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Coordination of Economic Activity: An Example

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Introduction

In this paper we consider issues of organization of economic activity, especially coordination of productive activity. We begin with a classic question; “Which economic activities are (or should be) coordinated by markets, and which by other means?” “Other means” usually refers to administrative or bureaucratic mechanisms; sometimes the word “hierarchies” is used. While this paper is based on a formal model in which this question can be asked, its main content is an example–a specialization of the formal model–in which efficient coordination means just-in-time production. The example has two cases. In each case market equilibria exist, and the First Welfare Theorem is valid. In Case 1 it is shown that the market mechanism cannot achieve efficient coordination. There are alternative mechanisms, which are described here as direct mechanisms; one of these does achieve efficient coordination. In Case 2 both types of mechanism achieve efficient coordination, but the market mechanism has higher informational costs. The two cases are the same except in one respect, the nature of demand for the final products. The example is highly simplified in order to make its point without the need for heavy analytic machinery. However, its simplicity may conceal the richness of the set of organizational mechanisms for coordinating economic activity, and the sensitivity with which choice among coordinating mechanisms depends on apparently small differences in the coordination problem at hand. The large variety of different coordination mechanisms that can be observed in the economy suggests that variety in the set of efficient coordination mechanisms corresponds to the variety of coordination problems.

It is a widely held view that a need for coordination of the activities of economic agents exists when different agents have different information, a condition that arises, at least in the production sector, from division of labor and specialization. When agents have private information incentives exist to use it to their advantage. The analysis of the example assumes that the agents will act as the mechanism prescribes, and not in their own self-interest, if those two behaviors are in conflict. If the market mechanism operated by selfless agents cannot bring about efficient coordination, then the market mechanism operating with rational agents acting on their private information in their own self-interest cannot bring about efficient coordination. Thus, the result in Case 1 of the example serves as an impossibility result–a benchmark for mechanism design when incentives are taken into account.

Questions about how economic activity should be coordinated have usually been addressed in the context of ‘theory of the firm.’ But that seems not to be a useful setting in which to study choice among coordinating mechanisms. A mechanism of coordination that is used within a firm may also be used across firms. One can observe a variety of coordinating mechanisms each commonly and persistently used in the economy, especially in the production sector, between, across and within firms. This phenomenon is not likely to be understood by way of theories in which the only coordinating mechanisms are firms and markets. The concept of 'firm' currently prevalent is essentially legal; a firm is seen as a collection of assets with an owner, an agent who has the legal right to control the use of the assets, and the right to the residual income. A firm that has more than one person in it generally has a mechanism for coordinating its internal activities, and generally a firm is a participant in external mechanisms that coordinate its actions with the actions of other economic units. A given firm can change its internal coordination mechanisms without changing its identity as a firm; the mechanisms that coordinate a firm's actions with those of others might also change without changing the identities of the firms involved. To clarify ideas, it is useful to have a concrete example in mind.

A manufacturer of tractors, General Tractors (GT) requires a transmission for each tractor it produces. The transmissions are supplied by another firm, GG, that makes them. It is important to coordinate the delivery of transmissions with the production schedule of tractors. At the same time another producer of tractors, UT, has an internal unit, UG, that produces transmissions. The same coordination problem exists inside UT between its tractor division and its transmissions division as exists between GT and its supplier. It may happen that the same coordinating mechanism is used in both cases, for the same reasons.

It is not infrequent that a firm will use another firm as a supplier of components, then at a later time set up an internal unit for the same purpose, and at a still later time shut down or sell the internal unit, returning again to using an outside supplier, all with essentially the same coordination mechanism. In such a case the coordination mechanism stays the same, but the firm structure changes. It is therefore likely that the problem of understanding which coordination mechanism is or should be used is different from understanding which firms do or should form. Because our focus is on understanding choice among coordination mechanisms, it seems best to approach that problem directly, without asking whether it is within or across firms.

It may be the case that the transmission's design is unique to the GT tractor, different from the transmissions used by other tractor manufacturers. When the design of the transmission is uniquely specific to the GT tractors, there cannot be competitive market, or perhaps any market, where that transmission is routinely bought and sold. It is common practice in this situation that the transaction between the transmission supplier and the tractor maker is coordinated by direct exchange of information and negotiation between the two parties. But it is also possible to use direct contact and negotiation between GT and GG to coordinate their actions even when the transmissions are standardized and there is a market for them. In that case there is the possibility of using either coordination mechanism.

The view of the market as a mechanism for coordinating economic activity is both old and familiar, beginning with Adam Smith. The fundamental modern formal expression of this view is in the general equilibrium model. In that model conditions for the existence of competitive equilibrium are established, as are conditions for the First Welfare Theorem. These results concern static competitive equilibrium. Specifically, they deal with allocations and prices that are verified to be competitive equilibria. But these results are not sufficient to ensure that efficient coordination of economic activity results from the market mechanism. In the example presented below general competitive equilibrium exists, and the First Welfare Theorem holds, but the market mechanism does not produce efficient coordination of production. To understand and justify our interpretation of this result it is useful to state some perhaps obvious things.

First, coordination refers to a relation between actions of economic agents. Thus, each of, say, two agents can take one of several actions, determining an array of individual actions taken. The array is coordinated if and only if it satisfies some given condition, expressing the idea that the two actions match. In the case of the Walrasian market mechanism, the action of a producer is determined by the rule that it maximizes profit over the set of actions available to that producer, given the prices of commodities; the action of a consumer is determined by the rule that it maximizes the consumer's utility subject to his budget constraint given the prices, initial endowment and shares of the profits of firms; the prices are determined by the requirement that excess demand at those prices is non-positive. Coordination means that the actions of all agents are feasible and jointly consistent.

In general, the action of each agent is determined by a behavior rule that is part of the coordinating mechanism. The joint action is coordinated if it satisfies a given requirement, for example, that it results in efficient production.

Second, a fully specified coordinating mechanism must produce a specific action outcome. If the mechanism is not fully specified, in which case it can result in several possible actions for some or all agents, then it must be the case that any selection of the action of each player must form an array that is coordinated.

For instance, consider the situation of two musicians who are to perform a certain duet together. For simplicity we suppose that they must start playing simultaneously. This is the meaning of coordination in this instance. If the coordinating mechanism were such that it allowed each player to start at any time within a two second interval, then the mechanism would not be fully specified. The players could start at the same time, in which case their actions would be coordinated, or they could not, in which case the actions would not be coordinated. A coordinating mechanism that resulted in this performance would not be an acceptable mechanism. This suggests that the appropriate requirement for coordination is that *every* array of actions of agents that is a result of the mechanism constitute a coordinated array.

The literature that seems closest to the problem posed here is in its earlier manifestations one that considered two organizations of production in firms; one in which two production units formed one vertically integrated firm; the other in which the two production units formed two firms, one buying inputs from the other in an intervening market. [see Grossman & Hart (1986)]. More recently another alternative has been introduced into the theoretical literature. A firm requiring inputs from another, its supplier, can enter into an on-going relationship with one or more firms to supply it with inputs. These relationships can be contractual or not, but they are the result of direct contacts between the manufacturer and the potential suppliers. Thus, there may be three alternative modes of organization to consider. When the third alternative is present the issues of whether a firm should or should not own a supplier, or whether a supplier should own a firm it supplies remain, but a broader range of questions having to do with the arrangements between the supplied firms and the supplying firms also arises. In this literature the analysis of investment and incentives is extended to the multi-firm network setting under broader assumptions about uncertainty and other matters [See Kranton and Minehart, 1999].

The question posed at the beginning of this paper should be addressed with the help of a formal model. That model should include a formal representation of economic activity, a criterion for assessing or comparing the extent to which activities are coordinated, and the mechanisms or techniques for coordinating economic decisions and actions. The class of coordination mechanisms considered should include markets, and a range of alternatives to markets. The main content of this paper is an example in which the question stated in the introduction is asked and answered. The example is simplified to such an extent that it can be explained and analyzed in its own terms. This permits us to avoid going through the technicalities of a formal model whose generality is not used in this paper. Therefore we do not present that model in detail, but instead we sketch its main features, and then go directly to the example.

Economic activity is production and exchange. Technology consists of *processes*, along the line of activity analysis models (linear or nonlinear). A process transforms objects of economic relevance; for this example they are *commodities*. Each process is associated with a *manager*, the actions of a manager are defined. Essentially a manager decides how to use his process. There can be several identical copies of a given process, each with its own manager, and each using its own *facility*. Just as in Koopmans' model [1957], we do not presuppose the existence of units like firms; there is no suggestion that the managers are in the same firm or other economic unit smaller than the entire economy.

The purpose of production is to satisfy some outside requirements, such as demands of economic agents outside the particular sector of production under consideration, or managers of other processes. In the general model these demands or requirements are represented by parameters that characterize them, such as parameters of utility or demand functions of consumers or the demands of other producers who are not operating the given technology. We assume here that the technology associated with a given manager is fixed and known, but that the outside environment is not known–only the set of possible outside environments is known. (It can be the case that the technology itself depends on parameters and that the organizational design must be determined knowing only the set of possible values of those parameters as well, rather than a particular value.)

INFORMATION

The <u>initial information</u> of a manager consists of:

- knowledge of the processes assigned to her;
- a subset of parameters that the manager can observe directly.

In addition to initial information, a manager may acquire information through communication. Each manager can send and receive messages to or from other managers, using communication channels. We consider two types of channels, called *direct channels* and *market channels*, respectively. Channels are directed; the channel used by manager A to send a message to manager B is different from the one used by B to send a message to A.

A direct channel carries addressed messages from one specific agent to another. The sender and receiver are known to one another. For each message, the sender knows who the receiver is and vice versa. As with any communication channel, sender and receiver share the language in which messages are composed. The structure and content of messages is otherwise restricted only by channel capacity. An agent who communicates with several other agents via direct channels can use her knowledge of who is at the other end to send a different message to each of the other agents. She can receive different messages from each of them.

The concept of a market channel requires some discussion. The term "market" is ubiquitous in economics. It has many different meanings. One can speak of the wheat market in Chicago, or the real estate market in Chicago, or the advertising market in Chicago, or the marriage market in Chicago, though these are very different things. If the term 'market' can be used, as it is sometimes, in a way that suggests that it applies to any situation in which two or more economic agents exchange something, or agree on a joint economic action, then the question we started with, ‘Which economic activities are, or should be, coordinated by markets and which by something else?’ has only one possible answer. For this question to be interesting, we need a class of things, which we may call *coordination mechanisms*, with (at least) two elements, one of which is called “market” and the other isn’t. Before describing the concepts used in the formal model of coordination mechanisms, we comment on some of the motivation for them.

It seems desirable to require that a coordination mechanism called the *market mechanism* should cover the case of a competitive market mechanism as it appears in general equilibrium models, such as the classical Arrow-Debreu model. It seems slightly ironic that there is no formal entity called “market” in that model; there is only “commodity”, “price” and the actions of agents. There is no communication among agents directly; communication goes through ‘the market’. In the classical model, this means the price determining, or price adjusting entity, for example, the Walrasian auctioneer. All messages are anonymous and consist of sets of commodity space vectors and price vectors.

But it is also suggestive to think about examples, such as the local supermarket for groceries, or a hardware store, or some types of farmers’ market or flea market. The supermarket stands ready to sell a variety of commodities to buyers, and buys what it has to sell from sellers. A buyer who goes to the supermarket can expect to buy what he wants, if he is willing to pay the posted market price. A seller who supplies the supermarket can offer to the buying agency of the supermarket what he wants to sell at a given price. There is no direct communication between those who buy from the supermarket and those who sell to the supermarket. The price at which goods are offered to buyers is the same for all buyers. Similarly, in a competitive situation, the price paid for a given product is the same to all sellers of that product. Furthermore, the supermarket cannot tell any buyer what to buy at the posted prices, nor can it instruct a seller what to sell. However, we must also take account of the fact that a supermarket or hardware store provides economic services that are not just informational such as storing and delivering commodities. Conceptually we can separate these functions from the purely informational function of acting as a communication link between buyers and sellers, concentrating our attention on only the latter function. With these remarks in mind we give the following concept of “market channel.”

A market is personified as an agent, called the market agent. Messages flow only between individual managers and the market agent. The form of messages is prescribed. In the case of a perfectly competitive market a message from a manager to the market agent is a supply or demand function, or in a more dynamic version, consists of bids and offers, while a message from the market agent to a manager is a vector of prices, the same for each manager. This is a static formulation. If, as is the case in our example, several time periods are involved, the market is viewed as finding a multi-period equilibrium at the beginning of the relevant time period, as in Debreu’s model [1959].

In some institutional situations that we would naturally think of as markets, buyer and seller do meet directly. For example, in a farmers’ market or a flea market this is usually the case. The basic distinction we draw here is whether or not there is bargaining between buyer and seller. In our local farmers’ market, where buyer and seller meet, the prices of the items for sale are posted. If a price is changed in the course of a market day, the new price is posted. This market can be described as one in which the seller–the farmer–is represented by an agent who transmits information-the price of each item for sale–from seller to buyer, and also accumulates information about demand of buyers. As in the case of the supermarket, we focus on the informational role of the seller rather than the economic service of storing and delivering the items for sale. In fact, it is often the case that the sales booth is not manned by the supplier of the items offered for sale, but by an agent who is not authorized to change the posted prices in response to individual buyers. Furthermore, the identity of buyers or sellers is not used in determining the market agent's response to messages received.

In the case of a competitive market, as modeled here, the market agent's message is obtained by aggregating the messages of managers to obtain the aggregate demand correspondence, and finding a price (if there is one) such that the aggregate excess demand at that price is (or contains) zero. If so, that price is the message sent from the market agent to all managers, i.e., posted.

There is another aspect of markets that should be addressed, namely its scope. Consider a somewhat idealized flea market in which the items offered for sale at each booth are unique to that booth. In other words, the commodity vector is partitioned into sub-vectors each associated with a subset of sellers, in this case a unique seller. While buyers (and possibly also sellers) might consider the entire vector of prices in calculating their decisions to supply or demand commodities, in this situation it may be the case that there is a different market agent for each segment of the market corresponding to the partition of the commodity vector into its components–in the simplified flea market, a single market agent for each booth. The case where the sub-vectors of the commodity vector corresponding to different 'local' markets can overlap is more complicated; it does not arise in our example.

We can also consider cases where the market is not perfectly competitive, but still a market, i.e., it is still the case that communication between buyers and sellers is mediated in a way that does not constitute direct personalized communication between an individual buyer and an individual seller. For example, a market agent might communicate to each seller the demand function for that seller's product as a function of the prices of the products of certain other sellers, without naming the buyers, or permitting the offered terms of sale to be different for different buyers. This would fit the case of non-discriminating duopolists, for example.

When the interaction between agents is *direct*, as in bilateral bargaining, we do not speak of a market between them. Thus, a farmers' market, or flea market in which the price of an item for sale is not posted, but can change from one buyer to another is not a market channel.

Coordination mechanisms

As we have said, the task of a manager is to choose the state of her process. The group of managers compute their decisions about actions in a joint distributed computation, each using his initial information, including the results of observation, and the information obtained as a result of communication. The joint computation, including the algorithm that it uses and the assignment of resources used to carry out the computation, constitute the coordination mechanism.

The algorithm used to compute the desired actions is represented by a modular network, as described in Mount and Reiter [1990]. A modular network consists of a directed graph and a set F of elementary functions. Functions in F can be 'located' at the vertices or nodes of the graph; they form the modules of the network. Certain vertices are input nodes, at which information from outside the graph may be fed into the computation. Computation proceeds by having the function located at a particular vertex calculate its value at the input it receives from preceding vertices or from outside the network. Each evaluation of a module– an elementary function–takes a unit of time. The output of computation appears at the output vertices of the network after a certain time which, if not finite, says that the computation cannot be carried out.

The assignment of processes to managers determines a partition of the set of (possibly replicated) processes. Therefore, certain sub-nets of the F -network that expresses the algorithm can be associated with the manager whose actions are computed by that sub-network. Furthermore, because the network's inputs are determined by the initial information, and the structure of initial information is also associated with the managers, the connections between sub-networks associated with different managers consist exclusively of communication channels. Suppose there are two managers each managing a process. Correspondingly there are two sub-nets, $\mathcal{N}_1, \mathcal{N}_2$ that compute the actions to be taken by the two managers, $\mathcal{N}_1$ associated with manager 1 and $\mathcal{N}_2$ with manager 2. The (directed) arcs of the graph $\mathcal{G}$ that connect the graph $\mathcal{G}_1$ with the graph $\mathcal{G}_2$ of $\mathcal{N}_2$ carry communications between the two managers. These arcs represent direct channels.

Alternatively, if a market agent were introduced between the two managers, two things would be different.

First, all communication between the managers would take place through the market agent. In the case of a competitive market, messages from managers to the market agent would depending on the specific mechanism, be about commodity vectors, e.g., bids or offers, or supply or demand functions, and messages from the market agent would be prices. That is, communication between the managers would be via a market channel.

Second, the nets $\mathcal{N}_1, \mathcal{N}_2$ would be replaced by nets that compute the messages required by the market channel, but the class F of elementary operations would have to be the same as in the case of an F-network with direct channels. When there is a competitive market agent between the two managers, each manager would compute her excess demand, say, by maximizing the profit on her process, taking the prices transmitted by the market agent parametrically. Her action would be a state of her process that results in a commodity vector yielding the maximum possible profit to her production unit, given all the data and the prices.

In a market with other structure, the behavior of the managers would be determined by the equilibria of the game that models the market. In either case, the task of manager i, which is to compute her action from the inputs to her sub-network, is represented by a sub-network. The inputs are the variables whose values come into the sub-network either from the environment or via communication lines into the sub-network.

The use of a competitive market channel usually, but not necessarily, entails the assumption that there are many production units using the same process, with a different manager in charge of each unit. When several managers are in charge of replicas of a given process, or set of processes, we say they are the same type. In that case, all managers of the same type carry out the same computation. On the other hand when the channels between sub-networks corresponding to different managers are direct channels, then the algorithm embodied in the network specifies the computations to be carried out by each manager, which might be different for each manager. Generally these computations will be different from those required when the channel is a market channel.

An *information technology* consists of the set of F-networks available, the restrictions on observation as expressed above, and the channels available for communication. In operating an

F-network a manager can use *agents* who, under her supervision, carry out the elementary steps of the computation specified by the network. The manager herself may be one of those agents. Each agent works for only one manager. An agent supervised by a given manager is subject to the same constraints on initial information as her manager. Thus, the vertices of the graph of the network are assigned to agents under certain restrictions described in Reiter [1995]. A network whose vertices are assigned to agents is called an *assigned F-network*. The simplest case is that in which all computations of a manager are carried out by that manager.

A coordination mechanism is an assigned F-network.

To summarize:

- There is a criterion for evaluating whether a given joint action is desirable. Coordination means that actions of individual producers (managers) combine to make up a desired joint action.
- The *information structure* is defined, specifying what each manager knows directly about the technology and the parameters characterizing other aspects of the environment.
- Each manager's action is to choose the value of the variable that represents how his process will be operated. The managers are viewed as computing their joint decision in a distributed computation in which each manager uses his own direct knowledge, and information received from others to carry out his part of the joint computation. This part of the formal model is an extension of the modular network model of computation [Reiter (1995)]. In the extended model, each elementary computational step is assigned to some agent to carry out, subject to some restrictions. A *coordination mechanism* is an assigned modular network.
- Different coordination mechanisms using the same basic information about the environment correspond to different distributed algorithms and may involve different communication channels. In this paper we consider two types of mechanism, the *market mechanism*, which uses *market channels*, and an algorithm for computing profit maximizing actions, and one using *direct channels*, and associated rules for computing individual actions.
- The *criterion for comparing coordination mechanisms is lexicographic.* The first desideratum is that the mechanisms under examination achieve coordinated joint action. Mechanisms that do achieve coordinated joint action are compared according to their *informational efficiency*.

We turn next to our example.

Example

This example has two parts. The first is a case in which the coordination of production specified by the performance criterion cannot be achieved using a market channel, while it can be achieved using a direct channel. Therefore, according to our lexicographic criterion for comparing coordination mechanisms, the first step is decisive; there is no need to go on to a comparison of information costs.

The second part is a variation of the first in which the desired coordination can be achieved using either a market channel or a direct channel, hence bringing the comparison of costs of the two mechanisms into play.

Technology

Suppose the technology T consists of two processes, P_1, P_2 each with two states, $S_1^i, S_2^i, i = 1,2$. In state S_1^1 P_1 produces one unit of good W(hite) using one unit of input B(lue) and uses the services of one unit of facility F_1 for half a week. In state S_2^1 P_1 produces one unit of G(reen) from one unit of R(ed), using the same facility for half a week.

For example, the facility F_1 consists of machines for manufacturing tractors. There are two types of tractor, a large heavy tractor that requires a heavy duty gear-train, and a smaller light tractor that uses a light duty gear train. To set up the machinery to produce a heavy tractor requires a period of time, and once the machinery is so configured it is not possible to produce light tractors until the machinery is reconfigured for that purpose.

In state S_1^2 P_2 produces one unit of B(lue) from a unit of raw material, using the services of one unit of facility F_2 for half a week, and in state S_2^2 P_2 produces one unit of R(ed) from raw material, using one unit of facility F_2 for half a week. To avoid cluttering up the calculations without adding generality, we assume that the raw material is free. There is one unit of each facility available. W and G are final products; B and R are intermediate products. There are two production units, one operating P_1 and the other P_2.

This technology operates in time. We assume that the unit of time is a half-week. Each process can operate in each half-week, and correspondingly, commodities are dated, the date being the half-week in which they become available. For purposes of exposition fix a time t_0 and consider the periods shown in Figure 1. These are $\left(t_0 - \frac{1}{2}, t_0\right), \left(t_0, t_0 + \frac{1}{2}\right), \left(t_0 + \frac{1}{2}, t_0 + 1\right)$ which we relabel $v = 1, v = 2, v = 3$, respectively. Then a typical commodity vector has the form $(W(2), W(3), G(2), G(3), B(1), B(2), R(1), R(2))$. The commodity space is further restricted by requiring the amount of each commodity to be a nonnegative integer. (Thus, we abuse the notation by using the same symbol for the name of the commodity and its quantity, and rely on the context to make clear which is intended). When a market channel is used, prices are introduced; the price vector corresponding to the commodity subspace is

$$(w(2), \omega(3), \gamma(2), \gamma(3), p(1), p(2), q(1), q(2)).$$

[Figure 1 goes here.]

The space of attainable states is

$$S^A(t_0) = \{s_j^i(v), i, j, \in \{1,2\}, v \in \{1,2,3\}\}.$$

A profile of states for the time period under consideration has the form

$$s(t_0) = (s^1(1), s^1(2), s^1(3), s^2(1), s^2(2), s^2(3)), \text{ where } s^i(v) \in \{s_j^1, i,j = 1,2\}.$$

We suppose that there is a representative end user who receives the final products. The end user can absorb at most two units of product in a given week. He pays $\omega(2)$ for a unit of $W(2)$ delivered at time $t_0 + \frac{1}{2}$ and $\omega(3)$ for a unit of $W(3)$ delivered at time $t_0 + 1$. The end user can refuse delivery of items he didn't order and pay nothing. Because only one unit of each commodity is possible, when the final commodities have prices, the payments $\omega(j)$, and $\gamma(j) j = 2,3$ coincide with those prices. We distinguish two cases, each leading to a different conclusion.

CASE 1. The end user does not distinguish time periods shorter than a week. In this case periods 2 and 3 together make up one time period in which the end user does not distinguish $W(2)$ from $W(3)$ nor $G(2)$ from $G(3)$. The effect of this will be to require $\omega(2) = \omega(3)$ and $\gamma(2) = \gamma(3)$.

CASE 2. The end user recognizes half-weeks, and thus distinguishes $W(2)$ from $W(3)$ and $G(2)$ from $G(3)$. (The effect will be to allow $\omega(2) \neq \omega(3)$ or $\gamma(2) \neq \gamma(3)$).

In CASE 1 the demand parameters are $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Buyers of W (resp. G) do not distinguish between units of W (resp. G) delivered at any time within the week for which they are desired. Demand for W and G is determined for each week independently, and the demand for week $(t, t+1)$ beginning at t is known at time $t - \tau, \geq \frac{1}{2}$.

Demand for final products delivered in any period is shown in Table 1 in terms of θ.

Table 1.

$\theta(t) = \theta_1$	1 unit each of W and G in week $(t, t+1)$
$\theta(t) = \theta_2$	2 units of W and 0 units of G in week $(t, t+1)$
$\theta(t) = \theta_3$	0 units of W and 2 units of G in week $(t, t+1)$

In CASE 2 the demand parameters are as shown in Table 2.

Table 2.

$\theta(t) = \theta_{11}$	1 unit of W and 0 units of G in period $(t, t+\frac{1}{2})$, or 0 units of W and 1 unit of G in period $(t+\frac{1}{2}, t+1)$.
$\theta(t) = \theta_{12}$	0 units of W and 1 unit of G in period $(t, t+\frac{1}{2})$, or 1 unit of W and 0 units of W in period $(t+\frac{1}{2}, t+1)$.
$\theta(t) = \theta_2$	2 units of W and 0 units of G in week $(t, t+1)$.
$\theta(t) = \theta_3$	0 units of W and 2 units of G in week $(t, t+1)$

We focus on the actions of a manager M_1 of process P_1 and a manager M_2 of process P_2. We can think of these managers as representatives of managers of their types.

Recall from the specification of the technology that to produce one unit of W in week $(t, t+1)$ requires either one unit of B delivered by time t, in which case one unit of W can be produced in the interval $(t, t+\frac{1}{2})$, or one unit of B delivered by time $(t+\frac{1}{2})$ in which case one unit of W can be produced in the interval $(t+\frac{1}{2}, t)$. Similarly for G and R. Satisfaction of demand in the time period indexed by t_0 is expressed by the condition that

$$W^d(t_0) = W^s(2) + W^s(3)$$
$$G^d(t_0) = G^s(2) + G^s(3)$$

where the superscript "d" indicates a quantity required or demanded and "s" indicates a quantity supplied. Demand quantities for the periods corresponding to t_0 are the functions shown in Table 1 in CASE 1 and in Table 2 in CASE 2. Supplies are the functions of $S(t_0)$ defined above.

To avoid an unnecessarily tedious exposition of the performance criterion and an equally tedious analysis of the example, we take note of two points:

- If inventories can be kept, then any pattern of demand can be met, provided initial inventories are suitable;
- If carrying inventories is costly, and if every admissible pattern of demand can be met without inventories, then no production plan that involves carrying inventories can be *efficient*.

The three patterns of demand shown in Table 1 can be met by M_1 the manager of the producer using process P_1 by choosing:

If $\theta(t) = \theta_1$	*then* $s^1(2) = s_1^1; s^1(3) = s_2^1$; or $s^1(2) = s_2^1; s^1(3) = s_1^1$
If $\theta(t) = \theta_2$	*then* $s^1(2) = s_1^1; s^1(3) = s_1^1$
If $\theta(t) = \theta_3$	*then* $s^1(2) = s_2^1; s^1(3) = s_2^1$

provided that the inputs are available in time. This can be assured by the following rule for the producer using P_2:

If $s^1(2) = s_1^1$, *then* $s^2(1) = s_1^2$
If $s^1(2) = s_2^1$, *then* $s^2(1) = s_2^2$
If $s^1(3) = s_1^1$, *then* $s^2(2) = s_1^2$
If $s^1(3) = s_2^1$, *then* $s^2(2) = s_2^2$

These are the desired actions derived from the performance criterion implied by the two simplifying points made above; that is, these actions optimize the outcome on the feasible set.

We turn now to comparison of the coordination mechanisms under consideration, and

compare two mechanisms, $\mathcal{D}$ and $\mathcal{M}$ for coordinating the actions of the managers of the two processes. The first mechanism uses a direct channel between the two managers; the second uses a market channel between the two managers.

We assume that transmission over each channel is instantaneous and costless. This simplification avoids having to keep track of transmission delays and the accounting that goes with them. With this simplification, we shall see that in CASE 1 the comparison of mechanisms reduces to verifying that the mechanism using a direct channel is *efficient* and that the market mechanism is not. Therefore the need to compare information processings costs does not arise. In CASE 2, both mechanisms are *efficient;* therefore the need to compare information processing costs does arise.

$\mathcal{D}$. Direct channels

$W^s(2), W^s(3), G^s(2), G^s(3)$

At time $t_0 - \tau$ manager M_1 learns the demand for the final products in period t_0. With that information M_1 can decide on her actions $s^1(2), s^1(3)$ thereby determining $W^s(2), W^s(3), G^s(2)G^s(3)$, and therefore also determining the input requirements $B^d(2), B^d(3), R^d(2), R^d(3)$. Availability of a direct channel whose capacity is sufficient to carry four one digit numbers allows M_1 to communicate these requirements directly to M_2, who can then determine his actions $s^2(1), s^2(2)$ and, if necessary, communicate them to M_1.

The availability of a direct channel allows the managers to achieve just-in-time delivery of intermediate products, and therefore to meet demand without carrying inventories of intermediate or final products, thus attaining the efficient outcome.

$\mathcal{M}$. **Market channels**

Under this mode of organization transactions between the managers of processes of type P_1 and those of type P_2 are mediated by a competitive market. (We do not model the transactions between producers of the final products and consumers of them explicitly, but these could also be mediated by a competitive market.) Using competitive market channels as the coordinating mechanism prescribes the behavior of the managers; each manager of a process must determine her action as the solution of a profit maximization problem.

Consider first the profit $\pi^{M_1} = \pi^1$ of the manager of process P_1 when the coordination mechanism is the market mechanism.

$$\pi^1 = \omega(2)W(2) + \omega(3)W(3) + \gamma(2)G(2) + \gamma(3)G(3) \\ -(p(1)B(1) + p(2)B(2) + q(1)R(1) + q(2)R(2)).$$

To derive the demand for $B(1), B(2), R(1), R(2)$ we maximize π^1 subject to the technological constraints

$$W(2) = B(1); W(3) = B(2); G(2) = R(1); G(3) = R(2),$$

$$W(j)G(j) = 0 = B(i)R(i), j = 2,3; i = 1,2, \text{ and}$$

$$W(j), G(j), B(i), R(i) \in \{0,1\} \textit{ for } i \in \{1,2\}, j \in \{2,3\}.$$

In addition there is the resource constraint

$$0 \leq W(2) + W(3) + G(2) + G(3) \leq 2.$$

Using the technological constraints we can write the profit function in the form

$$\pi^1 = (\omega(2) - p(1))B(1) + (\omega(3) - p(2))B(2)$$

$+(\gamma(2) - q(1))R(1) + ((\gamma(3) - q(2))R(2))$ (*)

with the constraints

$0 \leq B(1) + B(2) + R(1) + R(2) \leq 2,$

and

$B(j)R(j) = 0, for\ j = 1,2,\ and\ B(j),R(j) \in \{0,1\}.$ (**)

The technological and resource constraints define the feasible set in the four-dimensional subspace whose coordinates are $B(1), B(2), R(1), R(2)$. This set consists of the origin and the eight points whose coordinates have exactly one coordinate different from 0, and the four points that have exactly two coordinates different from 0.

We assume that demand is not satiated. Then the points having only one coordinate diffenent from 0 are not *efficient*. Then we can confine attention to the four points that have two coordinates different from 0, and, of course, those coordinates are each equal to 1.

Because of (**) two of the coordinates in the demand vector for inputs $(B(1), B(2), R(1), R(2))$ must be zero. Therefore we can abbreviate the notation, letting (B, R) represent demand for a unit of B in the first period and a unit of R in the second and (R, B) represents demand for a unit of R in the first period and a unit of B in the second, and so on. The demand correspondence for inputs of a manager of type 1 is shown in the next table as a function of certain intervals of prices.

	$\omega - p(2) > \gamma - q(2)$	$\omega - p(2) < \gamma - q(2)$	$\omega - p(2) = \gamma - q(2)$
$\omega - p(1) > \gamma - q(1)$	(B, B)	(B, R)	$(B, R), (B, B)$
$\omega - p(1) < \gamma - q(1)$	(R, B)	(R, R)	$(R, R), (R, B)$
$\omega - p(1) = \gamma - q(1)$	$(R, B), (B, B)$	$(R, R), (R, B)$	$(R, R), (R, B), (B, R), (B, B)$

Table 3

Recalling that the prices $p(i), q(i)$ are net of the cost of the input to process P_2, which we assumed to be zero, the profit function of the second type of manager is
$\pi^2 = p(1)B(1) + p(2)B(2) + q(1)R(1) + q(2)R(2).$

The constraints are the same as those displayed for process P_1 above. Then supply correspondence for the manager of process P_2 is shown in **Table 4.**

	$p(2) > q(2)$	$p(2) < q(2)$	$p(2) = q(2)$
$p(1) > q(1)$	(B,B)	(B,R)	$(B,R),(B,B)$
$p(1) < q(1)$	(R,B)	(R,R)	$(R,R),(R,B)$
$p(1) = q(1)$	$(R,B),(B,B)$	$(R,R),(B,R)$	$(R,R),(R,B),(B,R),(B,B)$

Table 4

For the coordination mechanism using the market channel to be efficient, it must have an (efficient) equilibrium for every state of demand θ. For the sake of the argument suppose π^2 is maximized at the point $(B(1),B(2),R(1),R(2)) = (1,0,0,1)$, which we also write (B,R), indicating that a unit of B is produced in the period $v = 1$, and a unit of R is produced in the period $v = 2$. Then the profit of $\dot{M}_2$ at (B,R) is

$$\pi^2(B,R) = p(1) + q(2).$$

If this maximizes π^2 on the feasible set, then the following three inequalities must hold.

$$p(1) + q(2) \geq p(1) + p(2)$$
$$p(1) + q(2) \geq q(1) + p(2)$$
$$p(1) + q(2) \geq q(1) + q(2)$$

It follows that

$$p(1) \geq q(1) \quad (1a)$$
$$q(2) \geq p(2) \quad (1b)$$
$$p(1) + q(2) \geq q(1) + q(2) \geq q(1) + p(2) \quad (1c)$$

If (B,R) is to be an equilibrium, it must also maximize π^1 on the feasible set. In CASE 1 $\omega(2) = \omega(3) =_{def} \omega$, and $\gamma(2) = \gamma(3) =_{def} \gamma$. If (B,R) maximizes π^1 on the feasible set, then the following three inequalities must hold.

$$\omega + \gamma - (p(1) + q(2)) \geq 2\omega - (p(1) + p(2))$$
$$\omega + \gamma - (p(1) + q(2)) \geq 2\gamma - (q(1) + q(2))$$
$$\omega + \gamma - (p(1) + q(2)) \geq \gamma + \omega - (p(1) + p(2))$$

which can be written

$$\gamma - (p(1) + q(2)) \geq \omega - (p(1)) + p(2)) \quad (2a)$$
$$\omega - (p(1) + q(2)) \geq \gamma - (q(1)) + q(2)) \quad (2b)$$
$$-(p(1) + q(2)) \geq -(q(1)) + p(2)) \quad (2c)$$

Thus, inequalities (1a) - - (1c) and (2a) - - 2(c) must hold. Together they imply

$$(p(1) + q(2) = (q(1) + q(2) = q(1) + p(2)$$

and hence

$$p(1) = q(1), p(2) = q(2)$$

Then it follows from (2a) that

$$\omega = \gamma.$$

Substituting in the expression for π^1 gives

$$\pi^1 = (\omega - p(1))(B(1) + R(1)) + (\omega - p(2))(B(2) + R(2))$$

Thus, π^1 takes the same value $2\omega - (p(1) + p(2))$ at each of the four nonzero feasible actions, i.e., those that produce (1,0,0,1), (0,1,1,0), (1,1,0,0) and (0,0,1,1) in the subspace with coordinates $B(1), B(2), R(1), R(2)$.

Turning now to π^2 after substitution

$$\pi^2 = p(1)(B(1) + R(1)) + p(2)(B(2) + R(2)).$$

This function takes the same value at every non-zero feasible action of M_2. Because not every pair consisting of a feasible action of M_1 and a feasible action of M_2 is *efficient,* it follows that the mechanism using a market channel does not assure *efficient* performance in CASE 1.

Note that there are competitive equilibria in this example. The price vector

$$(p(1), p(2), q(1), q(2), \gamma(1), \gamma(2), \omega(1), \omega(2))$$
$$= (p(1), p(2), p(1), p(2), \gamma(1), \gamma(2), \gamma(1), \gamma(2))$$

and either of the two commodity vectors

$$(W(1), W(2), G(1), G(2), B(1), B(2), R(1), R(2)) \;\; = (1, 0, 0, 1, 1, 0, 0, 1).$$

or

$$(W(1), W(2), G(1), G(2), B(1), B(2), R(1), R(2)) \;\; = (0, 1, 1, 0, 0, 1, 1, 0).$$

form competitive equilibria. In each case the markets for final goods and for intermediate goods clear. Both equilibria are efficient.

However, if manager M_1 chooses (1,0,0,1) while M_2 chooses (0,1,1,0), the market for final goods clears, but the market for intermediate goods does not. Similarly if the managers choose (0,1,1,0) and (1,0,0,1) respectively. In neither of these cases do we have a competitive equilibrium. Therefore the hypothesis of the First Welfare Theorem is not satisfied, and hence the theorem is vacuously true.

In CASE 2 the profit function of M_2 remains the same as in CASE 1. Hence equations (1a) - - (1c) remain valid. However, it is not required that $\omega(2) = \omega(3)$ or that $\gamma(2) = \gamma(3)$; in general neither of these equalities will hold. The profit function of M_1 in CASE 2 is

$$(\omega(2) - p(1))(B(1) + (\omega(3) - p(2))B(2) +$$
$$(\gamma(2) - q(1))R(1) + (\gamma(3) - q(2))R(2)$$

which is to be maximized subject to the constraints

$$B(1)R(1) = 0 = B(2)R(2)$$
$$0 \leq B(1) + R(1) + B(2) + R(2) \leq 2$$

where all variables are nonnegative integers.

In this case for each of the four possible states of final demand there is a competitive equilibrium with a unique associated production plan for each production unit. For example, the price vector

$$(\omega(2), \omega(3), \gamma(2), \gamma(3), p(1), p(2) q(1), q(2)) \ = (8, 4, 5, 6, 3, 2, 1, 3)$$

is part of the competitive equilibrium that leads to the commodity vector

$$(1, 0, 0, 1, 1, 0, 0, 1).$$

The price vector

$$(\omega(2), \omega(3), \gamma(2), \gamma(3), p(1), p(2) q(1), q(2)) \ = (8, 7, 4, 4, 3, 3, 1, 2)$$

is part of a competitive equilibrium with commodity vector

$$(1, 1, 0, 0, 1, 1, 0, 0).$$

To summarize, in CASE 1, because each producer of a given type has no information on which to distinguish among profit maximizing production plans, each could choose any of them. Thus, there is no assurance of efficient equilibrium. Even when there are several producers of each type, and the number of producers of each type is the same, and even if all producers of a given type choose the same production plan, equilibrium would not be assured. This is, of course, the case when there is just one producer of each type. Thus, the mechanism using a market channel cannot be guaranteed to achieve just-in-time production, and consequently cannot coordinate production without inventories. This result may be anticipated whenever the supply and demand correspondences are not single-valued at equilibrium. This in itself might not be considered too serious a difficulty except for the fact that in this case the multiplicity of values arises not from linearity in the technology, but from relations of timing. These are likely to be prevalent in the economy, and that prevalence indicates that the example is more typical than its simplified form might suggest. The practices associated with just-in-time production in the automobile industry provide a good example. There are many other important examples that attest to the prevalence in the economy of coordination mechanisms that use direct channels. Because the market mechanism cannot ensure just in time production, the criteria specified for comparing mechanisms tell us that informational costs, or informational efficiency properties, do not come into play.

In contrast, in CASE 2 the market mechanism is in the class of efficient mechanisms. Therefore, comparison of the cost or informational efficiency properties of the two mechanisms is in order.

Comparison of the information properties of $\mathcal{M}$ and $\mathcal{D}$ in CASE 2.

To compare the informational cost or efficiency of the two mechanisms requires that we construct the two networks that represent the computations to be carried out. For this we must specify the class $\mathcal{F}$ of elementary functions. For our example it is natural to consider the binary operations of arithmetic, together with the two unary functions, the identity, Id and the sign function, sgn. Thus, the networks used are (2,1)-networks, where $\mathcal{F} = \{add, subt, mult, Id, sgn\}$.

To begin with we consider the informational tasks under the assumption that the relevant networks are given, and that the communication channels exist.

In the direct mechanism the computational task of a manager of type M_1 is to observe the demand vector for final products, and having done so, to select from the four feasible plans a production plan that meets the demand. The computation involved is to choose a vector $W(2), W(3), G(2), G(3))$, to translate that choice into demand for inputs $(B^d(1), B^d(2), R^d(1), R^d(2))$, and then communicating that vector to M_2. This amounts to evaluating a (vector) identify function at the given demand point and sending the result to the other manager. This computation has no delay, and requires at most four cross links, i.e., there are four one dimensional direct channels over which these messages are transmitted.

The task of M_2 is also to evaluate identity functions at the points communicated to her by M_1. Because communication is assumed to be instantaneous and computation of the identity function is also without delay, this computation is without delay.

In the mechanism $\mathcal{M}$, a manager of type M_1 must find a profit maximizing production plan, and hence her demand for the intermediate products, given all the prices. From equation (*) we see that finding the profit maximizing demands for intermediate products reduces to the following computation. M_1 must first compute the four differences

$$\omega(2) - p(1) = a$$
$$\omega(3) - p(2) = b$$
$$\gamma(2) - q(1) = c$$
$$\gamma(3) - q(2) = d.$$

In addition, M_1 must compute the signs of the expressions

$$a + d - b + c$$
$$d - b$$
$$a - c$$
$$a + b - c + d$$

That is, in addition to computing four differences, M_1 must compute two sums and make the comparisons indicated among the eight resulting quantities. Verifying that the maximum profit is nonnegative follows from the signs of the differences and sums. Using (2,1) -networks it follows that if the manager carries out the computation alone, the delay would be eleven units of time, with an additional agent 6 units of time, and with 3 additional agents, the minimum attainable delay of 3 units of time. The calculation of M_2 is similar.

The calculation of the market agent, M_0 is to receive the excess demands of M_1 and M_2, and to select the eight prices to make excess demand equal to zero. When, as is usually the case, there are replications of the producers, M_0 must first aggregate excess demand, which involves executing $2(N - 1)$ adds when there are N managers of each type. Finally, the prices must be communicated from the market agent to all the managers. This could be done by broadcast channels, in effect posting the prices.

It is clear from these comparisons that in this example, the direct mechanism is informationally more efficient than the market mechanism. This seems to be in conflict with the conventional wisdom that market mechanisms are informationally more efficient than alternatives.

However, the calculation we made is based on the assumption that the communication channels exist. In the case of a direct mechanism each direct channel must be created. In industrial practice this usually involves activities of a purchasing agent, or team of them for the unit that is buying, and a sales agent or group of them for the producing unit that is selling. Typically each side invests resources to create and maintain a long term relationship. Negotiation between the two units often takes place repeatedly, even when a long term relationship is established between them, because contracts are usually for a specified period of time. Thus, although once a direct channel is established the cost of using it may be very small, the cost of setting it up and maintaining it can be significant.

Market channels exist in different institutional forms, including organized markets, like the Chicago Board of Trade, or in more decentralized manifestations, which combine informational functions with other service functions. An organized market may be costly to set up, but the costs of creating the market are usually borne by the market intermediaries, and not directly by the buyers and sellers. Buyers and sellers typically pay for each use of a market channel.

As remarked above, it sometimes happens that two producing units, one a supplier to the other, are in the same firm. It is often remarked that the firm could if it wished introduce an internal market channel between the units. It would be interesting to know how frequently coordination of the two units is done by market channels. When the two units are in the same firm it is likely that the cost of setting up and maintaining a direct channel between them is relatively small, while the cost of creating a genuine market channel is relatively large. Furthermore, to create and maintain an internal market may be more difficult in light of incentive issues when the channel is internal to a single firm. It should also be noted that the use of internal transfer prices is in itself not sufficient to constitute an internal market. A mechanism that looks like a competitive market can be modified so that it becomes a central planning mechanism that uses prices and competitive behavior rules. The modification needed is to allow the market agent to communicate prices and quantities to each producing unit directly. In CASE 1 that extension of the market agent's function would allow him to select one of the equilibria from the excess demand correspondence and communicate each manager's part of that equilibrium to that manager, in effect communicating the production plan. In the example a direct mechanism would bring about efficient coordination more cheaply. Thus, if the example were typical, except for incentive reasons, we should not expect to observe price coordinated central mechanisms in such cases. Indeed, casual observation suggests that we do not see such mechanisms very often. Usually incentive schemes are tacked on to direct coordination mechanisms in such cases. However, if the computations required to operate the direct mechanism were sufficiently more complex than the calculations involved in figuring out profit maximizing actions, the comparison might go the other way.

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