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Games with Espionage

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"There is no place where espionage is not possible."

- Sun Tzu, *The Art of War*, approximately 500BC.

Abstract

We consider extensive form and normal form games in which players decide on their strategies before the start of play and can purchase noisy information about their opponents' decisions concerning future response policies (i.e., *spy* on their opponents' decisions). This addition to the agent's optimization problem naturally changes the set of subgame perfect equilibria (SPE). For example, in the chain-store model, for sufficiently small costs of espionage, the population of Incumbents splits into a positive fraction that accommodates and a positive fraction that fights. For general 2x2 games in extensive form, the existence of equilibria with espionage turns out to depend on the difference between the Stackelberg equilibrium payoffs and the SPE payoffs. We characterize the set of equilibria with espionage as a subset of the set of correlated equilibria. Welfare and Pareto properties of such equilibria are also explored.

Key Words: Espionage, subgame perfect equilibria, information, timing.

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1 Introduction

In many real world interactions players decide what to do long before they have to play the chosen action - an army prepares for different situations in the battlefield years before the war begins; a government decides on its policy and reactions to various scenarios before starting negotiations; some people are born fighters and others peaceful (thus their choices are made even before they know they might play the game).

Once decisions are made in advance, espionage comes into mind. Suppose players F and S engage in a two-stage sequential game that prescribes F to be the first to play an action and S to be the second. If S decides on her reactions to F's move in the outset of the game, F might benefit by sending spies that will reveal the decisions made by S. In essence, if F spies on S, it is as if the order of actions is switched. Thus, employment of espionage depends on the existence of a "second-mover advantage." Even if espionage is costly and provides a noisy signal of S's decisions, player F may still profit by utilizing it.

We thus consider the case where a player can purchase information as the game proceeds. The motivation for this inquiry comes from the attempt to explain the employment of different institutions providing information in a variety of economic environments. To mention a few examples, investors can employ experts that report on different attributes of firms to allow better stock investments; in certain industries, engagement in industrial espionage is common practice; specialists are often hired to give forecasts before certain projects are undertaken (e.g., political authorities for defense projects, geologists before starting new settlements, etc.).

Despite the prevalence of espionage opportunities, it turns out that the mere *option* to spy may in fact reduce a player's profit, since it can allow the second player to exploit a first mover advantage in the game. In such cases the first player would prefer not to have the ability to spy, since this ability reduces his expected payoff in equilibrium. Nonetheless, not utilizing his spying capabilities may make him even worse off.

The role of espionage is, thus, unclear. If only a single player can spy, does he always profit? Are there games where all players profit when espionage is available? Even if not all players profit, maybe society as a whole profits; that is, the sum of payoffs of all the players might increase when espionage is possible. In such a case a social planner will provide tools for spying.

We provide an extension of the chain-store model where the option of espionage is available. Players 1 and 2 correspond to the Entrant and the Incumbent in the standard terminology. (i) First, Player 2 chooses her reaction to the move of Player 1. (ii) Then, Player 1 has an option to purchase an espionage device, which reveals the action chosen by Player 2 with some accuracy. Player 1 can choose not to utilize this option and to receive no information. (iii) Finally, Player 1 chooses an action. His move is announced to Player 2, who plays according to her chosen strategy.

There is a continuum of espionage devices, and the cost of a device indicates the accuracy of the information it provides (in particular, zero accuracy corresponds to no espionage and is costless). Thus, there is a tradeoff between the cost of a device and the information gain of the player using it.

It is easy to see that *any pure* subgame perfect equilibrium in the original game is also a subgame perfect equilibrium in the extended game, where the players do not utilize their option of spying. Indeed, if the opponent's strategy is pure, no information can be gained by way of costly espionage.

However, there are many cases where there are new subgame perfect equilibria. For example, in the chain-store model there is a subgame perfect equilibrium where the Incumbent accommodates or fights, both with positive probability, and the Entrant purchases an espionage device and enters or stays out according to the signal he receives from the device. This differs from reputational explanations (see, e.g., Kreps, Milgrom, Roberts, and Wilson [1982] and Fudenberg and Levine [1989, 1992]) both in assumptions and results. We do not assume anything about the distribution of types of Incumbents. Hence, the somewhat problematic assumption of "irrational Incumbents" is not needed in this model. Moreover, our results predict that a non-vanishing portion of the population of Incumbents will in fact accommodate.

In this equilibrium the payoff of the Entrant is smaller than his payoff in the subgame perfect equilibrium of the original game, but society as a whole profits when the cost of espionage is low enough. Nonetheless, it turns out that the device that the Incumbent purchases (i.e., the information acquired) does not depend on its cost. The cost of the device only influences the probability that the Incumbent will fight. However, if the cost of this device is too high, the Entrant will not profit by purchasing it, and there will be no subgame perfect equilibrium where the option to spy is used.

We generalize the chain-store example and characterize 2×2 extensive form games for which only one player profits from the existence of espionage and such games for which both players profit from the availability of espionage. These two classes turn out to be exhaustive. We also discover that for both classes, for sufficiently low costs of information, espionage provides an efficiency improvement.

Normal form games can be viewed as extensive form games with imperfect information and are thus a natural generalization of chain-store models. In general normal form games, a Nash equilibrium always exists when mixing between purchasing and not-purchasing information is allowed. However, if a player can choose only among certain information devices, equilibrium may not exist. It turns out that under certain convexity assumptions on the cost function, an agent will mix at most two devices: the null device and a non-trivial device. We also give conditions for existence of pure equilibria.

Since information devices allow for players to correlate their actions, the relation between equilibria with espionage and correlated equilibria seems a natural one. We provide a characterization of equilibria with espionage as a specific subset of the set of correlated equilibria. Namely, the subset of correlated equilibria in which Player 2 (who is the first to choose her strategy and has no ability to spy) is indifferent between the signals she can get with positive probability.

Games with endogenous timing have been tackled with in the Industrial Organization literature. Timing of output choice in the market determines the competition structure. Sequential choice corresponds to a Stackelberg game, where the first firm to make a choice is termed the Stackelberg leader and the second is termed the Stackelberg follower. Simultaneous choice of output corresponds to a Cournot competition. Mailath [1993] allows a firm with superior information to delay its quantity decision until the decision period of the less informed firm (so that decisions are made simultaneously). The unique stable equilibrium turns out to be one in which the informed firm moves first, even though the leader may earn lower ex-ante profits than it would earn if it was choosing quantities simultaneously with the follower. Sadanand and Sadanand [1996] generalized Mailath's results and showed that when there is demand uncertainty and firms endogenously choose entry timing, *relative* firm sizes *and* uncertainty jointly determine the equilibrium. Van-Damme and Hurkens [1996, 1997] study the endogenous timing problem in the context of commitment. In their model, players can see the actions of players who moved before them. Thus, a player can turn the underlying simultaneous game to a sequential game in which she is the first to move. A player will then choose an action early in the game if she has a "first-mover advantage." Our paper adds to this branch of literature in that the underlying game can be sequential and the change of turns is both probabilistic and costly. Thus, part of the optimization problem is the determination of how much resource is to be allocated for switching turns and exploiting the "second-mover advantage," if it exists.

In our model the cost of information is exogenous. There is a vast literature dealing with the value of information. Several authors (e.g., Hirshleifer [1971], Green and Stokey [1981], and Allen [1986]) studied the value of private information to a player. Others (e.g. Kamien, Tauman, and Zamir [1990] and the references therein) considered a situation in which an agent possesses information relevant to the players of a game in which he is not a participant. The value of information is then defined according to the amount this agent can achieve by behaving strategically. We view these theories as possible foundations for the cost function which we take as given.

The literature on espionage per-se appears to be very sparse. Matsui [1989] did consider the problem of espionage, but from a different angle. He considered the case of an infinitely repeated two-person game in which there is an exogenous small probability that one or both of the players will be perfectly informed of the other's supergame strategy at the outset of the game. The players have a chance to revise their strategies on the basis of this information before actual play begins. Matsui's main result is that any subgame perfect equilibrium pair of payoffs is Pareto efficient, provided that the probability of espionage is sufficiently small. Unsurprisingly, our model yields different predictions. In particular, not all subgame perfect equilibria with espionage are Pareto efficient.

We begin by analyzing a few motivating examples in Section 2. We then provide the general framework for our analysis in Section 3. In Section 4 we specify existence conditions for equilibria with non-trivial utilization of espionage. Section 5 contains the characterization of the set of equilibria with espionage via equivalence to a subset of the correlated equilibria. Section 6 concludes. Technical proofs are relegated to the Appendix, where we also discuss what are reasonable cost functions of information devices.

2 Examples

In this section we provide several examples that illustrate the main results of the paper. All the examples are of games in which each player has only 2 possible actions, and the information devices are symmetric – they report the correct action with some probability and the incorrect action otherwise.

We begin by studying games in extensive form. In Example 1 we study the standard chain-store model, and characterize when there is an additional subgame perfect equilibrium in which espionage is used, and when this new equilibrium is more efficient.

We then provide a game where both players benefit if Player 1 uses his ability to spy.

Next, we study games in normal form. We first study the Matching Pennies, and find necessary and sufficient conditions on the cost of devices for the game to have a pure-espionage equilibrium (i.e., an equilibrium that involves the use of only one information device). We conclude with the observation that there are games where a pure-espionage equilibrium need not exist. We also provide an asymmetric version of the matching pennies that has no pure-espionage equilibrium and, if the cost of information is too low, has no equilibrium without utilizing espionage either.[1]

[1] These observations are reminiscent of some of the results in the auditing literature (see, for example, Townsend [1979] or Mookherjee and Png [1989]).

EXAMPLE 1: Consider the following extensive-form game (the standard chain-store model):

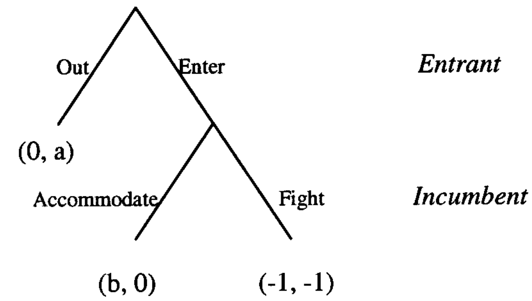

Figure 1

where a>0, b>0.

The game is played by an Entrant and an Incumbent. The Entrant decides whether to enter the market or stay out. If the Entrant enters, the Incumbent has to decide whether to Fight or Accommodate. The payoffs are as given in Figure 1. The first element of any payoff pair corresponds to the Entrant's payoff and the second element corresponds to the Incumbent's payoff.

It is well known that the unique subgame perfect equilibrium is comprised of the Entrant entering and the Incumbent accommodating, whereby the equilibrium payoff is (b, 0).

Suppose now that the Incumbent must decide on her reaction before the Entrant chooses whether or not to enter and that the Entrant can purchase information on the decision of the Incumbent. As mentioned in the Introduction, the pure subgame perfect equilibrium remains a subgame perfect equilibrium in the extended game. We now proceed to find another subgame perfect equilibrium where the Entrant uses his ability to spy.

Suppose that in equilibrium p is the probability with which the Incumbent accommodates and $\Phi(q)$ is the spying device purchased by the Entrant before entering: the Entrant receives the correct report with probability q. The cost of $\Phi(q)$ is $\varphi(q)$, $1/2 \leq q \leq 1$, which we assume to be twice differentiable, increasing, and convex in q. We assume that the cost of the null device is 0, thus $\varphi(1/2)=0$.

DEFINITION 1: *A device is* effective *if the Entrant plays a best reply against the report of the device.*

In the context of the chain-store model, if the report is "Fight" the Entrant stays out, while if the report is "Accommodate" the Entrant enters.

In Lemma 1 below we prove that in any 2×2 game in normal form, if a device is purchased in equilibrium, then it is effective.

We will now find the exact values of p and q that constitute an equilibrium with espionage. In such an equilibrium $0 < p < 1$ (else no espionage is needed). Since in equilibrium the Incumbent is indifferent between fighting and accommodating, and the Entrant receives a correct report with probability q, it follows that

$$q = (1+a)/(1+2a) > 1/2.$$

In particular, it follows that the espionage device that is purchased by the Entrant is *independent of its cost.* If the cost is very high using espionage cannot be profitable for the Entrant, but for sufficiently low costs of espionage, the quality of the purchased device is determined solely by the Entrant's payoff from choosing Out.

The Entrant maximizes his expected payoff with respect to p, thus solving:

(1) $$\max_q \{pqb + (1-p)[-(1-q)] - \varphi(q), \max\{0, pb - (1-p)\}\},$$

where the first term is his payoff if he purchases the device q, and the latter if he doesn't purchase any device. If φ is strictly convex then (1) has a unique solution. The F.O.C that corresponds to the first part in (1) implies that if an espionage device is purchased then

$$p = (\varphi'(q)-1) / (b-1).$$

Thus, there exists an equilibrium with espionage if and only if

(2) $$0 < p < 1,$$

(3) $$pqb + (1-p)[-(1-q)] - \varphi(q) \geq 0 \quad \text{and}$$

(4) $$pqb + (1-p)[-(1-q)] - \varphi(q) \geq pb-(1-p).$$

One can verify that $b > 1$ implies that $\varphi'(q) > b$ and inequality (3) implies (4), whereas if $b < 1$ then $\varphi'(q) < b$ and inequality (4) implies (3). Moreover, for every positive a and b for which (2) is satisfied, there is a cost function that satisfies (3) and (4).

Note that in an equilibrium with espionage the Entrant receives a payoff which is smaller than the payoff he receives in the subgame perfect equilibrium of the game without the option to spy. Nonetheless, for certain φ's, espionage provides an efficiency improvement. Indeed,

PROPOSITION 1: *Assume either a>b>1 or 0<a<b<1. There exists a cost function φ such that the payoffs corresponding to equilibria with espionage constitute a more efficient outcome than the payoffs corresponding to the subgame perfect equilibrium without espionage.*

Proof: The Entrant's payoff is

$$\{\frac{1+a}{1+2a}b[\varphi'(\frac{1+a}{1+2a}) - 1]/[b - 1] - \frac{a}{1+2a}[b - \varphi'(\frac{1+a}{1+2a})]/[b - 1]\} - \varphi(\frac{1+a}{1+2a}),$$

while the Incumbent's payoff is a-qa = a^2 / (1+2a).

Assume φ((1+a) / (1+2a)) is arbitrarily small. Using continuity, it suffices to show that the sum of the players' utilities is larger than b for φ'((1+a) / (1+2a)) = b. This is equivalent to a(b - 1)(a - b) > 0 and is satisfied if either a > b > 1 or 0 < a < b < 1. ^

NOTE: The payoffs corresponding to equilibria with espionage are not Pareto efficient. This stands in sharp contrast to the main message of Matsui [1989] that information leakage leads to Pareto efficient outcomes.

EXAMPLE 2: Both players profit when the Entrant uses his ability to spy.

Consider the following extensive-form game (we keep the notation of Entrant and Incumbent (instead of Players 1 and 2) in order to make the comparison with Example 1 more transparent):

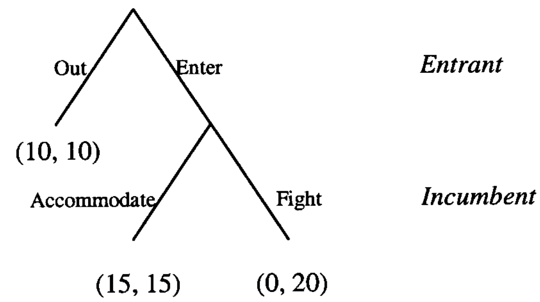

Figure 2

Without espionage, the unique SPE is comprised of the Entrant staying out and the Incumbent fighting upon entrance. The corresponding payoffs are (10, 10)

A similar analysis to that performed for the first example gives conditions for the existence of an equilibrium with espionage. Denote by p the equilibrium probability that the incumbent accommodates if the Entrant enters, and by $\Phi(q)$ the equilibrium device purchased by the Entrant. Then $q = 2/3$ and $p = 2 - \varphi'(2/3)/5$. One can verify that there is such an equilibrium if $5 < \varphi'(2/3) < 10$ and $\varphi(2/3) \leq 10/9$. It is clear that both Players get at least 10 in such an equilibrium (the Entrant has the alternative to stay out and get 10, while 10 is the lowest payoff in the game for the Incumbent), hence the ability to spy leads to a Pareto improvement over the SPE result.

The next examples are of 2×2 games in normal form. Player 1 is the row player and Player 2 is the column player. Player 2 chooses her actions first, and Player 1 has the option to purchase an information device before he has to choose his action.

EXAMPLE 3: Matching Pennies. We look at the standard 2×2 matching pennies game.

	Left	Right
Top	1, 0	0, 1
Bottom	0, 1	1, 0

If Player 2 assigns probability y to Left in equilibrium, Player 1 solves:

$$\max_q \{yq + (1-y)q - \varphi(q), \max\{y, 1-y\}\} = \max_q \{q - \varphi(q), \max\{y, 1-y\}\}.$$

The first term in the maximization refers to the payoff achieved by purchasing information and the second term corresponds to the maximal payoff achievable without purchasing information.

Denote by $\Phi(q^*)$ the information device purchased by Player 1 in an equilibrium with espionage (if such an equilibrium exists). Then q* is chosen to maximize the first term in the above problem. The first order condition implies that $1 = \varphi'(q^*)$ and q* does depend on the cost function.

For an equilibrium with espionage we need $\varphi'^{-1}(1) - \varphi(\varphi'^{-1}(1)) \geq 1/2$. Note that for this specific game, any $y \in [1 - \varphi'^{-1}(1) + \varphi(\varphi'^{-1}(1)), \varphi'^{-1}(1) - \varphi(\varphi'^{-1}(1))]$ is part of an equilibrium.

EXAMPLE 4: Non existence of pure-espionage equilibrium.

Consider the following zero-sum game:

	Left	Right
Top	1, -1	0, 0
Bottom	0, 0	2, -2

This is the Matching Pennies game with different payoffs to different matchings.
We claim that there is no pure-espionage equilibrium in this game.
The mixed equilibrium in the game without espionage is ((2/3, 1/3), (2/3,1/3)).
Suppose Player 2 plays a mixed strategy (y, 1-y). Player 1 then solves

$$\max_q\{yq+2(1-y)q-\varphi(q), \max\{y, 2(1-y)\}\}=$$

$$=\max_q\{2q-yq-\varphi(q), \min\{y, 2(1-y)\}\}.$$

For φ small enough (e.g., $\varphi(3/4)<1/3$), the mixed equilibrium in the game without espionage is no longer an equilibrium in the extended game. However, if information of quality $q>1/2$ is purchased, the payoff of Player 2 is $-yq-2(1-y)q=yq-2q$, which is maximized at $y=1$. Hence, for sufficiently low cost functions there is no equilibrium where Player 1 purchases a unique information device.

One could conceive that Example 4 is specific in that we restrict ourselves to symmetric devices – devices that report the correct report with some probability q, independently of the action Player 2 chose. It is not difficult to check that for the "Matching Pennies" (Example 3) there exists a cost function such that the extended game has no pure-espionage equilibrium, when we allow for a larger set of devices: all devices of the form $\Phi(q_1, q_2)$, where q_1=Prob(signal=L | action=L) and q_2=Prob(signal=R | action=R).

3 General Framework

We consider two-player games in normal form. Player 1 is the row player and Player 2 is the column player. We denote by $A = (a_{ij})$ and $B = (b_{ij})$ the payoff matrices of the two players. A *game in normal form with espionage* is a tuple $G = (A, B, S, Q, \varphi)$ where (i) A and B are $n\times m$ payoff matrices, (i) S is a finite set of signals, (iii) Q is a set of functions $q : J \rightarrow \Delta(S)$, where $\Delta(S)$ is the set of probability distributions over S. For each $q \in Q$ corresponds an information device $\Phi(q)$, which, when action j is chosen by Player 2, gives a (probabilistic) signal s with probability q(s, j). Finally, (iv) $\varphi : Q \rightarrow \mathbf{R}$ is the cost of information device $\Phi(q)$. In the sequel we will not distinguish between a function $q : J \rightarrow \Delta(S)$ and the corresponding device $\Phi(q)$.

The game is played as follows:

Stage 1 - Player 2 chooses an action j in $J = \{1, \ldots, m\}$.

Stage 2 - Player 1 purchases an espionage device $\Phi(q)$ from a set Q of available devices.

Stage 3 - Player 1 receives a signal s from a set of signals, where prob(s|j) = q(s, j).

Stage 4 - Player 1 chooses an action i in $I = \{1, \ldots, n\}$.

The payoff for the players is $(a_{ij}- \varphi(q), b_{ij})$.

A strategy for Player 2 is a mixed action y over J. A strategy for Player 1 is a pair (μ, x) where μ is a probability distribution over devices, and x=x(q, s) is a (measurable) function that assigns to each information device that was chosen at stage 2 and each signal that was received at stage 3 a probability distribution over I. If μ is concentrated on a single information device it is called a *pure-espionage strategy*. In that case we write x=x(s). Otherwise it is called a *mixed-espionage strategy*.

We can present this game as a game in extensive form as follows:

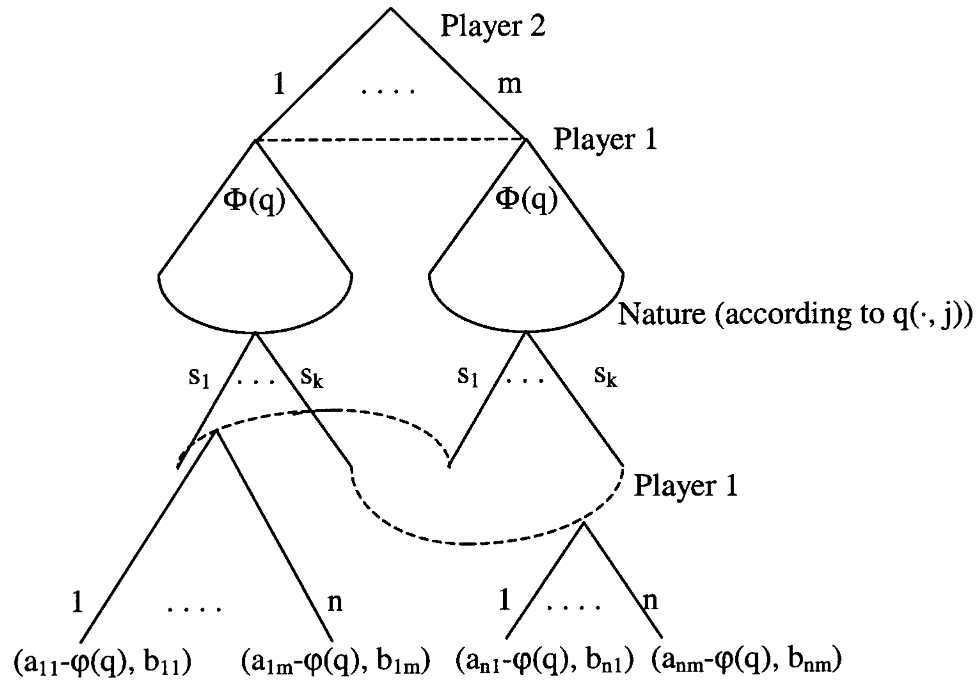

Figure 3

We denote by $\pi^l(y; q, x)$ the payoff to Player l when Player 2 plays the (mixed) strategy y, and Player 1 uses the pure-espionage strategy (q, x). Note that if $\varphi(q)$ is quasi-convex, then $\pi^l(y; q, x)$ is a quasi-concave function for any fixed $x=x(s)$.

Espionage equilibria are perfect Bayesian equilibria (PBE) of the extended game, as depicted in Figure 2. *Pure-espionage equilibria* are espionage equilibria in which Player 1 is using a pure-espionage strategy. Similarly, *mixed-espionage equilibria* are espionage equilibria in which Player 1 is using a mixed-espionage strategy.

In general, the cost function is a function from the set of Markov matrices to the real numbers. However, one might want to impose conditions on the cost function. For example, swapping two columns in the matrix does not change the information of Player 1 whatsoever, but changes the device we are dealing with. One would like the cost function to give the same cost to such two matrices.

In general, we would expect one information device to cost more than another if and only if it is "more informative." Blackwell [1950] defined a partial preference ordering on information devices (known also as *garbling* in the information theory literature).

DEFINITION 2: *Let P_1 and P_2 be two n×m Markov matrices. $P_1 \succ P_2$ if and only if there exists an m×m Markov matrix M such that $P_2 = P_1M$.*

Intuitively, P_2 is defined to be at least as good as P_1 if P_1 is a noisy distortion of P_2. Alternatively, P_2 is at least as good as P_1 if a player who receives information according to P_2 can pretend to be playing according to P_1 by ignoring some of his information. In particular, Player 1 will achieve at least as high a payoff with device P_2 as with device P_1, for *any* game.

Thus, we confine our discussion to the set of quasi-convex cost functions over Markov matrices that preserve the Blackwell relation. Since the determinant is a quasi-convex function over Markov matrices that preserves the Blackwell relation, this set is not empty. In the sequel we will see other functions in this set.

In Appendix B we provide a geometric interpretation of the Blackwell relation, and we prove that any quasi-convex function over n×2 Markov matrices preserves the Blackwell relation.

4 Existence of Equilibria with Espionage

It is easy to see that any pure equilibrium in the game (A, B) corresponds to a pure equilibrium in the extended game, where the option to spy is not used[2]. Moreover, any n×m game in normal form with espionage that satisfies (i) Q is compact and (ii) φ is quasi-concave, has an equilibrium in mixed-espionage strategies. Indeed, since the spaces of pure actions of both Players 1 and 2 are compact, it follows that the space of mixed-action combinations, which are probability measures over a compact set, is compact in the w^*-topology. The payoff function is continuous and quasi-concave, hence the best-

[2] One class of games that has been recently studied in the literature and is comprised of games that always possess a pure equilibrium is that of potential games (see Monderer and Shapley [1996]).

reply correspondence has non-empty and convex values, and its graph is closed. By Kakutani's fixed point theorem, an equilibrium in mixed strategies exists.

Note that since Player 2 plays *before* Player 1, the equilibrium strategy of Player 2 can be taken to be a mixed action, rather then a probability measure over mixed actions.

4.1 Chain Store Models

In this subsection we study chain-store models; that is, 2×2 games where Player 1 has an action that yields the players the same payoff, regardless of the action of Player 2. It is more convenient to present this game in extensive form. The general game without espionage is thus:

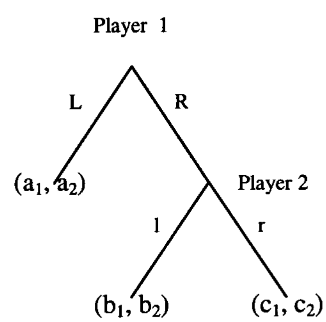

Figure 4

For simplicity of exposition, we assume that each player's payoffs differ across terminal nodes. W.l.o.g. we assume that $c_2 > b_2$.

We provide two theorems: one characterizes the conditions under which there exists an equilibrium with espionage, and the other characterizes the conditions under which this new equilibrium is more efficient than the SPE of the original game.

The proof of Theorem 1, which is rather tedious, is relegated to Appendix A.

THEOREM 1: *There exist cost functions for which an equilibrium with espionage exists if and only if the SPE is different from the Stackelberg equilibrium with Player 2 being the Stackelberg leader.*

Theorem 1 is rather intuitive. Divergence of the Stackelberg payoff from the SPE payoff implies that Player 2 would prefer to use a reaction which is sub-optimal for her in order to get Player 1 to choose an action that differs from that prescribed by the SPE. That is, Player 2 faces a trade-off between choosing a reaction policy that is optimal if realized (direct effect) and choosing a reaction policy that is sub-optimal, but causes Player 1 to choose a beneficial action (indirect effect). In the original game commitment is not possible and thus, according to the definition of SPE, no player chooses an action that is sub-optimal at any decision node. However, the existence of espionage allows Player 2 to commit herself (albeit probabilistically) to a sub-optimal reaction. Thus, as long as the costs of espionage are not extreme (low or high), espionage makes the trade-off between the direct effect and the indirect effect on Player 2's payoffs non-trivial. It is in these situations that an equilibrium with espionage arises.

Theorem 2 gives a characterization of when existence of espionage provides an efficiency improvement, as captured by the sum of the players' payoffs.

THEOREM 2: *Efficiency is affected by the option to spy according to the structure of the game being played:*

1. *If $b_1 < a_1 < c_1$, $a_2 > c_2$, and $a_1+a_2 < c_1+c_2$ then for small enough cost functions efficiency is improved in the PBE with espionage over the pure SPE in the original game, but the ability to spy never increases Pareto efficiency. If $a_1+a_2 > c_1+c_2$ then any PBE with espionage is inferior efficiency-wise to the pure SPE in the original game.*
2. *If $c_1 < a_1 < b_1$ and $b_2 > a_2$ then any PBE with espionage provides a Pareto improvement over the pure SPE in the original game. In particular, efficiency is increased.*

Proof: Case 1 is equivalent (up to an affine transformation of the players' payoffs) to that studied in Example 1 in Section 2.

In case 2, the unique pure SPE is (L, r) which yields the payoff pair of (a_1, a_2). Assuming that the cost function is arbitrarily small, in any equilibrium with espionage Player 2 gets an expected payoff that is a convex combination of a_2, b_2 and c_2, and is thus strictly higher than a_2. Individual rationality of Player 1 assures that in any equilibrium of the expanded game he receives at least a_1. Thus, any PBE with espionage constitutes a Pareto and efficiency improvement over (L, r). ^

4.2 Pure-Espionage Equilibrium

In this subsection we consider 2×2 normal form games with symmetric information devices: the signal space is assumed to be the action set of Player 2, and Q = [1/2, 1]. Results similar to the ones we present here can be derived for general information devices. The analysis is, however, more cumbersome.

We denote Player 1's pure actions by I = {Top (T), Bottom (B)} and Player 2's pure actions by J = {Left (L), Right (R)}.

We assume that the cost function $\varphi = \varphi(q)$, that depends on a single number $1/2 < q < 1$, satisfies $\varphi(1/2) = 0$, $\varphi'(q) > 0$, $\varphi''(q) > 0$. We also assume that no player has a dominating strategy. In particular, w.l.o.g. $a_{11}>a_{21}$ and $a_{22}>a_{12}$.

Recall that a device is efficient if Player 1 plays a best response against the signal he receives.

LEMMA 1: *If an information device is purchased in equilibrium, then it is effective.*

Proof: Assume player 2 chooses L with probability y. Denote by V(b, s) the expected payoff of Player 1 if he receives the signal s and plays the action b when the information device purchased is $\Phi(q)$ (for ease of notation we omit the dependence of V on q).

$$V(T, L) = \frac{qya_{11} + (1-q)(1-y)a_{12}}{qy + (1-q)(1-y)} - \varphi(q)$$

$$V(T, R) = \frac{(1-q)ya_{11} + q(1-y)a_{12}}{(1-q)y + q(1-y)} - \varphi(q)$$

$$V(B, L) = \frac{qya_{21} + (1-q)(1-y)a_{22}}{qy + (1-q)(1-y)} - \varphi(q)$$

$$V(B, R) = \frac{(1-q)ya_{21} + q(1-y)a_{22}}{(1-q)y + q(1-y)} - \varphi(q)$$

If V(T, L) - V(B, L) and V(T, R) - V(B, R) have the same sign or if either equals to zero, then Player 1 will profit more by playing the same action whatever signal he receives, foregoing the option to purchase information.

If V(T, L) > V(B, L) and V(T, R) < V(B, R) Player 1 profits most by following the signal he receives.

Since we restrict our discussion to $q \geq 1/2$, the two inequalities V(T, R) > V(B, R) and V(T, L) < V(B, L) cannot hold together. Indeed, these two inequalities hold if and only if:

$$qya_{11} + (1-q)(1-y)a_{12} < qya_{21} + (1-q)(1-y)a_{22} \text{ and}$$

$$(1-q)ya_{21} + q(1-y)a_{22} < (1-q)ya_{11} + q(1-y)a_{12}.$$

These two inequalities are equivalent to:

$$qy(a_{11}-a_{21}) < (1-q)(1-y)(a_{22}-a_{21}) \text{ and}$$

$$(1-q)y(a_{11}-a_{21}) > q(1-y)(a_{22}-a_{21}).$$

Since $a_{11}>a_{12}$ and $a_{22}>a_{21}$, dividing the two inequalities leads to $q/(1-q) < (1-q)/q$, which holds only if $q < 1/2$. ^

LEMMA 2: *Any equilibrium of the original game in which Player 2 uses a pure strategy, is an equilibrium in the extended game where espionage is not utilized.*

Proof: If Player 2 plays a pure strategy, Player 1 cannot profit by a costly purchase of information. The result follows from the definition of an equilibrium. ^

LEMMA 3: *If (A, B) possesses a fully mixed equilibrium (x_0, y_0), then it induces an equilibrium (without utilizing espionage) in the extended game G if and only if:*

$$2(a_{22} - a_{12})(a_{11} - a_{21}) / (a_{22} - a_{12} + a_{11} - a_{21}) < \varphi'(1/2).$$

Proof: Let (x_0, y_0) be a fully mixed equilibrium in (A, B). From the indifference condition of Player 1, $y_0 = (a_{22} - a_{12}) / (a_{22} - a_{12} + a_{11} - a_{21})$. If ($x_0$, y_0) does not induce an equilibrium without espionage in the extended game, then Player 1 is better off by purchasing some information device. By Lemma 1, a best reply of Player 1 involves playing a best response against the signal he receives.

Denote by $x^*=(x^*(s))$ the strategy of Player 1 that indicates his best replies against all the signals he can possibly receive. Then (x_0, y_0) induces an equilibrium without utilizing espionage in the extended game if and only if the concave function $f(q) = \pi^1(y_0; q, x^*))$ achieves it's maximum at $q \leq 1/2$. We therefore have,

$$0 \geq \partial\pi^1 / \partial q\ (y; 1/2, q^*) = ya_{11} - ya_{21} - (1-y)a_{12} + (1-y)a_{22} - \varphi'(1/2)$$

Substituting y for its value proves the claim. ^

In Appendix A we prove the following general result, which characterizes when a pure-espionage equilibrium exists.

Denote $\alpha = a_{11} - a_{12} + a_{21} - a_{22}$ and $\beta = b_{11} - b_{12} + b_{21} - b_{22}$.

THEOREM 3: *Let* (A, B) *be a 2×2 game in normal form. Assume that* $a_{11} > a_{12}$ *and* $a_{22} > a_{21}$.

i) *There exists a cost function* φ *such that the extended game* (A, B, J, [1/2, 1], φ) *has a non-trivial pure-espionage equilibrium (for a suitable* S*) if and only if one of the following holds*

   a) $\beta = 0$ *and* $b_{11} = b_{22}$.

   b) $\beta \neq 0$ *and* $1/2 < (b_{12} - b_{21}) / \beta \leq 1$.

ii) $\Phi(q)$ *is the information device purchased by Player 1 in equilibrium if and only if*

   a) $\beta q = b_{12}-b_{21}$.

   b) *If* $\alpha \neq 0$ *then* $\min\{ a_{11}-a_{21}, a_{22}-a_{12} \} < \varphi'(q) < \max\{ a_{11}-a_{21}, a_{22}-a_{12} \}$ *and*

   $\varphi(q) \leq \min\{ (1-y)q(a_{22}-a_{12}) - y(1-q)(a_{11}-a_{21}), yq(a_{11}-a_{21}) - (1-y)(1-q)(a_{22}-a_{12}) \}$

   where

(5) $$y = (\varphi'(q) + a_{12} - a_{22}) / \alpha.$$

   c) *If* $\alpha = 0$ *then* $\varphi'(q) \leq a_{22}-a_{12}$ *(with equality if* q=1) *and* $\varphi(q) \leq (q-1/2)(a_{22}-a_{12}) = (q-1/2)(a_{11}-a_{21})$.

In the proof we show that if $\alpha \neq 0$ then $y = (\varphi'(q) + a_{12} - a_{22}) / \alpha$ is the mixed action chosen by Player 2 in equilibrium. Note that the game in Example 3 satisfies $\alpha = \beta = 0$.

4.3 Mixed-Espionage Equilibrium

As mentioned before, any game with espionage has an equilibrium in mixed strategies. As we show now, when the players have only two actions, Player 1 randomizes in equilibrium between at most two different information devices, one of which is the trivial device.

PROPOSITION 2: *Let* $(y;\ \mu,\ x)$ *be a mixed-espionage equilibrium. Then there is* $q^* \in (1/2,1]$ *such that* μ *gives probability 0 to* $(1/2, q^*) \cup (q^*, 1]$.

Proof: Denote by $x^*(q, s)$ the best reply of Player 1 to the signal s. By Lemma 1, $x = x^*$ μ-a.s. Assume that the proposition does not hold. Then the average value of μ, conditioned on the interval (1/2, 1], is well defined. Denote this average by q^*. The joint distribution on pairs (b, s), where b is an action chosen by Player 2 and s is a signal reported to Player 1, is linear in the device purchased by Player 1. Hence both μ and q^* induce the same joint distribution over the space of these pairs, and therefore the same expected payoff for Player 1. Since φ is strictly convex, the cost of q^* is lower then the expected cost of the device chosen by μ, hence $(y;\ \mu,\ x)$ is not an equilibrium. $\wedge$

5 Espionage and Correlated Equilibria

Espionage ensures that players' actions are correlated. This suggests a comparison between the set of equilibria with espionage and the set of correlated equilibria. In this context, $\Phi(q)$ serves as a correlation device. In this section we set $Q = \{\ q : B \to \Delta(S)\ \}$. Thus we no longer consider only symmetric information devices, and we assume that Player 1 can purchase any conceivable device.

EXAMPLE 5: Consider the following example of a 3x3 game (Moulin and Vial [1978]):

	L	M	R
T	0, 0	1, 5	5, 1
I	5, 1	0, 0	1, 5
B	1, 5	5, 1	0, 0

The only Nash Equilibrium without espionage is {(1/3, 1/3, 1/3), (1/3, 1/3, 1/3)}.

Assume that the signal space is {"Not L", "Not M", "Not R"} and that Player 1 can purchase the following information device q with low cost:

q(Not L, L) = 0,   q(Not M, L) = 1/2,   q(Not R, L) = 1/2

q(Not L, M) = 1/2,   q(Not M, M) = 0,   q(Not R, M) = 1/2

q(Not L, R) = 1/2,   q(Not M, R) = 1/2,   q(Not R, R) = 0

This device allows Player 1 to rule out one of the actions that Player 2 *did not* choose. An equilibrium in that environment is for Player 2 to play (1/3, 1/3, 1/3) and for Player 2 to purchase information (up to a certain threshold) and play T, I or B, depending on whether the signal was "Not L", "Not M" or "Not R" respectively. The diagonal entries are not reached in equilibrium and the corresponding payoff pair, not including the cost of espionage, is then (3, 3), which corresponds to the optimal correlated equilibrium of this game.

NOTE: Such a construction cannot be replicated for 2x2 games. It follows from Theorem 4 below that no matter what the cost function is, one cannot get close to the best correlated equilibrium (10/3, 10/3) of the following classic example (Aumann [1974]):

	Left	Right
Top	5, 1	0, 0
Bottom	4, 4	1, 5

DEFINITION 3: *A* semi-correlated equilibrium distribution *of a game $(A=(a_{ij}),B=(b_{ij}))$ is a probability distribution p over the matrix cells such that*

1) *For every i and i':* $\sum_j p_{ij}a_{ij} \geq \sum_j p_{ij}a_{i'j}$.

2) *For every j and j':* $\sum_i p_{ij}b_{ij} \geq \sum_i p_{ij}b_{ij'}$.

3) *For every j and j' with* $\sum_i p_{ij}, \sum_i p_{ij'} > 0$,

$$\sum_i p_{ij}b_{ij} / \sum_i p_{ij} = \sum_i p_{ij'}b_{ij'} / \sum_i p_{ij'} .$$

REMARK: Conditions 1 and 2 are the standard conditions of correlated equilibrium – no player can profit by acting as if he received a different signal. Condition 3 is the condition of distribution equilibrium given by Sorin [1998] – the expected payoff of Player 1 is the same, given any signal that he receives.

Each strategy pair (y; μ, x) in the extended game induces a probability distribution $p=(p_{ij})$ on the matrix:

$$p_{ij} = \int_q \sum_s y_j q(s, j) x(q, s) d\mu(q).$$

p_{ij} is the probability that the cell (i, j) will be played.

THEOREM 4: $p=(p_{ij})$ is a probability distribution induced by an equilibrium in the extended game if and only if it is a semi-correlated equilibrium distribution of the original game.

Proof: Let $p=(p_{ij})$ be a probability distribution induced by an equilibrium in the extended game. Since in equilibrium Player 1 plays a best response given the signal he received, condition 1 in Definition 3 holds. Let j and j' be such that $\sum_i p_{ij}, \sum_i p_{ij'} > 0$. If either $\sum_i p_{ij} b_{ij} < \sum_i p_{ij} b_{ij'}$ or $\sum_i p_{ij} b_{ij} / \sum_i p_{ij} < \sum_i p_{ij'} b_{ij'} / \sum_i p_{ij'}$ then Player 2 would not play the action j with positive probability, and then $\sum_i p_{ij} = 0$. In particular, conditions 2 and 3 in Definition 3 hold for such j and j'.

Let p be a semi-correlated equilibrium distribution. Define $y \in \Delta(J)$ by $y_j = \Sigma_i p_{ij}$, and let the signal space be $S = I \cup J$. Define the following function $q : J \rightarrow \Delta(S)$. For every j such that $y_j = 0$, $q(j, j) = 1$ while for every j such that $y_j > 0$, $q(i, j) = p_{ij} / y_j = p_{ij} / \Sigma_i p_{ij}$. Finally, define $x : S \rightarrow I$ as follows: if $s \in B$ then x(s) is the action of Player 1 that yields Player 2 the lowest payoff when she plays s, whereas if $s \in A$, then x(s) = a.

In words, y prescribes Player 2 to play according to the marginal distribution of p. If she plays an action that has a positive probability according to y, then Φ(q) recommends

Player 1 to play some action, according to the distribution p. x prescribes Player 1 to follow the recommendation. If Player 2 plays an action that has 0 probability according to p, the action is reported deterministically to Player 1, and x prescribes Player 1 to "punish" player 2.

We will now see that since p is a semi-correlated equilibrium distribution, if the players play (y; q, x) then Player 2 cannot gain by deviating, and Player 1 cannot gain by playing a mixed-action different from x. We will then construct a cost function that forces Player 1 to purchase the information device $\Phi(q)$.

Indeed, assume that the players play the pure-espionage strategy (y; q, x). The probability distribution induced on the pairs of actions is exactly p. By condition 3 of Definition 3 Player 2 is indifferent between all actions j with $y_j > 0$, and by condition 2 he cannot profit by any deviation. By condition 1, Player 1 cannot profit by not following the recommendation of q.

We shall now construct a cost function φ that "forces" Player 1 to purchase the device $\Phi(q^*)$. Let conv(e, q*) be the convex hull of the null device and the device $\Phi(q^*)$. Let Q_0 be the set of devices that are Blackwell-inferior to some device in conv(e, q*). Q_0 is convex and closed. (see Appendix B for a geometrical representation of the Blackwell Relation). Define a cost function $\varphi_r(q) = r \, dist(q, Q_0)$, r times the Euclidean distance between q and Q_0. One can easily check that φ_r is quasi-convex and preserves the Blackwell relation.

For any device $\Phi(q)$ that is not in conv(e, q*) there exists r such that if φ_r is the cost function, then Player 1 cannot profit by purchasing the device $\Phi(q)$. By the compactness of the set of devices, there exists r sufficiently large such that if φ_r is the cost function, then Player 1 cannot profit by purchasing *any* device which does not correspond to a point in conv(e, q^*). Define $\varphi=\varphi_r$.

By the construction of φ, Player 1 cannot profit by purchasing any device outside Q_0 and the cost of all devices in Q_0 is 0. ^

6 Concluding Remarks

In this paper we have demonstrated the effects of players' option to purchase information on their opponents' decisions (i.e., to spy on their opponents). This alteration of the agents' optimization problem changes the set of predictions of the game. While pure equilibria of the original game remain equilibria in the extended game with espionage, the set of mixed equilibria may change for sufficiently small costs of information. Moreover, there may be additional mixed (perfect Bayesian) equilibria when espionage is available. Assuming the cost of information is quasi-convex, mixed equilibria in the extended game give weight to at most two information devices: the null information device and some non-trivial information device (provided player 2 has 2 possible actions).

The set of equilibria with espionage is closely linked to the set of correlated equilibria of the original game. A complete equivalence exists when looking at the subset of semi-correlated equilibria.

Our analysis concentrated mostly on 2×2 extensive form games and one shot 2×2 normal form games. The natural next step is to extend this study to multi-stage games with a sequence of players' decisions. The algorithm presented in Section 3 for transforming the game tree of the original game to that of the extended game can be used in the same way, but the analysis becomes much more complicated. This extension has economic relevance to the timing of decisions. Given that spying is possible only on policies that have already been determined, there might be a trade-off between committing oneself to policies early on in the game and waiting to a stage where the opponent's actions can be spied upon. A resolution of this trade-off can serve to determine the endogenous timing of policy meetings.

It is also worthwhile noting that espionage can be considered in the context of private information that is not related to the players' actions. That is, allowing players to purchase information on others' private signals or types could extend the standard models of games with incomplete information.

Another possible direction for extending this model is allowing for protection against espionage (folk wisdom suggests that this phenomenon occurs in army related enterprises, as well as in industrial/economic ones). Since espionage sometimes leads to a strict Pareto improvement, it is not clear that even if protection is very cheap, the game is equivalent to the original game without espionage. We do predict, though, that if protection is extremely costly, the game resembles the extension considered in this paper. The authors do not know how the current predictions change when protection costs are comparable to the costs of information. Nonetheless, it is not hard to construct examples in which both players invest in information purchase and protection, and payoffs are Pareto dominated by the equilibria payoffs of the original game.

Appendix A – Proofs

Proof of Theorem 1:

Since $c_2 > b_2$ action r of Player 2 is part of any SPE. If $a_1 < c_1$ then the SPE is (R, r) and the Stackelberg payoff is different if and only if $a_2 > c_2$ and $b_1 < a_1$. If $a_1 > c_1$ then the SPE is (L, r) and the Stackelberg payoff is different if and only if $b_2 > a_2$ and $a_1 < b_1$. To summarize, the condition of the Theorem holds if and only if one of the following conditions holds:

1. $b_1 < a_1 < c_1$ and $a_2 > c_2$; or
2. $c_1 < a_1 < b_1$ and $b_2 > a_2$.

We now go case by case according to the parameters of the game:

1. If $a_1 < \min\{b_1, c_1\}$ then R strictly dominates L for Player 1 and since $c_2 > b_2$, (R, r) with no spying is the unique SPE.
2. If $a_1 > \max\{b_1, c_1\}$ then L strictly dominates R for Player 1 and since $c_2 > b_2$, (L, r) with no spying is the unique SPE.
3. If $b_1 < a_1 < c_1$ and $a_2 < c_2$, since (c_1, c_2) strictly Pareto dominates all other feasible payoffs, in every SPE (with or without espionage) Player 2 chooses r with probability 1. But then employment of any non-trivial (and thus costly) espionage cannot be part of an equilibrium and (R, r) with no spying is the unique equilibrium.
4. If $c_1 < a_1 < b_1$ and $c_2 > a_2 > b_2$ the game is one of conflicting interests and the unique SPE is (L, r) with no espionage.
5. If $c_1 < a_1 < b_1$ and $a_2 > c_2 > b_2$ then in any equilibrium with espionage Player 2 should choose r with probability 1 (in order to induce Player 1 to play L). But then employment of any non-trivial (and thus costly) espionage cannot be part of an equilibrium and (L, r) with no spying is the unique equilibrium.
6. The case $b_1 < a_1 < c_1$ and $a_2 > c_2$ is equivalent to the chain store model presented in Example 1 and does qualify for an equilibrium with espionage for low enough cost functions as explicitly characterized in Section 2.
7. Example 2 is a special case of $c_1 < a_1 < b_1$ and $c_2 > b_2 > a_2$. In particular, equilibrium with espionage is possible. The general analysis is analogous to that of the chain store

model. Using the notation of Section 2, indifference of Player 2 in an equilibrium with espionage requires that

$$q^* = \frac{c_2 - a_2}{b_2 + c_2 - 2a_2}.$$

Player 1's problem is:

$$\max_q \{pqb_1+p(1-q)a_1+(1-p)qa_1+(1-p)(1-q)c_1-\varphi(q), \max\{a_1, pb_1+(1-p)c_1\}$$

for which the solution is $q^*(p)$. The pair $(p^*, \Phi(q^*(p^*)))$ is part of an equilibrium if:

$$q^*(p^*) = \frac{c_2 - a_2}{b_2 + c_2 - 2a_2} \quad .$$

Just like for case 6, this condition will be satisfied for small enough cost functions.

$\wedge$

Proof of Theorem 3:

Let $(y; q, x^*)$ be a pure-espionage equilibrium in an extended game $(A, B, J, [1/2,1], \varphi)$ with $q > 1/2$.

Step 1: Indifference condition of Player 2

In equilibrium y must be fully mixed (otherwise espionage is not needed), and therefore Player 2 must be indifferent between his actions. In particular,

$$qb_{11} + (1-q)b_{21} = qb_{22} + (1-q)b_{12},$$

or equivalently, $\beta q = b_{12}-b_{21}$. Since $1/2<q\leq 1$, it follows that if a pure-espionage equilibrium exists then one of conditions (i).a or (i).b holds.

Step 2: Player 1 maximizes his payoff at q

Player 1's payoff is given by

$$\pi^1(y; q,x^*) = yqa_{11} + y(1-q)a_{21} + (1-y)qa_{22} + (1-y)(1-q)a_{12} - \varphi(q).$$

Since Player 1 maximizes his payoff at q, either q=1 and $\partial\pi^1/\partial q(y;1,x^*)\geq 0$ or $1/2<q<1$ and $\partial\pi^1/\partial q(y;q,x^*)=0$. In both cases, if $\alpha\neq 0$ then $y\alpha = \varphi'(q)+a_{12}-a_{22}$. Therefore, $0<y<1$ if and only if $\min\{ a_{11}-a_{21}, a_{22}-a_{12} \} < \varphi'(q) < \max\{ a_{11}-a_{21}, a_{22}-a_{12} \}$ which proves the first part of (ii).b. If $\alpha=0$ then $\varphi'(q)\leq a_{22}-a_{12}$, with equality if $q<1$ and $\varphi'(q) = a_{22}-a_{12} = a_{11}-a_{21}$. Thus the first part of (ii).c is proved.

Step 3: Player 1 prefers to purchase the device q rather than not purchasing any device.

In a pure-espionage equilibrium Player 1 prefers to purchase the device q and follow its signal rather than to play always B. Hence

$$ya_{21} + (1\text{-}y)a_{22} \leq yqa_{11} + y(1\text{-}q)a_{21} + (1\text{-}y)qa_{22} + (1\text{-}y)(1\text{-}q)a_{12} - \varphi(q).$$

Therefore

(6) $$y(1\text{-}q)(a_{11} - a_{21}) + \varphi(q) \leq (1\text{-}y)q(a_{22} - a_{12}).$$

Similarly, if Player 1 prefers to purchase q rather than to play always T, we have

(7) $$(1\text{-}y)(1\text{-}q)(a_{22} - a_{12}) + \varphi(q) \leq yq(a_{11} - a_{21}).$$

If $\alpha \neq 0$ then the second part of (ii).b follows. If $\alpha=0$ we note that $a_{11}\text{-}a_{21} = a_{22}\text{-}a_{12}$, hence (6) and (7) translate to

$$\varphi(q) \leq q(a_{11}\text{-}a_{21}) - \max\{ y, 1\text{-}y\} (a_{11}\text{-}a_{21})$$

Since y has no constraint, by choosing y=1/2 we get the second part of (ii).c

Step 4: Sufficiency of (i).

Taking $\varphi(q)$ to be arbitrarily small, (6) and (7) translate to

(8) $$\frac{1-y}{y}\frac{q}{1-q} < \frac{a_{11}-a_{21}}{a_{22}-a_{12}} < \frac{1-y}{y}\frac{1-q}{q}.$$

If $\beta = 0$ let q be arbitrary. Choose a function φ such that y that is defined by (5) satisfies (8) (this is a constraint on $\varphi'(q)$), and $\varphi(q)$ is sufficiently small, so that (6) and (7) hold as well. Then $(y; q, x^*)$ is a pure-espionage equilibrium in the extended game. ^

Appendix B: The Blackwell Relation

In this Appendix we provide a geometric interpretation of the Blackwell relation for n×2 Markov matrices, and we use this representation to prove that any quasi-convex function over the space of n×2 Markov matrices preserves the Blackwell relation.

If $P \succ Q$, then any column of Q is a convex combination of P's columns.

Each n×m Markov matrix P defines a point p in $\mathbf{R}^{n(m-1)}$, and therefore, defines a convex set C(P) which corresponds to the matrices which are less informative than P. Thus, $P \succ Q$ if and only if $q \in C(P)$, where q is the point in $\mathbf{R}^{n(m-1)}$ that corresponds to Q. Note that in this case $C(Q) \subseteq C(P)$.

In particular, any 2×2 Markov matrix $P = \begin{pmatrix} a & 1-a \\ 1-b & b \end{pmatrix}$ corresponds to the point (a,1-b) in $\mathbf{R}^2$, and to the parallelogram defined as conv{(0,0), (1,1), (a,1-b), (1-a,b)}. Figure 5 captures this description graphically:

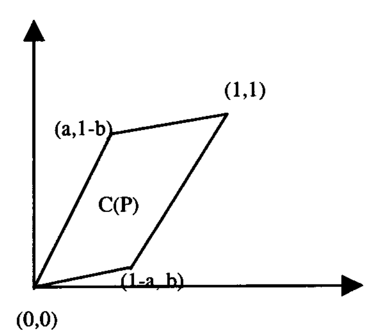

Figure 5

PROPOSITION 3: *Let f be a quasi- convex function over 2×2 Markov matrices that*

(i) vanishes on the trivial matrices(a=1-b=0 and a=1-b=1) and

(ii) gives the same value to matrices that differ in the order of columns.

Then f preserves Blackwell's relation.

Proof: Let f be a quasi-convex function over the space of 2×2 Markov matrices satisfying (i) and (ii) above. If $P \succ Q$ then Q lies in C(P). However, the two null matrices that correspond to the points (0,0) and (1,1) have f-value 0, whereas f(P) > 0. Therefore $f(P) \geq f(R)$ for every R in C(P). In particular, $f(P) \geq f(Q)$. ^

The proof of Proposition 3 can be extended to any n×2 matrix (since then C(P) is determined by a single point in $\mathbf{R}^n$). Nonetheless, these conditions are not sufficient for matrices of higher dimensions, as the following example illustrates.

EXAMPLE 6: Consider the space of 2×3 Markov matrices and let f be a linear function such that f(P)=1 where $P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$, f(Q) = 2 where $Q = \begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$, and f satisfies the two conditions of Proposition 3.

Since P, Q, and all their equivalents (and the trivial matrices) are not linearly related, such an extension exists.

Note that $P \succ Q$. Hence f does not preserve Blackwell relation.

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