theta})$. Since C' is monotonic, without loss of generality, ξ can be rewritten $q = (C')^{-1}[g(\hat{\theta})]$, where $g = C' \circ \xi$. The second best problem can thus be reduced to the search for an estimator g.

The collection of decentralized information $\hat{\theta}$ induces a moral hazard problem, since the agent's effort is not observable, and an adverse selection problem, since private information must be elicited. To take care of these constraints, we now describe a multi-stage mechanism.

1^o) A Principal is chosen at random in the set of agents to carry out the mechanism. The agents' types θ_i are drawn in the prior distribution $k(\theta_i)$ and remain hidden to the agents themselves.

2^o) The Principal draws at random a sample I containing $n < N$ individuals. Without loss of generality, sample members are relabeled from 1 to n. Non-sampled members are also relabeled, with index i running from $n+1$ to N.

3^o) Individuals pay the fixed cost F as soon as they become sample members. Each sample member i in I chooses an unobservable effort level $e_i \geq 0$. Non-sampled individuals do not pay the fixed cost and do not exert any effort.

4^o) Each sample member i receives an informative signal s_i, drawn in the probability distribution $h(s_i|\theta_i; e_i)$.

5^o) Sample members declare their type $\hat{\theta}_i = \hat{\theta}_i(s_i; e_i)$ to the Principal, according to a prespecified direct revelation mechanism.

6^o) The Principal chooses the production according to the statistical rule:

$$C'(q(\hat{\theta})) = g(\hat{\theta}),$$

7^o) Budget is balanced by means of taxes t_i paid by all agents $i = 1, \dots, N$. Since the non-sampled members do not report any private characteristic, they are indistinguishable from each other. Therefore, they pay the same tax denoted t_o, which is a function of the sample members' reports $\hat{\theta}$. This transfer function is defined by the *ex post* budget constraint: for all sample I, for all $\hat{\theta}$:

$$\sum_{i \in I} t_i(\hat{\theta}) + (N-n)t_o(\hat{\theta}) = NC[q(\hat{\theta})]. \tag{18}$$

Within the class described by 1^o to 7^o above, a mechanism is characterized by an array of functions (g, t); it is called *second best optimal* if it maximizes the sum of expected utilities under, *(i)* effort incentive constraints, *(ii)* type revelation constraints (where truthful revelation is a dominant strategy) and *(iii)*, if the functions g and t_i depend on verifiable declarations $(\hat{\theta}_1, \dots, \hat{\theta}_n)$ only. We obtain the following result.

Proposition 2. *Under Assumptions 1-6, a public decision rule $g(\hat{\theta}_1, ..., \hat{\theta}_n)$ which is second best optimal for all probability distributions of $\hat{\theta}_i$ and anonymous, must be the arithmetic mean, i.e., for all $\hat{\theta}$,*

$$C'(q(\hat{\theta})) = g(\hat{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i. \tag{19}$$

For proof, see the Appendix.

The proof of Proposition 2 shows, in a first step, that a second best optimal decision rule g must be an *unbiased* estimator of $\bar{\mu}$. We then add the democratic requirement that the decision rule should be anonymous, *i.e.*, that g should be invariant with respect to all permutations of its arguments. This is a natural consequence of the requirement of non-manipulability by the Principal: if g was not invariant with respect to permutations of its variables, the Principal could change the value of g just by relabeling the sampled agents.

Now, if unbiasedness and anonymity are required for all probability distributions of $\hat{\theta}_i$, then, a second step in the proof shows that the only possible estimator[6] g is the arithmetic mean, $g(\hat{\theta}) = (1/n)\sum_{i=1}^{n}\hat{\theta}_i$: we get a statistical form of Samuelson's rule.

4.2. Groves transfer schemes

Without loss of generality (by the Revelation Principle), we will require that the mechanism (q, t) induces truthful revelation of private forecasts $\hat{\theta}_i$ by sample members. If we then consider the non-manipulable and revealing mechanisms which fit the above described framework, we can, *without any loss of welfare*, achieve revelation by restricting further to the simple and robust subclass of Dominant Strategy mechanisms. In our framework, incentive compatible Dominant Strategy mechanisms are essentially pivotal mechanisms (see Green and Laffont (1979)).

Let $q(\hat{\theta})$ be the arithmetic average rule, as defined by (19). Then, truthful revelation of $\hat{\theta}$ by sample members can be obtained by imposing the following transfer to sample member i, for all i in I,

$$t_i(\hat{\theta}) = -\sum_{j\neq i, j\in I}\hat{\theta}_j q(\hat{\theta}) + nC[q(\hat{\theta})] + m, \tag{20}$$

where m is a constant. Expression (20) can be called a Groves transfer scheme. To prove the revelation property, we compute the expected utility of sample member i, denoted U_i, that is,

$$U_i(e) = E[\theta_i q(\hat{\theta}) - t_i(\hat{\theta}) \,|\, e] - \phi(e_i) - F. \tag{21}$$

By definition of $\hat{\theta}_i$, conditioning on signal s yields,

$$U_i(e) = E_s[\hat{\theta}_i q(\hat{\theta}) - t_i(\hat{\theta}) \,|\, e\,] - \phi(e_i) - F. \tag{22}$$

[6] We could relax the anonymity requirement and consider weighted statistics of the form $g=\sum_i \alpha_i\hat{\theta}_i$ with $\sum_i \alpha_i=1$. But due to agent symmetry and effort cost convexity, it would then be possible to show that the optimal choice of α_i is $1/n$.

Note that these computations can be performed with the help of agent i's subjective probability assessments on the distribution of other agents' signals. Substituting the Groves formula given by (20) in expression (22) easily yields,

$$U_i(e) = nE_s\Big[\Big(\frac{1}{n}\sum_{j=1}^{n}\hat{\theta}_j\Big)q(\hat{\theta}) - C[q(\hat{\theta})]\,\Big|\, e\Big] - m - \phi(e_i) - F. \tag{23}$$

We now reached a crucial step in the analysis. It is easy to check that the mechanism (q, t) defined by (18), (19) and (20) is revealing in dominant strategies: all sample members i have an incentive to report their true $\hat{\theta}_i$, whatever the declarations of other sample members are. But note, in addition, that if subjected to a Groves transfer (20), sample members would *unanimously* support the production rule which maximizes (23). The solution of this maximization problem happens to be the arithmetic average of sample members' declarations rule (19). Therefore, if representatives are subjected to the Groves transfer (20), they will have no reason to manipulate or distort the arithmetic production rule (19). Sampling procedures reduce the total cost of collective decision-making, and, coupled with Groves transfer schemes, ensure both honest representation and adherence of representatives to the pursuit of general interest. We summarize these findings in the following proposition.

Proposition 3. *For all sampled agent $i = 1, \dots, n$, if subjected to the Groves transfer scheme (20),*
(i), it is a dominant strategy for agent i to reveal his (her) information $\hat{\theta}_i$ truthfuly;
(ii), the arithmetic average rule (19) maximizes agent i's utility on the set of public production rules.
(iii), For all sample of size $n < N$, the budget is balanced.

Given that non-manipulability by the Principal leaves the arithmetic average production rule (19) as the only possible choice, there are no additional social costs associated with information revelation by means of a Groves mechanism, that is, no distortion due to informational rents, no budget deficit. Proposition 3 depends crucially on point *(iii)*, which is a consequence of the mechanism's definition and of (18). Non-sampled agents are here to ensure budget balance, and do not participate in the revelation process. Hence, an important virtue of representation seems to be that, while it excludes some individuals from the decision process, at the same time, by creating a pool of passive agents, it creates a degree a freedom which helps representatives to reach a social compromise. It follows that we are constrained to choose the number n of representatives strictly between 0 and N, for at least one agent must belong to the sample, and at least one agent must remain out of the sample. Then, according to Proposition 3, representatives will unanimously choose the decision which would be socially efficient if they collectively formed an exact reduced image of the whole population.

4.3. Hidden effort functions

We can now compute the effort of sample agents under the revelation mechanism (18)-(20). Let $\bar{\theta}_n = (1/n)\sum_1^n \hat{\theta}_i$. Substituting (19) in (23) yields,

$$U_i(e) = nE[\bar{\theta}_n^2 - (1/2)\bar{\theta}_n^2 \,|\, e] - m - \phi(e_i) - F,$$

since $q = \bar{\theta}_n$. Simple computations, based on the assumption that agent i knows that all signals s_j are stochastically independent, yield the following result.

Lemma 2. *Under Assumptions 1-6, and under the Groves mechanism (18)-(20), for all $i = 1, \dots, n$, a sample member's expected utility can be expressed as follows.*

$$U_i(e) = \frac{v(e_i)}{2n} - \phi(e_i) - F + M_i(e_{-i}), \tag{24}$$

where M_i is a function which depends on $e_{-i} = (e_j)_{j \neq i}$ and not on i's own effort e_i.

The function M_i also depends on agent i's beliefs about the other agents' mean preference types.

It is now possible to characterize individually optimal effort levels. Expression (24) shows that due to our symmetry assumptions, all sampled individuals will choose the same optimal effort level, denoted $e(n)$. Clearly, given (24), $e(n)$ maximizes $(v(e)/2n) - \phi(e)$ with respect to e. Define $\varepsilon(n)$ as the solution, if it exists, of the first order equation:

$$\frac{v'(\varepsilon(n))}{2n} = \phi'(\varepsilon(n)), \tag{25}$$

together with the second order condition,

$$\frac{v''(\varepsilon(n))}{2n} - \phi''(\varepsilon(n)) < 0. \tag{26}$$

Then, the equilibrium effort $e(n)$ of any sample member can be expressed as follows:

$$e(n) = \begin{cases} \varepsilon(n) & \text{if} \quad (1/2n)v(\varepsilon(n)) \geq \phi(\varepsilon(n)) \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

The effort function typically jumps downwards if n, the number of sample members, exceeds a certain threshold denoted $\tilde{n}$. The threshold $\tilde{n}$ (possibly not an integer) satisfies the following equation.

$$\frac{v(\varepsilon(\tilde{n}))}{2\tilde{n}} = \phi(\varepsilon(\tilde{n})). \tag{28}$$

It is the number of representatives above which a positive effort level is not profitable any more.

If n is viewed as a real variable, a simple application of the Implicit Function Theorem shows that the effort function $e(.)$ is decreasing. Differentiating (25), we find

$$\varepsilon' = \frac{2(\phi')^2}{v''\phi' - \phi''v'} < 0,$$

(given that under (25) and (26), $v''\phi' - \phi''v' < 2n\phi''\phi' - \phi''v' = 2n\phi''\phi' - \phi''(2n\phi') = 0$).

We have just proved,

Lemma 3. *The optimal effort function $e(n)$, defined by (27), is non-increasing with respect to n.*

The intuition for this result is straightforward. As the number of sample members increase, the manipulation power of each individual member decreases, in the sense that his (her) influence on the final outcome becomes weaker. It follows that the individual incentives to invest in information acquisition (effort) also weaken as n increases, and eventually drop to zero for n larger than $\tilde{n}$.

5. The optimal number of representatives

5.1. The second best welfare function

Substituting the effort function in the expression of expected welfare (*i.e.*, the expected total sum of utilities), yields the following result, after some computations.

Lemma 4. *Under Assumptions 1-6; under the budget constraint (18), if mechanism (q,t) is defined by the arithmetic average production rule (19) and the Groves transfers (20), then, the expected total sum of utilities, denoted $W(n)$, can be expressed by the formula,*

$$W(n) = \frac{1}{2}N\bar{\mu}^2 + n\Big[\frac{V(e(n))}{2n} - \phi(e(n)) - F\Big] - \Big[\frac{1}{n} - \frac{1}{N}\Big]\frac{NV(e(n))}{2}. \qquad (29)$$

where by definition,

$$V(e) = z^2 + v(e). \qquad (30)$$

For proof, see the appendix

Formula (29) deserves a comment. $W(n)$ is the sum of three terms; the first term $(N/2)\bar{\mu}^2$ is the expected welfare in a society in which all effort variables are zero (and the Planner implements $q = \bar{\mu}$).

The second term is n times the additional surplus of a sample member, *i.e.*, by definition,

$$S(e(n)) = (1/2n)V(e(n)) - \phi(e(n)), \qquad (31)$$

minus the fixed costs F. In expression (31), as well as in (29), $V(e)$ must be interpreted as the unconditional variance of preference types, *i.e.*, $V(e) = V[\hat{\theta}_i]$, and because of independence and Assumption 2, $V[\hat{\theta}_i] = V(\hat{x}) + V(\mu_i) = v(e) + z^2$.

The intriguing third term is exactly the total expected welfare loss due to sampling, which must be subtracted from total surplus (note that this term vanishes when $n = N$).

Lemma 5. *The expected per capita welfare loss due to sampling, denoted $L(n)$, can be expressed as follows:*

$$L(n) = \frac{1}{2}E\Big[\Big(\frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_i - \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i\Big)^2 \Big| e(n)\Big] = \Big[\frac{1}{n} - \frac{1}{N}\Big]\frac{V(e(n))}{2}. \tag{32}$$

For proof, see the appendix

With the above definitions, an elegant form of (29) is, in per capita terms,

$$(1/N)W(n) = (1/2)\bar{\mu}^2 + (n/N)[S(e(n)) - F] - L(n). \tag{33}$$

5.2. Properties of second best welfare as a function sample size

We can now turn to the study of $W(n)$ and of its optimum with respect to n. Recall that $\tilde{n}$ is defined by (28) above as the threshold above which effort jumps down to zero. It follows that W will possess a discontinuity at point $\tilde{n}$. The expressions of W and of its derivatives will be different on the intervals $[1, \tilde{n}]$ and $[\tilde{n}, N]$, if $\tilde{n}$ belongs to the admissible domain $[1, N]$. Hence, a local maximum of W can exist in the interior of $[1, \tilde{n}]$, but also in the interior of $[\tilde{n}, N]$.

Define n_O as the solution of the first order condition for a local maximum of W in the open interval $(1, \tilde{n})$, that is, if it exists, the solution of

$$-L'(n_O) = (1/N)[\phi(\varepsilon(n_O)) + F]. \tag{34}$$

On the other interval $[\tilde{n}, N]$, the *per capita* welfare function boils down to $(1/N)W(n) = (1/2)\bar{\mu}^2 - (n/N)F - (z^2/2n) + (z^2/N)$, which is strictly concave with respect to n. Define then n_A as the unique local maximum of W on the range $n > \tilde{n}$, that is, solving the first order condition,

$$n_A = z\sqrt{N/2F}. \tag{35}$$

Remark that according to (35), the optimal number of representatives is an increasing and concave function of N, an increasing function of the variance parameter z, and a decreasing function of the fixed cost of representation F.

5.3. Second best optimal sample size

The analysis will be divided into two broad cases. The first case is when second best welfare W, viewed as a function of the real variable n, is monotonically increasing with n on the whole domain $[1, N]$. This case corresponds to the idea of Direct Democracy, as explained below. The other case is when the maximum of W is reached at a point strictly smaller than N on $[1, N]$. This latter case will itself be divided in several subcases, in which Representative Democracy emerges as a second best. Recall that due to the sampling mechanism constraints, the second best optimal sample size, hereafter denoted n^{**}, must be an integer, with $1 \leq n^{**} \leq N - 1$.

Case 1: Direct Democracy.

Let us first define an economy as a *Second Best Direct Democracy* if W is monotonically increasing on the interval $[1, N]$. In this case, the second best optimum is clearly the maximal sample size $n^{**} = N - 1$.

If condition (17) holds, then $S(e(N)) > F$. (These two conditions are equivalent if and only if $z^2 = 0$.) This means that it is socially worth requiring some effort from everybody, and Lemma 6d (stated and proved in the appendix), shows that W is monotonically increasing with n. We conclude that in this case, the first best is approximately implementable by means of the maximal sample size $n^{**} = N-1$ (see Figure 2a). By continuity, equilibrium effort will then be close to $\varepsilon(N) = e^* > 0$, and the welfare loss with respect to first best will be close to zero, given that $L(N-1)$ is approximately equal to zero for large N. We call this case *Active Direct Democracy* (ADD), because, in essence, every agent exerts some effort and participates in the public decision process.

If on the contrary, $S(e(N)) < F$, then, condition (17) does not hold; yet, W can still be monotonically increasing. If this is the case, the second best optimal sample size is still $n^{**} = N - 1$ (see Figure 2b), but each agent will exert effort zero. For convenience, we call this case *Passive Direct Democracy* (PDD). It emerges if the inter-regional variance z^2 is large enough, since it is then socially profitable to sample many individuals, in order to better estimate the crucial parameter $\bar{\mu}$. Recall that under zero effort, representative i simply declares her *a priori* preference type μ_i to the Principal, who in turn chooses the second best production level $q^{**} = \frac{1}{(N-1)} \sum_1^{N-1} \mu_i \approx \hat{\mu}$.

A central result is then the following.

Proposition 4. *If Direct Democracy is a first best optimum, then, the first best is approximately implementable under asymmetric information, and the economy is an Active Direct Democracy,*
If $S(e(N)) < F$, and if in addition, the economy is a Second Best Direct Democracy, then, it must be a Passive Direct Democracy.

The proof of this result follows easily from the above discussion, and from Lemma 6 (see the appendix). Proposition 4 says that (17) is a sufficient condition for Direct Democracy in the second best world. But this condition is not necessary. In other words, Direct Democracy can also be second best optimal, in spite of the fact that it is not first best optimal. Second Best Direct Democracy, if it exists, is characterized by citizen passiveness when $\check{n} < N$.

[Insert FIGURE 2 around here]

When $z^2 = 0$, Proposition 4 can be strengthened as follows: ADD is second best optimal if and only if Direct Democracy is first best optimal; second best optimality is characterized by sampling if and only if the Reign of Tradition is first best optimal, and finally, PDD is impossible.

Case 2: Nonchalant Assembly.

Assume now not only that condition (17) does not hold, but also that $\tilde{n}$, defined by (28), satisfies $1 < \tilde{n} < N$. Then, if the global maximum of W is reached in the open interval $(\tilde{n}, N)$, that is, if $n^{**} = n_A$, the representatives are too numerous and therefore do not exert any effort. We call this situation an *Interior Nonchalant Assembly* (INA). See Figure 3a. There are other types of Nonchalant Assembly, corresponding to corner solutions: the *Corner Nonchalant Assembly* (CNA). See Figure 3b. They share the same qualitative features as INA.

[Insert FIGURES 3 around here]

These cases will happen when F is relatively small. Since individual efforts decrease when sample size increases, the second best is then to choose enough delegates (or representatives) to make sure that their manipulation power is completely diluted. The many representatives simply protect the citizen against public decision capture by an active minority. This is extremely useful under incomplete information, but has a social cost $n^{**}F$, which can be viewed as the cost of maintaining an assembly.

Case 3: Active Oligarchy.

The global maximum of W may also be reached at n_O, defined above by (34), with, $1 < n_O < \tilde{n}$. See Figure 3c. The optimal sample size is then $n^{**} = n_O$. The size of the sample is then (relatively) small, and as a result, each representative will exert some socially undesirable effort $\varepsilon(n_O) > 0$. The *active oligarchs* (AO) are responsible for a social loss; they increase the loss L, since a positive effort implies a positive v. Society would prefer the oligarchs to be less active, but the happy few have incentives to manipulate the public decision to suit their own convenience. Such a case will emerge for sufficiently high F and σ^2.

Case 4: Technocracy.

Finally, the second best welfare function W can be decreasing on its entire admissible domain $[1, N]$. See Figure 3d. Then, to minimize the social costs of sampling, the best choice is simply $n^{**} = 1$. This case will appear when fixed costs are high, and when preference dispersion is sufficiently low, more precisely when $z^2 + \sigma^2$ is small, for then necessarily, the value of effort $V(e)$ (which is bounded above by $z^2 + \sigma^2$) will also be small. *Technocracy* can be reached as an extreme form of Active Oligarchy, with a single active oligarch if $n_O < 1 < \tilde{n}$. In this case, we find an active form of technocracy, but it could happen as an extreme form of Nonchalant Assembly too (if $n_A < \tilde{n} < 1$), so that *passive technocrats* do also exist.

The technocracy case is not exactly dictatorship: If the variance of types is small, everybody more or less agrees with the production objective, and the decision chosen by the technocrat is close to everybody's preferred outcome with a very high probability. This situation corresponds approximately to a common value problem. The welfare losses are minimized by having to pay the fixed cost F only once.

5.4. When do the various cases happen? A simple illustrative example

We now turn to the study of a simple example. Let $\phi(e) = \alpha e$, $v(e) = \sigma^2 e$, with α nonnegative, and the effort variable constrained to belong to $[0, 1]$. The optimal effort function is therefore $e(n) = 1$ if $\sigma^2/2n \geq \alpha$ and $e(n) = 0$ otherwise. The threshold is naturally $\tilde{n} = \sigma^2/2\alpha$. The second best welfare function W can in this case be described as piecewise concave: It is a strictly concave function of n on $[0, \tilde{n}]$ and for $n > \tilde{n}$. A complete analysis of the example is cumbersome, because the frontier separating the AO and CNA cases is highly non-linear. Figure 4 depicts regions of the parameter space for which the various cases appear. It represents a mapping of the (σ^2, F) plane into the set of cases, with α and z^2 fixed.

[Insert FIGURE 4 about here]

The active and passive forms of direct democracy occur for small values of F. If we now fix F at a high enough value and increase the dispersion of preferences σ^2, we will first cross the INA region, then the Corner NA region. Further increases of the dispersion for fixed F will drive the economy to Active Oligarchy, and if σ^2 is high enough, the active Oligarchs will become so numerous that the region of Active Direct Democracy will eventually be reached. Following a horizontal direction, if for fixed σ^2, F is increased, the economy will eventually reach a form of technocracy.

6. Some empirical evidence, with political data

The above analysis suggests that the optimal number of representatives depends nonlinearly on the population size, the heterogeneity of preferences, and the costs of representation. To get a first view of the empirical relationships between N and n, we have regressed the total number of representatives (expressed in numbers of individuals) on the population size (expressed in millions of citizens) for a sample of 111 countries, which possess parliaments or representative assemblies. To fix ideas, the USA are in the sample, with $n = 535$ and $N = 260.341$. The complete data, which are extracted from The Europa World Year Book (1995), and some details of the estimations, are presented in the appendix. A first simple linear regression of the form $n = aN + b$, yields significant estimates of a and b, but with a poor R^2. By contrast, a surprisingly excellent adjustment is obtained when $log(n)$ is regressed on $log(N)$ and a constant: 74% of the variance of $log(n)$ is explained by $log(N)$ and the constant. Adding some exogenous variables, we find the following results,

$$\underset{}{log(n)} = \underset{(19.69)}{0.395}\ log(N) + \underset{(4.649)}{0.350}\ dev - \underset{(-5.59)}{154.10^{-6}}\ dens + \underset{(76.18)}{4.297}\ .$$

In the above regression, t-statistics are between brackets, *dev* is a dummy variable indicating a developed country, and *dens* is population density in the country, expressed in number of inhabitants per square kilometer. Population density is taken as a crude measure of population heterogeneity. Countries in which the density is high (*e.g.*, Japan) should have less heterogeneity than countries in which the density is much lower (*e.g.*, the USA). Accordingly, the sign of the coefficient on *dens* is negative. Moreover, we use the dummy variable *dev* to capture some differences in the costs of maintaining representatives. Accordingly, the sign of the coefficient on *dev* is positive (the richer a country, the larger its assembly). The adjusted R^2 for this regression is .78, and the global F-statistic is a highly significant 129.5.[7]

These results indicate that the number of representatives n increases less than proportionately with the size N of the population (in millions), since, according to our estimates, $n \approx 73.N^{0.4}$. The number of representatives does not seem to be determined by a constant sampling rate: it is as if costs did impede the increase of n in large countries.

[Insert FIGURE 5 about here]

Closer scrutiny of the above regression residuals show that eastern European countries in transition seem to have "too many" representatives, in the sense that they lie above the regression line. France and Italy have "too many" representatives too, whereas the United States and Israel are below the regression line. See figure 5, which is a plot of the fitted number of representatives (denoted REPREF) against the actual number (REPRE). France, Italy, and the USA are clearly outliers.

The constitutional history of the United States shows that the representation ratio has constantly decreased for more than 200 years. In a footnote, Tocqueville (1835, part I, chap. VIII, p 190) already notes the fact: the representation ratio decreased from 1 representative for every 33,000 inhabitants in 1792, to 1 over 48,000 in 1832. This trend has not been reversed since then, the ratio reaching the record low of 1 over 573,394 in 1990; and furthermore, the number of seats in the House has reached a ceiling of 435 in 1910, which has been fixed by statute in 1929 (see O'Connor and Sabato (1993), p. 191). According to our empirical results, the US Congress "should" have 935 members instead of 535. It is too soon to provide an explanation for these facts; and to understand the way adjustments in the number of seats are made, the processes of periodical reapportionment and redistricting in various countries should be studied more carefully. It is of course difficult to decide if the number of US representatives is inefficiently low, but the question has been posed a long time ago, by the opponents of the American Constitution:

> The very term, representative, implies, that the person or body chosen for this purpose, should resemble those who appoint them (...). Those who are placed instead of the people, should possess their sentiments and feelings, and be governed by their interests, or, in other words, should bear the strongest resemblance of those in whose room they are substituted. (...) Sixty-five men cannot be found in the United States, who hold the sentiments, possess the feelings, or are acquainted with the wants and interests of this vast country.
>
> — *Essays of Brutus*, III, 1787 (in Storing (1981), p. 123).

[7] This result is good, given the nature of the data and the very limited set of control variables. The country's land surface is not a significant explanatory variable, when introduced in the above regression. The ethnolinguistic fractionalization index (ELF), which measures the degree of ethnic heterogeneity of a country, yields disappointing results. When these variables are added, the coefficients on the other variables do not change much, and it is tempting to conclude that the estimation of the coefficient on log(N) seems robust.

7. Concluding remarks

We have modeled the recourse to representation for the choice of a public policy with the help of a class of direct revelation mechanisms based on agent sampling. We interpret exhaustive sampling as "direct democracy", the random choice of a single agent as "technocracy", and sampling of a strict subset as "assembly" or "committee."

The economy is characterized by informational asymmetries. Agents know neither their own preferences nor the preferences of others exactly. However, each agent can exert some unobservable effort to improve her valuation of public projects. In this economy, the first best optimum is, either a direct democracy, or is characterized as a "reign of tradition," in which decisions are made by a benevolent planner on the basis of *a priori* information.

Since a benevolent planner cannot be found, first best mechanisms are in general not implementable under asymmetric information. We have then shown that direct democracy is second best optimal if it is also first best optimal. In contrast, the use of a strict subset of representatives is second best optimal in cases in which the reign of tradition would have prevailed under a benevolent and omniscient planner. The results allow an analysis of the forces determining the optimal number of representatives. The role of these representatives is, not only to produce or transmit useful information for the purpose of public decisions, but also, by their mere number and diversity, to protect society against opportunistic manipulations by an active minority.

Appendix

Proof of Lemma 1

Using the fact that C is quadratic, and the independence of signals s_k, from expression (12), we derive,

$$W = E\Big\{(1/2N)(\sum_{k=1}^{N}\hat{\theta}_k)^2\Big|\mu, e\Big\} - \sum_{i=1}^{n}\phi(e_i) - nF$$
$$= (1/2N)\Big\{\sum_{j=1}^{N}\sum_{k\neq j}E(\hat{\theta}_k|\mu_k;e_k)E(\hat{\theta}_j|\mu_j;e_j) + \sum_{k=1}^{N}E(\hat{\theta}_k^2|\mu_k;e_k)\Big\} - \sum_{i=1}^{n}\phi(e_i) - nF.$$

Now since by definition, $\mu_k = E(\hat{\theta}_k|\mu_k;e_k)$ and $v(e_k) = E(\hat{\theta}_k^2|\mu_k;e_k) - \mu_k^2$ for all k, substitution in the above expression yields,

$$W = (1/2N)\Big[\Big(\sum_{k=1}^{N}\mu_k\Big)^2 + \sum_{k=1}^{N}v(e_k)\Big] - \sum_{i=1}^{n}\phi(e_i) - nF,$$

with $e_k = 0$ for all $k = n+1,\ldots,N$. Using then $v(0) = 0$ and rearranging, we finally find the first best welfare function $W^*(e,n)$ as a simple restatement of the above formula. *Q.E.D.*

Proof of Proposition 2

The proof will be provided in two steps. In Step 1, we prove that g optimal implies g is an unbiased estimator of $\bar{\mu}$. In Step 2, we prove that g anonymous and unbiased for all probability distributions of $\hat{\theta}_i$ implies that g is the arithmetic mean.

Step 1

The expected social welfare writes $W = NE[\bar{\theta}g - (1/2)g^2\,|\,e] - \sum_i \phi(e_i) - nF$, where $g : \mathbf{R}^n \to \mathbf{R}$ is the public decision rule, a random variable depending on $(\hat{\theta}_1, ..., \hat{\theta}_n)$ only, $\bar{\theta} = (1/N)\sum_{i=1}^{N}\theta_i$, and $e = (e_1,\ldots,e_n)$ is the vector of effort variables of agents $1,\ldots,n$.

A mechanism is a vector of functions (g,t), where $t = (t_0,t_1,\ldots,t_n)$ is a vector of transfers, each depending on $(\hat{\theta}_1, ..., \hat{\theta}_n)$ only, and t_0 being the transfer of all non-sampled agents $i > n$. These transfer functions must satisfy the budget constraint

$$\sum_{i=1}^{n}t_i + (N-n)t_0 = (N/2)g^2.$$

Without loss of generality, g can be rewritten $g = m+X$, where m is a constant and X is a random variable, depending on $(\hat{\theta}_1, ..., \hat{\theta}_n)$ only, and a zero mean, $E(X) = 0$. Then, the social welfare function writes, $W = NE[\bar{\theta}(m+X) \mid e] - (N/2)E[(m+X)^2 \mid e] - \Sigma_i \phi(e_i) - nF$, that is,

$$(1/N)W = (\bar{\mu}m - (1/2)m^2) + E(\bar{\theta}X \mid e) - (1/2)E(X^2 \mid e) - (1/N)\sum_i \phi(e_i) - (n/N)F,$$

using $\bar{\mu} = E(\bar{\theta} \mid e)$. Remark that W is additively separable with respect to variable m. We would like to make sure that m can be chosen independently of X and $(t_1, \ldots, t_n)$. To this end, we first show that the individually optimal effort variables do not depend on m.

Each $i = 1, \ldots, n$ chooses the effort e_i so as to maximize,

$$U_i(e) = E_s[\hat{\theta}_i g(\hat{\theta}) - t_i(\hat{\theta}) \mid \mu_i, e] - \phi(e_i) - F,$$

assuming truthful revelation of $\hat{\theta}_i$.

Using $g = m + X$, we find, $U_i(e) = \mu_i m + E_s(\hat{\theta}_i X \mid \mu_i, e) - E_s(t_i \mid \mu_i, e) - \phi(e_i) - F$. It follows that for all $i = 1, \ldots, n$, e_i maximizes,

$$E_s(\hat{\theta}_i X \mid \mu_i, e) - E_s(t_i \mid \mu_i, e) - \phi(e_i).$$

The solution of this problem is a best response correspondence $e_i \in \eta_i(X, t_i, e_{-i})$. Define the Cartesian product $\eta = \Pi_{i=1}^n \eta_i$. The effort incentive constraints of our problem write $e \in \eta(X, t, e)$, where η does not depend on t_0. Since non-sampled agents are not revealing information and do not exert effort, t_0 can always be chosen so as to balance the budget and it follows that the choice of t_i, $i = 1, \ldots, n$, is unconstrained, apart from the requirement that it should depend on $\hat{\theta}$ only. We conclude that the choice of effort is independent of m.

Let $\mathcal{X}_n$ denote the set of random variables X depending on $(\hat{\theta}_1, ..., \hat{\theta}_n)$ only.

A second best optimal public decision rule is a solution of the following relaxed problem,

maximize $(\bar{\mu}m - (1/2)m^2) + [E(\bar{\theta}X \mid e) - (1/2)E(X^2 \mid e) - (1/N)\sum_{i=1}^n \phi(e_i)]$,

with respect to $(m, X, t_1, \ldots, t_n, e)$, subject to the constraints,

$E(X \mid e) = 0$,
$X \in \mathcal{X}_n$, $t_i \in \mathcal{X}_n$, $i = 1, \ldots, n$, and
$e \in \eta(X, t_1, \ldots, t_n, e)$.

In addition, we should constrain g, or X, to be *revealing*, that is, to lead to truthful revelation of $\hat{\theta}_i$ by each representative i, but we relax the optimization problem by neglecting these constraints here: it will then be necessary to show that the relaxed solution can be chosen with the revelation property. This happens to be true (see subsection 4.2. and Proposition 3 in this paper), because without any additional social cost, Groves transfers can ensure honest revelation of their information by representatives, while non-sampled agents help balancing the budget.

A glance at the above program now shows that the constraints do not bear on the choice of m and therefore, a necessary condition for an optimal solution is $m = \bar{\mu}$. Given that $g = m + X$, we have $E(g) = \bar{\mu}$, that is, g is an unbiased estimator of $\bar{\mu}$.

Step 2

It remains to show that if g is anonymous and unbiased for all probability distributions of $\hat{\theta}_i$, then g is the arithmetic mean.

Anonymity means that for all permutation $\sigma : \{1, ..., n\} \to \{1, ..., n\}$, one has

$$g(\theta_1, ..., \theta_n) = g(\theta_{\sigma(1)}, ..., \theta_{\sigma(n)}).$$

Define ψ as follows, $\psi(\theta) = g(\theta) - (1/n)\sum_{i=1}^n \theta_i$. Then, g unbiased implies $E(\psi) = 0$.

Let $A^k = \{\theta \in \mathbf{R}^n \,|\, \theta \text{ has } k \text{ distinct coordinates}\}$, for all $k \in \{1, ..., n\}$. For instance, $A^1 = \{\theta \in \mathbf{R}^n \,|\, \theta = (\alpha, ..., \alpha), \alpha \in \mathbf{R}\}$, and $A^n = \{\theta \in \mathbf{R}^n \,|\, \theta = (\alpha_1, \alpha_2, ..., \alpha_n), \alpha_i \neq \alpha_j,\, i \neq j\}$. Clearly, $\mathbf{R}^n = A^1 \cup A^2 \cup \ldots \cup A^n$. We will prove the result by induction over $k \in \{1, \ldots, n\}$.

Let α be a real number and $\nu = \delta_\alpha$ the Dirac probability measure at α. Then, $E_\nu(\psi) = 0$ is equivalent to $\psi(\alpha, ..., \alpha) = 0$, for all α. This shows that ψ is identically zero on A^1.

Assume now that $\psi = 0$ on the domain $A^1 \cup A^2 \cup \ldots \cup A^{k-1}$, for $k \leq n$. Let $\alpha_1, ..., \alpha_k$, be k distinct points of $\mathbf{R}$, and consider the probability distribution $\nu = \sum_{i=1}^k p_i \delta_{\alpha_i}$. Since ψ is symmetric, its value at a given point θ of $\mathbf{R}^n$, the coordinates of which belong to the finite support of ν, depends only on the order of multiplicity of each α_j among the coordinates of θ. Define $\alpha = (\alpha_1, ..., \alpha_k)$ and let $Q = \{\, q = (q_1, ..., q_k) \in \mathbf{N}^k \,|\, \sum_j q_j = n\,\}$, where q_j is the number of times α_j appears as a coordinate. Define then

$$b_\alpha(q_1, ..., q_k) = \psi[\alpha_1, \ldots, \alpha_1, \alpha_2, \ldots, \alpha_2, \ldots, \alpha_k, \ldots, \alpha_k],$$

where α_j appears exactly q_j times among the arguments of ψ. Under ν, the probability distribution of q is multinomial with parameters p in the unit simplex $\Delta_k = \{(p_1, \ldots, p_k) \in \mathbf{R}_+^k \,|\, \sum_j p_j = 1\,\}$.

It follows that we have,

$$E_\nu(\psi) = \sum_{q \in Q} K(q) b_\alpha(q_1, ..., q_k) p_1^{q_1} \ldots p_k^{q_k},$$

where

$$K(q) = \frac{n!}{q_1! q_2! \ldots q_k!}.$$

Now, by assumption, $E_\nu(\psi) = 0$, and in its expression, all the terms in which the vector α happens to have strictly less than k distinct values are zero by our induction hypothesis. Define

$$Q(1) = \{q = (q_1, \ldots, q_k) \in Q \,|\, q_i \geq 1\,\}.$$

Thus, for all p in Δ_k,

$$E_\nu(\psi) = \sum_{q \in Q(1)} K(q) b_\alpha(q) (\Pi_{i=1}^k p_j^{(q_j - 1)})(\Pi_{i=1}^k p_i) = 0,$$

and it follows that the expression $M(p)$, defined as $M(p) = \sum_{Q(1)} K(q)b_\alpha(q)(\Pi_{i=1}^k p_j^{q_j-1})$ is equal to zero for all p in the interior of Δ_k.

From this, we will deduce that $b_\alpha(q) = 0$ for all $\alpha \in \mathbf{R}^k$ and all $q \in Q(1)$. Consider a particular $\hat{q}$ in $Q(1)$, satisfying $\hat{q}_j = 1$ for $j > k_1$, $\hat{q}_j = r$ for $k_r < j \leq k_{r-1}$, and finally $\hat{q}_j = s$ for $j \leq s$, where $0 < k_s \leq \ldots \leq k_r \leq \ldots \leq k_1 \leq k$.

Define then recursively the sets $R(1) = \{q \in Q(1) \,|\, q_j = 1 \text{ iff } j > k_1\}$, $R(r) = \{q \in R(r-1) \,|\, q_j = r, \text{ iff } k_{r-1} \geq j > k_r\}$, and $R(s) = \{q \in R(s-1) \,|\, q_j = s, \text{ iff } k_s \leq j\}$. Remark that $R(s) = \{\hat{q}\}$.

Define then the quantity,

$$M_r(p) = \sum_{q \in R(r)} K(q)b_\alpha(q)p_1^{q_1-r} \ldots p_{k_r}^{q_{k_r}-r},$$

where p belongs to Δ_{k_r}.

We now prove by induction over r that $M_r(p) = 0$ for all p in the interior of Δ_{k_r}, denoted $int(\Delta_{k_r})$.To do this, we first show that $M_1(p) = 0$ over $int(\Delta_{k_1})$. Since $M(p) = 0$ over $int(\Delta_k)$, let $p_j \to \pi_j$ for all $j \leq k_1$ and $p_j \to 0$ for all $j > k_1$, with $\pi = (\pi_1, \ldots, \pi_{k_1}) \in int(\Delta_{k_1})$. Then, all the terms in $M(p)$ such that $q_j > 1$ for $j > k_1$ go to zero, and by continuity, $M(p) \to M_1(\pi) = 0$. Remark, then, that since all terms in $M_1(\pi)$ are such that $q_j > 1$ if $j \leq k_1$, it is also true that for all $\pi \in int(\Delta_{k_1})$,

$$\sum_{q \in R(1)} K(q)b_\alpha(q)\pi_1^{q_1-2} \ldots \pi_{k_1}^{q_{k_1}-2} = 0.$$

Given this result, and using the same method as above, it is not difficult to show that if $M_r(p) = 0$ for $p \in int(\Delta_{k_r})$, then $M_{r+1}(p) = 0$ for $p \in int(\Delta_{k_{r+1}})$. It follows that $M_s(p) = 0$ for all p in $int(\Delta_s)$, or, recalling that $R(s) = \{\hat{q}\}$, we get,

$$K(\hat{q})b_\alpha(\hat{q})p_1^{\hat{q}_1-s} \ldots p_{k_s}^{\hat{q}_{k_s}-s} = 0,$$

which is equivalent to $b_\alpha(\hat{q}) = 0$ for all α. This result shows that we possess an algorithm to prove that $b_\alpha(q) = 0$ for any α in $\mathbf{R}^k$ such that the coordinates of α are distinct, and any q in $Q(1)$. Therefore, ψ is identically zero on the set $A^1 \cup \ldots \cup A^k$, and by our first induction hypothesis, it follows that ψ is identically zero on $\mathbf{R}^n$, that is,

$$g(\theta_1, \ldots, \theta_n) \equiv \frac{1}{n}\sum_{i=1}^n \theta_i.$$

Q.E.D.

Proof of Lemma 4

It will be convenient to compute first $W(n|\mu)$, the expected welfare, conditional on interim information μ, that is, by definition,

$$W(n\,|\mu) = E_s\Big\{E_\theta\Big\{(\sum_{i=1}^N \theta_i)q(\hat{\theta}(s)) - NC[q(\hat{\theta}(s))]\,\Big|\, s, \mu; e\Big\}\,\Big|\,\mu; e\Big\} - \sum_{i=1}^n \phi(e_i) - nF$$

Taking expectations with respect to θ conditional on s and substituting the arithmetic mean rule yields,

$$W(n\,|\mu) = E_s\Big\{\Big(\sum_{i=1}^{n}\hat{\theta}_i(s_i;e_i) + \sum_{j=n+1}^{N}\mu_j\Big)\Big(\frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i(s_i;e_i)\Big) - \frac{N}{2}\Big(\frac{1}{n}\sum_{i=1}^{n}\hat{\theta}(s_i;e_i)\Big)^2\,\Big|\,\mu;e\Big\} - \sum_{i=1}^{n}\phi(e_i) - nF.$$

Easy algebra then yields,

$$W(n\,|\mu) = \frac{1}{n}\Big(\sum_{i=1}^{n}\mu_i\Big)\Big(\sum_{j=n+1}^{N}\mu_j\Big) + \Big[\frac{1}{n} - \frac{N}{2n^2}\Big]E_s\Big\{\Big(\sum_{i=1}^{n}\hat{\theta}_i\Big)^2\,\Big|\,\mu;e\Big\} - \sum_{i=1}^{n}\phi(e_i) - nF.$$

Using signal independence and the fact that $e_i = e(n)$ for all i yields, after some computations,

$$E_s\Big\{\Big(\sum_{i=1}^{n}\hat{\theta}_i\Big)^2\,\Big|\,\mu;e\Big\} = nv(e(n)) + \Big(\sum_{i=1}^{n}\mu_i\Big)^2.$$

Substituting this result in the expression for W and rearranging terms yields, after some simplifications,

$$W(n\,|\mu) = H(n\,|\mu) + \Big[\frac{1}{n} - \frac{N}{2n^2}\Big]nv(e(n)) - n\Big[\phi(e(n)) + F\Big],$$

where

$$H(n\,|\mu) = \Big[\frac{1}{n} - \frac{N}{2n^2}\Big]\Big(\sum_{1}^{n}\mu_i\Big)^2 + \frac{1}{n}\Big(\sum_{i=1}^{n}\mu_i\Big)\Big(\sum_{j=n+1}^{N}\mu_j\Big).$$

We then compute the expected value of $H(n\,|\mu)$ with respect to the prior distribution P of the μ_is (see Assumption 2 above), using the fact that all μ_is are independent. This yields,

$$E_\mu H = (N\bar{\mu}^2/2) + z^2\Big[1 - \frac{N}{2n}\Big].$$

Now, by definition, $W(n) = E_\mu W(n\,|\mu)$. Using the results just obtained and rearranging terms again yields,

$$W(n) = \frac{N\bar{\mu}^2}{2} + n\Big[\frac{v(e(n)) + z^2}{2n} - \phi(e(n)) - F\Big] - \Big[\frac{1}{n} - \frac{1}{N}\Big]\frac{N[v(e(n)) + z^2]}{2},$$

which is the desired result. *Q.E.D.*

Proof of Lemma 5

The *per capita* first best expected surplus under $e_i = e(n)$ can be written $(1/2)E[\bar{\theta}_N^2|e(n)] = (\bar{\mu}^2/2) + (1/2N)V(e(n))$, where by definition, $\bar{\theta}_n = (1/n)\sum_{i=1}^{n}\hat{\theta}_i$, for all $n = 1,\ldots,N$. Under the arithmetic mean production rule $q = \bar{\theta}_n$, the per capita expected surplus can be written, $E[\bar{\theta}_N\bar{\theta}_n - (1/2)\bar{\theta}_n^2 \,|\, e(n)]$. The per capita expected welfare loss is the difference between these two terms, precisely,

$$L(n) = E\{(1/2)\bar{\theta}_N^2 - \theta_n\theta_N + (1/2)\theta_n^2 \,|\, e(n)\} = (1/2)E\{(\bar{\theta}_N - \bar{\theta}_n)^2 \,|\, e(n)\}$$
$$= (1/2)\{V(\bar{\theta}_N) + V(\bar{\theta}_n) - 2Cov(\bar{\theta}_N, \bar{\theta}_n)\} = (1/n)V(e(n)) - (1/N)V(e(n))$$

To understand the last statement, note that $V(\bar{\theta}_n) = (1/n)V(e(n))$ for all n. In addition, using the expression $\bar{\theta}_N = (n/N)\bar{\theta}_n + (1/N)\Sigma_{j=n+1}^{N}\hat{\theta}_j$ and the independence of the $\hat{\theta}_j$s, one finds,

$$Cov(\bar{\theta}_N, \bar{\theta}_n) = E[\bar{\theta}_N\bar{\theta}_n|e] - \bar{\mu}^2 = E[(n/N)\bar{\theta}_n^2|e] + E[(1/N)(\Sigma_{j=n+1}^{N}\hat{\theta}_j)\bar{\theta}_n|e] - \bar{\mu}^2$$
$$= \frac{n}{N}\frac{V(e(n))}{n} + \frac{n}{N}\bar{\mu}^2 + \frac{N-n}{N}\bar{\mu}^2 - \bar{\mu}^2 = \frac{V(e(n))}{N}.$$

This string of equations proves (32). *Q.E.D.*

Lemma 6.

(a), As a function of n, $S(\varepsilon(n))$ (defined above by (31)) is decreasing.

(b), W has an upward jump at point $\tilde{n}$, i.e., with obvious notations, $W(\tilde{n}^+) > W(\tilde{n}^-)$.

(c), the right hand derivative of W at $\tilde{n}$ is always smaller than the left hand derivative at $\tilde{n}$, i.e., with obvious notations, $(1/N)W'(\tilde{n}^-) > (1/N)W'(\tilde{n}^+)$.

(d), If $S(e(N)) > F$, then W is non-decreasing on the interval $[1, N]$.

Proof of Lemma 6

Proof of (a). Compute the derivative of $S(\varepsilon(.))$ as if n was a real variable. Taking envelope conditions (25) into account, $S'\varepsilon' = -(1/2n^2)V(\varepsilon(n)) < 0$.

Proof of (b). W has an upward jump of at point $\tilde{n}$, *i.e.*, with obvious notations,

$$W(\tilde{n}^+) = N\bar{\mu}^2/2 - nF + z^2[1 - (N/2n)] > W(\tilde{n}^-) = N\bar{\mu}^2/2 + \tilde{n}[S(\varepsilon(\tilde{n})) - F] - NL(\tilde{n}).$$

To prove the result, it is sufficient to delete the terms appearing on both sides, showing that the above inequality is equivalent to $v(\varepsilon(\tilde{n}))[1/\tilde{n} - 1/N] > 0$, which is clearly always true. (Use the fact that $S(\varepsilon(\tilde{n})) = (z^2/2\tilde{n}) > 0$.)

Proof of (c). The right hand derivative of W at $\tilde{n}$ is always smaller than the left hand derivative at $\tilde{n}$. To show this, first write,

$$\frac{1}{N}W'(\tilde{n}^-) = -\frac{v'\varepsilon'}{2}\left[\frac{1}{\tilde{n}} - \frac{1}{N}\right] + \frac{V(\varepsilon(\tilde{n}))}{2\tilde{n}} - \frac{1}{N}[\phi(\varepsilon(\tilde{n})) + F] > \frac{1}{N}W'(\tilde{n}^+) = -\frac{F}{N} + \frac{z^2}{2\tilde{n}^2}.$$

This inequality is equivalent, after simplification, to

$$\left[\frac{1}{\tilde{n}} - \frac{1}{N}\right]\left[\frac{v(\varepsilon(\tilde{n}))}{2\tilde{n}} - \frac{v'(\varepsilon)\varepsilon'(\tilde{n})}{2}\right] > 0,$$

which is true since $v'(\varepsilon) > 0$ and $\varepsilon' < 0$.
Proof of (d). Compute the derivative of W to find the expression,

$$(1/N)W'(n) = \frac{1}{N}\Big[S(e(n)) - F\Big] + \Big[1 - \frac{n}{N}\Big]\frac{V(e(n))}{2n^2} + \Big[\frac{1}{n} - \frac{1}{N}\Big]\frac{(-v'(e)e'(n))}{2}.$$

It is easy to check that since $S(e(n))$ is a decreasing function of n (point *(a)* above), and since $e' < 0$, then if $S(e(N)) > F$, the above expression of $(1/N)W'$ is a sum of positive terms on $[1, N]$. *Q.E.D.*

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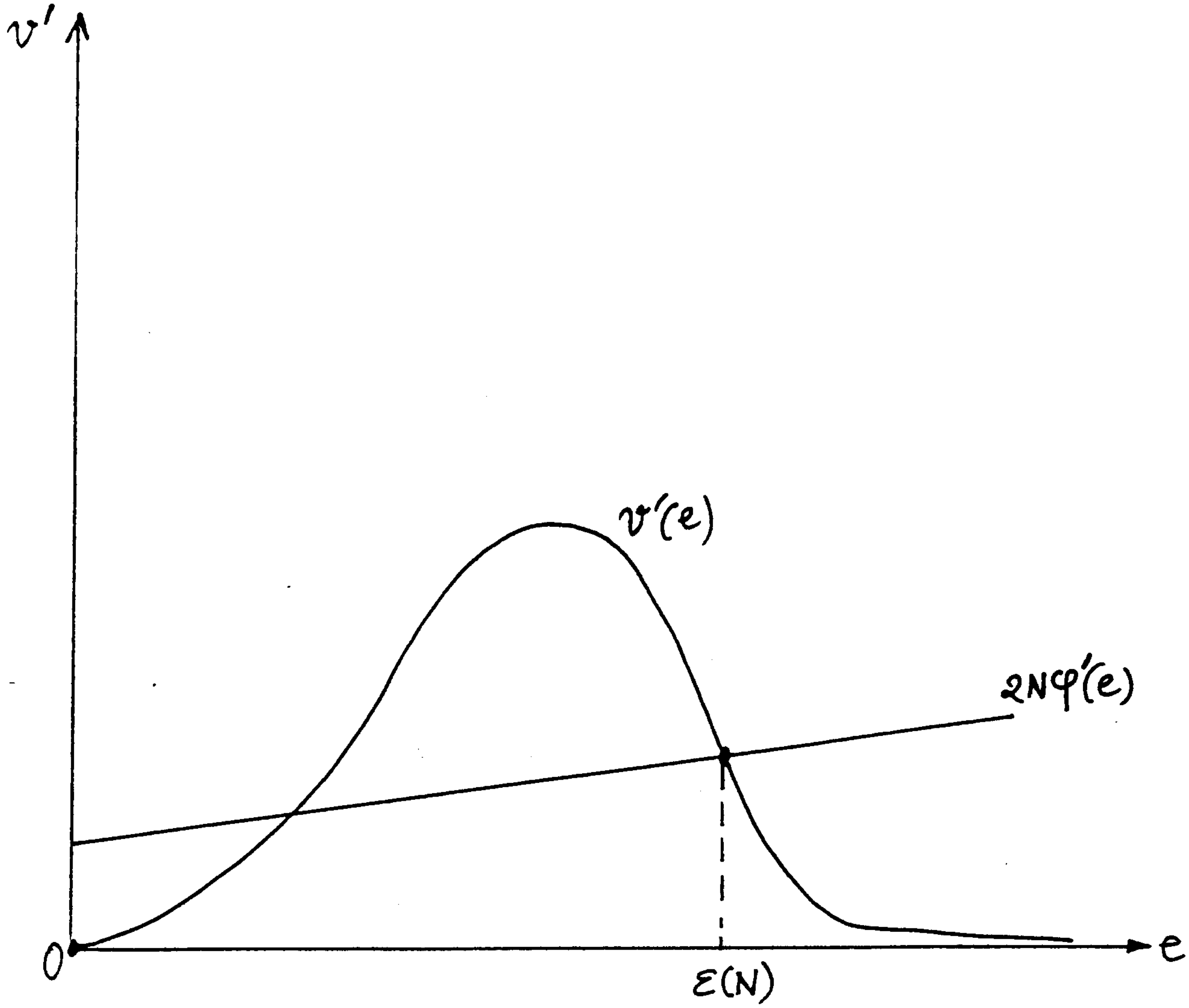


Figure 1

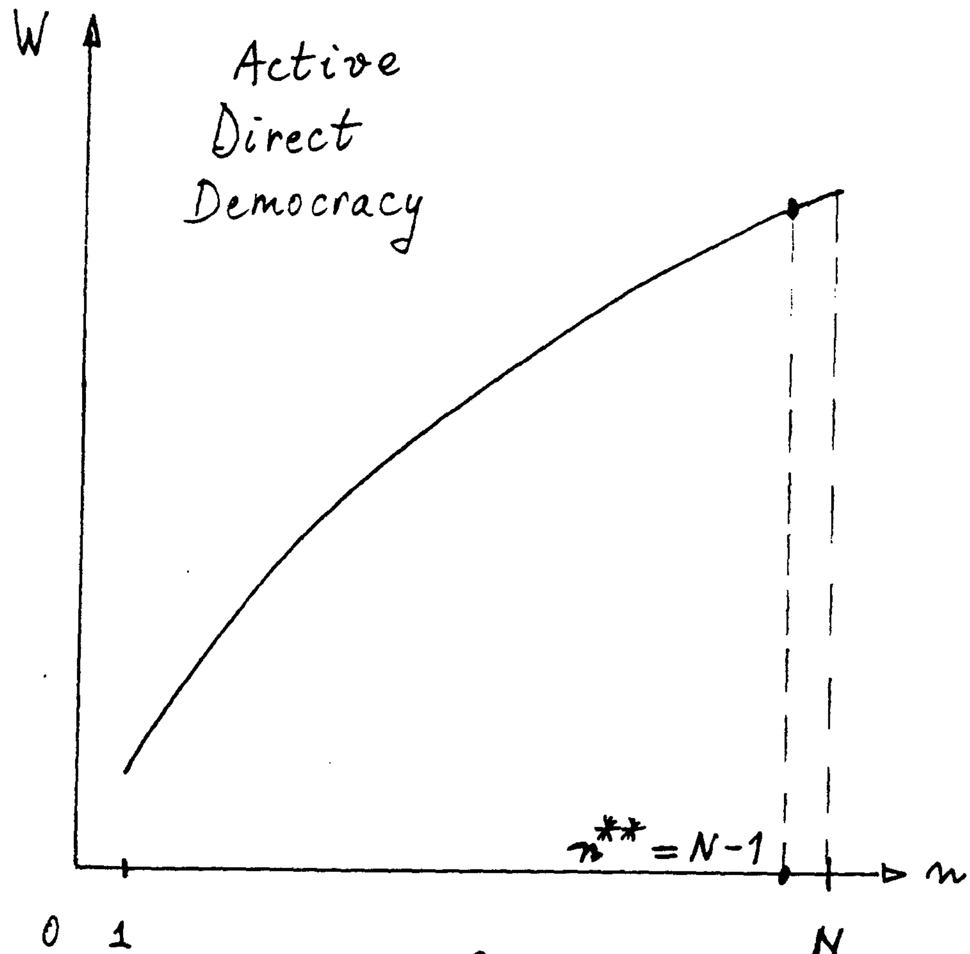
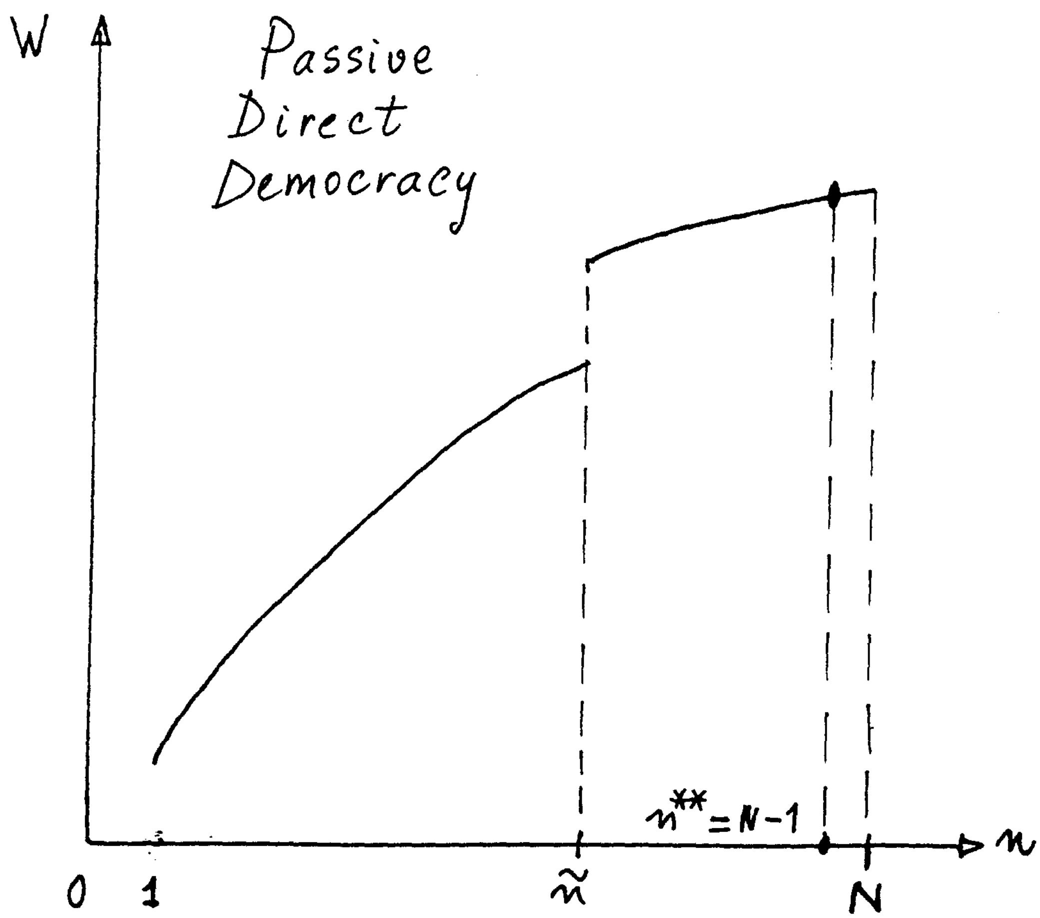

W
Active Direct Democracy
n** = N-1
n
0
1
N

Figure 2a


W
Passive Direct Democracy
n** = N-1
n
0
1
ñ
N

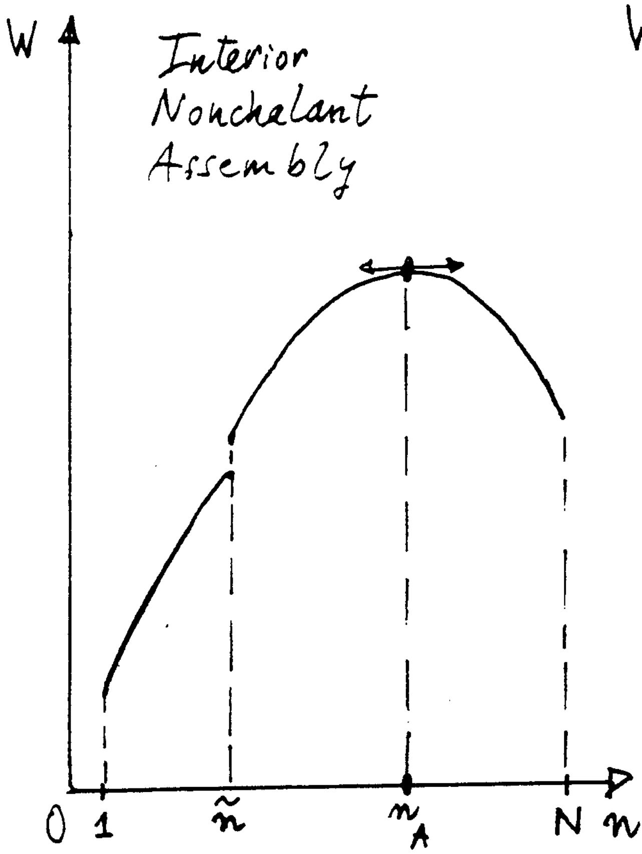


Figure 3a

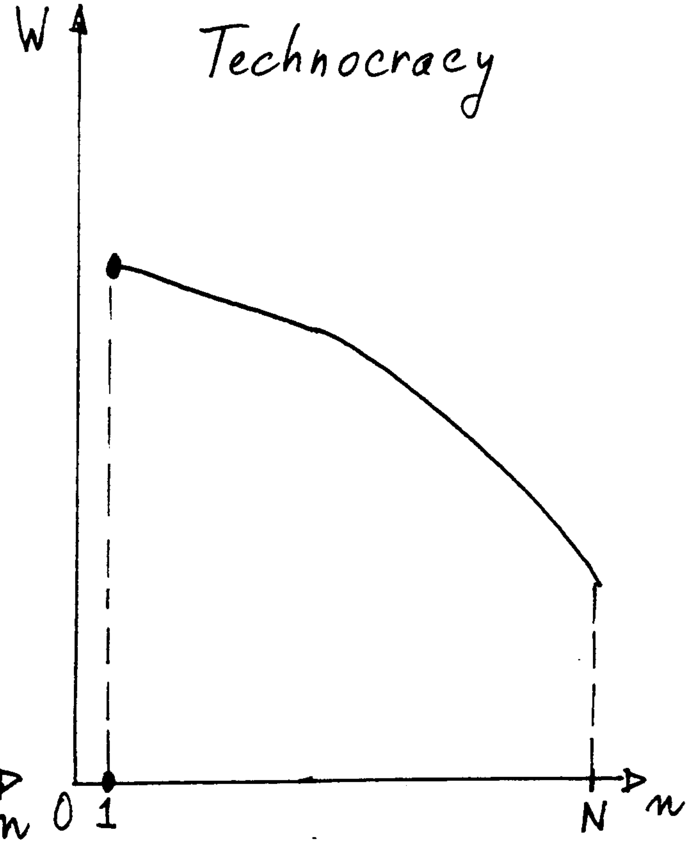


Figure 3d

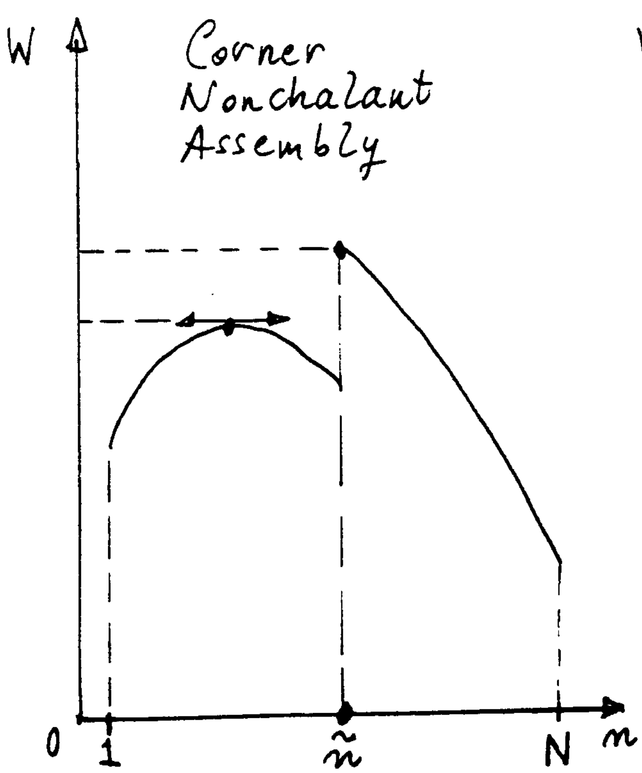


Figure 3b

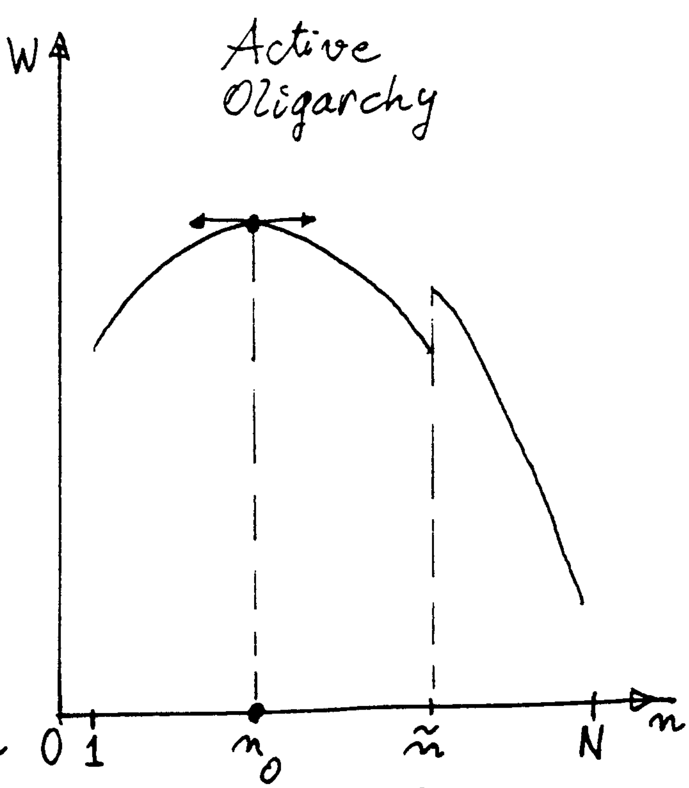


Figure 3c

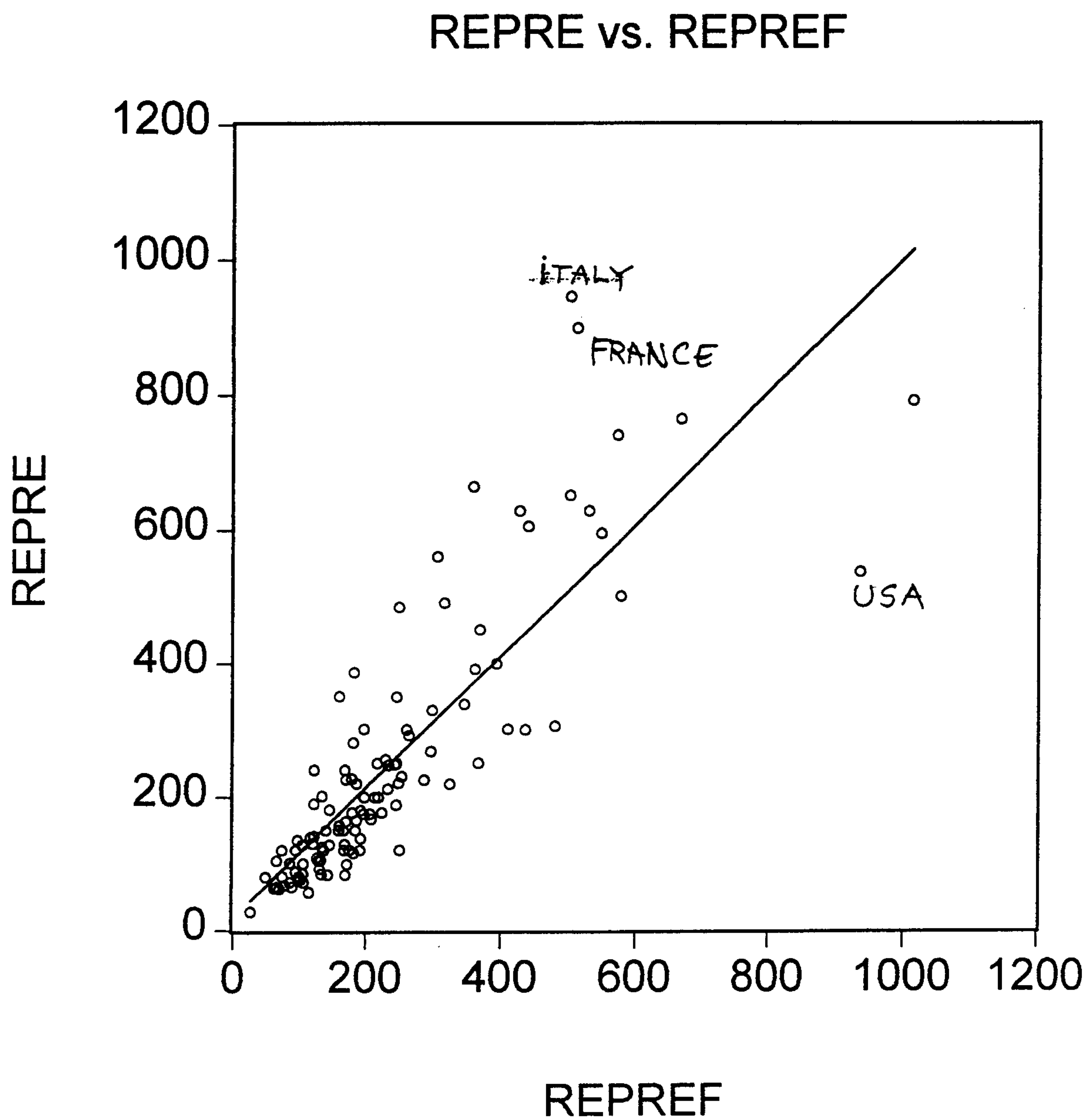

REPRE vs. REPREF
REPRE
REPREF
0
200
400
600
800
1000
1200
ITALY
FRANCE
USA

Figure 5

Appendix

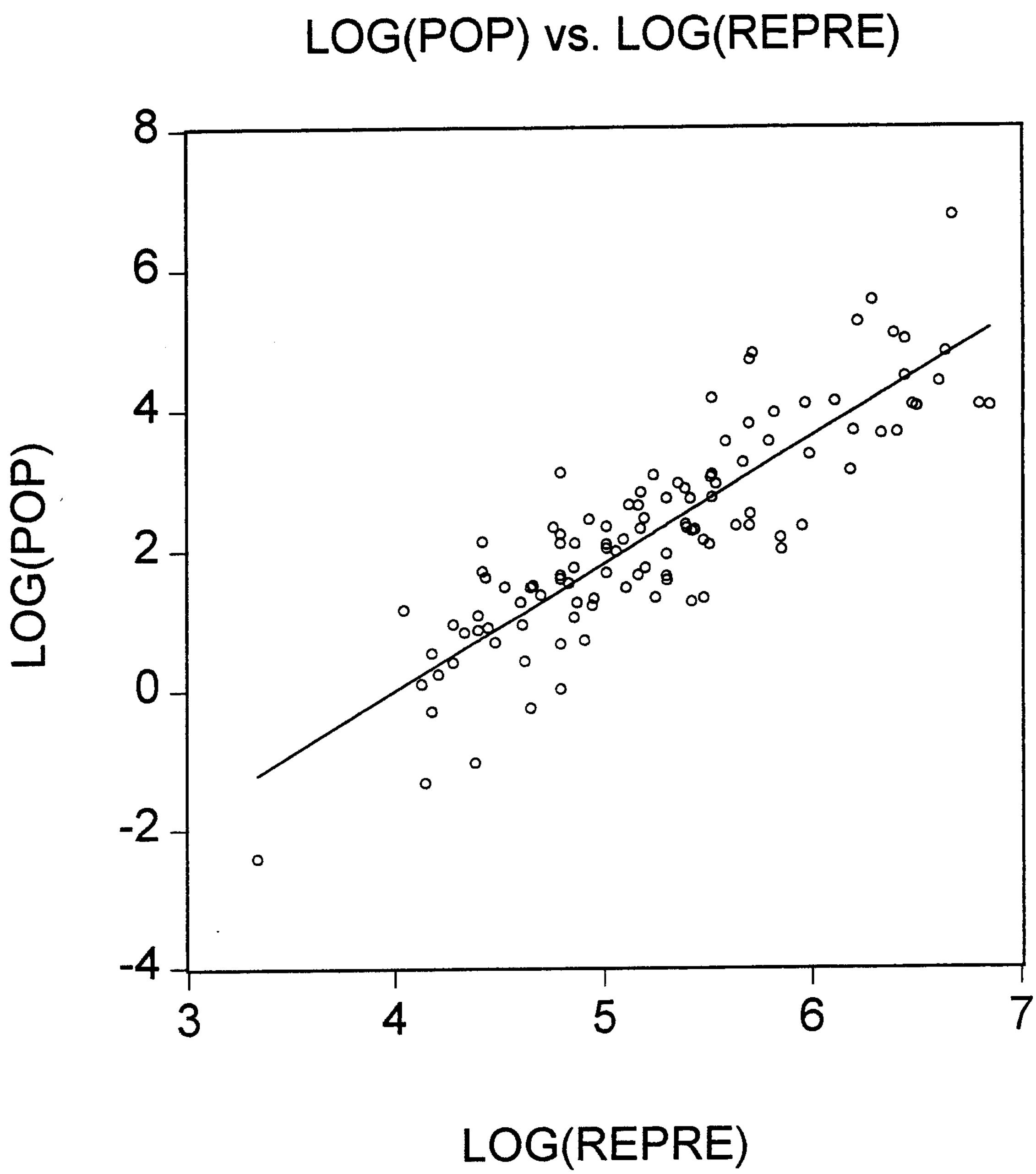


Figure 6.

Appendix TABLE 1 : the Data

Obs	COUNTRY	REP	POP	dev	DENSITY
1.0	albania	140.	3.3630000	0.	120.00000
2.0	angola	220.	10.609000	0.	9.0000000
3.0	argentina	329.	34.665000	0.	10.000000
4.0	armenia	190.	3.7600000	0.	130.00000
5.0	australia	219.	17.657000	1.	2.0000000
6.0	austria	247.	7.9880000	1.	100.00000
7.0	azerbaijan	350.	7.3910000	0.	90.000000
8.0	bangladesh	300.	111.40000	0.	930.00000
9.0	belgium	221.	10.146000	1.	310.00000
10.	benin	83.0	5.4750000	0.	50.000000
11.	bolivia	157.	7.2370000	0.	7.0000000
12.	bosnie-herz	240.	3.7070000	0.	72.500000
13.	brazil	594.	162.16170	0.	20.000000
14.	bulgaria	240.	8.4597000	0.	80.000000
15.	burk-faso	227.	9.6820000	0.	40.000000
16.	cambodia	120.	9.3080000	0.	60.000000
17.	cameroon	180.	11.540000	0.	30.000000
18.	canada	399.	28.753000	1.	3.0000000
19.	cl africa rep	85.0	2.4630000	0.	5.0000000
20.	chile	167.	14.026000	0.	20.000000
21.	colombia	267.	34.520000	0.	40.000000
22.	costa rica	57.0	3.1992300	0.	70.000000
23.	côte d'ivoire	175.	13.978000	0.	50.000000
24.	croatia	201.	4.7790000	0.	90.000000
25.	czech rep	281.	10.334000	0.	130.00000
26.	denmark	175.	5.1966400	1.	120.00000
27.	dominican r	150.	7.6080000	0.	160.00000
28.	egypt	664.	56.488000	0.	60.000000
29.	el salvador	84.0	5.0480000	0.	280.00000
30.	equ guinea	80.0	0.3560000	0.	13.000000
31.	estonia	101.	1.5069000	0.	30.000000
32.	fiji	104.	0.7711000	0.	42.000000
33.	finland	200.	5.0779000	1.	20.000000
34.	france	898.	58.060000	1.	110.00000
35.	gabon	120.	1.0117000	0.	4.0000000
36.	germany	740.	80.975000	1.	230.00000
37.	ghana	200.	15.400000	0.	80.000000
38.	greece	300.	10.368600	1.	80.000000
39.	grenada	28.0	0.0900000	0.	277.00000
40.	guatemala	116.	10.321900	0.	100.00000
41.	guyana	65.0	0.7395500	0.	3.0000000
42.	honduras	128.	5.7700000	0.	50.000000
43.	hungary	386.	10.277000	0.	110.00000
44.	iceland	63.0	0.2668000	1.	3.0000000
45.	india	790.	870.00000	0.	320.00000
46.	indonesia	500.	192.21650	0.	110.00000
47.	ireland	226.	3.5234000	1.	50.000000
48.	israel	120.	5.1959000	1.	280.00000
49.	italy	945.	57.138500	1.	200.00000
50.	jamaica	81.0	2.3742000	0.	240.00000
51.	japan	763.	124.45190	1.	330.00000
52.	jordan	120.	4.9360000	0.	50.000000
53.	kazakhstan	177.	16.763000	0.	6.0000000
54.	kenya	188.	21.444000	0.	50.000000
55.	rep of korea	299.	44.453000	1.	460.00000
56.	kyrgyr. rep	105.	4.4760000	0.	20.000000
57.	latvia	100.	2.5660000	0.	40.000000
58.	lebanon	128.	2.8380000	0.	400.00000
59.	lesotho	65.0	1.7000000	0.	70.000000
60.	lithuania	141.	3.7170000	0.	60.000000
61.	macedonia	120.	1.9370000	0.	80.000000

Appendix TABLE 1 Continued

62.	madagasca	138.	11.493000	0.	20.000000
63.	malawi	177.	10.033000	0.	110.00000
64.	malaysia	212.	19.047000	0.	60.000000
65.	mali	129.	8.1560000	0.	8.0000000
66.	mauritania	135.	2.0360000	0.	2.0000000
67.	mauritus	62.0	1.0980000	0.	560.00000
68.	mexico	628.	87.341000	0.	50.000000
69.	moldova	104.	4.3560000	0.	130.00000
70.	mongolia	76.0	2.2880000	0.	2.0000000
71.	mozambiq.	250.	15.583000	0.	20.000000
72.	namibia	72.0	1.4900000	0.	2.0000000
73.	nepal	255.	18.916000	0.	150.00000
74.	netherlands	225.	15.385000	1.	460.00000
75.	new zealand	99.0	3.5420000	1.	10.000000
76.	nicaragua	92.0	4.4010000	0.	40.000000
77.	niger	83.0	8.3610000	0.	7.0000000
78.	norway	165.	4.3250000	1.	10.000000
79.	pakistan	304.	122.80200	0.	170.00000
80.	panama	72.0	2.5830000	0.	40.000000
81.	papua guin	109.	3.9220000	0.	10.000000
82.	paraguay	125.	4.6430000	0.	10.000000
83.	peru	120.	22.454000	0.	20.000000
84.	philippines	250.	64.259000	0.	240.00000
85.	poland	560.	38.505000	0.	130.00000
86.	portugal	230.	9.8680000	1.	110.00000
87.	romania	484.	22.736000	0.	100.00000
88.	russian fed	628.	148.20000	0.	9.0000000
89.	senegal	120.	8.1520000	0.	40.000000
90.	singapore	81.0	2.9300000	1.	4990.0000
91.	slovakia	150.	5.3360000	0.	110.00000
92.	slovenia	88.0	1.9890000	0.	100.00000
93.	south africa	490.	40.436000	0.	30.000000
94.	spain	605.	39.143000	1.	80.000000
95.	sweden	349.	8.7450000	1.	20.000000
96.	switzerlan	200.	6.9690000	1.	180.00000
97.	tajikistan	181.	5.7510000	0.	40.000000
98.	tanzania	291.	25.635000	0.	30.000000
99.	thailand	391.	59.096000	0.	120.00000
100	trinidad tob	67.0	1.2600000	0.	250.00000
101	tunisia	163.	8.5720000	0.	60.000000
102	turkey	450.	61.183000	0.	80.000000
103	ukraine	338.	52.057000	0.	90.000000
104	united king	651.	58.192000	1.	240.00000
105	usa	535.	260.34100	1.	30.000000
106	uruguay	130.	3.5000000	0.	20.000000
107	usbekistan	250.	21.700000	0.	60.000000
108	venezuela	248.	20.712000	0.	30.000000
109	yemen	301.	12.302000	0.	30.000000
110	zambia	150.	8.0230000	0.	10.000000
111	zimbabwe	150.	10.402000	0.	30.000000

Appendix

TABLE 2: Data used in Figure 5

obs	REPRE	REPREF
1	140.0000	116.5868
2	220.0000	186.7147
3	329.0000	298.0242
4	190.0000	121.6537
5	219.0000	324.6420
6	247.0000	233.7584
7	350.0000	159.8645
8	300.0000	410.2312
9	221.0000	248.7456
10	83.00000	142.8710
11	157.0000	160.5790
12	240.0000	122.0489
13	594.0000	547.3777
14	240.0000	168.8849
15	227.0000	179.2334
16	120.0000	175.9231
17	180.0000	192.4005
18	399.0000	393.5460
19	85.00000	104.9308
20	167.0000	208.1355
21	267.0000	296.1602
22	57.00000	115.1933
23	175.0000	206.8961
24	201.0000	134.5685
25	281.0000	181.3772
26	175.0000	196.6386
27	150.0000	159.9693
28	664.0000	358.6611
29	84.00000	133.5474
30	80.00000	48.81159
31	101.0000	86.08679
32	104.0000	65.94561
33	200.0000	197.8740
34	898.0000	510.9844
35	120.0000	73.84460
36	740.0000	572.0825
37	200.0000	213.9777
38	300.0000	259.9298
39	28.00000	27.22371
40	116.0000	182.1325
41	65.00000	65.25682
42	128.0000	145.8639
43	386.0000	181.5395
44	63.00000	61.95313
45	790.0000	1014.950
46	500.0000	577.3525
47	226.0000	170.4827
48	120.0000	191.8434
49	945.0000	500.7784
50	81.00000	99.74511
51	763.0000	667.5882
52	120.0000	137.1398
53	177.0000	223.8030
54	188.0000	245.0057
55	299.0000	435.7038
56	105.0000	132.5518
57	100.0000	106.0698
58	128.0000	104.4260
59	65.00000	89.73226
60	141.0000	122.4142

Appendix

TABLE 2 continued

obs	REPRE	REPREF
61	120.0000	94.33473
62	138.0000	192.3865
63	177.0000	179.8243
64	212.0000	233.4376
65	129.0000	168.3186
66	135.0000	97.37300
67	62.00000	70.01481
68	628.0000	426.6977
69	104.0000	128.9344
70	76.00000	101.9668
71	250.0000	216.9735
72	72.00000	86.07430
73	255.0000	229.5988
74	225.0000	286.5182
75	99.00000	171.8929
76	92.00000	131.2651
77	83.00000	170.0035
78	165.0000	186.0045
79	304.0000	479.2468
80	72.00000	106.3469
81	109.0000	126.0043
82	125.0000	134.6911
83	120.0000	250.6560
84	250.0000	367.0825
85	560.0000	304.9671
86	230.0000	253.7234
87	484.0000	248.8116
88	628.0000	529.1467
89	120.0000	167.4590
90	81.00000	74.09024
91	150.0000	140.1266
92	88.00000	95.03411
93	490.0000	315.7431
94	605.0000	439.3112
95	349.0000	245.2730
96	200.0000	218.7787
97	181.0000	145.8984
98	291.0000	263.7183
99	391.0000	361.7566
100	67.00000	77.54023
101	163.0000	170.2905
102	450.0000	369.0160
103	338.0000	345.6714
104	651.0000	501.3092
105	535.0000	935.8255
106	130.0000	120.2779
107	250.0000	245.7783
108	248.0000	242.4122
109	301.0000	197.3225
110	150.0000	167.1774
111	150.0000	184.6689
112	NA	NA
113	NA	NA
114	NA	NA
115	NA	NA