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FLEXIBLE EXCHANGE RATES, FORWARD MARKETS, AND THE LEVEL OF TRADE

by

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One of the arguments made against a system of flexible exchange rates for regulating the balance of payments of countries is that the uncertainty posed by uncertain exchange rates will reduce the level of trade. Proponents of such a system have argued that traders are able to hedge their exchange risks by using forward exchange contracts or similar covering arrangements, and thus, traders can behave as if the exchange rates were certain. While there are numerous other arguments for and against a system of flexible exchange rates, the direct effect of uncertainty on trade is central to the choice between the systems. The purpose of this paper is to investigate, in the context of a simple two-period model, the effect of uncertain exchange rates on the output and exports of firms and to determine the role of forward markets in planning output and trade. The model utilized incorporates both the production and trade decisions of firms and an incomplete set of forward exchange and securities markets in which investors may trade. The model is based upon recent work by Hayne E. Leland (1973)(1974), Peter A. Diamond, Joseph E. Stiglitz, Stern Ekern and Robert Wilson, and Roy Radner.

It will be shown that in equilibrium all investors are indifferent to the covering decisions of firms and hence that the value of all firms is independent of the covering. In addition, all firms are able to plan their production and exports on the basis of the forward exchange rates and will behave as a multiproduct, "profit" maximizing firm using the fully-covered profit as a "planning-equivalent" even though the firm may not fully cover its exchange risks. The level of trade with a flexible exchange rate system thus is identical to that which would exist with fixed exchange rates equal to the forward rates, so the uncertainty regarding exchange rates has no effect on trade.

To be able to plan output, a firm does not need any information regarding the preferences or probability assessments of its shareholders, yet all shareholders will unanimously prefer the production levels that maximize the fully-covered profit. A voting process may be constructed that will lead to the equilibrium outputs, but such a process is not necessary, since a firm can investigate the preferences for various output levels by considering a hypothetical shareholder. The analysis of the hypothetical shareholder indicates unanimity on the part of all actual shareholders thus permitting the planning of outputs and exports.

The separation of control and ownership of firms creates the possibility of alternative criteria for choices of output levels with those criteria reflecting the preferences of managers or the managers' perception of what would be in the best interests of its shareholders. Managers may believe, for example, that they have more accurate expectations regarding exchange rates than have shareholders, and they may then be thought of as using their probability assessments to maximize the expected utility of profits for some utility function and some expectations regarding exchange rates. The output levels resulting from this criterion can be shown to be identical to those resulting from maximizing fully-covered profit, and thus managers of firms who use their own preferences for profit will be acting in the best interests of the stockholders of the firm. The separation of ownership and control thus does not affect trade if the managers maximize the expected utility of profit.

Central to these results is a forward exchange market, or its equivalent, for the currency of each country to which the firm exports. If investors are able to borrow and lend foreign currencies, the existence of forward markets are not necessary. The interest-rate-parity theorem of arbitrage is essentially a statement of the redundancy of forward exchange markets when foreign currency borrowing and lending is possible. If neither forward exchange markets nor borrowing and lending is possible, the indifference and unanimity results no longer hold.

The model and the investor equilibrium are presented in the next section, and the production planning of firms and the resulting levels of trade are considered in section II. The role of forward markets is investigated in Section III, and conclusions are offered in the final section.

I. Investor Equilibrium

A. Markets and Market Participants

Individual investors have the opportunity to trade in both foreign exchange markets and capital asset markets for the shares of firms. The firms will be identified by their home country, and their shares are assumed to be traded in a capital asset market in that country. Investors will initially be permitted to purchase shares in any of the capital asset markets which are assumed to be competitive in that the market participants do not believe that their portfolio decisions affect share prices. Transactions costs are taken to be zero and short sales are permitted.

Besides investing in shares, individuals may trade in the foreign exchange markets as may firms that seek to hedge their accounts receivables or payables in the forward market. A two-period model will be utilized, so three types of exchange rates are needed. The current spot exchange rate between country n and j is denoted by r_{nj} and is measured in units of currency n per unit of currency j. A forward exchange rate is denoted by ρ_{nj}, and firms and investors may sign contracts to exchange amounts of one currency for another in period 2 at the forward rate.[1] The future spot exchange rate in period 2 is uncertain and is denoted by $\rho_{nj}(\theta)$ where θ is a state of nature reflecting the actions of other traders, such as tourists and governments, and long-term capital movements as well as exogenous events. The model analyzed here does not provide an explanation of the determination of the future spot rates (see H. G. Grubel), since not all determinants of the balance of payments are considered. The model will however indicate the relationship between spot and forward rates.

Forward exchange markets are assumed to exist for every pair of currencies, and $\rho_{nn}(\theta) = \rho_{nn} = r_{nn} = 1$ for all θ and n. Two assumptions regarding the foreign exchange markets will be made. First, arbitrage opportunities within a market yield a zero return, so that $r_{nj} = 1/r_{jn}$ and $r_{nj} = r_{n\ell} r_{\ell j}$ for all n, j, and ℓ and the same is true for the forward rates and the future spot rates in every state θ. Investors are able to ~~borrow~~ and lend in foreign currencies, so arbitrage between spot and forward markets is permitted. Second, the foreign exchange markets are assumed to be competitive in the sense that investors do not believe that their forward trading or that of any firm will affect the exchange rates. For both the foreign exchange and capital asset markets the competitiveness assumption does not seem unreasonable.

B. Firms

A firm i located[2] in country ℓ is taken to sell both in its home country and in a set of foreign countries with the decisions being made in period 1 and the proceeds from its production and financial decisions being realized in the second period. To simplify the notation, let there by $j = 1,\ldots,J$ countries[3] and assume that all foreign sales are exports from the home country. The quantity of goods sold in country j by firm i located in country ℓ is denoted by q^j_{in}, and the price[4] in that country is given by an inverse demand function $p^j_{in}(q^j_{in})$. The firm may be thought of as making in period one a contract for delivery of q^j_{in} units at a price $p^j_{in}(q^j_{in})$ in period 2. The uncertainty faced by the firm is the spot exchange rate that will obtain at the time in the future (period 2) when the firm receives payment for its exports. The firm may hedge its uncertain revenue from each country at the forward exchange rate by selling currency j against the firm's home currency at the forward rate ρ_{nj}. Letting A^j_{in} be the sale ($A^j_{in} > 0$) or purchase ($A^j_{in} < 0$) of currency j against n, the total revenue $TR_{in}(\theta)$ is

$$(1) \qquad TR_{in}(\theta) = \sum_j [\rho_{nj}(\theta)(R^j_{in} - A^j_{in}) + \rho_{nj} A^j_{in}]$$

where $R^j_{in} = p^j_{in}(q^j_{in})q^j_{in}$ is the revenue in currency j and $A^n_{in} \equiv 0$.[5] To simplify the analysis all production will be assumed to take place in the home country and all factor inputs are assumed to be purchased in the home country. The cost of production will be represented by a cost function $C_{in}(q_{in})$ where $q_{in} = (q^1_{in},\ldots,q^J_{in})$ and payment to the factors of production is made in period 2.[6,7] The profit $\pi_{in}(\theta)$ realized in period 2 in state θ is

$$(2) \qquad \pi_{in}(\theta) = TR_{in}(\theta) - C_{in}(q_{in}).$$

The profit function is assumed to be strictly concave and twice differentiable in every state.

C. Investors

Investors are assumed to seek to maximize their expected utility of period-two return by investing in shares of firms and by trading in foreign currencies subject to a budget constraint. An investor k in country ℓ has initial wealth consisting of holdings of equity of firms, margin requirements on forward exchange contracts, and savings (or borrowing) in each currency. The investor's initial share of the equity of firm i in country n, which has a market value V_{in} in currency n will be denoted by $\bar{z}^{in}_{k\ell}$, and the investor's holding of currency j will be denoted by $\bar{y}^{j}_{k\ell}$ which has an initial value in the home currency of $r_{\ell j}\ \bar{y}^{j}_{k\ell}$ with $\bar{y}^{j}_{k\ell}$ >(<) 0 representing savings (borrowing). The investor may also hold a proportion $t^{nj}_{k\ell}$ of the forward sales of currency j against currency n where $t^{nj}_{k\ell}$ >(<) 0 represents a forward purchase (sale) of currency j against n. The total forward sales A_{nj} of currency j by firms in country n is $A_{nj} \equiv \sum_i A^{j}_{in}$, and the investor is committed to supply $\sum_j \bar{t}^{nj}_{k\ell}\ \rho_{nj}\ A_{nj}$ of currency n. For n = j, $A_{nn} = 0$, so $\bar{t}^{nn}_{k\ell}$ is not a decision variable and the summation with respect to j will be understood to be for j ≠ n. The investor will be assumed to deposit a margin payment of $\beta_{k\ell}(\sum_n r_{\ell n} \sum_j \bar{t}^{nj}_{k\ell}\ \rho_{nj}\ A_{nj})/s_\ell$ for his forward purchases or sales, where s_ℓ is the gross interest rate in the investor's home country. The margin requirement $\beta_{k\ell} \in [-1,1]$ is taken to be positive (negative) if $(\sum_n r_{\ell n} \sum_j \bar{t}^{nj}_{k\ell}\ \rho_{nj}\ A_{nj})$ >(<) 0 and is assumed to earn interest at a rate s_ℓ.[8] The initial wealth $W^{o}_{k\ell}$ of the investor is thus

$$(3)\quad W^{o}_{k\ell} = \sum_i \sum_n \bar{z}^{in}_{k\ell}\ V_{in}\ r_{\ell n} + \sum_j \bar{y}^{j}_{k\ell}\ r_{\ell j} + \beta_{k\ell}\ (\sum_n r_{\ell n} \sum_j\ \bar{t}^{nj}_{k\ell}\ \rho_{nj}\ A_{nj})/s_\ell$$

Given his initial wealth, the investor reallocates his portfolio by choosing new ownership proportions $z^{in}_{k\ell}$ and new foreign currency claims and holdings $(t^{nj}_{k\ell}, y^{j}_{k\ell})$ subject to the budget constraint

$$(4)\quad W^{o}_{k\ell} = \sum_i \sum_n z^{in}_{k\ell}\ V_{in}\ r_{\ell n} + \sum_j y^{j}_{k\ell}\ r_{\ell j} + \beta_{k\ell}\ (\sum_n r_{\ell n} \sum_j t^{nj}_{k\ell}\ \rho_{nj}\ A_{nj})/s_\ell$$

The return on the portfolio is his share of the profit of the firms plus the net proceeds on the forward transactions plus the value of savings or borrowing which earn a gross interest rate s_j in country j. The gain on the forward transactions is the home currency value of the forward purchases $\sum_j \rho_{\ell j}(\theta) \sum_n t_{k\ell}^{nj} A_{nj}$ less the payment for those purchases $\sum_n \rho_{\ell n}(\theta) \sum_j \rho_{nj} t_{k\ell}^{nj} A_{nj}$ minus the margin deposit $\beta_{k\ell}(\sum_n r_{\ell n} \sum_j t_{k\ell}^{nj} \rho_{nj} A_{nj})$. The net cash inflow in period two for forward transactions may be written as

$$(5) \quad \sum_j \sum_n \rho_{\ell n}(\theta) \, (\rho_{nj}(\theta) - \rho_{nj}) \, t_{k\ell}^{nj} A_{nj} + \beta_{k\ell} \, (\sum_n r_{\ell n} \sum_j t_{k\ell}^{nj} \rho_{nj} A_{nj}).$$

The total return $X_{k\ell}(\theta)$ in currency ℓ is then

$$(6) \qquad X_{k\ell}(\theta) = \sum_i \sum_n z_{k\ell}^{in} \pi_{in}(\theta) + \sum_j y_{k\ell}^{j} s_j \rho_{\ell j}(\theta)$$

$$+ \sum_j \sum_n \rho_{\ell n}(\theta) \, (\rho_{nj}(\theta) - \rho_{nj}) t_{k\ell}^{nj} A_{nj} + \beta_{k\ell} \, (\sum_n r_{\ell n} \sum_j t_{k\ell}^{nj} \rho_{nj} A_{nj}.$$

Each investor is assumed to have preferences for period 2 return represented by a utility function $U_{k\ell}(X_{k\ell}(\theta))$ that is restricted to be strictly increasing, strictly concave and twice differentiable in $X_{k\ell}(\theta)$.[9] Each individual is assumed to assess a probability distribution on θ, and expectation with respect to the assessed probability distribution will be denoted by $E_{k\ell}$. Investors may have different assessed distribution of the states of nature, but all are assumed to agree that there is no default risk on forward contracts or on borrowing commitments. The investor thus seeks to maximize

$$E_{k\ell} \, U_{k\ell} \, (X_{k\ell}(\theta))$$

with respect to $z_{k\ell}^{ij}$, $t_{k\ell}^{nj}$ and $y_{k\ell}^{j}$ subject to the budget constraint in (5). In equilibrium the proportions for all investors are restricted by the constraints

(7) $\sum_{\ell=1}^{J} \Sigma_{k=1}^{M} z_{k\ell}^{ij} = 1, \quad j=1,\ldots,J, \; i=1,\ldots,N,$

$$\sum_{\ell=1}^{J} \Sigma_{k=1}^{M} t_{k\ell}^{nj} = 1, \quad j=1,\ldots,J, \; n=1,\ldots,J, \; n \neq j.$$

D. Investor Equilibrium

To analyze the optimal portfolio decisions of the investor, solve the budget constraint in (5) for the savings $y_{k\ell}^{\ell}$ in the home currency and substitute into $X_{k\ell}(\theta)$ in (6) to obtain

(8) $$X_{k\ell}(\theta) = \sum_{i}\sum_{n} z_{k\ell}^{in} \left(\rho_{\ell n}(\theta)\pi_{in}(\theta) - s_{\ell} r_{\ell n} V_{in}\right)$$
$$+ \sum_{\substack{j \\ j\neq\ell}} y_{k\ell}^{j} \left(s_j \; \rho_{\ell j}(\theta) - r_{\ell j} s_{\ell}\right) + s_{\ell} W_{k\ell}^{o}$$
$$+ \sum_{j}\sum_{n} \rho_{\ell n}(\theta) \; (\rho_{nj}(\theta) - \rho_{nj}) \; t_{n\ell}^{nj} A_{nj}$$

The necessary optimality conditions for the investor are

(9) $$\frac{\partial E_{k\ell} U_{k\ell}}{\partial z_{k\ell}^{in}} = E_{k\ell}\{U_{k\ell}' \cdot (\rho_{\ell n}(\theta)\pi_{in}(0) - s_{\ell} r_{\ell n} V_{in})\} = 0, \quad i=1,\ldots,N, \; n=1,\ldots,J$$

(10) $$\frac{\partial E_{k\ell} U_{k\ell}}{\partial t_{k\ell}^{nj}} = E_{k\ell}\{U_{k\ell}' \cdot (\rho_{\ell n}(\theta)(\rho_{nj}(\theta) - \rho_{nj}) A_{nj})\} = 0, \quad n=1,\ldots,N, \; j=1,\ldots,J, \; j \neq n.$$

(11) $$\frac{\partial E_{k\ell} U_{k\ell}}{\partial y_{k\ell}^{j}} = E_{k\ell}\{U_{k\ell}' \cdot (s_j \rho_{\ell j}(\theta) - r_{\ell j} s_{\ell})\} = 0, \qquad j=1,\ldots,J, \; j \neq \ell$$

where $U_{k\ell}'$ denotes the first derivative with respect to $X_{k\ell}(\theta)$ and the arguments of $U_{k\ell}$ and $U_{k\ell}'$ are understood. The optimal decision will be denoted by a (^),[10] and the optimal investor decisions as well as the equilibrium firm values and forward exchange rates will be referred to as an investor equilibrium for a given level of the decisions of the firm. The investor equilibrium will be analyzed at its <u>ex post</u> position so that $\bar{z}^{in}_{k\ell} = \hat{z}^{in}_{k\ell}$, $\bar{t}^{nj}_{k\ell} = \hat{t}^{nj}_{k\ell}$, and $\bar{y}^{j}_{k\ell} = \hat{y}^{j}_{k\ell}$ for all i, n, j, and for all investors and the conditions in (7) will be satisfied.[11]

E. Arbitrage between Spot and Forward Markets

The interest rate parity theorem[12] may be verified by rewriting $\rho_{\ell j}(\theta)$ in (11) as $\rho_{\ell n}(\theta)\ \rho_{nj}(\theta)$, solving (11) for $E_{k\ell}(U'_{k\ell}\cdot\rho_{\ell j}(\theta))$, and by substituting into (10) to obtain

$$E_{k\ell}\{U'_{k\ell}\cdot(r_{\ell j}(s_\ell/s_j) - \rho_{\ell n}(\theta)\rho_{nj})A_{nj}\} = 0$$

Then solving (11) for $E_{k\ell}(U'_{k\ell}\cdot\rho_{\ell n}(\theta))$ and substituting above, yields

$$E_{k\ell}\{U'_{k\ell}\cdot A_{nj}[r_{\ell j}(s_\ell/s_j) - \rho_{nj}r_{\ell n}(s_\ell/s_n)]\} = 0$$

The term in brackets does not involve θ and marginal utility is positive, so in equilibrium

$$r_{\ell j}/s_j - \rho_{nj}r_{\ell n}/s_n = 0$$

which may be rewritten as

$$\rho_{nj} = r_{nj}\frac{s_n}{s_j} \qquad (12)$$

since $r_{\ell j}/r_{\ell n} = r_{nj}$. The interest-rate parity theorem is thus verified and all gains due to arbitrage between spot and forward markets will be eliminated at any investor equilibrium. The arbitrage mechanism has not been introduced explicitly in this model but instead the two components of that form of arbitrage,borrowing or saving in a foreign currency and a matching forward contract, have been utilized. The significance of the interest-rate parity theorem will be considered in Section IV.

II. Firm Production and Covering Decisions

A. Processes for Firm Decision Making

The process by which firms make productive and covering decisions has yet to be specified, and that process is subject to debate with firms being hypothesized to maximize their market value, to maximize the expected utility of their shareholders, or to maximize the preferences of their managers, for example. Leland (1974), Stiglitz, and Eken and Wilson have demonstrated that value maximization does not in general result in Pareto optimal resource allocations, so attention here will be focussed on the latter two processes.[13] The maximization of the expected utility of shareholders is perhaps the most reasonable criterion for a firm, but firms do not know the preferences, probability assessments, and portfolio holdings of their shareholders. Although a firm does not know the characteristics of its shareholders, it can consider a hypothetical shareholder. Such a shareholder, denoted by $k\ell$, will necessarily satisfy the conditions in (9), (10), and (11) for some utility function, some probability assessments, and some initial wealth. The firm may then determine how its decisions affect that shareholder. Given the assumption regarding the markets, the hypothetical investor acts as a price taker with respect to security prices and exchange rates and believes that the decisions of firms do not affect exchange rates but can affect the value of the firm.

In order to investigate an investor's preference for levels of covering by the firm, a change ΔA^j_{in} in covering at an investor equilibrium $(\hat{z}^{in}_{k\ell}, \hat{t}^{nj}_{k\ell})$, i=1,...,N, n=1,...,J, j=1,...,J for the output levels q^j_{in}, i=1,...,N, n=1,...,J, j=1,...,J. Given such a change in covering by a firm, suppose that each investor alters his currency purchases as follows:

(12) $$\Delta y^{j}_{k\ell} = -(\hat{z}^{in}_{k\ell} - \hat{t}^{nj}_{k\ell})\ \Delta A^{j}_{in}/s_j$$

$$\Delta y^{n}_{k\ell} = (\hat{z}^{in}_{k\ell} - \hat{t}^{nj}_{k\ell})\ \Delta A^{j}_{in}\ \rho_{nj}/s_n$$

The return $X_{k\ell}(\theta)$ in each state θ is then unchanged by the change in the firm's covering, since the change in the return on forward contracts is exactly offset by the change in the return on foreign currency borrowing and lending. If the investor's alterations in his currency purchases are feasible, the distribution of return optimal before the change can be duplicated after the change in covering. Then no investor has an incentive to alter his shareholding or his share of the forward contracts, so the value of the firm (and all other firms) must remain the same. To show that the investor's alterations in his currency purchases are feasible, the net change in purchases in (12) is

$$r_{\ell j}\ \Delta y^{j}_{k\ell} + r_{\ell n}\ \Delta y^{n}_{k\ell} = -(\hat{z}^{in}_{k\ell} - t^{nj}_{k\ell})\ \Delta A^{j}_{in}(r_{\ell j}/s_j - \rho_{nj}\ r_{\ell n}/s_n)$$

The arbitrage relationship $\rho_{nj} = r_{nj}\ s_n/s_j$ then implies that this term is zero. This holds for all investors at all output levels for firms, so the investor's portfolio does not change except for the above alterations in currency purchases. The investor is thus able to exactly offset or duplicate any covering of a firm. Consequently, the value of a firm is independent of its covering, and all investors are indifferent to the covering of any firm. This homemade covering result is stated as Proposition 1.

Proposition 1: At any investor equilibrium and any output levels of the firm, all investors are indifferent to the covering decisions of any firm, and hence the value of any firm is independent of the covering by any firm.[14]

An investor is not able to duplicate the return possibilities generated by the productive activities of firms, and hence, investors are not indifferent to the production levels of firms. The preferences for the level of q^j_{in} for a hypothetical investor may be determined by differentiating his optimal expected utility with respect to q^j_{in} to obtain

$$(13)\quad \frac{\partial E_{k\ell}U_{k\ell}}{\partial q^j_{in}} = E_{k\ell}\left\{U'_{k\ell}\cdot\left[\sum_m\sum_\gamma\left(\bar{z}^{\gamma m}_{k\ell} - \hat{z}^{\gamma m}_{k\ell}\right)\; s_\ell r_{\ell j}\;\frac{\partial V_{\gamma m}}{\partial q^j_{in}}\right.\right.$$

$$+\sum_m\sum_\gamma\left(\pi_{\gamma m}(\theta)\rho_{\ell m}(\theta) - s_\ell r_{\ell m}V_{\gamma m}\right)\frac{\partial \hat{z}^{\gamma m}_{k\ell}}{\partial q^j_{in}}$$

$$+\sum_m\sum_{\substack{\gamma\\ \gamma\neq m}} A_{\gamma m}\;\rho_{\ell\gamma}(\theta)\;\left(\rho_{\gamma m}(\theta)-\rho_{\gamma m}\right)\frac{\partial \hat{t}^{\gamma m}_{k\ell}}{\partial q^j_{in}}$$

$$+\sum_m\left(s_m\rho_{\ell m}(\theta) - s_\ell r_{\ell m}\right)\frac{\partial \hat{y}^m_{k\ell}}{\partial q^j_{in}}$$

$$\left.\left.+\;\rho_{\ell n}(\theta)\;\left(\rho_{nj}(\theta)\;MR^j_{in} - MC^j_{in}\right)\;\hat{z}^{in}_{k\ell}\right]\right\}$$

where $MR^j_{in} = p^{j'}_{in}(q^j_{in})q^j_{in} + p^j_{in}(q^j_{in})$ is marginal revenue in currency j and $MC^j_{in} = \partial C_{in}(q_{in})/\partial q^j_{in}$ is marginal cost. At an investor equilibrium $\bar{z}^{\gamma n}_{k\ell} = \hat{z}^{\gamma m}_{k\ell}$, so the first term in (13) is zero. The second, third, and fourth terms are zero from (9), (10), and (11), respectively, so (13) reduces to

(14) $$\frac{\partial E_{k\ell} U_{k\ell}}{\partial q^j_{in}} = E_{k\ell} \{U'_{k\ell} \cdot \rho_{\ell n}(\theta) \; (\rho_{nj}(\theta) \; MR^j_{in} - MC^j_{in})\} \; \hat{z}^{in}_{k\ell}, \; j=1,\ldots,J.$$

Solving for $E_{k\ell}\{U'_{k\ell} \cdot \rho_{\ell j}(\theta)\}$ and $E_{k\ell}\{U'_{k\ell} \cdot \rho_{\ell n}(\theta)\}$ from (11) and substituting into (14) yields

(15) $$\frac{\partial E_{k\ell} U_{k\ell}}{\partial q^j_{in}} = E_{k\ell} \{U'_{k\ell}\} \; \rho_{\ell n} \; (\rho_{nj} \; MR^j_{in} - MC^j_{in}) \; \hat{z}^{in}_{k\ell}, \; j=1,\ldots,J.$$

If the term in (15) is positive (negative) for a shareholder (an investor with $\hat{z}^{in}_{k\ell} > 0$), the shareholder prefers a small increase (decrease) in q^j_{in}. Since the term in parentheses does not involve any future spot exchange rate, the sign of (15) is the same for all shareholders. The expected utility of all shareholders will thus be maximized by the firm's output levels when the terms in (15) are zero or when

(16) $$\rho_{nj} \; MR^j_{in} - MC^j_{in} = 0 \quad j=1,\ldots,J.$$

The firm may thus act as a multiproduct firm using the forward exchange rates as planning equivalents, and in so doing will maximize the expected utility of all shareholders. This establishes the following stockholder unaminity theorem (see Leland (1973)(1974)).

Proposition 2: All shareholders have unanimous preferences for (small) changes in the output levels of firms, and the firm may maximize its shareholders' expected utility by acting as a multiproduct, profit maximizing firm using the forward exchange rates as planning equivalents.[15] The output levels are Pareto optimal.[16]

This proposition obtains because investors are able to offset any exchange rate risk created by the exports of the firms by trading in forward exchange markets and/or by borrowing and lending in foreign currencies. For example, if the firm has export revenue R^j_{in} in currency j, the investor can offset the exchange rate risk by borrowing a quantity $R^j_{in}\ \hat{z}^{in}_{k\ell}$ of currency j to cover the risk generated by exports. Proposition 1 reflects this homemade covering effect and thus implies that the firm can ignore the exchange rate uncertainty in planning its operations and use the forward rate as a "planning equivalent."[17]

Proposition 1 also supports the position of E. Sohmen that there is no "risk premium" associated with the uncertain exchange rate. Any divergence between the planning equivalent and the present spot exchange rate may be completely explained by interest rate differentials. If investors have expectations regarding the future spot rate that differ from the forward rate, they will adjust their portfolios accordingly, but the output of the firms will be unaffected.

The process by which the firm may determine the optimal levels of its outputs may be thought of as a voting process in which the firm submits its planned output levels to its shareholders for ratification. Given any output plan, the shareholders will be unanimous with respect to their preferences for small changes in the plan, and the process will terminate when (16) is satisfied.[18] Such a process is not necessary however because a firm can consider any hypothetical shareholder and can predict the response of that shareholder and hence all shareholders. To do so, the firm does not need to know anything about shareholders except that they maximize the expected utility of their portfolio return. The determination of the optimal output levels thus is an efficient process and respects the privacy of investors.

An alternative method of determining output levels would be to explicitly recognize the separation between the owners and the managers of firms. The managers of firms may have objectives reflecting their own preferences and probability assessments, and even if they do seek to act in the best interests of its shareholders, the management may seek to do so by accepting a criterion such as maximizing the expected utility of profit. The criterion of maximizing the expected utility of profit for some utility function and some probability assessments has been frequently used in the literature[19] as a surrogate for the maximization of the expected utility of investor's return, and if firms are not assumed to have performed the previous analysis, such a criterion might seem reasonable. Furthermore, the management of firms might believe that it has more accurate knowledge of the likelihood of various exchange rate levels obtaining, for example, and thus might wish to use its own probability assessments.

Suppose that a firm (in) has chosen a strictly concave utility function $\mathcal{U}_{in}$ and a probability distribution over the states of nature and seeks to maximize

$$E_{in}\,\mathcal{U}_{in}\,(\pi_{in}(\theta))$$

with respect to (q^j_{in}, A^j_{in}). The necessary optimality conditions are[20]

$$(17)\quad \frac{\partial E_{in}\mathcal{U}_{in}}{\partial q^j_{in}} = E_{in}\{\mathcal{U}'_{in}\cdot(\rho_{nj}(\theta)\, MR^j_{in} - MC^j_{in})\} = 0, \quad j=1,\ldots,J$$

(18) $\frac{\partial E_{in}U_{in}}{\partial q_{in}^{j}} = E_{in}\{\mathcal{U}'_{in}\cdot(\rho_{nj}-\rho_{nj}(\theta))\} = 0, \quad j=1,\ldots,J, \; j\neq n.$

Solving (18) for $E_{in}\{\mathcal{U}'_{in}\cdot\rho_{nj}(\theta)\}$ and substituting into (17) yields

$$E_{in}\{\mathcal{U}'_{in}\}\cdot(\rho_{nj}\cdot MR_{in}^{j}-MC_{in}^{j}) = 0, \quad j=1,\ldots,J.$$

Since expected marginal utility is positive, the term in parentheses must be zero and is identical to (16).[21] Consequently, a firm that maximizes the expected utility of profit will produce such that all shareholders prefer those output levels and none would urge the firm to make changes in those levels. This result holds for any utility function and probability assessments used by the firm and is again a form of a homemade covering result. The firm may compensate for the exchange rate risk created by exportation by selling (or buying) forward foreign currencies, and the amount of such forward transactions will depend on expectations and the utility function used. Since, however, investors are indifferent to the forward exchange transactions of firms, the dependence of the covering decisions on the expectations and preferences of management is unimportant.

Consequently, the firm may act as a maximizer of expected utility of profit or equivalently may act to directly maximize the expected utility of a shareholder, and the resulting output levels will be Pareto optimal and will maximize the expected utility of all shareholders. Both of these processes are efficient in the sense that they require no communication between firms and investors, and both respect the privacy of investors. The equivalence of the two processes in terms of the output levels of firms results because of the opportunity to trade foreign currency risks in a market. While the level of such covering by firms may differ for the two processes, the value of a firm will be identical, since that value depends on the real productive decision of the firm and not on the covering of the exchange rate risks as indicated in Proposition 1.

B. Firm Valuation

The value of a firm may be determined by eliminating the future spot rates from (9) by using (10) and (11) to obtain

$$E_{k\ell}\{U'_{k\ell}\}\ [\rho_{\ell n}(\sum_j \rho_{nj}\ R^j_{in} - C_{in}(\hat{q}_{in})) - s_\ell r_{\ell n} V_{in}] = 0, \tag{19}$$

where $\hat{q}_{in}$ is the vector of outputs satisfying (16). The term in brackets does not involve θ, so the value of the firm is

$$V_{in} = (\sum_j \rho_{nj}\ R^j_{in} - C_{in}(\hat{q}_{in}))/s_n. \tag{20}$$

The value of the firm is thus the fully-covered profit of the firm discounted at the risk-free rate of interest and is independent of the level of covering as previously indicated. The value of the firm does depend on the forward exchange rate as do the output levels, and the production planning condition in (16) may be used to eliminate the forward exchange rates from (20) yielding

$$V_{in} = \sum_j R^j_{in}\ (MC^j_{in}/MR^j_{in}) - C_{in}(\hat{q}_{in}))/s_n. \tag{21}$$

The value of the firm is determined completely by its revenue and cost functions. If, for example, the firm sells in only one country j and the market in that country is competitive, $R^j_{in}/MR^j_{in} = \hat{q}^j_{in}$ and the value of the firm is

$$V_{in} = \hat{q}^j_{in} \ (MC^j_{in} - AC_{in})/s_n$$

where $AC_{in} = C_{in}(\hat{q}^j_{in})/\hat{q}^j_{in}$ is average cost. The value of the firm will be positive if the marginal cost exceeds average cost at the optimal output, which is the usual valuation rule for a competitive firm facing a price $p^j_{in} = MC^j_{in}$.

C. Comparative Statics of Trade Policies

Within the context of the model a variety of governmental policies can be analyzed. For example, a tariff T_j imposed by country j on imports reduces the revenue of an importer to $(p^j_{in} \ (q^j_{in}) - T_j)q^j_{in}$ and the marginal revenue to $(MR^j_{in}-T_j)$. Such a tariff will reduce the imports of that country ik the same manner as in comparative statics results. Such a tariff can of course be offset by the exporting country with a per unit subsidy of $\tau_j \ \rho_{nj}$ from company n. A lump-sum subsidy or import license fee does not affect the quantity of imports or exports directly but does reduce the value of the firm by the amount of the fee. If the cost of the import license is great enough, a firm will not export. Similarly, the exports of a firm are inde-ependent of the profits tax rate in the firm's home country.

III. The Role of Forward Markets

With the results of Propositions 2, one can compare fixed and flexible exchange rate systems. A fixed exchange rate may be considered as a special case of a flexible exchnage rate system in which all market participants have degenerate probability assessments with $\rho_{\ell n}(\theta) = \rho_{\ell n} = r_{\ell n}(s_\ell/s_n)$ for all ℓ and n. The outputs resulting with a fixed exchange rate system will be exactly those obtaining with a flexible exchange rate system, and thus the "fully-covered" level of foreign exchange earned under both systems would be identical. Consequently, in the context of this model the two systems may be considered to be equivalent. This equivalence is insured by the existence of forward exchange rates or by the opportunity to borrow and lend in foreign currencies. Either permits the firm to adopt one of the processes in Section III and hence to use the forward rate for planning purposes.

These results are valid for divirgent expectations regarding the future spot rate as well as for divergent risk preferences indicating that expectations and risk preferences do not affect output. Investor expectations and risk preferences affect the portfolio decisions of investors but not those of firms. The role of forward markets in taking into account these divergent investor characteristics may be seen by writing (11) as

$$(22) \qquad E_{k\ell}(\rho_{\ell j}(\theta)) - s_\ell r_{\ell j}/s_j = - \mathrm{cov}_{k\ell}(U'_{k\ell}, \rho_{\ell j}(\theta))/E_{k\ell}U'_{k\ell}$$

where $\mathrm{cov}_{k\ell}$ is the covariance of $U'_{k\ell}$ and $\rho_{\ell j}(\theta)$. The left side of (22) is the difference between the expected future spot rate and the actual forward rate. If the investor believes that the expected future spot rate will be greater (less) than the forward rate, the investor will adjust his portfolio so that the covariance between marginal utility and the future spot rate will be negative (positive). For example, if the left-side of (22) is positive, the investor will choose a portfolio that has positive net claims to currency j through share ownership, savings, and forward sales. Rewriting (21) as follows

$$E_{k\ell}(\rho_{\ell j}(\theta)) + \mathrm{cov}_{k\ell}(U'_{k\ell}, \rho_{\ell j}(\theta))/E_{k\ell}U'_{k\ell} = \rho_{\ell j}$$

indicates that all investors in country ℓ will adjust their portfolios so that the expected future spot rate plus the normalized (by $E_{k\ell}U'_{k\ell}$) covariance is equal to the forward rate.

A similar interpretation may be given to (9) which may be written as

$$(23)\quad E_{k\ell}(\rho_{\ell n}(\theta)\ \pi_{in}(\theta)) - r_{\ell n}V_{in}s_{\ell} = \mathrm{cov}_{k\ell}(U'_{k\ell}, \rho_{\ell n}(\theta)\ \pi_{in}(\theta))/E_{k\ell}U'_{k\ell}\ .$$

The left-side of (23) is the difference between expected profit and the (opportunity) cost of purchasing the firm, and if this difference is positive (negative), the investor will arrange his portfolio so that the covariance between the home currency values of the profits and marginal utility is positive.[22] For example, if the portfolio is composed only of savings in country ℓ and shares of one firm that sells in only one country $j \neq \ell$, the investor will hold (sell short) shares of the firm if his expectation of profit is greater (less) than the opportunity cost. In equilibrium, expected profit plus the normalized covariance must be the same for all investors.

In order to further investigate the role of forward exchange markets, a variety of assumptions regarding the existence and structure of such markets will be considered. Initially, assume that forward markets do not exist, but that investors are able to borrow and lend in foreign currencies and that firms may borrow and lend in foreign currencies. Then, it may be demonstrated that all shareholders are indifferent to the firm's borrowing and lending, since investors can again exactly duplicate the covering actions of the firms. Similarly, the unanimity results regarding production decisions are unchanged by the absence of forward markets.

The forward exchange market and the borrowing and lending markets in foreign currencies are thus redundant in the sense that if only one existed the resulting equilibrum would be the same.[23] The interest rate parity theorem is essentially a statement of this redundancy. If a country adopted a law prohibiting an investor from borrowing or lending in foreign markets, exports would be unaltered. Consequently, for countries whose currencies are not traded in forward markets the unanimity results for production planning still result as long as borrowing and lending in that currency is permitted. If a forward market does not exist and borrowing and lending is prohibited, unanimity does not result in general.[24]

In addition to market segmentation resulting from the actions of governments and the absence of forward markets and their equivalents, investors themselves may create a <u>de facto</u> form of market segmentation. While the investors who trade in foreign currencies are also likely to trade in the secutiries of firms, there are many investors who are willing to invest in securities of firms but who do not consider trading in foreign currencies. The indifference and unanimity results still obtain for the shareholders in the former group but not those in the latter group. For the latter group the sign of the term in (14), for example, depends on the preferences and probability assessments of an investor and hence production planning is complicated when the unanimity results do not hold.

As an example that such investor segmentation does not necessarily affect output, consider a firm in country ℓ that sells only in country $j \neq \ell$. The conditions in (9) and (14) for an investor in country ℓ who does not consider forward transactions or foreign borrowing or lending are

$$(9a) \qquad E_{k\ell}\{U'_{k\ell} \cdot [\rho_{\ell j}(\theta)\ R^j_{i\ell} - C_{i\ell} - s_\ell V_{i\ell}]\} = 0$$

$$(15a) \qquad \frac{\partial E_{k\ell} U'_{k\ell}}{\partial q^j_{i\ell}} = E_{k\ell}\{U_{k\ell}\}\ [(MR^j_{i\ell}/R^j_{i\ell})\ (C_{i\ell} + s_\ell V_{i\ell}) - MC^j_{i\ell}]\ \hat{z}^{i\ell}_{k\ell}\ .$$

The term in brackets in (15a) is independent of θ, and all such investors will work to have this term equal zero. All such investors are unanimous with respect to their output preferences, but all investors who consider forward transactions or forward borrowing or lending may unanimously prefer a different level of output. If the output satisfying (16) is produced, then (15a) is zero and all shareholders have their optimal expected utility.

Another type of market segmentation will exist if investors in one country do not consider trading in the security markets in other countries, so that the investor equilibrium conditions in (9) hold only for $j = \ell$. If those investors trade in foreign currencies, all investors are again indifferent to the covering decisions of firms and all shareholders in a firm are unanimous with respect to their preferences for input-output changes. The value of the firms and the forward exchange rates are also unchanged. The only essential effect of such segmentation is that these results must be developed by looking at the secutiries market in each country instead of a single market. If the investors also do not trade in foreign currencies, then the unanimity and indifference results do not obtain.

IV. Conclusions

Forward exchange markets permit investors to perform their own covering of the uncertain returns from share ownership, and consequently, firms are able to plan their outputs on the basis of a fully covered marginal revenue. These results must be qualified to the extent to which there are imperfections in the markets such as a risk that forward contracts will not be executed, if interest rates are different for borrowing and lending, if the interest paid on margin deposits does not equal the risk-free rate, or if there is a default risk on individual portfolios. The results also do not hold for additional forms of uncertainty that cannot be offset in the securities and forward markets. In the absence of such imperfections the forward markets duplicate the borrowing and lending markets as indicated by the interest rate parity theorem. Either of these markets permits firms to plan outputs so as to maximize the expected utility of its shareholders without knowing any characteristics of its shareholders.

Footnotes

1. Other forms of forward exchange transactions, such as swaps, will not be considered, since they simply duplicate the forward exchange markets.

2. The term "located" is intended to indicate that the firm converts its revenue into the currency of that country and distributes its profits to its investors in that currency.

3. To simplify the notation, each firm will be assumed to trade in each country, and the number of firms N and investors M in each country will be taken to be the same.

4. All firms are assumed to price in terms of the currency of the foreign country. An alternative strategy, not considered here, would be to price only in the home currency thus strifting the currency risk to the customers of the firm.

5. Firms will not be considered to perform arbitrage operations or to deal in the forward markets not involving their country. Firms are also assumed to not borrow and lend in foreign currencies. The significance of this assumption will be considered in Section IV.

6. The model may readily be generalized to the case in which a firm has production units in a foreign country. For example, if firm i has a production unit in a country j and $C^j_{in}(q^j_{in})$ is the cost for that unit, then the profit from country j in terms of the currency of the home country is

$$\rho_{nj}(\theta)\ (R^j_{in} - C^j_{in}(q^j_{in}) - A^j_{in}) + \rho_{nj}\ A^j_{in}\ .$$

The results presented below also hold for this case.

7. The model may be easily generalized to the case in which the firm has a production function $F_{in}(q_{in}, b_{in}) = 0$ where b_{in} is a vector of factor inputs. Then extending the model to the case in which the factor inputs are imported does not alter the results.

8. A margin requirement is assumed to be placed only on individuals and not on firms although the results would be the same if a margin were required for all forward trades.

9. The utility function may be state dependent and may be an indirect utility function derived from optimal consumption decisions after θ has occurred in period 2.

10. The condition in (10) for $t^{nj}_{k\ell}$ is equivalent to that for $t^{jk}_{k\ell}$ as can be seen by letting $\rho_{nj} = 1/\rho_{jn}$ and noting that $\rho_{\ell n}(\theta)\ (\rho_{nj}(\theta) - \rho_{nj}) = (1/\rho_{jn})(\rho_{\ell j}(\theta)\ \rho_{jn} - \rho_{\ell n}(\theta))$.

Consequently, an optimal solution can be taken to have $t^{nj}_{k\ell} - t^{jn}_{k\ell}$ as the investor's share of the net forward sales of currency j against n.

11. The equilibrium conditions for investors involve NJ + J(J-1)(the $z^{ij}_{k\ell}$, $t^{nj}_{k\ell}$, and $Y^j_{k\ell}$) decision variables for each investor plus NJ values of the firms and the J(J-1) values of the forward exchange rates for a total of M(NJ + J(J-1)+J + J(J-1)+NJ. The equilibrium is determined by the M(NJ + J + J(J-1)) equations in (9), (10), and (11) plus the MJ budget constraints plus the NJ + J(J-1) constraints

in (7). The J(J-1) supply and demand conditions for the forward exchange for each pair of countries also must be satisfied giving a total of M(NJ + J(J-1) + J) + NJ + J(J-1) + J(J-1) equations. The supply-demand condition for the forward exchange of country j with country n is $A_{nj} = \sum_k \sum_\ell \hat{t}^{nj}_{k\ell} A_{nj}$, but simce $\sum_k \sum_\ell t^{nj}_{k\ell} = 1$ from (7), each of the supply-demand conditions is implied leaving M(NJ + J(J-1) + J) + NJ + J(J-1) equations. As indicated in footnote 10 only J(J-1)/2 of the equations in (10) are independent and J(J-1)/2 of the $\hat{t}^{nj}_{k\ell}$ may be eliminated. As indicated in Section III, the remaining conditions in (10) are implied by (11).

12. The interest rate parity theorem is discussed in detail by S. C. Tsiang.

13. The criterion of value maximization also does not lead to optimal output levels that are independent of investor characteristics as do the other two criteria.

14. The same result obtains if firms may borrow and lend in foreign currencies, which justifies leaving such possibilities out of the model.

15. Proposition 2 may be referred to as an <u>ex post</u> unanimity results (see Leland(1973)(1974),Ekern and Wilson, and Radner) which obtains because for each state θ the marginal returns with respect to a firm's decisions may be expressed as a linear combination of the returns on the foreign currency in that state. The unanimity result is <u>ex post</u> in the sense that it holds at the optimal investment levels for investors. A necessary condition for this

result is that the securities markets and the foreign exchange markets are competitive. The competitive assumption for securities markets is standard and simply requires that investors believe that their portfolio decisions do not affect the values of firms. Competitive foreign exchange markets means that investors believe that neither their foreign exchange trading nor the exports and covering by an individual firm affects the exchange rates. Proposition 1 is stronger than Proposition 2 because of the existence of a forward market or foreign currency borrowing and lending and that proposition may be viewed as a general result on investor indifference to the decisions of firms in forward markets. By rearranging his portfolio, an investor may offset any decision in the forward market made by any firm and thus is indifferent to the covering. Ex ante unaminity results (Leland (1973)) for output levels may be obtained if the additional assumption is made that investors believe that the value of firms is independent of output, but such an assumption does not seem to be warranted.

16 To demonstrate that the output levels are Pareto optimal, consider the problem of maximizing the expected utility of one investor subject to constrained levels of expected utility for other investors. Thus, for investor $k\ell$ the program is

$$\max E_{k\ell}\, U_{k\ell}(X_{k\ell}(\theta))$$

$$\text{subject to } E_{\alpha\beta}\, U_{\alpha\beta}\, (X_{\alpha\beta}(\theta)) = U^{o}_{\alpha\beta}, \quad \alpha\beta \neq k\ell$$

where $U^{o}_{\alpha\beta}$ is a fixed level of expected utility. The necessary condition for Pareto optimality for q^{j}_{in}, for example is

$$E_{k\ell}\left\{U'_{k\ell}\cdot\frac{\partial X_{k\ell}(\theta)}{\partial q^j_{in}}\right\} - \sum_{\alpha\beta\neq k\ell} \lambda^{\alpha\beta}_{k\ell}\, E_{\alpha\beta}\left\{U'_{\alpha\beta}\cdot\frac{\partial X_{\alpha\beta}(\theta)}{\partial q^j_{in}}\right\} = 0$$

where $\lambda^{\alpha\beta}_{k\ell}$ is a multiplier corresponding to the constrained expected utility of investor $\alpha\beta$. From (13), (15), (9), (10), and (11), this condition is satisfied for every investor.

17. Production may thus be planned independently of any security market considerations. The results of Leland (1974) indicate that output will in general be a function of the value of the firm. In the model considered here, the output levels are independent of the value of the firm, (except that the forward exchange rates and firm values are determined in equilibrium) because of the existence of the forward exchange market or equivalently the possibility of borrowing or lending in foreign currencies.

18. For,example, a gradient process would converge to the optimal output levels.

19. See D. P. Baron (1970) and Leland (1972), for example.

20. The optimal output levels are assumed to be positive.

21. The expression in (23) may be reduced to (20) by writing the covariance as the sum of the covariances of the individual terms and then substituting for those covariances from (22) and from (10).

22. This result has also been derived by W. Ethier.

23. This may also be seen by noting that the condition in (11) for ℓn and nj imply (10) for jn.

24. If a firm exports to only one country, then (9) may be used in conjunction with (13) to demonstrate unanimity as in (9a), (14a), and (15a) below.

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