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# Credit Market Frictions and the Reallocation Process*

by

Gadi Barlevy

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**Abstract**

In a seminal paper, Davis and Haltiwanger (1990) demonstrate that recessions are associated with an increase in job reallocation, at least in the manufacturing sector. The conventional view has interpreted this as evidence of "cleansing" effects: less productive jobs are destroyed in recessions, and resources are reallocated towards more productive uses. Thus, recessions serve to improve allocative efficiency. This paper shows that when credit market frictions are introduced, the result can be reversed. That is, the most efficient jobs are destroyed in recessions, resources are reallocated towards less productive uses, and misallocation is exacerbated.

**JEL Classification Numbers**: E24, E32, E44

**Key Words**: Credit Market Imperfections, Job Reallocation, the Cleansing effect

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## Introduction

One of the most active areas of research in macroeconomics in recent years has centered on how business cycles affect the allocation of resources in the economy. This research was motivated by empirical work starting with Davis and Haltiwanger (1990) which demonstrated that recessions are associated with more intense reallocation of jobs across plants, at least in the manufacturing sector.[1] In light of these facts, macroeconomists have spent great effort in trying to understand the reallocation process and what it has to say about the nature and welfare implications of business cycles. The prevailing view to emerge from this research program is that recessions help to allocate resources more efficiently across production units. In the jargon of this literature, the increase in reallocation during recessions represents a "cleansing" of inefficient production arrangements, which frees up resources to be used more efficiently elsewhere. This view is conveyed, for example, in Aghion and Saint Paul (1991), Hall (1991), Mortensen and Pissarides (1994), and Caballero and Hammour (1994, 1996). To be sure, this literature does not argue that recessions and reallocation are inherently desirable. Clearly, recessions can be associated with other adverse effects. Likewise, if job reallocation is socially costly, an increase in reallocation may be inefficient despite its inherent benefits. But a common theme in this literature is that the increase in reallocation during recessions leads to a better allocation of resources across production units. This increase in allocative efficiency is a desirable consequence of recessions, even though it might ultimately be offset by other considerations.[2]

While there is relatively little empirical work on whether recessions serve to reallocate resources into more productive uses, the evidence that does exist is not entirely favorable to the cleansing view. Griliches and Regev (1995) examine average labor productivity for manufacturing plants in Israel and find only weak composition effects within industries, some of which contradict

[1] For a survey of the empirical literature on reallocation in the manufacturing sector, see Davis, Haltiwanger, and Schuh (1996). For a critique on whether these results extend to other sectors, see Foote (1998).

[2] The idea that recessions serve to allocate resources more efficiently has an even longer standing in the economics literature. Early examples include the 'liquidationist' view set forth in Schumpeter (1939) which argues that recessions provide a disciplining force that punishes unproductive investment. More recently, the sectoral shifts literature starting with Lilien (1982) has argued that recessions are associated with efficient reallocation, but the emphasis in this literature has been on reallocation across sectors rather than across plants. See also Rogerson (1987), Hamilton (1988), and Phelan and Trejos (1996).

the cleansing hypothesis. Bailey, Bartelsman, and Haltiwanger (1998) examine similar data for the U.S. and again find weak composition effects, but these are generally consistent with cleansing. Using an altogether different approach, Barlevy (1998a) looks at wage data from individual workers to make inference about changes in productivity over the cycle. He finds that recessions are more likely to shift workers into less productive rather than more productive jobs, in contrast with the cleansing effect. For specific industries, initial work by Bresnahan and Raff (1991) appeared to find evidence of cleansing effects in the automobile industry during the Great Depression through the entry and exit of plants. However, they find no evidence of reallocation towards more productive plants among those plants which survive the Depression, and in subsequent work Bresnahan and Raff (1998) question whether the automobile plants which survived the Great Depression did so because they were more productive. Bertin, Bresnahan, and Raff (1996) investigate the blast-furnace industry during this same time period and find no evidence of cleansing at all.

This evidence is quite puzzling, since agents employed in relatively unproductive jobs should have great incentive to disband and move into more productive arrangements during recessions. Why is it, then, that the cleansing effect is so muted, and, if anything, less productive jobs appear to thrive during hard times? This paper attempts to provide one answer to this question.[3] In particular, it argues that the presence of credit market frictions can reverse the cleansing effect, allowing the less efficient jobs to thrive during recessions. To show this, I develop a general equilibrium model in which, in the absence of credit market frictions, the standard cleansing effect arises: recessions destroy those jobs which produce the least amount of social surplus, and workers are reallocated into jobs which generate more social surplus. The reason that cleansing effects arise in this context is that production decisions are subject to participation constraints. Jobs which generate less surplus are more likely to violate the participation constraint for at least one of the agents during recessions; simply put, there is not enough surplus on such jobs during recessions to keep both parties interested in preserving the

[3] In a related paper, Barlevy (1998b) develops a model which introduces a "sullying" effect that works against the cleansing effect. In particular, he argues that recessions hamper the ability of workers to search on-the-job and move into better matches; as a result, fewer good matches are created during recessions, even as the worst jobs at the bottom are cleansed. But this explanation cannot account for all the above evidence. In particular, the data suggests the worst jobs at the bottom are not destroyed during recessions, a fact which is inconsistent with cleansing regardless of whether it occurs simultaneously with sullying.

match. But once I introduce credit market frictions into this framework, production decisions become subject to additional constraints from financial contracts used to finance production. In contrast with the standard participation constraints, there is no obvious reason why these financial constraints are more likely to be binding for the least efficient jobs during recessions. It is therefore possible that introducing additional constraints on agents initiating production can reverse the cleansing effect.

The key insight of the model is that it shows incentive constraints arising from credit market imperfections can work in the opposite direction as participation constraints, i.e. they are more likely to bind during recessions for the *most* efficient jobs rather than the *least* efficient jobs.[4] Thus, credit market frictions can reverse the cleansing effect, further exacerbating the misallocation of resources during recessions. To see why this is so, consider an economy populated by entrepreneurs who (1) choose what type of technology to use for production, and (2) bargain with workers who operate these technologies over how to split the output that is produced. The argument proceeds in two steps. First, in an interior equilibrium, entrepreneurs must be indifferent between paying a large cost to develop a superior technology and a small cost to develop an inferior one. Projects with high setup costs will therefore be associated with more revenue in equilibrium. But when revenues are large, workers can more effectively threaten to hold out unless they receive higher wages. For conventional bargaining games such as Nash bargaining, jobs with higher setup costs will pay higher wages to workers. Thus, in equilibrium, both technologies generate the same profits for entrepreneurs, but workers employed on the more costly technologies earn higher wages. This outcome is the first step in the argument — jobs with higher setup costs generate more total surplus in equilibrium. This same intuition has been discussed previously in Pissarides (1994) and Acemoglu (1998). The second step is the one which involves credit market imperfections. Suppose entrepreneurs have to borrow all of the funds to finance their investment. In addition, entrepreneurs face a moral hazard problem when investing the funds that they borrow: they can either use the money to build a unit of

[4] This result differs from Caballero and Hammour (1998), who also develop models of reallocation with credit market imperfections. Under their specification, 'financial constraints' apply to all existing jobs equally regardless of productivity, and become binding first for the least productive projects among newly initiated projects. Monge (1998) likewise presents a model in which incentive constraints bind for the least efficient jobs first. The reason for this is that their models assume exogenous differences in job productivity. The present model differs from both of these papers because the distribution of surplus across jobs arises endogenously.

capital and pay back their creditors, or they can steal their funds, use them to finance private consumption, and default on their debt. Since entrepreneurs must be indifferent between projects in an interior equilibrium, the payoff to building a production technology is the same for both types of projects. But entrepreneurs who borrow more have more incentive to abscond with the funds they borrow, simply because there is more to steal. As a result, when a negative productivity shock hits the economy, profits decline equally on both projects, and good jobs which require more resources will violate the incentive constraint first.

The fact that recessions reallocation resources into less productive jobs is analogous to the type of selection we observe in evolutionary biology. During harsh climactic episodes, the animals that are most likely to survive are not majestic beasts, but cockroaches.[5] The reason is not because cockroaches are inherently more efficient at finding food or more productive at generating offspring than other animals, but because cockroaches require little resources to sustain themselves. Likewise, since creditors are skittish about extending large amounts of credit during recessions, the jobs which survive recessions are those jobs which require the least amount of resources to sustain themselves, not the jobs which are necessarily the most productive or efficient. Since the logic above dictates that jobs which require the most resources must also generate the most amount of social surplus in equilibrium, we would expect that recessions kill off good jobs, freeing up resources to sustain bad jobs.

The theory developed in this paper can provide an explanation for why reallocation during recessions channels resources into less productive jobs rather than more productive jobs. But the model also makes a contribution by illustrating a new channel through which credit markets amplify productivity shocks. The conventional analysis on credit markets, as laid out in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), focuses on internal equity and net-worth considerations. In these models, a small productivity shock reduces the amount of internal funds available to entrepreneurs, forcing them to finance their projects by borrowing more on the credit market. External finance distorts the incentives of entrepreneurs to invest, deterring productive investment and further reducing output. My model presents an additional

[5]The durability of the cockroach is legendary. Biologists even speculate that cockroaches would be able to outlast a nuclear holocaust. See Gordon (1996).

channel by which credit markets amplify productivity shocks: at a given level of external financing, shocks affect the composition of projects that entrepreneurs develop in equilibrium, because the more socially efficient projects are also more vulnerable to financing constraints. This consideration is distinct from the internal equity channel emphasized is previous work; in fact, I generate this effect in a model where entrepreneurs are born penniless and must finance *all* of their investment externally, regardless of aggregate conditions. Thus, not only do productivity shocks force firms to borrow more funds externally, but they change the type of projects that firms who borrow externally can develop in equilibrium. Both of these effects will exacerbate the damage inflicted by productivity shocks on the economy.

The paper is organized as follows. Section 1 develops the basic setup. Section 2 demonstrates the conventional cleansing effect in the absence of credit market imperfections. Section 3 introduces credit market imperfections and shows how the cleansing effect can be reversed. Section 4 concludes.

## 1. Basic Setup

This section describes the essential ingredients of the model, which borrows from Acemoglu (1998). The key players in the model are entrepreneurs, who produce intermediate goods which are subsequently used to produce a final good. As discussed in the Introduction, the model relies on two assumptions. First, different intermediate goods are associated with different setup costs to initiate production. Second, production requires the entrepreneur to hire a worker, and the worker and firm engage in Nash bargaining over the revenue generated by the match. This section imbeds these two assumptions into a general equilibrium model. To simplify the analysis, I consider an economy with overlapping generations of entrepreneurs, so that investment decisions in different periods are undertaken by different agents. Allowing for longer horizons would introduce additional complications, although, as I discuss in the Conclusion, the results described here might still carry over to longer horizons under certain condtions.

There are two types of agents in the model, workers and entrepreneurs. There is a continuum of mass 1 of each type of agent. Both types of individuals live for two periods and have identical preferences defined over consumption and leisure. Agents can supply labor only in their first period of life, and care only about consumption in their second period of life. These assumptions help simplify the derivation, but are not essential. In addition, I impose the more restrictive assumptions that preferences are linear, so that the preferences for an individual born at time $t$ are given by

$$u(c_{t+1}, \ell_t) = c_{t+1} - b\ell_t$$

where $c_{t+1}$ denotes consumption of final goods in the second period of life, $\ell_t$ denotes the fraction of time the worker spends working in period $t$, and $b$ is the marginal value of leisure.

While agents have identical preferences, they differ in their initial endowments. Workers are endowed with one unit of labor each. Entrepreneurs, on the other hand, are endowed with no labor. Instead, they are endowed with blueprints on how to produce capital, which can in turn be operated by workers to produce intermediate goods. In other words, entrepreneurs cannot supply their own labor, but can use their endowed knowledge to create capital and hire labor to operate it.

Production in this economy occurs in three stages. In the first stage, entrepreneurs build units of capital. After the capital is installed, entrepreneurs hire workers to operate the capital and produce intermediate goods. Lastly, the intermediate goods are combined to produce a final consumption good. I assume there are only two types of intermediate goods, indexed by $j \in \{g, b\}$. Each entrepreneur chooses the type of specific capital to build, which determines the type of intermediate good he can produce in the subsequent period. Entrepreneur $i$ can choose from a set $\Omega_i \subseteq \{g, b\}$ of projects he can invest in. In general, I will allow $\Omega_i = \{g, b\}$ for all $i \in [0, 1]$, i.e. any entrepreneur can produce either type of capital. The reason I introduce this notation is that for certain instances, the only possible equilibrium involves job rationing. Since price-taking entrepreneurs fail to take into account constraints on the aggregate number of jobs, it will be necessary in such instances to introduce an exogenous rationing rule $\Omega_i$ that limits the choice set of some of the entrepreneurs.

Each entrepreneur can produce only one unit of capital. Ignoring the production function for capital momentarily, consider an entrepreneur who has already installed a unit of capital which can be used to produce good $j$. Production of intermediate goods is Leontieff, i.e. the one unit of capital can be combined with one unit of labor to yield one unit of intermediate good $j$. For simplicity, I assume production eats up all of the capital, i.e. there is 100% depreciation. Let $1_{ij}$ denote an indicator that equals 1 if entrepreneur $i$ produces capital of type $j$, and 0 otherwise. The total supply of intermediate good $j$ is then given by

$$Y_j = \int_0^1 1_{ij} \, di$$

Intermediate goods cannot be consumed directly, but they can be converted into final consumption goods. Let a lower case $y_j$ denote the quantity of intermediate good $j$ employed by a final goods producer. The amount of final goods that can be produced from given quantities of intermediate goods is characterized by a constant returns to scale CES production function

$$\begin{aligned} f(y_g, y_b) &= z\left(\tfrac{1}{2}y_g^\rho + \tfrac{1}{2}y_b^\rho\right)^{\frac{1}{\rho}} \\ &= 2^{-\frac{1}{\rho}} z\left(y_g^\rho + y_b^\rho\right)^{\frac{1}{\rho}} \end{aligned}$$

where $z$ is a measure of aggregate productivity and $\rho \in (0,1)$ is the elasticity of substitution between intermediate inputs. One unit of good $g$ and one unit of good $b$ together yield $z$ units of the final good. This technology is readily available to all agents in the economy. Since the technology exhibits constant returns to scale, I can assume final goods are produced by a single agent. Moreover, since profits will be zero for this producer in equilibrium, the identity of the agent who produces these goods is irrelevant. For convenience, I normalize the price of the final good to equal 1.

Finally, I return to the production technology for capital. Producing one unit of capital for production of intermediate good $b$ requires $k_b$ units of the final good. Producing one unit of capital for production of intermediate good $g$ requires the same $k_b$ units of the final good, but the process of converting the capital towards production of good $g$ requires an additional $k < \frac{1}{2}$ units of each type of intermediate good. The cost of a project of type $g$ is therefore given by

$$k_g = k_b + zk$$

Note that the difference in setup costs between the two projects is proportional to $z$, so that whenever aggregate productivity improves, the cost of producing intermediate goods of type $g$

increases. The reason for choosing this specification will become clear in the subsequent discussion. Also, it should be noted that this specification implies a chicken-and-egg paradox: final goods must be produced before any new intermediate goods can be produced, but intermediate goods must be produced before any new final goods can be produced. This paradox can be resolved either by assuming an infinite history, or by assuming that enough goods are endowed from heaven at the initial date so that production can take off.

Timing in the model is as follows. At the beginning of each period $t$, a new cohort of entrepreneurs is born. Each entrepreneur chooses whether to produce a unit of capital, and if so, which type. Since entrepreneurs have no funds of their own when they start out, they must borrow the funds to finance their production. If the entrepreneur builds capital in period $t$, he can hire a worker in period $t+1$ to operate the capital and produce intermediate good $j$. This good is then sold on the market at a price $p_j$. In particular, the good will be sold to either the final goods producer or to an entrepreneur in the next cohort who is producing capital for sector $g$ and requires both intermediate goods. The revenue from selling the intermediate good is used by the entrepreneur to pay his worker and his creditor, with the remaining part going to finance his own consumption.

The above paragraph describes the life of an entrepreneur. Workers, by contrast, spend their first period of life working for entrepreneurs from the previous cohort who already have existing capital. Since they only want to consume in the second period, they need to transfer their earnings into consumption in the following period. I allow them do so in two ways. First, there exists a storage technology which converts one unit of consumption into $R$ units of consumption in the following period, so agents can simply buy final goods today and store them.[6] Alternatively, they can lend their earnings to entrepreneurs in their own cohort who wish to finance projects for the subsequent period, and then use their returns next period to finance consumption. In what follows, I will assume that workers are willing to supply more funds than entrepreneurs require in equilibrium, so that no-arbitrage requires the interest rate on borrowed

[6] While I refer to this technology as storage, there is nothing requiring $R \leq 1$. In other words, it is possible for the storage technology to be productive (e.g. planting a seed yields a tree), so that an agent can consume more than he originally stored.

funds to equal $R$.[7] To summarize, then, output today is used to pay wages for workers, who in turn lend out their wages to finance capital that will be used to produce output tomorrow. These dynamics are illustrated graphically in Figure 1 for the reader's convenience.

The second key assumption of the model is that the labor market operates with frictions. In particular, I assume workers are randomly assigned to entrepreneurs after entrepreneurs invest in capital. This assumption is meant to capture an underlying search process in which workers bump into entrepreneurs at random. As long as search costs are sufficiently large, workers will agree to work for whichever entrepreneur they first encounter. After production takes place and the intermediate good is sold, the worker and entrepreneur negotiate over how to split the revenue $p_j$. I assume the two engage in a bargaining game of the type described in Rubinstein (1982) in which the two parties make alternating offers until an offer is accepted. It is crucial for the model that wages cannot be contracted in advance of the match: entrepreneurs must choose which intermediate good to produce and workers must decide whether to supply their labor prior to the wage being set. This feature introduces the possibility of inefficient production: either the entrepreneur or the worker can hold up the production of an intermediate good which generates positive surplus to both of them, simply because the two cannot credibly commit on how to split the surplus in a way to satisfy both parties.

Binmore, Rubinstein, and Wolinsky (1985) demonstrate that as the length of time between offers in the Rubinstein game goes to zero, the unique subgame perfect equilibrium of the game is just the Nash bargaining rule, i.e.

$$w_j = \alpha p_j$$

where $\alpha \in (0, 1)$ reflects the relative bargaining strength of the worker in the underlying bargaining game. The wage is increasing in $p_j$, so a project which generates more gross revenue pays a higher wage. Hence, workers are better off in jobs with higher setup costs. Entrepreneurs, on the other hand, are indifferent between producing the two goods, since they are free

[7] I could have equally assumed that the economy in question is a small open economy where any agent could borrow and lend at a rate $R$, in which case the question of whether domestic funds can finance investment again becomes moot. Likewise, I could have assumed lenders and borrowers bargain over the terms of loan contracts, as in Caballero and Hammour (1998). If entrepreneurs had all of the bargaining power in credit markets, they would offer to pay creditors no more than $R$, regardless of supply.

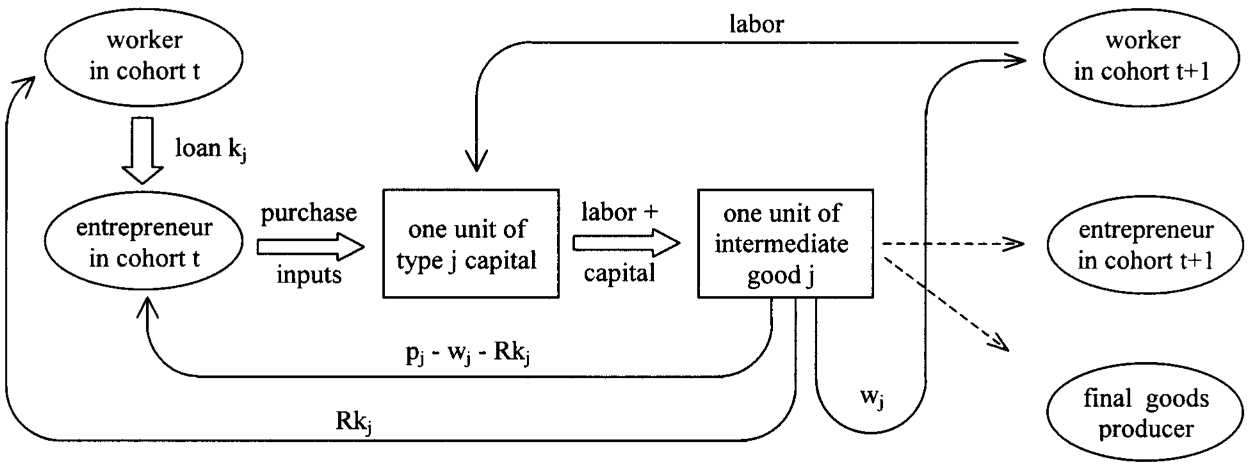



Figure 1: The Production Process for Final Goods

to choose either project. Adding the surplus for both entrepreneurs and workers, we have that jobs with higher setup costs yield more social surplus in equilibrium. For this reason, I follow Pissarides (1994) and Acemoglu (1998) in indexing jobs by $g$ and $b$, to reflect the fact that jobs with higher costs are good jobs which pay higher wages and generate more social surplus.

The specification of wages completes the description of the economy. I can now define an equilibrium for this economy in which the interest rate on loans is equal to the return to storage $R$. An equilibrium consists of a set of quantities of intermediate goods *produced* by entrepreneurs $\{Y_{gt}, Y_{bt}\}$ , a set of quantities of intermediate goods *purchased* by final goods producers $\{y_{gt}, y_{bt}\}$, and a set of prices $\{p_{gt}, p_{bt}\}$ such that

1. $y_{gt}$ and $y_{bt}$ solve the maximization problem of the final goods producer

$$\max_{y_g, y_b} f(y_{gt}, y_{bt}) - p_{gt}y_{gt} - p_{bt}y_{bt} \tag{1.1}$$

2. Each entrepreneur chooses the most profitable project given the rationing rule $\Omega_i$, subject to participation constraints for both the entrepreneur and worker:

$$\max_{j \in \Omega_i} \{p_{j,t+1} - w_{j,t+1} - Rk_j\} \tag{1.2}$$

$$\begin{aligned} \text{s.t.} \quad p_{j,t+1} - w_{j,t+1} &\geq Rk_j & (1.3) \\ Rw_{j,t+1} &\geq b & (1.4) \end{aligned}$$

3. The market for intermediate goods clears, i.e. demand for each intermediate good equals supply:

$$Y_{jt} = y_{jt} + kY_{g,t+1} \tag{1.5}$$

4. The supply of funds exceeds the demand for funds:

$$\sum_{j \in \{g,b\}} Y_{j,t-1}w_{jt} > \sum_{j \in \{g,b\}} Y_{jt}k_j \tag{1.6}$$

which insures the interest on loans must be $R$.

From (1.2), it follows that the wage in either sector will be at least $\frac{b}{R}$. As long as the economy is in full employment in every period (i.e. every entrepreneur produces an intermediate good), a sufficient condition to insure (1.6) is for $\frac{b}{R} > k_g$. This insures that all workers are paid sufficiently high wages to provide funds for the next cohort of entrepreneurs to finance their projects. Moreover, this condition can be entirely consistent with additional parameter restrictions discussed in subsequent sections.

## 2. Equilibrium with Perfect Credit Markets

In this section, I characterize equilibrium when loan contracts are fully enforceable. My analysis proceeds in two steps. I begin by solving for an equilibrium assuming that the labor participation constraints (1.4) are not binding for either intermediate good. This approach is useful because it demonstrates that fluctuations in $z$ have no effect on the allocation of resources in this economy unless labor participation constraints are binding in equilibrium for at least one of the goods. I then derive the conditions under which labor participation constraints become binding, and discuss how equilibrium changes once these constraints are taken into account. To preview the results, labor participation constraints imply that a negative productivity shock will tend to shift resources towards good jobs, leading to a more efficient allocation of resources. Thus, in the absence of credit market imperfections, the model is consistent with the standard cleansing view of recessions.

### 2.1. Equilibrium when Neither Labor Participation Constraint is Binding

In the definition of an equilibrium, the decision problem for an entrepreneur (1.2) is stated as subject to two participation constraints: the entrepreneur must earn positive profits on his investment, and the worker must be willing to operate the capital and forgo leisure. In this subsection, I restrict attention to equilibria in which the second participation constraint is not binding for either intermediate good. I return to the question of whether these constraints are binding in equilibrium in the next subsection. In what follows, I assume $\Omega_i = \{g, b\}$ for all entrepreneurs $i$. I begin my analysis with the following proposition. The proof of this as well as all other propositions is delegated to an appendix.

**Proposition 1**: Suppose neither labor participation constraint is binding in equilibrium, and consider an arbitrary period $t$. There exists a $z_0$ such that if $z_t < z_0$, no production in period $t$ is the unique equilibrium outcome (i.e. $Y_{jt} = y_{jt} = 0$ in any equilibrium where neither labor participation constraint is binding), and no production is not an equilibrium outcome if $z_t > z_0$.

Although the detailed proof of Proposition 1 is delegated to an Appendix, there is a graphical counterpart to the proof which illustrates the argument. Consider Figure 2 below. The axes denote $(1-\alpha)p_j$, which are the shares of revenue that go to the entrepreneur from producing each of the two intermediate goods. In equilibrium, the maximization problem of the final goods producer requires that $p_j \geq \frac{\partial f}{\partial y_j}$. I therefore trace out the set

$$\Gamma(z) = \left\{ (1-\alpha) \cdot \left( \frac{\partial f}{\partial y_b}, \frac{\partial f}{\partial y_g} \right) \middle| \text{ for all pairs } (y_g, y_b) \text{ at a given } z \right\}$$

Since the production function $f(y_g, y_b)$ exhibits constant returns to scale, the derivatives of $f$ with respect to $y_j$ depend only on the ratio $\frac{y_b}{y_g}$, and $\Gamma(z)$ is a one-dimensional curve. As demonstrated in the Appendix, $\Gamma(z)$ is downward sloping and convex. An equilibrium in which both goods are produced must correspond with a point on the curve $\Gamma(z)$, since the maximization problem of the final goods producer requires that $p_j = \frac{\partial f}{\partial y_j}$ for $y_j > 0$. By the same logic, an equilibrium in which neither good is produced must correspond with a point on or above this curve. Since neither labor participation constraint is binding by assumption, the only relevant constraints are the nonnegative profit conditions $(1-\alpha)p_j \geq Rk_j$. These constraints appear as horizontal and vertical lines in Figure 2. The rectangle labeled $K(z)$ represents the set of prices for which neither constraint is satisfied, i.e. $(1-\alpha)p_j \leq Rk_j$ for both $j$, while the region labeled $K'(z)$ denotes the set of prices for which both constraints are satisfied, i.e. $(1-\alpha)p_j \geq Rk_j$ for both $j$. For no production to be an equilibrium, it is necessary and sufficient that $K \cap \Gamma \neq \emptyset$, i.e. there exists a pair of prices on or above the curve $\Gamma$ for which profits are nonpositive in both sectors. At such prices, entrepreneurs are not willing to supply any intermediate goods, and since $p_j \geq \frac{\partial f}{\partial y_j}$, setting $y_j = 0$ solves the maximization problem for the final goods producer. Figure 2 illustrates a case in which a no-production equilibrium exists. As $z$ rises, the curve

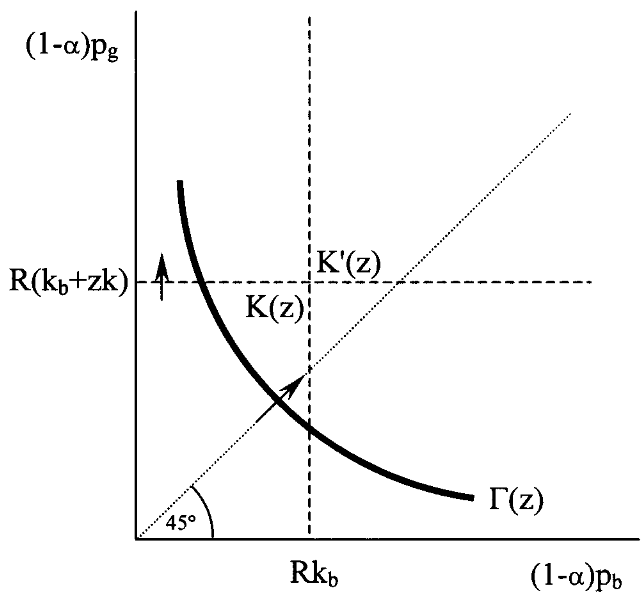



Figure 2: A No-Production Equilibrium ($K \cap \Gamma \neq \varnothing$)

$\Gamma(z)$ shifts outwards along the 45° line and the line given by $Rk_g$ shifts upward. These changes are depicted by the arrows in Figure 2. The Appendix shows formally that the curve $\Gamma(z)$ shifts outward at a faster rate than the line $Rk_g$ shifts upwards. Hence, for $z$ sufficiently large, $\Gamma(z)$ will lie entirely outside $K(z)$, and a no-production equilibrium will no longer be possible, since for all prices on $\Gamma(z)$, entrepreneurs will wish to produce at least one type of intermediate good. Hence, no production is an equilibrium only for low $z$.

While the above argument relies on graphical intuition, Proposition 1 also has a straightforward economic interpretation. When $z_t$ is small, the production technology for final goods is inefficient: more than one unit of the final good is required (because of required setup costs for capital) to produce a unit of the final good. Hence, no production will take place. However, because of distortions introduced through the bargaining process, no production will continue to be an equilibrium even when the production technology is efficient: entrepreneurs refuse to engage in production because they receive too small a share of the revenue production generates. Previous work, such as Caballero and Hammour (1996), has tended to attribute great importance to the inefficiency caused by this hold-up problem. I will ignore the possibility of entrepreneurs holding up production by assuming $z_t > z_0$ for all $t$. This assumption is only natural for this economy, since once production stops for one period, it ceases forever: with no final goods produced today, no new capital can be built for production tomorrow. This type of economic collapse is far too persistent. The restriction that $z_t > z_0$ leaves us with the question of what equilibrium looks like for higher values of $z_t$. This is taken up in the next proposition:

**Proposition 2**: Consider an arbitrary sequence $\{z_t\}$ where $z_t > z_0$ for all $t$, and suppose neither labor participation constraint is binding in equilibrium. Then equilibrium prices and quantities are uniquely determined. Moreover, the quantities of intermediate goods produced and used in final good production are constant and independent of $z_t$, i.e. $Y_{jt} = Y_j$ and $y_{jt} = y_j$ for $j \in \{g, b\}$.

A few remarks are in order about the equilibrium in Proposition 2. First, it is an interior equilibrium, i.e. both intermediate goods are produced. Since entrepreneurs are free to choose between the two goods, this implies profits are the same in both sectors. Second, this equilibrium is inefficient. This is because entrepreneurs are indifferent between the two goods, but workers earn higher wages producing good $g$. The social surplus associated with good $g$ is therefore greater than with good $b$, implying more consumption goods could be produced from the same amount of labor by shifting resources to produce good $g$ instead of good $b$. Finally, the composition of output in equilibrium is independent of $z$. That is, if neither labor participation constraint is binding in equilibrium, productivity shocks have no effect on the allocation of resources across jobs. This result stands in contrast with Pissarides (1994). He also considers a model with two jobs which differ in setup costs, but he finds changes in $z$ affect the composition of jobs, even though labor participation constraints are never binding. The difference between the two models is that Pissarides assumes building capital for good $g$ requires $k$ units of the *final good*, whereas my specification assumes it requires $k$ units of *intermediate goods*. If building capital requires final goods, a change in $z$ has no effect on setup costs. At the same time, an increase in $z$ changes the prices of intermediate goods; specifically, the difference between the prices of the two intermediate goods increases, since both prices increase by the same proportion but $p_g > p_b$. As a result, an increase in $z$ raises profits from producing good $g$ more than profits from producing good $b$. To restore equilibrium, more resources must be shifted into good $g$ until profits are once again equalized. For this reason, Pissarides finds that an increase in $z$ is associated with a reallocation towards good $g$. By contrast, if building capital requires intermediate goods, an increase in $z$ raises both the setup cost and the price of good $g$. Since the difference in costs between the two sectors changes at the same rate as the difference in prices, no reallocation of resources occurs. The effect described in Pissarides is thus sensitive to changes in the production structure. In fact, with a slightly different production structure, I can get the difference in setup costs to increase by more than the difference in prices, causing a reallocation of resources in exactly the opposite direction as in Pissarides.[8] In the absence of empirical evidence as to which difference is more sensitive to the cycle, I choose a specification in which this effect is neutral. With this effect suppressed, aggregate fluctuations have no effect on the allocation of resources in the economy as long as labor participation constraints are not binding.

[8] In particular, this will be the case if capital in sector $g$ requires only intermediate goods, but capital in sector requires only final goods.

### 2.2. Equilibrium when Labor Participation Constraints are Binding

The previous subsection illustrates that in this economy, aggregate fluctuations can affect the allocation of resources in this economy only if labor participation constraints are binding. This raises two questions. First, under what conditions are labor participation constraints binding in equilibrium? Second, when labor participation constraint are binding, how do aggregate shocks affect the allocation of resources across jobs? This subsection tackles these two questions. The next proposition establishes when labor participation constraints are binding in equilibrium.

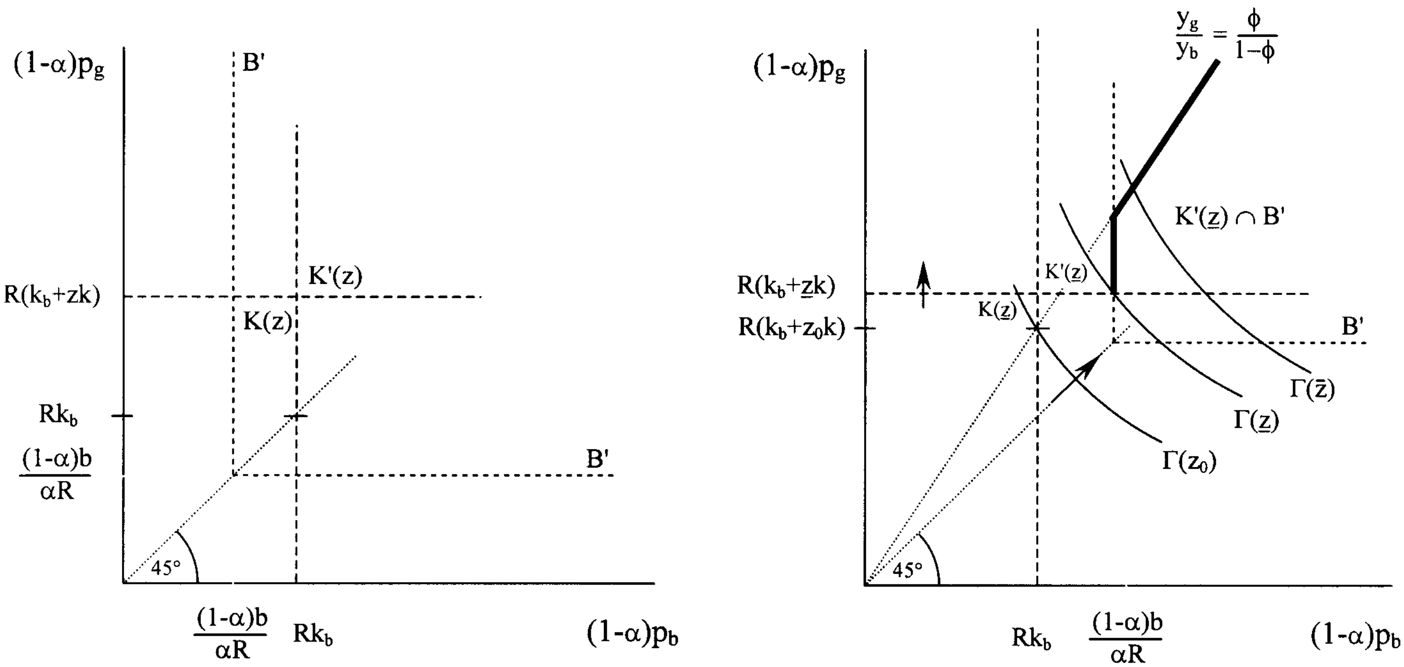
**Proposition 3**:

1. If $b \leq \frac{\alpha R^2 k_b}{1-\alpha}$, then labor participation constraints are never binding in equilibrium for $z_t > z_0$, and the unique equilibrium outcome is given by Proposition 2.

2. If $b > \frac{\alpha R^2 k_b}{1-\alpha}$, then there exist two numbers $\underline{z}$ and $\overline{z}$ where $\overline{z} > \underline{z} > z_0$ such that

   - No production is the unique equilibrium in period $t$ if $z_t < \underline{z}$.
   - For a given period $t$, the labor participation constraint is not binding in equilibrium if $z_t > \overline{z}$. Hence, if $z_t > \overline{z}$ for all $t$, the unique equilibrium is given by Proposition 2.

Once again, there is a graphical counterpart to the proof of Proposition 3, which is presented in Figure 3. Substituting in the Nash bargaining rule, the labor participation constraints become $(1-\alpha)p_j \geq \frac{(1-\alpha)b}{\alpha R}$. Graphically, these constraints appear as horizontal and vertical lines which intersect on the 45° line. The region labeled $B'$ denotes the set of prices which satisfy both labor participation constraints. The first panel of Figure 3 illustrates the case where $b < \frac{\alpha R^2 k_b}{1-\alpha}$, which once rearranged becomes $\frac{(1-\alpha)b}{\alpha R} < Rk_b$. When this condition holds, $K'(z) \subset B'$ for all $z$. Hence, labor participation constraints are automatically satisfied whenever profits are nonnegative. The second panel of Figure 3 illustrates the case in which $b > \frac{\alpha R^2 k_b}{1-\alpha}$. As can be seen in the figure, $B'$ will no longer contain $K'(z)$. The effects of changes in $z$ for this case are illustrated in Figure 3. Just as before, as $z$ rises, the curve $\Gamma(z)$ shifts outwards along the 45° line and the line $Rk_g$ shifts upward. A no-production equilibrium exists as long as




Case (i): $b < \dfrac{\alpha R^2 k_b}{1 - \alpha}$

Case (ii): $b > \dfrac{\alpha R^2 k_b}{1 - \alpha}$

Figure 3: Equilibrium with Labor Participation Constraints

$K'(z) \cap B' \cap \Gamma(z) = \emptyset$; when this condition is satisfied, it will always be possible to find a set of prices on or above the curve $\Gamma(z)$ for which no entrepreneur will be able to produce intermediate goods, since at least one of the constraints will be binding for each good. Since $\Gamma(z)$ shifts out at a faster rate than the line $Rk_g$, there exists a $\underline{z}$ such that $K'(z) \cap B' \cap \Gamma(z) = \emptyset$ only if $z < \underline{z}$. Hence, no-production equilibria exist only for low values of $z$. Turning to higher values of $z$, I show in the Appendix that any equilibrium with production must be an interior equilibrium; but since entrepreneurs are free to choose which good to produce, this requires profits to be the same for both goods. This will be the case only for a particular ratio of intermediate goods, i.e. only if $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$ for some constant $\phi \in \left(0, \frac{1}{2}\right)$ defined in the Appendix. Graphically, this condition traces out a ray which emanates from the origin and cuts through the point $(Rk_b, Rk_b + Rkz_0)$ as depicted in Figure 3. For any pair of prices above this ray, profits from good $g$ will exceed profits from good $b$, and vice versa for any pair of prices below this ray. An equilibrium with production must lie both on this ray and on the curve $\Gamma$, i.e. the equilibrium pair of prices is given by the intersection of these two curves. At $\overline{z}$ this intersection lies on the edge of $K' \cap B'$, and since $\Gamma$ shifts out at a faster rate than $K'$, this intersection will lie in the interior of $K' \cap B'$ for all $z > \overline{z}$. In this case, both labor participation constraints will be satisfied in equilibrium, as claimed.

The economic intuition behind Proposition 3 is as follows. First, labor participation constraints can only be binding if workers care enough about leisure, i.e. $b$ must be sufficiently large. The other important parameter is $z$. For low values of $z$, a unit of labor cannot generate enough final goods to induce workers to give up their leisure. In this case, no production is the unique equilibrium. For sufficiently large $z$, a unit of labor is sufficiently productive so that workers in both sectors are willing to forgo leisure, and neither labor participation constraint will be binding. Proposition 3 leaves unanswered the question of what happens if $z_t$ assumes a value between $\underline{z}$ and $\overline{z}$. But, as can be inferred from Figure 3, an equilibrium fails to exist for $z_t \in (\underline{z}, \overline{z})$. The reason is that for $z > \underline{z}$, the only candidate equilibria are those in which both intermediate goods are used in the production of final goods. But if $\Omega_i = \{g, b\}$ for all $i$, this requires profits to be equal for both goods, so the equilibrium will correspond with the intersection of $\Gamma(z)$ and the ray defined by $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$. For $z \in (\underline{z}, \overline{z})$, this intersection lies outside the set $B'$. In particular, the pair of prices in the set $\Gamma(z)$ which equalize profits across sectors violate the labor participation constraint in sector $b$, so entrepreneurs will not be able to produce this good. But this contradicts the fact that in any equilibrium with production, both goods must be produced.

The solution to this existence problem is to introduce a rationing rule which allows for prices in $\Gamma(z)$ not on the ray generated by $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$ to be an equilibrium. For $z_t \in (\underline{z}, \overline{z})$, the only prices in $\Gamma(z)$ which satisfy both constraints (i.e. which lie in $K' \cap B'$) are associated with greater profits from good $b$ than from good $g$. We therefore need a rule which prevents some of the entrepreneurs from producing good $b$ even though it generates higher profits in equilibrium.[9] The following rationing rule satisfies this requirement and allows the maximum number of entrepreneurs to produce good $b$:

$$\Omega_{it} = \begin{cases} \{g, b\} & \text{if } i \leq (1-\phi_z)(1-2kY_{g,t+1}) + kY_{g,t+1} \\ \{g\} & \text{else} \end{cases} \tag{2.1}$$

where $\phi_z \in [0,1]$ is defined by

$$\frac{\partial f(\phi_z, 1-\phi_z)}{\partial y_b} = \frac{b}{\alpha R}$$

A few comments are in order about this rationing rule. First, the cutoff level $(1-\phi_z)(1-2kY_{g,t+1}) + kY_{g,t+1}$ is chosen so that if all entrepreneurs with $i$ below the cutoff produce good $b$ and all remaining entrepreneurs produce good $g$, the labor participation constraint will be exactly binding in sector $b$ when the prices of the intermediate goods are equal to their marginal products. Second, the rationing rule is in part determined endogenously, since the number of entrepreneurs who can produce good $b$ depends on the production of good $g$ will in the next period. Armed with this rule, I can now characterize an equilibrium when labor participation constraints are binding. I begin with the case when productivity is constant over time.

**Proposition 4**: Suppose $b > \frac{\alpha R^2 k_b}{1-\alpha}$, $\Omega_i$ is given by (2.1), and $z_t = z$ for all $t$. Then

1. If $z < \underline{z}$, no production is the unique equilibrium.

[9]The necessity of unequal profits at low levels of productivity and the use of rationing rules to resolve the existence of equilibrium are also discussed in Matsuyama (1998).

2. If $z \in (\underline{z}, \overline{z})$, prices and quantities are uniquely determined in equilibrium. Equilibrium quantities $Y_{jt}$ and $y_{jt}$ are constant over time, i.e. $Y_{jt} = Y_j$,and $y_{jt} = y_j$ for $j \in \{g, b\}$. Moreover, the levels of output in sector $g$ depend negatively on the level of $z$, i.e. $Y_g'(z) < 0$ and $y_g'(z) < 0$.

3. If $z > \overline{z}$, the rationing rule is not binding, and the unique equilibrium is given by Proposition 2. Specifically, $Y_g$ and $y_j$ are independent of $z$.

Proposition 4 shows that when labor participation constraints are binding, the level of aggregate productivity $z$ does affect the allocation of resources. The intuition for this is as follows. As more resources are devoted to producing good $b$, the price of good $b$ falls, as does the wage for workers producing good $b$. At low levels of aggregate productivity $z$, too many entrepreneurs producing good $b$ will cause wages to be so low as to violate the labor participation constraint. At the same time, jobs producing good $g$ will not violate this constraint; this is because good $g$ generates more surplus than good $b$ in equilibrium, enough surplus to keep both the worker and the entrepreneur interested in preserving the match even when this is not the case in sector $b$. As a result, some entrepreneurs have no choice but to produce good $g$. For this reason, lower levels of aggregate productivity will be associated with a more efficient allocation of resources into jobs which generate more social surplus.[10] Interestingly, entrepreneurs who shut down less productive jobs do so quite reluctantly in the model, since these projects are the ones which generate high profits to the entrepreneurs even though they generate less total surplus. This is consistent with casual evidence that entrepreneurs are often very reluctant to shut down operations even though they are socially inefficient.

Proposition 4 compares the allocation of resources across different levels of productivity $z$; that is, it states that less productive economies will tend to allocate their resources more efficiently. But the same intuition extends to aggregate fluctuations in $z$ within a single economy. The easiest way to demonstrate this is to consider a simple process in which $z_t$ oscillates deterministically between two levels of productivity. This case is taken up in the next proposition:

[10] This statement is only true locally near $\overline{z}$. As $z$ continues to fall, enough resources will be shifted to producing good $g$ until the efficient level is achieved. For still lower values of $z$, too many resources are shifted into good $g$, causing a misallocation of resources. But since too little of good $g$ is produced relative to the efficient level for $z$ close to $\overline{z}$, reallocation will necessarily increase welfare locally.

**Proposition 5**: Suppose $b > \dfrac{\alpha R^2 k_b}{1-\alpha}$, $\Omega_i$ is given by (2.1), and that $z_t = Z$ for $t$ even, $z_t = z$ for $t$ odd, where

$$\underline{z} < z < \overline{z} < Z$$

Then the unique equilibrium is associated with fluctuations in the allocation of resources, specifically

$$Y_{gt} = \begin{cases} Y_g(Z) & \text{if } t \text{ even} \\ Y_g(z) & \text{if } t \text{ odd} \end{cases}$$

and

$$y_{gt} = \begin{cases} y_g(Z) & \text{if } t \text{ even} \\ y_g(z) & \text{if } t \text{ odd} \end{cases}$$

where $Y_g(z) > Y_g(Z)$ and $y_g(z) > y_g(Z)$.

Proposition 5 demonstrates the familiar cleansing effect of recessions. In aggregate downturns, bad jobs are "destroyed" because they fail to generate enough surplus to keep both parties interested in preserving the match. This frees up resources to be reallocated towards good jobs, which pay relatively higher wages and generate more social surplus.

To sum up, in the absence of credit market frictions, the model generates the familiar cleansing effect which has been discussed in the previous literature. It generates these cleansing effects through labor participation constraints. That is, a decline in aggregate productivity pulls down wages on the least efficient jobs, up to the point at which workers refuse to participate in production at the given wage. Their refusal to participate forces a reallocation of resources from the least efficient jobs to more efficient jobs, where workers are willing to provide their labor services. Consequently, even though recessions tend to reduce welfare by reducing the amount of final consumption goods which the economy can produce, the decline in welfare is partly offset by an improvement in allocative efficiency as bad jobs are cleansed and resources are better allocated.

## 3. Credit Market Imperfections

When credit markets operate without frictions, as in the previous section, the decision to initiate production is subject only to participation constraints. Since these constraints become binding for the least efficient jobs first when aggregate productivity falls, the jobs which get destroyed during recessions are necessarily those which generate the least amount of surplus. But once we add credit market frictions, production decisions become subject to additional constraints. In particular, if entrepreneurs can default on the funds which they borrow because of poor enforcement, credit contracts must be incentive compatible or else lenders will refuse to extend credit to entrepreneurs. But unlike participation constraints, incentive constraints will not necessarily become binding for the least efficient jobs first during a recession. To the contrary: this section demonstrates that incentive constraints make the most efficient jobs more vulnerable to aggregate fluctuations. Thus, once we modify traditional models to include credit market imperfections, the cleansing effect can be reversed.

To model credit market imperfections, I assume that when an entrepreneur borrows funds to finance the construction of new capital, he can either use the funds to build capital and pay back his creditor, or embezzle the funds and use them to finance his own consumption. That is, if the entrepreneur develops a unit of capital, his assets can be seized and liquidated in order to repay the debt. But if the entrepreneur hides or uses the money to finance consumption, he cannot be punished. This could reflect limited liability constraints, or the inherent weakness of courts in verifying conflicting claims by disputing parties. This gives entrepreneurs the option to avoid repaying the debt and consuming the funds that they borrow. Such a moral hazard problem would clearly be less significant if I allowed for longer horizons, since reputation concerns could be used to punish entrepreneurs who default on their loans. But as long as reputation concerns are not sufficiently strong to deter agents from defaulting during recessions, the results should continue to hold.

In this environment, lenders will only extend credit to those whom they expect will not default on the loan. This means that it must be in the best interests of the entrepreneur to produce an intermediate good rather than steal the funds and consume them. Formally, an entrepreneur can borrow funds to finance project $j$ if the following incentive constraint is satisfied:

$$(1-\alpha)p_j - Rk_j \geq Rk_j \tag{3.1}$$

Note that the value of stealing the $k_j$ funds is equal to $Rk_j$, since entrepreneurs can only consume their funds in the second period of life, and so must store their bounty for one period before eating it. Thus, the only modification of credit market imperfections to the previous framework is to introduce an additional constraint on the decision to initiate production (1.2); not only do the worker and entrepreneur have to agree to engage in production, which is reflected in the standard participation constraints, but the contract must also be incentive compatible so that the entrepreneur has incentive to go ahead with the project instead of defaulting on the loan.

As in the previous section, I begin by first ignoring labor participation constraints and assuming they are not binding in equilibrium. The next proposition provides conditions for whether the remaining incentive constraints are binding or not in equilibrium, and is analogous to Proposition 3 for the case of perfect credit markets:

**Proposition 6**: Suppose (1.2) is subject to (3.1), and neither labor participation constraint is binding in equilibrium. Then there exists two numbers $z'$ and $z''$ where $z'' > z' > z_0$, such that

1. No production is the unique equilibrium in period $t$ if $z_t < z'$.
2. For a given period $t$, neither incentive constraint is binding in equilibrium if $z_t > z''$. Hence, if $z_t > z''$ for all $t$, the unique equilibrium is given by Proposition 2.
3. If $\Omega_i = \{g, b\}$ for all $i$, there exists no equilibrium if $z_t \in (z', z'')$.

The cutoff $z''$ is potentially infinite. In particular, the cutoff $z'' < \infty$ if and only if

$$k < \frac{(1-\alpha)\left(1+\left(\frac{\phi}{1-\phi}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}}}{2^{\frac{1+\rho}{\rho}}R} \tag{3.2}$$

The key difference between the above proposition and Proposition 3 is that the cutoff $\overline{z}$ is always finite, while the cutoff $z''$ is potentially infinite. The reason for this is that when labor is

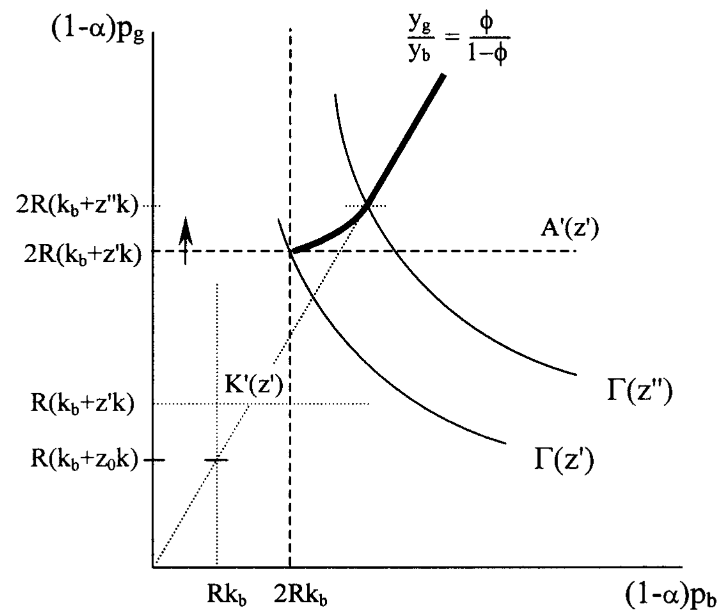



Figure 4: Equilibrium with Incentive Constraints

sufficiently productive, workers will always be willing to forgo the $b$ units of utility they obtain from leisure. Hence, participation constraints cease to be binding for $z$ large. But with incentive constraints, as $z$ increases, the incentive for entrepreneurs producing good $g$ to steal the funds they borrow rises, since entrepreneurs need to borrow more funds and can therefore steal more from their creditors. Thus, incentive constraints will be satisfied at high values of $z$ only if the amount of funds that entrepreneurs must borrow to produce good $g$ does not grow too rapidly with $z$. Condition (3.2) insures this is in fact the case.

The graphical counterpart of Proposition 6 for the case where $z'' < \infty$ is illustrated in Figure 4. The incentive constraints (3.1) are given by horizontal and vertical lines at $2Rk_j$, respectively, and the set $A'(z)$ denotes the set of prices which satisfy both incentive constraints. As can be seen from Figure 4, $A' \subset K'$, so if the incentive constraints for good $j$ are satisfied, the entrepreneurs' participation constraints are automatically satisfied as well. Noting the set $A$ is equivalent to a set $K$ with an interest rate $2R$ instead of $R$, the existence of $z'$ trivially follows from the same argument which establishes the existence of $z_0$. Next, if $\Omega_i = \{g, b\}$ for all $i$, we know that equilibrium with production requires profits to be equal in both sectors. Once again, then, equilibrium will occur at the intersection of $\Gamma$ and the ray defined by $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$. At $z''$, the line $Rk_g$ cuts through this intersection. Hence, for $z = z''$, the intersection lies in the set $A'$. Moreover, since $\Gamma$ shifts out at a faster rate than $Rk_g$ shifts upward, then this intersection continues to lie in the set $A'$ for $z > z''$, so that the candidate equilibrium satisfies both incentive constraints. But for $z \in (z', z'')$, this intersection lies outside the set $A'$. It is therefore impossible to simultaneously achieve equal profits and for incentive constraints to be satisfied for both goods. In contrast with Proposition 3, for $z \in (z', z'')$, prices in $\Gamma(z)$ which satisfy both incentive constraints (i.e. which also lie in $A'$) are associated with greater profits from good $g$ than from good $b$. This is because, as noted earlier, prices above the ray defined by $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$ are associated with higher profits from good $g$. This time, then, the jobs which need to be rationed during recessions are good jobs, not bad jobs. I now define the following rationing rule, which allows the maximum number of entrepreneurs to produce good $g$:

$$\Omega_{it} = \begin{cases} \{g, b\} & \text{if } i \leq \phi_z(1 - 2kY_{g,t+1}) + kY_{g,t+1} \\ \{b\} & \text{else} \end{cases} \tag{3.3}$$

where $\phi_z \in [0,1]$ is defined by

$$\frac{\partial f(\phi_z, 1-\phi_z)}{\partial y_g} = \frac{2R(k_b + zk)}{1-\alpha}$$

Since good jobs are being rationed, a negative productivity shock would reallocate resources into bad jobs rather than good jobs. Hence, the cleansing effect of the previous section is reversed. This is demonstrated formally in the next proposition:

**Proposition 7**: Suppose neither labor participation constraint is binding in equilibrium, $\Omega_i$ is given by (3.3), condition (3.2) is satisfied, and $z_t = Z$ for $t$ even, and $z_t = z$ for $t$ odd, where

$$z' < z < z'' < Z$$

Then there is a unique equilibrium in which the allocation of resources also oscillates, i.e.

$$Y_{gt} = \begin{cases} Y_g(Z) & \text{if } t \text{ even} \\ Y_g(z) & \text{if } t \text{ odd} \end{cases}$$

and

$$y_{gt} = \begin{cases} y_g(Z) & \text{if } t \text{ even} \\ y_g(z) & \text{if } t \text{ odd} \end{cases}$$

and where $Y_g(Z) > Y_g(z)$ and $y_g(Z) > y_g(z)$.

Proposition 7 shows that incentive constraints work in the opposite direction as participation constraints: they are more likely to bind for the more efficient jobs rather than the less efficient jobs. Consequently, instead of cleansing out the less efficient jobs and redirecting resources into more efficient uses, recessions now cleanse out the more efficient jobs and redirect resources into less efficient uses. The intuition behind this result is as follows. As long as entrepreneurs can freely choose which good to develop, profits will be the same from both goods. Hence, the incentives for an entrepreneur to engage in production are the same for both goods. But for entrepreneurs who commit to producing good $g$, the incentive to steal is greater, since these projects require more resources and offer more to steal. Thus, good jobs will violate the incentive constraint first. Entrepreneurs who are unable to borrow funds to finance capital for good $g$ have no choice but to invest in good $b$, and resources are shifted into producing this good. The driving force behind this result is the reluctance of creditors to extend large amounts of credit to any one project during recessions. But the types of projects which require more credit are precisely those which generate more surplus and higher wages. This pattern could potentially account for why empirical evidence seems to go against the cleansing effect: the reason why less productive firms continue to survive during recessions is because they are less expensive to maintain. Bresnahan and Raff (1998) lend some credence to this view, since they find that the most important determinant for exit among automobile plants during the Great Depression was whether a plant faced large unavoidable costs to continue production, more so than differences in productivity across plants. It is unclear, however, whether plants with such costs were in fact forced to shut down because of the unavailability of credit to finance these costs. Future work will hopefully establish whether recessions in fact shift credit towards less efficient uses as predicted by the model, and how much this channel can account for the absence of cleansing effects in the data.

Finally, I return to the question of whether labor participation constraints are binding, and how they affect the nature of equilibrium. As the above analysis suggests, there is a tension between participation constraints and incentive constraints during recessions: each tends to reallocate resources to production of different goods. Which effect dominates turns out to depend on the value of leisure $b$: if this parameter is sufficiently large, workers require high wages in order to forgo leisure, and participation constraints will become binding first before incentive constraints kick in. Alternatively, if this parameter is low, workers are willing to forgo leisure even at low wages, and the incentive constraints become binding first. This is summarized in the following Proposition:

**Proposition 8**: Suppose condition (3.2) holds. Then

1. There exist two numbers $z^*$ and $z^{**}$ where $z^{**} \geq z^* > z_0$ such that

   a. No production is the unique equilibrium in period $t$ if $z_t < z^*$

b. For a given period $t$, neither participation constraints nor incentive constraints are binding if $z_t > z^{**}$

2. There exists a $b^*$ such that $z^{**} > z^*$ for all $b \neq b^*$. Suppose $b \neq b^*$, and let $z_t = Z$ for $t$ even, and $z_t = z$ for $t$ odd, where

$$z^* < z < z^{**} < Z$$

Then there exists no equilibrium unless jobs are rationed. In particular,

a. If $b < b^*$, jobs producing good $g$ must be rationed. The following rationing rule allows the maximum number of entrepreneurs to produce good $g$:

$$\Omega_{it} = \begin{cases} \{g, b\} & \text{if } i \leq \phi_z(1 - 2kY_{g,t+1}) + kY_{g,t+1} \\ \{b\} & \text{else} \end{cases} \tag{3.4}$$

where $\phi_z \in [0, 1]$ is defined by

$$\frac{\partial f(\phi_z, 1 - \phi_z)}{\partial y_g} = \max\left[\frac{2R(k_b + zk)}{1 - \alpha}, \frac{b}{\alpha R}\right]$$

This rule implies a unique equilibrium, for which $Y_g(Z) > Y_g(z)$ and $y_g(Z) > y_g(z)$

b. If $b > b^*$, jobs producing good $g$ must be rationed. (2.1) allows the maximum number of entrepreneurs to produce good $b$. This rule implies a unique equilibrium, for which $Y_g(Z) < Y_g(z)$ and $y_g(Z) < y_g(z)$

In other words, for low values of $b$, the incentive constraints dominate and recessions exacerbate the misallocation of resources, but for high values of $b$, the participation constraints dominate and recessions ameliorate the misallocation of resources. The offsetting effects of these two forces could explain why cleansing effects appear to be so muted in the data: the traditional forces which operate toward cleansing could be offset by financing considerations which plague the most efficient jobs, and the resulting changes in the composition of jobs are relatively minor.

## 4. Conclusion

Ever since Schumpeter originated the notion of creative destruction, it has been recognized that the process of reallocating resources is painful but necessary for an economy to grow.

Economists studying industry evolution have demonstrated that firm and plant turnover is a crucial for industry productivity growth. For example, Olley and Pakes (1996) document that entry and exit accounted for a most of the productivity growth in the telecommunications industry following deregulation in the early 1980s. It is not surprising, therefore, that when Davis and Haltiwanger (1990) demonstrated that recessions were associated with an increase in reallocation of jobs across plants, the natural interpretation of their results was that this increase in reallocation contributed to productivity growth and improved the allocation of resources across production sites. Cleansing during recessions is also the most natural implication of the simplest models with heterogeneity in productivity: when agents are allowed to choose whether to shut down or not, it is obvious that the least productive will shut down first during recessions. For this reason, it was almost taken for granted that the cleansing effect was true, even though little empirical work has investigated whether the reallocation observed during recessions led to an improvement in productivity. But as discussed in the Introduction, the evidence that does exist seems to defy this conventional wisdom: there is very little evidence of cleansing in the data, and some of evidence even suggests reallocation during downturns exacerbates rather than improves the allocation of resources.

This paper presents a stylized model which shows that reallocation can work against the cleansing effects, shifting resources into less productive rather than more productive arrangements. The reason for this result is that the model introduces additional incentive constraints which arise because of credit market imperfections. In contrast with the regular participation constraints, these constraints will not necessarily bind for the least efficient jobs first. The paper develops a general equilibrium model in which, for certain parameter restrictions, the most efficient jobs are also the most vulnerable to aggregate conditions. To ease the exposition, the model relies on certain simplifications, some of which are helpful in generating this prediction. For example, the model assumes short time horizons, which precludes creditors from punishing entrepreneurs who default by holding back credit. However, the same result could probably be generated with long horizons as well. Albequerque and Hopenhayn (1997) and Monge (1998) both consider the problem of optimal contracts in dynamic environments in which default is potentially desirable for the entrepreneur. One could introduce projects which require fixed investment to sustain production every period into their framework, and allow different projects to have different fixed costs. As long as entrepreneurs can choose which projects to develop, the same results would most likely continue to hold: in an interior equilibrium, the return on both projects must be the same, so that the threat of punishment will be the same for both types of projects. But projects with higher setup costs would be associated with a greater temptation to default, and so would be the first to be destroyed. Another simplification in the model concerns the interest rate. In my framework, the interest rate $R$ is determined exogenously by the storage technology. But without this feature, creditors could potentially use interest rates to design contracts which are incentive compatible. Once again, the results described in this paper will probably continue to hold. Lenders who extend larger amounts of credit will also require higher interest rates. But if these interest rates are too high, entrepreneurs will have incentive to default. Hence, once again, it is likely that projects with large setup costs will be the most vulnerable to recessions. Although the model is rather stylized, its implications are probably more robust than would appear at first.

Finally, it is worth emphasizing that the model predicts credit markets could amplify underlying productivity shocks by forcing a reallocation into less productive jobs. This channel is distinct from the net-worth effect channel, which was formalized in Bernanke and Gertler (1989). In the latter, recessions reduce the net worth of entrepreneurs, forcing them to finance a larger fraction of their projects through outside funding instead of internally. But if projects are financed externally, the incentives of entrepreneurs become distorted, and production falls below the optimal level. This argument does not depend on composition: the effects would continue to hold even if production was carried out by a single entrepreneur. By contrast, the channel discussed in this paper is an argument about changes in composition: during recessions, projects which generate more surplus are more likely to violate an incentive constraint, forcing resources to be reallocated towards other projects in equilibrium. This raises a natural question as to how significant this amplification is. Carlstrom and Fuerst (1997) calibrate a version of the Bernanke and Gertler model to assess the amplification affects that channel. A similar exercise could be carried out for the present model.

## Appendix

**Lemma**: Suppose $\Omega_i = \{g, b\}$ for a positive measure of entrepreneurs. Then there is no equilibrium in which only one type of intermediate good is sold to final goods producers.

**Proof**: Suppose not, i.e. $y_{jt} = 0$ for some $j$ but $y_{j't} > 0$ for $j' \neq j$. Since $y_{gt}$ and $y_{bt}$ must solve the maximization problem for the final goods producer, it follows that

$$\frac{\partial f}{\partial y_{jt}} \leq p_{jt} \tag{4.1}$$

with equality if $y_{jt} > 0$. Differentiating $f$ with respect to $y_{jt}$ yields

$$\frac{\partial f}{\partial y_{jt}} = 2^{-\frac{1}{\rho}} z \left(\frac{y_{jt}}{y_t}\right)^{\rho - 1}$$

where $y_t = \left(y_{gt}^{\rho} + y_{bt}^{\rho}\right)^{\frac{1}{\rho}}$. Since $\rho < 1$, $\dfrac{\partial f}{\partial y_{jt}} = \infty$. Hence, $p_{jt} = \infty$. Since $y_{j't} > 0$, $p_{j'} = \dfrac{\partial f}{\partial y_{j't}} = 2^{-\frac{1}{\rho}} z < \infty$. This implies an entrepreneur could earn infinite profits from producing good $j$ but only finite profits from producing good $j'$. The positive measure of entrepreneurs for which $\Omega_i = \{g, b\}$ will produce good $j$, which implies $y_{jt} > 0$, a contradiction.

**Proof of Proposition 1**: For ease of notation, I suppress time subscripts unless necessary. The proof proceeds in several steps. First, I show there exists a $z_0$ such that $Y_j = 0$ is an equilibrium if and only if $z < z_0$. I then demonstrate that this equilibrium is unique for $z < z_0$.

Since labor participation constraints are assumed not to bind, a no-production equilibrium involves two conditions. First, no entrepreneur wishes to produce either good $j$, so

$$(1 - \alpha) p_j \leq R k_j$$

Second, since $Y_j = 0$ implies $y_j = 0$, then $y_g = y_b = 0$ must solve the maximization problem of the final goods producer. Since the profit function is globally concave, this will be true if and only if

$$\frac{\partial f}{\partial y_j} - p_j \leq 0$$

for all $(y_g, y_b) \in R_+$. Substituting in for the derivative of $f$ and combining the two constraints implies that a no-production equilibrium exists if and only if for all $(y_g, y_b) \in R_+$,

$$\begin{aligned} 2^{-\frac{1}{\rho}} z (1 - \alpha) \left(1 + \left(\frac{y_g}{y_b}\right)^{-\rho}\right)^{\frac{1-\rho}{\rho}} &\leq R k_g \\ 2^{-\frac{1}{\rho}} z (1 - \alpha) \left(1 + \left(\frac{y_g}{y_b}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}} &\leq R k_b \end{aligned}$$

Let $\theta = \frac{y_g}{y_b}$, and define $x_j = (1-\alpha)\frac{\partial f}{\partial y_j}$. Then

$$\begin{aligned} x_g &= 2^{-\frac{1}{\rho}} z (1-\alpha) \left(1+\theta^{-\rho}\right)^{\frac{1-\rho}{\rho}} \\ x_b &= 2^{-\frac{1}{\rho}} z (1-\alpha) \left(1+\theta^{\rho}\right)^{\frac{1-\rho}{\rho}} \end{aligned}$$

Consider the graph $\Gamma(z) = \{(x_b, x_g) \mid \theta \in (0,\infty)\}$ defined for a given $z$. This graph traces a downward sloping convex curve in $(x_b, x_g)$ space. To see this, note that

$$\frac{dx_g}{dx_b} = \frac{dx_g/d\theta}{dx_b/d\theta} = -\left(\frac{1+\theta^{-\rho}}{1+\theta^{\rho}}\right)^{\frac{1}{\rho}} < 0$$

and

$$\frac{d^2 x_g}{dx_b^2} = \frac{d\left(dx_g/dx_b\right)/d\theta}{dx_b/d\theta} = \frac{\theta^{-\rho}\left(1+\theta^{\rho}\right)^{\frac{2(\rho-1)}{\rho}}\left(1+\theta^{-\rho}\right)^{\frac{1}{\rho}}}{1-\rho} > 0$$

Let $K(z) = [0, Rk_b] \times [0, Rk_g]$. It follows that a no-production equilibrium exists if and only if $K(z) \cap \Gamma(z) \neq \emptyset$. For ease of notation, I omit the reference to $z$ when referring to each set.

The set $K \cap \Gamma$ is isomorphic to the set $\{\theta \in (0,\infty) \mid (x_b, x_g) \in K\}$. Hence, $K \cap \Gamma = \emptyset$ if and only if $\{\theta \in (0,\infty) \mid (x_b, x_g) \in K\} = \emptyset$. Since $\Gamma$ is downward sloping and each $x_j$ is monotonic in $\theta$, the set $\{\theta \in (0,\infty) \mid (x_b, x_g) \in K\}$ is a closed interval $\left[\underline{\theta}, \overline{\theta}\right]$ where the endpoints of the interval are given by the solutions to the two equations

$$\begin{aligned} x_g(\underline{\theta}) &= Rk_b + zk \\ x_b(\overline{\theta}) &= Rk_b \end{aligned}$$

This system can be rewritten as

$$\begin{aligned} \left(1+\underline{\theta}^{-\rho}\right)^{\frac{1-\rho}{\rho}} &= \frac{2^{\frac{1}{\rho}} Rk_b}{z(1-\alpha)} + \frac{2^{\frac{1}{\rho}} Rk}{(1-\alpha)} \\ \left(1+\overline{\theta}^{\rho}\right)^{\frac{1-\rho}{\rho}} &= \frac{2^{\frac{1}{\rho}} Rk_b}{z(1-\alpha)} \end{aligned}$$

Since $0 < \rho < 1$, the LHS in the first equation is monotonically decreasing in $\underline{\theta}$, while the LHS in the second equation is monotonically increasing in $\overline{\theta}$. Hence, $\underline{\theta}(z)$ is increasing in $z$ and $\overline{\theta}(z)$ is decreasing in $z$. Thus, if $K \cap \Gamma = \emptyset$ for some $z'$, then $K \cap \Gamma = \emptyset$ for any $z > z'$, and if $K \cap \Gamma \neq \emptyset$ for some $z'$, then $K \cap \Gamma \neq \emptyset$ for any $z < z'$. It remains to show that a $z_0$ exists. To see this, note that as $z \to 0$, $\underline{\theta} \to 0$ and $\overline{\theta} \to \infty$, so $K \cap \Gamma \neq \emptyset$ for $z = 0$. Next, for $z = \frac{2^{\frac{1}{\rho}} Rk_b}{1-\alpha}$, $\overline{\theta} = 0$ while $\underline{\theta} > 0$, so $K \cap \Gamma = \emptyset$ for $z = \frac{2^{\frac{1}{\rho}} Rk_b}{1-\alpha}$. Since both $\underline{\theta}$ and $\overline{\theta}$ are continuous in $z$, there exists a $z_0$ such that $\overline{\theta}(z_0) = \underline{\theta}(z_0)$. It follows that no production is an equilibrium if and only if $z < z_0$

To prove that no production is the unique equilibrium, suppose $z < z_0$. From the lemma, we know that either $y_j = 0$ for both $j$ or $y_j > 0$ for both $j$. We need to rule out the case where $y_j > 0$ for both $j$. To do this, note that $y_j > 0$ implies $p_j = \frac{\partial f}{\partial y_j}$. Let $K' = [Rk_b, \infty) \times [Rk_g, \infty)$. The case where $y_j > 0$ can be an equilibrium only if $K' \cap \Gamma \neq \emptyset$, or else entrepreneur would not be willing to supply both intermediate goods. But since $\Gamma$ is downward sloping, $K \cap \Gamma \neq \emptyset$ implies $K' \cap \Gamma = \emptyset$. Hence, if $z < z_0$, any equilibrium involves $y_j = 0$ for both $j \in \{g, b\}$. Now, if $y_j = 0$, it will be impossible to produce capital in period $t$, since capital requires final goods. This implies $Y_{g,t+1} = 0$. Since $Y_{jt} = y_{jt} + kY_{g,t+1}$, it follows that $Y_{jt} = 0$ for both $j = 0$ in any equilibrium when $z_t < z_0$.

**Proof of Proposition 2**: From the proof of Proposition 1, $z_t > z_0$ implies $K \cap \Gamma = \emptyset$. Hence, there always exists at least one good for which profits are positive. Since $\Omega_i = \{g, b\}$ for all $i$, this implies that if an equilibrium exists, all entrepreneurs will produce intermediate goods, i.e. $Y_{gt} + Y_{bt} = 1$. Of this amount, $kY_{g,t+1}$ of each intermediate good is sold to entrepreneurs in the next cohort who produce good $g$. The remaining $1 - 2kY_{g,t+1}$ entrepreneurs sell their output directly to the final goods producer. Since $k < \frac{1}{2}$ and $Y_{g,t+1} \leq 1$, this implies that $y_{gt} + y_{bt} > 0$. But from the lemma, this requires $y_{jt} > 0$ for both $j \in \{g, b\}$.

Since $\Omega_i = \{g, b\}$ for all $i$, $Y_{jt} > 0$ for both $j$ only when entrepreneurs are indifferent between projects, i.e.

$$(1-\alpha)(p_{gt} - p_{bt}) = R(k_g - k_b)$$

or

$$p_{gt} - p_{bt} = \frac{zRk}{1-\alpha}$$

Since both $y_{jt} > 0$, the maximization problem of the final goods producer implies $p_{jt} = \dfrac{\partial f}{\partial y_{jt}}$, which yields the following equation:

$$\frac{y_{gt}^{\rho-1} - y_{bt}^{\rho-1}}{\left(y_{gt}^{\rho} + y_{bt}^{\rho}\right)^{\frac{\rho-1}{\rho}}} = \frac{Rk}{1-\alpha}$$

This equation is independent of $z_t$. Substituting in the fact that $y_{gt} + y_{bt} = 1 - 2kY_{g,t+1}$ yields

$$\frac{y_{gt}^{\rho-1} - (1 - 2kY_{g,t+1} - y_{gt})^{\rho-1}}{\left(y_{gt}^{\rho} + (1 - 2kY_{g,t+1} - y_{gt})\,\rho\right)^{\frac{\rho-1}{\rho}}} = \frac{Rk}{1-\alpha}$$

This LHS of the above equation is monotonic in $y_{gt}$ and yields the solution

$$y_{gt} = \phi\,(1 - 2kY_{g,t+1})$$

where $\phi \in \left(0, \frac{1}{2}\right)$ is a constant which depends on the parameters $k$, $R$, $\alpha$, and $\rho$.

Next, from the market clearing condition for intermediate good $g$, we have

$$\begin{aligned} Y_{gt} &= y_{gt} + kY_{g,t+1} \\ &= \phi\,(1 - 2kY_{g,t+1}) + kY_{g,t+1} \\ &= \phi + k\,(1 - 2\phi)\,Y_{g,t+1} \end{aligned}$$

which implies a linear mapping from $Y_{gt}$ to $Y_{g,t+1}$. This mapping is illustrated in Figure A1. This mapping has a slope of

$$\frac{dY_{g,t+1}}{dY_{gt}} = \frac{1}{k(1-2\phi)} > 1$$

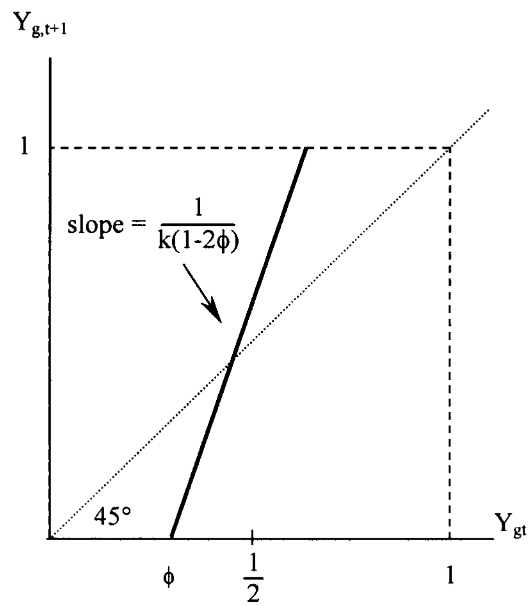



Figure A1: The Dynamic Relationship governing Output of Good g and has a fixed point given by

$$Y_{gt} = Y_{g,t+1} = \frac{\phi}{1 - k(1 - 2\phi)}$$

which is less than $\frac{1}{2}$ for $k, \phi < \frac{1}{2}$. This fixed point implies the following quantities:

$$\begin{aligned} Y_g &= \frac{\phi}{1 - k(1 - 2\phi)} && (4.2) \\ Y_b &= \frac{1 - \phi - k(1 - 2\phi)}{1 - k(1 - 2\phi)} && (4.3) \\ y_g &= \frac{\phi(1 - k)}{1 - k(1 - 2\phi)} && (4.4) \\ y_b &= \frac{(1 - \phi)(1 - k)}{1 - k(1 - 2\phi)} && (4.5) \end{aligned}$$

At the prices $p_j = \frac{\partial f}{\partial y_j}$ evaluated at the above $y_j$, we know by construction that profits are equal for both goods. Moreover, since profits are increasing in $z$ and profits are equal to 0 at $z = z_0$, we know that profits are strictly positive. Hence, at these prices, entrepreneurs are willing to supply $Y_g$ and $Y_b$ intermediate goods. Hence, this fixed point constitutes an equilibrium.

To see that this is the unique equilibrium quantities and prices, suppose $Y_{gt} \neq \frac{\phi}{1 - k(1 - 2\phi)}$ for some $t$. Applying the mapping $Y_{gt} \to Y_{g,t+1}$ repeatedly, we have that in finite time, we reach a point where either $Y_{gt} = 0$ and $Y_{bt} = 1$ or $Y_{gt} = 1$ and $Y_{bt} = 0$. In either case, there exists a period $t$ and an intermediate good $j \in \{g, b\}$ such that $y_{jt} = 0$ in equilibrium. But at the same time, $y_{gt} + y_{bt} = 1 - 2kY_{g,t+1} > 0$. But this contradicts the lemma, which implies that either $y_{jt} = 0$ for both $j$ or $y_{jt} > 0$ for both $j$.

**Proof of Proposition 3**: The proof proceeds in three steps. It will be useful in what follows to substitute in the Nash bargaining rule into the labor participation constraint and rewrite it as

$$\alpha p_j \geq \frac{b}{R}$$

The first step in the proof shows that the equilibrium in Proposition 2 automatically satisfies the labor participation constraint if $b \leq \frac{\alpha R^2 k_b}{1 - \alpha}$. For suppose the entrepreneur participation constraint is satisfied in sector $j$, i.e. $(1 - \alpha)p_j \geq Rk_j$. Multiplying both sides by $\frac{\alpha}{1 - \alpha}$ yields

$$\alpha p_j \geq \frac{\alpha R k_j}{1 - \alpha}$$

Since $b \leq \frac{\alpha R^2 k_b}{1 - \alpha}$, the fact that $k_j \geq k_b$ for both $j$ implies that $\alpha p_j \geq \frac{b}{R}$, so the labor participation constraint is automatically satisfied whenever the entrepreneur participation constraint is satisfied.. Hence, the labor participation constraint is redundant for $z_t > z_0$, implying that the unique equilibrium is given by Proposition 2.

The second step in the proof deals with the existence of $\underline{z}$. Define $K'$ as before, and define the set $B' = \left[\frac{(1 - \alpha)b}{\alpha R}, \infty\right) \times \left[\frac{(1 - \alpha)b}{\alpha R}, \infty\right)$. I first show there exists a $\underline{z}$ such $K' \cap B' \cap \Gamma \neq \emptyset$ if and only if $z \geq \underline{z}$, and then use this to make statements about the nature of equilibrium. Once again, I can construct the set $\{\theta \in (0,\infty) \mid (x_b, x_g) \in K' \cap B'\}$. This set is empty if and only if the set $K' \cap B' \cap \Gamma$ is empty. The first set will be a (possibly empty) interval $[\underline{\theta}, \overline{\theta}]$. For $b > \dfrac{\alpha R^2 k_b}{1-\alpha}$, the endpoints of the interval are given by the solutions to the two equations:

$$\begin{aligned} x_b(\underline{\theta}) &= \frac{(1-\alpha)b}{\alpha R} \\ x_g(\overline{\theta}) &= \max\left(Rk_b + zRk, \frac{(1-\alpha)b}{\alpha R}\right) \end{aligned}$$

Substituting in for $x_j(\theta)$ yields

$$\begin{aligned} \left(1+\underline{\theta}^{\rho}\right)^{\frac{1-\rho}{\rho}} &= \frac{2^{\frac{1}{\rho}} b}{\alpha R z} \\ \left(1+\overline{\theta}^{-\rho}\right)^{\frac{1-\rho}{\rho}} &= \begin{cases} \dfrac{2^{\frac{1}{\rho}} b}{\alpha R z} & \text{if } z < \dfrac{(1-\alpha)b}{\alpha R^2 k} - \dfrac{k_b}{k} \\ \dfrac{2^{\frac{1}{\rho}} R k_b}{z(1-\alpha)} + \dfrac{2^{\frac{1}{\rho}} R k}{(1-\alpha)} & \text{else} \end{cases} \end{aligned}$$

In both equations, the RHS is a monotonic and continuous function of $z$. Since $\rho \in (0,1)$, the LHS in the first equation is monotonically increasing in $\underline{\theta}$, while the LHS in the second equation is monotonically decreasing in $\overline{\theta}$. Hence, $\underline{\theta}(z)$ is decreasing in $z$ and $\overline{\theta}(z)$ is increasing in $z$. Taking limits as $z$ approaches 0 and $\infty$ establishes the existence of a $\underline{z}$ for which $\underline{\theta} = \overline{\theta}$. Hence, $K' \cap B' \cap \Gamma$ is empty if and only if $z < \underline{z}$.

Likewise, I can define a new set $H(z) = \left[0, \dfrac{(1-\alpha)b}{\alpha R}\right] \times \left[0, \max\left(Rk_g, \dfrac{(1-\alpha)b}{\alpha R}\right)\right]$ and apply the same argument on the set $\{\theta \in (0,\infty) \mid (x_b, x_g) \in H\}$ to show that the set $H \cap \Gamma = \emptyset$ if and only if $z > \underline{z}$. Turning to equilibrium, final goods can be produced in equilibrium if and only if $K' \cap B' \cap \Gamma \neq \emptyset$, since the lemma implies that final goods must be produced using both intermediate goods. Hence, if an equilibrium exists for $z_t < \underline{z}$, it must be a no-production equilibrium. Since $H \cap \Gamma \neq \emptyset$ for $z < \underline{z}$, it follows that there exist prices such that there is a binding participation constraint for each good at such prices, so no entrepreneur can supply intermediate goods and is thus solving (1.2).

To rank $z_0$ and $\underline{z}$, recall from Proposition 1 that $K' \cap \Gamma = \emptyset$ if $z < z_0$. Since $K' \cap \Gamma \cap B' \neq \emptyset$ if $z > \underline{z}$, it follows that $\underline{z} \geq z_0$. To show the inequality is strict, consider $z = z_0$. The proof of Proposition 1 establishes that $K'(z_0) \cap \Gamma(z_0) = \{(Rk_b, R(k_b + z_0 k))\}$, i.e. the intersection of the two sets at $z = z_0$ consists of a single point. But for $b > \dfrac{\alpha R^2 k_b}{1-\alpha}$, it follows that at this point, $x_b < \dfrac{(1-\alpha)b}{\alpha R}$, implying $(Rk_b, R(k_b + z_0 k)) \notin B'$. Hence, $K'(z_0) \cap B' \cap \Gamma(z_0) = \emptyset$, so $\underline{z} > z_0$.

The third and last step of the proof involves the existence of $\overline{z}$. Since the lemma implies both intermediate goods will be produced in any equilibrium with production, it must be the case that profits are equal in both sectors. From the proof of Proposition 2, we know that profits will be equal in both sectors if and only if

$$\begin{aligned} y_{gt} &= \phi\left(1 - 2kY_{g,t+1}\right) \\ y_{bt} &= (1-\phi)\left(1 - 2kY_{g,t+1}\right) \end{aligned}$$

Recall that the set $\Gamma(z) = \{(x_b, x_g) \mid \theta \in (0, \infty)\}$ is parameterized by the ratio $\theta = \frac{y_{gt}}{y_{bt}}$. When profits are equal in both sectors, this ratio will equal $\frac{\phi}{1-\phi}$. So consider the set $\Gamma(z)$ evaluated at this value of $\theta$ for different levels of $z$. Since $x_j$ is monotonically increasing in $z$, there exists a $\overline{z}$ such that $x_b\left(\frac{\phi}{1-\phi}\right) > \frac{(1-\alpha)b}{\alpha R}$ if and only if $z > \overline{z}$. For $b > \frac{\alpha R^2 k_b}{1-\alpha}$, this implies $x_b\left(\frac{\phi}{1-\phi}\right) > Rk_b$. Since $x_j - Rk_j$ is the same in both sectors at this value of $\theta$, it also follows that $x_g\left(\frac{\phi}{1-\phi}\right) > Rk_g$. Finally, since $\phi < \frac{1}{2}$, we have $x_g > x_b$. Hence, $x_g\left(\frac{\phi}{1-\phi}\right) > \frac{(1-\alpha)b}{\alpha R}$. Hence, if $z_t > \overline{z}$, the pair $(x_b, x_g)$ evaluated at $\theta = \frac{\phi}{1-\phi}$ is contained in the set $K' \cap B'$, i.e. both participation constraints are satisfied at this equilibrium. Since $K' \cap B' \cap \Gamma \neq \emptyset$ only if $z_t \geq \underline{z}$, it follows that $\overline{z} \geq \underline{z}$. Hence, if $z_t > \overline{z}$, any equilibrium must involve production in period $t$ since $z_t > \underline{z}$, and if an equilibrium does exist, the labor participation must not be binding in period $t$ in that equilibrium, establishing the claim.

Finally, I need to show $\overline{z} > \underline{z}$. At $z_t = \underline{z}$, we have $K' \cap B' \cap \Gamma = \left\{\left(\frac{(1-\alpha)b}{\alpha R}, \max\left[Rk_g, \frac{(1-\alpha)b}{\alpha R}\right]\right)\right\}$, i.e. the set contains a single point. Now, if $x_g = \frac{(1-\alpha)b}{\alpha R}$, then $x_g/x_b = 1$, which implies $\theta = 1$. But we know that at $z_t = \overline{z}$, the set $K' \cap B' \cap \Gamma$ includes the point which corresponds with $\theta = \frac{\phi}{1-\phi} < 1$. So it must be the case that $x_g = Rk_g$. But this implies profits are equal to zero in sector $g$ in equilibrium, which is only true for $z = z_0$. But I already proved $\underline{z} > z_0$, so again we have a contradiction. Hence, $\overline{z} > \underline{z}$.

**Proof of Proposition 4**: Consider $z < \underline{z}$. The no-production equilibrium from Proposition 3 continues to be an equilibrium even after introducing a rationing rule, since the rationing rule allows each entrepreneur to choose from a subset of his original choice, but allows any entrepreneur to choose not to produce. Hence, the same prices can still be used to sustain a no-production equilibrium. I can rule out additional equilibria with production, since $\Omega_i$ still allows a positive measure of entrepreneurs to choose between two projects, implying that the lemma still holds. This implies that in any equilibrium with production, both intermediate goods must be produced. But the previous proof established that this is impossible for $z_t < \underline{z}$, since $K' \cap B' \cap \Gamma = \emptyset$.

Next, consider $z \in (\underline{z}, \overline{z})$. To sustain a no-production equilibrium, we must have participation constraints being violated in both sectors, i.e. $H \cap \Gamma \neq \emptyset$. But from the proof of Proposition 3, $H \cap \Gamma = \emptyset$ if $z > \underline{z}$. So we can rule out no production as an equilibrium. Next, Proposition 2 tells us that the unique $\theta$ for which profits are equal in both sectors is $\theta = \frac{\phi}{1-\phi}$. But since, by the construction in Proposition 3, $x_b\left(\frac{\phi}{1-\phi}\right) \geq \frac{(1-\alpha)b}{\alpha R}$ if and only if $z \geq \overline{z}$, from which it follows that for $z \in (\underline{z}, \overline{z})$, $\left(x_b\left(\frac{\phi}{1-\phi}\right), x_g\left(\frac{\phi}{1-\phi}\right)\right) \notin K' \cap B'$. Hence, profits cannot be equal across sectors in equilibrium for $z \in (\underline{z}, \overline{z})$. Now, we know that $x_b - Rk_b > x_g - Rk_g$ if and only if $\theta > \frac{\phi}{1-\phi}$. So we need to determine whether for $z < \overline{z}$, the set $[\underline{\theta}, \overline{\theta}]$ lies above or below $\frac{\phi}{1-\phi}$. By continuity, it is sufficient to determine this for any particular value $z < \overline{z}$. Take $z = \underline{z}$. At this value, we know from Proposition 3 that $K' \cap B' \cap \Gamma$ consists of a single point, i.e. $x_b = \frac{(1-\alpha)b}{\alpha R}$ and $x_g = \max\left[Rk_g, \frac{(1-\alpha)b}{\alpha R}\right]$. If $x_g = \frac{(1-\alpha)b}{\alpha R}$, then $\underline{\theta} = \overline{\theta} = 1 > \frac{\phi}{1-\phi}$. If $x_g = Rk_g$, then $x_g - Rk_g = 0$. But since $(x_g, x_b) \in K'$, then $x_b - Rk_b \geq 0$, and since we know $\underline{\theta} = \overline{\theta} \neq \frac{\phi}{1-\phi}$, it must be the case that $x_b - Rk_b > 0$. But then $x_b - Rk_b > x_g - Rk_g$, which implies $\underline{\theta} = \overline{\theta} > \frac{\phi}{1-\phi}$. It follows that the interval $[\underline{\theta}, \overline{\theta}]$ lies above $\frac{\phi}{1-\phi}$ for $z < \overline{z}$, implying that profits must be greater in sector $b$ in equilibrium. Since $x_b - Rk_b > x_g - Rk_g \geq 0$, any entrepreneur with $\Omega_i = \{g, b\}$ will produce good $b$. Given the rationing rule, this implies

$$\begin{aligned} Y_{bt} &= kY_{g,t+1} + (1-\phi_z)(1-2kY_{g,t+1}) \\ y_{bt} &= (1-\phi_z)(1-2kY_{g,t+1}) \end{aligned}$$

Define $\psi$ so that in equilibrium,

$$y_{gt} = \psi(1-2kY_{g,t+1})$$

It follows that $\psi \in [0, \phi_z]$, and that $\theta = \frac{\psi}{1-\phi_z} \leq \frac{\phi_z}{1-\phi_z}$ in equilibrium. By definition, $\phi_z$ is determined so that $\frac{\partial f(\phi_z, 1-\phi_z)}{\partial y_b} = \frac{b}{\alpha R}$. But this says $\frac{\phi_z}{1-\phi_z} = \overline{\theta}(z)$. Since $x_g(\overline{\theta}) - Rk_g > 0$ for $z > \underline{z}$, and since $x_g(\theta)$ is increasing in $\theta$, this implies that for all values of $\psi$, $x_g - Rk_g > 0$. Hence, all entrepreneurs with $\Omega_i = \{g\}$ produce good $g$, so that

$$\begin{aligned} Y_{gt} &= kY_{g,t+1} + \phi_z(1-2kY_{g,t+1}) \\ y_{gt} &= \phi_z(1-2kY_{g,t+1}) \end{aligned}$$

This characterizes the unique quantities associated with equilibrium. We can verify this is in fact an equilibrium, since $\theta = \overline{\theta}$ implies that at these prices, $K' \cap B' \cap \Gamma \neq \emptyset$ and all participation constraints are satisfied. Entrepreneurs are sure to solve (1.2) by construction. Turning to the determination of $\phi_z$, we have

$$\left(1 + \left(\frac{\phi_z}{1-\phi_z}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}} = \frac{2^{\frac{1}{\rho}} b}{\alpha R z}$$

which shows that $\phi_z$ is decreasing in $z$. Now, for $z \in (\underline{z}, \overline{z})$, the fact that $x_b = \frac{(1-\alpha)b}{\alpha R}$ implies $x_g \geq \frac{(1-\alpha)b}{\alpha R}$, since $(x_b, x_g) \in K' \cap B'$ when $x_b = \frac{(1-\alpha)b}{\alpha R}$. This implies $\theta \leq 1$, which implies $\phi_z < \frac{1}{2}$ for $z > \underline{z}$. Hence, just as in the proof of Proposition 2, the mapping of $Y_{gt}$ into $Y_{g,t+1}$ is upward sloping with a slope greater than 1, so the unique equilibrium must be the fixed point

$$\begin{aligned} Y_g &= \frac{\phi_z}{1-k(1-2\phi_z)} \\ Y_b &= \frac{1-\phi-k(1-2\phi_z)}{1-k(1-2\phi_z)} \\ y_g &= \frac{\phi_z(1-k)}{1-k(1-2\phi_z)} \\ y_b &= \frac{(1-\phi_z)(1-k)}{1-k(1-2\phi_z)} \end{aligned}$$

Since $Y_g$ and $y_g$ are both increasing in $\phi_z$, both are decreasing in $z$, as claimed.

For the final part of the proposition, suppose $z > \overline{z}$. Since $\dfrac{\phi_z}{1-\phi_z} = \overline{\theta}$ and $\dfrac{\phi}{1-\phi} \in (\underline{\theta}, \overline{\theta})$ for $z > \overline{z}$, then $\dfrac{\phi_z}{1-\phi_z} < \dfrac{\phi}{1-\phi}$ and $\phi > \phi_z$. Since $z > \overline{z} > \underline{z}$, we can rule out no production as an equilibrium.. Given the rationing rule, we must have either profits being equal for both goods or profits are higher for good $b$. Suppose the latter is the case. Then all entrepreneurs with $\Omega_i = \{g, b\}$ produce good $b$. But given the rationing rule,

$$\theta = \frac{y_{gt}}{y_{bt}} \leq \frac{\phi_z}{1-\phi_z} < \frac{\phi}{1-\phi}$$

which implies profits in sector $g$ are greater than in sector $b$, a contradiction. Hence, profits must be equal in both sectors in equilibrium. This requires

$$\begin{aligned} Y_{bt} &= kY_{g,t+1} + (1-\phi)(1-2kY_{g,t+1}) \\ &< kY_{g,t+1} + (1-\phi_z)(1-2kY_{g,t+1}) \end{aligned}$$

which is feasible under the rationing rule. At this value of $\theta$, $(x_b, x_g)$ is in the interior of $K' \cap B'$, so both labor participation constraints are satisfied, and the equilibrium is the same as in Proposition 2.

**Proof of Proposition 5**: Applying the arguments from the proof of Proposition 4, we know that if $z_t = z$ then

$$y_{gt} = \phi_z(1-2kY_{g,t+1})$$

which implies

$$Y_{gt} = kY_{g,t+1} + \phi_z(1-2kY_{g,t+1})$$

and if $z_t = Z$ then

$$y_{gt} = \phi(1-2kY_{g,t+1})$$

which implies

$$Y_{gt} = kY_{g,t+1} + \phi(1-2kY_{g,t+1})$$

where $\phi_z > \phi$.

Choose $t$ so that $z_t = z$. Substitute in for $Y_{g,t+1}$ from the equation in period $t+1$ to obtain the following two-period ahead mapping:

$$\begin{aligned} Y_{gt} &= \phi_z + k(1-2\phi_z)Y_{g,t+1} \\ &= \phi_z + k(1-2\phi_z)\left[\phi + k(1-2\phi)Y_{g,t+2}\right] \\ &= \phi_z + k(1-2\phi_z)\phi + (1-2\phi_z)(1-2\phi)k^2 Y_{g,t+2} \end{aligned}$$

This describes a linear mapping from $Y_{gt}$ to $Y_{g,t+2}$ with slope equal to $\left[(1-2\phi_z)(1-2\phi)k^2\right]^{-1} > 1$, and with a fixed point given by

$$Y_g(z) = \frac{\phi_z + \phi k - 2k\phi\phi_z}{1-(1-2\phi_z)(1-2\phi)k^2} \in (0,1)$$

and likewise

$$Y_g(Z) = \frac{\phi + \phi_z k - 2k\phi\phi_z}{1-(1-2\phi_z)(1-2\phi)k^2} \in (0,1)$$

Once again, since the slope of the mapping is greater than 1, any value of $Y_g(z)$ or $Y_g(Z)$ other than the fixed point will shoot off towards either 0 or 1, which is inconsistent with equilibrium since both goods must be produced. This establishes uniqueness. Moreover, since $\phi_z > \phi$ and $k < \frac{1}{2}$, it follows that $\phi_z + \phi k < \phi + \phi_z k$, which implies $Y_g(z) > Y_g(Z)$. Finally,

$$\begin{aligned} y_g(z) &= \phi_z(1 - 2kY_g(Z)) \\ y_g(Z) &= \phi(1 - 2kY_g(z)) \end{aligned}$$

and since $\phi_z > \phi$ and $Y_g(z) > Y_g(Z)$, then $1 - 2kY_g(Z) > 1 - 2kY_g(z)$, so $y_g(z) > y_g(Z)$.

**Proof of Proposition 6**: Suppose (3.1) is satisfied. Then $(1-\alpha)p_j - Rk_j > Rk_j > 0$, so (3.1) implies (1.3). Thus, the only relevant constraints for the entrepreneur are (3.1). Since the incentive constraint $(1-\alpha)p_j > 2Rk_j$ is identical to the participation constraint $(1-\alpha)p_j > R'k_j$ where $R' = 2R$, the existence of $z'$ follows from the existence of $z_0$ from Proposition 1.

Next, fix $\theta = \dfrac{\phi}{1-\phi}$. Since $x_j$ is monotonically increasing in $z$, it follows that there exists a $z_1$ such that $x_b\left(\dfrac{\phi}{1-\phi}\right) > 2Rk_b$ if and only if $z > z_1$, so the incentive constraint is satisfied for good $b$. Writing out this condition for good $g$ yields the following condition:

$$\left[2^{-\frac{1}{\rho}}(1-\alpha)\left(1+\left(\frac{\phi}{1-\phi}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}} - 2Rk\right] z > 2Rk_b$$

If the expression in brackets is positive, which occurs if and only if condition (3.2) holds, then there also exists a $z_2$ such that $x_g\left(\dfrac{\phi}{1-\phi}\right) > 2Rk_g$ if and only if $z > z_2$. Let $z'' = \max(z_1, z_2)$. Since in any equilibrium with $z > z^*$, profits must be equal and $\dfrac{y_{gt}}{y_{bt}} = \dfrac{\phi}{1-\phi}$, it follows that for $z > z''$, neither incentive constraint will be binding in equilibrium. Since labor participation constraints are not binding by assumption, the analysis of Proposition 2 applies. If the expression in brackets is nonpositive, then $x_g\left(\dfrac{\phi}{1-\phi}\right) < 2Rk_g$ for all $z$, and the incentive constraints will bind for good $g$ when profits are the same for both goods. In this case, define $z'' = \infty$, since in this case there exists no $z > z''$.

To prove $z' > z_0$, note that (3.1) implies (1.3), so that $K' \cap \Gamma \neq \emptyset$ for $z = z'$. Since $K' \cap \Gamma$ only for $z \geq z_0$, it follows that $z' \geq z_0$. To prove the inequality is strict, note that at $z = z_0$, $K' \cap \Gamma =$

$\{(Rk_b, R(k_b + zk))\}$, i.e. it contains a single point, which is associated with a ratio $\theta = \frac{y_g}{y_b} = \frac{\phi}{1-\phi}$. At this ratio, $x_j = Rk_j < 2Rk_j$. For all other values of $\theta = \frac{y_g}{y_b} \neq \frac{\phi}{1-\phi}$, there exists at least one $j \in \{g, b\}$ such that $x_j(\theta) < Rk_j < 2Rk_j$. Hence, at $z = z_0$, for all $\theta \in [0, \infty)$, there exists a $j$ such that $x_j < 2Rk_j$. But by definition, at $z'$, there exists a $\theta$ such that $x_j(\theta) > 2Rk_j$ for both $j$. It follows that $z' > z_0$.

Next, since it is possible to satisfy both incentive constraints for $z > z''$ at the ratio $\theta = \frac{\phi}{1-\phi}$, it follows that $z'' \geq z'$. To show that the inequality is strict, consider the case where $z = z'$, where both incentive are exactly binding: then there is single pair of $x_g$ and $x_b$ which satisfy both constraints, and at this pair both constraints are exactly binding: $x_g = 2R(k_b + z'k)$ and $x_b = 2Rk_b$. This pair is associated with a particular ratio $\theta'$. But then $\frac{x_g}{x_b} = \frac{k_b + z'k}{k_b} > \frac{k_b + z_0 k}{k_b}$. Since $\frac{x_g}{x_b}$ is monotonically decreasing in $\theta$, and since $\frac{x_g}{x_b} = \frac{k_b + z_0 k}{k_b}$ when $\theta = \frac{\phi}{1-\phi}$, it follows that $\theta' < \frac{\phi}{1-\phi}$. Hence, at $z = z'$, we have $x_g\left(\frac{\phi}{1-\phi}\right) < 2Rk_g$. By construction, then, $z'' > z'$.

Finally, suppose $z \in (z', z'')$ in some period $t$. Since $z > z'$, no production is not an equilibrium. Then if an equilibrium does exist, some production must take place. As in Proposition 1, since $z > z_0$, profits must be strictly positive for at least one good in equilibrium, all entrepreneurs will produce some good, so that $Y_g + Y_b = 1$. This implies $y_g + y_b = 1 - 2kY_{g,t+1} > 0$. By the lemma, this requires $y_j > 0$. Since $\Omega_i = \{g, b\}$ for all $i$, this requires profits to be equal for both goods. But profits are equal if and only if $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$. Since $z < z''$, $x_g\left(\frac{\phi}{1-\phi}\right) < 2Rk_g$, no entrepreneur can obtain credit to finance good $g$ in equilibrium. But this implies $y_g = 0$, a contradiction. Hence, no equilibrium exists.

**Proof of Proposition 7**: Define $A(z) = [0, 2Rk_b] \times [0, 2Rk_g]$ and $A'(z) = [2Rk_b, \infty) \times [2Rk_g, \infty)$. By the arguments above, since $z_t > z'$ for all $t$, no production cannot be an equilibrium given that $A \cap \Gamma = \emptyset$ for $z > z'$. Since the rationing rule leaves a positive mass of entrepreneurs with a choice of which good to produce, the lemma applies, and so it must be the case that $y_{jt} > 0$ for both $j$ in any candidate equilibrium. But if both goods are produced, that must mean (3.1) will be satisfied for both goods. In this case, the participation constraints (1.3) hold as strict inequalities, implying that all entrepreneurs will wish to produce goods, i.e. $Y_{gt} + Y_{bt} = 1$, and $y_{gt} + y_{bt} = 1 - 2kY_{g,t+1}$.

Consider first a period $t$ for which $z_t = Z$. Since $Z > z''$, we know that $x_g\left(\frac{\phi}{1-\phi}\right) > 2Rk_g$. By construction, $\phi_z$ in the rationing rule (3.3) is defined so that $x_g\left(\frac{\phi_z}{1-\phi_z}\right) = 2Rk_g$. Since $x_g$ is decreasing in $\theta$, it follows that $\frac{\phi_z}{1-\phi_z} > \frac{\phi}{1-\phi}$, so $\phi_z > \phi$. Now, suppose profits are unequal across goods. Given the rationing rule, it must be the case that profits in good $g$ exceed those in good $b$. Then any entrepreneur with $i \leq \phi_z(1 - 2kY_{g,t+1}) + kY_{g,t+1}$ will choose good $g$. Since profits are strictly positive in equilibrium, all remaining entrepreneurs choose good $b$. But in this case, the ratio $\frac{y_g}{y_b} = \frac{\phi_z}{1-\phi_z} > \frac{\phi}{1-\phi}$. But at such a ratio, profits from good $b$ exceed those in good $g$, a contradiction. Hence, profits must be equal in equilibrium. This can only be the case if $\frac{y_g}{y_b} = \frac{\phi}{1-\phi}$. Since $\phi < \phi_z$, it is possible under this rationing rule for enough entrepreneurs to choose good $g$ while the remainder choose good $b$. Hence, the unique equilibrium quantities must satisfy the conditions

$$y_{gt} = \phi(1 - 2kY_{g,t+1})$$

and

$$Y_{gt} = kY_{g,t+1} + \phi(1 - 2kY_{g,t+1})$$

Next, consider a period $t$ for which $z_t = z$. Since $z < z''$, we know that $x_g\left(\frac{\phi}{1-\phi}\right) < 2Rk_g$. Hence, profits in equilibrium when $z_t = z$ must be unequal, or else good $g$ will not be produced. Given the rationing rule, it must be the case that profits are higher from good $g$. Hence, all entrepreneurs with $i \leq \phi_z(1 - 2kY_{g,t+1}) + kY_{g,t+1}$ will produce $g$, and all remaining entrepreneurs produce good $b$. This implies $\frac{y_g}{y_b} = \frac{\phi_z}{1-\phi_z}$. By the same argument as above, it now follows that $\frac{\phi_z}{1-\phi_z} < \frac{\phi}{1-\phi}$. Hence, profits at this allocation will in fact be higher for good $g$. Hence, the unique equilibrium quantities must satisfy the conditions

$$y_{gt} = \phi_z(1 - 2kY_{g,t+1})$$

and

$$Y_{gt} = kY_{g,t+1} + \phi_z(1 - 2kY_{g,t+1})$$

where $\phi_z > \phi$.

The remainder of the proof follows from Proposition 5. In particular, the unique equilibrium corresponds with the fixed points of the two-period mapping, and are given by

$$Y_g(z) = \frac{\phi_z + \phi k - 2k\phi\phi_z}{1 - (1 - 2\phi_z)(1 - 2\phi)k^2} \in (0, 1)$$

and

$$Y_g(Z) = \frac{\phi + \phi_z k - 2k\phi\phi_z}{1 - (1 - 2\phi_z)(1 - 2\phi)k^2} \in (0, 1)$$

Since $\phi_z > \phi$ and $k < \frac{1}{2}$, it follows that $\phi_z + \phi k > \phi + \phi_z k$, which implies $Y_g(Z) > Y_g(z)$. Finally,

$$\begin{aligned} y_g(z) &= \phi_z(1 - 2kY_g(Z)) \\ y_g(Z) &= \phi(1 - 2kY_g(z)) \end{aligned}$$

and since $\phi_z < \phi$ and $Y_g(z) < Y_g(Z)$, then $1 - 2kY_g(Z) > 1 - 2kY_g(z)$, so $y_g(z) < y_g(Z)$.

**Proof of Proposition 8**: Let $m_j = \max\left(2Rk_j, \frac{(1-\alpha)b}{\alpha R}\right)$, and define the sets $M(z) = [0, m_g] \times [0, m_b]$ and $M'(z) = [m_g, \infty) \times [m_b, \infty) = A'(z) \cap B'$. A no-production equilibrium exists if and only if $M \cap \Gamma \neq \emptyset$, and an equilibrium with production exists only if $M' \cap \Gamma \neq \emptyset$. An equilibrium in which no constraints are binding exists if and only if $\left(x_b\left(\frac{\phi}{1-\phi}\right), x_g\left(\frac{\phi}{1-\phi}\right)\right) \in M'$. The standard arguments used in previous propositions can be applied to prove the existence of $z^*$ and $z^{**}$, as well as the fact that $z^* > z_0$. Condition (3.2) is necessary to insure that $z^{**}$ is finite. This establishes the first part of the proof.

Next, define

$$b^* = \alpha 2^{-\frac{1}{\rho}} R z'' \left(1 + \left(\frac{\phi}{1-\phi}\right)^{\rho}\right)^{\frac{1-\rho}{\rho}}$$

where $z''$ is defined in Proposition 6. The cutoff $b^*$ is chosen so that $\frac{(1-\alpha)b^*}{\alpha R}$ will equal $x_b\left(\frac{\phi}{1-\phi}\right)$ at $z = z''$. Suppose first $b < b^*$, and consider $z = z''$. By definition of $z''$, we know $x_j\left(\frac{\phi}{1-\phi}\right) = 2Rk_j$. Hence, both incentive constraints are satisfied at the point when profits are equal. In addition, we know that

$$x_b\left(\frac{\phi}{1-\phi}\right) = \frac{(1-\alpha)b^*}{\alpha R} > \frac{(1-\alpha)b}{\alpha R}$$

Since $x_g\left(\frac{\phi}{1-\phi}\right) > x_b\left(\frac{\phi}{1-\phi}\right)$, it follows that $x_g\left(\frac{\phi}{1-\phi}\right) > \frac{(1-\alpha)b}{\alpha R}$. Hence, $M' \cap \Gamma \neq \emptyset$ at $z = z''$. Since $M' \cap \Gamma \neq \emptyset$ if and only if $z \geq z^{**}$ by definition of $z^{**}$, it follows that $z'' \geq z^{**}$. But since incentive constraints are both satisfied at $\theta = \frac{\phi}{1-\phi}$ only if $z \geq z''$, it must also be the case that $z^{**} \geq z''$. Hence, $z^{**} = z''$.

Next, I need to show $z'' = z^{**} > z^*$. Consider the case where $z = z^*$. At this point, we know $M' \cap \Gamma$ contains a single point, specifically $(m_b, m_g)$. There are two cases to consider: either $2Rk_b \geq \frac{(1-\alpha)b}{\alpha R}$ or $2Rk_b < \frac{(1-\alpha)b}{\alpha R}$. In the first case, $x_b = 2Rk_b$ and $x_g = m_g = 2Rk_g$ since $2Rk_g > 2Rk_b \geq \frac{(1-\alpha)b}{\alpha R}$. Hence, $z^* = z'$ and the statement follows from Proposition 6. Alternatively, suppose $2Rk_b < \frac{(1-\alpha)b}{\alpha R}$. Then once again, at $z^*$, $M' \cap \Gamma$ contains a single point, and at this point we have $x_b = m_b = \frac{(1-\alpha)b}{\alpha R} < \frac{(1-\alpha)b^*}{\alpha R}$. But at $z''$, $M' \cap \Gamma$ contains the point $x_b\left(\frac{\phi}{1-\phi}\right) = \frac{(1-\alpha)b^*}{\alpha R}$. Hence, $z^* \neq z'' = z^{**}$. Since $z^{**} \geq z^*$, the statement follows.

Finally, from Proposition 7 we know that for $z < z''$, all prices in $\Gamma(z)$ which lie in $A'$ and thus in $M'$ are those for which profits are greater from good $g$, i.e. for which $x_g - Rk_g > x_b - Rk_b$. Hence, in equilibrium, good jobs must be rationed, and it is necessary that in equilibrium, profits from producing good $g$ exceed those from producing good $b$. Given the rationing rule (3.4), we can apply the arguments of Proposition 7 to show that there is a unique equilibrium outcome, given by the fixed point of the two-period ahead mapping, i.e.

$$Y_g(z) = \frac{\phi_z + \phi k - 2k\phi\phi_z}{1 - (1-2\phi_z)(1-2\phi)k^2} \in (0,1)$$

and

$$Y_g(Z) = \frac{\phi + \phi_z k - 2k\phi\phi_z}{1-(1-2\phi_z)(1-2\phi)k^2} \in (0,1)$$

where $\phi_z < \phi$ so that $Y_g(z) < Y_g(Z)$ and $y_g(z) < y_g(Z)$. This proves part 2a.

Now consider the case where $b > b^*$, and consider the productivity level $z^*$. At this level of productivity, $M' \cap \Gamma$ consists of a single pair $(x_g, x_b) \in \Gamma(z^*)$ which is associated with a particular ratio $\theta^*$. Since this point lies in $M'$, it follows that

$$\begin{aligned} x_b &= m_b = \max\left(\frac{(1-\alpha)b}{\alpha R}, 2Rk_b\right) \\ &> \frac{(1-\alpha)b^*}{\alpha R} \\ &\geq 2Rk_b \end{aligned}$$

where the last inequality follows from the definition of $b^*$. Hence, $x_b = m_b = \dfrac{(1-\alpha)b}{\alpha R}$. At the same time, $x_g = \max\left(\dfrac{(1-\alpha)b}{\alpha R}, 2Rk_g\right)$. There are two cases to consider. First, suppose $x_g = \dfrac{(1-\alpha)b}{\alpha R} \geq 2Rk_g$. Then $\theta^* = 1 > \dfrac{\phi}{1-\phi}$. Second, suppose $x_g = 2Rk_g > \dfrac{(1-\alpha)b}{\alpha R}$. The claim is that once again, $\theta^* > \dfrac{\phi}{1-\phi}$. To see this, suppose not. Then there are two cases to consider: either $\theta < \dfrac{\phi}{1-\phi}$ or $\theta = \dfrac{\phi}{1-\phi}$. Suppose first $\theta < \dfrac{\phi}{1-\phi}$. Given that $x_g = 2Rk_g$, then $(x_b, x_g) \in A'$, from which it follows that $z^* > z''$, since Proposition 6 establishes that $(x_b, x_g) \in A'$ for $\theta < \dfrac{\phi}{1-\phi}$ only for $z > z''$. But if $z^* > z''$, it also follows that at $z^*$, $\left(x_b\left(\dfrac{\phi}{1-\phi}\right), x_g\left(\dfrac{\phi}{1-\phi}\right)\right) \in A'$. Since $x_g$ is decreasing in $\theta$, $x_g\left(\dfrac{\phi}{1-\phi}\right) < x_g(\theta^*) = 2Rk_g$. This contradicts the fact that $(x_b(\theta), x_g(\theta)) \in A'$. So $\theta^* \geq \dfrac{\phi}{1-\phi}$. Suppose then that $\theta = \dfrac{\phi}{1-\phi}$. Since $x_g\left(\dfrac{\phi}{1-\phi}\right) = 2Rk_g$, it follows that $z = z''$. But at $z = z''$, we know $x_b\left(\dfrac{\phi}{1-\phi}\right)$ should be equal to $\dfrac{(1-\alpha)b^*}{\alpha R}$. Instead, $x_b\left(\dfrac{\phi}{1-\phi}\right) = \dfrac{(1-\alpha)b}{\alpha R} > \dfrac{(1-\alpha)b^*}{\alpha R}$. Once again, a contradiction. It follows that $\theta > \dfrac{\phi}{1-\phi}$. Since $z^* \leq z^{**}$ and the set $M' \cap \Gamma$ is isomorphic to the interval $[\underline{\theta}(z), \overline{\theta}(z)]$, where $\underline{\theta}(z)$ is decreasing in $z$ and $\overline{\theta}(z)$ is increasing in $z$, the fact that $\underline{\theta}(z^*) = \overline{\theta}(z^*) > \dfrac{\phi}{1-\phi}$ implies $[\underline{\theta}(z), \overline{\theta}(z)] > \dfrac{\phi}{1-\phi}$ for all $z < z^{**}$. This establishes that $z^{**} > z^*$, since at $z^{**}$, the pair $(x_g, x_b)$ which corresponds with $\dfrac{\phi}{1-\phi}$ is contained in $M' \cap \Gamma$. Moreover, it follows that profits must be higher from good $b$ than from good $g$ in equilibrium for $z \in (z^*, z^{**})$.

Given the rationing rule (2.1), we can apply the arguments of Propositions 4 and 5 to show that there is a unique equilibrium outcome, given by the fixed point of the two-period ahead mapping, i.e.

$$Y_g(z) = \frac{\phi_z + \phi k - 2k\phi\phi_z}{1-(1-2\phi_z)(1-2\phi)k^2} \in (0,1)$$

and

$$Y_g(Z) = \frac{\phi + \phi_z k - 2k\phi\phi_z}{1 - (1 - 2\phi_z)(1 - 2\phi)k^2} \in (0, 1)$$

where $\phi_z > \phi$ so that $Y_g(z) > Y_g(Z)$ and $y_g(z) > y_g(Z)$. This proves part 2b, as well as the fact that for $b \neq b^*$, $z^{**} > z^*$.

## References


[1] Acemoglu, Daron, 1998. "Good Jobs versus Bad Jobs: Theory and Some Evidence" Mimeo, MIT.

[2] Aghion, Phillippe and Gilles Saint Paul, 1991. "On the Virtue of Bad Times: An Analysis of the Interaction between Economic Fluctuations and Productivity Growth" CEPR Discussion Paper #578.

[3] Albequerque, Rui and Hugo Hopenhayn, 1997. "Optimal Dynamic Lending Contracts with Imperfect Enforceability" Mimeo, University of Rochester.

[4] Bailey, Martin, Eric Bartelsman, and John Haltiwanger, 1998. "Labor Productivity: Structural Change and Cyclical Dynamics" Mimeo, University of Maryland.

[5] Barlevy, Gadi, 1998a. "Is There a Cleansing Effect of Recessions? Evidence from Wage Data" Mimeo, Northwestern University.

[6] Barlevy, Gadi, 1998b. "The Sullying Effect of Recessions" Mimeo, Northwestern University.

[7] Bertin, Amy, Tim Bresnahan, and Daniel Raff, 1996. "Localized Competition and the Aggregation of Plant-Level Increasing Returns: Blast Furnaces, 1929-1935" *Journal of Political Economy*, April, 104(2), p241-66.

[8] Bernanke, Ben and Mark Gertler, 1989. "Agency Costs, Net Worth, and Business Fluctuations" American Economic Review, March, 79(1), p14-31.

[9] Binmore, Kenneth, Ariel Rubinstein, and Asher Wolinsky, 1986. "The Nash Bargaining Solution in Economic Modelling" *Rand Journal of Economics*, Summer , 17(2), p176-88.

[10] Bresnahan, Tim and Daniel Raff, 1991. "Intra-industry Heterogeneity and the Great Depression: The American Motor Vehicles Industry, 1929-1935" *Journal of Economic History*, June, 51(2), p317-31.

[11] Bresnahan, Tim and Daniel Raff, 1998. "Plant Shutdown Behavior During the Great Depression and the Structure of the American Motor Vehicle Industry" *Journal of Economic History*, December, 58(4).

[12] Caballero, Ricardo and Mohamad Hammour, 1994. "The Cleansing Effect of Recessions" *American Economic Review*, December, 84(5), p1350-68.

[13] Caballero, Ricardo and Mohamad Hammour, 1996. "On the Timing and Efficiency of Creative Destruction" *Quarterly Journal of Economics*, August, 111(3), p805-52.

[14] Caballero, Ricardo and Mohamad Hammour, 1998. "Improper Churn: Social Costs and Macroeconomic Consequences" Mimeo, MIT.

[15] Carlstrom, and Timothy Fuerst, 1997. "Agency Costs, Net Worth, and Business Fluctuations: a Computable General Equilibrium Analysis" American Economic Review, 87, p893-910.

[16] Cooper, Russel and John Haltiwanger, 1993. "The Aggregate Implications of Machine Replacement: Theory and Evidence" *American Economic Review*, June, 83(3), p181-186.

[17] Davis, Steven and John Haltiwanger, 1990. "Gross Job Creation and Destruction: Microeconomic Evidence and Macroeconomic Implications." *NBER Macroeconomics Annual.* Cambridge, Mass: MIT Press.

[18] Davis, Steven, John Haltiwanger, and Scott Schuh, 1996. *Job Creation and Destruction.* Cambridge, MA: MIT Press.

[19] Foote, Christopher, 1998. "Trend Employment Growth and the Bunching of Job Creation and Destruction" *Quarterly Journal of Economics*, August, 113(3), p809-34.

[20] Gordon, David George, 1996. *The Compleat Cockroach.* Ten Speed Press, Berkeley, CA.

[21] Griliches, Zvi and Haim Regev, 1995. "Firm Productivity in Israeli Industry: 1979-1988" *Journal of Econometrics*, January, 65(1), p175-203.

[22] Hall, Robert, 1991. "Labor Demand, Labor Supply, and Employment Volatility" *NBER Macroeconomics Annual.* Cambridge, Mass: MIT Press.

[23] Kiyotaki, Nobuhiro and John Moore, 1997. "Credit Cycles" Journal of Political Economy, 105, p211-248.

[24] Matsuyama, Kiminori, 1998. "Endogenous Inequality" Mimeo, Northwestern University, December.

[25] Monge, Alexander, 1998. "Recursive Bank-Entrepreneur Relationships, Creation and Destruction of Firms and Aggregate Dynamics" Mimeo, University of Chicago.

[26] Mortensen, Dale and Christopher Pissarides, 1994. "Job Creation and Job Destruction in the Theory of Unemployment" *Review of Economic Studies*, July, 61(3), p397-415.

[27] Olley, and Ariel Pakes, 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry" *Econometrica*, November, 64(6), p1263-97.

[28] Phelan, Chris and Alberto Trejos, 1996. "On the Aggregate Effects of Sectoral Reallocations" Mimeo, Northwestern University.

[29] Pissarides, Christopher, 1994. "Search Unemployment with On-the-Job Search" *Review of Economic Studies*, July, 61(3), p457-75.

[30] Rogerson, Richard, 1987. "An Equilibrium Model of Sectoral Reallocation" *Journal of Political Economy*, August, 95(4), p824-34.

[31] Rubinstein, Ariel, 1982. "Perfect Equilibrium in a Bargaining Model" *Econometrica*, January, 50(1), p97-109.

[32] Schumpeter, Joseph, 1939. *Business Cycles.* New York: McGraw-Hill.