Redistributing Income under Proportional Representation[1]

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Abstract

Although majoritarian decision rules are the norm in legislatures, relatively few democracies use simple majority rule at the electoral stage, adopting instead some form of multiparty proportional representation. Moreover, aggregate data suggest that average income tax-rates are higher and distributions of post-tax income flatter, in countries with proportional representation than in those with majority rule. While there are other differences between these countries, this paper explores how variations in the political system per se influence equilibrium redistributive tax-rates and income distributions. A three-party proportional representation model is developed in which taxes are determined through legislative bargaining among successful electoral parties, and the economic decision for individuals is occupational choice. Political-economic equilibria for this model and for a two-party, winner-take-all, majoritarian system are derived and compared.

1 Introduction

This paper concerns the redistribution of income through political choice of the tax system. The paper is in part motivated by two observations. The first is that while almost all of the extant theoretical literature on the topic presumes some form of two-party majority rule political system for determining the redistributive tax-rate (eg [8], [10], [11], [12], [13]), most Western political systems use some form of proportional representation system with more than two parties. And the second is that the countries with proportional representation typically exhibit higher average tax-rates and flatter distributions of post-tax income than those using (essentially) two-party majority rule. Figures 1 and 2, reproduced from Atkinson, Rainwater and Smeeding [1], illustrate this observation with data from the mid-1980s.

[Figures 1 and 2 here]

Figures 1a and 1b describe the bottom and top deciles, respectively, of personal post-tax incomes as a percentage of the median income by country; Figure 2 describes the entire distribution for the US, France (FR) and Sweden (SW), again in terms of percentage deviations from the median in each country. With the exception of the US, the UK, and France, all the countries represented are proportional representation polities; the US and UK are basically two-party, winner-take-all plurality rule systems, and France uses a run-off electoral scheme.[1] Although far from conclusive, these data are distinctly suggestive. Of course, there are many other differences between these countries and to conclude that the electoral system per se accounts for the variation would be premature. Nevertheless it is of some interest to study the implications of different political systems on policy choice.[2]

In what follows, I build a relatively simple model of a political economy. The main demands for such a model are, first, that it exhibit a tradeoff between the level of output and its distribution and, second, that the polity

[1] Under the run-off system, many parties compete for votes in a first round election; if some party wins a strict majority then that party is the winner, otherwise the top two vote-getters run against each other in a second round election under simple plurality rule. Loosely speaking, then, the French system is intermediate between two-party plurality rule and more-than-two party proportional representation.

[2] In a very recent contribution, Birchfield and Crepaz [5] present an empirical study focusing explicitly on "the impact of political institutions on income inequality" among 18 OECD countries, concluding that majoritarian institutions lead to greater inequality than do more "consensual" structures.

is tightly connected to the economy. In the usual median voter models, the first desideratum is introduced by assuming individuals have a labor/leisure tradeoff while the second is reflected in the incentives of two competitive and vote-maximizing parties. With a proportional representational polity involving more than two parties, however, vote-maximizing is not a plausible objective to assume. This is because typically no party can attract an absolute majority of votes and, therefore, final policy choices are the consequence of some sort of legislative bargaining process. And an essential feature of any political model involving legislative bargaining is that parties have policy preferences over the whole range of possible outcomes.

Once parties are presumed to have policy preferences, there is then an issue concerning the source and structure of such preferences. Ideally, parties' policy preferences would be derived from some underlying theory of party organization (see [13] for an example). Here, however, I simply assume (and justify more fully later on) that parties are "ideological" in that they seek to maximize the ex post average consumption of members of particular economic groups. Clearly if, as in the two-party median voter model, individuals are assumed to be differentiated only by their respective willingness to tradeoff labor for leisure, there is no structural basis for the existence of multiple economic groups. So the basic economic model is one of occupational choice in which individuals have differential endowments of labor ability. There is then one party per occupation and party preferences are well-defined, distinct and rooted in the economy. Of course, not all occupations are represented by distinct parties in the real world, nor are all parties in proportional representation systems based on economic groupings. What matters here is less the empirical match of parties to occupations and more the existence of multiple parties with incentives and constraints derived from the economy; the assumption that parties are the products of distinct occupations captures this.

In the model, there is a given symmetric distribution of types (endowments of ability) and national output is determined by the endogenous allocation of types across three occupations - employer, employee and voluntary unemployed – and income is redistributed via an affine tax system subject to a balanced budget constraint.[3] The tax-rate is determined through the political process; the focus here is on proportional representation and, as

[3]It is worth remarking here that the symmetry assumption on the distribution of types does not imply a similar equilibrium distribution of income.

suggested above, the political process has two stages. In the first stage, three parties compete for votes in an election under a pure proportional representation electoral system; in the second stage the tax-rate is chosen as an equilibrium outcome of a noncooperative bargaining game. The implications of two assumptions about parties' electoral credibility are examined. Since parties have known policy preferences there is a nontrivial issue of the extent to which parties can commit credibly to the electorate to pursue objectives other than their given preferences. I look at the extremes: either there is no commitment possible at the electoral stage, in which case the only action in the election involves voter behavior; or full commitment is possible, in which case the parties have a real decision to make regarding the platforms they offer to the electorate. It turns out, however, that the (appropriately defined) equilibrium outcome on tax-rates is the same in both cases. And once the tax-rate is fixed, individuals sort into occupations, income is generated and redistributed. All agents are farsighted and have rational expectations. Finally I compare the equilibrium outcome in the model to that predicted with a two-party majority rule system.

The main result is a sufficient (but certainly not necessary) condition for the motivating empirical observation: if the cost of entering the workforce at all is sufficiently low, then proportional representation polities tend to adopt higher redistributive tax-rates than two-party majoritarian systems. Given such a cost, the result further implies that national income is lower, (voluntary) unemployment is higher, and the distribution of post-tax income is flatter when taxes are chosen through a proportional representation rather than a majoritarian system.

The intuition underlying the result is as follows. Under competitive two-party majority rule, the pivotal voter is defined by the voter with median income in the electorate at large, irrespective of that voter's (equilibrium) choice of occupation, but under the proportional representation system with legislative bargaining, the pivotal voter is (loosely speaking) defined by the voter with average employee income among only those types who choose to be employees ex post. Because the latter is endogenous, depending in part on the chosen tax-rate, it is not transparent whether the critical type is higher or lower than the median type. In particular, while the immediate impact of a marginal increase in the tax-rate over that chosen by the median voter is to lower net consumption (utility) of higher type voters, it also induces a change in the distribution of types across occupations that raises the average type of employee. When the cost of working is not too high, the positive impact on average employee income due to the induced change in the distribution of employee types dominates the negative impact on this income due to the increased tax burden.

A second comparative static result worth emphasizing concerns the response of the two political-economic systems to an exogenous shift in productive economic capacity. Under some conditions, a marginal improvement in productivity induces a decrease in the equilibrium redistributive tax-rate under majority rule but an increase in the tax-rate under proportional representation. When costs of working are sufficiently low, however, the converse of this claim cannot obtain in the model, although there are conditions under which both political systems respond with a higher tax-rate leading, inter alia, to relatively more redistribution.

2 Economics

Individuals in the economy are distinguished by their productivities, defined in terms of endowments of (homogenous) efficiency units of labor. Let $\theta \in \Theta = (0, \bar{\theta})$ denote a generic individual's endowment, or type, where $\overline{\theta}$ is finite. There is a very large finite number of individuals, approximated by a continuum of individuals with total population normalized to one. Assume the distribution of types within the population is described by a smooth, strictly quasi-concave symmetric density, $g(\cdot)$, with mean $\hat{\theta}$ equal to median θ_m and support equal to Θ; assume further that $\theta g(\theta)$ is nondecreasing in θ on Θ. Given the interpretation of type as a natural ability (rather than a wage rate, for example), the symmetry assumption on its distribution is fairly natural in a single generation model without human capital accumulation, as here. The assumption that $\theta g(\theta)$ is nondecreasing in θ means that the distribution cannot be too spiked about its mean and is essentially technical.

Every individual has risk-neutral preferences over consumption of a homogenous commodity with price normalised to one; let $y_j(\cdot, \theta)$ and $x_j(\cdot, \theta)$, respectively denote the gross earned income and consumption of an individual of type θ in occupation j, both measured in units of the consumption good. Individuals select into one of three possible occupations: employer ($j = e$), employee ($j = l$), and (voluntarily) unemployed ($j = d$). Employers use labor input under a given smooth technology, F, to produce the consumption good. Specifically, an employer of type θ using L efficiency units of labor produces an amount of consumption good $F(L, \theta)$, where F is assumed to be at least thrice differentiable and strictly increasing in both arguments. F is further assumed strictly concave in L, convex in the employer's type with $\partial^2 F/\partial L \partial\theta > 0$ for all strictly positive θ and $F(L, 0) = F(0, \theta) = 0$ all L, θ. It is also convenient to assume $\lim_{\theta\to 0} \partial F/\partial\theta = 0$ and $\partial^3 F/\partial L \partial L \partial\theta \leq 0$. Thus labor is productively employed in the technology F and higher type employers are capable of extracting more output from a given level of labor input than lower type employers. Employees supply their labor endowment to employers inelastically at a competitively determined wage rate, w. Then the gross earned income of an employer of type θ hiring total labor L at wage rate w is $y_e(L, w, \theta) = F(L, \theta) - wL$, and that for an employee of type θ working at wage rate w is $y_l(w, \theta) = w\theta$. Unemployed individuals earn no income: $y_d(\cdot, \theta) = 0$ for all $\theta \in \Theta$ (hence the notation d for "dependent").[4]

Assume there is a fixed cost, $c > 0$, for going to work either as an employer or as an employee and that there is no direct cost for not working. Throughout, the cost c is implicitly assumed sufficiently small that there is always a positive measure of types who find it worthwhile to work. All individuals receive a common lump-sum transfer financed by a proportional tax on the earned income of those working. Let $t \in [0, 1]$ denote the tax-rate and let $b(t)$ denote the lump-sum transfer. So, given a tax-rate t on earned income, consumption for an individual of type θ in occupation $j \in \{e, l, d\}$ is given by:

$$x_e(L, t, w, \theta) = (1 - t)[F(L, \theta) - wL] + b(t) - c \tag{1}$$

$$x_l(t, w, \theta) = (1 - t)\theta w + b(t) - c \tag{2}$$

$$x_d(t, \theta) = b(t). \tag{3}$$

For any given tax and wage rate pair (t, w), let $\lambda_j(t, w)$ denote those types in Θ choosing occupation j. And for any $\theta \in \lambda_e(t, w)$, let $L(w, \theta)$ denote the value of labor input L that maximizes $x_e(L, t, w, \theta)$; clearly, under the assumptions on F, $L(w, \theta)$ is uniquely defined and independent of t for given w, and strictly increasing in θ at any (t, w).

[4] Having seen an earlier version of this paper, Michel LeBreton referred me to a paper by Didier Laussel and himself [9] in which they study a very similar model of occupational choice. The main focus of their paper, however, is quite different from that here.

Definition 1 *For any given tax-rate $t \in [0,1]$, a sorting equilibrium at t is a nonnegative wage rate $w^* = w^*(t)$ such that:*
(1) $\int_{\lambda_e(t,w^*)} L(w^*,\theta)g(\theta)d\theta = \int_{\lambda_l(t,w^*)} \theta g(\theta)d\theta$,
(2) $\forall \theta \in \Theta$, $\forall j, j' \in \{e,l,d\}$, $\theta \in \lambda_j(t,w^*)$ *implies* $x_j(\cdot,\theta) \geq x_{j'}(\cdot,\theta)$.

Condition (1) requires labor demand equal labor supply, and condition (2) requires that no type can switch occupations and increase its consumption (utility).

Finally, assume throughout that the budget balances:

$$b(t) = t\left[\int_{\lambda_e(t,w)} y_e(L(w,\theta),w,\theta)g(\theta)d\theta + \int_{\lambda_l(t,w)} y_l(w,\theta)g(\theta)d\theta\right], \qquad (4)$$

where, since the population size is normalized to one, $b(t) = \int_\Theta b(t)g(\theta)d\theta$.

Proposition 1 *For all $t \in [0,1)$, there exists a unique sorting equilibrium at t, $w^*(t) = w^*$. The equilibrium is characterized by an ordered pair of types $(\theta_1(t,w^*), \theta_2(t,w^*))$ such that:*
$\lambda_d(t,w^*) = (0, \theta_1(t,w^*))$;
$\lambda_l(t,w^*) = [\theta_1(t,w^*), \theta_2(t,w^*)]$; *and*
$\lambda_e(t,w^*) = (\theta_2(t,w^*), \bar{\theta})$.

(Formal proofs for this and all subsequent results are relegated to an Appendix.)

Hereafter, for any tax-rate t I shall be concerned only with behavior in the associated sorting equilibrium, $w^*(t)$. So it is convenient to write $x_e(L(w^*(t),\theta),t,w^*(t),\theta) \equiv x_e(t,\theta)$ and $x_l(t,w^*(t),\theta) \equiv x_l(t,\theta)$. Figure 3 illustrates a typical sorting equilibrium for given $t \in [0,1)$. And note that in any sorting equilibrium, earned income is strictly increasing in type on $[\theta_1(t,w^*),\bar{\theta})$ and constant (at zero) on $(0,\theta_1(t,w^*))$.

[Figure 3 here]

Uniqueness of the sorting equilibrium at any tax-rate implies that individuals' induced preferences over tax-rates are well-defined. The next two results both help identify the structure of these induced preferences and are of independent interest.

Lemma 1 *$w^*(t)$ is differentiable, nonlinear and strictly increasing in t.*

Because taxes are levied proportionately on employer income, $y_e(\cdot)$, a parametric increase in the tax-rate leaves employers' labor demands unaffected. However, the lowest types of (pre-tax increase) employer now prefer to be employees and, similarly, the very lowest types of (pre-tax increase) employee prefer to be unemployed. On balance, the fall in labor supply at the lower end of the distribution due to more types choosing unemployment exceeds the increase at the upper end due to some employers becoming employees; thus the supply of labor falls relative to the demand and wages rise to clear the market.

Several of the results below depend in part on the relative size of the second derivative, d^2w^*/dt^2, which in turn depends on details of the production function F and the distribution of types g. Although not an assumption on the primitives of the model, the following appropriately summarizes the required restrictions on F and g. For any tax rate t, let $\epsilon(t)$ and $\tilde{\epsilon}(t)$, respectively, denote the tax-elasticities of the equilibrium and the marginal equilibrium wage rates:

$$\epsilon(t) = \frac{dw^*(t)}{dt}\frac{t}{w^*(t)} \text{ and } \tilde{\epsilon}(t) = \frac{dw_t^*(t)}{dt}\frac{t}{w_t^*(t)},$$

where $w_t^*(t) \equiv dw^*(t)/dt$. Then assume that for all $t \in [0, 1)$,

$$-2t \leq (1-t)\tilde{\epsilon}(t) \leq [(1-t)\epsilon(t) + t]. \quad (5)$$

In effect, (5) requires that the function $w^*(t)$ is never "too" concave or "too" convex at any t. It turns out that $(1-t)\tilde{\epsilon}(t)$ is finite for all t and, therefore, by Lemma 1, (5) surely holds for extreme values of t. The assumption that it also holds for intermediate values is not unreasonable and is maintained hereon.

Lemma 2 *Given (5), the equilibrium level of transfer payment, $b(t)$, is strictly concave on $[0, 1]$ with interior argmax.*

Define $\mu \in \Theta$ to be the type earning the average income when the tax-rate is zero: recalling population size is normalized to one,

$$\mu \equiv \{\theta \in \Theta |\ y_j(\cdot, \theta) = Y(0, w^*(0))\}.$$

Since incomes are strictly increasing convex in type on $[\theta_1(0, w^*(0)), \bar{\theta})$ and $y_d(\theta) = 0$ for all $\theta \in (0, \theta_1(0, w^*(0)))$, if (as assumed here) the distribution of types is symmetric about θ_m then $y_{j'}(\cdot, \mu) > y_j(\cdot, \theta_m)$; that is, the equilibrium income distribution is skewed to the right when the distribution of types is symmetric. Now for any $\theta \in \Theta$ and any $j \in \{e, l, d\}$, let $t_j(\theta)$ denote the most preferred tax-rate of type θ in occupation j. That this is well-defined is the content of the following result (where, notationally, singleton argmax sets are identified with their element).

Proposition 2 *(1) For any $\theta \in \Theta$, $x_d(t, \theta)$ is strictly concave in t with $t_d(\theta) = \arg\max b(t)$.*

(2) For any $\theta \in \Theta$, $x_l(t, \theta)$ is strictly concave in t and there exists a type $\nu_l > \mu$ such that $t_l(\theta) > 0$ if and only if $\theta \in [0, \nu_l)$; furthermore, $t_l(\theta)$ is strictly decreasing on $[0, \nu_l)$ with $t_l(0) = \arg\max b(t)$.

(3) For any $\theta \in \Theta$, $x_e(t, \theta)$ is strictly quasi-concave in t and there exists a type $\nu_e < \mu$ such that $t_e(\theta) > 0$ if and only if $\theta \in [0, \nu_e)$; furthermore, $t_e(\theta)$ is strictly decreasing on $[0, \nu_e)$ with $t_e(\theta) < t_l(\theta)$.

The reason for $\nu_e < \mu < \nu_l$ in the proposition is that, while aggregate income falls with increases in the tax-rate, this is the net effect of an increase in the pre-tax earned income of workers and a decrease in the pre-tax earned income of employers, both effects being due to the equilibrium wage adjustment associated with the tax-change. Thus there are some worker-types earning more than average income at $t = 0$ who nevertheless prefer some redistribution and, conversely, some employer-types earning less than average income at $t = 0$ who most prefer a zero tax-rate.

Hereafter, assume the following innocuous assumption on (implicitly) the technology and the distribution of types:

$$\int_0^{\theta_1(0,w^*(0))} g(\theta)d\theta \leq \int_{\theta_2(0,w^*(0))}^{\bar{\theta}} g(\theta)d\theta < 1/2 \ \& \ \theta_1(t_d(0), w^*(t_d(0))) < \theta_m. \tag{6}$$

This assumption insures that in any realizable sorting equilibrium a majority of the population never chooses either to be unemployed or to be employers and that, when there are no taxes, at least as high a proportion of types are employers as are unemployed.

3 Politics

The tax-rate is a political decision. The central model assumes proportional representation at the electoral stage followed by a noncooperative bargaining process to determine the final policy decision at the legislative stage. Moreover, there are three policy motivated political parties, one for each occupation, and parties are assumed to be unitary actors. Having analysed this model I compare the results to those derived from a two-party majority rule political system, the description of which is deferred until necessary.

Assume that there are three parties, $\mathcal{E}$, $\mathcal{L}$, $\mathcal{D}$, representing the three occupations, e, l, d, respectively. Parties are assumed to have policy preferences; for each party $\mathcal{J} \in \{\mathcal{E}, \mathcal{L}, \mathcal{D}\}$ and any tax-rate t, let $u_{\mathcal{J}}(t)$ denote the party's payoff from t, where $u_{\mathcal{J}} : [0,1] \to \Re$. For the moment, assume that for each party $\mathcal{J}$, $u_{\mathcal{J}}$ is strictly quasi-concave on $[0,1]$ with most preferred policy $t_{\mathcal{J}} \equiv \arg\max u_{\mathcal{J}}(t)$ and assume further that $t_{\mathcal{D}} > t_{\mathcal{L}} > t_{\mathcal{E}}$. Later, these party preferences are specified explicitly in terms of economic payoffs and the assumptions made here justified formally.

At the electoral stage, each party offers a platform (defined momentarily) to the electorate simultaneously and voters vote for at most one party. Because party preferences are given and common knowledge, it is likewise common knowledge that in the absence of any commitment mechanism, parties' legislative behavior will reflect these preferences irrespective of any electoral positioning. Consequently, it is necessary to specify whether or not such credible commitment is possible and the form it takes. Both assumptions – existence and absence of credible commitments by parties - are considered and shown to yield the same principal result. However, it is easier to begin by assuming that *no* commitments to pursue preferences other than their respective true preferences are credible. Thus there is no loss in generality in assuming at the outset that, for each party $\mathcal{J}$, $\mathcal{J}$'s electoral platform is given by the function $u_{\mathcal{J}}(t)$; let $u = (u_{\mathcal{E}}, u_{\mathcal{L}}, u_{\mathcal{D}})$ denote the list of party platforms. (The reason for defining party platforms as preferences rather than more simply as, say, tax-rates is discussed below.)

Each party's representation, or weight, in the legislature is given by its vote share. The implemented tax-rate is the outcome of a legislative bargaining game. There are several ways to model the bargaining process (e.g. [2], [3]) and I adopt the simplest model (see [4]). Fix an exogenously given status quo tax-rate, t_0 (considered further later on). Given the list of electoral platforms, u and a status quo policy t_0 let $v_{\mathcal{J}}(t_0, u)$ denote the vote share of party $\mathcal{J} \in \{\mathcal{E}, \mathcal{L}, \mathcal{D}\}$. If $v_{\mathcal{J}}(\cdot)$ exceeds 1/2 for some party $\mathcal{J}$, then that party implements its most preferred policy (i.e. $t_{\mathcal{J}}$). If no party receives an overall majority, then one party is selected randomly to propose a tax-rate; the probability party $\mathcal{J}$ is chosen is exactly $v_{\mathcal{J}}(t_0, u)$.[5] If at least one other party agrees to the proposal, then that proposal is the final decision, otherwise the status quo t_0 is implemented.

Before going on, it is worth emphasizing that the motivation for specifying parties' electoral platforms in terms of preferences over the set of feasible tax-rates, $[0,1]$, derives from the (typical) necessity of a nondegenerate legislative bargaining stage to determine the final policy choice. Under two-party plurality rule, one party generically wins a clear plurality. Consequently, it suffices to know the tax-rate that each party would implement conditional on winning to infer the payoff consequences of voting for one party over another. Indeed, the specification of an electoral commitment in this case is also straightforward: assume each party is bound to implement its platform if elected. On the other hand, with more than two parties and proportional representation, knowledge only of the tax-rate a party would implement if it were able to form a majority government alone is not enough - typically the final policy choice is the outcome of a bargaining process in which parties must compromise to some extent. And parties' willingness to compromise depends in part on their preferences over all feasible tax-rates, not just on their most preferred rate. So in this instance, an electoral commitment must be a commitment to a preference schedule and not simply to a point. Details of the commitment model are deferred until after results for the no commitment case are developed.

Now consider equilibrium behavior at the legislative stage. A *legislative strategy* for party $\mathcal{J}$ is a pair $\sigma_{\mathcal{J}} = (\tau_{\mathcal{J}}, \psi_{\mathcal{J}})$. Given (under the no-commitment assumption) that party electoral platforms are essentially fixed at u, $\tau_{\mathcal{J}} : [0,1] \to [0,1]$ describes $\mathcal{J}$'s proposal of a tax-rate conditional on being chosen to propose and conditional on the status quo policy, and $\psi_{\mathcal{J}} : [0,1]^2 \to [0,1]$ describes $\mathcal{J}$'s acceptance probability of a proposal offered by a party other than $\mathcal{J}$, conditional on that proposal, say t', and the status quo tax-rate. Given a status quo policy t_0 and the list of electoral plaforms u, a (subgame perfect) *legislative equilibrium* is a triple of mutual best-response (relative to u) legislative strategy pairs $(\sigma^*_{\mathcal{E}}, \sigma^*_{\mathcal{L}}, \sigma^*_{\mathcal{D}})$ such that

[5] The assumption that recognition probabilities are given by vote shares has some empirical support: see [7].

$\sigma^*_{\mathcal{J}}$ is weakly undominated and sequentially rational, all $\mathcal{J}$. It is not hard to see that legislative equilibria always exist and are generically unique. Let $\sigma^*(t_0, u)$ denote the legislative equilibrium conditional on t_0 and on the parties' electoral policy platforms u and, for any party $\mathcal{J}$, let $t(\tau^*_{\mathcal{J}}(t_0))$ denote the legislative equilibrium outcome conditional on $\mathcal{J}$ being selected to make a proposal.

Lemma 3 *For any status quo policy t_0, there exists a unique legislative equilibrium $\sigma^*(t_0, u)$. Let party $\mathcal{J}$ be selected to propose a tax-rate, $\tau^*_{\mathcal{J}}(t_0)$. Then the legislative equilibrium outcome $t(\tau^*_{\mathcal{J}}(t_0))$ is given by $t_{\mathcal{J}}$ if $v_{\mathcal{J}}(t_0, u) > 1/2$ and, if no party has a simple majority, $t(\tau^*_{\mathcal{J}}(t_0))$ is given by:*

(1) If $t_0 \le t_{\mathcal{E}}$ and $\mathcal{J} \in \{\mathcal{E},\mathcal{L}\}$ then $t(\tau^*_{\mathcal{J}}(t_0)) = t_{\mathcal{J}}$, and if $\mathcal{J} = \mathcal{D}$ then $t(\tau^*_{\mathcal{J}}(t_0)) = \arg\max[u_{\mathcal{D}}(t)|u_{\mathcal{L}}(t) \ge u_{\mathcal{L}}(t_0)] \le t_{\mathcal{D}}$;

(2) If $t_{\mathcal{E}} < t_0 < t_{\mathcal{L}}$ then $t(\tau^*_{\mathcal{J}}(t_0)) = t_0$ if $\mathcal{J} = \mathcal{E}; t(\tau^*_{\mathcal{J}}(t_0)) = t_{\mathcal{L}}$ if $\mathcal{J} = \mathcal{L}$;and $t(\tau^*_{\mathcal{J}}(t_0)) = \arg\max[u_{\mathcal{D}}(t)|u_{\mathcal{L}}(t) \ge u_{\mathcal{L}}(t_0)] \le t_{\mathcal{D}}$ if $\mathcal{J} = \mathcal{D}$;

(3) If $t_0 = t_{\mathcal{L}}$ then $t(\tau^*_{\mathcal{J}}(t_0)) = t_{\mathcal{L}}$ for all parties $\mathcal{J}$.

Symmetric outcomes obtain for $t_0 > t_{\mathcal{L}}$.

This lemma (the proof of which is straightforward and omitted) is an application of the standard agenda-setter model [14]. To save on notation, where there is no ambiguity write σ^* for $\sigma^*(t_0, u)$, leaving the arguments implicit.[6]

Consider the electoral stage of the political process. Individuals can vote for at most one party and I assume voters of the same type use the same strategy. Thus a *voting strategy* is a map

$$\pi : \Theta \times [0, 1] \times \{u\} \to \triangle^2,$$

where $\triangle^2 = \{(\pi_{\mathcal{E}}, \pi_{\mathcal{L}}, \pi_{\mathcal{D}}) \in [0, 1]^3 | \sum \pi_{\mathcal{J}} = 1\}$ is the two-dimensional simplex and $\pi(\theta, t_0, u) \in \triangle^2$ is the vector of probabilities that an individual of type θ votes for candidate ($\mathcal{E}$,$\mathcal{L}$,$\mathcal{D}$) given the status quo t_0 and the candidate platforms u. Occasionally, write $\pi(\mathcal{J}|\theta, t_0, u)$ to denote the probability type θ votes for party $\mathcal{J}$ given t_0 and u.

[6] It is worth noting here that the rationale for introducing a status quo tax-rate at the bargaining stage is not only that there always exists such a status quo (t_0 might be zero, for instance), but also that it supports a unique legislative equilibrium. Had the bargaining model been an infinite stage, stochastic alternating offers model, as in Baron [3] for example, there would be no guarantee of uniqueness of equilibrium (because the preferences are not necessarily strictly concave), in which case solving for equilibrium voting behaviour would at least require an equilibrium selection.

Equilibrium voting behavior is required to be weakly undominated and to reflect rational expectations regarding any economic consequences from the legislative deliberations following the election. For any tax-rate and sorting equilibrium $(t, w^*(t))$ and any individual of type θ, let $\xi(t,\theta)$ denote the individual's maximum consumption level conditional on $(t, w^*(t))$; i.e. for every occupation $j \in \{e, l, d\}$, $\xi(t,\theta) \geq x_j(t,\theta)$. Recalling that the assumption of a continuum of individuals is understood as an approximation to there being a very large finite number of agents, say N, any individual of type θ contributes a proportion $1/N \approx 0$ to the vote shares. Therefore, in view of Lemma 3, a strategically rational individual evaluates his or her voting strategy according to

$$E[\xi(t,\theta)|\pi(\theta,\cdot),\pi_{-\theta},\sigma^*] = \sum_{\mathcal{J}}[v_{\mathcal{J}}(t_0,u) + \frac{\pi_{\mathcal{J}}(\theta,\cdot)}{N}]\xi(t(\tau^*_{\mathcal{J}}(t_0)),\theta),$$

where $\pi_{-\theta}$ denotes the restriction of π to $\Theta\backslash\{\theta\}$. Now define a *voting equilibrium* to be a symmetric strategy π^* such that, for all $\theta \in \Theta$ and any (t_0, u), $\pi^*(\theta, t_0, u)$ is weakly undominated and maximizes the expected payoff $E[\xi(t,\theta)|\pi(\theta,\cdot),\pi^*_{-\theta},\sigma^*]$.

Lemma 4 *If π^* is a voting equilibrium then, for all $\theta \in \Theta$, any individual of type θ votes with positive probability only for a party that offers the highest available sorting equilibrium consumption level for his or her type, conditional on that party being selected to make a proposal at the legislative stage.*

Thus weakly undominated and strategically rational voting by individuals is observationally equivalent to sincere voting over the set of possible equilibrium economic outcomes $\{t(\tau^*_{\mathcal{E}}(t_0)), t(\tau^*_{\mathcal{L}}(t_0)), t(\tau^*_{\mathcal{D}}(t_0))\}$. This property of voting equilibria, however, does not pin down how an individual chooses when the best alternative is not unique (for instance, if $\xi(t(\tau^*_{\mathcal{D}}(t_0)),\theta) = \xi(t(\tau^*_{\mathcal{L}}(t_0)),\theta) \geq \xi(t(\tau^*_{\mathcal{E}}(t_0)),\theta)$). To close the model in this respect, hereafter assume tie-breaking by sincere myopic preference; that is, every individual breaks ties on the basis of his or her (induced) preferences over the set $\{t_{\mathcal{D}}, t_{\mathcal{L}}, t_{\mathcal{E}}\}$.[7]

Definition 2 *Fix a status quo policy $t_0 \in [0,1]$. A proportional representation political equilibrium (prpe) for t_0 is a list $p^*(t_0) = (u, \pi^*, \sigma^*, w^*)$ of party*

[7] The set of individuals indifferent over any pair of tax-rates in this set is negligible, and so ignored.

platforms, u, a voting equilibrium π^* *(with tie-breaking by sincere myopic preference), a legislative equilibrium* $\sigma^* = (\sigma^*_{\mathcal{E}}, \sigma^*_{\mathcal{L}}, \sigma^*_{\mathcal{D}})$*, and a sorting equilibrium* $w^*(t(\tau^*_{\mathcal{J}}(t_0)))$ *for each possible final legislative policy outcome* $t(\tau^*_{\mathcal{J}}(t_0))$*.*

In a prpe for t_0, all agents have rational expectations about the final policy outcome and make (weakly undominated) decisions accordingly. The specified voting behavior insists that individuals vote on the basis of legislative outcomes rather than on the basis of electoral platforms per se and, as demonstrated above, the identified strategy is (up to tie-breaking) the only one consistent with optimizing over the set of weakly undominated strategies.

It is now useful to be explicit about parties' preferences over tax-rates; i.e. to specify $u_{\mathcal{J}} : [0,1] \rightarrow \Re$. As with the legislative bargaining game, there are a variety of possibilities and the one adopted here is to assume each party seeks to maximize the ex post average consumption of their respective occupations. In effect, each party is controlled by an "ideological" leadership seeking to promote the interests (as consumption) of the average member of the occupation it represents. In particular, the leadership is in principle willing to trade off occupational membership for occupational consumption. This does not seem to be farfetched; for example, it is reasonable to argue that historically European socialist parties supported policies that lead to both higher unemployment and higher incomes for the employed. Similarly, more pro-business parties often advocate policies supporting the business community while not being apparently concerned with the composition of that community. And it is important to note that since the composition of occupations is endogenous in the model, there is no reason to presume that maximizing the average consumption of an occupational member necessarily coincides with maximizing the average consumption of those who in fact vote for the party in an election.[8]

Formally, assume that for all tax-rates $t \in [0,1]$,

$$\begin{aligned} u_{\mathcal{E}}(t) &= [\int_{\theta_2(t,w^*(t))}^{\bar{\theta}} x_e(t,\theta) g(\theta) d\theta] / \int_{\theta_2(t,w^*(t))}^{\bar{\theta}} g(\theta) d\theta \\ &= b(t) + (1-t)\hat{y}_e(t) - c \end{aligned}$$

[8]An alternative specification of party preferences which leads to the same conclusions is that each party maximizes the average consumption of individuals in its *core constituency*, defined to be that set of types who in equilibrium choose the same relevant occupation at every tax-rate in the set $[t_e(\bar{\theta}), t_d(0)]$. Under this specification there is clearly no issue regarding trading off membership against mean consumption.

where $\hat{y}_e(t) = Y_e(t, w^*)/[1 - G(\theta_2(t, w^*(t)))]$ is the mean employer income in the sorting equilibrium at t;

$$u_{\mathcal{L}}(t) = [\int_{\theta_1(t,w^*(t))}^{\theta_2(t,w^*(t))} x_l(t,\theta)g(\theta)d\theta] / \int_{\theta_1(t,w^*(t))}^{\theta_2(t,w^*(t))} g(\theta)d\theta$$
$$= b(t) + (1-t)w^*\hat{\theta}_l(t) - c$$

where $\hat{\theta}_l(t) = E[\theta|\theta \in (\theta_1(t, w^*(t)), \theta_2(t, w^*(t)))]$ is the mean worker type in the sorting equilibrium at t; and

$$u_{\mathcal{D}}(t) = [\int_0^{\theta_1(t,w^*(t))} x_d(t,\theta)g(\theta)d\theta] / \int_0^{\theta_1(t,w^*(t))} g(\theta)d\theta$$
$$= b(t).$$

By Proposition 1, these preferences are well-defined. By Lemma 2, $u_{\mathcal{D}}(\cdot)$ is strictly concave in t with $\arg\max u_{\mathcal{D}}(t) \equiv t_{\mathcal{D}} = \arg\max b(t)$. Since $y_d(\cdot) = 0$, Proposition 1 and (6) imply $\hat{y}_e(t)$ strictly greater than mean income at t. So by Proposition 2, $u_{\mathcal{E}}(\cdot)$ is strictly decreasing in t on $[0, 1]$, and so strictly quasi-concave in t, with $\arg\max u_{\mathcal{E}}(t) \equiv t_{\mathcal{E}} = 0$ (although in equilibrium there can be employer-types with strictly positive most preferred tax-rates). The concavity properties of $u_{\mathcal{L}}(\cdot)$, however, are not so immediate. The complication in this case is that the effect of a change in tax-rate can be decomposed into the sum of two parts: a change in the average consumption level given the set of types choosing to be employees, and a change in the set of types choosing that occupation given the average consumption. While both parts are strictly quasi-concave in t, their sum may not be so. Let $V(t) \equiv [1 - \frac{1-t}{w^*}\frac{dw^*}{dt}]$.

Lemma 5 *Both $u_{\mathcal{D}}(t)$ and $u_{\mathcal{E}}(t)$ are strictly quasi-concave in t and, if $\frac{d\hat{\theta}_l(t)}{dt} \geq \frac{d^2\hat{\theta}_l(t)}{dt^2}[\frac{1-t}{1+V(t)}]$ for all $t \in (0, 1)$, $u_{\mathcal{L}}(t)$ is also strictly quasi-concave in t. Moreover, $0 = t_{\mathcal{E}} < t_{\mathcal{L}} < t_{\mathcal{D}} = \arg\max b(t)$.*

The sufficient condition in the lemma is considerably stronger than necessary to insure quasi-concavity of the employee party maximand. Moreover, it is not an assumption on primitives. However, its role is to insure that once the earned income-increasing effect of a change in tax-rate through changes in the composition of the occupation exactly offsets the consumption-reducing effect of a change in tax-rate at any given occupational composition, the former does not dominate the latter; and this seems a sensible property of the economy.

By Proposition 2, Lemma 5, and $g(\cdot)$ having full support on Θ, there exists a unique pair of types $\alpha, \beta \in \Theta$ such that $\alpha < \beta$, $\xi(t_{\mathcal{D}}, \alpha) = \xi(t_{\mathcal{L}}, \alpha)$ and $\xi(t_{\mathcal{L}}, \beta) = \xi(t_{\mathcal{E}}, \beta)$. To avoid trivialities with party $\mathcal{D}$ or $\mathcal{E}$ invariably commanding a strict majority in the electorate, assume hereafter that

$$\alpha < \theta_m < \beta. \tag{7}$$

Proposition 3 *Assume (7). For any status quo policy t_0 there exists a unique prpe, $p^*(t_0)$. Moreover, in equilibrium all parties receive votes.*

It is worth noting here that it is quite possible for there to exist a positive measure of types that vote (in equilibrium at t_0) for a party representing the interests of an occupation that these types do not choose once the final tax-rate is determined. For example, suppose the equilibrium outcome if $\mathcal{D}$ is the proposer, say t', exceeds that if $\mathcal{L}$ is the proposer, say t''; then, in the associated sorting equilibria, $\theta_1(t', \cdot) > \theta_1(t'', \cdot)$ and there is a γ strictly between these two marginal types whose consumption as a dependent under t' equals type γ's consumption as an employee under t''. Hence, all types $\theta \in (\theta_1(t'', \cdot), \gamma)$ vote for $\mathcal{D}$ but choose to be workers if $\mathcal{L}$ is the proposer rather than $\mathcal{D}$, and all types $\theta \in (\gamma, \theta_1(t'. \cdot))$ vote for $\mathcal{L}$ but choose to be dependents if $\mathcal{D}$ is the proposer rather than $\mathcal{L}$. (And similarly, there can be an interval of types that vote for $\mathcal{E}$ but choose to be workers if $\mathcal{L}$ is the proposer rather than $\mathcal{E}$, and an adjacent interval of types that vote for $\mathcal{L}$ but choose to be employers if $\mathcal{E}$ is the proposer rather than $\mathcal{L}$.) Figure 4 illustrates.

[Figure 4 here]

4 Political economic equilibrium

When the final tax-rate is set through the proportional representation political process, the status quo tax policy, t_0, matters. Given the status quo is unexplained in the model, this is somewhat unsatisfactory. So rather than simply considering equilibrium outcomes relative to a status quo t_0, I look for a status quo tax-rate t_0 such that the set of prpe equilibrium outcomes relative to t_0 consists exclusively of t_0 itself. Such a tax-rate (if one exists) can reasonably be taken as the long-run, or stable, outcome. Formally, for any status quo t_0 and induced prpe $p^*(t_0)$, let $\mathcal{T}(p^*(t_0))$ denote the set of possible equilibrium tax-rate outcomes. Then a tax-rate t_0 is said to be *prpe-stable* if $\mathcal{T}(p^*(t_0)) = \{t_0\}$. Given this definition, the following result is immediate from Lemma 3 and Proposition 3.

Proposition 4 *There exists a unique prpe-stable tax-rate, $t_{\mathcal{L}}$: $\mathcal{T}(p^*(t_{\mathcal{L}})) = \{t_{\mathcal{L}}\}$.*

When no party can credibly claim to pursue any objective other than their true preferences, Proposition 4 says that the final legislative decision on the tax-rate under the proportional representation system here is the tax-rate most preferred by the party representing the workers, party $\mathcal{L}$. I am interested in comparing this outcome with that generated with a two-party majority rule system. Before going on to do this, however, I make good on the claim that Proposition 4 goes through when parties can commit to other objectives, which in turn makes parties' electoral behavior nontrivial.

As argued above, to close the model it is is necessary to assume that parties can commit to preference schedules over feasible tax-rates and not just to a most-preferred rate. This is because final policy decisions are equilibrium outcomes to the legislative bargaining process, and equilibrium bargaining strategies depend on party preferences. So assume parties can, at the electoral stage of the political process, credibly commit to pursue any preference ordering from the set of all continuous and strictly quasi-concave functions on $[0, 1]$, denoted $\mathcal{U}$. An *electoral strategy* for each party $\mathcal{J}$ is a mapping from the set of possible status quo policies into the set of feasible preferences,

$$\varphi_{\mathcal{J}} : [0, 1] \to \mathcal{U}.$$

Parties make their choices simultaneously. Let $\tilde{u}(t_0) = (\varphi_{\mathcal{E}}(t_0), \varphi_{\mathcal{L}}(t_0), \varphi_{\mathcal{D}}(t_0))$ denote the list of party platforms offered the voters at t_0. Given party platforms $\tilde{u}(t_0)$, behavior constituting a *(commitment) proportional representation political equilibrium* is exactly as specified in Definition 2 with the preferences $\tilde{u}(t_0)$ replacing the "true" preferences u throughout; in particular, legislative equilibrium strategies $\sigma^*(t_0, \tilde{u})$ and voting behavior $\pi^*(\cdot, t_0, \tilde{u})$ are all relative to the preferences (and associated most preferred tax-rates) to which parties commit themselves at the electoral stage; for example, the domain of the voting strategy is now $\Theta \times [0, 1] \times \mathcal{U}^3$. To complete the definition of a (commitment) prpe assume that, given the platforms of the other two parties, each party $\mathcal{J}$ chooses platform $\varphi_{\mathcal{J}}(t_0)$ from $\mathcal{U}$ to maximize its expected final equilibrium payoff relative to its true preferences, $u_{\mathcal{J}}$; that is, letting $\tilde{u}^*_{-\mathcal{J}}(t_0)$ denote the list of other parties' platforms, party $\mathcal{J}$ solves

$$\max_{f \in \mathcal{U}} E[u_{\mathcal{J}}(t)|\ f, \tilde{u}^*_{-\mathcal{J}}(t_0), \sigma^*(t_0, f, \tilde{u}^*_{-\mathcal{J}}(t_0)), \pi^*(\cdot, t_0, f, \tilde{u}^*_{-\mathcal{J}}(t_0))]$$

where the expectation is over which party gets to make the legislative proposal after the election. Then the list $\tilde{u}^*(t_0) = (\varphi^*_{\mathcal{E}}(t_0), \varphi^*_{\mathcal{L}}(t_0), \varphi^*_{\mathcal{D}}(t_0))$ is part of a (commitment) prpe if and only if it is a list of weakly undominated mutual best responses.

As with the no commitment model, equilibrium behavior and induced outcomes with commitment depend in general upon the ruling status quo policy. But again, extending the idea of prpe-stable tax-rate to the commitment case yields the same prediction.

Proposition 5 *Fix true party preferences u. Then there exists a unique (commitment) prpe-stable tax-rate,* $t_{\mathcal{L}}$*.*

In view of Propositions 4 and 5, the rest of the analysis focuses on the unique stable equilibrium outcome and makes no further reference to equilibria with or without commitment.

5 Comparative statics: political system and technology

The canonic model of two-party competition under majority rule presumes plurality maximizing candidates. In the current setting, Proposition 2 insures the equilibrium outcome under this assumption involves both parties converging on the median type's most preferred tax-rate, whatever the status quo tax-rate happens to be. By (7), the median must be a worker, so in particular the majority rule equilibrium outcome is $t_l(\theta_m)$. As remarked earlier, the equilibrium distribution of income is skewed to the right; therefore Proposition 2 implies $t_l(\theta_m) > 0$. The same median voter conclusion obtains in a model in which the two parties, say $\mathcal{A}$ and $\mathcal{B}$, have strictly quasi-concave preferences over tax-rates with most preferred rates, $t_{\mathcal{A}}$ and $t_{\mathcal{B}}$ respectively such that

$$t_{\mathcal{A}} \leq t_l(\theta_m) \leq t_{\mathcal{B}},$$

and parties can commit to a preference schedule (in this case, to a tax-rate to impose if elected) and choose electoral platforms to maximize their expected final payoffs (eg [6]). The remaining cases of given preferences and no commitment, and of given preferences with commitment but most-preferred rates on one side of the median, are uninteresting in the present model and hard to motivate, so I ignore them.

The interesting question here concerns the sign of $[t_{\mathcal{L}} - t_l(\theta_m)]$. Although an unequivocal result is unavailable, the following is true. Recall that the distribution of types, $g(\cdot)$, is presumed symmetric about θ_m.

Proposition 6 *There exists a cost of working $\bar{c} > 0$ such that, for all $c \leq \bar{c}$, $t_{\mathcal{L}} > t_l(\theta_m)$.*

Because an individual's type essentially reflects that individual's natural ability in the model, the symmetry assumption on the distribution of types seems plausible. And given symmetry, the argument for Lemma 2 and the result imply that so long as the fixed cost of entering the workforce in some capacity is not excessive, then national income is lower, (voluntary) unemployment is higher, and post-tax income is flatter when taxes are chosen through a proportional representation political system rather than through a two-party plurality rule system.

An intuition underlying the result is offered in the Introduction. Essentially, because party $\mathcal{L}$ is concerned only with employees' consumption it responds (loosely speaking) to the preferences of the average worker (with respect to consumption), and the average worker does not usually coincide with the median individual. Moreover, the average worker is endogenous and the party $\mathcal{L}$ takes this into account in choosing which platform to support. The two most important effects of an increase in tax-rate for $\mathcal{L}$ are an increase in average worker type as a result of induced changes in occupational choice, and an offsetting shift in consumption due to higher taxes. The sufficient condition on the fixed cost c in Proposition 6 is precisely to insure that the second, offsetting, effect is relatively small.

Finally, consider the implications of an improvement in the technology available to the economy. Specifically, assume the production function used by any employer of type θ is given by $kF(L, \theta)$. I am interested in how the political choice of tax-rates responds to an incremental shift in the parameter k at $k = 1$. Although in general this comparative static is equivocal in the model, some results are available. Let $\eta > 0$ denote the elasticity of the market clearing wage-rate with respect to k, evaluated at $k = 1$: $\eta \equiv \frac{dw^*}{dk}\frac{k}{w^*}\Big|_{k=1}$.

Lemma 6 *For $t \in [0, 1)$, $[d\theta_1(t, w^*)/dk]_{k=1} < 0$ and $[d\theta_2(t, w^*)/dk]_{k=1} \gtreqless 0$ as $\eta \gtreqless 1$.*

A parametric outward shift in the production possibility set, therefore, induces more types to enter the workforce via an increase in the equilibrium wage-rate, but leaves the net effect on the composition of types choosing to be employers equivocal: the direct effect is to increase the set of types choosing to be employers, but there is also a general increase in demand for labor which pushes up the wage-rate, thus reducing the incentive to become an employer at the margin. Which of these two effects dominates depends essentially on the change in aggregate intramarginal demand for labor, as reflected in the elasticity, η. If η is less than one, then the change in aggregate intramarginal demand for labor does not induce an increase in the market clearing wage-rate sufficient to offset the incentive at the margin to switch from being an employee to an employer; and conversely when η exceeds one.

Recall that w^* depends on k and so write $V(t, k) \equiv V(t)$, evaluated at k.

Proposition 7 *Assume $\eta \leq 1$ and $[dV(t, k)/dk]_{k=1} \leq 0$. Then the equilibrium tax-rate under both political systems increases with an outward shift in the production possibility frontier: $[dt_{\mathcal{L}}/dk]_{k=1} > 0$ and $[dt_l(\theta_m)/dk]_{k=1} > 0$.*

A marginal improvement in technology results in a net increase in demand for labor that in turn leads to a marginal increase in the equilibrium wage-rate at the given tax-rate. While this induces an increase in pretax worker income which reduces the most preferred tax-rate, it also leads to an increase in the marginal benefit from redistribution through taxes that counters such a reduction. On balance, it turns out that, under the hypotheses of the proposition, the latter effect dominates the disincentive for employees to support higher taxes at the margin and employees most preferred tax-rates marginally increase. Since the median type is, in equilibrium, an employee the comparative static for the majority rule polity follows immediately. And under proportional rule, again given the hypotheses of the proposition, the set of types choosing to be employees in equilibrium shifts to the left with an increase in productivity; thus not only do all employees prefer higher tax-rates, the average employee type falls with an increase in productivity which, by Proposition 2(2), leads to a rise in the average employee's most preferred tax-rate independently of any other change.

Whether or not the sufficient conditions for Proposition 7 obtain is an empirical issue, depending on the details of the technology and the distribution of types.[9] Should the conditions fail, then it can be checked that the most preferred tax-rate of sufficiently high types of employee can fall with a marginal increase in k. In particular, suppose $\eta > 1$ (but we maintain the assumption on $V(t,k)$), then *either* $[dt_l(\theta)/dk]_{k=1} \geq 0$ for all types $\theta \in \Theta$ (with strict inequality for $\theta < \nu_l$), *or* there exists some type $\kappa < \bar{\theta}$ such that $[dt_l(\theta)/dk]_{k=1} > 0$ for all $\theta \in (0,\kappa)$ and $[dt_l(\theta)/dk]_{k=1} \leq 0$ for all $\theta \in (\kappa,\bar{\theta})$ (with strict inequality for at least some positive measure of types). And in the latter case, it is possible (when $\kappa < \theta_m$) for the equilibrium tax-rate under proportional representation to increase, and that under two-party majority rule to decrease, with an outward shift in the production possibility frontier; the converse of this statement is not possible, however. Figure 5 illustrates the possibility.

[Figure 5 here]

6 Conclusion

The observation with which the paper began is that countries using some form of proportional representation political system with more than two parties typically exhibit higher average tax-rates and flatter distributions of income than those using simple majority rule with two parties. A sufficient (but not necessary) condition for the observation to hold in the equilibrium model developed here with a symmetric distribution of talents, is that the fixed cost of earning an income is not too high. A further result is that under both majoritarian and proportional representation systems, an outward shift in the production possibility frontier for the economy leads, under some plausible conditions, to higher chosen tax-rates; in the absence of these conditions, however, the two political systems can lead to different qualitative predictions on how tax-rates vary with technical change.

It is a commonplace to observe that "institutions matter" for the allocation of economic resources. Recognizing this, however, is not by itself very

[9] It is worth noting an early cross-national empirical study in this context: Wilensky [15] finds that per capita GDP and the proportion of GDP allocated to welfare spending are positively correlated.

useful without an understanding of how they matter. The model developed here, albeit very stylized, is intended to develop some insight into the mutual interplay between the political and economic incentives induced by two different collective decision schemes – proportional representation with legislative bargaining and simple majority rule with winner-take-all legislative decision making. It turns out that in the proportional representation system, the political incentives driving party behavior are largely governed by the individual with average employee income and this individual is endogenously identified in equilibrium. On the other hand, in the majority rule system, political incentives are shaped exclusively by the interests of the individual with median income in the electorate as a whole, and the identity of this individual (if not his or her income) is fully determined by the exogenous distribution of productive abilities (types). So political "institutions matter" because the institutional differences are reflected in differences in the incentives of political agents to appeal to particular groups of voters who typically have distinct economic opportunities and, therefore, distinct preferences over economic policy.[10]

7 Appendix: proofs

Proof of Proposition 1: First show that any sorting equilibrium must partition the type space in the way described. To do this, let $t \in [0, 1)$ and suppose $w = w(t)$ is a sorting equilibrium. The distribution of types has continuous support on Θ, and (2) and (3) give $x_d(t, \theta)$ is constant, and $x_l(t, w, \theta)$ is strictly increasing, in θ. Consequently, since $x_l(t, w, 0) < x_d(t, 0)$, Definition 1(2) implies there must exist a unique type θ_1 such that $x_l(t, w, \theta_1) = x_d(t, \theta_1)$ and $\lambda_d(t, w) = (0, \theta_1)$, with

$$\theta_1 = c/[(1 - t)w]. \tag{8}$$

Consider any employer, $\theta \in \lambda_e(t, w)$. Recalling that θ's income maximizing demand for labor is $L(w, \theta)$, (1) and the Envelope Theorem imply

$$\frac{\partial x_e(L(w, \theta), t, w, \theta)}{\partial \theta} = (1 - t)F_\theta(L, \theta)|_{L=L(w,\theta)} > 0.$$

[10]It is an open and important problem, however, to identify the extent to which political institutions continue to matter in the long run when there is free entry into the political arena.

And by assumption, $F_{\theta\theta} \geq 0$ and $F_{L\theta} > 0$, so $x_e(L(w,\theta),t,w,\theta)$ is convex in θ; also $F(L,0) = 0$ with $\lim_{\theta\to 0} \partial F/\partial\theta = 0$. Therefore, $L(w,0) = 0$; $x_e(L(w,0),t,w,0) = x_l(t,w,0) = b(t) - c$; and

$$\lim_{\theta\to 0} \partial x_e(\cdot,\theta)/\partial\theta < \lim_{\theta\to 0} \partial x_l(\cdot,\theta)/\partial\theta.$$

Hence there is a unique type θ_2 such that $x_l(t,w,\theta_2) = x_e(L(w,\theta_2),t,w,\theta_2)$ and $\lambda_e(t,w) = (\theta_2,\overline{\theta})$, with θ_2 implicitly defined by

$$F(L(w,\theta_2),\theta_2) - wL(w,\theta_2) = w\theta_2. \tag{9}$$

Moreover, by convexity of $x_e(L(w,\theta),t,w,\theta)$ and θ_2 unique, Definition 1(2) requires $\theta_2 > \theta_1$. Therefore, $\lambda_e(t,w) = (\theta_2,\bar{\theta})$ and $\lambda_l(t,w) = (\theta_1,\theta_2)$, as claimed. Now establish existence and uniqueness. Given any pair $(t,w) \in [0,1) \times \Re_{++}$, aggregate labor demand is

$$\int_{\lambda_e(t,w)} L(w,\theta)g(\theta)d\theta = \int_{\theta_2(t,w)}^{\bar{\theta}} L(w,\theta)g(\theta)d\theta \tag{10}$$

where $\theta_2(t,w)$ is the type defined by (9) for (t,w). Differentiating RHS(10) wrt w yields

$$\int_{\theta_2(t,w)}^{\bar{\theta}} L_w(w,\theta)g(\theta)d\theta - L(w,\theta_2(t,w))g(\theta_2(t,w))\frac{\partial\theta_2(t,w)}{\partial w}. \tag{11}$$

By assumptions on $F(\cdot)$, it is easy to check $L_w(\cdot) < 0$. Differentiating through (9), writing $\theta_2 = \theta_2(t,w)$ to save notation and collecting terms,

$$\frac{\partial\theta_2(t,w)}{\partial w} = \frac{L(w,\theta_2) + \theta_2}{F_\theta(L(w,\theta_2),\theta_2) - w}. \tag{12}$$

The argument for (9) and for θ_2 unique implies that in (θ,x_j) space, the graph of $x_e(L(w,\theta),t,w,\theta)$ cuts that of $x_l(t,w,\theta)$ from below at θ_2; hence,

$$[\partial x_e(L(w,\theta),t,w,\theta)/\partial\theta - \partial x_l(t,w,\theta)/\partial\theta]|_{\theta=\theta_2} = (1-t)[F_\theta(L(w,\theta_2),\theta_2) - w] > 0.$$

Therefore $\partial\theta_2/\partial w > 0$, in which case expression (11) is strictly negative; ie aggregate labor demand is strictly decreasing in w. Aggregate labor supply is

$$\int_{\lambda_l(t,w^*)} \theta g(\theta)d\theta = \int_{\theta_1(t,w)}^{\theta_2(t,w)} \theta g(\theta)d\theta. \tag{13}$$

Substituting from (8) and differentiating RHS(13) wrt w yields

$$\theta_2(t,w)g(\theta_2(t,w))\frac{\partial\theta_2(t,w)}{\partial w}+g(\theta_1(t,w))\frac{c^2}{(1-t)^2w^3}. \tag{14}$$

Since (12) is strictly positive, (14) is strictly positive also. Hence aggregate labor supply is strictly increasing in w. Therefore, since labor supply is strictly less than demand at $w=0$ and strictly greater than demand for w sufficiently large (by, for any $t<1$, $\lim_{w\to 0}\theta_1(t,w)=\bar{\theta}$, $\lim_{w\to\infty}\theta_1(t,w)=0$ and $\lim_{w\to\infty}\theta_2(t,w)=\bar{\theta}$), there exists a unique wage-rate, $w^*=w^*(t)$ equilibrating labor supply and demand. And by construction, $w^*(t)$ is a sorting equilibrium. This completes the proof.□

Proof of Lemma 1: By Proposition 1, for any $t\in(0,1)$, $w^*(t)$ is unique and implicitly defined to be w^* such that

$$\int_{\theta_2(t,w^*)}^{\bar{\theta}} L(w^*,\theta)g(\theta)d\theta-\int_{\theta_1(t,w^*)}^{\theta_2(t,w^*)}\theta g(\theta)d\theta\equiv 0. \tag{15}$$

By (8) and (9) respectively, $\theta_1(t,w)$ and $\theta_2(t,w)$ are differentiable in t and w, and $L(w,\cdot)$ is differentiable in w. So differentiability of w^* in t on $(0,1)$ follows from the Implicit Function Theorem. Writing $\theta_i=\theta_i(t,w^*)$ to save on notation, implicitly differentiating through (15) and collecting terms, we obtain

$$\frac{dw^*}{dt}=\frac{w^*\theta_1^2g(\theta_1)}{(1-t)A(t,w^*)}, \tag{16}$$

where

$$A(t,w^*)\equiv[F(L(w^*,\theta_2),\theta_2)g(\theta_2)\frac{\partial\theta_2}{\partial w}+\theta_1^2g(\theta_1)-w^*\int_{\theta_2(t,w^*)}^{\bar{\theta}}L_w(w^*,\theta)g(\theta)d\theta]$$

and we have substituted for $\partial\theta_1/\partial t$ and $\partial\theta_1/\partial w$, computed from (8), and used (9). From the argument for Proposition 1, $\partial\theta_2/\partial w>0$ and $L_w(\cdot)<0$. Hence $A(t,w^*)>0$ and the lemma follows.□

Proof of Lemma 2: The balanced budget condition (4) and the labor market clearing condition imply $b(t)=t\int_{\theta_2(t,w^*)}^{\bar{\theta}}F(L(w^*,\theta),\theta)g(\theta)d\theta$. Clearly $b(t)>0$ for $t\in(0,1)$ and $\lim_{t\to 0}b(t)=\lim_{t\to 1}b(t)=0$. To prove the lemma, therefore, it suffices to show that $b(\cdot)$ is strictly concave on $[0,1]$. And to do this it turns out easier to disaggregate total income. So let $Y_i(t,w^*)$ denote the aggregate income of occupation $i \in \{e,l\}$ at (t,w^*), and let $Y(t,w^*) = Y_e(t,w^*) + Y_l(t,w^*)$. Then

$$\begin{aligned} b(t) &= t[Y_e(t,w^*(t)) + Y_l(t,w^*(t))] \\ &= t\left[\int_{\theta_2(t,w^*)}^{\bar{\theta}} [F(L(w^*,\theta),\theta) - w^* L(w^*,\theta)]g(\theta)d\theta + w^* \int_{\theta_1(t,w^*)}^{\theta_2(t,w^*)} \theta g(\theta)d\theta\right] \end{aligned}$$

and

$$b''(t) = 2\frac{dY(t,w^*)}{dt} + t\frac{dY^2(t,w^*)}{dt^2}.$$

By definition, $\frac{dY(t,w^*)}{dt} = [\frac{dY_e(t,w^*)}{dt} + \frac{dY_l(t,w^*)}{dt}]$ where

$$\frac{dY_e(t,w^*)}{dt} \equiv \int_{\theta_2(t,w^*)}^{\bar{\theta}} \frac{\partial y_e(\cdot)}{\partial w}\frac{dw^*}{dt} g(\theta)d\theta - y_e(L(w^*,\theta_2),\theta_2)g(\theta_2)\frac{\partial\theta_2}{\partial w}\frac{dw^*}{dt}$$

and

$$\frac{dY_l(t,w^*)}{dt} \equiv \frac{dw^*}{dt}\int_{\theta_1(t,w^*)}^{\theta_2(t,w^*)} \theta g(\theta)d\theta + w^*[\theta_2 g(\theta_2)\frac{\partial\theta_2}{\partial w}\frac{dw^*}{dt} - \theta_1 g(\theta_1)(\frac{\partial\theta_1}{\partial w}\frac{dw^*}{dt} + \frac{\partial\theta_1}{\partial t})].$$

By definition, $y_e(L(w^*,\theta_2),\theta_2) = y_l(w^*,\theta_2) = \theta_2 w^*$. Further, (15) holds in equilibrium and, by the Envelope Theorem,

$$\frac{\partial y_e(\cdot)}{\partial w} = [F_L(\cdot) - w^*]L_w(\cdot) - L(\cdot) = -L(\cdot).$$

So substituting and collecting terms,

$$[\frac{dY_e(t,w^*)}{dt} + \frac{dY_l(t,w^*)}{dt}] = -w^*\theta_1 g(\theta_1)(\frac{\partial\theta_1}{\partial w}\frac{dw^*}{dt} + \frac{\partial\theta_1}{\partial t}). \tag{17}$$

Now, differentiating (8) appropriately, substituting into RHS(17) and collecting terms gives

$$\frac{dY(t,w^*)}{dt} = -\frac{\theta_1^2 g(\theta_1)}{1-t}V(t), \tag{18}$$

where $V(t) \equiv [1 - \frac{1-t}{w^*}\frac{dw^*}{dt}]$. Since $A(t, w^*) > \theta_1^2 g(\theta_1) > 0$, (16) implies $V(t) > 0$. So, $dY(t, w^*)/dt < 0$. Using the upper bound of assumption (5), it is easily checked that $V'(t) \geq 0$. Therefore, differentiating RHS(18) with respect to t and taking account of the assumption that $\theta g(\theta)$ is nondecreasing in θ, yields $d^2Y(t, w^*)/dt^2 < 0$. The result follows.□

Remark. The conclusion that $dY(t, w^*)/dt < 0$ follows almost immediately from differentiation of $Y(t, w^*) = \int_{\theta_2(t,w^*)}^{\bar{\theta}} F(L(w^*, \theta), \theta)g(\theta)d\theta$. The gain from taking the indirect approach above is entirely in signing the second derivative of aggregate income, $d^2Y(t, w^*)/dt^2$.

Proof of Proposition 2: (1) Since $x_d(t, \theta) = b(t)$, Lemma 2 immediately gives $x_d(t, \theta)$ strictly concave in t with $t_d(\theta) = \arg\max b(t)$ for all θ.

(2) Consider $x_l(t, \theta)$. Differentiating w.r.t. t and collecting terms yields

$$\frac{dx_l(t, \theta)}{dt} = b'(t) - \theta w^* V(t).$$

By earlier arguments, $V(t) > 0$ and $V'(t) \geq 0$. Differentiating a second time, therefore, Lemma 2 implies $x_l(t, \theta)$ strictly concave in t, and $t_l(\theta)$ is implicitly defined by the first-order condition, $dx_l(t, \theta)/dt = 0$; it follows immediately that $t_d(\theta) > t_l(\theta)$. Now

$$b'(t) = Y(t, w^*) + t[dY(t, w^*)/dt] \tag{19}$$

and $dY(t, w^*)/dt < 0$ (see (18)). Therefore, by Lemma 1 and the definition $y_l(w^*, \theta) = \theta w^*$, the equation $dx_l(t, \theta)/dt = 0$ implies there exists a type $\nu_l > \mu$ such that

$$\begin{aligned} \theta \leq \nu_l \quad &\Rightarrow [dx_l(t, \theta)/dt]_{t=0} \geq 0 \\ &\Rightarrow t_l(\theta) > 0 \text{ if } \theta < \nu_l. \end{aligned}$$

Further, since the second term of the derivative $dx_l(t, \theta)/dt$ is decreasing in θ, $b''(t) < 0$ implies $t_l(\theta) > t_l(\theta')$ for $\theta < \theta' < \nu_l$. On the other hand, $\theta \geq \nu_l$ implies $[dx_l(t, \theta)/dt]_{t=0} \leq 0$, in which case $t_l(\theta) = 0$.

(3) Now consider $x_e(t, \theta)$. First assume that $x_e(t, \theta)$ is indeed strictly quasi-concave in t. Then differentiating w.r.t. t and using the Envelope Theorem gives $t_e(\theta)$ implicitly defined by

$$\begin{aligned} \frac{dx_e(t, \theta)}{dt} &= b'(t) + (1 - t)\frac{dy_e(L(w^*, \theta), w^*, \theta)}{dt} - y_e(L(w^*, \theta), w^*, \theta) \\ &= b'(t) - (1 - t)\frac{dw^*}{dt}L(w^*, \theta) - y_e(L(w^*, \theta), w^*, \theta). \end{aligned}$$

By Lemma 1 and (19),there exists a type $\nu_e < \mu$ such that

$$\begin{aligned} \theta \leq \nu_e \quad &\Rightarrow [dx_e(t,\theta)/dt]_{t=0} \geq 0 \\ &\Rightarrow t_e(\theta) > 0 \text{ if } \theta < \nu_e. \end{aligned}$$

If $\theta \geq \nu_e$ then $[dx_e(t,\theta)/dt]_{t=0} \leq 0$, in which case $t_e(\theta) = 0$. The first-order condition $[dx_e(t,\theta)/dt] = 0$ for $\theta < \nu_e$ directly implies $t_d(\theta) > t_e(\theta) > 0$. And since $\partial y_e(L(w^*,\theta), w^*, \theta)/\partial\theta = F_\theta(\cdot) > 0$ and $L_\theta(\cdot) > 0$, the second and third terms of the first order condition strictly decrease in θ. So by $b''(t) < 0$, $t_e(\theta) > t_e(\theta')$ for $\theta < \theta' < \nu_e$. It remains to check $x_e(t,\theta)$ strictly quasi-concave in t.

To show quasi-concavity, note that the first order condition immediately gives $[dx_e(t,\theta)/dt] < 0$ for all $\theta \geq \nu_e$, so quasi-concavity is assured for these types. Furthermore, for all θ and all $t > \arg\max b(t)$, the first order condition also implies $[dx_e(t,\theta)/dt] < 0$. Let $\theta < \nu_e$ and $t \leq \arg\max b(t)$. To save on notation, write $w_t^* = dw^*(t)/dt$, $y_e'(\cdot,\theta) = dy_e(L(w^*,\theta), w^*, \theta)/dt$, etc, and differentiate the first order condition to yield

$$\begin{aligned} \frac{d^2 x_e(t,\theta)}{dt^2} &= b''(t) + (1-t)y_e''(\cdot,\theta) - 2y_e'(\cdot,\theta) \\ &= b''(t) - (1-t)[(w_t^*)^2 L_w(w^*,\theta) + w_{tt}^* L(w^*,\theta)] + 2w_t^* L(w^*,\theta). \end{aligned}$$

By assumption, for all (L,θ), F is thrice differentiable in both arguments, $F(L,0) = F(0,\theta) = 0$, and $\lim_{\theta\to 0} \partial F/\partial\theta = 0$. Hence,

$$\lim_{\theta\downarrow 0} L_w(w^*,\theta) = \lim_{\theta\downarrow 0} L(w^*,\theta) = 0.$$

Therefore, by Lemma 2, d^2x_e/dt^2 continuous in θ implies

$$\lim_{\theta\downarrow 0} \frac{d^2 x_e(t,\theta)}{dt^2} = b''(t) < 0,$$

and so $x_e(t,\theta)$ is strictly concave in $t \leq \arg\max b(t)$ for θ sufficiently small. By the Envelope Theorem,

$$\frac{d^2 x_e(t,\theta)}{dt d\theta} = -(1-t)w_t^{*2} L_\theta(w^*,\theta) - F_\theta(L(w^*,\theta),\theta) < 0.$$

Moreover, by Young's Theorem, $\frac{d}{dt}[dy_e/d\theta] = \frac{d}{d\theta}[dy_e/dt]$ and so

$$\frac{d}{dt}\left[\frac{d^2 x_e(t,\theta)}{dt d\theta}\right] = -L_\theta(w^*,\theta)[2w_t^* + (1-t)w_{tt}^*] - (1-t)w_t^{*2} L_{\theta w}(w^*,\theta) \leq 0,$$

with the inequality following from the lower bound of (5) and the assumption that $F_{LL\theta} \leq 0$. Together, the previous two inequalities state that, at any $t \leq \arg\max b(t)$, the slope dx_e/dt is strictly decreasing in θ and the rate at which it decreases is no slower for higher than for lower values of t. Because $x_e(t, \theta)$ is strictly concave in $t \leq \arg\max b(t)$ for θ sufficiently small, these facts, with the previous observations on the strict quasi-concavity of $x_e(t, \theta)$ in t for all θ and $t > \arg\max b(t)$, yield $x_e(t, \theta)$ strictly quasi-concave in t for all θ. □

Proof of Lemma 4: We have to show that if π^* is a voting equilibrium then, for all $\theta \in \Theta$, all t_0, u, and all $\mathcal{J} \in \{\mathcal{E},\mathcal{L},\mathcal{D}\}$, $\pi^*(\mathcal{J}|\theta, t_0, u) > 0$ implies

$$\forall \mathcal{J}' \neq \mathcal{J},\ \xi(\tilde{t}_{\mathcal{J}}(\sigma^*), \theta) > \xi(\tilde{t}_{\mathcal{J}'}(\sigma^*), \theta).$$

Suppose the contrary. Then (without loss of generality) for some pair (t_0, u) and some type θ, $\xi(\tilde{t}_{\mathcal{L}}(\sigma^*), \theta) > \xi(\tilde{t}_{\mathcal{D}}(\sigma^*), \theta)$ but $\pi^*_{\mathcal{D}}(\theta, \cdot) > 0$. Now let $\pi \neq \pi^*$ be such that: $\pi_{\mathcal{D}}(\theta, \cdot) = 0$, $\pi_{\mathcal{L}}(\theta, \cdot) = \pi^*_{\mathcal{L}}(\theta, \cdot) + \pi^*_{\mathcal{D}}(\theta, \cdot)$, $\pi_{\mathcal{E}}(\theta, \cdot) = \pi^*_{\mathcal{E}}(\theta, \cdot)$ and $\pi_{-\theta} = \pi^*_{-\theta}$. Then

$$\begin{aligned}
&E[\xi(t, \theta)|\pi(\theta, \cdot), \pi_{-\theta}, \sigma^*] - E[\xi(t, \theta)|\pi^*(\theta, \cdot), \pi^*_{-\theta}, \sigma^*] \\
&= [\frac{\pi_{\mathcal{L}}(\theta, \cdot)}{N} - \frac{\pi^*_{\mathcal{L}}(\theta, \cdot)}{N}]\xi(\tilde{t}_{\mathcal{L}}(\sigma^*), \theta) + [\frac{\pi_{\mathcal{D}}(\theta, \cdot)}{N} - \frac{\pi^*_{\mathcal{D}}(\theta, \cdot)}{N}]\xi(\tilde{t}_{\mathcal{D}}(\sigma^*), \theta) \\
&= \frac{\pi^*_{\mathcal{D}}(\theta, \cdot)}{N}[\xi(\tilde{t}_{\mathcal{L}}(\sigma^*), \theta) - \xi(\tilde{t}_{\mathcal{D}}(\sigma^*), \theta)] > 0.
\end{aligned}$$

Hence $\pi^*(\mathcal{J}|\theta, t_0, u)$ cannot maximize $E[\xi(t, \theta)|\pi(\theta, \cdot), \pi^*_{-\theta}, \sigma^*]$, contradicting the supposition.□

Proof of Lemma 5: The claims regarding $u_{\mathcal{E}}(t)$ and $u_{\mathcal{D}}(t)$ have already been established. Consider $u_{\mathcal{L}}(t)$. The first- and second-order derivatives with respect to t are, respectively (where the dependency of $w^*(\cdot)$ and $V(\cdot)$ on t are suppressed and I write $\hat{\theta}'_l(t) \equiv d\hat{\theta}_l(t)/dt$ etc.),

$$u'_{\mathcal{L}}(t) = b'(t) - \hat{\theta}_l(t)w^*V + (1 - t)w^*\hat{\theta}'_l(t) \tag{20}$$

and

$$u''_{\mathcal{L}}(t) = b''(t) - 2\hat{\theta}'_l(t)w^*V - \hat{\theta}_l(t)[\frac{dw^*}{dt}V + w^*\frac{dV}{dt}] + (1 - t)w^*\hat{\theta}''_l(t). \tag{21}$$

¿From earlier arguments, $d\theta_1(t)/dt > 0$ and $d\theta_2(t)/dt > 0$; hence, $\hat{\theta}_l'(t) > 0$. By $g(\cdot)$ symmetric and Proposition 1, the income distribution is skewed to the right; so (6) implies $\hat{\theta}_l(0) \leq \mu$; by Proposition 2, $0 < t_l(\theta) < \arg\max b(t)$ for all $\theta < \nu_l$ and $\nu_l > \mu$. Hence $\hat{\theta}_l'(t) > 0$ for all $t \in [0, 1)$ implies $\lim_{t \to 0} u_{\mathcal{L}}'(t) > 0$. Therefore, for any maximizer $t_{\mathcal{L}}$ of $u_{\mathcal{L}}(t)$, $t_{\mathcal{L}} > 0$. Now let t be any stationary point of $u_{\mathcal{L}}(t)$. Then $u_{\mathcal{L}}'(t) = 0$, and we can substitute for $\hat{\theta}_l(t)$ from (20) into (21) and collect terms to yield

$$\begin{aligned} u_{\mathcal{L}}''(t) &= b''(t) - b'(t)[\frac{dw^*}{dt}V + w^*\frac{dV}{dt}]/[w^*V] \\ &- \hat{\theta}_l'(t)w^*[1 + V + (1-t)\frac{dV/dt}{V}] + (1-t)w^*\hat{\theta}_l''(t). \end{aligned}$$

By previous arguments, each term on the RHS of this expression, with the possible exception of the last, is strictly negative. But by assumption, $\hat{\theta}_l'(t) \geq \hat{\theta}_l''(t)[\frac{1-t}{1+V(t)}]$; hence, $u_{\mathcal{L}}''(t) < 0$. Therefore any stationary point is a maximum and, since $\lim_{t \to 0} u_{\mathcal{L}}'(t) > 0$, $u_{\mathcal{L}}(t)$ is strictly quasi-concave as required. Finally, $t_{\mathcal{L}}$ unique and (20) give $t_{\mathcal{L}} < t_{\mathcal{D}}$.□

Proof of Proposition 3: Suppose first that $t_0 \neq t_{\mathcal{L}}$. By Proposition 1 and Lemma 3, it suffices to check there is a unique equilibrium voting strategy, $\pi^*(\cdot, t_0)$. By Lemma 3 and $t_0 \neq t_{\mathcal{L}}$, there are three possible final tax-rate outcomes from the legislative bargaining process, ordered by

$$t_{\mathcal{D}} \geq \tau_{\mathcal{D}}^*(\cdot) > \tau_{\mathcal{L}}^*(\cdot) = t_{\mathcal{L}} > \tau_{\mathcal{E}}^*(\cdot) \geq t_{\mathcal{E}}.$$

By Proposition 2, Lemma 5, and $g(\cdot)$ having full support on Θ, there exists a unique pair of types $\alpha', \beta' \in \Theta$ such that $\alpha' < \beta'$, $\xi(\tau_{\mathcal{D}}^*, \alpha') = \xi(\tau_{\mathcal{L}}^*, \alpha')$ and $\xi(\tau_{\mathcal{L}}^*, \beta') = \xi(\tau_{\mathcal{E}}^*, \beta')$. And Proposition 2 further implies that

$$\begin{aligned} &\forall \theta \in (0, \alpha'),\ \forall \mathcal{J} \neq \mathcal{D},\ \xi(\tau_{\mathcal{D}}^*, \theta) > \xi(\tau_{\mathcal{J}}^*, \theta) \\ &\forall \theta \in (\alpha', \beta'),\ \forall \mathcal{J} \neq \mathcal{L},\ \xi(\tau_{\mathcal{L}}^*, \theta) > \xi(\tau_{\mathcal{J}}^*, \theta) \\ &\forall \theta \in (\beta', \bar{\theta}),\ \forall \mathcal{J} \neq \mathcal{E},\ \xi(\tau_{\mathcal{E}}^*, \theta) > \xi(\tau_{\mathcal{J}}^*, \theta). \end{aligned}$$

Therefore, by Lemma 4, any equilibrium voting strategy π^* must satisfy the following properties: $\forall \theta \in (0, \alpha')$, $\pi^*(\mathcal{D}|\theta, t_0, u) = 1$; $\forall \theta \in (\alpha', \beta')$, $\pi^*(\mathcal{L}|\theta, t_0, u) = 1$; and $\forall \theta \in (\beta', \bar{\theta})$, $\pi^*(\mathcal{E}|\theta, t_0, u) = 1$. And although α' [respectively, β'] might in some cases be free to randomize between $\mathcal{L}$ and $\mathcal{D}$ [respectively, $\mathcal{L}$ and $\mathcal{E}$], the set $\{\alpha', \beta'\}$ has measure zero; so π^* as described is unique. Finally, by definition of α' and β', it is apparent that all parties receive votes under π^*; in particular, Proposition 2 and (7) imply $v_{\mathcal{J}}(t_0, u) < 1/2$ for $\mathcal{J} \in \{\mathcal{E}, \mathcal{D}\}$.

Now let $t_0 = t_{\mathcal{L}}$. Then by Lemma 3 all individuals are indifferent over which party gets to make the legislative proposal. So by the tie-breaking condition imposed on equilibrium voting behavior, if π^* is an equilibrium voting strategy, $v_{\mathcal{D}}(t_0, u) = \int_0^{\alpha} g(\theta) d\theta < 1/2$ and $v_{\mathcal{E}}(t_0, u) = \int_{\beta}^{\bar{\theta}} g(\theta) d\theta < 1/2$ where α and β are defined in (7). Therefore neither $\mathcal{D}$ nor $\mathcal{E}$ can receive a strict majority of votes and the specified behavior constitutes an equilibrium. The proposition follows.□

Proof of Proposition 5: Let $\tilde{u}(t_0) \in \mathcal{U}^3$ be any list of platforms to which the parties are committed in an election when the status quo is t_0. By Proposition 3 there is a unique prpe for t_0 relative to $\tilde{u}(t_0)$, say $\tilde{p}(t_0)$, with equilibrium outcomes $\mathcal{J}(\tilde{p}(t_0))$ defined, mutatis mutandis, by Lemma 3. To prove the proposition, therefore, it suffices to show, first, that if $t_0 \neq t_{\mathcal{L}}$ then there is no commitment prpe with $\mathcal{J}(\tilde{p}(t_0)) = \{t_0\}$ and, second, that if $t_0 = t_{\mathcal{L}}$ then there exists a commitment prpe $\tilde{p}(t_{\mathcal{L}})$ and, for any such prpe, $\mathcal{J}(\tilde{p}(t_{\mathcal{L}})) = \{t_{\mathcal{L}}\}$.

Without loss of generality, suppose $t_0 < t_{\mathcal{L}}$ and let $\tilde{u}(t_0) = (\tilde{u}_{\mathcal{E}}, \tilde{u}_{\mathcal{L}}, \tilde{u}_{\mathcal{D}}) \in \mathcal{U}^3$ be any list of equilibrium platforms to which the parties are committed. Clearly, all parties must receive a strictly positive vote share in equilibrium. Let $\tilde{s}_{\mathcal{J}} = \arg\max \tilde{u}_{\mathcal{J}}$. Because $t_{\mathcal{D}} > t_{\mathcal{L}} > t_{\mathcal{E}} = 0$, Lemma 3 and the presumption that parties are committed to their respective electoral platforms at the legislative bargaining stage imply that if $\tilde{u}(t_0)$ is an equilibrium list of platforms, then necessarily $1 > \tilde{s}_{\mathcal{D}} \geq \tilde{s}_{\mathcal{L}} \geq \tilde{s}_{\mathcal{E}} \geq 0$. Therefore, by Lemma 3(3), the commitment assumption implies that if $\tilde{s}_{\mathcal{L}} = t_0$, then $E[u_{\mathcal{L}}(t(\tilde{\tau}_{\mathcal{J}}(t_0)))| \tilde{p}(t_0)] = u_{\mathcal{L}}(t_0)$ surely. Fixing $(\tilde{u}_{\mathcal{E}}, \tilde{u}_{\mathcal{D}})$, consider a platform $\bar{u}_{\mathcal{L}} \in \mathcal{U}$ such that $\bar{s}_{\mathcal{L}} = \arg\max \bar{u}_{\mathcal{L}} = \tilde{s}_{\mathcal{L}} + \delta \in (t_0, t_{\mathcal{L}}]$ and $\bar{u}_{\mathcal{L}}(t_0) = \bar{u}_{\mathcal{L}}(t_{\mathcal{L}})$; such a platform exists by definition of $\mathcal{U}$ and the supposition that $t_0 < t_{\mathcal{L}}$ and, by strict quasi-concavity, $u_{\mathcal{L}}(t_0) < u_{\mathcal{L}}(\bar{s}_{\mathcal{L}})$. Then, in obvious notation,

$$E[u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{J}}(t_0)))| (\tilde{u}_{\mathcal{E}}, \bar{u}_{\mathcal{L}}, \tilde{u}_{\mathcal{D}}), \bar{\sigma}, \bar{\pi}] = \sum_{\mathcal{J}} \bar{v}_{\mathcal{J}} u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{J}}(t_0))).$$

Therefore, since $\sum_{\mathcal{J}} \bar{v}_{\mathcal{J}} = 1$ by definition,

$$\begin{aligned} &E[u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{J}}(t_0)))| (\tilde{u}_{\mathcal{E}}, \bar{u}_{\mathcal{L}}, \tilde{u}_{\mathcal{D}}), \bar{\sigma}, \bar{\pi}] - E[u_{\mathcal{L}}(t(\tilde{\tau}_{\mathcal{J}}(t_0)))| \tilde{p}(t_0)] \\ &= \sum_{\mathcal{J}} \bar{v}_{\mathcal{J}} [u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{J}}(t_0))) - u_{\mathcal{L}}(t_0)]. \end{aligned}$$

Because all parties receive a strictly positive vote share at $\tilde{u}(t_0)$, Lemma 4 and $\delta > 0$ sufficiently small give $\bar{v}_{\mathcal{J}} > 0$ for all parties $\mathcal{J}$. So Lemma 3, choice of $\bar{u}_{\mathcal{L}}$ and $\tilde{s}_{\mathcal{D}} \geq \tilde{s}_{\mathcal{L}} = t_0 \geq \tilde{s}_{\mathcal{E}}$ imply:

$$\begin{aligned} &u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{D}}(t_0))) \in (u_{\mathcal{L}}(t_0), u_{\mathcal{L}}(t_{\mathcal{L}})], \\ &u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{E}}(t_0))) = u_{\mathcal{L}}(t_0), \text{ and} \\ &u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{L}}(t_0))) \in (u_{\mathcal{L}}(t_0), u_{\mathcal{L}}(\bar{s}_{\mathcal{L}})]. \end{aligned}$$

Hence, $\sum_{\mathcal{J}} \bar{v}_{\mathcal{J}}[u_{\mathcal{L}}(t(\bar{\tau}_{\mathcal{J}}(t_0))) - u_{\mathcal{L}}(t_0)] > 0$ in which case, if $t_0 < t_{\mathcal{L}}$ and $\tilde{u}(t_0)$ is part of a commitment prpe for t_0, then $\arg\max \tilde{u}_{\mathcal{L}} > t_0$. By Lemma 3, therefore, $\mathcal{T}(\tilde{p}(t_0)) \neq \{t_0\}$.

Suppose $t_0 = t_{\mathcal{L}}$. Then evidently party $\mathcal{L}$ choosing $\varphi_{\mathcal{L}}(t_0) = u_{\mathcal{L}}$ is a best response to any platform selected by the other two parties. And since all parties' true preferences are strictly quasi-concave with $t_{\mathcal{D}} > t_{\mathcal{L}} > t_{\mathcal{E}}$, any best response by party $\mathcal{D}$ to $(\varphi_{\mathcal{L}}(t_0), \varphi_{\mathcal{E}}(t_0)) = (u_{\mathcal{L}}, \varphi_{\mathcal{E}}(t_{\mathcal{L}}))$ has $\arg\max \varphi_{\mathcal{D}}(t_0) \geq t_0 = t_{\mathcal{L}}$; and similarly for party $\mathcal{E}$. By Lemma 3(3), therefore, $(\varphi^*_{\mathcal{E}}(t_0), \varphi^*_{\mathcal{L}}(t_0), \varphi^*_{\mathcal{D}}(t_0)) = (u_{\mathcal{E}}, u_{\mathcal{L}}, u_{\mathcal{D}})$ can support a commitment prpe. And since any commitment prpe for $t_0 = t_{\mathcal{L}}$ necessarily has $\arg\max \varphi_{\mathcal{L}}(t_0) = t_{\mathcal{L}}$, we have $\mathcal{T}(\tilde{p}(t_{\mathcal{L}})) = \{t_{\mathcal{L}}\}$ for all such prpe.□

Proof of Proposition 6: By Proposition 2, $t_l(\theta_m)$ is implicitly defined by the equation,

$$b'(t_l(\theta_m)) - \hat{\theta} w^*(t_l(\theta_m)) \left[1 - \frac{(1 - t_l(\theta_m))}{w^*(t_l(\theta_m))} \left. \frac{dw^*}{dt} \right|_{t_l(\theta_m)} \right] = 0.$$

Similarly, by Lemma 5, $t_{\mathcal{L}}$ is implicitly defined by the equation,

$$b'(t_{\mathcal{L}}) - \hat{\theta}_l(t_{\mathcal{L}}) w^*(t_{\mathcal{L}}) \left[1 - \frac{(1 - t_{\mathcal{L}})}{w^*(t_{\mathcal{L}})} \left. \frac{dw^*}{dt} \right|_{t_{\mathcal{L}}} \right] + (1 - t_{\mathcal{L}}) w^*(t_{\mathcal{L}}) \left. \frac{d\hat{\theta}_l}{dt} \right|_{t_{\mathcal{L}}} = 0.$$

Since $b(t)$ is strictly concave and $d\hat{\theta}_l/dt > 0$, these two equations imply that a sufficient (but not necessary) condition for $t_{\mathcal{L}} > t_l(\theta_m)$ is $\hat{\theta}_l(t_{\mathcal{L}}) \leq \theta_m$. By (8), (9) and (15) both $\theta_1(t, w^*(t))$ and $\theta_2(t, w^*(t))$ are decreasing in c. In particular, for any $t \in [0, 1)$, $\lim_{c \to 0} \theta_1(t, w^*(t)) = 0$ and, by (6), $\lim_{c \to 0} \theta_2(t, w^*(t)) > \theta_m$. Therefore, since $g(\cdot)$ is symmetric, $\theta_m = \hat{\theta}$ and there exists some $\bar{c} > 0$ such that $\hat{\theta}_l(t_{\mathcal{L}}) = \theta_m$ and, for all $c \leq \bar{c}$, $\hat{\theta}_l(t_{\mathcal{L}}) < \theta_m$.□

Proof of Lemma 6: From (8), $sgn[d\theta_1(t, w^*)/dk]_{k=1} = -sgn[dw^*/dk]$. Let $L(w^*, k, \theta)$ denote maximizing labor demand when output is $kF(L, \theta)$. Then differentiating through (15) and collecting terms gives

$$\frac{dw^*}{dk} = \frac{\int_{\theta_2}^{\bar{\theta}} L_k(w^*, k, \theta)g(\theta)d\theta - [L(w^*, k, \theta_2) + \theta_2]g(\theta_2)\frac{\partial\theta_2}{\partial k}}{[L(w^*, k, \theta_2) + \theta_2]g(\theta_2)\frac{\partial\theta_2}{\partial w} - \theta_1 g(\theta_1)\frac{\partial\theta_2}{\partial w} - \int_{\theta_2}^{\bar{\theta}} L_w(w^*, k, \theta)g(\theta)d\theta}.$$

By earlier arguments, the denominator of the above expression is strictly positive. Routine manipulation of the first-order condition defining $L(w^*, k, \theta)$ gives $L_k(w^*, k, \theta) > 0$, and implicit partial differentiating through (9) (mutatis mutandis) yields

$$\frac{\partial\theta_2}{\partial k} = \frac{-F(L(w^*, k, \theta_2), \theta_2)}{[kF_\theta(L(w^*, k, \theta_2), \theta_2) - w^*]}. \tag{22}$$

By earlier arguments and $k \geq 1$, $\partial\theta_2/\partial k < 0$. Hence $dw^*/dk > 0$, implying $d\theta_1/dk < 0$. Now consider the total derivative, $d\theta_2/dk$. Totally differentiating through (9) yields

$$\frac{d\theta_2}{dk} = \frac{\partial\theta_2}{\partial k} + \frac{\partial\theta_2}{\partial w}\frac{dw^*}{dk}.$$

Using (12), (9) and (22), therefore,

$$\left.\frac{d\theta_2}{dk}\right|_{k=1} = \frac{\partial\theta_2}{\partial w}\left[\frac{dw^*}{dk} - w^*\right]_{k=1}.$$

Since $\partial\theta_2/\partial w > 0$, the lemma follows.□

Proof of Proposition 7: First show that $[dt_l(\theta)/dk]_{k=1} > 0$ for all $\theta < \nu_l$. Recall the first-order condition implicitly defining $t_l(\theta)$ for $\theta < \nu_l$,

$$\frac{dx_l(t, \theta)}{dt} = \frac{db(t, k)}{dt} - \theta w^*(t, k)V(t, k) = 0 \tag{23}$$

where we have emphasized the dependency of b and w^* on both t and k. Since the second-order condition is satisfied (Proposition 2),

$$sgn\left.\frac{dt_l(\theta)}{dk}\right|_{k=1} = sgn\left[\frac{d^2b}{dtdk} - \theta(\frac{dw^*}{dk}V + w^*\frac{dV}{dk})\right]_{k=1}. \tag{24}$$

By definition, $b(t,k) = tk\int_{\theta_2}^{\bar{\theta}} F(L(w^*,k,\theta),\theta)g(\theta)d\theta$. Hence,

$$\begin{aligned}\frac{d^2b}{dtdk} &= [t\frac{dY}{dt} + Y] + k[t\frac{d^2Y}{dtdk} + \frac{dY}{dk}] \\ &= \frac{db}{dt}.\frac{1}{k} + k[t\frac{d^2Y}{dtdk} + \frac{dY}{dk}]\end{aligned}$$

where $Y \equiv \int_{\theta_2}^{\bar{\theta}} F(L(w^*,k,\theta),\theta)g(\theta)d\theta$. As in the proof of Lemma 2, it is convenient to recognize that aggregate labor costs and aggregate employee income are identical and decompose

$$\begin{aligned}Y(\cdot) &\equiv Y_e(\cdot) + Y_l(\cdot) \\ &= \int_{\theta_2(\cdot)}^{\bar{\theta}} [F(L(w^*,k,\theta),\theta) - w^*L(w^*,k,\theta)]g(\theta)d\theta + \int_{\theta_1(\cdot)}^{\theta_2(\cdot)} w^*\theta g(\theta)d\theta.\end{aligned}$$

Doing the calculus (and suppressing the arguments of functions where there is no ambiguity),

$$\frac{dY_e}{dk} = \int_{\theta_2}^{\bar{\theta}} \{[F_L - w^*][L_w\frac{dw^*}{dk} + L_k] - L(w^*,k,\theta)\frac{dw^*}{dk}\}g(\theta)d\theta - y_e(\cdot,\theta_2)g(\theta_2)\frac{d\theta_2}{dk};$$

and

$$\frac{dY_l}{dk} = \frac{dw^*}{dk}\int_{\theta_1}^{\theta_2} \theta g(\theta)d\theta + y_l(\cdot,\theta_2)g(\theta_2)\frac{d\theta_2}{dk} - y_l(\cdot,\theta_1)g(\theta_1)\frac{d\theta_1}{dk}.$$

As in the proof for Lemma 2, use the Envelope Theorem, the identity of (aggregate) labor costs and employee income, and the fact that, in equilibrium, $y_e(\cdot,\theta_2) = y_l(\cdot,\theta_2)$ to obtain

$$\left.\frac{dY}{dk}\right|_{k=1} = -w^*\theta_1 g(\theta_1)\left.\frac{d\theta_1}{dk}\right|_{k=1} > 0,$$

with the inequality following by Lemma 6. Because $d\theta_1/dk$ is not available explicitly, it is easiest to use RHS(18) to get $d^2Y/dtdk$; doing this yields

$$\frac{d^2Y}{dtdk} = \frac{-[\frac{d\theta_1}{dk}\theta_1(2+\theta_1 g'(\theta_1))V + \theta_1^2 g(\theta_1)\frac{dV}{dk}]}{1-t}.$$

By assumption, $\frac{dV}{dk}\big|_{k=1} \leq 0$. And with the maintained assumption on the distribution $g(\cdot)$, therefore, Lemma 6 implies $d^2Y/dtdk > 0$. Hence, at $k = 1$,

$$\frac{d^2b}{dtdk} > \frac{db}{dt}.$$

Therefore, using (23),

$$\left[\frac{d^2b}{dtdk} - \theta(\frac{dw^*}{dk}V + w^*\frac{dV}{dk})\right]_{k=1} > \left[\theta w^* V - \theta(\frac{dw^*}{dk}V + w^*\frac{dV}{dk})\right]_{k=1}$$
$$= \theta w^* \left[(1-\eta)V(t,1) - \frac{dV}{dk}\bigg|_{k=1}\right].$$

Because $\eta \leq 1$ and $\frac{dV}{dk}\big|_{k=1} \leq 0$ by assumption, the RHS of this expression is positive. Hence, by (24), $[dt_l(\theta)/dk]_{k=1} > 0$ for all $\theta < \nu_l$ as was to be shown.

The majority rule equilibrium tax-rate is $t_l(\theta_m)$, so the preceding argument immediately gives $[dt_l(\theta_m)/dk]_{k=1} > 0$ as claimed. And, under the hypotheses of the proposition, $[d\theta_1/dk]_{k=1} < 0$ and $[d\theta_2/dk]_{k=1} < 0$ by Lemma 6. Therefore $[d\hat{\theta}_l/dk]_{k=1} < 0$ in which case, by Proposition 2(2) and $[dt_l(\theta)/dk]_{k=1} > 0$ for all $\theta < \nu_l$, $[dt_{\mathcal{L}}/dk]_{k=1} > 0$ also.□

Remark: If $\eta > 1$ then, even with $\frac{dV}{dk}\big|_{k=1} \leq 0$, for sufficiently high types the right hand side of the inequality above can be negative, permitting $[dt_l(\theta)/dk]_{k=1} < 0$ for θ sufficiently high. On the other hand, for sufficiently low types, the strict inequality implies that we must have $[dt_l(\theta)/dk]_{k=1} > 0$ for θ sufficiently low and any finite value of the elasticity, η. Together, these observations justify the claim made in the text that the two political systems might induce qualitatively different responses to an improvement in technical efficiency.

References

[1] Atkinson, A.B., L. Rainwater, and T. Smeeding (1995), Income Distribution in European Countries, *Dept. of Applied Economics, University of Cambridge, DAE Working Paper 9535.*

[2] Austen-Smith, D. and J.S. Banks (1988), Elections, Coalitions and Legislative Outcomes, *American Political Science Review, 82, 405-422.*

[3] Baron, D. (1991), A Spatial Bargaining Theory of Government Formation in Parliamentary Systems, *American Political Science Review, 85, 137-164.*

[4] Baron, D. and D. Diermeier (1997), Dynamics of Parliamentary Systems: Elections, Governments and Parliaments, *Graduate School of Business Discussion Paper, Stanford University.*

[5] Birchfield, V. and M.L. Crepaz (1998), The Impact of Constitutional Structures and Collective and Competitive Veto Points on Income Inequality in Industrialized Democracies, *European Journal of Political Research,* 34, 175-200.

[6] Calvert, R. (1985), Robustness of the Multidimensional Voting Model:Candidates' Motivations, Uncertainty, and Convergence, *American Journal of Political Science*, 29, 69-95.

[7] Diermeier, D. and A. Merlo (1999), An Empirical Investigation of Coalitional Bargaining Procedures, *Working Paper, Department of Economics and Department of Politics, New York University.*

[8] Krusell, P. and J-V. Rios-Rull (1997), On the Size of Government: Political Economy in the Neoclassical Growth Model, *Federal Reserve Bank of Minneapolis, Research Department Staff Report 234.*

[9] Laussel, D. and M. LeBreton (1995), A General Equilibrium Theory of Firm Formation Based on Individual Unobservable Skills, *European Economic Review,* 39, 1303-1319.

[10] Meltzer, A.H. and S.F. Richard (1981), A Rational Theory of the Size of Government, *Journal of Political Economy*, 89(5), 914-27.

[11] Perotti, R. (1993), Political Equilibrium, Income Distribution, and Growth, *Review of Economic Studies*, 60(4), 755-76.

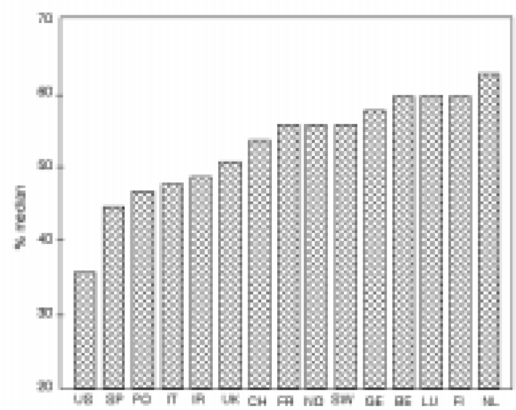
[12] Piketty, T. (1995), Social Mobility and Redistributive Politics, *Quarterly Journal of Economics*, 110(3), 551-84.

[13] Roemer, J.E. (1999), The Democratic Political Economy of Progressive Income Taxation, *Econometrica,* 67(1), 1-20.

[14] Romer, T. and H. Rosenthal (1978), Political Resource Allocation, Controlled Agendas and the Status Quo, *Public Choice,* 33, 27-43.

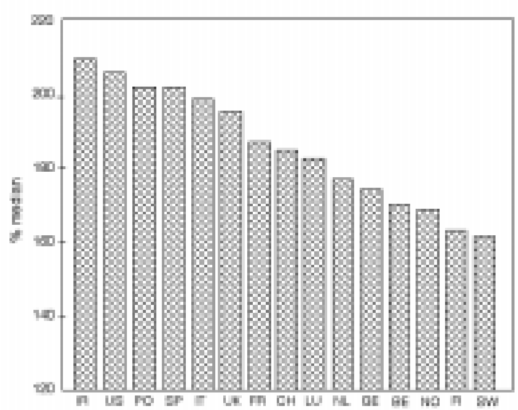
[15] Wilensky, H.L. (1975), *The Welfare State and Inequality*, Berkeley:University of California Press.

Figure 1a
Bottom decile as a percentage of the median



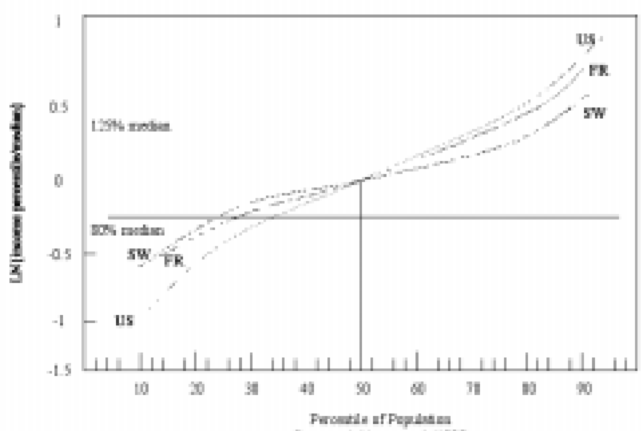
Source: Atkinson et al (1995)

Figure 1b
Top decile as a percentage of the median



Source: Atkinson et al (1995)

Figure 2: Relative Incomes at Different Percentiles



Source: Atkinson et al (1995)

Figure 3
Sorting equilibrium to 0 < t < 1

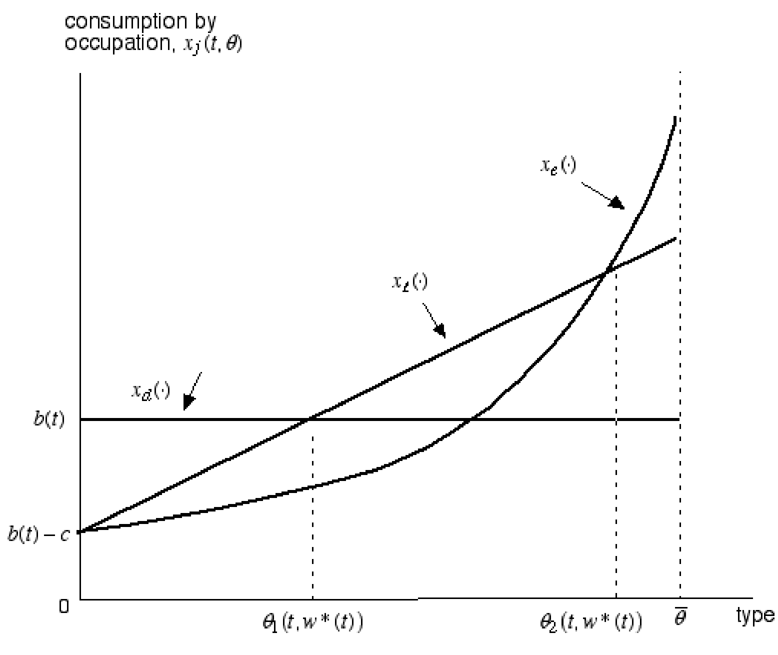


Figure 4

Some types vote against the party representing their occupation

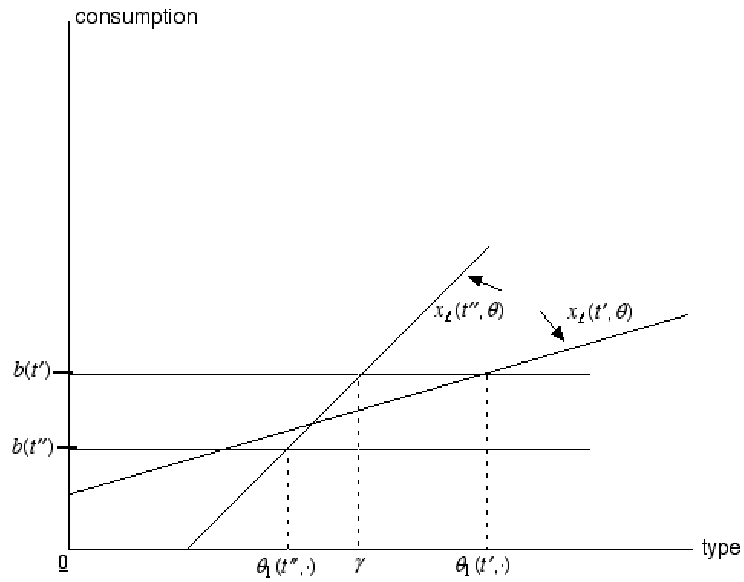


Figure 5

If $\eta > 1$, majority and proportional polities can respond differently to a change in technical productivity.