Discussion Paper No. 1268

Enforceable Contracts under Generalized Information of the Court

by
Francesco Squintani
Northwestern University*

August 1999

http://www.kellogg.nwu.edu/research/math

Abstract

Bernheim and Whinston (1997) (henceforth BW) formalize court's verifiability as a correspondence mapping actually played actions into events (i.e. sets of actions) verified by the court. Their normal-form analysis restricts attention to partitional product correspondences. They define any element in the partition a "complete" enforceable contract. After motivating the discussion of non-partitional and non-product correspondences by means of simple examples, we show that the BW approach may fail to capture all feasible outcomes for product non-partitional correspondences, and that is valid against all partitional non-product ones only if one allows for a joint liability regime. Even in the case of joint liability regimes, the BW approach may be extended only to deal with non-product *or* non-partitional correspondences. Therefore, a definition of enforceable contract that is independent of the players' payoffs may not capture all feasible outcomes.

---

* Department of Economics, 2003 Sheridan Rd. Evanston, IL, 60208-2009.
E-mail: squinzio@nwu.edu. I thank Eddie Dekel, Steven Matthews, Juuso Valimaki, and Joel Watson. The usual caveat applies.

1 Introduction

Before playing a game, players can sign a contract to rule out some actions. In order to punish a player violating the contract, the court must be able to verify the actions taken. The court's information structure may be formalized as a correspondence mapping each action profile actually played into an event (i.e. a set of action profiles) verified by the court. If the correspondence maps some action profiles to non-singleton sets, then verifiability is imperfect, and some contracts are not enforceable.

Bernheim and Whinston 1997 (henceforth BW) consider partitional and product information structures. They define any set in the partition as a "complete" (enforceable) contract, and derive all enforceable outcomes by calculating all the equilibria of the game restricted to each of these contracts. A nice property of their approach is that the definition of enforceable and complete contract is function of legal and physical characteristics of the game only, and is independent of the players' private motives (i.e. the payoff functions).

In fact, one would like to separate the judicial aspect of the problem, represented by the court's information, from the private incentives aspect represented by the players' payoffs. Given an economic interaction, the information structure may be determined by a legal scholar, and may be modified by a legislator. The private motives, instead, depend on the particular players involved in the interaction, and may not always be observed by an external party. In particular, a legal scholar would like to have a definition of enforceable contract that does not depend on the players' private motives, and that is only function of observable legal and physical characteristics of the scenario.

After showing simple economic examples in which the court's information structure is not partitional or product, we test whether in these cases the BW approach still captures all outcomes that may be supported signing a suitable contract. While the BW procedure may fail against a non-partitional correspondence, it is still valid against non-product correspondences if one allows for a joint liability regime: upon verifying a violation, the court may punish all players who are not able to show that they complied with the contract.

We therefore consider joint liability first. After extending the BW procedure to derive all feasible outcomes against all non-partitional (product) correspondences, we unexpectedly find out that the extension fails against some non-product, non-partitional information structures. In such environments, a simple example shows that a principle that defines enforceable contracts independently of the payoff functions may not capture all feasible outcomes.

We then consider a regime that allows only for individual liability. While the original BW approach may select solutions that are in fact unfeasible (and is thus too fine), the extension that accounts for non-partitional information may fail to capture all feasible outcomes (and is thus too coarse). That failure occurs even in environments characterized by non-product, partitional information structures.

In the second section we review the BW model for normal form games. In the third section we generalize court information. In the fourth section, we analyze the model for joint liability, and in the fifth section for individual liability.

2 Review of BW Normal-Form Model

Consider a normal-form game $G = (I, A, u)$, where I is the set of players, A is the finite action space, and u the utility functions. Before choosing actions in A, the players can sign a contract $C \in 2^A$ (where 2^A is the set of all the subsets of A), so as to forbid the players to take any action profile $\hat{a} \notin C$. Say that each player is unlimitedly liable for breaking the contract: if the court can verify that she violated the contract, it will punish her in an arbitrarily harsh manner.

Represent the court's information structure by the correspondence $P : A \rightarrow 2^A$; if the players played profile a, the court cannot distinguish it from any other action profile contained in $P(a)$. The correspondence P is assumed to be partitional, product, and truthful. Formally, P is *partitional* if the range of P is a partition of A, P is *product* if each player is verified independently of the others (i.e. $\exists(P_1, \cdots, P_I)$ with $P_i : A_i \rightarrow 2^{A_i}$ such that $\forall a \in A, P(a) = \times_{i=1}^{I} P_i(a_i)$), and P is *truthful* if $\forall a \in A, a \in P(a)$. The court cannot punish the violator unless it concludes that the violation has occurred,[1] thus some contracts may be ineffective at constraining the choice of the players, as they rule out actions that cannot be verified by the court.

Specifically, assume that the players convene to play an action profile a and sign the contract C such that $a \in C$. Player i deviates from the profile a, by taking the action b_i forbidden by the contract $C : (b_i, a_{-i}) \notin C$. Say that b_i a *verifiable violation* of (C, a) if $P(b_i, a_{-i}) \cap C = \emptyset$. If the profile (b_i, a_{-i}) is played, and $P(b_i, a_{-i}) \cap C \neq \emptyset$, then the court cannot rule out that an action allowed by the contract C was played instead, and will not punish player i. Moreover, the contract C may not always be enforced, as it prescribes i not to play b_i when her opponents play a_{-i}, and yet, when i plays b_i, she will not be punished.

When interacting in an environment described by the game G and the court information P, the players proceed as follows. Before playing the game G, they may agree to sign an (enforceable) contract C^*, and propose to coordinate on an action profile $a^* \in C^*$. Each player i knows that if she takes a verifiable violation b_i, she will be punished and her utility will be $-\infty$ instead of $u_i(b_i, a^*_{-i})$. Formally, define $u_i|_{C^*}(b_i, a^*_{-i}) = -\infty$ if b_i is a verifiable violation of (C^*, a^*), and $u_i|_{C^*}(b_i, a_{-i}) = u_i(b_i, a^*_{-i})$ otherwise. While playing $G|_{C^*}$ (the game G under the contract C^*), the player i will deviate from a^* if and only if that strictly increases her utility $u_i|_{C^*}$.

BW propose a simple procedure to find a solution for this environment.

Principle 1 (Berhneim and Whinston 1997) *First consider the court's information structure P : each of the elements $P(a)$ in the range of P is said to be an enforceable complete contract C^*. Secondly, derive the games $G|_{C^*} := (I, C^*, u|_{C^*})$ by taking the game G and restricting u to each complete contract C^*, and calculate the pure strategy equilibria a^* of each game $G|_{C^*}$.*

The traditional definition of complete contracts is that a single action is prescribed to

[1] BW model does not require the court to punish a violator only if it assesses that the violation has occurred with probability 1. In fact, the correspondence P may be derived also from p-belief operators (see Monderer and Samet 1989), where p is arbitrary. The court concludes that a violation has occurred whenever it assesses that probability larger than p.

each player in each state of the world. Such definition allows for non-enforceable contracts and motivates incomplete contracts on the basis of imperfect verifiability. BW reinterpret a contract as "complete" when it is enforceable and makes use of the court's verification power in a complete manner.

3 Generalized Court Information

Non-partitional information structures occur mainly because some evidence may be conclusive to prove a claim, whereas the contrary evidence may not be conclusive to prove the contrary claim. Consider a simple revisitation of Example 2 in Okuno-Fujiwara, Postlewaite and Suzumura (1990). Two oligopolists may or may not invest to reduce marginal costs. It is very simple to verify in court that the marginal cost is low, for instance by running the production line very fast. Running the line slowly however does not demonstrate that it cannot run faster, and thus that the costs of production are high. Formally $P(H) = \{H, L\}$ and $P(L) = L$.

It is straightforward to see that, in order to analyze non-partitional information, we cannot use Principle 1. In the above example, if an oligopolist plays L the court will be able to verify that she played L, but if an oligopolist plays H, the court will not be able to verify that H has been played, but it will conclude that H or L has been played. The only enforceable contracts are $\{H\}$ and $\{H, L\}$. Yet, the court's partition is $P(L) = \{L\}$ and $P(H) = \{H, L\}$: according to Principle 1, $\{L\}$ would be an enforceable complete contract. However, if an oligopolist prefers to play H, when her opponent plays L, there is no way to achieve the outcome (L, L). Note also that the minimal enforceable contract that allows action H is $\{H\}$ and the minimal enforceable contract that allows action L is $\{H, L\}$. If one accepts the BW approach, she obtains that the list of complete contracts need not be a partition and may be nested.

Non-product information structures arise when the court cannot verify the action taken by each player independently. For example, in many partnership problems, the court can only verify whether the partnership achieved the result for which it was formed or whether it did not. In the latter case, it cannot tell which partner did not cooperate.

Whether the partnership contract is enforceable or not thus depends on the regime of liability. If each player is only liable for her own actions (individual liability), she may not be punished when violating the partnership contract, which is therefore not enforceable.

However, if the contract is signed under a regime of *joint liability*, when a contract violation is verified, the court will punish *all* partners who are unable to show their innocence. This feature may make the partnership contract enforceable. Consider for example a group of farmers signing a partnership contract to jointly produce high quality grocery goods. If any of partners violate a health regulation, after the contaminated groceries are sold in the market, all the partners in the cooperative will be jointly liable, so that each of them will comply with the partnership contract requiring to produce high quality.

If all players are punished jointly upon verification of an unidentified violation, it is straightforward to show that when P is partitional, Principle 1 captures all enforceable outcomes, even if P is not product.

An appealing feature of Principle 1 is the fact that the definition of enforceable and complete contracts is independent of the players' payoff functions. In the remainder of this note, we shall extend the BW approach in relation to the issues surfaced in the above discussion, and try to maintain the analytical separability between P and u. We consider first a regime allowing for joint liability, and then we study a regime that rules it out.

4 Joint Liability

4.1 The Solution Concept

Given the definition of verifiable deviation of the previous section, and the consequent introduction of the contracted games $G|_C$, one may readily appreciate that a general solution concept for the framework consists of a straightforward extension of the Nash Equilibrium concept, that we call *Enforceable Equilibrium*.

Definition 1 *Given the game* $G = (I, A, u)$, *and the court information structure* $P : A \rightarrow$

2^A *the pure strategy profile* a^* *is called an* enforceable equilibrium[2] *whenever* $\forall i, \forall a_i \in A_i$, *at least one of the following holds:*

- $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ (self-enforcement)
- $a^* \notin P(a_i, a^*_{-i})$ (legal enforcement).

Due to the pre-game agreement interpretation of the above solution, it also makes sense to say that the parties will be able to coordinate on a Pareto-efficient enforceable equilibrium.

The parties consider self-enforcing and legally-enforceable incentives *together* before coordinating on the equilibrium action profile. They will support the equilibrium by signing a suitable supporting contract so as to activate the legally enforced incentives. If any player deviates, the supporting contract will be brought to court in order to verify the deviator and punish her. Whether a contract is enforceable may be determined only ex-post, after the enforceable equilibria have been calculated. The enforceability of a contract crucially depends on the players' private incentives.

Definition 2 *Given the game* G, *the information* P, *and the Enforceable Equilibrium* a^*, *any contract* C^* *such that* $a^* \in C^*$ *and* $\forall i, \forall a_i \in A_i$,

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}) \quad \text{or} \quad [(a_i, a^*_{-i}) \notin C^* \ \text{ and } \ P(a_i, a^*_{-i}) \cap C^* = \emptyset], \tag{1}$$

is defined a supporting contract *of* a^*.

If a^* is an enforceable equilibrium, it is always the case that $\{a^*\}$ is a supporting contract of a^* (in particular it is the minimal supporting contract). Moreover, once the players have coordinated on an enforceable equilibrium, it is payoff irrelevant which particular contract they sign to support it: the supporting contract is an effective deterrent, so no player is ever brought to court and punished for violating it.[3]

[2] The definition is given for pure strategies to be consistent with the rest of the paper, however a mixed strategies enforceable equilibrium is easily defined to be σ^* s.t. $\forall i, \forall a_i^* \in Supp(\sigma^*), \forall a_i \in A_i$, $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ or $a^* \notin P(a_i, a^*_{-i})$ and that gives directly the existence of the (mixed strategies) enforced equilibrium with standard Nash 1951 construction.

[3] The enforceable equilibrium concept determines both the contract the parties will sign before the interaction, and the action profile they agree to play. The concept of enforceable equilibrium can thus be

4.2 Extending the BW Approach

In order to extend Principle 1 to generalized court information under a joint liability regime, the first step is to extend BW definition of contract-enforceability. That extension should not depend on the payoff functions, but only on the court's information structure.

Definition 3 *A contract* $C \subseteq A$ *is defined* always enforceable *if*

$$\forall a \in C, \forall i, \forall (b_i, a_{-i}) \notin C, \quad P(b_i, a_{-i}) \cap C = \emptyset \tag{2}$$

For any $a \in C$, if player i deviates by taking the action $b_i : (b_i, a_{-i}) \cap C = \emptyset$, the court will verify the deviation.

As in BW, we focus on the smallest enforceable contracts, which they define as "complete". It is straightforward to see that if an action profile is not supported by any minimal contract, it may not be supported by larger contracts either. The next Lemma, shows that one can well define a minimal enforceable contract for each action profile. Moreover, in the proof, one finds a simple algorithm to derive the minimal contract starting from the action profile.

Lemma 1 *For any information structure* P, *for any action profile* $a \in A$, *there exists a unique* minimal *(in terms of set inclusion) always enforceable contract* $C(a)$ *such that* $a \in C(a)$.

Proof. For any player i, consider b_i s.t. $a \in P(b_i, a_{-i})$, then, for any C enforceable with $a \in C$, it must be that $(b_i, a_{-i}) \in C$: if not Condition (2) is contradicted. Fixing this (b_i, a_{-i}), the same condition must hold for any b_k s.t. $(b_i, a_{-i}) \in P(b_i, a_{-ik}, b_k)$, and for any b_k s.t. $a \in P(b_i, a_{-ik}, b_k)$, and so on. Thus any C enforceable such that $a \in C$ contains the unique minimal contract $C(a)$ defined as follows: set $C_0 = \{a\}$, $\forall n \geq 1$ define $C_n = f(C_{n-1})$ iteratively using the correspondence $f : 2^A \rightarrow 2^A$ defined as

$$f(C) = \cup_{i=1}^{I} \cup_{\{a \in C, b_i \in A_i, \ s.t. \ P(b_i, a_{-i}) \cap C \neq \emptyset\}} (b_i, a_{-i}). \tag{3}$$

thought of as a bridge between the concept of contract and that of equilibrium.

Define $C(a)$ to be the fixed point C_N such that $C_N = f(C_N)$: the fixed point always exists because the correspondence is non-decreasing and 2^A is finite, also $C(a)$ is unique since f is well defined. ■

Let $\mathcal{C}$ denote $\{C(a)|a \in A\}$, the list of minimal always enforceable contracts. Principle 1 can now be revised as follows.

Principle 2 *First, consider the court information structure P : for each action profile $a \in A$, determine the minimal always enforceable contract $C(a)$, using Lemma 3. Secondly, derive the games $G(a) := (I, C(a), u|_{C(a)})$ by taking the game G and restricting u to each $C(a)$, and calculate the pure strategy equilibria a^* of each game $G(a)$.*

4.3 Product (Non-Partition) Information

First we show that the product structure of P is inherited by the list of minimal always enforceable contracts $\mathcal{C}$.

Proposition 1 *If the information P is product, then $C(a)$ is a product set, for any $a \in A$.*

Proof. If the information is product, $P = \times_{i=1}^{I} P_i$, the restriction of P on the i player $P(a_i, a_{-i})|_i$ is equal to $P_i(a_i)$, for any a_{-i}. Thus $a \in P(b_i, a_{-i})$ iff $a_i \in P_i(b_i)$. So the function in Condition (3) satisfies: $f(C) = \cup_{i=1}^{I} \cup_{\{a \in C, b_i \in A_i, \ s.t. \ P(b_i) \cap C|_i \neq \emptyset\}} (b_i, a_{-i})$. Thus $f(C)|_i := \cup_{\{a \in C, b_i \in A_i, \ s.t. \ P(b_i) \cap C|_i \neq \emptyset\}} (b_i)$ and $f(C) = \times_{i=1}^{I} f(C)|_i$. Therefore $C(a)$ is a product set. ■

Most importantly, Principle 2 is vindicated by the following result.

Proposition 2 *For any game $G = (I, A, u)$, if the information structure P is* product, *then Principle 2 gives a general solution according to Definitions 1 and 2.*

Proof. We need to show that the pair $(a^*, C(a^*))$ is a solution according to Definitions 1 and 2 if and only if a^* is a Nash Equilibrium of $G|_{C(a^*)}$ and $C(a^*)$ is a minimal always enforceable contract.

For the if part, consider an always enforceable contract $C(a^*)$ s.t. a^* is a Nash Equilibrium of $G|_{C(a^*)}$. Clearly, $C(a^*)$ satisfies the second requirement of Condition (1) in Definition 2, and a^* satisfies the first requirement, finally $a^* \in C(a^*)$.

For the only if part, we need to show that if a^* is a pure strategy equilibrium of a contract C^*, s.t. (a^*, C^*) satisfy Condition (1) then a^* is also an equilibrium of the minimal always enforceable contract $C(a^*)$. For any contract C, player i and opponent profile a_{-i}, consider the restriction $C|_{a_{-i}}$: it is the list of actions available to player i under the contract C, when her opponents play a_{-i}. Say that $C(a^*)|_{a_{-i}} \subseteq C^*|_{a_{-i}}$. If for any i, $a_i^* \in \arg\max_{a_i \in C^*|_{a_{-i}}} u_i(a_i, a_{-i}^*)$, then also $a_i^* \in \arg\max_{a_i \in C(a^*)|_{a_{-i}}} u_i(a_i, a_{-i}^*)$. Say that $C(a^*)|_{a_{-i}} \not\subseteq C^*|_{a_{-i}}$. As a^*, satisfies the first requirement of Condition (1), $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$, $\forall a_i \in C^*|_{a_{-i}^*}$. Since the information structure is product, $P(a_i, a_{-i})|_i = P_i(a_i)$, $\forall a_{-i} \in C^*|_{a_i}$ and $\forall a_{-i} \in C(a)|_{a_i}$. Now $\forall a_i \in C(a^*)|_{a_{-i}^*} \setminus C^*|_{a_{-i}^*}$, $a_i \in P_i(a_i^*)$ or else the minimality of $C(a^*)$ is violated, and so, $\exists b_i \in C^*|_{a_{-i}^*}$ s.t. $u_i(b_i, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$, if not $a_i \in C^*|_{a_{-i}^*}$, finally $u_i(a_i^*, a_{-i}^*) \geq u_i(b_i, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$. Therefore, it is concluded that $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$, $\forall a_i \in C(a^*)|_{a_{-i}^*}$. ■

4.4 Partitional (Non-Product) Information.

When P is a partition of A, the list of minimal always enforceable contracts is a non-coarser partition of A.

Proposition 3 *If the information P is partitional, then $C(a) \subseteq P(a)$, $\forall a \in A$, and $\mathcal{C}$ is a partition of A.*

Proof. For the first claim, observe that, being P a partition, $\forall i, \forall b_i$, if $a' \in P(b_i, a_{-i}) \cap C$ then $P(a') = P(b_i, a_{-i})$. So the function 3 is such that $f(\{a\}) \subseteq P(a)$, and that, if $C \subseteq P(a)$, $f(C) \subseteq P(a)$, $\forall C \subseteq A$. Thus its fixed point $C(a) \subseteq P(a)$.

Say that $\exists a'' \in (C(a) \cap C(a'))$ then, by the minimality of $C(a)$ and of $C(a')$, and the construction in Lemma 1, there must exist a k and a m and finite sequences $\{C_n\}_{n=1}^k$ and $\{C_n\}_{n=k}^m \in I^k$ s.t. $C_0 = \{a\}$, $C_m = \{a'\}$, $a'' \in C_k$, and $\forall n \leq k$, $C_n = f_a(C_{n-1})$; $\forall n \geq k$, $C_n = f_{a'}(C_{n+1})$. So there exist $\hat{a}, a^{k+1} \in C_{k+1}$, possibly $\hat{a} = a^{k+1}$, and i, b_i s.t. $a'' = (b_i, a_{-i}^{k+1})$ and $\hat{a} \in P(a'') \cap C_{k+1}$. As $a'' \in P(a'')$, and $a'' \in C_k$, $a'' \in P(a'') \cap C_k$. Since P is a partition, $P(\hat{a}) = P(a'')$. Thus $a'' \in P(\hat{a}) \cap C_k$. By the first part of the Proof, $P(\hat{a}) = P(a^{k+1})$, so $a'' \in P(a^{k+1}) \cap C_k$. Now relabel $a^{k+1} = (b_i, a''_{-i})$, and it is proven that $a^{k+1} \in f_a(C_k)$.

Repeat the construction for a^n for $n = k+2, \cdots m$. As $C_m = \{a'\}$, it must be that $a^m = a'$. Thus $a' \in f_a^m(\{a\})$ and analogously $a \in f_{a'}^m(\{a'\})$. Moreover, by the definition of f in the Proof of Lemma 1, $\forall n, f_a^n(\{a'\}) \subseteq f_a^{m+n}(\{a\})$ and $f_a^n(\{a\}) \subseteq f_{a'}^{m+n}(\{a'\})$. As f_a and $f_{a'}$ are non-decreasing, $C(a)$ is the fixed point of f_a, and $C(a')$ is the fixed point of $f_{a'}$, it follows that $C(a) = C(a')$. ■

Remark 1 *Somewhat unexpectedly, the list of the minimal payoff-independent enforceable contracts can be a finer partition than $P(a)$. For example, let $A = \{S, D\} \times \{s, d\}$, and $P(S,s) = P(D,d) = \{(S,s),(D,d)\}$, $P(S,d) = \{(S,d)\}$, $P(D,s) = \{(D,s)\}$. Applying Lemma 1, the list of the minimal contracts is such that $\forall a \in A, C(a) = \{a\}$.*

Principle 2 still captures all enforceable equilibria.

Proposition 4 *For any game $G = (I, A, u)$, if the information structure P is* partitional, *then Principle 2 gives a general solution according to Definitions 1 and 2.*

Proof. The first two parts of the Proof of Proposition 2 apply also here. To show the contrapositive of the only if part, say that the profile $a \in C(a^*)$ is not equilibrium under the minimal always enforceable contract $C(a^*)$ That is $\exists i, b_i$ s.t. $u_i(b_i, a_{-i}) > u_i(a)$ and $(b_i, a_{-i}) \in C(a^*)$. As $C(a^*)$ is minimal, $\exists a' \in P(b_i, a_{-i}) \cap C(a^*)$. Since the information structure is partitional, $P(b_i, a_{-i}) = P(a') = P(a)$ by Lemma 3. Therefore Condition (1) cannot be satisfied for any contract C s.t. $(b_i, a_{-i}) \notin C$ and $a \in C$. ■

4.5 Non-Product and Non-Partition Information.

This subsection will prove that there exist a game and a non-partitional, non-product information structure for which the parties can agree on an action profile a^* by signing a suitable supporting contract $C(a^*)$, and thereby Pareto improve any solution a of Principle 2.

P	B	N
H	(H,B); (H,N)	(H,B); (H,N)
L	(L,B); (L,N)	A,A

G	B	N
H	3,3	0,2
L	4,0	1,1

Example 1 *We want to show that there exist a* non-product and non-partitional *information structure P, and a game $G = (I, A, u)$, with an action profile a^*, such that (a^*, C^*) is a solution for (G, P) according to Definitions 1 and 2, and a^* Pareto dominates all the equilibria of its minimal always enforceable contract $C(a^*)$.*

Consider the above court information structure P. By making use of the construction in the Proof of Lemma 1, we calculate $C(H, B)$, the minimal always enforceable contract for the action profile (H, B). First $(H, N) \in C(H, B)$ as $(H, B) \in P(H, N)$, then also $(L, N) \in C(H, B)$ as $(H, N) \in P(L, N)$ and finally $(L, B) \in C(H, B)$ as $(L, N) \in P(L, B)$. That is, $C(H, B) = A$.

Now consider the game G. As $C(H, B)$ is equal to A, also $G(H, B)$, the game restricted to $C(H, B)$, is equal to A. The only equilibrium of $G(H, B)$ is thus (L, N).

However, the profile (H, B) is an enforceable equilibrium, as it satisfies the first clause of Definition 1 for the profile (H, N) and the second clause for the profile (L, B). Intuitively, player 2 prefers B to N and player 1 cannot play L because that would be verified, and she would be punished.

Since (H, B) Pareto dominates (L, N), we can meaningfully conclude that Principle 2 does not yield a general solution.

5 Individual Liability

Suppose that the players have signed the contract C and agreed to coordinate on the action profile $a \in C$. Even if the deviation b_i is verifiable, the court may not be able to identify i as the violator, and thus to punish player i. Since verifiability is not enough to guarantee contract enforceability, we introduce the concept of punishability. We define a deviation b_i *punishable* when $P(b_i, a_{-i})|_i \cap C|_i = \emptyset$ and $\forall j \neq i : P(b_i, a_{-i})|_j \cap C|_j \neq \emptyset$.

We can simply modify the definition of Enforceable Equilibrium, to account for punishable deviations.

Definition 4 *Given the game $G = (I, A, u)$, and the court information structure $P : A \rightarrow 2^A$, the pure strategy profile a^* is called* strongly enforceable equilibrium *whenever*

$\forall i, \forall a_i \in A_i$, *at least one of the following holds:*

- $u_i(a^*) \geq u_i(a_i, a^*_{-i})$ (self-enforcement)
- $\forall J \subseteq I \setminus \{i\}, \forall c_J \in A_J, \quad (c_J, a^*_{-J}) \notin P(a_i, a^*_{-i})$ (legal enforcement).

Now we modify the definition of *always* enforceable contract.

Definition 5 *A contract $C \subseteq A$ is defined to be always strongly enforceable if*

$$\forall a \in C, \forall i, \forall (b_i, a_{-i}) \notin C, [\forall a' \in C, \forall J \subseteq I \setminus \{i\}, \forall c_J \in A_J, \quad (c_J, a'_{-J}) \notin P(b_i, a_{-i})] \qquad (4)$$

As can be easily proved by extending the Proof of Lemma 1, for any action profile a there exist a unique minimal always strongly enforceable $C(a)$ such that $a \in C(a)$. In the same way, mutatis mutandis, Propositions 1, and 2 are still proven to hold.

The message of Example 1 may be strengthened when ruling out joint liability: we may in fact construct a similar example even with partitional, non-product P.

	B	N
H	3,3	0,2
L	4,0	1,1

	B	N
H	(H,B), (H,N), (B,L)	(H,B), (H,N), (B,L)
L	(H,L)	(H,B), (H,N), (B,L)

Example 2 *Consider the above game and information partition. Take the action profile (H, B), and calculate the minimal always strongly enforceable contract from Definition 5: the contract must include (H, N) as $(H, B) \in P(H, N)$, then it must include (L, N) as $(H, N) \in P(L, N)$. Finally also (L, B) must be included: consider Definition 5, call $a = (H, B)$, and $b_i = L$, notice that calling $a' = (L, N)$ and $c_j = B$, it turns out that $(b_i, a_{-i}) = (c_j, a'_{-j})$, and so $(c_j, a'_{-j}) \in P(b_i, a_{-i})$. Thus the minimal always strongly enforceable contract for (H, B) is A, and the only equilibrium of A is (L, N).*

However, the profile (H, B) is a strongly enforceable equilibrium. In fact player 2 prefers B to N and player 1 cannot play L because she would be verified, and punished, finally, (H, B) Pareto dominates (L, N).

Proposition 3 is also weakened: when P is a partition of A, $\mathcal{C}$ is still a partition of A, but it can be either non-finer or non-coarser. The proof that $\mathcal{C}$ is a partition of A is an extension of the Proof of the second part of Proposition 3. To show that $\mathcal{C}$ may be a coarser partition than P, consider Example 2: $\forall a \in A$, the minimal payoff-independent strongly enforceable contract $C(a)$ is equal to A. In 2-player games, it can be proven that the list of the minimal contracts is always a non-finer partition than $P(a)$. However, in n-player games, a finer partition may occur. For example: let $A_i = \{B_i, C_i\}, i = 1, 2, 3$, set $P(C_1, C_2, C_3) = P(B_1, B_2, B_3) = \{(B_1, B_2, B_3), (C_1, C_2, C_3)\}$ and $P(a) = \{a\}$ for any other a. The list of minimal contracts is $\mathcal{C} = A$.

6 Short Discussion on Non-Partitional Information

A common criticism with non-partitional information structures is that it is unclear whether they are compatible with the epistemic knowledge of the players in a game. In this paper, non-partitional information structure is not imputed to the players, but to the court, an external institution that needs to present conclusive evidence for all the statements she presents in a sentence. As Shin (1993) remarks, such provability requirement may break the *Know That You Don't Know* axiom. Thus, as argued by Geanakoplos (1989), the information structure need not be a partition.

Our players instead follow epistemic knowledge, and, moreover, they know the legal procedure that constrains the court. Thus, it is ex-ante common knowledge among the players that the court has non-partitional information. It is therefore logically consistent to require the players to calculate ex-ante utility compounding the interim utility from the non-partitional information structure, and it makes sense to find an ex-ante equilibrium concept constructed in the same way.

That is not the case when the non-partition is derived from imperfect information processing by the players. In that instance, as underlined by Brandenburger, Dekel, and Geanakoplos (1992), the ex-ante expected utility calculation is problematic, because ex-ante the information processing pathology has not occurred yet. Moreover, they argue, an ex-ante equilibrium solution depending on the ex-post non-partitional information is meaningless. As its derivation requires common knowledge of each other's information structure, each player must also know her own information structure, but then she can solve it into a partition.

References

[1] Brandenburger A. Dekel E. and J. Geanakoplos [1992]: "Correlated Equilibrium with Generalized Information Structures" *Games and Economic Behavior* **4**: 182-201

[2] Bernheim B. D. and M. D. Whinston [1997]: "Incomplete Contracts and Strategic Ambiguity" *American Economic Review* **88**: 902-32..

[3] Dekel E. and F. Gul [1997]: "Rationality and Knowledge in Game Theory" in Kreps D. M. and F. K. Wallis (Eds.) *Advances in Economic Theory, Seventh World Congress.* Cambridge, Cambridge University Press

[4] Fudenberg D. and J. Tirole [1991]: *Game Theory* Cambridge MA, MIT press

[5] Holmstrom B. [1982]: "Moral Hazard in Teams" *The Bell Journal of Economics* **13**: 324-340

[6] Geanakoplos J. [1989]: "Game Theory without Partitions, and Applications to Speculations and Consensus" *Cowles Foundation for Research at Yale University* mimeo

[7] Lippman B. L. and D. J. Seppi [1995]: "Robust Inference in Communication Games with Partial Provability" *Journal of Economic Theory* **66**: 370-405

[8] Milgrom P. and J. Roberts [1986] "Relying on the Information of Interested Parties" *RAND Journal of Economics* **17**: 18-32

[9] Monderer D. and D. Samet [1989]: "Approximating Common Knowledge with Common Beliefs" *Games-and-Economic-Behavior* **1**: 170-90

[10] Nash J. [1950]: "Equilibrium Points in n-Person Games" *Proceedings of the National Academy of Science* **36**: 48-49

[11] Okuno-Fujiwara M. A. Postlewaite and K. Suzumura [1990]: "Strategic Information Revelation" *Review of Economic Studies* **57**: 25-47

[12] Seidmann D. [1997]: "Can the Right of Silence Benefit Innocent Suspects who tell the Truth?" *University of Newcastle-upon-Tyne* mimeo

[13] Shin H. S. [1993] "Logical Structure of Common Knowledge" *Journal of Economic Theory* **60**: 1-13