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The Equivalence of Price and Quantity Competition with Incentive Scheme Commitment

by

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Abstract

We consider a two stage differentiated products duopoly model (with linear demand and constant marginal cost). In the first stage profit maximizing owners choose incentive schemes in order to induce their managers to exhibit a certain type of behavior. In the second stage the managers compete either in prices or in quantities. In contrast to Singh and Vives (1984), we show that if the owners have sufficient power to manipulate the incentives of their managers, the equilibrium outcome is the same regardless of whether the firms compete in prices or in quantities. Basing the manager's objective function on a convex combination of own profit and the difference between own profit and the rival firm's profit is sufficient for the equivalence result to hold.

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1. Introduction

Singh and Vives (1984) prove the somewhat surprising result that in a differentiated products duopoly with constant marginal costs, all else being equal, the equilibrium under price competition differs from the equilibrium under quantity competition. In particular, they show that in Cournot competition[1], quantities are lower and prices are higher than in Bertrand competition, regardless of whether the goods are substitutes or complements. This difference stems from the fact that the perceived elasticity of demand when a firm takes it's rival's price as constant is larger than the perceived elasticity of demand when a firm takes it's rival's quantity as constant. In other words, the optimal reaction of the manager differs depending on whether he believes his opponent to be playing a price strategy, in which case he imagines his opponent's price to be fixed, or a quantity strategy, in which case he imagines his opponent's quantity to be fixed.

The difference between Cournot and Bertrand equilibria *for the same demand system* constitutes a major puzzle for economists seeking to understand the strategic interactions of firms. The polar case is, of course, products that are perfect substitutes. In this case we find the so-called Bertrand Paradox, that quantity competition yields positive gross margins, while price competition leads to prices at the marginal cost level. For differentiated products, price competition is generally "more aggressive" than quantity competition, yielding higher quantities and lower prices (Singh and Vives 1984).

The problem of understanding the equilibrium behavior in a duopoly market is apparently even more complicated as demonstrated by the work of Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985), who show in a variety of models that the owner of a firm may want to distort the preferences of its managers (the people responsible for making the price or quantity decision) away from pure

[1] Throughout this paper we use the term "Cournot competition" when the firms compete by setting quantities and "Bertrand competition" when the firms compete by setting prices.

profit maximization in order to commit to a certain competitive posture. Fershtman and Judd (1987) and Sklivas (1987) (hereafter FJS) consider two stage duopoly models where in the first stage profit maximizing firms choose compensation schemes for managers that are a linear combination of profits and sales. In the second stage, the managers, knowing both compensation schemes, compete in a duopoly. Sklivas shows that in Cournot competition owners will choose to have managers put less than full weight on their costs, while in price competition with differentiated, substitute goods, firms will put more than unitary weight on their costs in evaluating their managers. This is equivalent to putting positive weight on both profits and sales in the first case, and putting positive weight on profits but negative weight on sales in the second. The results of Fershtman and Judd are similar in character.

Fumas (1991) and Miller and Pazgal (1997) consider the role of relative performance evaluation using the natural variation on the FJS-style model where managers are compensated based on a linear combination of the firm's own profit and the profit of its rival. They study a two stage game where in the first stage the owners decide on the weight to be put on the profits of the rival firm and in the second stage the managers of the firms compete in a duopoly game. The equilibrium of this game is shown to have lower prices and higher quantities in the case of Cournot competition and higher prices and lower quantities in the case of Bertrand competition as compared to the equilibrium in games without ex ante commitment to relative performance. Hence, the equilibria under price and quantity competition are shifted closer together once the role of incentive scheme commitment is recognized.

In this paper we consider a general class of two-stage games in which in the first stage owners choose a vector of incentive parameters for their managers and in the second stage the managers compete (either in prices or in quantities) in a differentiated products duopoly environment with linear demand and constant marginal cost. We show that if the owners have sufficient power to manipulate the incentives of their managers, the equilibria under price and quantity competition are *equivalent.* Specifically, for any price competition (respectively quantity competition) equilibrium there exists a quantity competition equilibrium (price competition equilibrium) that induces the same equilibrium prices and quantities, although the equilibrium incentive parameters will almost certainly differ.

Section 2 describes the model and proves the equivalence result. Section 3 shows how the equivalence result holds in the relative performance model of Miller and Pazgal (1997) but does not hold in a model based on Fershtman and Judd (1987). Section 4 discusses issues regarding the in generalization of the result to nonlinear demand and incentive systems. Section 5 concludes.

2. The Equivalence of Price and Quantity Competition

While the FJS, Fumas, and Miller and Pazgal models differ in the specific mechanism by which the profit-maximizing owners of the firms commit their managers to a certain competitive postures, the two stage games they consider share a basic structure. In the second stage the managers, their incentives set by the owners in the first stage, compete in a duopoly game. The manager's optimization problem therefore defines his[2] reaction function such that for any strategy s_j played by his rival, the manger chooses the optimal strategy, s_i. Graphically, if we plot s_i on the vertical axis and s_j on the horizontal axis, the reaction function is such that the isoquants of manager i's incentive function have vertical tangents through all (s_j, s_i) on the reaction function. We will refer to this condition as the *Manager's Optimality Condition.*

In the first stage of the competition the owners choose an incentive scheme for the managers. That is, given the second stage equilibrium induced by the owners' choice of incentive parameters, the owners choose the incentive parameters in order

[2]Throughout the paper, owners will be referred to as "she" and managers as "he".

to maximize their profit. Graphically, this is equivalent to, whenever possible[3], owner i choosing the incentive parameters such that the isoprofit curve of firm i through the second stage equilibrium point is tangent to the reaction function of firm j. That is, in choosing the incentive parameters, owner i is choosing its most preferred point off of the reaction curve of manager j. We will refer to this as the *Owner's Optimality Condition.*

In this section, we show that if owners have sufficient power to manipulate the incentives of their managers, then for any equilibrium of the two-stage price setting game (respectively, quantity setting game), there exists an equilibrium of the two-stage quantity setting game (price setting game) that induces the same equilibrium prices and quantities (although the equilibrium values of the incentive parameters may differ).

Finally, we consider a two stage game. In the first stage the owner of each firm chooses a vector of incentive parameters γ_i, from a feasible set of incentive parameter values Γ_i, that will be used to determine the compensation of her own manager. Following Fershtman and Judd, when we say "owner" we mean an individual or a group whose sole purpose is to maximize the profits of the firm. "Manager" refers to an agent that the owner hires to make real time operating decisions.

At the beginning of the second stage the compensation schemes of the managers (expressed by γ_1, γ_2) are revealed and become common knowledge. Then, the two managers engage in some form of duopoly competition.

In this paper, we consider the linear differentiated products demand system given by

$$q_1 \quad = \quad a_1 - b_1 p_1 + z_1 p_2$$

[3]We say *whenever possible* since the owner may not have sufficient power to manipulate the incentives of her manager for this condition to hold. In this case, the owner selects the most preferred (highest profit) point off of her opponent's reaction curve possible, given the power to set incentives that she possesses.

$$q_2 = a_2 - b_2 p_2 + z_2 p_1.$$

Firms are assumed to have constant marginal cost equal to $c_i > 0$[4]. In the absence of incentive scheme commitment, the managers' reaction functions would be linear in such a game, regardless of whether the firms compete in prices or in quantities. We wish to retain this property, and so we restrict the incentive parameters available to the owners according to the following condition.

Linearity: The incentive parameters available to the owners are such that the managers' incentive reaction functions are linear in the strategy of the other firm.

This condition is satisfied by all of the models mentioned in the introduction. A general specification of a model satisfying the linearity condition would be one where owner i chooses weights $(\gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4}) \in \Gamma_i \subset \Re^4$ such that the manager is compensated according to the following function:

$$\alpha_i + \beta_i \left[\gamma_{i1} \left(p_i q_i\right) - \gamma_{i2} \left(c_i q_i\right) - \left(\gamma_{i3} \left(p_j q_j\right) - \gamma_{i4} \left(c_j q_j\right)\right)\right].$$

The equilibrium concept we employ is subgame perfection. That is, we require that in the second stage the managers choose a strategy that maximizes their incentive functions for any given vector of incentive parameters, taking the strategy of their opponent as given. In the first stage, we require that the owners choose the incentive parameters in order to maximize their profit, given the choice of incentive parameters of their opponent and the equilibrium that will result when the managers play the second stage game. The next subsections describes the game in greater detail.

2.1. The Second Stage

We assume that the linearity condition holds and that in the first stage the owners have selected the incentive parameters $\gamma_i \in \Gamma_i$, $i = 1, 2$. The manager's incentive

[4] In order to guarantee production of positive quantities in equilibrium we need to assume that the marginal cost of production is small enough. Specifically, we assume that the marginal cost of firm i obeys $c_i < \frac{b_j a_i - z_j a_j}{b_i b_j - z_i z_j}$ where $j \neq i$.

function $M_i(\gamma_i, s_i, s_j)$ depends on the incentive parameters chosen by his own owner, the strategy the manager chooses, as well as the strategy chosen by his opponent. In price competition, the strategic variables are prices, while in quantity competition, they are quantities. The manager's reaction function $R_i(\gamma_i, s_j)$ which maps pairs (γ_i, s_j) to strategies s_i , is the solution to

$$R_i(\gamma_i, s_j) \in \arg\max_{s_i} M_i(\gamma_i, s_i, s_j).$$

Since $M_i(\gamma_i, s_i, s_j)$ is differentiable, this is equivalent to $R_i(\gamma_i, s_j)$being such that $\frac{\partial M_i(\gamma_i, R_i(\gamma_i, s_j), s_j)}{\partial s_i} = 0$ and the appropriate second order conditions for a maximum hold.

A **second stage equilibrium** is a vector (s_1^*, s_2^*) such that $s_i^* = R_i\left(\gamma_i, s_j^*\right)$, $i = 1, 2$, $j \neq i$. That is, a second stage equilibrium is a pair (s_1^*, s_2^*) of strategies such that the reaction functions of the two managers intersect. An (second stage) **equilibrium outcome** under (s_1^*, s_2^*) is a price $p^* = (p_1^*, p_2^*)$ and quantity $q^* = (q_1^*, q_2^*)$ such that p^* and q^* are the price and quantity that occur when the equilibrium pair (s_1^*, s_2^*) are played.

2.2. The First Stage

In the first stage of the game, owners choose the incentive parameters in order to maximize their profits given the choice of incentive parameters of their opponents and the equilibrium these will induce in the second stage. That is, owner i chooses γ_i^* in order to solve the following program

$$\begin{aligned} \gamma_i^* &\in \arg\max_{\gamma_i} (p_i^* - c_i)\, q_i^* \\ s.t.\ s_j^* &= R_j\left(\gamma^{j*}, s_{-j}^*\right)\ \ j = 1, 2 \\ &\quad \text{and } p^*, q^* \text{ is the equilibrium outcome under } (s_1^*, s_2^*)\ . \end{aligned}$$

Since the profit function is differentiable, the owner's choice of γ_i^* is equivalent to her choosing γ_i^* in order to shift her own reaction function so that it intersects with the opposing manager's reaction function at the point along the opponent's reaction function that maximizes its profit. If the owner has sufficient power to manipulate the reaction function of its manager, then this amounts to choosing the point along its opponent's reaction function such that its profit isoquant is tangent to the opponent's reaction function at that point. With this graphical intuition in place, we are ready to state the main result.

2.3. The Main Result

We begin by considering conditions under which there is a corresponding equilibrium in the two stage quantity competition game that has the same equilibrium outcome as a given equilibrium of the price setting game. As noted above, the tangency condition in the first stage will only hold if the owners have sufficient power to choose the incentives of their managers. Specifically, the owners must have enough power to shift the reaction curve of their own managers to the point along their opponent's reaction curve where their own profit is maximized, i.e. their profit isoquant is tangent to the reaction curve of their opponent. For the reminder of the section we assume the owner's have at least this much power.

(TPE) Tangency in Price Equilibrium: *Assume that in any price equilibrium, owners have enough control over their manager's incentives in the price setting game that the profit isoquant of owner i is tangent to the reaction curve of manager j at the equilibrium p^* for all $i \neq j$.*

For example, power to manipulate either the slope or the intercept of manager $i's$ reaction curve over a broad enough range will be sufficient to satisfy (TPE).

Any point in the (q_1, q_2) space can be mapped to a point in the (p_1, p_2) space using the equations characterizing the demand system. This identifies the quantity (q_1, q_2) with the corresponding prices (p_1, p_2) that would clear the market and cause these quantities to be sold. Since the demand system is linear, lines in the quantity space map into lines in the price space and vice versa. So, the reaction function resulting from quantity competition $R_i^q\left(\gamma_i^{q*}, q_{-i}^*\right)$which resides in the quantity space can be translated into the price space by applying the transformation given by the demand system.

Consider the following condition:

(CQI) Control of Quantity Incentives: *Owners 1 and 2 have sufficient power to manipulate the incentives of their managers so that if p^* is the equilibrium outcome price under price competition, then there exist incentive parameters γ_i^{q*} such that the quantity competition reaction function $R_i^q\left(\gamma_i^{q*}, q_{-i}^*\right)$, when transformed into the price space, is identical to the price competition reaction function $R_i^p\left(\gamma_i^{p*}, p_{-i}^*\right)$ that induces the p^* equilibrium outcome.*

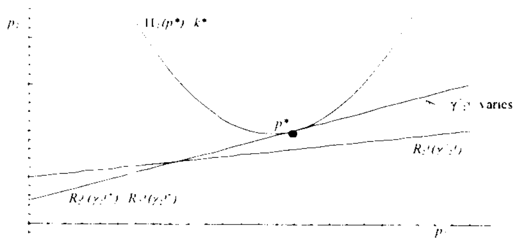
Assumptions (TPE) and (CQI) are sufficient for the first half of the equivalence result.

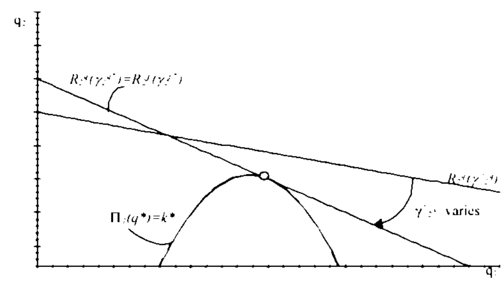
Theorem 1: *Let (p^*, q^*) be the second stage equilibrium outcome that results from price competition. If (TPE) and (CQI) hold, then there is a corresponding equilibrium in the two stage quantity competition game that induces (p^*, q^*) as an equilibrium outcome.*

Proof: Consider the two stage price competition game. At the equilibrium, the owners choose incentive parameters $(\gamma_1^{p*}, \gamma_2^{p*})$ such that the managers' reaction functions intersect at p^*. Since (TPE) holds, at this point the reaction curve of manager i is tangent to the profit isoquant of owner j, $i \neq j$. If (CQI) holds, there exist γ_i^{q*} such that the quantity reaction curves defined by γ_i^{q*}, when projected into the price space, intersect at p^* and coincide with the price reaction curves.

Consider the quantity reaction curves defined by γ_i^{q*}, $i = 1, 2$, in the quantity space. Since they coincide with the equilibrium price reaction curves in the price space, and the equilibrium price reaction curves intersect at p^* in the price space, the quantity reaction curves defined by γ_i^{q*}, $i = 1, 2$, intersect at q^* in the quantity space. Thus, the Manager's Optimality Condition is satisfied at q^* for both managers. Furthermore, the fact that the profit isoquant of owner i, $i = 1, 2$, is tangent to the reaction curve of manager $j \neq i$ in the price space implies that the tangency still holds in the quantity space, since the tangent is just the slope of the quantity reaction function in the price space and both are translated to the quantity space via the demand system. Thus γ_i^{q*} is an optimal incentive parameter choice for owner i, $i = 1, 2$, implying the Owner's Optimality Condition is satisfied. Therefore, γ_i^{q*} are equilibrium choices of the incentive parameters which induce p^*, q^* as the equilibrium outcome. ■

The intuition the proof of Theorem 1 is relatively straightforward and is illustrated in Figures 1a and 1b. The proof shows that if there exists a γ_i^{q*} such that the quantity reaction curve, when translated into the price space, corresponds with the price reaction curve at the equilibrium γ_i^{p*}, then the geometric relationships that make γ_i^{p*} satisfy the Manager's and Owner's Optimality Conditions translate into the price space in a manner that preserves these relationships. If the Manager's Optimality Condition and Owner's Optimality Condition hold in the quantity space, then γ_i^{q*}induces an equilibrium of the quantity setting game. Hence γ_i^{q*} induces an equilibrium in the quantity game that has the same equilibrium outcome as the γ_i^{p*} equilibrium in the price game.





Theorem 1 shows that (TPE) and (CQI) holding simultaneously are sufficient for there to be a quantity equilibrium outcome corresponding to any price equilibrium outcome. We can state the analogous assumptions about tangency of the quantity equilibrium, (TQE) and control of the price incentives, (CPI). These assumptions will be sufficient for the symmetric result about equilibrium outcomes of the quantity competition.

Theorem 2: *Let (p^*, q^*) be the second stage equilibrium outcome that results from quantity competition. If (TQE) and (CPI) hold, then there is a corresponding equilibrium in the two stage price competition game that induces (p^*, q^*) as an equilibrium outcome.*

Proof: Mutatis Mutandis as theorem 1.■

Theorems 1 and 2, taken together, provide sufficient conditions for the equivalence of the set of equilibrium outcomes under price and quantity competition.

Corollary 1: *Suppose (TQE), (TPE), (CQI), and (CPI) hold. Then the set of equilibrium outcomes under the two stage price and quantity competition are identical.*

Proof: Follows directly from Theorems 1 and 2.

For the remainder of the paper, we will use (TE) to refer to the case when (TQE) and (TPE) hold, and (CI) to refer to the case when (CQI) and (CPI) hold.

The assumptions (TE) and (CI) explicitly state conditions under which owners have sufficient power to commit their managers to a certain type of behavior. The (TE) assumptions require that owners have sufficient control over incentives *for a given type of competition*, and that they can choose their favorite point off of their opponent's reaction curve through their choice of incentive parameters. The (CI) assumptions require that owners have sufficient control over the incentives of their managers in one type of competition (say quantity) to get them to mimic the behavior they exhibit in an equilibrium of the other type of competition (say price). Thus the (CI) assumptions, would generally require owners to exercise control over both the slope and the intercept of the manager's reaction curves, while the (TE) assumptions seem to require control only over the slope or the intercept.

Theorems 1 - 2 and Corollary 1 state that if the owners have sufficient control over incentives in the sense of (TE) and (CI), then the equilibria are equivalent. This is somewhat paradoxical, since one's initial reaction is to say "if they have that much control, can't they do even better?" However, the logic of the proof shows where this intuition fails. We start by taking an equilibrium in one type of two-stage competition as given. We then show that if assumptions (TE) and (CI) hold, then the owners can replicate the equilibrium conditions (tangency of the isoprofit to the other manager's reaction curve, and the equilibrium price being at the intersection of the reaction curves) in the other type of competition by manipulating the incentives of the managers. By starting with an equilibrium in one type of competition and replicating it in the other type of competition, we are not trying to identify the absolute best outcome for the owners. Rather, we are asking whether the conditions that make the original equilibrium an equilibrium can be replicated in the other type of competition. If (TE) and (CI) hold, the answer to this question is yes.

For another approach, consider the following. The reason why simple price and quantity competition (with no incentive scheme commitment) fail to have the same equilibrium is that the incentives given to a manager when he takes his opponent's price as fixed in choosing a strategy are different than the incentives given to a manager when he takes his opponent's quantity as fixed. However, we assume that owners here have sufficient power over incentives (reaction functions) to mitigate these differences. When the reason why the equilibria differ is removed by admitting incentive scheme commitment, they coincide.

3. Applications

In this section we consider two different types of incentive scheme commitment mechanisms. First, we consider the model of Miller and Pazgal (1997), where in the first stage owners commit their managers to a specific attitude toward relative performance and in the second stage the managers compete in either prices or quantities. We show that owners having power to control the weight managers give to their performance relative to the competition is sufficient for (TE) and (CI) to hold. Consequently the equilibria under price and quantity competition are identical.

Second, we consider a model based on Fershtman and Judd (1987), where owners choose the weight managers put on revenues. We show that in this case (CI) fails to hold, and thus the equilibria do not coincide.

3.1. Commitment to Relative Performance

In the model of commitment to an incentive scheme based on relative performance suggested by Miller and Pazgal (1997), owners give managers the incentive to maximize not only the firm's own profit in isolation, but managers are also compensated based on their performance relative to the competitors. This is equivalent to having incentives to maximize a convex combination of own profit and the difference between own profit and the rival's profit.

The demand structure we will use is the following symmetric one:

$$q_1 = 1 - p_1 + zp_2 \tag{3.1}$$

$$q_2 = 1 - p_2 + zp_1. \tag{3.2}$$

In this demand structure we have suppressed all of the parameters except for the differentiation parameter z. Furthermore, we will assume that firms have zero marginal cost[5].

We begin by deriving the equilibrium where the managers compete in prices in the second stage. We let θ_i^p be the weight owner i places on the profit of the other firm in the manager's compensation scheme. In this environment, the managers' objective functions are given by

[5] We make these assumptions for expositional purposes. All of the results hold for a general linear demand structure with (possibly different) constant marginal costs.

$$m_i^p = (1 - p_i + zp_j)\, p_i - \theta_i^p (1 - p_j + zp_i)\, p_j, \; i = 1, 2, \; j \neq i.$$

The corresponding reaction functions are

$$p_i(p_j) = \frac{1}{2}\left(1 + z(1 - \theta_i^p)\, p_j\right), \; i = 1, 2, \; j \neq i.$$

and the second stage equilibrium prices are thus

$$p_i = \frac{-z + \theta_i^p z - 2}{-4 + z^2 - \theta_i^p z^2 - \theta_j^p z^2 + \theta_j^p z^2 \theta_i^p}, \; i = 1, 2, \; j \neq i.$$

Letting x_i be the profit earned by firm i, the owner's objective functions are given by

$$x_i = (-z + \theta_i^p z - 2)\frac{-\theta_i^p z^2 + \theta_j^p z^2 \theta_i^p - \theta_i^p z - z - 2}{\left(-4 + z^2 - \theta_i^p z^2 - \theta_j^p z^2 + \theta_j^p z^2 \theta_i^p\right)^2}, \; i = 1, 2, \; j \neq i. \quad (3.3)$$

This yields the owners' reaction functions

$$\theta_i^p = \left(-1 + \theta_j^p\right)(z + 2)\frac{z}{-z^2 + \theta_j^p z^2 - 2\theta_j^p z + 2z + 4}, \; i = 1, 2, \; j \neq i. \quad (3.4)$$

The system of equations defined by (3.4) has two solutions, $\left\{\theta_1^p = \frac{z+2}{z}, \theta_2^p = \frac{z+2}{z}\right\}$, and $\left\{\theta_1^p = \frac{z}{z-2}, \theta_2^p = \frac{z}{z-2}\right\}$. However, the $\left\{\theta_1^p = \frac{z+2}{z}, \theta_2^p = \frac{z+2}{z}\right\}$ solution does not satisfy the second order conditions, and we discard it. The $\left\{\theta_1^p = \frac{z}{z-2}, \theta_2^p = \frac{z}{z-2}\right\}$ solution yields the following equilibrium:

$$\begin{aligned} q_1^p &= q_2^p = \frac{1}{4}z + \frac{1}{2}, \\ p_1^p &= p_2^p = \frac{1}{4}\frac{z-2}{z-1}, \\ x_1^p &= x_2^p = \frac{1}{16}\frac{z^2-4}{z-1}, \\ \theta_1^p &= \theta_2^p = \frac{z}{z-2}. \end{aligned}$$

Direct computation of the equilibrium when the managers compete in quantities shows that the same equilibrium prices and quantities obtain, with the owners putting weight $\theta_i^q = \frac{z}{z+2}$, $i = 1, 2$,on the profits of the rival firm in the objective function of the managers. We now show how the logic of the proof of Theorem 1 can be used to demonstrate this equivalence.

Inverting the demand system given by (3.1) and (3.2) yields the inverse demand system

$$\begin{aligned} p_1 &= \frac{q_1 - 1 + zq_2 - z}{z^2 - 1} \\ p_2 &= \frac{q_2 - 1 + zq_1 - z}{z^2 - 1}. \end{aligned}$$

Letting θ_i^q be the weight assigned to the profit of firm j by firm i , the manager's objective functions when the firms compete in quantities are given by:

$$m_i^q = \left(\frac{q_i - 1 + zq_j - z}{z^2 - 1}\right) q_i - \theta_i^q \left(\frac{q_j - 1 + zq_i - z}{z^2 - 1}\right) q_j, \ i = 1, 2, \ j \neq i.$$

Differentiating yields reaction functions

$$q_i = \frac{1}{2}\left[1 + z + z\left(\theta_i^q - 1\right) q_j\right], \ i = 1, 2, \ j \neq i.$$

Next, we use the demand system to translate the quantity reaction curves to the price space. Manager i's reaction curve maps to:

$$p_i = \frac{1 + zp_j - \theta_i^q z + z\theta_i^q p_j}{2 + \theta_i^q z^2 - z^2}, \ i = 1, 2, \ j \neq i. \tag{3.5}$$

In price competition, the equilibrium prices are given by $p_1^p = p_2^p = \frac{1}{4}\frac{z-2}{z-1}$. Substituting these values into (3.5) and solving for θ_1^q and θ_2^q yields $\theta_1^q = \theta_2^q = \frac{z}{z+2}$. Hence, when $\theta_1^q = \theta_2^q = \frac{z}{z+2}$, the translated quantity reaction curves intersect at the price equilibrium prices $p_1^q = p_2^q = \frac{1}{4}\frac{z-2}{z-1}$. In the quantity space, they intersect at $q_1^q = q_2^q = \frac{1}{4}z + \frac{1}{2}$. Since these quantities lie on the reaction curves of the managers, they form a second stage equilibrium of the game. Thus in quantity competition when $\theta_1^q = \theta_2^q = \frac{z}{z+2}$ the Manager's Optimality Condition holds at the equilibrium prices and quantities from the price competition.

It remains to show that $\theta_1^q = \theta_2^q = \frac{z}{z+2}$ form an equilibrium of the first stage of the game. To do this, we must verify that the Owner's Optimality Condition holds. The slope of (3.5) is given by $z\frac{1+\theta_1^q}{2+\theta_i^q z^2 - z^2}$. The slope of manager i's equilibrium price reaction curve is given by $\frac{1}{2}z\left(1-\frac{z}{z-2}\right)$. We set these equal and solve for θ_i^q in order to find the value of θ_i^q that makes the slope of the translated quantity reaction curve the same as the optimal price reaction curve. This yields $\theta_i^q = \frac{z}{z+2}$. Hence the Owner's Optimality Condition holds as well, and $\theta_1^q = \theta_2^q = \frac{z}{z+2}$ form an equilibrium for the two stage quantity competition game that induces the same prices and quantities as the two stage price competition game. Thus the ability of owners to control the weight their managers put on relative performance is sufficient to satisfy (TE) and (CI) and as a result there is a quantity competition equilibrium that induces the same outcome as the price competition equilibrium. Since the same argument shows there exists a price competition equilibrium corresponding to any quantity competition equilibrium, the equilibria under price and quantity competition coincide.

It is interesting to note that while in general (CI) will require control over the slope and intercept of the reaction curves, this example shows that it is not necessary to have *independent* control over both slope and intercept of the reaction curves.

3.2. A Fershtman and Judd Style Model

In the example of the previous section, ex ante commitment to compensate managers based on relative performance provided sufficient power to control incentives for the equivalence result to hold. One may wonder if a similar result holds in a Fershtman and Judd style model, where owners commit their managers to behavior that maximizes a linear combination of profits and sales. The answer to this question is no.

Fershtman and Judd consider a model where managers maximize

$$a_i \pi_i + (1 - a_i) R_i$$

where π_i is the profit earned by firm i and R_i is its revenue. Each firm is assumed to have constant marginal cost equal to c_i, and we assume the cost is small enough to allow for the existence of equilibria. Consider the simplified linear demand system of the previous section, and focus on firm 1. (The symmetric arguments hold for firm 2.) In price competition managers seek to maximize:

$$(1 - p_1 + zp_2)(p_1 - a_1^p c_1)$$

Differentiating this with respect to p_i yields price reaction curve

$$p_1 = \frac{1}{2}(1 + a_1^p c_1 + zp_2).$$

which has slope $\frac{1}{2}z$.

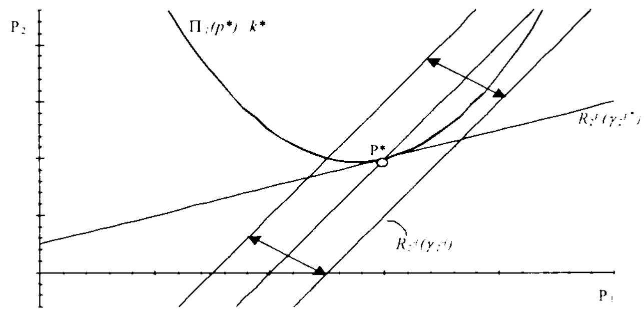
The reaction curve for manager 1 in the quantity game is given by:

$$q_1 = \frac{1}{2}\left(1 - a_1^q c_1 + z + a_1^q c_1 z^2 - zq_2\right).$$

Transforming the above result to the price space yields:

$$p_1 = -\frac{\frac{1}{2} + \frac{1}{2}zp_2 + \frac{1}{2}a_1^q c_1 - \frac{1}{2}a_1^q c_1 z^2}{-1 + \frac{1}{2}z^2}. \tag{3.6}$$

The slope of this curve is always equal to $\frac{z}{-2+z^2}$. Since this is independent of a_i^q, there is no value of a_i^q that will make the slope of (3.6) equal $\frac{1}{2}z$, the slope of the price reaction curve. Since changing the incentive parameters can only shift the reaction curve in a parallel manner but cannot affect the slope, (CQI) fails to hold, and there is no equilibrium in the quantity game that induces the same equilibrium outcome as the equilibrium of the price game (see figure 2). Further computation verifies that the same problem causes (CPI) to fail, and direct computation confirms that the unique equilibria in the two games are, in fact, different.

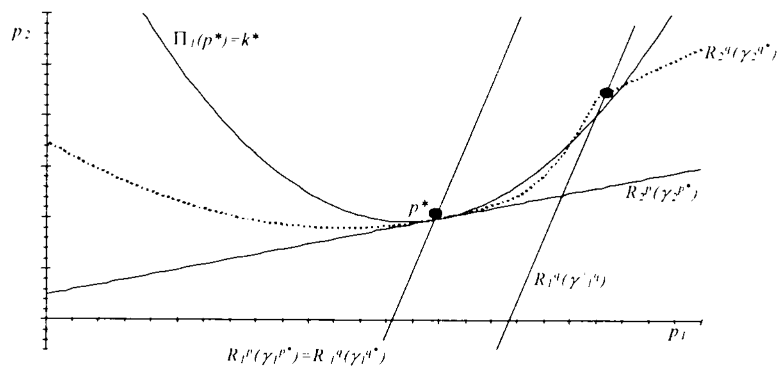


4. Generalizations

The model presented here considers only the special case of linear demand and constant marginal cost. However, the intuitive argument presented above suggests that the results hold in a broader class of demand structures. Since the crucial step in the argument involves having "sufficient power to manipulate the reaction curves of the managers" such that the quantity reaction curves (in the price space) pass through the equilibrium prices from the price competition and are tangent to the profit isoquants of the other firms at this point, it suggests that all that is needed for the argument to go through is for the profit isoquants and reaction functions to be differentiable at the equilibrium prices.

However, when we depart from the world of reaction curves that are linear under all possible values of the incentive parameters, the result presented here might fail to hold. The intuition is illustrated in figure 3. It may be possible for the quantity isoquants, when translated to the price space, to intersect at the price equilibrium and be tangent to the profit isoquant of the other firm at the equilibrium price, but to also cross the profit isoquant away from the equilibrium. In this case, there will be a point along the quantity reaction curve that yields a higher profit than the price equilibrium. Hence this price vector and the corresponding quantity vector will not generally be an equilibrium outcome in the quantity game.

On the other hand, the results presented here easily generalize to oligopolies with more than two firms. The second stage optimality condition and first stage tangency condition do not depend on the competition taking place in two dimensions for their validity. Thus the arguments presented here hold in linear differentiated products oligopolies with any number of firms.



5. Discussion

In this paper we prove that for linear demand differentiated products duopolies with constant marginal cost, the equilibrium outcomes under price and quantity competition coincide if owners have sufficient power to control the incentives of their managers. We show how a two stage model where in the first stage owners choose the weight that their managers will give to their performance relative to that of their rivals and in the second stage managers compete with these incentives is sufficient for the equivalence result to hold. While a model where owners control only the relative weight managers give to profits and sales is not sufficient, since owners do not have the power to manipulate the slope of their managers' reaction curves.

What do these results mean for the owners and managers in an industry and for economists trying to understand them? For economists, these results mean that if owners have power to control the incentives of their managers, we should expect that they will use it to mitigate the differences between price and quantity competition, making price competition less aggressive and quantity competition more aggressive. Thus, from the point of view of economists, once incentive scheme commitment is recognized, the differences between price and quantity competition equilibria are expected to be less pronounced than the original economic models predict.

From the point of view of owners and managers in an industry, the results presented here do not imply that they need not be concerned with whether they are competing in prices or quantities. As the relative performance example shows, the sign of the optimal weight put on relative performance will generally depend on whether the firms compete in prices or quantities. Thus, it is crucial for owners to know what kind of strategies their managers fundamentally employ, since they cannot design an optimal incentive scheme without such knowledge.

However, while owners and managers still need to know whether they compete in prices or quantities, incentive scheme commitment implies that in terms of profits, the firms should be indifferent between an industry where the firms compete in prices and one in which they compete in quantities, since once this is known firms will adjust the incentives of the managers accordingly, and the ultimate equilibrium outcome will be the same regardless of the type of competition.

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