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INFORMATIONAL ORIGINS OF POLITICAL BIAS
TOWARDS CRITICAL GROUPS OF VOTERS

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Abstract. We show how nonsymmetric politicization can arise in a democracy where voters are distributed across several ex-ante symmetric sectors. The voters are uncertain about the administrative ability of an elected official. They observe the quality of her performance, which depends on her ability and her effort. The official can allocate her efforts symmetrically or nonsymmetrically across sectors. We show the existence of a nonsymmetric equilibrium, in which the official allocates more effort to administering one critical sector, because voters in other sectors rationally respond less to what they observe about the quality of to her administration.

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Informational Origins of Political Bias Towards Critical Groups of Voters

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1. Introduction

Politicians understand that some groups of voters are more politically active than others, and they put more effort into getting the votes of these groups. It is not hard to think of sociological characteristics that might account for such differences in politicization, such as more education, better grass-roots organization, more interest in public goods, lower costs of voting. In this paper, I will analyze an example to show how such differences in politicization could occur in a rational equilibrium even among groups of voters who are otherwise the same.

The basic idea of this paper is as follows. If an elected official is expected to put more effort into improving local public goods that benefit a particular critical group of voters, then voters in this critical group will feel better able to evaluate the official's ability. So voters in this critical group are more likely to vote for or against for the official depending on her performance. Thus, to get more votes, the official should put more effort into improving local public goods that benefit this critical group.

The basic insight described above is derived from a model of Lohmann (1998), who argued that special interests can get better treatment from an elected official when they are better able to measure the quality of the official's performance than other voters. When there is uncertainty about the overall quality of the official's ability, voters who are less able to get information about her ability may rationally abstain, because of the swing-voter's curse of

Feddersen and Pesendorfer (1996) (see also Austen-Smith and Banks, 1997, Feddersen and Pesendorfer, 1997, and Myerson, 1998c). Our substantive contribution in this paper is only to show that voters' differences in ability to measure an elected official's performance may be an endogenous result of rational effort-allocation decisions by the elected official, in response to this expected pattern of abstention among some groups of voters. The example considered in this paper may also have some methodological interest, as an illustration of how of large voting games can be tractably modeled as Poisson games (see Myerson, 1998a, 1998b, 1998c).

2. Basic assumptions of the model

In this model, there is uncertainty about the abilities of a particular elected official, who may be either Effective or Ineffective as an administrator. Suppose that, at the beginning of this official's term in office, everyone agreed that her probability of being an Effective administrator was 1/2. But during her term in office, individual voters may get different information about her administrative performance, which will cause them to update their beliefs about whether her administrative type is Effective or Ineffective.

The voters live in five different sectors, numbered from 1 to 5, and the elected official has to decide how to allocate her administrative effort across these five sectors. Let α_i denote the fraction of her effort that the official devotes to sector i . So $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$ must be a vector of nonnegative numbers that sum to one. That is, the elected official must choose her effort allocation vector α subject to the constraints

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 1, \text{ and } \alpha_i \geq 0 \quad \forall i \in \{1, 2, 3, 4, 5\}.$$

Voters cannot directly observe the official's effort or ability, but each voter observes a Good or Bad signal which depends on the quality of public goods in his sector. For each voter in sector i , the probability of observing a Good signal is

$$P(\text{Good}|\text{Effective}, \alpha_i) = 0.2 + 0.5\alpha_i \text{ if the official's ability is Effective,}$$

$$P(\text{Good}|\text{Ineffective}, \alpha_i) = 0.1 + 0.25\alpha_i \text{ if the official's ability is Ineffective.}$$

Thus, a voter's probability of getting a Good signal increases linearly in the official's local effort α_i , and the probability of getting a Good signal from any given effort level is twice as high if the official is an Effective administrator than if she is Ineffective. We assume that each voter's signal is conditionally independent given the official's overall ability and allocation of effort.

The five sectors have equal expected size, but there is some uncertainty about the exact size of the voting population in each sector. To be precise, we assume that the total number of voters in each sector is an independent Poisson random variable with mean $n/5$, where n is some large positive number. To analyze large games, we will consider limits of equilibria as $n \rightarrow \infty$.

In the next election, the elected official will face a challenger whose abilities are also unknown, with equal probability of being Effective or Ineffective. Each voter, knowing only his own signal, wants to maximize the probability that the winner of the next election will be Effective. The official wants to maximize her probability of being re-elected.

3. Rational voting without abstention assuming all effort in the critical sector

We are look for an equilibrium in which the official is expected to treat sector 1 as a

critical sector and put all her effort there. So let us now analyze the voters' decision problems when they assume that the official's effort-allocation vector is

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (1, 0, 0, 0, 0).$$

We assume that voters must vote Yes or No on the question of the official's re-election, and she will be re-elected to a second term in office if there are more Yes votes than No votes in the election. In the case of a tie, we assume that the official will be re-elected with probability 1/2, based on the result of a coin toss. In this section, we assume that voters cannot abstain; the possibility of abstention will be introduced in Section 5.

The voters' equilibrium behavior will be described by some strategy function σ . Given any possible vote v in $\{\text{Yes}, \text{No}\}$, any possible signal t in $\{\text{Good}, \text{Bad}\}$, and any possible sector i in $\{1, 2, 3, 4, 5\}$, we let $\sigma(v|t, i)$ denote the probability that a voter would cast vote v if he is in sector i and gets signal t . When abstention is ruled out, we must have

$$\sigma(\text{Yes}|t, i) + \sigma(\text{No}|t, i) = 1, \forall t \in \{\text{Good}, \text{Bad}\}, \forall i \in \{1, 2, 3, 4, 5\}.$$

Given any ability type θ in $\{\text{Effective}, \text{Ineffective}\}$, we let $\tau(v|\theta)$ denote the expected fraction of the voters who will cast v -votes when the official's is type is actually θ . With the effort allocation vector $\alpha = (1, 0, 0, 0, 0)$, for each v in $\{\text{Yes}, \text{No}\}$, the formulas for likelihoods of Good and Bad signals imply:

$$\begin{aligned} \tau(v|\text{Effective}) &= (1/5)(.70\sigma(v|\text{Good}, 1) + .30\sigma(v|\text{Bad}, 1)) + \\ &\quad + \sum_{j>1} (1/5)(.20\sigma(v|\text{Good}, j) + .80\sigma(v|\text{Bad}, j)), \\ \tau(v|\text{Ineffective}) &= (1/5)(.35\sigma(v|\text{Good}, 1) + .65\sigma(v|\text{Bad}, 1)) + \\ &\quad + \sum_{j>1} (1/5)(.10\sigma(v|\text{Good}, j) + .90\sigma(v|\text{Bad}, j)). \end{aligned}$$

In the Poisson voting game, conditional on the official's type θ , the numbers of Yes and No

votes are independent Poisson random variables with means $n\tau(\text{Yes}|\theta)$ and $n\tau(\text{No}|\theta)$ respectively.

For any permissible vote v in $\{\text{Yes}, \text{No}\}$, we may say that a vote v is pivotal if adding one v -vote would change the winner. We let $L(v)$ denote the event that a v -vote is pivotal. This event occurs with probability $1/2$ when numbers of Yes and No votes are equal (because then adding one v vote would break a tie), and it also occurs with probability $1/2$ when there is exactly one less v vote than the other vote (because then adding one v vote would make a tie). Assuming large n , Myerson (1998b) has shown that the conditional probability of a vote v being pivotal when the official's is type θ can be approximated by the formula:

$$(1) \quad P(L(v)|\theta) \approx \frac{\exp\left(-n\left(\sqrt{\tau(\text{Yes}|\theta)} - \sqrt{\tau(\text{No}|\theta)}\right)^2\right)}{4\sqrt{\pi n\sqrt{\tau(\text{Yes}|\theta)\tau(\text{No}|\theta)}}} \left(\frac{\sqrt{\tau(\text{Yes}|\theta)} + \sqrt{\tau(\text{No}|\theta)}}{\sqrt{\tau(v|\theta)}} \right).$$

(Here the approximate equality \approx indicates that the ratio of the two sides goes to 1 as n goes to infinity.) Notice that the pivotal vote v appears only once in the denominator of the last part of equation (1), and thus:

if $\tau(\text{Yes}|\theta) < \tau(\text{No}|\theta)$, then $P(L(\text{Yes}|\theta)) > P(L(\text{No}|\theta))$,

if $\tau(\text{Yes}|\theta) > \tau(\text{No}|\theta)$, then $P(L(\text{Yes}|\theta)) < P(L(\text{No}|\theta))$.

That is, the vote v that is likely to get less than a majority in the election is more likely to be pivotal, because the event of a v -vote being pivotal includes outcomes where the v -votes are slightly less than half the total votes but does not include outcomes where the v -votes are more than half the total votes.

When the strategy σ is fixed or convergent as $n \rightarrow \infty$, these pivot probabilities satisfy the simpler formula

$$\log(P(L(v)|\theta)) \approx -\left(\sqrt{\tau(\text{Yes}|\theta)} - \sqrt{\tau(\text{No}|\theta)}\right)^2.$$

This rate at which the natural logarithm of the pivot probabilities in state θ go to ∞ as n becomes large may be called the pivot magnitude in state θ , and may be denoted by $\mu(\theta)$.

On election day, by Bayes's rule, each voter thinks, given his sector i and signal t , that the probability that the official's type is Effective is

$$P(\text{Effective}|\text{Good}, 1) = .5 \times .70 / (.5 \times .70 + .5 \times .35) = 0.667$$

$$P(\text{Effective}|\text{Bad}, 1) = .5 \times .30 / (.5 \times .30 + .5 \times .65) = 0.316$$

$$P(\text{Effective}|\text{Good}, j) = .5 \times .20 / (.5 \times .20 + .5 \times .10) = 0.667, \text{ for } j \in \{2,3,4,5\},$$

$$P(\text{Effective}|\text{Bad}, j) = .5 \times .80 / (.5 \times .80 + .5 \times .90) = 0.471, \text{ for } j \in \{2,3,4,5\}.$$

So Good signals are equally favorable to the official in all sectors, but Bad signals are stronger evidence against the official in sector 1 than elsewhere. Of course, we also have

$$P(\text{Ineffective}|t,i) = 1 - P(\text{Effective}|t,i), \quad \forall t, \forall i.$$

A voter's utility gain from casting a Yes vote for the official's re-election can be defined to be +1 if the vote is pivotal when the official is Effective, -1 if the vote is pivotal when the official is Ineffective, and 0 otherwise. Thus, the expected utility gain of voting Yes is, for a voter with signal t in sector i ,

$$\begin{aligned} U(\text{Yes}|t,i) &= \\ &= P(\text{Effective}|t,i) P(L(\text{Yes})|\text{Effective}) - P(\text{Ineffective}|t,i) P(L(\text{Yes})|\text{Ineffective}). \end{aligned}$$

A voter's utility gain from casting a No vote can be defined to be +1 if the vote is pivotal when the official is Ineffective, -1 if the vote is pivotal when the official is Effective, and 0 otherwise. Thus, the expected utility gain of voting No is, for a voter with signal t in sector i ,

$$U(\text{No}|t,i) = \\ = P(\text{Ineffective}|t,i) P(L(\text{No})|\text{Ineffective}) - P(\text{Effective}|t,i) P(L(\text{No})|\text{Effective}).$$

Then the voter should rationally vote Yes or No depending on whether $U(\text{Yes}|t,i)$ or $U(\text{No}|t,i)$ is greater.

If there were no other voters then a voter's vote would always be pivotal, and then any voter in any sector would want to vote Yes if and only if he got a Good signal, because

$$P(\text{Effective}|\text{Good},i) > 1/2 \quad \text{and} \quad P(\text{Effective}|\text{Bad},i) < 1/2 \quad \forall i.$$

So a hypothesis of "sincere voting" suggests that all voters with Good signals would vote Yes and all voters with Bad signals would vote No. But this strategy would yield

$$\begin{aligned} \tau(\text{Yes}|\text{Effective}) &= .30, & \tau(\text{No}|\text{Effective}) &= .70, \\ \tau(\text{Yes}|\text{Ineffective}) &= .15, & \tau(\text{No}|\text{Ineffective}) &= .85. \end{aligned}$$

The pivot magnitudes would then be

$$\mu(\text{Effective}) = -(\sqrt{.30} - \sqrt{.70})^2 = -0.083 > \mu(\text{Ineffective}) = -(\sqrt{.15} - \sqrt{.85})^2 = -0.286.$$

That is, the pivot probabilities would go to zero as $e^{-.083n}$ when the official's type Effective, but the pivot probabilities would go to zero as $e^{-.286n}$ when the official's type is Ineffective. So this difference in magnitudes means that the probability of a vote being pivotal would be vastly smaller when the official is Ineffective than when Effective, with a likelihood ratio that goes to 0 as $n \rightarrow \infty$. Thus, every voter would prefer to vote Yes if all other voters were expected to vote sincerely, because when a vote is pivotal it would be almost sure that the official was Effective. So we must conclude that sincere voting is not an equilibrium!

But everyone voting Yes also is not an equilibrium, because then pivot probabilities would be the same in both states, and so voters would prefer to vote sincerely. So we should

look for an equilibrium somewhere between sincere voting and everyone-voting-Yes. That is, we should look for an equilibrium in which all voters with Good signals vote Yes, but some voters with Bad signals do not vote No. Among the voters with Bad signals, the voters in the critical sector 1 have stronger evidence for Ineffectiveness of the official than voters in the other sectors, because

$$P(\text{Effective}|\text{Bad},1) = 0.316 < 0.471 = P(\text{Effective}|\text{Bad}, j), \quad \forall j > 1.$$

So let us consider strategy functions in which the voters with Bad signals who do not vote No are not in the critical sector 1. That is, while abstention is being ruled out, let us consider strategy functions of the form

$$\sigma(\text{Yes}|\text{Good}, i) = 1, \quad \sigma(\text{No}|\text{Good}, i) = 0, \quad \text{for all } i \text{ in } \{1,2,3,4,5\},$$

$$\sigma(\text{Yes}|\text{Bad}, 1) = 0, \quad \sigma(\text{No}|\text{Bad}, 1) = 1,$$

$$\sigma(\text{Yes}|\text{Bad}, j) = \rho, \quad \sigma(\text{No}|\text{Bad}, j) = 1-\rho, \quad \text{for } j \text{ in } \{2,3,4,5\}$$

where ρ is some number between 0 and 1.

With such σ , the expected fractions voting Yes and No in each state are

$$\tau(\text{Yes}|\text{Effective}) = .70/5 + (.20 + .80\rho)4/5 = 0.30 + 0.64\rho,$$

$$\tau(\text{No}|\text{Effective}) = .30/5 + .80(1-\rho)4/5 = 0.70 - 0.64\rho,$$

$$\tau(\text{Yes}|\text{Ineffective}) = .35/5 + (.10 + .90\rho)4/5 = 0.15 + 0.72\rho,$$

$$\tau(\text{No}|\text{Ineffective}) = .65/5 + .90(1-\rho)4/5 = 0.85 - 0.72\rho.$$

If the limiting pivot magnitude $\mu(\theta)$ were different when the official is Effective than when she is Ineffective, then for all large n the voters would infer that the event of being pivotal was overwhelming evidence in favor of the official's type that yields the higher pivot magnitude, and so voters would not mix between Yes and No as σ requires. So a limit of

equilibria σ as $n \rightarrow \infty$ must satisfy the magnitude equation $\mu(\text{Effective}) = \mu(\text{Ineffective})$, that is,

$$\begin{aligned} & -\left(\sqrt{\tau(\text{Yes}|\text{Effective})} - \sqrt{\tau(\text{No}|\text{Effective})}\right)^2 / \\ & = -\left(\sqrt{\tau(\text{Yes}|\text{Ineffective})} - \sqrt{\tau(\text{No}|\text{Ineffective})}\right)^2. \end{aligned}$$

This equation holds for σ and τ as above when $\rho = 0.404$, which yields

$$\begin{aligned} \tau(\text{Yes}|\text{Effective}) &= 0.559, & \tau(\text{No}|\text{Effective}) &= 0.441, \\ \tau(\text{Yes}|\text{Ineffective}) &= 0.441, & \tau(\text{No}|\text{Ineffective}) &= 0.559. \end{aligned}$$

That is, in the limit of these equilibria, the expected vote shares for the Yes and No sides must differ identically but in opposite directions in the two possible states, with an expected majority of Yes for the official when she is Effective, but with an expected majority of No against the official when she is Ineffective. (For the relationship of this result with the Condorcet jury theorem, see Myerson, 1998c.)

For finite n , the equilibrium σ can be characterized by the need to make noncritical voters willing to randomize between voting Yes and No when they get a Bad signal. That is, an equilibrium of this form must satisfy the equation

$$\begin{aligned} (2) \quad & .471 \times P(L(\text{Yes})|\text{Effective}) - (1-.471) \times P(L(\text{Yes})|\text{Ineffective}) \\ & = (1-.471) \times P(L(\text{No})|\text{Ineffective}) - .471 \times P(L(\text{No})|\text{Effective}). \end{aligned}$$

because Bayes's rule gave us $P(\text{Effective}|\text{Bad}, j) = 0.471$ when $j \neq 1$. This equilibrium condition is satisfied by values of ρ that converge to 0.404 as $n \rightarrow \infty$. When $n=100$, for example, we can use formula (1) for pivot probabilities (in terms of τ , which in turn depends on ρ as shown above for such σ) to find that the equilibrium value of ρ is (to three decimal places) $\rho = 0.400$. For any n , the value of ρ that satisfies this equation (2) will also yield:

$$.316 \times P(L(\text{Yes})|\text{Effective}) - (1-.316) \times P(L(\text{Yes})|\text{Ineffective})$$

$$< (1-.316) \times P(L(\text{No})|\text{Ineffective}) - .316 \times P(L(\text{No})|\text{Effective}),$$

so voters in sector 1 who get bad signals will strictly prefer to vote No; and

$$.667 \times P(L(\text{Yes})|\text{Effective}) - (1-.667) \times P(L(\text{Yes})|\text{Ineffective})$$

$$> (1-.667) \times P(L(\text{No})|\text{Ineffective}) - .667 \times P(L(\text{No})|\text{Effective}),$$

so voters who get good signals in all sectors will strictly prefer to vote Yes. Thus, our strategy σ with this value of ρ is indeed a full equilibrium for the voters.

4. The official's choice of effort allocation

The argument in the previous section showed that, if the official is expected to allocate all her effort in the critical sector 1, then a voting equilibrium σ would have voters in noncritical sectors voting Yes for the official's re-election with a probability ρ that is approximately 0.404 when the expected population size n is large, while all others are voting "sincerely." Now we need to verify that, when voters are expected to vote in this way, the official should rationally allocate all her effort to the critical sector 1.

So consider what would happen if the official deviated from expectations and distributed her effort more equally across sectors. We are assuming that this redistribution of effort is not directly observable by the voters, but it would have an effect on the expected number of Bad signals in each sector. Allocating some effort away from the critical sector would increase the expected number of Bad signals in the critical sector and would decrease the expected number of Bad signals in the noncritical sectors. Because we have assumed that the expected number of Bad signals is the same linear function of the official's effort in each

sector, changing her effort allocation will not change the expected total number of Bad signals in all sectors together. So reallocating some effort away from the critical sector would essentially transfer some Bad signals from the noncritical sectors, where (according to σ) some voters with Bad signals do not vote against the official, into the critical sector, where (according to σ) all voters with Bad signals vote against the official. So by allocating effort to the noncritical sectors, the official can only lose votes and decrease her probability of re-election. Thus, it is rational for the official to allocate all effort to the critical sector 1, as our equilibrium with $\alpha = (1,0,0,0,0)$ requires.

To formalize the argument in the preceding paragraph, we can let ε denote the total effort that the official allocates to the noncritical sectors (that is, $\varepsilon = 1 - \alpha_1$). Then with the voting equilibrium σ as above, the expected fractions of the vote are

$$\begin{aligned}\tau(\text{Yes}|\text{Effective}) &= .30 + .64\rho - .10\varepsilon\rho, & \tau(\text{No}|\text{Effective}) &= .70 - .64\rho + .10\varepsilon\rho, \\ \tau(\text{Yes}|\text{Ineffective}) &= .15 + .72\rho - .05\varepsilon\rho, & \tau(\text{No}|\text{Ineffective}) &= .85 - .72\rho + .05\varepsilon\rho.\end{aligned}$$

So increasing ε above zero could only decrease the expected number of Yes votes and increase the expected number of No votes. Thus we have constructed a nonsymmetric equilibrium where sector 1 is treated as a special critical sector.

Of course, there is also a symmetric equilibrium where the official distributes her effort equally across all sectors, $e_i = 1/5$ for all i in $\{1,2,3,4,5\}$, and voters in all sectors use the same voting strategy. It can be shown that, for large n , this symmetric equilibria involves a voting strategy of the form

$$\begin{aligned}\sigma(\text{Yes}|\text{Good}, i) &= 1, & \sigma(\text{No}|\text{Good}, i) &= 0, \\ \sigma(\text{Yes}|\text{Bad}, i) &\approx .355, & \sigma(\text{No}|\text{Bad}, i) &\approx .645, \quad \forall i \in \{1,2,3,4,5\}.\end{aligned}$$

5. Equilibrium analysis with abstention

Now let us drop the unrealistic assumption that voters cannot abstain on question of this official's re-election. That is, suppose now that voter's have three options in the election: to vote Yes, No, or Abstain.

The equilibria constructed in the preceding sections fails because the voters who were supposed to randomize between voting Yes and No would prefer to Abstain instead. The expected utility gain from Abstaining is of course zero,

$$U(\text{Abstain}|t,i) = 0,$$

because an Abstention cannot change the outcome of the election. The expected utility gains for voting Yes and No (defined formally in Section 3 above) satisfy

$$\begin{aligned} U(\text{Yes}|t,i) + U(\text{No}|t,i) &= \\ &= P(\text{Effective}|t,i)(P(L(\text{Yes})|\text{Effective}) - P(L(\text{No})|\text{Effective})) \\ &\quad + P(\text{Ineffective}|t,i)(P(L(\text{No})|\text{Ineffective}) - P(L(\text{Yes})|\text{Ineffective})). \end{aligned}$$

But the voting equilibrium σ in Section 3 generated expected vote totals such that

$$\tau(\text{Yes}|\text{Effective}) > \tau(\text{No}|\text{Effective}),$$

$$\tau(\text{Yes}|\text{Ineffective}) < \tau(\text{No}|\text{Ineffective}),$$

which with the pivot-probability formula (1) imply that

$$P(L(\text{Yes})|\text{Effective}) < P(L(\text{No})|\text{Effective}),$$

$$P(L(\text{Yes})|\text{Ineffective}) > P(L(\text{No})|\text{Ineffective}).$$

Thus we must have

$$U(\text{Yes}|t,i) + U(\text{No}|t,i) < 0.$$

So any type of voter who is willing to randomize between voting Yes or No must strictly

prefer to Abstain. (This result is the swing voter's curse of Feddersen and Pesendorfer, 1996).

We can still construct a nonsymmetric equilibrium where the official puts all effort into sector 1, but now the voters with Bad signals in noncritical sectors randomize between voting No and Abstaining. That is, assuming that the official is expected to choose the effort allocation

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (1, 0, 0, 0, 0),$$

we can find a voting equilibrium σ of the form

$$\begin{aligned} \sigma(\text{Yes}|\text{Good}, i) &= 1, & \sigma(\text{No}|\text{Good}, i) &= 0, & \sigma(\text{Abstain}|\text{Good}, i) &= 0, & \forall i \in \{1,2,3,4,5\}, \\ \sigma(\text{Yes}|\text{Bad}, 1) &= 0, & \sigma(\text{No}|\text{Bad}, 1) &= 1, & \sigma(\text{Abstain}|\text{Bad}, 1) &= 0, \\ \sigma(\text{Yes}|\text{Bad}, j) &= 0, & \sigma(\text{No}|\text{Bad}, j) &= 1-\rho, & \sigma(\text{Abstain}|\text{Bad}, j) &= \rho, & \forall j \in \{2,3,4,5\}. \end{aligned}$$

With such σ , the expected fractions voting on each side of the question are

$$\begin{aligned} \tau(\text{Yes}|\text{Effective}) &= .70/5 + .20(4/5) = .30, \\ \tau(\text{No}|\text{Effective}) &= .30/5 + .80(1-\rho)4/5 = .70 - .64\rho, \\ \tau(\text{Yes}|\text{Ineffective}) &= .35/5 + .10(4/5) = .15, \\ \tau(\text{No}|\text{Ineffective}) &= .65/5 + .90(1-\rho)4/5 = .85 - .72\rho. \end{aligned}$$

As n becomes large, the limiting equilibrium value of ρ must generate the same pivot magnitudes given either state of the official's ability, because otherwise a pivot event would be overwhelming evidence for or against her effectiveness. So the limiting value of ρ must satisfy

$$\begin{aligned} \mu(\text{Effective}) &= -\left(\sqrt{\tau(\text{Yes}|\text{Effective})} - \sqrt{\tau(\text{No}|\text{Effective})}\right)^2. \\ &= \mu(\text{Ineffective}) = -\left(\sqrt{\tau(\text{Yes}|\text{Ineffective})} - \sqrt{\tau(\text{No}|\text{Ineffective})}\right)^2. \end{aligned}$$

This equation is satisfied when $\rho = .815$, which yields

$$\tau(\text{Yes}|\text{Effective}) = .30, \quad \tau(\text{No}|\text{Effective}) = .178,$$

$$\tau(\text{Yes}|\text{Ineffective}) = .15, \quad \tau(\text{No}|\text{Ineffective}) = .263,$$

$$\mu(\text{Effective}) = -(\sqrt{.30}-\sqrt{.178})^2 = -0.0158 = \mu(\text{Ineffective}) = -(\sqrt{.15}-\sqrt{.263})^2.$$

With such a voting equilibrium σ , the official wants to put all effort in the critical sector 1, to move as many Bad signals out of sector 1 as possible, because voters in sector 1 always vote against her when they get Bad signals, whereas voters in other sectors sometimes abstain when they get Bad signals.

When $n=100$, the actual voting equilibrium has $\rho = 0.817$, to satisfy the equation

$$(1-.471) \times P(L(\text{No})|\text{Ineffective}) - .471 \times P(L(\text{No})|\text{Effective}) = 0,$$

so that voters in noncritical ($j>1$) sectors with Bad signals are indifferent between voting no and abstaining. With this n and ρ , the pivot-probability formula (1) also gives us

$$.471 \times P(L(\text{Yes})|\text{Effective}) - (1-.471) \times P(L(\text{Yes})|\text{Ineffective}) = -0.00349,$$

so that voters with bad signals in noncritical sectors do not want to vote Yes.

We also find

$$(1-.316) \times P(L(\text{No})|\text{Ineffective}) - .316 \times P(L(\text{No})|\text{Effective}) = 0.00393$$

$$> 0 > .316 \times P(L(\text{Yes})|\text{Effective}) - (1-.316) \times P(L(\text{Yes})|\text{Ineffective}) = -0.00753,$$

so that voters with Bad signals in sector 1 will strictly prefer to vote No; and

$$.667 \times P(L(\text{Yes})|\text{Effective}) - (1-.667) \times P(L(\text{Yes})|\text{Ineffective}) = 0.00162$$

$$> 0 > (1-.667) \times P(L(\text{No})|\text{Ineffective}) - .667 \times P(L(\text{No})|\text{Effective}) = -0.00498.$$

so that voters with Good signals in any sector will strictly prefer to vote Yes, as the equilibrium σ requires. Thus we have a nonsymmetric equilibrium in which all effort goes

into sector 1.

As before, there also exists a symmetric equilibrium where the official distributes her effort equally across all sectors, such that $\alpha_i = 1/5$ for all i in $\{1,2,3,4,5\}$. In this symmetric equilibrium, voters in all sectors use the same voting strategy, which for large n satisfies

$$\sigma(\text{Yes}|\text{Good}, i) = 1, \quad \sigma(\text{No}|\text{Good}, i) = 0, \quad \sigma(\text{Abstain}|\text{Good}, i) = 0,$$

$$\sigma(\text{Yes}|\text{Bad}, i) = 0, \quad \sigma(\text{No}|\text{Bad}, i) \approx .283, \quad \sigma(\text{Abstain}|\text{Bad}, i) \approx .717, \quad \forall i \in \{1,2,3,4,5\}.$$

REFERENCES:

- D. Austen-Smith and J. Banks, "Information aggregation, rationality, and the Condorcet jury theorem," American Political Science Review 90:34-45 (1996).
- T. Feddersen and W. Pesendorfer, "The swing voter's curse," American Economic Review 86:408-424 (1996).
- T. Feddersen and W. Pesendorfer, "Voting behavior and information aggregation in elections with private information," Econometrica 65:1029-1058 (1997).
- S. Lohmann, "An information rationale for the power of special interests," American Political Science Review (December 1998).
- R. Myerson, "Population uncertainty and Poisson games," to appear in International Journal of Game Theory (1998a).
- R. Myerson, "Large Poisson games," to appear in Journal of Economic Theory (1998b).
- R. Myerson, "Extended Poisson games and the Condorcet jury theorem," to appear in Games and Economic Behavior (1998c).