Discussion Paper No. 1252

On the Possibility of Stock Market Crashes In the Absence of Portfolio Insurance*

by

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January 29, 1999

http://www.kellogg.nwu.edu/research/math

Abstract

Policymakers and academic economists have placed most of the blame for the October 1987 stock market crash on hedging strategies by portfolio insurers, which dictated selling stocks as soon as prices fell. The fact that the practice of buying and selling stocks as portfolio insurance has virtually disappeared since then has given many comfort that a replay of the 1987 crash, when prices fell so much so quickly, is unlikely. This note argues with this view by developing a model in which crashes are possible in the absence of portfolio insurance. In our model, a crash is driven by panic selling among rational but uninformed traders.

JEL Classification Numbers: G14, D82, D84

Key Words: Giffen Effect, Portfolio Insurance, Hedging Demand, Market Crashes

* We thank Anat Admati, John Campell, Russell Cooper, Eddie Dekel, Drew Fudenberg, Christopher Good, Martin Hellwig, John Leahy, Andrew Metrick, and Antonio Rangel for their helpful suggestions on a previous version of the paper. We also thank participants in seminars at Harvard, Chicago GSB, Tel Aviv University, and the NBER Complementaries Workshop for their comments. All correspondence should be sent to Gadi Barlevy, Department of Economics, Northwestern University, Evanston, IL 60208.

Introduction

Whenever stock prices begin to slip, intense public speculation begins over whether another stock market crash could be looming on the horizon. While no credible economist would claim to accurately predict the future, it is probably true that many economists today would be willing to assure the public that another crash as spectacular as the one that occurred in October 1987, when the Dow Jones index declined by 23% in a single day, is unlikely to be replayed. This optimism is rooted in the subsequent economic analysis of the 1987 crash and its causes. In particular, the 1987 crash has been blamed primarily on then popular hedging strategies dictated by portfolio insurance models; but the use of direct buying and selling of stocks for dynamic hedging has been largely displaced by the use of options. For example, the Brady Commission (1988) which investigated the causes of the 1987 crash singled out portfolio insurance as a major factor contributing to the downward pressure on stock prices, noting that portfolio insurance models dictated large selling positions as soon as stock prices started to decline. Despite initial skepticism on whether such a relatively small volume of trade could trigger a sharp fall in prices, an influential paper by Gennotte and Leland (1990) argued convincingly that portfolio insurance could have been the main culprit behind the crash. They develop a model in which, without hedging demand by portfolio insurers, crashes are impossible. But in that same model, a relatively small amount of hedging demand produces discontinuities in the price function, and thus large jumps in prices in response to small changes in underlying fundamentals. They further demonstrate that crashes are particularly likely if other traders in the market are unaware that there are agents who are emplying such hedging strategies. Subsequent work by Jacklin, Kleidon, and Pfleiderer (1992) gave further credence to the argument that uncertainty over the extent of strategic hedging demand could generate significant price crashes. The fact that the use of stocks as hedging instruments has now all but disappeared in favor of options as hedging instruments has therefore instilled a sense confidence that markets are no longer as vulnerable to such dramatic price crashes in which stock prices fall so much so quickly.

This note argues with the above conventional wisdom, and demonstrates that crashes of the type described by Gennotte and Leland can occur quite naturally even in the absence of hedging strategies. The reason that hedging strategies generate discontinuities in Gennote and Leland's model is that they introduce an upward sloping component into the demand for assets. This is because portfolio insurance dictates to sell when the price is low, and buy when the price is high. As a result, a small price decline causes a selloff of stocks among portfolio insurers, which drives prices down further. What we show is that fully rational investors who are uninformed can behave the same way as portfolio insurers — selling at low prices and buying at high prices — as long as a sufficient number of informed investors are also present in the market. The reason is that uninformed traders extract information from prices. If they observe a low price, they can become pessimistic about stocks and choose to get out of the market, while if they observe a high price, they can become optimistic and enter the market. Their actions can therefore mimic the behavior of portfolio insurers. Gennotte and Leland rule out such behavior among uninformed traders in their model because they adopt convenient functional form assumptions which are common in the literature on information in financial markets but are also quite restrictive. In particular, their assumptions impose linearity in the model, which makes it impossible to generate an upward sloping demand curve when there is only one asset, a fact formally demonstrated by Admati (1985).[1] Using an alternative set of assumptions described in Barlevy and Veronesi (1998), we demonstrate that once we move away from linearity, it is possible to get the same type of effects as in Gennotte and Leland without having to assume exogenous hedging strategies. Thus, portfolio insurance is not a necessary precondition for a crash. Thus, our example suggests that a repeat of the 1987 crash is possible, with panic selling among rational but uninformed traders putting the same type of pressure on prices as portfolio insurance did back in 1987.

Ours is not the first paper to develop price crashes in the absence of portfolio insurance. Bulow and Klemperer (1994) and Madrigal and Scheinkman (1997) both develop models of crashes, i.e. models in which the equilibrium price function exhibits regions where prices

[1] Admati also demonstrates that upward sloping demand are possible with multiple assets. However, this upward sloping demand curve does not generate price crashes in her framework. We return to Admati's results below.

fall sharply in response to a small change in the environment. These papers rely on very different environments than the one in Gennotte and Leland to generate crashes, making it difficult to relate them to one another. But, more fundamentally, we argue that our example of crashes is more realistic and relevant than the stylized market environments described in these two papers. For example, Bulow and Klemperer develop price crashes in the context of a sequential auction for a scarce number of commodities in which agents have different valuations for the good and commodities are sold one at a time. A crash occurs when the valuation between two agents is sufficiently large so that the price of the next commodity being auctioned has to fall before another agent is willing to buy it. This sequential auctioning of goods one at a time is essential for generating a crash, but it is not a good representation of the way in which assets are traded in actual stock markets. In Madrigal and Scheinkman, a price crash is due to strategic manipulation by a monopolistic and fully informed market maker who finds such a pricing scheme optimal. For this explanation to account for stock market crashes, we would have to believe that a decline in price such as the one observed in 1987 is intentionally caused by a market maker, something which Madrigal and Scheinkman themselves appear to be skeptical about. Our explanation, by contrast, assumes a standard Walrasian market for assets in which prices equate supply and demand. Thus, it is a more empirically relevant theory of price crashes in the absence of portfolio insurance than these previous papers. Moroever, there is a ring of truth to our explanation, at least with regards to the original 1987 crash. The Brady Commission (1988) argued that along with portfolio insurance, an important factor in the downward pressure on stock prices was the massive redemptions on mutual funds by private investors, as well as selling by mutual funds managers themselves. This theme is reflected in a newspaper article marking the ten year anniversary of the crash by Wyatt (1997), which notes ominously that the growth of mutual funds over the past decade could significantly magnify the effect of uninformed traders in future crashes. Our model explains why this type of behavior might in fact be rational, and helps to reconcile it with the earlier result in Gennotte and Leland (1990) that portfolio insurance is a necessary condition for crashes in their framework.

The paper is organized as follows. Section 1 describes the basic layout of the model. Section 2 relates our model to previous work, and constructs an example of a discontinuous price function driven only by the demand of uninformed traders. Section 3 concludes.

1. The Model

Our specification borrows from previous work in Barlevy and Veronesi (1998). There are two assets in the economy. The first is a risky asset, whose price is denoted by P and which yields a random payoff of θ to its owner, where θ is binomial:

$$\theta = \begin{cases} \overline{\theta} & \text{with probability } \rho \\ \underline{\theta} & \text{with probability } 1-\rho \end{cases}$$

The supply of this asset is exogenous and set equal to 1. The second asset is a safe asset (call it money) whose price is normalized to 1 and which yields a fixed payoff of $1+R$. To reduce on notation, we set $R=0$.

The economy is populated by a continuum of agents, so that no agent can affect the price through his own trades. There are two types of agents in this economy: a fraction z who are informed, i.e. who know the value of θ, and the remaining fraction $1-z$ who are uninformed and know only the distribution of θ. Barlevy and Veronesi (1998) derive z endogenously by assuming that agents can learn θ at a cost and showing that it is possible that not all agents will choose to aquire information in equilibrium. But since the source of asymmetric information is not relevant for our discussion, we suppress this in our current discussion.

Both informed and uninformed agents are endowed with one unit of money which they must allocate between the two assets. These agents are also assumed to be risk-neutral. This assumption allows us to solve for equilibrium in closed form, but is not on its own necessary for the results, a point we return to in the Conclusion. Since risk-neutral agents would take infinite positions whenever the payoff to one asset exceeded the other, we need to also assume agents cannot spend more than their original endowment to keep demand finite.

As is standard in models of this type, we need to introduce some source of noise into the price function so that not all information is revealed through prices. The convention in this literature is to assume the presence of noise traders, whose trades are driven by exogenous reasons rather than strategic considerations that depend on prices. Noise trade in our framework consists of two components, both of which insure that prices are not fully informative. First, we assume that noise traders wish to spend a constant amount of wealth w on assets. Thus, if the price of the asset is equal to P, they purchase $\frac{w}{P}$ shares of the asset. In particular, w is sufficiently large to keep the market "liquid" and prevent prices from falling in the bad state of the world. That is, we assume

$$w > \overline{\theta} \tag{1.1}$$

This assumption implies that for any price $P \leq \overline{\theta}$, noise traders can afford to buy up the entire net supply of assets (which recall was set fixed at 1), and prevents the price of the asset from falling in the bad state of the world, since noise traders can absorb the entire net supply of assets and keep the price propped up. Without this assumption, high prices would become fully revealing, since prices would always collapse in the bad state of the world.

In addition to this liquidity component, we need to add a stochastic component so that price function is not invertible in θ. Towards this end, we assume that out of this quantity $\frac{w}{P}$ of the asset purchased by noise traders, a random amount x of assets must be sold off for exogenous reasons. That is, demand from the noise sector is given by

$$x^0 = \frac{w}{P} - x$$

In principle, we can allow x to assume any distribution in the interval $[0, \infty)$. The exponential distribution generates the results we desire, and also turns out to be particularly simple algebraically. Thus, we assume

$$x \sim \exp(\mu) \tag{1.2}$$

To recap, total demand in the market for assets is the sum of the demand of the risk-neutral agents and of noise traders. The total supply of assets is equal to 1. Agents trade given their information, and the market clears so that supply equals demand. Formally, a rational expectations equilibrium in this market is defined by a set of functions $\left(P,\ x^I,\ x^U\right)$, where $P(x, \theta)$ gives the price of the asset as a function of liquidity trading and the value of the asset, $x^I(\theta, P)$ gives the demand of informed traders for the asset when the value of the asset is θ and the price of the asset is P, and $x^U(P; P(\cdot,\cdot))$ is the demand of uninformed traders for the asset when the price of the asset is P, conditional on the equilibrium price function $P(\cdot,\cdot)$. This set of functions comprises an equilibrium if

1. **Utility Maximization**: x^I and x^U solve the maximization problem of the agents conditional upon their information:

$$\max_{x^i} \frac{E_i[\theta]}{P} x^i + (1 - x^i)$$

2. **Market Clearing** For all pairs (x, θ), the price $P(x, \theta)$ equates supply and demand, i.e.

$$x^0 \ + \ z \cdot x^I(\theta, P(x, \theta)) \ + \ (1 - z) \cdot x^U(P(x, \theta); P(\cdot,\cdot)) = 1$$

2. Stock Market Crashes

We now turn to the possibility of stock market crashes. Gennotte and Leland define a crash as a discontinuity in the equilibrium price function, so that prices appear to jump in response to a small change in one of the underlying variables. As Gennotte and Leland point out, such discontinuities can arise because of the presence of an upward sloping part in the demand curve for assets. The possibility of upward sloping demand curves for assets, or informational Giffen effects, has been discussed at length in previous literature. Grossman (1978) shows that in rational expectations models without noise, Giffen effects can be ruled out for most reasonable utility functions. But once noise is introduced, Grossman's result fails to hold. This is illustrated in Admati (1985), who constructs an example of Giffen effects with multiple assets. Her example relies on partial correlations across asset payoffs. To see the intuition behind her results, suppose the returns on two assets are negatively correlated. If we increase the price of one asset *while holding the price of the other asset fixed*, uninformed traders will become more bullish on the asset whose price we have increased. The reason is that this type of ceteris paribus experiment conveys two important pieces of information to the trader: not only is the relevant asset selling at a higher price, but the other asset is not selling at a high price. Since the two assets are negatively correlated, it stands to reason that the asset whose price has increased will yield a high return, making it more desirable. However, with only one asset, this potential for correlation disappears, and Admati shows that Giffen effects are not possible. Since Gennotte and Leland use the same model as Admati, they obtain the same result: without exogenous hedging strategies, the demand for the asset is downward sloping, and stock market crashes do not arise.

However, it turns out that the reason Giffen effects do not arise with only one asset in these models is due to functional form assumptions rather than to deep economic reasoning. In particular, both Admati and Gennotte and Leland assume agents have exponential utility, and that both θ and x are normally distributed. Under these assumptions, the demand of uninformed traders for the asset is linear in the price of the asset. But once demand is linear, Giffen effects are by necessity global, i.e. if a small increase in price leads an uninformed agent to demand more of an asset, then any increase in price would lead him to demand more of the asset. Such a demand schedule would be truly perverse. With non-linear demand, it is possible for Giffen effects to arise at certain prices but not at all prices, i.e. Giffen effects can be local rather than global. In our example, demand for the asset is inherently nonlinear in price: an infinitesimally small change in price causes informed traders to shift from investing nothing in risky assets to investing all of their endowment in risky assets. This nonlinearity turns out to be sufficient to allow for an upward sloping demand curve, as we demonstrate below. The fact that we can generate such upward sloping demand should not be entirely surprising. After all, we know from previous work on asymmetric information in goods markets such as Wilson (1979, 1980) that demand can be upward sloping if average quality is sufficiently responsive to price. But while the analogy between these settings and asset markets is immediate, it has not been sufficiently appreciated in previous work on asset markets. By developing a formal illustration of how the same principles extend to asset markets, we hope to make this point clear, and draw attention to the fact that hedging demand strategies are not necessary to generate discontinuities and crashes.

To construct an upward sloping demand curve for uninformed traders in our framework, we need two key ingredients. First, we need uninformed traders not to buy the asset when the price is low. To do this, it must be the case that the payoff to holding the asset in the bad state of the world is sufficiently low. To make this as stark as possible, we consider the extreme case in which $\underline{\theta} = 0$, so that the asset has no value in the bad state of the world. Under our distributional assumptions for x, this will imply agents stay away from the asset at sufficiently low prices. Second, we need uninformed traders to invest in the asset when the price is sufficiently high. This is done by assuming that ρ is sufficiently large. In this case, the asset will appear attractive at intermediate prices, and agents will invest their wealth. But this will only hold at intermediate prices; when prices are sufficiently high, i.e. when $P > \overline{\theta}$, none of the traders (with the exception of noise traders) will buy the asset. As long as these two conditions hold, and provided that the fraction of informed traders z is sufficiently large, there exists an equilibrium with Giffen effects and discontinuous price functions. Formally,

Proposition: Suppose $\underline{\theta} = 0$. Then there exists a value $\rho^* \in (0, 1)$ and a value $z^* \in (0, 1)$ such that if $\rho > \rho^*$ and $z > z^*$, there will be an equilibrium with Giffen effects, i.e.

the demand of the uninformed traders will be increasing with price for some values of P. Moreover, the price function in this equilibrium is discontinuous, so that market crashes are possible.

The technical details of the proof are delegated to an Appendix. Essentially, we conjecture that an equilibrium exists in which the demand for an uninformed trader exhibits two cutoff prices, $\underline{P}$ and $\overline{P}$, so that an uninformed trader invests all of his wealth in money for prices below $\underline{P}$ and above $\overline{P}$, but invests all of his wealth in assets for prices between these two cutoffs. We then solve for the equilibrium price function given this demand curve among uninformed traders. Finally, we confirm that for $\underline{\theta} = 0$ and ρ sufficiently large, the optimal demand for an uninformed trader given this price function does in fact exhibit two cutoff prices as conjectured. Hence, there exists an equilibrium with Giffen demand among uninformed traders. Moreover, we can show that this upward sloping demand curve generates a discontinuity in the price function. Both the equilibrium price function and uninformed demand for the asset are illustrated graphically in Figure 1.

We now describe the distinguishing features of this equilibrium. For prices above $\overline{\theta}$, only noise traders are willing to hold the asset, since holding the asset delivers a strictly lower payoff than money in both states of the world. Once the price dips below $\overline{\theta}$, informed traders shift their wealth into the asset in the good state of the world, driving prices up. At price $\overline{P}$, uninformed traders purchase the risky asset in both states of the world, again driving prices up. At still lower prices, uninformed traders become pessimistic and shift all of their wealth back to money. But because informed traders hold on to the asset in the good state of the world but not in the bad state, the price function drops more steeply in the bad state, creating a region of prices which can only arise in the good state of the world and are thus fully revealing. Below these prices there is a discrete jump in the price function, since the shift of wealth out of assets by uninformed traders causes downward pressure on prices. This discontinuity is exactly analogous to the discontinuity described in Gennotte and Leland, except that in their model, the discontinuity is due to an upward sloping demand from exogenous hedging strategies, while in our model the upward sloping demand curve is due

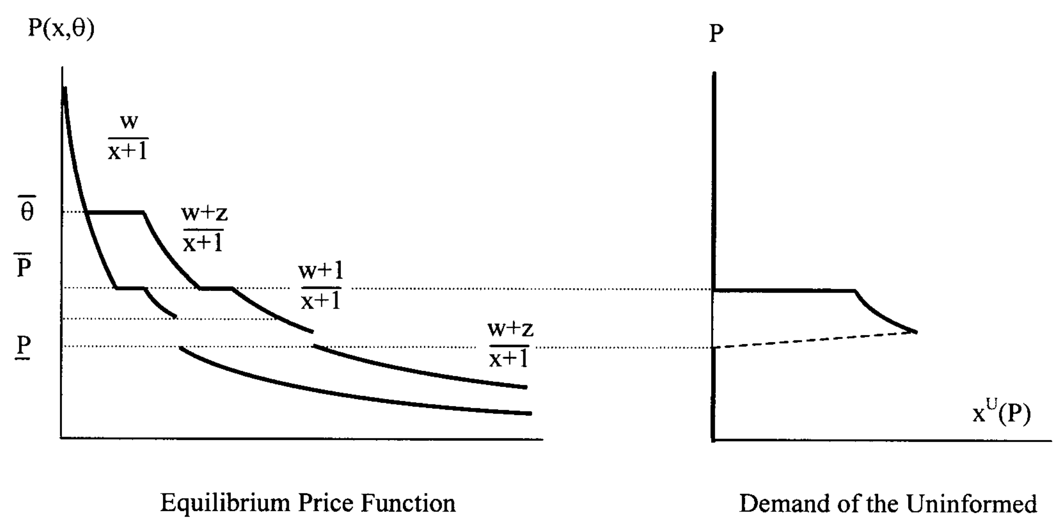

Figure 1: Equilibrium Price and Demand

to the behavior of rational but uninformed traders. The magnitude of the crash, i.e. the percentage decline in price between the top and the bottom parts of the price function at the point of discontinuity, depends on the state of the world. When $\theta = \overline{\theta}$, the discontinuity represents a decline of $\frac{1-z}{1+w}$ percent from the price just above the threshold where the discontinuity occurs. In the bad state of the world, the discontinuity represents a decline of $\frac{1-z}{1+w-z}$ percent. In both cases, the magnitude of the potential crash is decreasing in the fraction of informed traders z. Thus, when more uninformed traders are present in the market, the magnitude of the crash becomes larger. This prediction is important, since increasing participation over the past decade has probably been concentrated among small investors who are less informed about the fundamentals, suggesting that the magnitude of another crash now could in fact be quite significant.

A few final remarks on the equlibrium are in order. First, uninformed traders drop out of the market at prices below $\underline{P}$, and only noise traders, and in the good state of the world informed traders, remain in the market. Of course, this last feature arises only because we assume that the value of the asset in the bad state of the world equals 0. If it was bounded away from zero, then for sufficiently low prices, i.e. $P < \underline{\theta}$, uninformed agents would once again shift all of their wealth into the asset. Second, because informed agents act differently in different states of the world, certain prices may become partially revealing in equilibrium. This feature is ruled out under the particular functional form in Gennotte and Leland, since the demand for assets is essentially separable between θ and x. But it demonstrates that price crashes can involve more complicated information structures than suggested by the Gennote and Leland model; in particular, our model predicts that prices become less informative after a crash than immediately prior to it.[2]

[2]Interestingly, Madrigal and Scheinkman (1997) also suggest that prices just above the level which triggers the crash is fully informative, while the price after the crash is less informative. However, in their framework, the fact that prices are less informative causes the crash. In our framework, the fact that traders act differently under the two states of the world causes prices to become more informative in the region where the crash occurs.

3. Conclusion

We end our discussion with two observations. First, although the assumptions we employ to generate Giffen effects and market crashes are quite stylized, they are not essential for our result. We chose these assumptions only because they allow us to derive a closed form solution and demonstrate our result analytically. But, as the analogy with upward sloping demand curves from other asymmetric information models suggest, Giffen effects arise under a broad class of assumptions. For example, Giffen effects do not require that agents be risk-neutral. We were able to demonstrate numerically that in the original framework of Gennotte and Leland, where agents have exponential utility and noise trade x is normally distributed, Giffen effects can arise when θ is distributed binomially instead of normally. Hence, risk-neutrality is not essential. It is less clear whether a bimodal distribution for θ is important for generating Giffen effects. We suspect that this too does not play a pivotal role, but solving for an equilibrium when θ has a continuous distribution that does not imply normality is quite complicated. Still, our intuition is that the normal-linear model employed in previous work and which rules out price crashes is the exceptional case, rather than the model we develop here. That is, price discontinuities and market crashes are probably common even in the absence of upward sloping hedging demand.

Second, while our model demonstrates that price crashes are possible even in the absence of portfolio insurance, it does not absolve portfolio insurance from blame for the 1987 crash. Nor does it deny the mechanisms outlined in the other papers we discuss which generate crashes without relying on portfolio insurance. Rather, our example merely illustrates that uninformed traders can effectively behave like portfolio insurers, and generate price crashes in much the same fashion. The role of nervous mutual fund holders was noted in the original Brady Commission report (1988) on the causes of the 1987 crash. The report notes that "to the market, their [mutual fund companies] behavior looked much like that of the portfolio insurers, that is, selling without primary regard to price." The fact that stock market participation has increased over the past decade, particularly among small investors who are more likely to be uninformed about economic fundamentals in real time, should lead us to reevaluate the possibility of a future crash in stock prices. Even though portfolio insurance has been largely displaced by option trading, making the market less vulnerable to downward pressures from hedging demand when prices begin to decline, the stock market today may be as ripe for a significant decline in prices as it was back in 1987. This fact that uninformed traders on their own can cause market crashes has not been fully appreciated, both because portfolio insurance provided a convenient target for blame as well as a tradition of functional form assumptions among theoretical economists which rule out such effects. Our example should hopefully clarify this issue, and shift the focus away from dynamic hedging strategies as the only possible source of market crashes.

Appendix

Proof of the Proposition: Note that in any equilibrium, the demand of an informed trader must equal

$$x^I(\theta, P) = \begin{cases} 0 & \text{if} \quad P > \theta \\ \dfrac{1}{P} & \text{if} \quad P < \theta \end{cases} \tag{3.1}$$

Suppose that the demand of uninformed traders follows a two cutoff rule:

$$x^U(P; P(\cdot,\cdot)) = \begin{cases} 0 & \text{if} \quad P < \underline{P} \\ \dfrac{1}{P} & \text{if} \quad P \in (\underline{P}, \overline{P}) \\ 0 & \text{if} \quad P > \overline{P} \end{cases} \tag{3.2}$$

where $\underline{P} < \overline{P} \leq \overline{\theta}$. Given this demand curve, we can solve out for the price function $P(x, \theta)$ from the market clearing conditon. For example, for $P > \overline{\theta}$, only noise traders purchase the asset, so market clearing requires $\frac{w}{P} - x = 1$, so $P(x, \theta) = \frac{w}{x+1}$ for $x + 1 < \frac{w}{\overline{\theta}}$. The same procedure can be used to compute the entire price function:

$$P(x, \underline{\theta}) \quad = \quad \begin{cases} \dfrac{w}{x+1} & \text{if} \quad x + 1 < \dfrac{w}{\overline{\overline{P}}} \\ \overline{P} & \text{if} \quad x + 1 \in \left[\dfrac{w}{\overline{\overline{P}}}, \dfrac{w+1-z}{\overline{P}}\right] \\ \dfrac{w+1-z}{x+1} & \text{if} \quad x + 1 \in \left(\dfrac{w+1-z}{\overline{P}}, \dfrac{w}{\underline{P}}\right) \\ \dfrac{w}{x+1} & \text{if} \quad x + 1 \geq \dfrac{w}{\underline{P}} \end{cases} \tag{3.3}$$

$$P(x, \overline{\theta}) \quad = \quad \begin{cases} \dfrac{w}{x+1} & \text{if} \quad x + 1 < \dfrac{w}{\overline{\overline{\theta}}} \\ \overline{\theta} & \text{if} \quad x + 1 \in \left[\dfrac{w}{\overline{\theta}}, \dfrac{w+z}{\overline{\theta}}\right] \\ \dfrac{w+z}{x+1} & \text{if} \quad x + 1 \in \left(\dfrac{w+z}{\overline{\theta}}, \dfrac{w+z}{\overline{P}}\right) \\ \overline{P} & \text{if} \quad x + 1 \in \left[\dfrac{w+z}{\overline{P}}, \dfrac{w+1}{\overline{P}}\right] \\ \dfrac{w+1}{x+1} & \text{if} \quad x + 1 \in \left(\dfrac{w+1}{\overline{P}}, \dfrac{w+z}{\underline{P}}\right) \\ \dfrac{w+z}{x+1} & \text{if} \quad x + 1 > \dfrac{w+z}{\underline{P}} \end{cases} \tag{3.4}$$

For z sufficiently close to 1, $\frac{\underline{P}}{\overline{P}} < \frac{w}{w+1-z} < \frac{w+z}{w+1}$, and the intervals above are well-defined. Using Bayes' rule and the above price function, we can compute the conditional expectation $E(\theta \mid P(\cdot,\cdot) = P)$:

$$E(\theta \mid P_z(\cdot,\cdot) = P) = \begin{cases} \rho\overline{\theta} & \text{if} \quad P > \overline{\theta} \\ \overline{\theta} & \text{if} \quad P = \overline{\theta} \\ \dfrac{\rho\overline{\theta}}{\rho + (1-\rho)\exp\left[\frac{\mu z}{P}\right]} & \text{if} \quad P \in \left[\dfrac{w+1-z}{w}\underline{P}, \overline{\theta}\right) \\ \overline{\theta} & \text{if} \quad P \in \left(\underline{P}, \dfrac{w+1-z}{w}\underline{P}\right) \\ \dfrac{\rho\overline{\theta}}{\rho + (1-\rho)\exp\left[\frac{\mu z}{P}\right]} & \text{if} \quad P \leq \underline{P} \end{cases} \tag{3.5}$$

To satisfy utility maximization, an uninformed trader must invest all of his wealth in the asset if $E(\theta \mid P(\cdot,\cdot) = P) > P$, in money if this condition is reversed. We now confirm that for ρ sufficiently large, the demand schedule for an uninformed trader which satisfies these conditions given the price function above involves two cutoffs. Hence, there exists an equilibrium with two cutoffs which generates the above price function, and thus involves a discontinuity.

To prove this claim, note that (1) for $P > \overline{\theta}$, $E(\theta \mid P(\cdot,\cdot) = P) = \overline{\theta} < P$, so uninformed traders invest their wealth in money; (2) for $P = \overline{\theta}$, $E(\theta \mid P(\cdot,\cdot) = P) = \overline{\theta} = P$, and any allocation of wealth is consistent with utility maximization; and (3) for $P \in \left(\underline{P}, \frac{w+1-z}{w}\underline{P}\right)$, $E(\theta \mid P(\cdot,\cdot) = P) = \overline{\theta} > \overline{P} > P$ for z sufficiently large, and an uninformed trader would invest all of his wealth in the asset. Now consider the remaining values of P. Define

$$h(P;\rho) = \frac{1}{P}\left[\frac{\rho\overline{\theta}}{\rho + (1-\rho)\exp\left[\frac{\mu z}{P}\right]}\right] \tag{3.6}$$

For these values of P, the condition $E(\theta \mid P(\cdot,\cdot) = P) \gtreqless P$ implies $h(P;\rho) \gtreqless 1$. Differentiating h with respect to P, we have

$$\text{sign}\left(\frac{\partial h}{\partial P}\right) = -\text{sign}\left(\rho P + (1-\rho)(P - \mu z)\exp\left[\frac{\mu z}{P}\right]\right) \tag{3.7}$$

Differentiating $\rho P + (1-\rho)(P - \mu z)\exp\left[\frac{\mu z}{P}\right]$ with respect to P yields

$$\rho + (1-\rho)e^{\frac{\mu z}{P}}\left[1 - \frac{\mu z}{P} + \left(\frac{\mu z}{P}\right)^2\right] \tag{3.8}$$

which is positive since $1 - a + a^2 > 0$ for any real number a. Next,

$$\lim_{P\to 0} \rho P + (1-\rho)(P-\mu z)\exp\left[\frac{\mu z}{P}\right] = -\infty \tag{3.9}$$

$$\lim_{P\to \infty} \rho P + (1-\rho)(P-\mu z)\exp\left[\frac{\mu z}{P}\right] = \infty \tag{3.10}$$

so by continuity there exists a value of P for which $\rho P + (1-\rho)(P-\mu z)\exp\left[\frac{\mu z}{P}\right] = 0$. It follows that $h(P;\rho)$ is single peaked for a given ρ, and $h(P,\rho) = 1$ at most twice. Next, define a function $\rho^*(P)$ for $P \in \left(0, \overline{\theta}\right]$ such that

$$h(P, \rho^*(P)) = 1 \tag{3.11}$$

$\rho^*(P)$ is well defined, since

1. $h(P,0) = 0$
2. $h(P,1) = \dfrac{\overline{\theta}}{P}$
3. $\dfrac{\partial h}{\partial \rho} > 0$

Define $\rho^*(0) = \lim_{P\to 0} \rho^*(P) = 1$. Since $h(P,\rho)$ is continuous, it follows that $\rho^*(P)$ is continuous; since it is defined over a compact space $[0,\overline{\theta}]$, it attains a minimum. Define $\rho^* = \inf_{P\in[0,\overline{\theta}]} \rho^*(P)$. Now, suppose $\rho > \rho^*$. Then there exists a $P < \overline{\theta}$ such that $h(P,\rho) > 1$. This is because there exists a P such that $h(P,\rho^*) = 1$ and $\dfrac{\partial h}{\partial \rho} > 0$. But $h(P;\rho) < 1$ for $P = \overline{\theta}$ and $h(P;\rho) \to 0 < 1$ for $P \to 0$. By continuity, there must exist at least two P for which $h(P) = 1$. But since we just argued that there can be at most two values, there must be exactly two. Thus, there exist two values $\underline{P}$ and $\overline{P}$ such that $h(P) > 1$ for $P \in (\underline{P}, \overline{P})$ and $h(P) < 1$ for $P \in [\underline{P}, \overline{P}]$. This demonstrates that given the price function above, the optimal demand for the uninformed is in fact a dual cutoff rule, as originally assumed.

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