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Government Turnover in Parliamentary Democracies

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ABSTRACT

In this paper we consider a dynamic model of government formation and termination in parliamentary democracies that accounts for the following phenomena: (1) Cabinet terminations due to replacement or early election (2) Cabinet reshuffles (3) Minority and Surplus governments; (4) the relative instability of minority governments.

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1 Introduction

The distinctive characteristic of parliamentary democracies is the fact that the executive derives its mandate from and is politically responsible to the legislature. This has two consequences. First, unless one party wins a majority of seats, the government is not determined by an election alone, but is the result of an elaborate bargaining process among the parties represented in the parliament. Second, parliamentary governments may lose the confidence of the parliament at any time, which leads to their immediate termination.

The following is a list of prominent empirical regularities about the formation and termination of parliamentary governments.[1]

1. Governments frequently terminate before the end of the legislative period. While most governments are immediately replaced by a new cabinet, 45% of all governments terminate in an early election (Diermeier and Stevenson n.d.).

2. Cabinets frequently reshuffle the allocation of cabinet posts and other government positions during their lifetime (Laver and Shepsle 1996).

3. Minimal winning governments are not the norm. Only 39% of all governments formed are minimal winning. Minority governments occur in about 37% of all government formations, surplus governments in 24% (Strom 1990; Laver and Schofield 1990).[2]

4. Minority governments are, on average, less stable than other governments (Strom 1990), but some minority governments survive until the next regular election. Moreover, if a minority government terminates, it is frequently replaced by another minority government even after an early election.

[1] For recent overviews of the large empirical literature on government formation and termination see Laver and Schofield 1990, Strom 1990, and Warwick 1994.

[2] There is substantial cross-country variation in the types of governments formed. Germany almost always has had minimal winning coalitions, but of the 20 Danish governments between 1945 and 1987 18 were minority governments. In Italy surplus governments form in 45% of all cases.

While these regularities are well documented empirically, no theoretical model exists that can simultaneously explain all of them. Recently, a series of non-cooperative models have been proposed to account for the first regularity. These models interpret cabinets as equilibria in a legislative bargaining process.

Lupia and Strom (1995) consider a one round bargaining model with outside options or "events". They focus on a particular type of events related to electoral prospects based on public opinion polls. That is, events are interpreted as common knowledge information about what would happen if parliament were dissolved and an election held immediately. Parties with favorable electoral prospects may either realize their advantage at the pools or extract benefits through bargaining with parties that would loose seats in an early election. Whether gains are realized in a new government coalition ("a replacement") or in the calling of early elections (a "dissolution") depends on the relative magnitude of election and negotiation related transaction costs.

Baron (1998) proposes a dynamic model of government stability using an infinite horizon version of the legislative bargaining model proposed by Diermeier and Feddersen (1998). They show that the very fact that ruling coalitions need to maintain the confidence of the chamber, allows them to capture almost all the rents from legislation. This result holds if sufficiently many bargaining periods are left and future payoffs are not discounted too heavily. Baron demonstrates that if this condition is not satisfied, governments may fall because some ruling party's current reservation value may be too high compared to the future benefits of maintaining the current government. In his model any government termination is a replacement.

In line with the previous literature we interpret cabinets as the outcome of a legislative bargaining process. Like Baron, but in contrast to Lupia and Strom, we resort to a dynamic bargaining model with random events. In contrast to Baron we also allow for the dissolution of parliament and cabinet reshuffles. Our model accounts for the occurrence of minority and surplus coalitions and the relative instability of minority governments. Most of the traditional literature views minority governments as pathologies.[3] Strom (1990), however, suggests that minority governments may be the result of rational calculations by parties for whom government participation would be too costly. While there is some empirical support for this view, to date there is no satisfactory theoretical account for minority governments.[4]

To build a model that allows for minority governments we first need to clarify what we mean by a "government". Following Laver and Shepsle (1990, 1996) we identify a government with control over government ministries. Thus, a party that supports a minority government on critical votes but does not hold any cabinet portfolios is not part of the government, but is only part of the supporting coalition. In our model this assumption has two consequences. First, holding a ministry implies political control of the bureaucracy. This is important, since in parliamentary democracies the effective power to draft and implement public policy rests with the civil service.[5] Since only members of the government can sufficiently control and verify the implementation of policies, bargains on policy with parties outside of the cabinet are not credible. It follows that only the members of a government decide on policies.[6]

Second, membership in the cabinet allows control over government posts. While some of these posts have direct influence over policy, others are better interpreted as perks that are valued by all actors and thus can be freely distributed. Examples are well-paid positions on boards of state-owned businesses or the national television. These government posts can be interpreted as transferable benefits or "money" that can be allocated by the cabinet. Since money can be exchanged for policy concessions, governing coalitions can bargain efficiently

[3]For an overview see chapter 1 of Strom (1990).

[4]Laver and Shepsle (1990) propose a structure-induced equilibrium model of minority governments, but, as demonstrated by Austen-Smith and Banks (1990), their existence result does not generalize. Baron (1998) considers a model where minority governments could be sustained, but they are never chosen in equilibrium.

[5]See Laver and Shepsle (1994) for supporting empirical evidence.

[6]Laver and Shepsle (1990) and Austen-Smith and Banks (1990) consider models where each minister is a dictator on his policy dimension. In our model policies are the result of efficient cabinet bargaining.

on policies. These benefits may also be allocated to parties in the supporting coalition in exchange for their votes. Given that appointments to government jobs are easily verifiable, cabinets may thus buy parliamentary support by allocating money to opposition parties. However, since only the cabinet controls the allocation of perks, transfers can only be made from the government to outside parties. To summarize, a government is a set of parties that can efficiently bargain over policy and the distribution of perks. Perks may also be allocated to parties that are not members of the government. In particular, they can be used to sustain minority governments.

Distributive benefits, however, are not only important to buy support for a minority government. They are also critical for our model of coalition bargaining. In our paper we use a version of efficient proto-coalition bargaining (Baron and Diermeier 1998). The main advantage of this approach is that the policy chosen by any coalition depends only on the parties' policy preferences. It does not depend on the details of the bargaining process or the location of the status quo policy. So, each proto-coalition is uniquely associated with a policy: the policy it would implement if it were in government. To implement efficient bargaining, we allow the parties to make transfers of office-holding benefits. For any given bargaining procedure we can then calculate the pay-offs each party would receive in a proto-coalition. This induces preferences over proto-coalitions for the actor that is entitled to propose a potential government (a so-called "formateur"). Proposing a government then is a simple one-person optimization problem.

There is a long and distinguished tradition in the study of multi-party coalitions that has focused on minimal winning coalitions as the "natural" outcome of a government formation game (e.g. Laver and Schofield 1990, ch.5, Baron 1998). In the one period version of our model we derive exactly the opposite conclusion: a minimal winning coalition is *never* chosen. Depending on the status quo we will either see minority or supermajority governments. The intuition here is that in efficient bargaining the size of the "pie" depends on the coalition members. If the formateur's share is increasing in the pie size, then he will prefer either to go alone (minority government) or to include as many other parties as possible (supermajority government). But then, why do we observe minimal winning coalitions?

The reason minimal winning governments occur at all lies in the need to maintain existing governments in the presence of changes in the political and economic environment. To demonstrate this insight we need to consider a two-period model of coalition bargaining with both public opinion and policy shocks. Ruling coalitions may need to reshuffle office benefits among the member parties to preserve their governments, because a shift in the status quo may increase the outside options of some coalition member. But reshuffles may be expensive. Thus, in situations were in the one-period model a supermajority government would have been chosen, the formateur in a two period model now may prefer a minimal winning coalition, since the risk of a costly reshuffle in the future is lower. While in cases where a minority government would have formed, the formateur may now prefer to form a minimal winning coalition, because the price for current and future outside support may be too high. In equilibrium, all types of government can occur, including minority governments with a high risk of terminating in the second period.

2 The Model

We consider a two-period spatial model of government formation and termination in a parliamentary democracy which builds on the framework developed by Baron and Diermeier (1998). Let $N = \{1, ..., n\}$ denote the set of parties in a parliamentary democracy and assume that $n = 3$. Each party $i \in N$ has time-separable quasi-linear preferences over policy outcomes $x \in \Re^2$ and distributive benefits $y_i \in \Re$. We assume that the per-period utility function of party i, $i = 1, 2, 3$, is given by

$$U_i(x, y_i) = u_i(x) + y_i$$

where

$$u_i(x) = -(x_1 - z_1^i)^2 - (x_2 - z_2^i)^2$$

and the parties ideal points $z^i = (z_1^i, z_2^i) \in \Re^2$, $i = 1, 2, 3$, are located symmetrically. Without further loss of generality, we normalize the parties ideal points so that $z^1 = (0, 0)$, $z^2 = (1, 0)$, and $z^3 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$. This specification captures the intuition that parties care both about policy outcomes and the benefits from holding office. We normalize aggregate transfers to be zero in each period (i.e., $\sum_{i \in N} y_i = 0$), and assume that utility in the second period is discounted at a common discount factor $\beta \in [0, 1]$.

Period 1 begins after a general election (not modeled here), which determines the parties relative shares in the parliament $\pi = (\pi_1, \pi_2, \pi_3)$. We assume that $\pi \in \Pi = \{(\pi_1, \pi_2, \pi_3) : \pi_i \in (0, \frac{1}{2}), \pi_i \neq \pi_{i'}, \text{ and } \sum_{i \in N} \pi_i = 1\}$. This assumption implies that no single party has a majority of seats, but any two-party coalition is winning under majority rule. Also given in period 1 is a *default policy* $q \in Q = \{z^1, z^2, z^3\}$. This is the policy that is implemented if no government forms in that period.[7] It determines each party's payoff in period 1 if such an event occurs. Our assumption about Q captures the intuition that the default policy may be particularly favorable to one of the parties. If $q = z^j$, we refer to party j as the party *favored* by the default policy.

Let $s \equiv (q, \pi) \in S = Q \times \Pi$ denote the *state* of the political system in period 1, which is summarized by the default policy and the distribution of parliamentary seat shares among the parties.

At the beginning of period 1, the head of state chooses one of the parties to try to form a government. We refer to the selected party $\Bbbk \in N$ as the *formateur*. We assume that the head of state is non-strategic and each party $i \in N$ is selected to be a formateur with probability equal to its seat share π_i.[8] The formateur then chooses a *proto-coalition* $D \in \Delta_\Bbbk$, where $\Delta_\Bbbk$ denotes the set of subsets of N which contain $\Bbbk$. (For example, if party 1 is the formateur, then $\Delta_1 = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$.) Intuitively, a proto-coalition is a set

[7]The default policy can be interpreted, for example, as the current state of the economy, or the policy that would be implemented by a caretaker government.

[8]For an empirical justification of this assumption see Diermeier and Merlo (1999).

of parties that agree to talk to each other about forming a government together. After the proto-coalition is chosen, D selects a set of non-negative transfers to parties outside the proto-coalition, $t(D,s) = (t_j(D,s))_{j \in N \setminus D} \in \Re_+^{|N \setminus D|}$. These transfers can be interpreted as payments to non-coalition parties to sustain the proposed government coalition.

Given D and t, the parliament votes to approve the formateur's proposal under majority rule. If the proposal is defeated, the default policy is implemented and each party $i \in N$ receives a period 1 payoff of $U_i(q,0)$. If the formateur's proposal is accepted, the members of D bargain over a policy $x(D,s) \in \Re^2$ and transfers with respect to coalition members $r(D,s) = (r_j(D,s))_{j \in D} \in \Re^{|D|}$. The bargaining procedure is such that for as long as no agreement is reached, each party in D is independently selected to make a proposal with probability $\frac{1}{|D|}$. An agreement entails unanimous approval of the proto-coalition members.[9] If the members of D do not reach an agreement on a common policy and vector of transfers, then the government formation attempt fails and each party $i \in N$ receives a period 1 payoff of $U_i(q,0)$. If instead an agreement is reached, then D forms the government and each party $i \in D$ receives a period 1 payoff of $U_i(x(D,s), r_i(D,s))$ while each party $j \notin D$ receives a period 1 payoff of $U_j(x(D,s), t_j(D,s))$.

At the beginning of period 2 a new default policy $q' \in Q = \{z^1, z^2, z^3\}$ is realized. We assume that the default policy follows a Markov process with transition probabilities $\Pr[q' = z^i | q = z^i] = \lambda$ and $\Pr[q' = z^j | q = z^i] = \frac{1-\lambda}{2}$, $i \neq j = 1,2,3$. Also at the beginning of period 2, the parties receive a common signal about the seat share each party would receive if the current parliament were dissolved and an early election called. Let $\pi' = (\pi'_1, \pi'_2, \pi'_3)$ denote the vector of the new shares. We assume that $\pi' = \pi + \varepsilon$, where $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is a random vector that takes the value $(0,0,0)$ with probability ρ and takes each of the values $(-2e, e, e)$, $(e, -2e, e)$, or $(e, e, -2e)$ with probability $\frac{1-\rho}{3}$, and e is small.[10] In particular, we

[9] We assume that bargaining takes no time and hence there is no within period discounting.

[10] We may think of e as one parliamentary seat. That is, the case where $e = (0,0,0)$ corresponds to a shift in public support that is not large enough to change the allocation of seats in the chamber.

assume that it is still the case that $\pi' \in \Pi$, and that $E[\pi'] = \pi$. This assumption captures the fact that in multiparty parliamentary democracies it is very unlikely that one party could gain (or lose) significant shares in a short period of time. In the case of an early election each party incurs a small cost of dissolution δ with $\delta \to 0$.

Let $s' \equiv (q', \pi', \pi) \in S' = Q \times \Pi \times \Pi$ denote the state of the political system in period 2, which is summarized by the default policy in that period, the distribution of seat shares parties would receive if an early election were called, and the current seat distribution.

If a government formed in period 1, then after observing s', the incumbent government can renegotiate its agreement. Renegotiation is similar to government formation, with the incumbent government being the chosen proto-coalition. Hence, first the government may choose a set of period 2 transfers to the parties outside the government coalition, $t(D, s') = (t_j(D, s'))_{j \in N \setminus D} \in \Re_+^{|N \setminus D|}$. Given $t(D, s')$, a vote is then taken to determine whether the incumbent government still has the confidence of a parliamentary majority to continue its ruling. If the government retains the confidence of the parliament, it then bargains over a policy $x(D, s')$ and transfers $r(D, s') = (r_j(D, s'))_{j \in D} \in \Re^{|D|}$ for period 2. If an agreement is reached, then D continues as a government and period 2 payoffs to the parties are determined as a function of $x(D, s')$, $r(D, s')$ and $t(D, s')$. If D fails to reach an agreement or loses the confidence of the parliament, then D terminates.

If the incumbent government terminates or no government formed in period 1, then the parliament decides under majority rule whether to dissolve and call early election or to continue. In the event an early election is called, a new government formation process begins with the head of state selecting a formateur with probabilities π'. If no early election is called, a new government formation process begins with the head of state selecting a formateur with probabilities π. Like in period 1, the outcome of the government formation process determines the period 2 payoffs to the parties. In particular, if a government D' forms, then each party $i \in D'$ receives a period 2 payoff of $U_i(x(D', s'), r_i(D', s'))$ while each party $j \notin D'$ receives a period 2 payoff of $U_j(x(D',s'), t_j(D',s'))$. If instead no government forms, then each party $i \in N$ receives a period 2 payoff of $U_i(q',0)$.At the end of period 2 a regularly scheduled election takes place and any incumbent government has to resign.

3 Results

Since the model we consider is a game with complete information and a finite horizon, we focus on the characterization of its subgame perfect equilibrium using backwards induction. Our characterization is presented in a series of lemmata which illustrate the main properties of the equilibrium of each subgame. A proposition containing the main result of the paper concludes the analysis.

The first two lemmata pertain to government formation in period 2. While this analysis is necessary to characterize subgame-equilibria for the case where the period 1 government has terminated (or where no government formed in the first period), it also captures the base-line one-period version of our model. We can then see why a dynamic model with shocks is necessary to account for the empirical regularities mentioned in the introduction.

Lemma 1: *Suppose a government formation process begins in period 2 and D' is chosen as the proto-coalition. Then for any $s' \in S'$ and for any $D' \subseteq N$, D' forms the government. Furthermore, the chosen policy is*

$$x(D',s') = \frac{1}{|D'|}\sum_{i \in D'} z^i$$

and transfers are equal to

$$r_i(D',s') = -\frac{1}{|D'|}\sum_{\substack{j \in D' \\ j \neq i}} u_j(q') + \frac{|D'|-1}{|D'|}u_i(q'),\ i \in D'$$

and

$$t_j(D',s') = 0,\ j \in N \backslash D'.$$

Proof: All proofs are relegated to an appendix.

Lemma 1 generalizes the Baron-Diermeier model of government formation bargaining to the case of minority governments. In contrast to Baron-Ferejohn models of coalition formation (Baron and Ferejohn 1989, Baron 1991) bargaining is efficient. That is, the negotiating parties will choose the policy that maximizes the proto-coalition's aggregate utility. Transfers are used to implement the optimal policy. It follows that in contrast to Baron-Ferejohn models, the policy choice of any coalition is independent of the default policy and does not depend on the details of the bargaining procedure. So, while the allocation of transfers $r(D, s')$ reflects bargaining power, the chosen policy only depends on the ideal points of the coalition members.[11] Note also that no government coalition needs to make transfers to parties outside the coalition.

The second lemma characterizes which government coalitions can form in period 2, if government formation is necessary. To answer this question, for any state in period 2 and for any formateur, we have to solve the formateur's maximization problem:

$$\max_{D' \in \Delta_{\Bbbk}} U_{\Bbbk}(x(D', s'), r_{\Bbbk}(D', s')).$$

Let $D'_{\Bbbk}(s')$ denote the solution to this optimization problem.

Lemma 2: *For any government formation process in period 2 with $\pi, \pi' \in \Pi$, and for any $i, j, \Bbbk \in N$, $i \neq j \neq \Bbbk \neq i$.*

(i) If $q' = z^{\mathbf{k}}$, $D'_{\Bbbk}(z^{\mathbf{k}}, \pi', \pi) = \{1, 2, 3\}$ forms the government.

(ii) If $q' = z^i$, $D'_{\Bbbk}(z^i, \pi', \pi) = \{\Bbbk\}$ forms the government with the external support of party j.

Note that Lemma 2 implies that in a one period model a minimal winning coalition government never forms. Only minority or surplus governments can form. This is exactly the

[11] This insight is general for a broad classes of quasi-linear preferences with strictly quasi-concave preferncces on policy (see Baron and Diermeier 1998). We use quadratic preferences for simplicity.

opposite conclusion of previous models of government formation (e.g. Baron 1998, Laver and Schofield 1990). The reason is that if a formateur can extract transfers from any other party during proto-coalition bargaining, then it is optimal to extract payments from all other parties. This leads to surplus governments if the formateur is favored by the status quo.[12] Otherwise, the formateur chooses to form a minority government. This follows, because at least one other party is indifferent between the status quo and the formateur's ideal point. So, the formateur can obtain that party's support for free. Whether a formateur selects a minority government or a supermajority government thus depends on the other parties' willingness to make transfers in exchange for a policy compromise. In either case a minimal winning coalition is dominated by some other choice.

Using Lemma 2 we can compute each party i's expected continuation payoff for any s' if a new government needs to be formed in period 2. Since continuation values depend on whether an early election has been called, we denote party i's expected continuation payoff by $W_i(q', \tilde{\pi})$, where $\tilde{\pi} \in \{\pi, \pi'\}$.

First consider the case where the second period default policy is the party's ideal point, i.e. $q' = z^i$. Then, with probability $\tilde{\pi}_i$ party i is chosen as formateur yielding i a payoff of $U_\ell(x(\{1,2,3\}, z^\ell), r_\ell(x(\{1,2,3\}, z^\ell)) = \frac{1}{3}$. With probability $1 - \tilde{\pi}_i$ some other party j is chosen as formateur in which case i receives the payoff $U_\ell(x(\{j\}), 0) = -1$. Hence,

$$W_i(z^i, \tilde{\pi}) = \frac{4}{3}\tilde{\pi}_i - 1.$$

Second, suppose that the default policy is another party j's ideal point. With probability $\tilde{\pi}_i$ party i is chosen as formateur, yielding the party the payoff $U_i(x(\{i\}, z^j), r_i(x(\{i\}, z^j)) = 0$, while with probability $\tilde{\pi}_j$ party j is chosen. In this case j forms a supermajority government yielding party i the payoff $U_i(x(\{1,2,3\}, z^j), r_i(x(\{1,2,3\}, z^j)) = -\frac{2}{3}$. Finally, with probability $1 - \tilde{\pi}_i - \tilde{\pi}_j$ the third party h is chosen as formateur yielding party i the payoff

[12] For another framework that generates surplus governments, see Baron and Diermeier (1998).

$U_i(x(\{h\}), 0) = -1$. Hence,

$$W_i(z^j, \tilde{\pi}) = \frac{1}{3}\tilde{\pi}_j + \tilde{\pi}_i - 1$$

We can now use these continuation values to the conditions under which the parliament would decide on early elections in period 2.

Lemma 3: *Suppose the parliament has to decide whether to dissolve and call early elections in period 2. For any $q' \in Q$, whenever $\pi' \neq \pi$, an early election is called.*

Lemma 3 states that if changes in public opinion shifts lead to changes in the parties' expected seat share, then a majority of parties is better off voting for early elections. The likelihood of this event is equal to $1 - \rho$.

It also follows that each party's continuation value $W_i(s')$ depends only on π'. Thus for any party $i \in N$ and $q' = z^h$ with $h \in N$, we can simply write

$$W_i(z^h, \pi') = \frac{1}{3}\pi'_h + \pi'_i - 1 \tag{1}$$

which corresponds to each party's continuation value for terminating the incumbent government during the renegotiation stage and for failing to form a government in period 1.

The next lemma pertains to the renegotiation stage of an incumbent government and characterizes the conditions under which a government would prematurely terminate.

Lemma 4: *Suppose $D \subseteq N$ is the incumbent government at the beginning of period 2.*
(i) If D is a majority government, then for any $s' \in S$, D remains in power throughout period 2. Furthermore, the chosen policy is

$$x'(D, s') = \frac{1}{|D|}\sum_{i \in D} z^i$$

and transfers are equal to

$$r'_i(D, s') = -\frac{1}{|D|}\sum_{\substack{j \in D \\ j \neq i}} W_j(s') + \frac{|D| - 1}{|D|} W_i(s'),\ i \in D$$

and

$$t'_j(D, s') = 0, \; j \in N \backslash D.$$

(ii) Suppose D is a minority government with $D = \{i\}$ and let $j, l \in N$, $i \neq j \neq l \neq i$ with $\pi'_j \leq \pi'_l$. Then $\{i\}$ terminates if and only if

$$q' = z^i \text{ and } \pi'_l < \frac{2}{3}\pi'_i$$

Otherwise $\{i\}$ remains in power throughout period 2. In this case, the chosen policy is

$$x'(\{i\}, s') = z^i$$

and transfers are equal to

$$r'_i(D, s') = -W_j(s') + u_j(z^i),$$

$$t'_j(\{i\}, s') = W_j(s') - u_j(z^i) \text{ and } t'_\ell(\{i\}, s') = 0.$$

Cabinet renegotiation is conducted under the same rules as proto–coalition bargaining. But in contrast to the one-period case characterized in Lemma 1, minority cabinets now do not receive support from some other party for free. Rather, they need to transfer benefits to the cheaper of the two outside parties. This follows, because each party now has a chance to be selected as the formateur and to collect the benefits from proposing the period 2 proto-coalition.

The price of support depends on each party's continuation values. Since the order of continuation values for the outside parties depends only on the parties' seat shares, the governing party would form a supporting coalition with the smaller of the two parties. However, the price of gaining support from even the cheapest party may still leave the party that forms the minority government worse off than its continuation value. For this to hold two conditions must be satisfied. First, both outside parties must be rather large. This follows because the necessary transfers are increasing in the continuation values which, given (1), are increasing in seat share. Second, the minority party's ideal point must equal the period

2 default policy. In this case, both outside parties are strictly worse off by implementing the minority government ideal policy.

The situation is different for majority cabinets, since these governments do not need outside support. However, depending on each governing parties continuation values, office benefits need to be reallocated among the coalition members. As Lemma 4 indicates, this is always possible for any π' and s'. By reshuffling office benefits incumbent governments can adjust for changes in the political and economic environment without terminating. The model can thus account for government terminations (fact 1) the relative instability of minority governments (fact 4), and the frequent occurrence of reshuffles (fact 2), provided we can show that these types of governments are actually chosen in period 1.

Lemma 4 allows us to qualify some of the results found in the literature. For example, as Lupia and Strom (1995) we find that in equilibrium governments may terminate in early elections and replacements. However, once we allow for efficient bargaining and reshuffles, the Lupia and Strom framework can no longer generate early terminations. If a government commands a majority of seats—the only case considered by Lupia and Strom—reshuffles can always be used to capture any changes in the bargaining environment within the current coalition. Once efficient bargaining is not possible, however, as in the case of bargaining between the minority government and the parties in its supporting coalition, governments may fall. Lemma 5 implies that a necessary condition for any government to fall is a change in the default policy q. That is, if the default policy is perfectly persistent, (i.e., $\lambda = 1$) no government can terminate. Contrary to Lupia and Strom, changes in expected seat share (i.e. public opinion shocks) are not sufficient for cabinet terminations, unless efficient reshuffles are restricted or transaction costs are assumed.

Using Lemma 4, for any given s' and for any government coalition D, we can compute the expected payoff to each party $i \in N$ in period 2 following a renegotiation by the government coalition

$$V_i(D, s') = \begin{cases} U_i(x(D, s'), r_i(D, s')) & \text{if } i \in D \text{ and } D \text{ remains in power} \\ U_i(x(D, s'), t_i(D, s')) & \text{if } i \notin D \text{ and } D \text{ remains in power} \\ W_i(s') & \text{if } D \text{ terminates} \end{cases} \tag{2}$$

The next lemma pertains to the outcome of the government formation process in period 1 for a given proto-coalition.

Lemma 5: *Suppose at the beginning of period 1 D is chosen as the proto-coalition.*

(i) For any $s \in S$ and for any $D \subseteq N$, D forms the government and chooses policy

$$x(D, s) = \frac{1}{|D|} \sum_{i \in D} z^i.$$

(ii) For any $s \in S$, if D is a majority government period 1 transfers are equal to

$$r_i(D, s) = -\frac{1}{|D|} \sum_{\substack{j \in D \\ j \neq i}} u_j(q) + \frac{|D| - 1}{|D|} u_i(q),\ i \in D$$

and

$$t_j(D, s) = 0,\ j \in N \backslash D.$$

(iii) Suppose $D = \{i\}$. For any $s \in S$, $j \neq i \neq \ell \neq i \in N$, period 1 transfers are equal to

$$r_i(\{i\}, s) = -\beta E[W_j(s') - V_j(\{i\}, s') | s] = -t_j(\{i\}, s),\ \textit{and}\ t_\ell(\{i\}, s) = 0,$$

if and only if

$$q = z^l \textit{ and } \pi_j > \pi_l.$$

Otherwise

$$r_i(\{i\}, s) = t_j(\{i\}, s) = t_\ell(\{i\}, s) = 0.$$

When combined with Lemma 4, Lemma 5 implies that no government coalition ever changes its policy in the second period. With respect to majority coalitions we have the same result as in Lemma 1. However, as in Lemma 4, minority governments may need to "buy" the external support of other parties to form the government. The government needs to make a positive transfer in the case where it expects to switch supporting coalitions in the second period. This situation can only occur if the smaller of the two out-parties is advantaged (and thus more expensive). Since the price of support in the second period depends only on each party's seat share, the smaller party will then be included in the supporting coalition. But since that party is advantaged in period 1 it is too expensive to be included in the supporting coalition in the first period, where the governing party relies on the disadvantaged party. Since the period 1 supporting party expects to be excluded in period 2, it needs to be compensated in period 1. The price of support depends on the difference in continuation values between forming the minority government and starting a new formation process in period 2.

By combining the results of the five previous lemmata, for any given s, we can compute the expected utility of each party $i \in N$ in period 1 conditional on each possible government coalition forming in period 1

$$V_i(D, s) = \begin{cases} U_i(x(D, s), r_i(D, s)) + \beta E[V_i(D, s')|s] & \text{if } i \in D \\ U_i(x(D, s), t_i(D, s)) + \beta E[V_i(D, s')|s] & \text{if } i \notin D. \end{cases}$$

These calculations represent the basis for the main result of the paper, stated in the following proposition, which characterizes the outcome of the government formation process in period 1. This characterization hinges on the solution of the following maximization problem faced by the formateur in period 1, for any state and for any formateur:

$$\max_{D \in \Delta_{\mathbf{k}}} V_{\mathbf{k}}(D, s)$$

Let $D_{\mathbf{k}}(s)$ denote the solution to this optimization problem.

Proposition 1: *Consider the government formation process at the beginning of period 1 and let* $\mathbb{k} \in N$ *denote the identity of the formateur.*

(i) Suppose $q = z^{\mathbf{k}}$ *and let* π *be such that* $\pi_j < \pi_l$ *Then, there exists a critical function* $p^*(\beta)$

$(\frac{dp^*(\beta)}{d\beta} < 0)$ *such that*

$$D_{\Bbbk}(z^{\Bbbk}, \pi) = \left\{ \begin{array}{c} \{\Bbbk, j\} \text{ iff } \pi_\ell > p^*(\beta) \\ D_{\Bbbk}(z^{\Bbbk}, \pi) = \{1, 2, 3\} \text{ otherwise} \end{array} \right\}$$

Further, there exist a critical value b^* *such that if* $\beta < b^*$, $D_{\Bbbk}(z^{\Bbbk}, \pi) = \{1, 2, 3\}$ *for all* π_ℓ.

(ii) Suppose $q = z^\ell$ *with* $j \neq \ell \neq \Bbbk \neq j$. *Then if* $\max\{\pi_j, \pi_l\} > \frac{2}{3}\pi_{\Bbbk}$, *there exists a critical function* $p_l^*(\beta, \lambda)$ $(\frac{\partial p_l^*(\beta,\lambda)}{\partial \beta} < 0, \frac{\partial p_l^*(\beta,\lambda)}{\partial \lambda} > 0)$ *such that*

$$D_{\Bbbk}(z^l, \pi) = \left\{ \begin{array}{c} \{j, \Bbbk\} \text{ iff } \pi_j > \pi_l \text{ and } \pi_l > p_l^*(\beta, \lambda) \\ \{\Bbbk\} \text{ otherwise,} \end{array} \right\}$$

where $\{\Bbbk\}$ *receives the external support of* j *and never terminates. Moreover, there exist critical functions* $b_1^*(\lambda)$ $(\frac{db_1^*(\lambda)}{d\lambda} < 0)$ *and* $b_2^*(\lambda)$ $(\frac{db_2^*(\lambda)}{d\lambda} > 0)$, *such that for all* π_l, *if* $\beta < b_1^*(\lambda)$, $D_{\Bbbk}(z^l, \pi) = \{\Bbbk\}$, *while if* $\beta > b_2^*(\lambda)$, $D_{\Bbbk}(z^l, \pi) = \{j, \Bbbk\}$.

On the other hand, if $\max\{\pi_j, \pi_l\} < \frac{2}{3}\pi_{\Bbbk}$ *there exists a critical function* $p_j^*(\beta, \lambda, \pi_l)$ $(\frac{\partial p_j^*(\beta,\lambda,\pi_l)}{\partial \beta} < 0, \frac{\partial p_j^*(\beta,\lambda,\pi_l)}{\partial \lambda} > 0$, *and* $\frac{\partial p_j^*(\beta,\lambda,\pi_l)}{\partial \pi_l} < 0)$ *such that*

$$D_{\Bbbk}(z^l, \pi) = \left\{ \begin{array}{c} \{j, \Bbbk\} \text{ iff } \pi_j > \pi_l \text{ and } \pi_j > p_j^*(\beta, \lambda, \pi_l) \\ \{\Bbbk\} \text{ otherwise,} \end{array} \right\}$$

where $\{\Bbbk\}$ *receives the external support of* j *and terminates with probability* $\frac{1-\lambda}{2}$. *Moreover, there exist critical functions* $b_3^*(\lambda)$ $(\frac{db_3^*(\lambda)}{d\lambda} < 0)$ *and* $b_4^*(\lambda)$ $(\frac{db_4^*(\lambda)}{d\lambda} < 0)$, *such that for all* π_j,*if* $\beta < b_3^*(\lambda)$, $D_{\Bbbk}(z^l, \pi) = \{\Bbbk\}$, *while if* $\beta > b_4^*(\lambda)$, $D_{\Bbbk}(z^l, \pi) = \{j, \Bbbk\}$.

To illustrate the results presented in Proposition 1, consider Figures 1 and 2 for the case $\beta = 1$ and $q = z^1$. First consider the case where party 1 is the formateur. From Lemma 2, we know that the solution to the one-period optimization problem is for party 1 to form the surplus government coalition $\{1, 2, 3\}$. Dynamic considerations, however, play an important role, since party 1's choice in period 1 also affects its period 2 payoff. Lemma 4 implies that the coalition $\{1, 2, 3\}$ would persist in period 2, but that depending on the default policy in period 2 office benefits may need to be reshuffled among the governing parties. In particular, while the one-period payoff resulting from choosing coalition $\{1,2,3\}$ dominates the payoff induced, for example, by the choice of $\{1,2\}$, the opposite is true with respect to two-period payoffs. This is the case since, loosely speaking, in the renegotiation stage, party 1 would only have to compensate one party as opposed to two parties to prevent them from leaving the current coalition with the expectation of obtaining a higher payoff in a new government formation process. Since a party's future prospects improve with its share, these considerations are particularly relevant when party 1's coalition partners are relatively "big". Hence, as the discount factor increases, if either party 2 or party 3 controls more than $\frac{1}{3}$ of the parliamentary seats, party 1 chooses to team up with the smaller of the two and form a minimal winning government rather than a surplus government.

Next, consider the case where party 2 is the formateur. Again, from Lemma 2 we know that the solution to the one-period optimization problem would be for party 2 to form the minority government $\{2\}$ with the external support of party 3. When dynamic considerations are taken into account, however, the next best alternative from a static point of view (i.e., forming a minimal winning government with party 3) may dominate. To see why, recall from Lemma 5 that, if $\{2\}$ forms the government and continues in period 2, it will seek the support of party 1 if and only if π_3 is greater than π_1. But then it has to compensate party 3 in period 1 for the fact that in the second period party 3 will not receive any transfers. The size of this compensation is increasing in the discount factor β. Hence, if the parties are sufficiently forward looking, then party 2 chooses to include party 3 in the government coalition rather than elicit its external support via expensive transfers. Thus minimal winning, supermajority and minority governments can occur in equilibrium which accounts for fact (3).

Note also that party 2 may choose to form a minority government even though it knows of the risk to fall in period 2. This will be the case where all three parties are of similar size. In this case both other potential supporting parties would be too expensive to buy off in period

2, or to include in a minimal winning coalition. Hence, unstable minority governments can form on the equilibrium path. This accounts for fact (4).

Proposition 1 thus implies that the stability and the relative occurrence of different types of governments are closely connected. Minimal winning coalitions are relatively cheaper to maintain than either minority or surplus coalitions. These considerations affect which kind of governments form. In the case of minority coalitions, the necessity to pay off an outside party that is willing to support the government may bring the government down or induce a minority government not to form. The price of support is determined by the outside party's continuation value that depends on its relative seat share and the state of the world. If both potential outside partners are expensive, a formateur may choose to form a minority government that is destined to fall (if the formateur's future prospects look favorable) or form a stable minimal winning government instead. In the case of surplus governments, the need to keep all members of the government in the coalition may induce a surplus government not to form. While it is always possible to make transfers within the government coalition to maintain a surplus government, if the coalition partners are large relative to the formateur party, a formateur may choose to form a minimal winning coalition instead.

4 Conclusion

Most game-theoretic models coalition formation in parliamentary democracies predict minimal winning coalitions. In this paper we propose a bargaining model of parliamentary governments where all types of governments (minimal winning, minority, and surplus) may form. Moreover, minority and surplus coalitions are not rare exceptions, but may be chosen for all parameter values. Indeed, contrary to the prevailing view that minimal winning coalitions are the natural outcome of coalition bargaining, our model implies the opposite. In the single-period version of our model, minimal winning coalitions would never form. The party favored by the default policy would always choose a surplus coalition, and a non-favored party would always form a minority government. Minimal winning coalitions only occur when dynamic considerations are important. That is, minimal winning coalitions are chosen, because it may be too expensive for a formateur to maintain surplus or minority coalitions over time, especially if the future state of the world is likely to favor a different party.

Our model also accounts for the fact that majoritarian governments (minimal winning and surplus) are considerably more stable than minority governments. While majoritarian governments always survive until the next regularly scheduled election, possibly by reshuffling government positions among the members of the government coalition, minority governments can terminate in early elections or be replaced by a new (minority !) government in a vote of no-confidence. Indeed, minority governments may still form even though all parties know that there exist states of the world where a minority government would fall for sure in the next period.

While our model can account for these basic regularities, it is too stylized to explain cross-country differences in government formation and stability. Why are there so many surplus coalitions is Italy, but none in Denmark or Germany? Why are minority governments unheard of in Germany, but are the norm in Denmark? What explains the differences in average government duration between countries? To answer these questions our model would need to be "augmented" to capture some of the institutional details of government formation that may account for cross-country differences. The model presented here represents a first step toward addressing these questions in a systematic fashion.

5 Appendix

Proof of Lemma 1: Suppose first that D' obtains the confidence of the parliament given transfers $t_j(D', s')$, $j \in N \backslash D'$. For any policy $x \in \Re^2$ coalition D' may choose to implement, the requirement that all coalition members have to agree defines a "cake" to be allocated among the coalition partners if they agree on that policy

$$c(x; D', s') = \sum_{i \in D'} [u_i(x) - u_i(q')] - \sum_{j \in N \backslash D'} t_j(D', s').$$

Given the bargaining procedure specified, the unique stationary subgame perfect equilibrium outcome of the bargaining game which determines how the cake is allocated (and hence the transfers within the coalition), is that parties immediately agree on a split of the cake such that each party $i \in D'$ receives an equal share

$$\frac{1}{|D'|} c(x; D', s')$$

(see, Binmore (1987) for the two-player case and Merlo and Wilson (1998) for the general case).

Since each party wants to maximize its share of the cake, it immediately follows that all parties in D' unanimously agree to select the policy that maximizes the size of the cake:

$$x(D', s') = \frac{1}{|D'|} \sum_{i \in D'} z^i.$$

Hence, if $c(x(D', s'); D', s') \geq 0$, D' forms the government and each party $i \in D'$ receives a payoff equal to

$$u_i(q') + \frac{1}{|D'|} c(x(D', s'); D', s')$$

or equivalently,

$$u_i(x(D', s')) + r_i(D', s'),$$

where

$$r_i(D', s') = -\frac{1}{|D'|} \sum_{\substack{j \in D' \\ j \neq i}} u_j(q') + \frac{|D'| - 1}{|D'|} u_i(q') - \frac{1}{|D'|} \sum_{j \in N \backslash D'} t_j(D', s').$$

If instead $c(x(D', s'); D', s') < 0$, then the government formation attempt fails and each party $i \in N$ receives a payoff of $U_i(q', 0)$. These results hold for minority coalitions as well as minimal winning and surplus coalitions for any $s' \in S'$.

Consider now how transfers to parties outside the proto-coalition are determined. If D' is a majority coalition, then it does not need the support of any other party outside the proto-coalition to obtain the confidence of the parliament. Hence, if D' is a majority,

$$t_j(D', s') = 0,\ j \in N \backslash D'.$$

This implies that $c(x(D', s'); D', s') > 0$, and hence D' forms the government.

If instead D' is a minority coalition (i.e., $D' = \{i\}$, $i \in N$), it needs the external support of at least another party to obtain the confidence of a parliamentary majority. Let $D' = \{\Bbbk\}$, $\Bbbk \in N$, and note that $x'(\{\Bbbk\}, s') = z^{\mathbf{k}}$. To obtain the support of party $j \neq \Bbbk$, $\{\Bbbk\}$ needs to pay that party at least

$$\max\{0, u_j(q') - u_j(z^{\mathbf{k}})\}.$$

Since the external support of one party is enough to obtain the confidence of the parliament, $\{\Bbbk\}$ pays the least amount possible to at most one party. Note that for any $q' \in Q$, there always exists some party $j \in N$, $j \neq \Bbbk$, for which $u_j(q') - u_j(z^{\mathbf{k}}) = 0$. Hence, if D' is a minority,

$$t_j(D', s') = 0,\ j \in N \backslash D'.$$

This implies that $c(x(D', s'); D', s') > 0$, and hence D' forms the government. Q.E.D.

Proof of Lemma 2: By Lemma 1, for any $\Bbbk \in N$, any $q' \in Q$, and any $\pi, \pi' \in \Pi$,

$$\begin{aligned}
U_{\mathbf{k}}(x(\{1,2,3\}, s'), r_{\mathbf{k}}(x(\{1,2,3\}, s')) &= u_{\mathbf{k}}(\frac{z^1 + z^2 + z^3}{3}) - \frac{1}{3}\sum_{\substack{i \in N \\ i \neq \mathbf{k}}} u_i(q') + \frac{2}{3}u_{\mathbf{k}}(q'), \\
U_{\mathbf{k}}(x(\{i,\Bbbk\}, s'), r_{\mathbf{k}}(x(\{i,\Bbbk\}, s')) &= u_{\mathbf{k}}(\frac{z^i + z^{\mathbf{k}}}{2}) - \frac{1}{2}u_i(q') + \frac{1}{2}u_{\mathbf{k}}(q'), \\
U_{\mathbf{k}}(x(\{j,\Bbbk\}, s'), r_{\mathbf{k}}(x(\{j,\Bbbk\}, s')) &= u_{\mathbf{k}}(\frac{z^j + z^{\mathbf{k}}}{2}) - \frac{1}{2}u_j(q') + \frac{1}{2}u_{\mathbf{k}}(q'),
\end{aligned}$$

and

$$U_{\Bbbk}(x(\{\Bbbk\}, q'), r_{\Bbbk}(x(\{\Bbbk\}, q')) = u_{\Bbbk}(z^{\mathbf{k}}),$$

$i, j \in N$, $i \neq j \neq \Bbbk$.

Next, note that for any $i, j, \Bbbk \in N$, $i \neq j \neq \Bbbk$, $u_{\Bbbk}(\frac{z^1+z^2+z^3}{3}) = -\frac{1}{3}$, $u_{\Bbbk}(\frac{z^i+z^{\mathbf{k}}}{2}) = u_{\Bbbk}(\frac{z^j+z^{\mathbf{k}}}{2}) = -\frac{1}{4}$, $u_i(z^{\mathbf{k}}) = u_j(z^{\mathbf{k}}) = -1$, and $u_{\Bbbk}(z^{\mathbf{k}}) = 0$.

(i) If $q' = z^{\mathbf{k}}$, then

$$\begin{aligned} U_{\Bbbk}(x(\{1,2,3\}, z^{\mathbf{k}}), r_{\Bbbk}(x(\{1,2,3\}, z^{\mathbf{k}})) &= \frac{1}{3}, \\ U_{\Bbbk}(x(\{i,\Bbbk\}, z^{\mathbf{k}}), r_{\Bbbk}(x(\{i,\Bbbk\}, z^{\mathbf{k}})) &= \frac{1}{4}, \\ U_{\Bbbk}(x(\{j,\Bbbk\}, z^{\mathbf{k}}), r_{\Bbbk}(x(\{j,\Bbbk\}, z^{\mathbf{k}})) &= \frac{1}{4}, \end{aligned}$$

and

$$U_{\Bbbk}(x(\{\Bbbk\}, z^{\mathbf{k}}), r_{\Bbbk}(x(\{\Bbbk\}, z^{\mathbf{k}})) = 0,$$

which establishes the first part of the lemma.

(ii) If $q' = z^i$, $i \neq \Bbbk$, then

$$\begin{aligned} U_{\Bbbk}(x(\{1,2,3\}, z^i), r_{\Bbbk}(x(\{1,2,3\}, z^i)) &= -\frac{2}{3}, \\ U_{\Bbbk}(x(\{i,\Bbbk\}, z^i), r_{\Bbbk}(x(\{i,\Bbbk\}, z^i)) &= -\frac{3}{4}, \\ U_{\Bbbk}(x(\{j,\Bbbk\}, z^i), r_{\Bbbk}(x(\{j,\Bbbk\}, z^i)) &= -\frac{1}{4}, \end{aligned}$$

and

$$U_{\Bbbk}(x(\{\Bbbk\}, z^i), r_{\Bbbk}(x(\{\Bbbk\}, z^i)) = 0.$$

Furthermore, party j is willing to support government $\{\Bbbk\}$ since

$$U_j(z^i, 0) = U_j(z^{\mathbf{k}}, 0),$$

which proves the second part of the lemma. Q.E.D.

Proof of Lemma 3: Suppose at the beginning of period 2 there is no incumbent government or the incumbent government has terminated. It is easy to see that for any party $i \in N$, for any realization of ε such that $\pi_i' > \pi_i$, $W_i(z^\ell, \pi') > W_i(z^\ell, \pi)$ and hence party i votes in favor of an early election. Obviously, if $\pi = \pi'$, then $W_i(z^\ell, \pi') = W_i(z^\ell, \pi)$ for all $i \in N$, and hence no party votes in favor of an early election. Therefore, for any realization of ε such that $\pi' \neq \pi$, since it is always the case that two parties gain at the expenses of the third party, a majority strictly prefers to dissolve the parliament and call early elections. Q.E.D.

Proof of Lemma 4: The argument is analogous to the one presented in the proof of Lemma 1 with $W_i(s')$ instead of $u_i(q')$.

Suppose first that D has maintained the confidence of the parliament given transfers $t_j(D, s')$, $j \in N\backslash D'$. Then the cake to be allocated among the coalition partners, if they agree on policy $x \in \Re^2$, is

$$c(x; D, s') = \sum_{i \in D} [u_i(x) - W_i(s')] - \sum_{j \in N\backslash D} t_j(D, s').$$

Hence, by the same argument given in the proof of Lemma 1,

$$x(D, s') = \frac{1}{|D|} \sum_{i \in D} z^i$$

and if $c(x(D, s'); D, s') \geq 0$, D stays in power and each party $i \in D$ receives transfers equal to

$$r_i(D, s') = -\frac{1}{|D|} \sum_{\substack{j \in D \\ j \neq i}} W_j(s') + \frac{|D| - 1}{|D|} W_i(s') - \frac{1}{|D|} \sum_{j \in N\backslash D} t_j(D, s').$$

If instead $c(x(D, s'); D, s') < 0$, then the renegotiation attempt fails and each party $i \in N$ receives its expected continuation payoff $W_i(s')$.

(i) Since a majority coalition does not need the support of any other party outside the government coalition we have

$$t_j(D, s') = 0, \; j \in N\backslash D'.$$

This implies that $c(x(D, s'); D, s') > 0$, and hence D remains the government.

(ii) Let $D = \{i\}$. To obtain the support of some party $h \neq i$, $\{i\}$ needs to pay that party at least

$$\max\{0, W_h(q', \pi') - u_h(z^i)\}.$$

Note that for any $s' \in S$ and any $h \in N$, $W_h(s') - u_h(z^i) > 0$. Hence, to stay in power $\{i\}$ needs to make a positive transfer either to party j or to party ℓ. Since the external support of one party is enough to obtain the confidence of the parliament, $\{i\}$ seeks the support of the cheapest outside party. Party j is cheaper if and only if

$$W_\ell(q, \pi') - u_\ell(z^i) \geq W_j(q, \pi') - u_j(z^i)$$

which given (1) and symmetry reduces to

$$\pi'_\ell \geq \pi'_j.$$

Since $\pi'_\ell \geq \pi'_j$, $\{i\}$ needs to make the transfer

$$t_j(\{i\}, q', \pi') = W_j(q', \pi') - u_j(z^i)$$

to stay in power, in which case party i's period 2 payoff is equal to

$$U_i(z^i, -(W_j(q', \pi') - u_j(z^i))).$$

Alternatively, $\{i\}$ could terminate, in which case party i would receive a period 2 expected payoff equal to

$$W_i(q', \pi').$$

Obviously, $\{i\}$ chooses to make the transfer and stay in power if and only if

$$U_i(z^i, -(W_j(q', \pi') - u_j(z^i))) \geq W_i(q', \pi')$$

if and only if

$$u_i(z^i) + u_j(z^i) \geq W_i(q', \pi') + W_j(q', \pi')$$

Given (1) and $q' = (z^h, \pi')$ for some $h \in N$ this reduces to

$$0 \geq \frac{2}{3}\pi'_h + \pi'_i + \pi'_j - 1$$

or

$$\pi'_l \geq \frac{2}{3}\pi'_h$$

Note, that this inequality is always satisfied for $h \neq i$. In the case where $h = l$ it reduces to $\frac{1}{3}\pi'_l \geq 0$, while for $h = j$ it follows directly from $\pi'_l \geq \pi'_j$. This leaves the case where $h = i$, in which case the condition is binding. This establishes (ii). Q.E.D.

Proof of Lemma 5: The proof of the first part of the lemma is analogous to the proof of Lemma 4. Suppose first that D obtains the confidence of the parliament given transfers $t_j(D, s)$, $j \in N \backslash D$. Then,

$$c(x; D, s) = \sum_{i \in D} [u_i(x) - u_i(q)] - \sum_{j \in N \backslash D} t_j(D, s)$$

and

$$x(D, s) = \frac{1}{|D|} \sum_{i \in D} z^i.$$

To show that D wants to form that government, however, it is no longer sufficient to show that $c(x; D, s) \geq 0$, since if the government formation attempt fails, each party $i \in N$ obtains a total payoff equal to

$$U_i(q, 0) + \beta E[W_i(s')|s].$$

Hence, D forms the government if and only if for all $i \in D$,

$$U_i(x(D, s), r_i(D, s)) + \beta E[V_i(D, s')|s] \geq U_i(q, 0) + \beta E[W_i(s')|s].$$

First consider the case where D is a majority. If D is a majority coalition, then it does not need the support of any other party outside the proto-coalition. Hence,

$$t_j(D, s) = 0, \; j \in N \backslash D.$$

This implies that for all $i \in D$, $U_i(x(D,s), r_i(D,s)) > U_i(q,0)$. Next observe that Lemma 4 implies that for all $s' \in S$, and all $i \in D$, $V_i(D,s') > W_i(s')$, which establishes the result. Hence, D forms the government and transfers within the government coalition in period 1 are equal to

$$r_i(D,s) = -\frac{1}{|D|}\sum_{\substack{j\in D\\ i\neq i}} u_j(q) + \frac{|D|-1}{|D|}u_i(q),\ i \in D$$

This completes the proof of parts (i) and (ii) of the lemma.

Next consider the case where D is a minority and let $D = \{i\}$. To obtain the confidence of a parliamentary majority, $\{i\}$ has to compute the minimum transfer required by one of the other two parties to elicit their support. For each party $h \neq i$, since the failure of the government formation attempt yields that party an expected payoff equal to

$$U_h(q,0) + \beta E[W_h(s')|s],$$

such transfer is equal to

$$\max\{0, (u_h(q) - u_h(z^i)) + \beta E[W_h(s') - V_h(\{i\},s')|s]\}.$$

Since q' and π' are independent, both $W_h(s')$ and $V_h(\{i\},s')$ are linear in π' (see equations (1) and (2)), and $\pi' = \pi + \varepsilon$ with $E(\varepsilon) = 0$,

$$\begin{aligned} E[W_h(s') - V_h(\{i\},s')|s] &= E_{q'}[E_{\pi'}[W_h(q',\pi') - V_h(\{i\},q',\pi')|\pi]|q] \\ &= E_{q'}[W_h(q',\pi) - V_h(\{i\},q',\pi)|q] \end{aligned}$$

and hence we can rewrite the expression for the transfer as

$$\max\{0, (u_h(q) - u_h(z^i)) + \beta E_{q'}[W_h(q',\pi) - V_h(\{i\},q',\pi)|q]\}.$$

Let $j, \ell \in N$, $i \neq j \neq \ell \neq i$. First, suppose that $q = z^i$. Then $u_j(q) - u_j(z^i) = u_\ell(q) - u_\ell(z^i) = 0$. Without loss of generality, assume that $\pi_j < \pi_\ell$. Then, Lemma 4 implies that for all $q' \in Q$, $W_j(q',\pi) - V_j(\{i\},q',\pi) = 0$, whether $\{i\}$ remains in power or terminates in period 2. Hence,

$$t_j(\{i\},s) = 0 \text{ and } t_\ell(\{i\},s) = 0.$$

Together with the fact that for all $q' \in Q$, $V_i(\{i\}, q', \pi) \geq W_i(q', \pi)$ (and the inequality is strict for all period 2 states where $\{i\}$ remains in power—see Lemma 4), this implies that

$$U_i(z^i, 0) + \beta E_{q'}[V_i(\{i\}, q', \pi)|q = z^i] > U_i(z^i, 0) + \beta E_{q'}[W_i(q', \pi)|q = z^i],$$

and hence $\{i\}$ forms the government in period 1.

Suppose now that $q = z^\ell$. Then $u_j(q) - u_j(z^i) = 0$ while $u_\ell(q) - u_\ell(z^i) = 1$. If $\pi_j < \pi_\ell$, Lemma 4 implies that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) = 0$, whether $\{i\}$ remains in power or terminates in period 2. Hence, again it is the case that

$$t_j(\{i\}, s) = 0 \text{ and } t_\ell(\{i\}, s) = 0,$$

and

$$U_i(z^i, 0) + \beta E_{q'}[V_i(\{i\}, q', \pi)|q = z^\ell] > U_i(z^\ell, 0) + \beta E_{q'}[W_i(q', \pi)|q = z^\ell],$$

so that $\{i\}$ forms the government in period 1.

If instead $\pi_j > \pi_\ell$, Lemma 4 implies that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) \geq 0$, (since if $\{i\}$ remains in power in period 2, it obtains the external support of party ℓ). Hence, in this case, to obtain the support of party j in period 1, $\{i\}$ needs to pay that party

$$\beta E_{q'}[W_j(q', \pi) - V_j(\{i\}, q', \pi)|q = z^\ell].$$

To obtain the support of party ℓ in period 1, $\{i\}$ needs to pay that party

$$u_\ell(z^\ell) - u_\ell(z^i) = 1,$$

since for all $q' \in Q$, $W_\ell(q', \pi) - V_\ell(\{i\}, q', \pi) = 0$. Note that this amount is always larger than what $\{i\}$ would have to pay party j. This follows from the fact that for all $q' \in Q$, $W_j(q', \pi) - V_j(\{i\}, q', \pi) < 1$. Thus, if $\pi_j > \frac{2}{3}\pi_i$ (i.e., $\{i\}$ would never terminate and thus $W_j(q', \pi) - V_j(\{i\}, q', \pi) > 0$ for all q'), we have

$$t_j(\{i\}, z^\ell, \pi) = \beta\left(\pi_j - \frac{1}{6}\pi_\ell(1 - 3\lambda) + \frac{1}{6}(1 - \lambda)\right) \text{ and } t_\ell(\{i\}, z^\ell, \pi) = 0,$$

whereas if ($\pi_j < \frac{2}{3}\pi_i$) (i.e., $W_j(z^i,\pi) - V_j(\{i\},z^i,\pi) = 0$ since $\{i\}$ would terminate if $q' = z^i$),

$$t_j(\{i\},z^\ell,\pi) = \beta\left(\pi_j(\frac{2+\lambda}{3}) + \pi_\ell\frac{\lambda}{3}\right) \text{ and } t_\ell(\{i\},z^\ell,\pi) = 0.$$

The last thing we need to show is that $\{i\}$ wants to form the government. This is always the case if

$$U_i(z^i, -t_j(\{i\},z^\ell,\pi)) + \beta E_{q'}[V_i(\{i\},q',\pi)|q = z^\ell] > U_i(z^\ell,0) + \beta E_{q'}[W_i(q',\pi)|q = z^\ell],$$

which is always true since for all $q' \in Q$, $V_i(\{i\},q',\pi) \geq W_i(q',\pi)$ (see Lemma 4), and since $t_j(\{i\},z^\ell,\pi) < 1$ implies

$$U_i(z^i, -t_j(\{i\},z^\ell,\pi)) = -t_j(\{i\},z^\ell,\pi) > -1 = U_i(z^\ell,0).$$

Q.E.D.

Proof of Proposition 1: (i) Suppose first that $q = z^{\mathbf{k}}$. Then by Lemma 5, (using the normalization $\pi_{\mathbb{k}} = 1 - \pi_j - \pi_\ell$), for any $\pi \in \Pi$,

$$\begin{aligned}
V_{\mathbb{k}}(\{1,2,3\},z^{\mathbf{k}},\pi) &= \frac{1}{3} + \beta\left(\frac{1}{3} - \pi_j - \pi_\ell\right)\\
V_{\mathbb{k}}(\{j,\mathbb{k}\},z^{\mathbf{k}},\pi) &= \frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right)\\
V_{\mathbb{k}}(\{\ell,\mathbb{k}\},z^{\mathbf{k}},\pi) &= \frac{1}{4} + \beta\left(\frac{1}{4} - \pi_\ell - \frac{1}{2}\pi_j\right)
\end{aligned}$$

and if $\pi_\ell > \frac{2}{3}\pi_{\mathbb{k}}$ (i.e., $\{\mathbb{k}\}$ receives the external support of party j and always survives in the second period),

$$V_{\mathbb{k}}(\{\mathbb{k}\},z^{\mathbf{k}},\pi) = 0 + \beta\left(-\pi_j(\frac{7-3\lambda}{6}) - \pi_\ell(\frac{1-3\lambda}{6}) - \frac{\lambda}{3}\right),$$

while if $\pi_\ell < \frac{2}{3}\pi_{\mathbb{k}}$) (i.e., $\{\mathbb{k}\}$ receives the external support of party j and terminates in the second period if $q' = \mathbb{k}$),

$$V_{\mathbb{k}}(\{\mathbb{k}\},z^{\mathbf{k}},\pi) = 0 + \beta\left(-\pi_j(\frac{7+\lambda}{6}) - \pi_\ell(\frac{1+7\lambda}{6}) + \frac{\lambda}{3}\right).$$

Note that then $\Bbbk$ prefers $\{j, \Bbbk\}$ to $\{\ell, \Bbbk\}$ iff $\pi_j < \pi_\ell$. Then

$$V_{\Bbbk}(\{1,2,3\}, z^{\mathbf{k}}, \pi) > V_{\Bbbk}(\{j, \Bbbk\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$\frac{1}{3} + \beta\left(\frac{1}{3} - \pi_j - \pi_\ell\right) > \frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right)$$

if and only if

$$\pi_\ell > \frac{1+\beta}{6\beta} := p^*(\beta).$$

Since $\pi_\ell \in (0, \frac{1}{2})$, this implies that if $\beta < b^* = \frac{1}{2}$, $\Bbbk$ prefers $\{1,2,3\}$ to $\{\Bbbk, j\}$ for all $\pi \in \Pi$ where $\pi_j < \pi_\ell$. If $\beta > b^* = \frac{1}{2}$, then $\Bbbk$ prefers $\{\Bbbk, j\}$ to $\{1,2,3\}$ for $\pi_\ell > p^*(\beta)$, and it prefers $\{1,2,3\}$ to $\{\Bbbk, j\}$ for $\pi_\ell < p^*(\beta)$. Note that $p^*(\beta)$ is a decreasing function of β.

Next, we show that for all $\pi \in \Pi$ where $\pi_j < \pi_\ell$, $\Bbbk$ prefers $\{j, \Bbbk\}$ to $\{\Bbbk\}$. Note that if it were ever the case that $\Bbbk$ prefers $\{\Bbbk\}$ to $\{j, \Bbbk\}$, it would have to be true that the payoff gain in period 2 is large enough to compensate the payoff loss in period 1. Hence, set $\beta = 1$ (i.e., the best case for $\{\Bbbk\}$). For $\pi_\ell > \frac{2}{3}\pi_{\Bbbk}$,

$$V_{\Bbbk}(\{j, \Bbbk\}, z^{\mathbf{k}}, \pi) > V_{\Bbbk}(\{\Bbbk\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$\frac{1}{4} + \frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell > -\pi_j\left(\frac{7-3\lambda}{6}\right) - \pi_\ell\left(\frac{1-3\lambda}{6}\right) - \frac{\lambda}{3}$$

if and only if

$$\pi_\ell < \frac{\pi_j(1-3\lambda) + (3+2\lambda)}{2+3\lambda}$$

which is always true since the right hand side is greater than $\frac{1}{2}$. For $\pi_\ell < \frac{2}{3}\pi_{\Bbbk}$,

$$V_{\Bbbk}(\{j, \Bbbk\}, z^{\mathbf{k}}, \pi) > V_{\Bbbk}(\{\Bbbk\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$\frac{1}{4} + \frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell > -\pi_j\left(\frac{7+\lambda}{6}\right) - \pi_\ell\left(\frac{1+7\lambda}{6}\right) + \frac{\lambda}{3}$$

if and only if

$$\pi_j > \frac{\pi_\ell(2-7\lambda)+2\lambda-3}{1+\lambda}$$

which is always true since the right hand side is negative.

(ii) Now suppose that $q = z^\ell$. Then by Lemma 5, (using the normalization $\pi_{\mathbb{k}} = 1-\pi_j-\pi_\ell$), for any $\pi \in \Pi$,

$$\begin{aligned}
V_{\mathbb{k}}(\{1,2,3\}, z^\ell, \pi) &= -\frac{2}{3} + \beta\left(\frac{1}{3} - \pi_j - \pi_\ell\right) \\
V_{\mathbb{k}}(\{j, \mathbb{k}\}, z^\ell, \pi) &= -\frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right) \\
V_{\mathbb{k}}(\{\ell, \mathbb{k}\}, z^\ell, \pi) &= -\frac{3}{4} + \beta\left(\frac{1}{4} - \pi_\ell - \frac{1}{2}\pi_j\right).
\end{aligned}$$

Suppose first that $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{3}\pi_{\mathbb{k}}$. Note that this is the case where $\{\mathbb{k}\}$ would not have to pay party j in period 1 to receive its support and would always survives in the second period. Using Lemma 5 we have,

$$V_{\mathbb{k}}(\{\mathbb{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta\left(\pi_j + \pi_\ell(\frac{1-3\lambda}{6}) - (\frac{1-\lambda}{6})\right),$$

while for $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{3}\pi_{\mathbb{k}}$ (i.e., $\{\mathbb{k}\}$ would not have to pay party j in period 1 to receive its support and would terminate in the second period if $q' = \mathbb{k}$), we have

$$V_{\mathbb{k}}(\{\mathbb{k}\}, z^{\mathbf{k}}, \pi) = 0 + \beta\left(-\pi_j(\frac{4-\lambda}{3}) - \pi_\ell(\frac{2-\lambda}{3}) + (\frac{1-\lambda}{6})\right),$$

On the other hand suppose that $\pi_j > \pi_\ell$ and $\pi_j > \frac{2}{3}\pi_{\mathbb{k}}$. In this case $\{\mathbb{k}\}$ would need to pay party j in period 1 to receive its support and would always survive in the second period). Thus,

$$V_{\mathbb{k}}(\{\mathbb{k}\}, z^{\mathbf{k}}, \pi) = -\beta\left(\pi_j - \pi_\ell(\frac{1-3\lambda}{6}) + (\frac{1-\lambda}{6})\right) + \beta\left(-\pi_\ell(\frac{5+3\lambda}{6}) - (\frac{1-\lambda}{6})\right),$$

while for $\pi_j > \pi_\ell$ and $\pi_j < \frac{2}{3}\pi_{\mathbb{k}}$ (i.e., $\{\mathbb{k}\}$ would need to pay party j in period 1 to receive its support and would terminate in the second period if $q' = \mathbb{k}$)

$$V_{\mathbb{k}}(\{\mathbb{k}\}, z^{\mathbf{k}}, \pi) = -\beta\left(\pi_j(\frac{2+\lambda}{3}) + \pi_\ell\frac{\lambda}{3}\right) + \beta\left(-\pi_\ell(\frac{7+\lambda}{6}) - \pi_j(\frac{5-5\lambda}{6}) + (\frac{1-\lambda}{6})\right).$$

We first show that for all $\pi \in \Pi$, $\Bbbk$ prefers $\{j, \Bbbk\}$ to $\{\ell, \Bbbk\}$ and to $\{1, 2, 3\}$.

$$V_{\mathbf{k}}(\{j, \Bbbk\}, z^{\ell}, \pi) > V_{\mathbf{k}}(\{\ell, \Bbbk\}, z^{\ell}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right) > -\frac{3}{4} + \beta\left(\frac{1}{4} - \pi_\ell - \frac{1}{2}\pi_j\right)$$

if and only if

$$\pi_j < \pi_\ell + \frac{1}{\beta}$$

which is always true since the right hand side is greater than 1. Furthermore,

$$V_{\mathbf{k}}(\{j, \Bbbk\}, z^{\ell}, \pi) > V_{\mathbf{k}}(\{1, 2, 3\}, z^{\ell}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right) > -\frac{2}{3} + \beta\left(\frac{1}{3} - \pi_j - \pi_\ell\right)$$

if and only if

$$\pi_\ell > \frac{\beta - 5}{6\beta}$$

which is always true since the right hand side is negative.

Next, we compare $\Bbbk$'s payoffs if it chooses $\{j, \Bbbk\}$ versus $\{\Bbbk\}$ in the different regions of the parameter space Π. Suppose that $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{3}\pi_{\mathbf{k}}$. Note that these restrictions imply that $\pi_\ell \in (\frac{2}{7}, \frac{1}{2})$ and $\pi_j \in (0, \frac{1}{2})$. Now

$$V_{\mathbf{k}}(\{j, \Bbbk\}, z^{\ell}, \pi) > V_{\mathbf{k}}(\{\Bbbk\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right) > \beta\left(\pi_j + \pi_\ell(\frac{1 - 3\lambda}{6}) - (\frac{1 - \lambda}{6})\right)$$

if and only if

$$\pi_j < \frac{-\pi_\ell\beta(8 - 6\lambda) + \beta(5 - 2\lambda) - 3}{24\beta}$$

which is never true since the right hand side is negative. To see this note that

$$-\pi_\ell\beta(8-6\lambda)+\beta(5-2\lambda)-3<0$$

if and only if

$$\pi_\ell > \frac{\beta(5-2\lambda)-3}{\beta(8-6\lambda)}$$

which is always true since the right hand side is smaller than $\frac{2}{7}$. Hence, for $\pi_j < \pi_\ell$ and $\pi_\ell > \frac{2}{3}\pi_{\Bbbk}$, $\Bbbk$ prefers $\{\Bbbk\}$ to $\{j,\Bbbk\}$.

Next suppose that $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{3}\pi_{\Bbbk}$. Note that these restrictions imply that $\pi_\ell \in (\frac{1}{4},\frac{1}{3})$ and $\pi_j \in (\frac{1}{6},\frac{2}{7})$. Now

$$V_{\Bbbk}(\{j,\Bbbk\},z^\ell,\pi) > V_{\Bbbk}(\{\Bbbk\},z^{\Bbbk},\pi)$$

if and only if

$$-\frac{1}{4}+\beta\left(\frac{1}{4}-\pi_j-\frac{1}{2}\pi_\ell\right) > \beta\left(-\pi_j(\frac{4-\lambda}{3})-\pi_\ell(\frac{2-\lambda}{3})+(\frac{1-\lambda}{6})\right)$$

if and only if

$$\pi_j > \frac{\pi_\ell 2\beta(2\lambda-1)-\beta(2\lambda+1)+3}{4\beta(1-\lambda)}$$

which is never true since the right hand side is greater than $\frac{2}{7}$. To see this note that

$$\frac{\pi_\ell 2\beta(2\lambda-1)-\beta(2\lambda+1)+3}{4\beta(1-\lambda)} > \frac{2}{7}$$

if and only if, for $\lambda \geq \frac{1}{2}$,

$$\pi_\ell > \frac{\beta(15+6\lambda)-21}{14\beta(2\lambda-1)}$$

which is always true since the right hand side is negative, and for $\lambda < \frac{1}{2}$,

$$\pi_\ell < \frac{21-\beta(15+6\lambda)}{14\beta(1-2\lambda)}$$

which is always true since the right hand side is greater than $\frac{1}{3}$. Hence, for $\pi_j < \pi_\ell$ and $\pi_\ell < \frac{2}{3}\pi_{\Bbbk}$, $\Bbbk$ prefers $\{\Bbbk\}$ to $\{j,\Bbbk\}$. When combined with the previous findings, this result implies that whenever $\pi_j < \pi_\ell$, $\{\Bbbk\}$ forms the government with the support of party j.

Next consider the case where $\pi_j > \pi_\ell$ and $\pi_j > \frac{2}{3}\pi_{\mathbb{k}}$. Note that these restrictions imply that $\pi_j \in (\frac{2}{7}, \frac{1}{2})$ and $\pi_\ell \in (0, \frac{1}{2})$. Now

$$V_{\mathbb{k}}(\{j, \mathbb{k}\}, z^\ell, \pi) > V_{\mathbb{k}}(\{\mathbb{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right) > -\beta\left(\pi_j - \pi_\ell(\frac{1-3\lambda}{6}) + (\frac{1-\lambda}{6})\right) + \beta\left(-\pi_\ell(\frac{5+3\lambda}{6}) - (\frac{1-\lambda}{6})\right)$$

if and only if

$$\pi_\ell > \frac{3 - \beta(7-4\lambda)}{2\beta(1+6\lambda)} =: p_\ell^*(\beta, \lambda)$$

which for $\beta < b_1^*(\lambda) = \frac{3}{8+2\lambda}$ is never true (since the right hand side is greater than $\frac{1}{2}$), whereas for $\beta > b_2^*(\lambda) = \frac{3}{7-4\lambda}$ is always true (since the right hand side is negative). Note that $b_1^*(\lambda)$ is a decreasing function of λ, $b_2^*(\lambda)$ is an increasing function of λ, and $p_\ell^*(\beta, \lambda)$ is decreasing in β and increasing in λ. Hence, for $\pi_\ell < \pi_j$ and $\pi_j > \frac{2}{3}\pi_{\mathbb{k}}$, if $\beta < b_1^*(\lambda)$, then $\{\mathbb{k}\}$ forms the government with the external support of party j; if $\beta > b_2^*(\lambda)$, then $\{\mathbb{k}, j\}$ forms the government; and if $b_1^*(\lambda) < \beta < b_2^*(\lambda)$, then for $\pi_\ell < p_\ell^*(\beta, \lambda)$, $\{\mathbb{k}\}$ forms the government with the external support of party j, whereas for $\pi_\ell > p_\ell^*(\beta, \lambda)$, $\{\mathbb{k}, j\}$ forms the government.

Finally consider the case where $\pi_j > \pi_\ell$ and $\pi_j < \frac{2}{3}\pi_{\mathbb{k}}$. Note that these restrictions imply that $\pi_j \in (\frac{1}{4}, \frac{1}{3})$ and $\pi_\ell \in (\frac{1}{6}, \frac{2}{7})$. Now

$$V_{\mathbb{k}}(\{j, \mathbb{k}\}, z^\ell, \pi) > V_{\mathbb{k}}(\{\mathbb{k}\}, z^{\mathbf{k}}, \pi)$$

if and only if

$$-\frac{1}{4} + \beta\left(\frac{1}{4} - \pi_j - \frac{1}{2}\pi_\ell\right) > -\beta\left(\pi_j(\frac{2+\lambda}{3}) + \pi_\ell\frac{\lambda}{3}\right) + \beta\left(-\pi_\ell(\frac{7+\lambda}{6}) - \pi_j(\frac{5-5\lambda}{6}) + (\frac{1-\lambda}{6})\right)$$

if and only if

$$\pi_j > \frac{-\pi_\ell\beta(8+6\lambda) - \beta(1+2\lambda) + 3}{6\beta(1-\lambda)} =: p_j^*(\beta, \lambda, \pi_\ell)$$

which for $\beta < b_3^*(\lambda) = \frac{21}{37+12\lambda}$ is never true (since the right hand side is greater than $\frac{1}{3}$), whereas for $\beta > b_4^*(\lambda) = \frac{18}{23+9\lambda}$ is always true (since the right hand is smaller than $\frac{1}{4}$). To show that this is the case, note that

$$\frac{-\pi_\ell\beta(8+6\lambda)-\beta(1+2\lambda)+3}{6\beta(1-\lambda)}>\frac{1}{3}$$

if and only if

$$\pi_\ell<\frac{3(1-\beta)}{2\beta(4+3\lambda)}$$

which, for $\beta<\frac{21}{37+12\lambda}$, is always true since the right hand side is greater than $\frac{2}{7}$. Furthermore,

$$\frac{-\pi_\ell\beta(8+6\lambda)-\beta(1+2\lambda)+3}{6\beta(1-\lambda)}<\frac{1}{4}$$

if and only if

$$\pi_\ell>\frac{-\beta(5+\lambda)+6}{4\beta(4+3\lambda)}$$

which, for $\beta>\frac{18}{23+9\lambda}$, is always true since the right hand side is smaller than $\frac{1}{6}$. Note that both $b_3^*(\lambda)$ and $b_4^*(\lambda)$ are decreasing function of λ, and $p_j^*(\beta,\lambda,\pi_\ell)$ is decreasing in β, increasing in λ, and decreasing in π_ℓ. Hence, for $\pi_\ell<\pi_j$ and $\pi_j<\frac{2}{3}\pi_{\Bbbk}$, if $\beta<b_3^*(\lambda)$, then $\{\Bbbk\}$ forms the government with the external support of party j; if $\beta>b_4^*(\lambda)$, then $\{\Bbbk,j\}$ forms the government; and if $b_3^*(\lambda)<\beta<b_4^*(\lambda)$, then for $\pi_j<p_j^*(\beta,\lambda,\pi_\ell)$, $\{\Bbbk\}$ forms the government with the external support of party j, whereas for $\pi_j>p_j^*(\beta,\lambda,\pi_\ell)$, $\{\Bbbk,j\}$ forms the government. Q.E.D.

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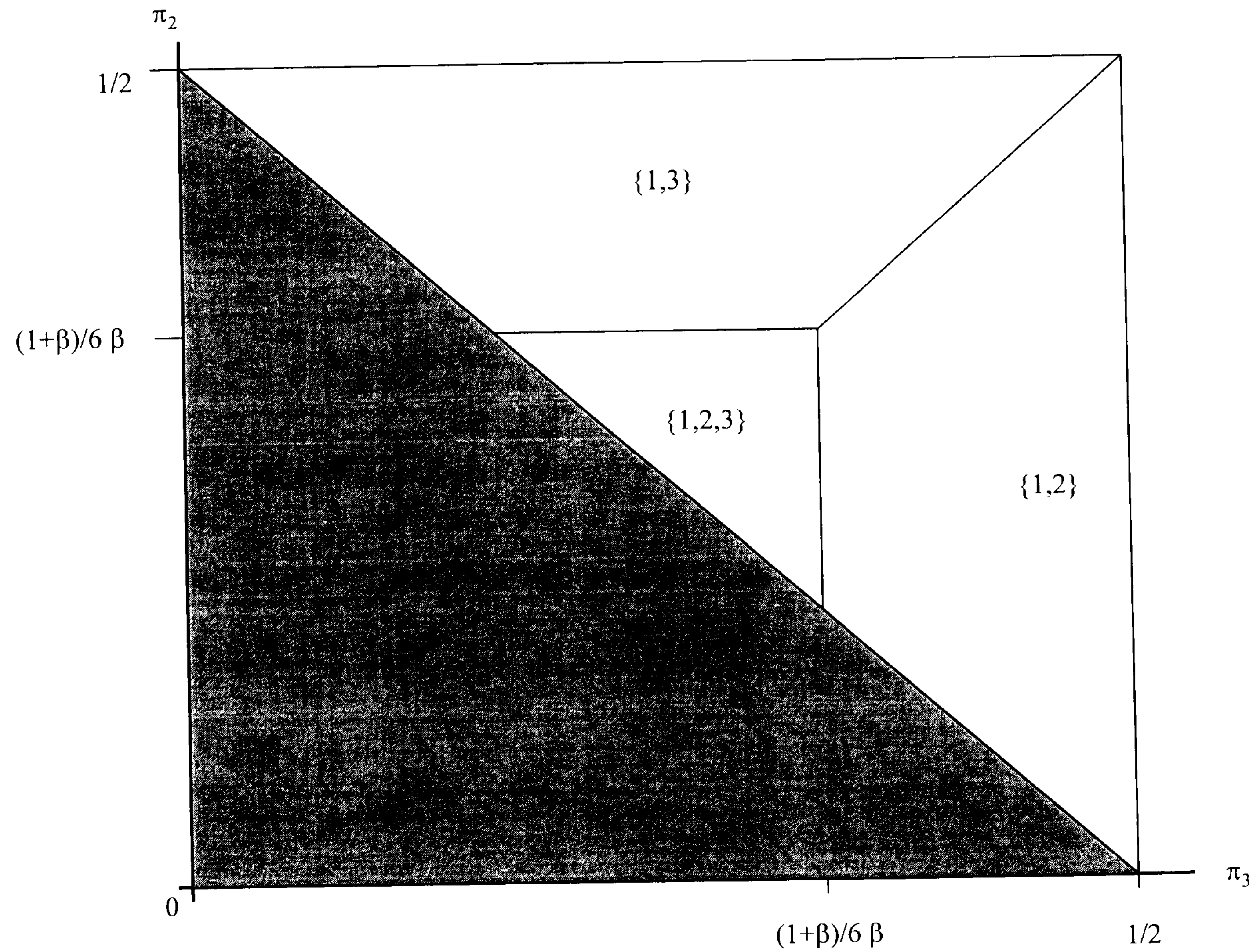
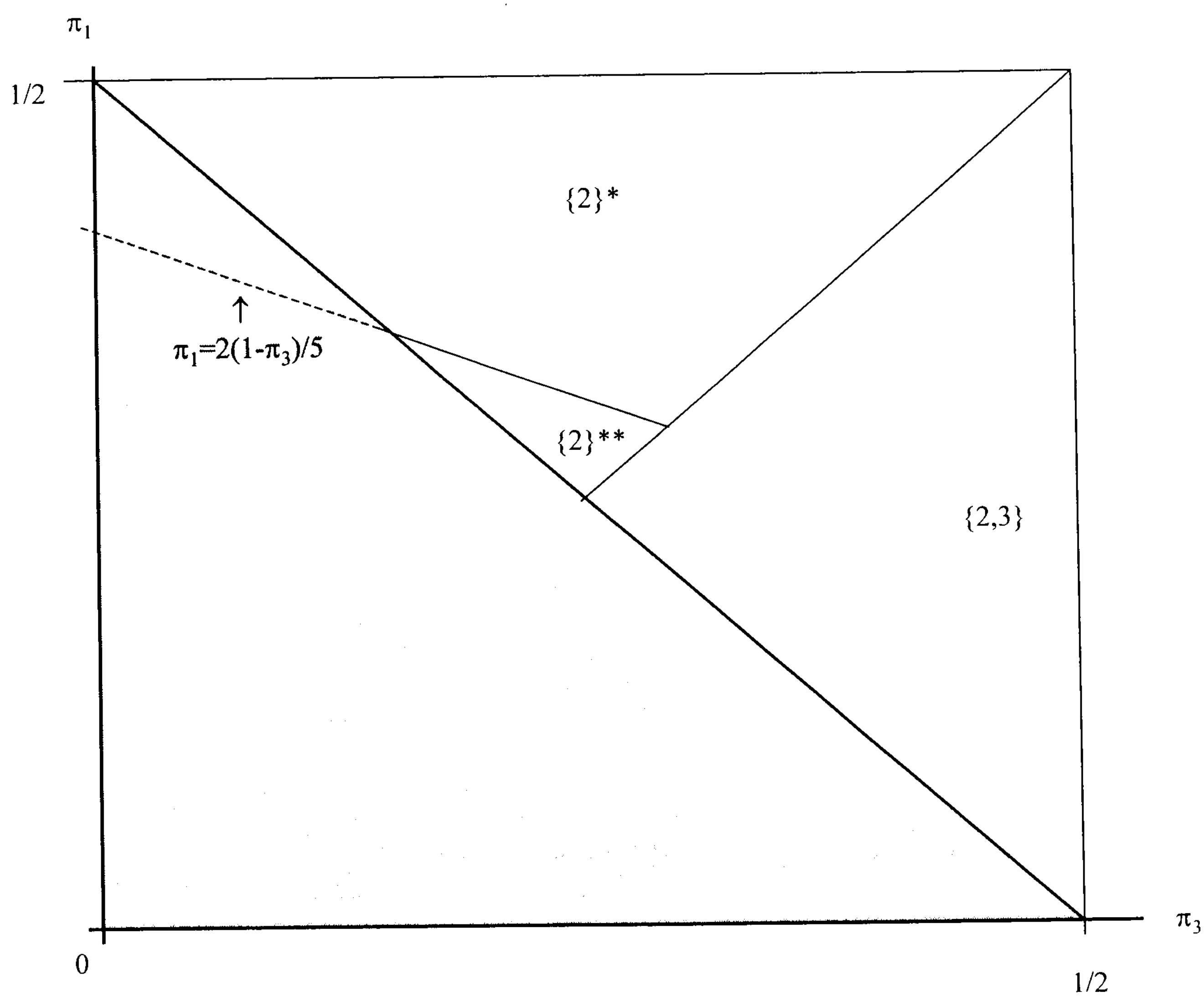
Figure 1
Coalition formation if the formateur is the favored party



In this example party 1 is the formateur, $q=z^1$, and $\beta = 1$. The area where {1,2,3} is chosen increases as β decreases until at $\beta=1/2$ it covers the entire upper triangle (i.e. the parameter space Π)

Figure 2
Coalition formation if the formateur is not the favored party



In this example party 2 is the formateur, $q=z^1$, and $\beta=1$; in the region indicated by * {2} always survives in period 2, while in the region indicated by ** {2} terminates with probability $(1-\lambda)/2$.