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DYNAMICS OF PARLIAMENTARY SYSTEMS: SELECTIONS, GOVERNMENTS, AND PARLIAMENTS

David P. Baron and Daniel Diermeier

Stanford University and Northwestern University

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Abstract

This paper presents a theory of parliamentary systems that incorporates electoral, government formation, and legislative institutions and focuses on the strategic opportunities inherent in those institutions. The electoral system is proportional representation, and a party is selected as formateur based on its representation in parliament. Parties are assumed to be unable to commit credibly to the policies they will implement once in government. Since the policy chosen by a government in one period becomes the status quo for the next period, a current government can strategically position the status quo to affect both the outcome of the next election and subsequent government formation. When parties have both policy and officeholding preferences, elections are not moderating; i.e., they do not contribute to policy centrality or stability. Policies can be outside the Pareto set, and governments as well as policies change with each election. Those governments are formed by minimal winning coalitions.

Dynamics of Parliamentary Systems: Elections, Governments, and Parliaments

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In many parliamentary systems governments and their policies change frequently. Budge and Keman (1990), for example, found that the average duration of governments was 1.9 years in nineteen countries over the 1950-1983 period. These changes in governments, whether as a result of a regularly-scheduled election or a government failure, often are accompanied by changes in policies. Although parliament chooses and implements policies, the government with the support of a parliamentary majority can in principle choose any policy it pleases. The government's control over policy choice is the consequence of two features of parliamentary systems, one institutional and the other operational. The institutional feature is that since parliament selects the government, the executive and a parliamentary majority are closely aligned. The operational feature is that the government controls the legislative agenda.[1] In such a system citizens' control is through elections. Elections, however, also provide an incentive to position current government policy to improve a government party's electoral prospects.

In the context of a model of such an unconstrained parliamentary system that includes elections, government formation, and legislation, this paper explores explanations for government and policy change based not on exogenous forces but instead on the strategic opportunities present in electoral competition and government formation. The focus is on both governments and their characteristics and on the policies chosen by those governments. With respect to governments, the issues addressed are whether governments are minimal winning coalitions, single-party, surplus, or consensus and whether those governments are formed by parties that have strong or weak bargaining positions. With respect to policies, the focus is on the incentives underlying the selection of multi-dimensional policies and on

[1] Silk and Walters (1995, p. 109) note that in the United Kingdom almost all government bills are enacted, and in 1992-3 all were passed. Baron (1998), Diermeier and Feddersen (1996), and Huber (1996) model agenda formation in parliamentary systems as controlled by the government.

their stability. The issue of stability is intertwined with elections and the strategies of parties in anticipation of elections and subsequently in government formation and legislative choice. For example, does the prospect of an election contribute to policy stability or does a government party select policies that advantageously position itself for the next election — and disadvantage opposition parties? That is, do elections in proportional representation parliamentary systems contribute to policy stability and the adoption of moderate policies?

In parliamentary systems, policy is the result of strategies implemented in electoral, government formation, and legislative institutions. Governments themselves result from a formation process that depends on the representation of parties in the parliament as determined by elections in which voters anticipate the policies that would be chosen by the governments that might form. A political equilibrium encompasses these considerations and consists of a legislative equilibrium, a government formation equilibrium, and an electoral equilibrium. This paper characterizes such a political equilibrium for a parliamentary system with a proportional representation electoral system and a government formation process in which a formateur is selected based on party representation in parliament where parties have both policy and officeholding preferences. A political equilibrium is determined relative to the status quo, so the policy implemented by a government in one period becomes the status quo for the following period. This introduces a natural dynamic to the political equilibrium, where the strategies of both voters and parties depend on the status quo and on the governments and policies anticipated in the future.

The issues addressed include the performance of parliamentary systems in terms of policies and their stability, governments and their continuity, and elections and how they represent voter preferences. As an example, consider the role of elections and political parties. The government, in principle, can implement any policy it wants provided the government parties agree on it. This suggests that substantial changes in policies would result when one government replaces another. Elections could moderate these policy swings, since parties that choose extreme policies may be punished by voters in the next election. This might be expected to discipline government parties and lead them to choose more centrally-located policies. This, however, is not the case in our model. If parties cannot credibly commit to the policies they will implement, elections need not be moderating. Instead, elections provide an opportunity for a current government to position the status quo to advantage at least one government party in the next election and to disadvantage the opposition party. When a government party strategically positions the status quo to its advantage, the government in the next period and its policy will differ from the current government and policy. Moreover, the strategically-chosen policies are more extreme than the subsequent policies. These results point to the importance of credible policy commitments by parties. How political parties generate and maintain credibility thus should be a focus of future research on parliamentary systems.

Government formation and policy choice are the result of bargaining among parties. The dimensions over which the parties bargain are of two types. One is policy such as defense, health care, international trade, privatization and regulation, and foreign policy. The other is transfers that help achieve bargains on governments and policies. The transfers can go in two directions. The party attempting to form a government may offer transfers to a potential coalition member or it may seek a transfer from a potential coalition member in exchange for policy concessions to that member.

In a majority-rule institution in which agenda control matters, government formation and policy choice depend on the status quo (Romer and Rosenthal 1977). Since the status quo policy remains in place if the parties fail to form a government, the status quo affects the bargaining positions of the parties. If the choice space is unidimensional, and the agenda setter is chosen probabilistically each period, status quo dependence is mitigated and the policy moves toward the core, the median legislator's ideal point (Baron 1996). In multi-dimensional policy spaces, however, the core in general does not exist, and thus policy would not be expected to converge.

The bargaining theory of Baron and Ferejohn (1989) is multidimensional and pertains to purely distributive policies; i.e., the distribution of benefits among legislative districts or constituents. The predicted outcomes are strongly majoritarian, and the distribution depends importantly on which player is the agenda-setter. This is thus a theory of fluidity in legislative choice. In the parliamentary model considered there, policies are fluid for two reasons. First, electoral competition determines the representation of parties which affects which party will be selected as formateur. Second, even if the formateur was in the previous government, change can occur if the formateur can strike a better bargain with a different government partner.

Austen-Smith and Banks (1988) provide a theory of government formation and unidimensional policy choice in a parliamentary system in which there is no status quo.[2] In their model, parties bargain over governments, and the formateur chooses a unidimensional policy and provides nonnegative transfers among the parties. This paper takes this perspective several steps further by considering multi-dimensional policies, by not restricting transfers among the parties, and providing an explicitly dynamic theory of policy choice.

The model incorporates a two-dimensional policy space with three parties. The parties have preferences over both policy and officeholding, and in the basic model their ideal policies are symmetrically located in the policy space. The electoral system is proportional representation with the formateur selected by a monarch or president based on the representation of parties in parliament. Parties are assumed to be unable to commit to the policies they will implement if in government, so citizens vote based on the governments and policies they anticipate. Governments are formed through a bargaining process in which the formateur makes a proposal consisting of a government, a policy to be implemented once the government has been formed, and transfers among the coalition partners. The formateur seeks the best bargain possible and the other parties accept or reject the proposal based on their reservation opportunities. If government formation is unsuccessful, a caretaker government maintains the status quo until the next election.

Strategic models of elections in spatial environments are usually considered under two assumptions about candidate behavior. The traditional approach based on Downs (1957) is to assume that a candidate's objective is to win elections. Austen-Smith and Banks' model of multi-party competition with proportional representation is based on this premise. An alternative approach is to assume that candidates care also, or exclusively, about policy outcomes. Wittman (1977, 1983) and Calvert (1985) analyze the consequences of this assumption in two-candidate elections. Assuming some degree of policy motivation seems especially appropriate in closed-list electoral systems or if candidates are members of disciplined parties.[3] In both cases electoral competition may best be viewed as competition

[2] They assume that if each of the three parties fails to form a government a caretaker government takes office and all parties have zero utility. This reflects the parties' officeholding preferences. If the parties had policy preferences, the caretaker government would have to provide transfers to the parties to make them all equally well-off.

[3] In closed-list systems the electorate casts its ballot not for individual candidates but

among parties rather than individual candidates. Associated with policy motivation is a credibility problem. A party could strategically announce a platform intended to increase its seat share in parliament, but once in office, it has an incentive to act according to its policy preferences rather than its announced platform. Rational voters will recognize this incentive and base their votes on the party's policy preferences rather than its announced platform.[4]

One institution not incorporated in the model is a government termination procedure such as a confidence motion or voluntary dissolution. A confidence motion may be initiated by the government to maintain coalition discipline (Huber 1996) or by the opposition to force dissolution of the government. Government termination must result from exogenous changes that are not fully taken into account *ex ante* and hence which generate *ex post* action by the parties. For example, in Baron (1998), Diermeier and Feddersen (1996), and Lupia and Strom (1995) uncertainty about reservation values can result in government termination.[5] In the model considered here there is no uncertainty, and hence there is no loss in generality in ignoring government termination procedures.

The theory generates results with respect to government formation and policy choice, electoral competition and representation, and policy and government stability. With respect to government formation the following results obtain. First, governments are formed based on bargaining positions and preference alignment. A formateur forms a government with that other party which is the more disadvantaged by the status quo, provided that that other party's ideal point is not too distant from that of the formateur. Second, if no party has a majority of seats, majoritarian (minimal winning) governments result when the status quo is not extreme. Consensus governments that include all parties occur only if the status quo is extreme. Consensus governments thus are more likely after a crisis or a shock that causes the status quo to be extreme relative to party preferences. Third, unless the status quo is favorable to the other parties, a party that obtains a majority in an election will form a consensus or a surplus government rather than a single-party government.

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for party lists. In systems with disciplined parties, the party leadership can enforce the party line by threats of deselection or withholding campaign funding.

[4] In a repeated voting game credible platform announcements can be sustained endogenously as subgame-perfect equilibria (Alesina 1988).

[5] Huber (1997) and Smith (1996) provide models of government termination incorporating incomplete information about pivotal voters.

With respect to policy choice, the results include the following. First, for a majoritarian government or a super-majority government the policy maximizes the aggregate utility of the government parties. Second, for a consensus government that includes all parties, the policy is at the centroid of the parties' preferences. Third, the formateur makes policy concessions to its government partner(s) in exchange for transfers.

With respect to the electoral equilibrium, the results for a single-period model include the following. First, for each status quo in the proportional representation system there exists a unique strong Nash equilibrium outcome. Second, some voters do not vote for the party closest to their ideal point but instead vote based on the distribution of possible government policies. Representation in parliament thus need not closely reflect voter preferences. Third, when the parties will form majoritarian governments, two parties have an implicit pre-election coalition in which if selected as the formateur each will form a government with the other. Third, when the parties will form majoritarian governments, the electoral equilibrium has one large and two small parties with the large party the one that is the most advantaged by the status quo. Fourth, depending on the status quo a party can receive a majority of the votes in an election. Fifth, if the status quo is extreme, each party will form a consensus government, and hence they split the vote.

With respect to dynamics, if parties and voters are myopic, governments change each period. After the first period no party represented in parliament receives a majority of seats and only majoritarian governments form. When parties and voters are not myopic, however, parties take into account the effects of their current government policy, which becomes next period's status quo, on the future electoral outcomes, government formation and policy choice. The resulting strategic dynamics yield results on changes in policy and government composition. First, policies are never stable; i.e., a new government changes the policy from the status quo. Moreover, if one of the current government parties is the formateur in the next period, its government partner changes. Second, if the parties are not highly patient, a consensus government will not form in the final period of a two-period model. Third, although a formateur could position the status quo so that it would receive a majority in the next period, it prefers not to do so unless the gains to officeholding are very large relative to policy preferences. Fourth, the policies of majoritarian governments are more centrally located in the second period than in the first period. Elections thus do not moderate policy changes when parties are unable to commit credibly to the policies they will implement if in government.

THE MODEL

Actors and Timing

The actors in the model are political parties and voters. The number of political parties is assumed to be fixed at three. Voters act only in the electoral stage and cast their votes for one of the three parties. The parties form governments and choose policies in the parliament.

A period is the length of an interelection period and consists of three stages, as illustrated in Figure 1. The first stage involves an election that determines the seat shares of the parties in the parliament. The second stage involves government formation, and the third stage is legislative and involves the choice of a policy in the parliament.

Policies and Transfers

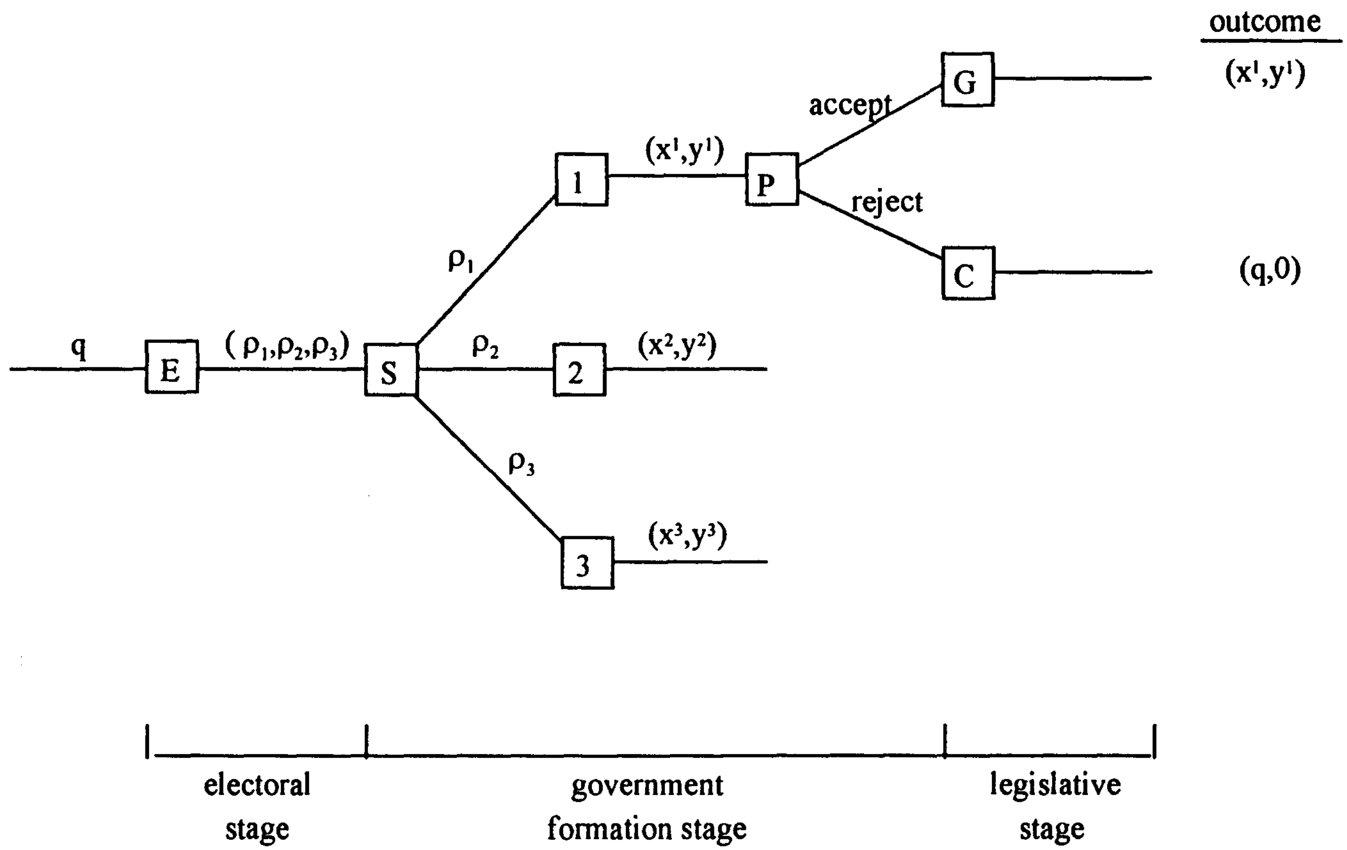
Parliament chooses a policy $x \in \Re^2$ in a two-dimensional space, and the parties can make a one-time transfer $y \in \Re^3$ among themselves. Policies are continuing, so when a government is formed the status quo is the policy in place under the previous government, and the policy chosen by the new government is the status quo for the following period. A continuing policy provides dynamics to the parliamentary system.

The transfers among the parties are assumed to be one-time, so if a government is not formed in a period, no transfers are made. The transfers are part of the bargain struck in forming the government and allow government formation and policy choice to be unrestricted. The transfers can be positive or negative, and, as will be shown below, allowing unrestricted transfers moderates the policy choices of governments.

The transfers may take a variety of forms. First, they may involve increases or decreases in extra-parliamentary patronage positions available to party members. Second, they may represent changes in perquisites that over time the parties have acquired. Third, they may represent exchanges between interest groups that support the parties; e.g., a labor union might pledge not to strike in exchange for policy concessions to the party with which it is aligned. Fourth, they may represent transfers of government resources to party stallwarts who would vote for that party in any case. Fifth, the parties can be thought of as having an

Figure 1

Single-Period Model



E election	i party i
S selection of formateur	G government
P potential government partner(s)	C caretaker government

account of government entitlements from which they can make transfers, and in a period the account balance can increase or decrease.[6] In none of these interpretations can the transfers be used to obtain votes in the electorate. The model thus does not include a model of electoral campaigns.

At a more abstract level transfers of distributive benefits serve as a medium of exchange that allows parties with preferences over both policy and benefits to bargain efficiently. Restricting transfers to be positive thus implicitly assumes some inefficiency in the government formation process. As we show below, in a model where actors have quadratic policy preferences it is efficient for parties to make policy concessions to the point at which the policy is located at the centroid of the parties' ideal points. Parties that make policy concessions need to be compensated by transfers of distributive benefits according to their bargaining strength. If the formateur can make a take it or leave it offer, his party will capture most of the rents. With counteroffers, however, more equal distributions of benefits result. The nature of the bargaining process, however, does not affect the policy chosen by the new government.

Governments and Policies

Parliamentary governments are typically identified by the parties that hold portfolios. This is not a fully adequate definition, however, since minority governments form and remain in office with the support of other parties (Strom 1990). Consequently, governing and government are not coincident. Since our model pertains to policy choice, its focus is on governing. A government is thus a coalition C of parties represented in parliament that constitutes a strict parliamentary majority and enact policy. In some countries minority governments are common (Strom 1990), and in those situations this model is to be interpreted as focusing on the supporting coalition that constitutes a majority to enact legislation rather than on which parties are represented in the cabinet. In parliament the government parties choose a policy and make transfers, so the policy choice and the transfers must be subgame perfect.

[6] To formalize the entitlements or extra-ministerial patronage interpretations, suppose that each party i has an entitlement $y_i^o, i = 1, 2, 3$, in each period. Then, a transfer $y_j^i > 0$ from party i to j increases party j's entitlements for the current period to $y_j^o + y_j^i$ and decreases party i's entitlements to $y_i^o - y_j^i$. A transfer $y_i^j < 0$ from party j to i increases i's current entitlement and decreases j's current entitlement.

Portfolios are not included in this model but may be interpreted as the influence parties have over the policy in its various dimensions. If one wishes to interpret the model in terms of portfolios, then cabinets should be thought of as arenas in which ministers bargain over policies based on the preferences of the parties to which they belong.[7] This is thus a model of cabinet government and ministerial bargaining rather than a model of ministerial delegation as in Austen-Smith and Banks (1990) and Laver and Shepsle (1990)(1996).

Government Formation

In the government formation process a party is selected as the formateur and proposes a government, a policy, and transfers; i.e., a proposal p^i by party i is a triple $p^i = (C^i, x^i, y^i)$. The proposal is subject to a government formation institution in which if accepted by parties with a majority in parliament the proposed government is installed. If the proposal is rejected, a caretaker government is assumed to take office. The caretaker does not change the policy, so the status quo remains in place and no transfers among the parties are made.

Government formation in the basic model involves a single attempt. This is not a restrictive assumption for two reasons. First, in a single-period model the equilibrium policies are independent of the number of stages in the government formation process, although the transfers are affected. Since voters' preferences are only over policies, the electoral equilibrium depends only on which governments would be formed. Second, in a multiperiod model there will be another government formation opportunity after the next election with a (possibly) different formateur. Nevertheless, the robustness of the results to the nature of the government formation process are investigated below.

Which party is selected as the formateur is assumed to be governed by a proportionality rule with the probability of selection equal to the party's seat share in the parliament, unless a party has a majority in which case it is selected as the formateur. In a study of parliamentary governments in twelve countries for the post-war period Diermeier and Merlo (1998) found strong support for a proportionality rule for the selection of the formateur with a sizable premium for selecting the incumbent (the formateur of the previous government). Their data do not support a rule under which the party with the most seats in parliament is selected with certainty. Since the government formation process and hence legislative

[7] Note that a ministerial allocation interpretation is strained in any model with a unidimensional policy.

outcomes are responsive to seat shares, there is a direct link between a vote in the election and policy outcomes.

Elections

The electoral system is assumed to be proportional representation with a minimum vote share m required for representation. If the vote shares ρ_i of all parties are at least m, their seat shares are ρ_i. If party i's vote share is less than m, then it is not represented in parliament and the other parties have seat shares $\rho'_\ell = \frac{\rho_\ell}{1-\rho_i}$, $\ell = j, k, \ell \neq i$. Since voters have preferences over policies and policies are the result of parliamentary action led by the government, voters vote prospectively based on their expectations about which governments will form and about their policy choices. Since the parties are unable to commit credibly to the policies they will choose once a government has been formed, voters form expectations over which policies will be chosen in equilibrium in the possible subgames commencing after the election.

Preferences

The preferences of party i are represented by a quasilinear utility function $U^i(x, y)$ given by

$$U^i(x, y) = u^i(x) + y_i, \quad i = 1, 2, 3, \tag{1}$$

where $u^i(x)$ represents party i's policy preferences, x is the policy, and $y = (y_1, y_2, y_3)$ is the vector of transfers between the formateur and party $i, i = 1, 2, 3$, where $y_j = 0$ if party j is not represented in parliament.[8] These preferences may be thought of as those of the party leaders, and for the sake of tractability the policy preferences are assumed to be quadratic with $z^i \in \Re^2$ denoting party i's ideal policy; i.e.,

$$u^i(x) = -(x_1 - z_1^i)^2 - (x_2 - z_2^i)^2. \tag{1a}$$

Thus, parties not only prefer policies closer to their ideal points but they are more averse to policy changes the farther those changes are from their ideal point. In the basic model

[8] Austen-Smith and Banks' model has the feature that the "ideal points" the parties use in government formation are chosen rather than exogenous. That is, the preferences of the parties are for the "*difference* between their electoral positions and the final policy outcome." They justify this assumption in terms of reputational considerations stemming from voter punishments for government policies that differ from the parties' electoral promises. A difficulty with this interpretation is that voters are assumed to punish even the party that is not in government and hence not responsible for the choice of the government policy.

the parties' ideal points are assumed to be symmetrically located so that the governments that form are based on institutional characteristics and strategies rather than on preference alignments.

The specification in (1) allows parties to have preferences for both policy and for officeholding, where the latter is reflected in the transfers within the government coalition. To facilitate comparisons the transfers y are assumed to aggregate to a fixed amount Y; i.e., $\sum_{i=1}^{3} y_i = Y$. The aggregate transfers Y are assumed to be small relative to the policy utility $u^i(x)$, so the gains to officeholding help achieve the bargain rather than providing the principal incentive to be the formateur.

The transfers are not made to constituents to obtain their votes nor do the transfers affect taxes, so voters are unaffected by the transfers. Voters thus have preferences only over the policy enacted by the government. The preferences of a voter with ideal point z are represented by a utility function $u(x; z)$,

$$u(x; z) = -(x_1 - z_1)^2 - (x_2 - z_2)^2,$$

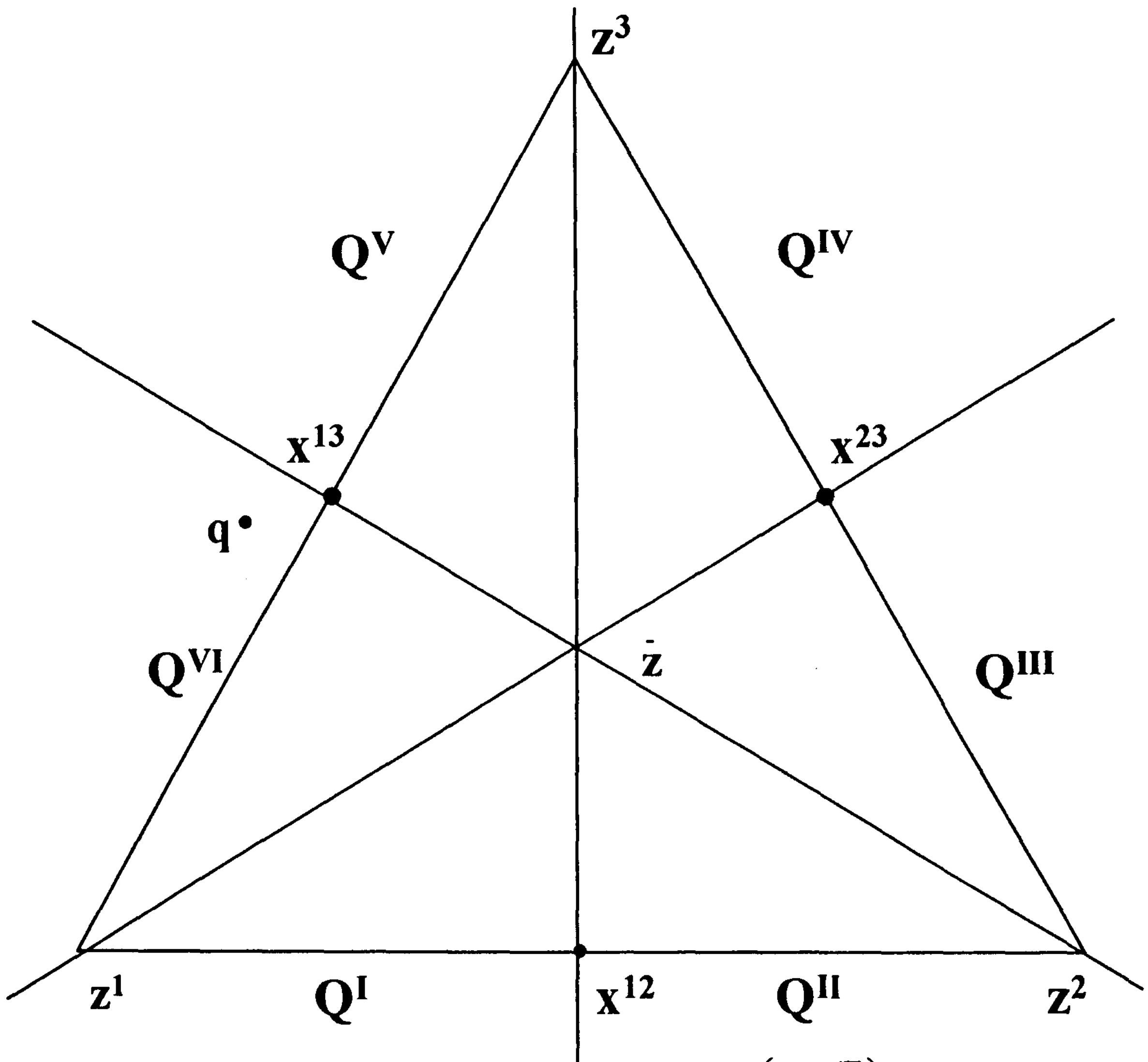
where $z \in Z$, the space of voters' ideal points. Since the model does not involve uncertainty or incomplete information, at the time of the election voters understand which governments will be formed given an election outcome and which policies the possible governments will choose. Since the government formation process involves the choice of the formateur with probabilities proportional to the seat shares of the parties, voters cast their votes taking into account the effect of their votes on the probability distribution of policies.

GOVERNMENT FORMATION AND LEGISLATION

Majoritarian Governments

This section considers a single-period model in which no party has a majority and characterizes the majoritarian (two-party) governments formed and their policy choices. In the government formation and legislative stages the proposal made by the formateur depends on the status quo $q \in \Re^2$, and to facilitate the analysis, suppose that the three parties have ideal points symmetrically located at the vertices of an equilateral triangle as illustrated in Figure 2. Let the policy space be divided into six regions $Q^j, j = I, \ldots, VI$, given by the intersections of the half spaces defined by the lines through the ideal points and

Figure 2
Symmetric Configuration of Party Preferences and Government Policies



N.B. $\mathbf{z}^1 = (0,0)$; $\mathbf{z}^2 = (1,0)$; $\mathbf{z}^3 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

the centroid $\bar{z}$ of the triangle. For example, $Q^I \equiv \{(q_1, q_2) \in \Re^2 \mid q_1 \leq \frac{1}{2}(z_1^1 + z_1^2),\ q_2 \leq z_2^1 + \frac{1}{\sqrt{3}}(q_1 - z_1^1)\}$.

A party $j, j \neq i$, will accept the proposal by party i if and only if

$$u^j(x^i) + y_j^i \geq u^j(q), \tag{2}$$

where i's government formation proposal $p^i = (C^i, x^i, y^i)$ is $C^i = \{i, j\}$ and $y^i = (y_1^i, y_2^i, y_3^i)$. Party i's optimal policy-transfer (x^{ij}, y^i) is given by

$$\begin{aligned} (x^{ij}, y^i) \in \arg\max_{x, y_j}\ & u^i(x) + Y - y_j \\ s.t.\ & u^j(x) + y_j \geq u^j(q), \end{aligned} \tag{3}$$

where $y_k^i = 0, k \neq i, j$, and $y_i^i = Y - y_j^i$. In obtaining j's support, i will provide the minimum transfers to j, so the equality in (2) will hold or

$$y_j = u^j(q) - u^j(x). \tag{4}$$

Using (4) the choice in (3) is

$$x^{ij} \in \arg\max_x u^i(x) + Y + u^j(x) - u^j(q). \tag{5}$$

The optimal policy thus maximizes the aggregate policy preferences of the government members, and with quadratic preferences this is the midpoint of the contract curve of the two parties; i.e.,

$$x^{ij} = \frac{1}{2}(z^i + z^j), \tag{6}$$

which is independent of the status quo q. The transfer in (4) depends on q; i.e., $y_j^i = u^j(q) - u^j(x^{ij})$.

This result is more general. For any coalition C with a majority of the seats in parliament, the optimal policy x^C for any formateur $i \in C$ is

$$x^C = \frac{1}{\mid C \mid} \sum_{k \in C} z^k. \tag{7}$$

The policy is independent of which member of C is the formateur. Note also that the policy in (7) and the transfer in (4) are independent of the seat shares of the parties, since government formation and policy choice require only a decisive coalition. The policy and transfer within the government are also independent of the aggregate transfer Y.

To interpret the policy choice in terms of the allocation of ministries, suppose that party $i \in C$ holds the ministry for policy dimension 1 and party $j \in C$ holds the ministry for policy dimension 2. Since each party cares about the policy choice of its government partner, the ministries will bargain over the policies to be implemented. The result of that bargain will be the policy in (7) and the transfers in (4).

The utility W^i of the formateur i is

$$W^i(x^{ij}) = u^i(x^{ij}) + Y + u^j(x^{ij}) - u^j(q), \tag{8}$$

and the utility $W^j(x^{ij})$ of i's government partner j is

$$W^j(x^{ij}) = u^j(q). \tag{9}$$

The utility W^k of the out party is

$$W^k(x^{ik}) = u^k(x^{ij}).$$

The following proposition establishes that a formateur will form a majoritarian government with the other party whose ideal point is farther from the status quo; i.e., the party with the weaker bargaining position. The robustness of this result is then investigated, and the set of status quos for which majoritarian governments will form is characterized.

Proposition 1: In a one-period model with three parties with ideal points located at the vertices of an equilateral triangle, the formateur i forms a majoritarian government with the party whose ideal point is farther from the status quo q; i.e., with party j if and only if $u^j(q) < u^k(q), j, k \neq i$, and is indifferent between the two other parties if $u^j(q) = u^k(q)$.

Proof: From (8) the difference in the utilities $W^i(x^{i\ell})$ of the formateur i for governments with parties $\ell = j, k$ is

$$W^i(x^{ij}) - W^i(x^{ik}) = u^i(x^{ij}) + u^j(x^{ij}) - u^j(q) - u^i(x^{ik}) - u^j(x^{ik}) + u^k(q).$$

By symmetry, $u^i(x^{ij}) = u^i(x^{ik})$ and $u^j(x^{ij}) = u^k(x^{ik})$, which implies

$$W^i(x^{ij}) - W^i(x^{ik}) = u^k(q) - u^j(q).$$

If $u^j(q) < u^k(q)$, i will form a government with party j, and if $u^j(q) = u^k(q)$, party i is indifferent between the other parties. **Q.E.D.**

Proposition 1 identifies the equilibrium governments and policies as a function of the status quo. As illustrated in Figure 2, the policies x^{ij} in (6) are on the opposite side from the status quo of the lines through the out party's ideal point and the centroid $\bar{z}$ of the parties' preferences.[9] This results because the formateur seeks the best bargain it can, and that bargain is obtained with the party that is in the weaker bargaining position; i.e., the party that is disadvantaged by the status quo. The formateur thus prefers to form a government with a policy that is on the opposite side of the Pareto set from the status quo. If the status quo is in the intersection of two regions in Figure 2, two parties are equidistant from the status quo. The formateur then is indifferent between the other two parties. For example, if party 3 is the formateur and $q \in Q^I \cap Q^{II}$, party 3 may form a government with party 1 at policy x^{13} or with party 2 at policy x^{23}. Note also that if party 1 is the formateur in two consecutive periods both the governments and the policies change.

To interpret the equilibrium, suppose that party j is farther from the status quo than is party k. Then, from (9) j receives its reservation value $u^j(q)$, but Proposition 1 and symmetry imply that $u^j(x^{ij}) > u^j(q)$, so $y^i_j = u^j(x^{ij}) - u^j(q) < 0$. Government formation thus has the following interpretation: the formateur proposes to form a government with the party that is in the worse bargaining position, i.e., is the more disadvantaged by the status quo, and proposes a policy that is advantageous relative to the status quo. In so doing, party 1 demands a transfer that leaves its partner no better off than it would be if it rejected the proposal and a caretaker government maintained the status quo. The transfers thus are from the government partner to the formateur, but as the next section indicates, this property is due to the simplifying assumption that there is only one round of government formation before a caretaker government takes office. The next section examines the robustness of these results for majoritarian governments, and the following section characterizes the sets of status quos such that the formateur forms a consensus government with both other parties.

[9] Proposition 1 implies that the government partner j could be worse off than the out party k if $u^k(x^{ij}) > u^k(q)$. As will be shown below, if this condition is satisfied, party 1 prefers to form a consensus government rather than a majoritarian government.

Robustness

This section addresses the robustness of the results for majoritarian governments with regard to nonnegative transfers, asymmetric preference configurations, different intensities of policy preferences, and an alternative government formation process. The supporting analysis is presented in Appendix A. Only electoral outcomes in which no party has a majority are considered.

Restricting transfers to be nonnegative does not alter the basic results established above for the case of unrestricted transfers. The analysis in Appendix A demonstrates the following. First, the formateur forms a government with the party that is the more disadvantaged by the status quo. Second, if a majoritarian government is formed, the policy chosen yields the government partner its reservation value. Third, a consensus government is never formed unless both parties prefer the formateur's ideal point to the status quo. Fourth, the formateur never makes policy concessions to its government partner. Fifth, the equilibrium transfers are zero. Sixth, when transfers are restricted to be nonnegative, the policies are more extreme, i.e., farther from the centroid of the parties' preferences, than are the policies when the transfers are unrestricted. Unrestricted transfers thus result in greater policy compromises than do restricted transfers.

If ideal points are not symmetrically located as in Figure 2, both the status quo and the degree of preference alignment among the parties affect which governments form and the policies they choose. If two parties are equally distant from the status quo, the formateur will form a government with the party with which its preferences are better aligned. This is reminiscent of the concept of connected parties in a unidimensional policy space (Axelrod 1970). The theory presented here modifies this intuition by taking into account the bargaining positions of the parties as determined by the status quo. Governments thus are formed by parties with aligned preferences unless a party with more extreme policy preferences is particularly disadvantaged by the status quo. An extreme party thus is not an attractive government partner unless it is also in a weak bargaining position and hence is willing to make substantial transfers in exchange for policy concessions.[10]

[10] For example, communist parties in a number of countries have refused to join governments, and the traditional explanation is that they preferred to remain out of government for ideological and electoral reasons. The result here indicates that their extreme policy

The specification of policy preferences in $(1a)$ implies that parties have the same intensity of preferences for each policy dimension. If policy intensities differ among parties and between policy dimensions, the policies chosen by a government differ from those in (6) and the sets of status quos that give rise to particular governments change. The basic intuitions remain unchanged, however. Suppose that party 2 has a greater intensity of preferences for the policy represented by dimension 1 than the policy represented by dimension 2. Then, if party 1 forms a government with party 2, the policy will be closer to the ideal point of party 2 than to the ideal point of party 1. This affects which government partner party 1 prefers, and party 2 is a less attractive partner for status quos near party 1's ideal point and a more attractive partner for status quos far from party 1's ideal point.

The basic model incorporates one round of bargaining over government formation. Government formation, however, could involve multiple rounds of formation attempts. As noted above this does not affect the policies the resulting governments choose, but it does affect the transfers and hence which governments form as a function of the status quo. Thus, the regions identified in Figures 2 and 3 below have different shapes. The basic intuition about which governments form remains unchanged, however. The government is formed with the other party that is the more disadvantaged by continuing to a subsequent round of government formation attempts. Moreover, multiple rounds of government formation increase the reservation values of the potential government partners, and hence the transfers can go from the formateur to the government partner.

The bargaining over government formation could also take different forms. One such form is a two-stage process in which the formateur first forms a proto-coalition and then the members of the coalition bargain over the division of the gain. Suppose, for example, that party 1 forms a proto-coalition with party 2 and the two parties bargain sequentially where in each round a party is selected to make a proposal to its partner with probability equal to its seat share in the coalition. As indicated in Appendix A, this process results in the same governments, equilibrium policies, and vote shares as with the government formation process in the basic model.

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positions may also have kept them from being in government. If the status quo is far from their ideal points, however, they may be invited to join the government. If a distant status quo is the result of a prior policy choice by a conservative government, a communist party would be invited to join a coalition government by a leftist party.

Consensus Governments

A consensus government includes all parties, and hence from (7) the government policy is the centroid $\bar{z}$ of the ideal points of the parties. The attractiveness of a consensus government is that by making policy concessions to both of the other parties the formateur can extract transfers from both. This, however, is at the cost of a policy that is less attractive to the formateur than if it formed a majoritarian government. If party 1 is the formateur and forms a consensus government, its utility W^{1C} is

$$\begin{aligned} W^{1C} &= u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^2(q) - u^3(q) \\ &= 3u^1(\bar{z}) + Y - u^2(q) - u^3(q), \end{aligned} \tag{10}$$

where the second equality follows from symmetry. If party 1 forms a majoritarian government and $q \in Q^I \cup Q^{II}$, by Proposition 1 it will be with party 3, and its utility W^{13} is, using symmetry,

$$\begin{aligned} W^{13} &= u^1(x^{13}) + u^2(x^{13}) + Y - u^3(q) \\ &= 2u^1(x^{13}) + Y - u^3(q). \end{aligned}$$

Party 1 prefers to form a consensus rather than a majoritarian government if and only if

$$W^{1C} - W^{13} = 3u^1(\bar{z}) - 2u^1(x^{13}) - u^2(q) \geq 0. \tag{11}$$

Consequently, if in the optimal majoritarian government the out party 2 is very disadvantaged by the status quo (i.e., $u^2(q)$ is very negative), party 1 prefers to form a consensus government so as to extract transfers from party 2 as well.

To be more explicit, consider without loss of generality the case in which the parties' ideal points are normalized at $z^1 = (0,0)$, $z^2 = (1,0)$, and $z^3 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$. The condition in (11) then is

$$W^{1C} - W^{13} = -\frac{1}{2} - u^2(q) \geq 0. \tag{12}$$

This condition defines a disk $D^2 \equiv \{q \mid -\frac{1}{2} \leq u^2(q)\}$ with center at z^2 and radius $\frac{1}{\sqrt{2}}$ such that for $q \in D^2 \cap (Q^I \cup Q^{II})$, party 1 prefers to form a majoritarian government with party 3, and for $q \notin D^2 \cap (Q^I \cup Q^{II})$ party 1 prefers to form a consensus government.[11] That is, if the status quo q is sufficiently distant from party 3 (i.e., $q \in Q^I \cup Q^{II}$) and from party 2

[11] This establishes the claim in footnote 10.

($q \notin D^2$), a consensus government is attractive because the other parties are both in weak bargaining positions and the formateur is able to extract sufficient transfers to more than compensate for the less favorable policy $\bar{z}$. As illustrated in Figure 3, the formateur forms a consensus government if the status quo is extreme.

To complete the characterization, for $q \in Q^I \cup Q^{II}$ party 2 will form a consensus government rather than a majoritarian government with party 3 if and only if $q \notin D^1 \equiv \{q \mid -\frac{1}{2} \leq u^1(q)\}$. If party 3 is the formateur, it will form a consensus government rather than a majority government with party 2 (3) if and only if $q \notin D^1 \cap Q^I$ ($q \notin D^3 \cap Q^{II}$). If $q \in Q^I \cap Q^{II}$, party 3 is indifferent between forming a majoritarian government with party 1 and one with party 2. Then, for $q \notin D^{12} \cap (Q^I \cup Q^{II})$, where D^{12} is a disk with radius $\frac{1}{\sqrt{2}}$ and center at $(\frac{1}{2}, 0)$, party 3 will not form a majoritarian government with either party 1 or party 2.[12]

For some q, all three parties will form a consensus government if selected as the formateur. That is, if $q \notin D^1 \cup D^2 \cup D^3$, each party will form a consensus government. To illustrate this, consider a status quo q on the line from z^3 through the centroid $\bar{z}$ as illustrated in Figure 3. If on that line q is farther than a distance of $\frac{1}{2}$ from the Pareto set, each of the three parties will form a consensus government if selected as the formateur. For a status quo that is closer than that point, all three parties form majoritarian governments. In particular, if a consensus government were formed in the previous period with policy $\bar{z}$, the government in the current period would be majoritarian. These results are summarized

[12] To provide a comparison with the model of Austen-Smith and Banks, consider the case in which the utility of each party is $\bar{u}$ if no government is formed in period n. In this case the formateur is indifferent to which party is its partner. If the formateur (party 1) forms a consensus government, its utility, using symmetry, is

$$W^{1z} = 3u^1(\bar{z}) + Y - 2\bar{u}.$$

If party 1 forms a majoritarian government, its utility using symmetry is

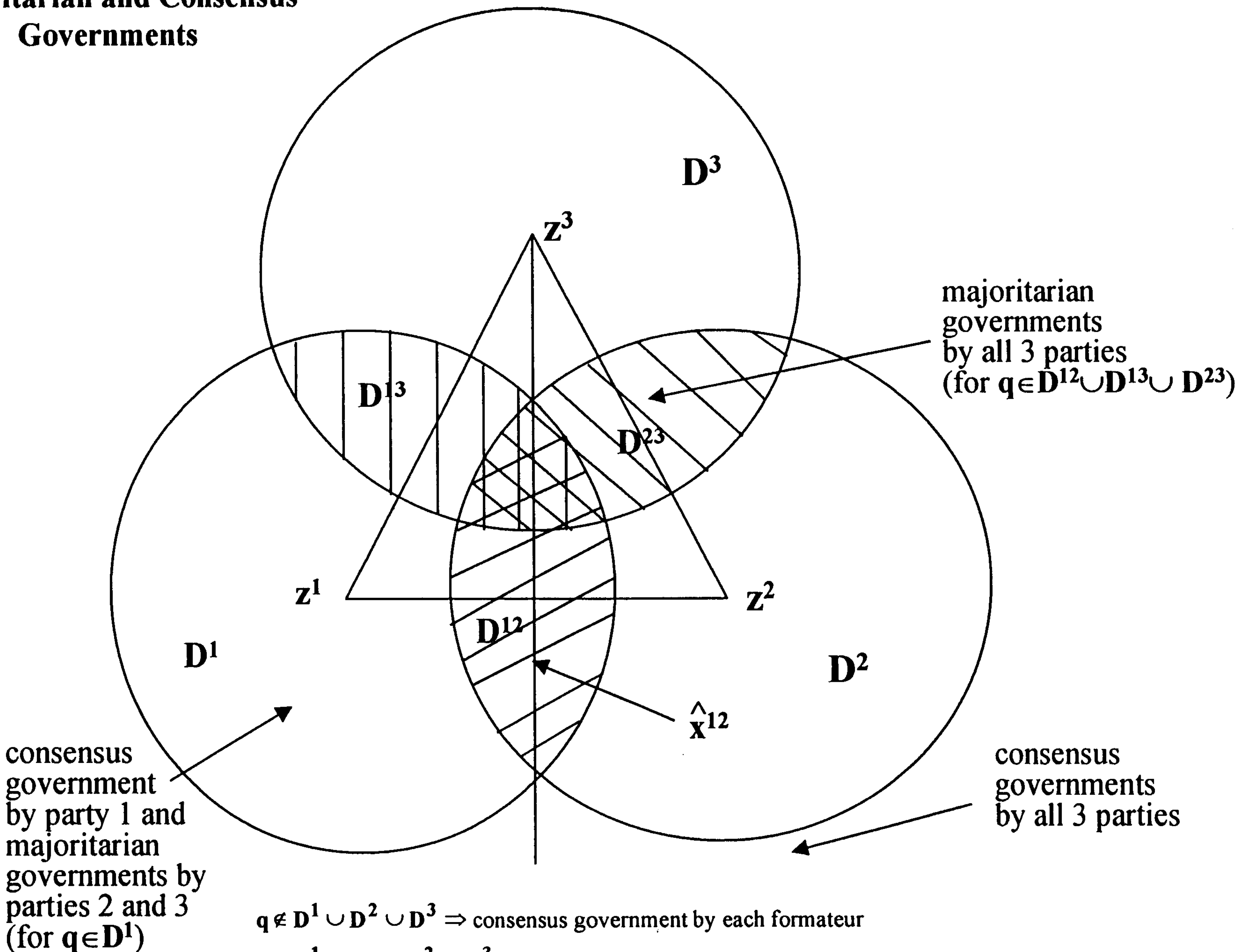
$$W^1 = 2u^1(x^{12}) + Y - \bar{u}.$$

Then,

$$W^{1z} - W^1 = 3u^1(\bar{z}) - 2u^{12} - \bar{u}.$$

Since $u^1(\bar{z}) = -\frac{1}{3}$ and $u^1(x^{12}) = -\frac{1}{4}$, party 1 prefers to form a consensus government if and only if $-\bar{u} \geq \frac{1}{2}$. This is the same condition as in (12).

Figure 3
Majoritarian and Consensus Governments



$q \notin D^1 \cup D^2 \cup D^3 \Rightarrow$ consensus government by each formateur

$q \in D^1$ and $q \notin D^2 \cup D^3 \Rightarrow$ consensus government by party 1 as:

Proposition 2: If no party has a majority in parliament, the formateur i forms a consensus government only if both the other parties are substantially disadvantaged by the status quo; i.e., one party j is more disadvantaged by the status quo and the other party k and $q \notin D^k$. If $q \notin D^1 \cup D^2 \cup D^3$, all three parties form consensus governments.

Figure 3 summarizes the types of governments that form when no party has a majority.

Majority Parties

Next consider the case of a party with a majority of the seats in parliament. A majority party will form one of three types of governments — surplus (with one other party), consensus (with both of the other parties), or single-party with a policy at its ideal point. If party 1 has a majority and forms a single-party government with policy z^1, its utility is $W^{1M} = Y$. If it forms a consensus government with policy $\bar{z}$, its utility is $\bar{W}^1 = -1 + Y - u^2(q) - u^3(q)$. For $q \notin D^2 \cup D^3$, $u^j(q) < -\frac{1}{2}, j = 2, 3$, so $\bar{W}^1 > W^{1M}$. When $q \notin D^2 \cup D^3$, a majority party then forms a consensus government with policy $\bar{z}$ rather than implement its own ideal policy. For example, if $q = z^1$, party 1 will form a consensus government.

If $q \in D^2 \cup D^3$, the majority party 1 could form a surplus or a single-party government. Proposition 1 indicates that a government will be formed with the party that is the more disadvantaged by the status quo. Let that be party 3; e.g., $q \in Q^V \cup Q^{VI}$. Then, using (8)

$$W^{1M} - W^{13}(x^{13}) = \frac{1}{2} + u^3(q).$$

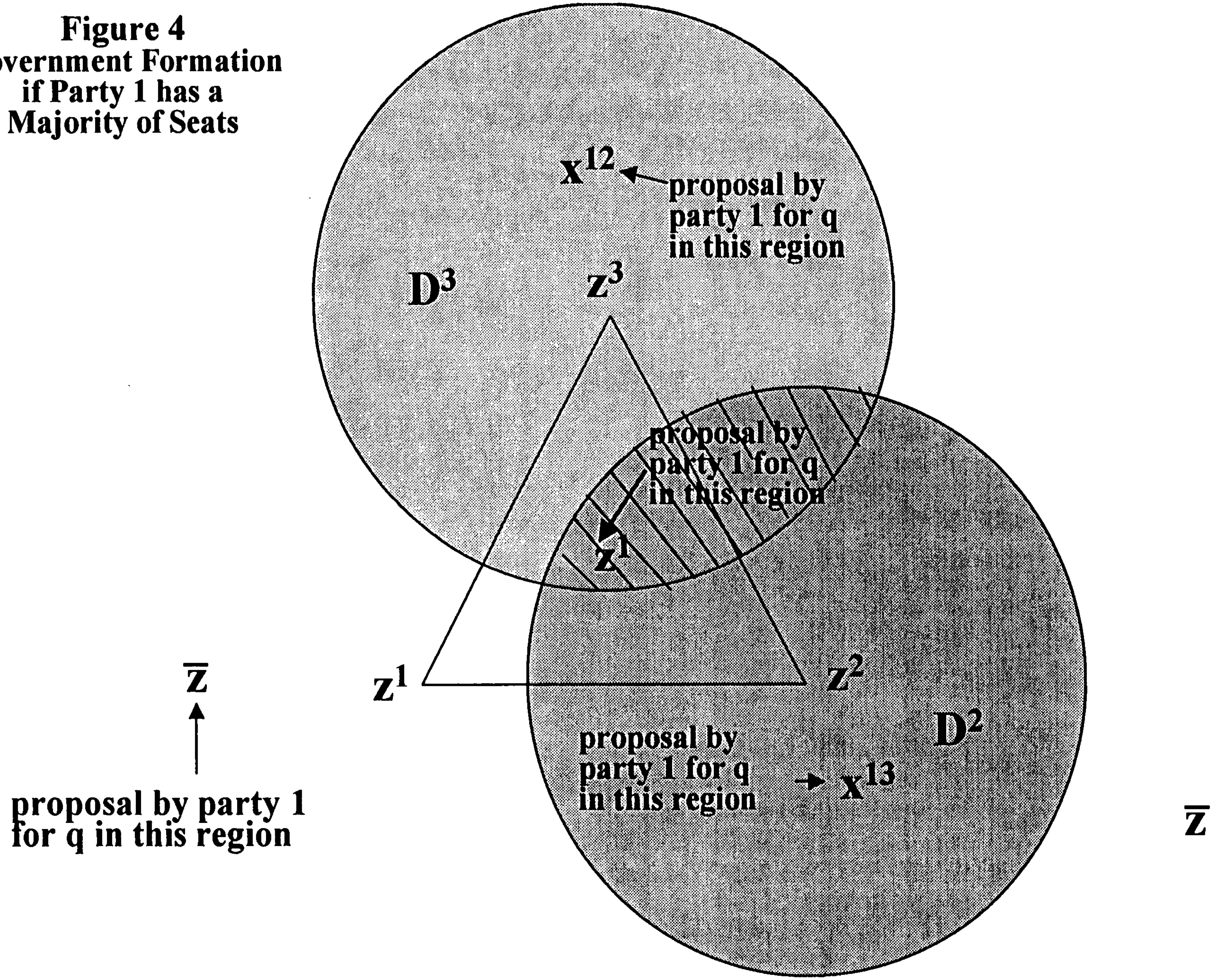
Consequently, if $q \notin D^3$, the majority party 1 forms a surplus government rather than a single-party government. Proposition 3 summarizes the governments a majority party forms, and Figure 4 illustrates those governments.

Proposition 3: Suppose party 1 has a majority of seats in parliament. Then, party 1 forms (1) a consensus government if and only if $q \notin D^2 \cup D^3$, (2) a surplus government if and only if $q \in D^2 \cup D^3$, and (3) a single-party government otherwise.

ELECTORAL EQUILIBRIA

Since parties are unable to commit credibly to policies, they do not compete electorally through their policies. That is, since no electoral promise is credible, and voters have policy

Figure 4
Government Formation
if Party 1 has a
Majority of Seats



preferences, their preferences for parties are derived from the policies the parties would choose if selected as the formateur. Parties are thus instrumental. From the previous section for a given status quo only finitely many alternatives may result as government policies. Since voters are exclusively concerned with policy outcomes, they will base their voting decision on this set of possible government policies. From the point of view of voters the parties represent different bargaining positions in parliament.

Voters can influence the policy outcome in three ways. First, they may be *representation pivotal*. That is, their vote determines which parties are represented in the legislature. Second, they may be *coalition pivotal*; i.e., their vote determines the set of decisive coalitions in parliament. For instance, an additional vote for party j may give j a majority of seats. As indicated in the analysis of the government formation stage, this may lead to different policy outcomes than if no party has a majority. Finally, voters may be *selection pivotal*. If no party commands a majority of seats, then given a proportional selection rule for the formateur and an electoral system with proportional representation, an additional vote for party j increases the likelihood that j will be selected as the formateur. Note that if no party receives a majority of seats, each voter is at least selection pivotal.

To facilitate the characterization of strong Nash electoral equilibria, a large, but finite (and even) number N of voters is considered.[13] The voters are also assumed to be symmetrically and uniformly distributed about the Pareto set of the parties, so no party has a natural electoral advantage. To simplify the analysis, we assume that there are no voters who are exactly indifferent between any two parties' ideal policies. The minimum vote threshold M for representation in the chamber satisfies $\frac{N}{4} > M \geq 1$ with $m = M/N$.

Electoral games commonly have many Nash equilibria associated with different outcomes. Under proportional representation iterated weak dominance does not help select among the equilibria (Austen-Smith and Banks 1988). In our model the electoral equilibria depend on the status quo, and for a given q there can be a continuum of equilibria. As we will show, however, for each status quo there exists a *unique* strong Nash equilibrium policy outcome. That is, there is a unique policy, or a lottery over policies, that is robust

[13] Alternatively, the approach in Alesina and Rosenthal (1996) of extending the strong Nash equilibrium concept to a continuum of voters may be employed. To shorten the exposition, a finite number of voters is assumed in the proof of Proposition 4.

to deviations by groups of voters. Moreover, the set of Nash equilibria that support an outcome can be characterized.

It is straightforward to show that there is no electoral equilibrium with two parties each having half the votes.[14] If no party controls a majority of seats and hence the parliament contains all three parties, the parliament is said to be a "minority parliament." Otherwise, the parliament is a "majority parliament." Since in the absence of a majority party, formateurs are chosen proportionally to seat shares, a minority parliament induces a lottery over government policies. Let $x^i(q), i = 1,2,3$, denote the policy party i will implement if it is selected as formateur. Note that in a minority parliament, a vote for say party 1 increases the probability that the government will be formed by party 1 with policy $x^1(q)$ and decreases the probability that $x^2(q)$ or $x^3(q)$ will result.

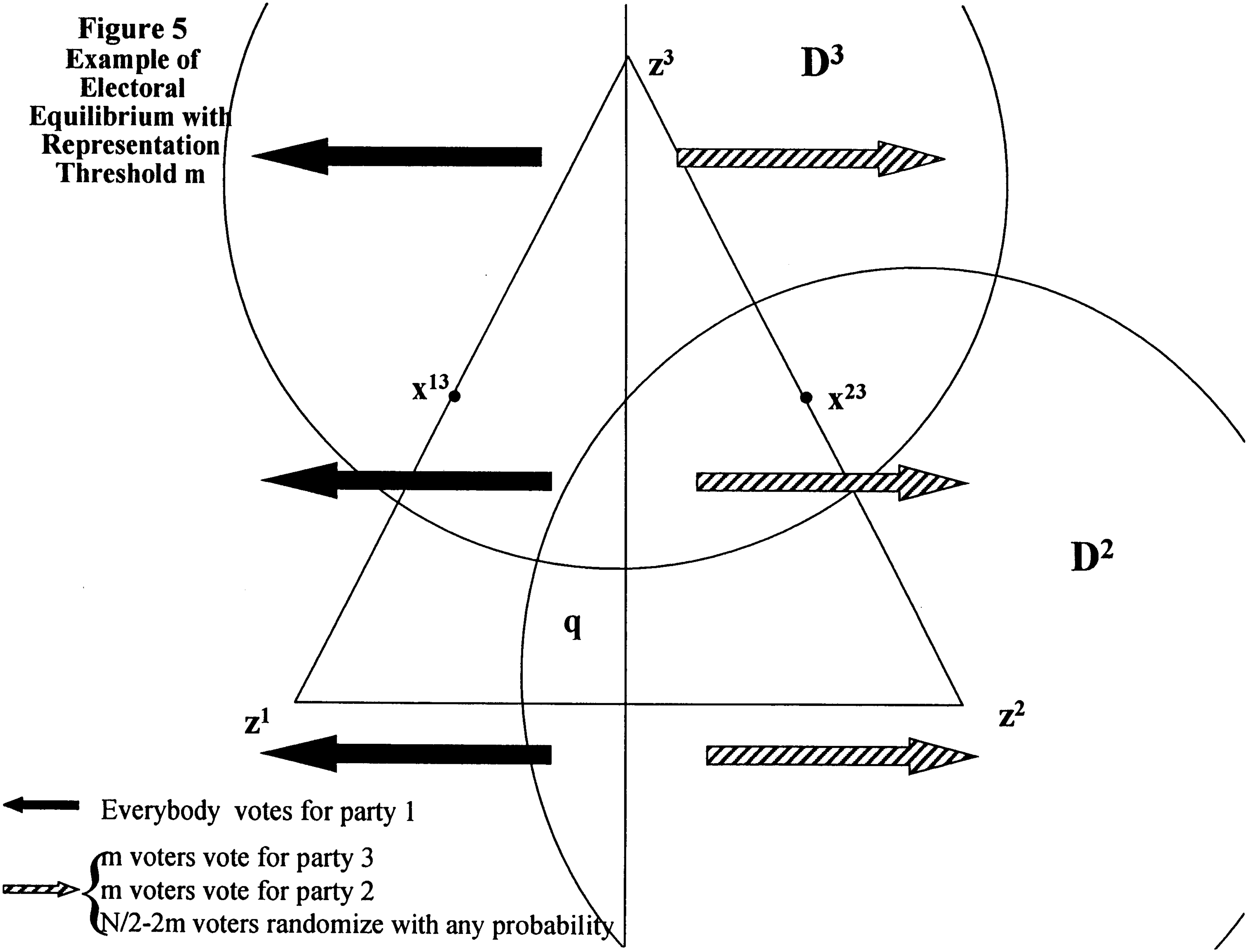
The result of the electoral stage is summarized in the following proposition, which is proven in Appendix B.

Proposition 4: (A) If given q there exists exactly one party j that as formateur will propose a consensus government in a three-party parliament, every strong voting equilibrium leads to a three-party majority parliament where the majority party j forms a consensus government with policy $\bar{z}$. (B) If given q no party as formateur will propose a consensus government, then all strong electoral equilibria lead to a minority parliament with an even lottery over some pair (x^{ik}, x^{ij}) in (6) as the unique equilibrium outcome $(j \neq k \neq i)$. (C) If given q all parties would form a consensus government, all voters are indifferent to how they cast their vote. (D) If in a minority parliament party i as formateur would randomize between two policies x^{ij} and x^{ik} and the other two parties would form governments with each other, the unique strong equilibrium yields $\rho_i = m$ and equal vote shares for the other two parties.

The electoral equilibria have the following properties and interpretations. First, as indicated in Figure 5 some voters do not vote sincerely; i.e., for the party with the closest ideal point. Figure 5 illustrates the voting equilibrium for Proposition 4 (B) and indicates

[14] To see this, suppose that parties 1 and 2 have half the vote. Whichever is selected as formateur will propose the policy x^{12}. Anticipating this, voters near z^3 prefer to vote for party 3, and at least m have such preferences. For example, if q is such that party 3 would form a government with party 2, party 3 would receive half the vote. Thus, there is no electoral equilibrium in which two parties each have half the vote.

Figure 5
Example of Electoral Equilibrium with Representation Threshold m



that some voters near the ideal point of party 3 vote for party 1. This results because voters care about policy and not about parties per se. Representation in parliament thus does not reflect the distribution of preferences in the electorate.[15] Second, voters vote strategically based on their beliefs about the governments each party will form if selected as the formateur. These beliefs are consistent with the strategies the parties will choose once in parliament. In minority parliaments all voters are selection pivotal, but not necessarily representation pivotal. In majority parliaments, individual voters may never be pivotal. Third, the equilibrium vote shares of the parties depend on the status quo because the subsequent government policies depend on the status quo. Since the voters have preferences over policies, their voting strategies implicitly depend on the status quo. There is thus a mapping from the status quo to election outcomes and to governments and their policies. Fourth, both minority and majority parliaments may occur. The majority party, however, always proposes a consensus government with a policy at the centroid of the distribution of voters' preferences. Thus, the same conditions that lead to a majority parliament also lead to a consensus government. Fifth, the party closest to the status quo will be over-represented in parliament unless all parties would propose $\bar{z}$. Depending on the location of the status quo it will either be the majority party or in minority parliaments have the largest seat share. But this does not mean that policy outcomes will be biased towards the largest party in parliament. A majority party, for instance, always proposes the centroid of all voters' ideal points. Indeed, it is because the largest party has no incentive to exploit its seat share advantage in the government formation process that creates an incentive for voters to over-represent it in parliament. Sixth, voters try to locate the government policy as close to the center of voter preferences as possible. So, if there exits a unique party that will implement a policy at the centroid, that party will receive a majority of the votes. If not, voters will balance representation in parliament to the extent that *ex ante* (that is before a formateur is chosen) government policies will be located as centrally as possible. If the status quo is extreme ($q \notin D^1 \cup D^2 \cup D^3$), all three parties form consensus governments, and their expected vote shares are equal. Thus, a crisis that results in an extreme status quo is followed by a consensus government.

To illustrate the strategies that constitute a strong Nash equilibrium, consider a status

[15] Austen-Smith and Banks reach the same conclusion for the equilibrium they study.

quo such that in a minority parliament each party would form a majoritarian government with party 1 choosing x^{12} and parties 2 and 3 choosing x^{23}. Then, as indicated in Figure 5, $\frac{N}{2}$ voters with $z_1 \le \frac{1}{2}$ vote for party 1 with probability one, and for voters with $z_1 \ge \frac{1}{2}$, M voters vote for parties 2 and 3 with probability one, and the remaining voters with $z_1 \ge \frac{1}{2}$ randomize among parties 2 and 3. That is, if party 1 has half the votes, the other two parties have vote shares ρ^o and $\frac{1}{2} - \rho^o$, where $\frac{1}{2} - m \ge \rho^o \ge m$. The seat (and vote) shares are summarized in Table 1.

Table 1

Equilibria With a Minority Parliament

	Vote Shares		
Status Quo	Party 1	Party 2	Party 3
Q^I	$\frac{1}{2}$	ρ^o	$\frac{1}{2} - \rho^o$
Q^{II}	ρ^o	$\frac{1}{2}$	$\frac{1}{2} - \rho^o$
Q^{III}	$\frac{1}{2} - \rho^o$	$\frac{1}{2}$	ρ^o
Q^{IV}	$\frac{1}{2} - \rho^o$	ρ^o	$\frac{1}{2}$
Q^{V}	ρ^o	$\frac{1}{2} - \rho^o$	$\frac{1}{2}$
Q^{VI}	$\frac{1}{2}$	$\frac{1}{2} - \rho^o$	ρ^o

As indicated in Table 1, the parties disadvantaged by the status quo are also disadvantaged in the election and receives the lowest vote share. Comparing Figure 4 and Table 1 indicates that one of the parties disadvantaged by the status quo will be in the government. That is, a party that is in the worst bargaining position is an attractive government partner, as established in Proposition 1.

Similarly, comparing Table 1 with Figure 2, the party nearest the status quo receives half the vote. Voters over-represent the party closest to the status quo because this party will be included in the government only if it is the formateur. That is, as shown in Proposition 1, the party advantaged by the status quo is not an attractive government partner for either of the other two parties, so the only way voters can obtain a policy on that party's contract curve is if it is selected as formateur. Voters thus balance the possible government policies by increasing that party's vote share. For example, if $q \in Q^I$, parties 2 and 3 will form governments with each other with policy x^{23} and party 1 will form a government with party 2 with policy x^{12}. To balance these two possible policy outcomes, voters give party 1 half the vote.

For status quos that lead to majoritarian governments, two parties form a government with each other when selected as formateur. This may be interpreted as a pre-election coalition induced by government formation incentives rather than by a pre-election commitment. The implicit pre-election coalition is that each of the two parties, if selected, will form a government with the other as partner, but since parties cannot commit to their post-election actions each of those parties may also accept a government formation offer from the third party. Voters anticipate the formation of this implicit pre-election coalition, however, and balance the election outcome to give the coalition only half the votes.

Since voters who randomize can do so with any probabilities, there is a continuum of electoral equilibria each of which yields the same distribution over policies. If all voters who randomize among a set of parties were to do so with equal probabilities, the vote shares ρ^o and $\frac{1}{2} - \rho^o$ from Table 1 equal one-quarter. Comparing Table 1 with Figure 2 and 3 indicates that in this case one of the smallest parties is always in a majoritarian government.[16]

DYNAMICS

Exogenous Dynamics

Electoral equilibria are fully determined by the status quo. That is, given the location of the status quo, the distribution over government policies can be determined. For extreme status quos this distribution is degenerate: the centroid of voter preferences is chosen as the policy with probability one. For less extreme status quos there is an even lottery over two polices. This defines a natural dynamic of policy change, where the current period's government policy determines next period's status quo.

[16] Austen-Smith and Banks also obtain this result. In this model, it is due to the bargaining position determined by the status quo, whereas in Austen-Smith and Banks it is because the smallest party is the formateur only if the other two parties fail to form a government.

To examine the dynamics, consider first the case in which parties and voters are myopic and care only about the current period. Then, as the political system moves through time Proposition 4 implies the following Corollary.

Corollary 4: If parties and voters are myopic, the following holds for all periods after the first: (1) There are only minority parliaments. (2) All realized government policies are of the form x^{ij} $(j \neq k \neq i)$. (3) Government policies change every period.

To see this, recall that no matter where the status quo is located at the beginning of the first period, the government will implement a policy x^{ij}, z^i, or $\bar{z}$. For each of these as the status quo for the next period a minority parliament results in all future periods, and the governments are majoritarian with policies x^{ij}. This follows directly from Table 1.

A different scenario would incorporate exogenous shocks to the status quo, for instance due to fluctuations in the economic environment. Consider the case in which the shocks follow a distribution that is unimodal around the chosen government policy. The random draws of realized status quos then generate a stochastic process that generates a distribution over electoral outcomes and government policies. The condition in Proposition 4 determines whether a formateur will form a majoritarian government or a consensus government. For example, if in Figure 4 the status quo q is in $D^2 \cap Q^{VI}$ but not in D^1 or the corresponding disk for party 3, parties 2 and 3 will form a majoritarian government with policy x^{23} and party 1 will form a government with $\bar{z}$. The electoral equilibrium is then as identified in the previous section – party 1 receives a strict majority.

Majority parliaments thus can result from a random status quo. However, Corollary 4 implies that they occur only after a large shock to the status quo, such as an economic or military crisis. The more typical case is a minority parliament with a majoritarian government.

These results were derived under the assumption that all actors are myopic and do not take into account the future consequences of their current actions. The next section analyzes the case of strategic dynamics and demonstrates, for example, that although a majority parliament can result from a random status quo, it will not result from a strategically-positioned status quo.

Strategic Dynamics: Government Formation

Since the policy enacted in one period becomes the status quo in the next period, the formateur has an opportunity to position itself strategically through the policy its government implements. Positioning the status quo for the next period has two effects. First, it affects the electoral equilibrium. Second, it affects which governments form in the next period and their policies. Those governments may be majoritarian or consensus and the parliament may be minority or majority, and those cases are considered next in the context of a two-period model.

The basic intuition underlying the equilibria is that the formateur in period 1 responds to three types of incentives. First, the formateur would like to choose an efficient policy for period 1; i.e., at the centroid of the single-period contract curve with its government partner(s). Second, the formateur prefers to position the period-one policy, and hence the status quo for period two, favorably for the period-two election. Third, the formateur prefers to position the period-one policy so that if it is selected as formateur in period 2 it will be able to strike a favorable bargain with its period-two government partner(s).

We characterize the equilibrium case by case. That is, we first characterize the optimal policy choice for a proposer under the assumption that she will choose a majoritarian government and then characterize the cases under which this supposition holds. The policy associated with a majoritarian government will lead to a minority parliament in the next period. We show, however, that unless Y is very large, a proposer will prefer to choose a policy that leads to a minority parliament. That is, proposers in period 1 have no incentive to choose a current policy that will lead to a majority of seats for their party in the next period. Thus, the finding that the dynamics of parliamentary systems encourage minority parliaments holds for exogenous and strategic dynamics.

For tractability we will only consider symmetric equilibria. That is, voters who in equilibrium are indifferent between voting for two parties are assumed to randomize with the same probability. Note that given Proposition 4 (D) for any status quo located on the lines in Figure 2 such as $q^1 \in Q^I \cap Q^{II}$, the unique electoral outcome is $\rho_3 = m$ and $\rho_1 = \rho_2 = \frac{1-m}{2}$.

Majoritarian Governments

As demonstrated above, in (the final) period 2 majoritarian governments implement policies at the centroid of the preferences of the government parties. The effect of the status quo q^1 on period 2 is summarized by the continuation values $v^i(q^1), i = 1,2,3$, which are the expected utilities of the parties in the electoral, government formation, and legislative equilibrium of the period-2 subgame commencing with a status quo q^1.

Which policy party 1 will choose in period 1 depends on the initial status quo q^0. Suppose that $q^0 \in Q^V \cup Q^{VI}$. In the single-period case this would imply a $q^1 \in Q^I \cap Q^{II}$. We first show that this conclusion holds also for the strategic dynamics case. That is, for the optimal proposal $\hat{x}_1^{12}(\delta) = \frac{1}{2}$ in (18) below.

To show that (18) is the optimal proposal for a majoritarian government, it is necessary to show that in forming a government with party 2, party 1 has a strict incentive to choose a period-one policy in $Q^I \cap Q^{II}$; i.e., on the line from z^1 through $\bar{z}$. If party 1 chooses a period-one policy with $x_1 < \frac{1}{2}$, in period 2 parties 2 and 3 will form governments with policy x^{23} and party 1 will form a government with policy x^{13}. The vote shares are then $\rho_1 = \frac{1}{2}$ and $\rho_2 = \rho_3 = \frac{1}{4}$. Party 1 then chooses the policy x^ℓ according to:

$$x^\ell \in \arg\max_x u^1(x) + \delta v^1(x) + u^2(x) + v^2(x), \tag{13}$$

where, for example,

$$v^2(x) = \frac{1}{2}u^2(x^{13}) + \frac{1}{4}(u^2(x^{23}) + Y + u^3(x^{23}) - u^3(x)) + \frac{1}{4}u^2(x).$$

Then, (13) is

$$x^\ell \in \arg\max_x u^1(x) + \left(1 + \frac{\delta}{4}\right)u^2(x) - \frac{3\delta}{4}u^3(x).$$

It is straightforward to show that the optimal policy has $x_1 = \frac{8-\delta}{4(4-\delta)} > \frac{1}{2}$, which is a contradiction. Consequently, $x_1 \geq \frac{1}{2}$.

In a similar manner if party 1 chooses a period-one policy with $x_1 > \frac{1}{2}$, parties 1 and 3 will form governments with each other and party 2 will form a government with party 3. The vote shares are then $\rho_1 = \rho_3 = \frac{1}{4}$ and $\rho_2 = \frac{1}{2}$. Proceeding as above, the optimal policy has $x_1 = \frac{2-\delta}{4-\delta} < \frac{1}{2}$. Consequently, the optimal period-one policy for party 1 has $x_1 = \frac{1}{2}$.

Proposition 4 then implies an electoral equilibrium where party 3 has a vote share m, and parties 1 and 2 split the remaining votes, so $\rho_1 = \rho_2 = \frac{1-m}{2}$. In period 2 both parties 1 and 2 will form a government with party 3 with respective policies x^{13} and x^{23} given in (6), and party 3 will randomize between these policies. The continuation value $v^1(q^1)$ for party 1 is then

$$v^1(q^1) = \frac{1-m}{2}\Big(u^1(x^{13}) + Y + u^3(x^{13}) - u^3(q^1)\Big) + \frac{1-m}{2}u^1(x^{23}) + \frac{m}{2}(u^1(x^{23}) + u^1(q^1)), \tag{14}$$

where the three terms correspond to party $\ell = 1, 2, 3$ as formateur, respectively. The other continuation values are

$$v^2(q^1) = \frac{1-m}{2}\Big(u^2(x^{23}) + Y + u^3(x^{23}) - u^3(q^1)\Big) + \frac{1-m}{2}u^2(x^{13}) + \frac{m}{2}(u^2(x^{13}) + u^2(q^1)) \tag{15}$$

$$v^3(q^1) = (1-m)u^3(q^1) + \frac{m}{2}(u^3(x^{13}) + Y + u^1(x^{13}) - u^1(q^1) + u^3(x^{23}) + Y + u^2(x^{23}) - u^2(q^1)). \tag{16}$$

As in period 2, the equilibrium government formation strategies in period 1 are chosen after the period-one election and hence are independent of the period-one seat shares. Suppose that party 1 is selected as the formateur in period 1, and suppose that party 2 is the government partner. If the period-two policies are constant in q^1, the period-one proposal $\hat{x}^{12}$ by party 1 to party 2 is[17]

$$\begin{aligned} (\hat{x}^{12}, y_2^1) \in \arg\max_{x, y_2} u^1(x) + \delta v^1(x) + Y - y_2 \\ s.t.\ u^2(x) + \delta v^2(x) + y_2 \geq u^2(q^0) + \delta v^2(q^0), \end{aligned} \tag{17}$$

where $\delta \in [0,1]$ is a (common) discount factor and q^0 is the status quo at the beginning of period 1. Substituting the continuation values from (14) and (15) and y_2 from the equality in the constraint in (17) yields

$$\hat{x}^{12} \in \arg\max_x \Big(1 + \delta\frac{m}{2}\Big)u^1(x) + \Big(1 + \delta\frac{m}{2}\Big)u^2(x) - \delta\Big(\frac{1}{2} - m\Big)u^3(x).$$

For the case of ideal points as in Figure 2, the optimal policy $\hat{x}^{12}$ is

$$\hat{x}^{12}(\delta) = \Big(\frac{1}{2}, -\frac{\delta(1-m)\sqrt{3}}{4 - 2\delta(1-2m)}\Big), \tag{18}$$

and the transfer y_2^1 is

$$y_2^1 = u^2(q^0) + \delta v^2(q^0) - u^2(\hat{x}^{12}(\delta)) - \delta v^2(\hat{x}^{12}(\delta)).$$

[17] The set of $q^1 \in Q^I \cap Q^{II}$ such that the period-two policies are constant in q^1 is characterized below.

The policy in (18) is on the line from z^3 through the centroid and is outside the Pareto set for $\delta \in (0,1]$. If this policy is in the cross-hatched region in Figure 3, the equilibrium in period 2 will be that each party as formateur will form a majoritarian government, and from the previous section the vote shares are $\rho_1 = \rho_2 = \frac{1-m}{2}$ and $\rho_3 = m$.

The policy $\hat{x}^{12}(\delta)$ in (18) is chosen outside the Pareto set so as to place party 3 in an unfavorable position for period 2. Party 3 has the both the minimal number of seats, and if either party 1 or 2 is selected as the formateur in period 2, it extracts transfers from party 3. The formateur (party 1) in period 1 also favorably positions itself, since it will receive a $\frac{1-m}{2}$ share of the seats.

The policy component $\hat{x}_2^{12}(\delta)$ in (18) is a strictly decreasing function of the discount factor. Consequently, political patience (high δ) results in more extreme policies, since it is more important to the period-1 formateur to position itself favorably for period 2. The policy $\hat{x}_2^{12}(\delta)$ is also strictly increasing in the vote share m of the smallest party.

The optimal policy in (18) for party 1 in period 1 will be an equilibrium; i.e., generate the subgame equilibrium on which the $v^i(q^i)$ in (14)-(16) are based, if $q^1 = \hat{x}^{12}(\delta)$ is in $D^1 \cap D^2$ and in D^{12} in Figure 3. This requires that $\frac{1}{2} \geq \frac{\delta(1-m)\sqrt{3}}{2(2-\delta(1-2m))}$. Consequently, if

$$\delta \leq \delta^*(m) \equiv \frac{2}{1-2m+\sqrt{3}(1-m)}, \tag{19}$$

the policy in (18) is an equilibrium. For example, if $m = 0$, $\delta^*(0) = 0.732$. For $m \geq \frac{\sqrt{3}-1}{2+\sqrt{3}} = 0.1962$, the inequality in (19) is satisfied for all δ. If the inequality in (19) is satisfied, there is an equilibrium for $q^0 \in Q^V \cup Q^{VI}$ in which if selected as the formateur in period 1, party 1 forms a government with party 2 with the policy $\hat{x}^{12}(\delta)$ in (18). For $\delta > \delta^*(m)$ the policy in (18) would result in all parties forming consensus governments in period 2. Party 1 prefers to form a majoritarian government in period 2, so it will form a period-one government with policy $\hat{x}^{12}(\delta^*(m)) = (\frac{1}{2}, -\frac{1}{2})$. This is proven in Appendix C. Consequently, for all $\delta \in [0,1]$, the governments in period 2 are majoritarian.

Proposition 5: Suppose that q^0 is such that party 1 as formateur forms a period-one majoritarian government. The equilibrium policy is $\hat{x}^{12}(\delta)$ in (18) for $\delta \leq \delta^*(m)$ and $\hat{x}^{12}(\delta^*(m))$ for $\delta \geq \delta^*(m)$. The electoral equilibrium in period 2 is $\rho_1 = \rho_2 = \frac{1}{2}(1-m)$ and $\rho_3 = m$, and the government formation-legislative equilibria in period 2 then are that party 1 (2) forms a government with party 3 with policy $x = x^{13}$ (x^{23}), and party 3 randomizes equally among the policies x^{13} and x^{23}.

What remains to be shown is that party 1 would indeed prefer party 2 as its coalition partner if $q^0 \in Q^V \cup Q^{VI}$. Defining $\hat{x}^{13}$ analogously to $\hat{x}^{12}$ in (18) and letting $\hat{W}^{1\ell}(q^0), \ell = 2, 3$, be the maximand in (17) at the optimal policy in period 1, it is straightforward to show that

$$\hat{W}^{12}(q^0) - \hat{W}^{13}(q^0) = (1+\delta)(u^2(q^0) - u^3(q^0)),$$

so the party that is the more disadvantaged by the status quo is preferred as a government partner. Consequently, the same regions in Figure 2 identify which majoritarian government partner the period-one formateur chooses.

Consensus Government in Period 1

The previous analysis applied to the case where the proposer would choose a majoritarian government. Consider next the possibility of one party forming a consensus government in period 1 with policy $\bar{z}$. Then, Proposition 1 indicates that in period 2 the formateur i will form a majoritarian government at either the policy $x^{ij}, j \neq i$, or $x^{ik}, k \neq i, j$, given in (6). To analyze this case, it is necessary to characterize the subgame equilibria if $\bar{z}$ is rejected and if it is approved. If $\bar{z}$ is rejected, then $q^1 = q^0$, and the period-2 equilibrium will be either $\bar{z}$ by all three parties if $q \notin D^1 \cup D^2 \cup D^3$, $\bar{z}$ by one party and x^{ij} by the other two parties (e.g., $q^0 \notin D^2 \cup D^3$ and $q^0 \in D^1$), or x^{ij} by all three parties.

Consider first a q^0 such that if $q^1 = q^0$, i.e., the formateur's proposal in period 1 were rejected, all parties would propose $\bar{z}$ in period 2, which requires that $q^0 \notin D^1 \cup D^2 \cup D^3$. The continuation values $v^i(q^0)$ are

$$\begin{aligned}
v^1(q^0) &= \frac{1}{3}\left[u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^2(q^0) - u^3(q^0)\right] + \frac{2}{3}u^1(q^0) \\
v^2(q^0) &= \frac{1}{3}\left[u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^1(q^0) - u^3(q^0)\right] + \frac{2}{3}u^2(q^0) \qquad (20) \\
v^3(q^0) &= \frac{1}{3}\left[u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^2(q^0) - u^1(q^0)\right] + \frac{2}{3}u^3(q^0),
\end{aligned}$$

where the period-two vote shares are $\rho_i = \frac{1}{3}, i = 1, 2, 3$.

Now, consider party 1's choice of government in period 1. If there is a consensus government in period 1 with policy $\bar{z}$, the period-two electoral equilibrium is $\rho_i = \frac{1}{3}, i =$

1, 2, 3, and all parties form majoritarian governments. The continuation values $\bar{v}^i$ are then

$$\bar{v}^i = \frac{1}{3}\left[u^i(x^{ij}) + Y + u^j(x^{ij}) - u^j(\bar{z})\right] + \frac{1}{3}u^i(\bar{z}) + \frac{1}{3}u^i(x^{kj}), \;\; i = 1,2,3, j \neq i, k \neq j. \quad (21)$$

The two-period expected utility $\bar{W}^1$ if party 1 is selected as the formateur in period 1 and forms a consensus government with policy $\bar{z}$ and continuation values in (21) is

$$\bar{W}^1 = u^1(\bar{z}) + \delta\bar{v}^1 + u^2(\bar{z}) + \delta\bar{v}^2 + u^3(\bar{z}) + \delta\bar{v}^3 + Y - u^2(q^0) - \delta v^2(q^0) - u^3(q^0) - \delta v^3(q^0). \quad (22)$$

If in period 1 party 1 were instead to form a majoritarian government we can use equations (14) to (16) given the respective continuation values $\hat{v}^i(\cdot)$ upon substituting $q^1 = \hat{x}^{12}(\delta)$. The expected utility $\hat{W}^1$ of party 1 if selected as the formateur in period 1 is then

$$\hat{W}^1 = u^1(\hat{x}^{12}(\delta)) + \delta\hat{v}^1(\hat{x}^{12}(\delta)) + u^2(\hat{x}^{12}(\delta)) + \delta\hat{v}^2(\hat{x}^{12}(\delta)) + Y - u^2(q^0) - \delta v^2(q^0). \quad (23)$$

Party 1 will form a consensus government in period 1 if and only if $\bar{W}^1 \geq \hat{W}^1$. The difference in utilities is, using symmetry, for $\delta \leq \delta^*(m)$,[18]

$$\begin{aligned}\hat{W}^1 - \bar{W}^1 &= 2u^1(\hat{x}^{12}(\delta)) + 2\delta\hat{v}^1(\hat{x}^{12}(\delta)) - 3u^1(\bar{z}) - 3\delta\bar{v}^1 + u^3(q^0) + \delta v^3(q^0)\\ &= 1 + \frac{m\delta}{2} - \frac{\delta}{3} - \frac{(2-\delta(1-2m))^2 + 3(1-m)^2\delta^2}{(2-\delta(1-2m))^2} + \frac{3}{4}\delta(1-m)\frac{(2+\delta m)^2}{(2-\delta(1-2m))^2}\\ &+ \left(1 + \frac{2\delta}{3}\right)u^3(q^0) - \frac{\delta}{3}u^2(q^0) - \frac{\delta}{3}u^1(q^0) + \delta Y\left(\frac{1}{3} - m\right).\end{aligned} \quad (24)$$

Inspection indicates that unless q^0 is far from the Pareto set or Y is very (compared to the policy cost) negative (since m is smaller than $\frac{1}{3}$ in all parliamentary systems), the formateur in period 1 will form a majoritarian government.

As demonstrated in Appendix D, this conclusion is robust to the location of the initial status quo. These results are summarized in the following proposition.

[18] If $\delta \geq \delta^*(m)$, the difference $\hat{W}^1 - \bar{W}^1$ is

$$\hat{W}^1 - \bar{W}^1 = (1-m)\left(1 - \frac{\sqrt{3}}{2}\right) - \frac{\delta}{3} - \frac{\delta}{3}u^1(q^0) + u^3(q^0)\left(1 + \frac{2}{3}\delta\right) + \delta Y\left(\frac{1}{3} - m\right).$$

Proposition 6: The period 1 formateur forms a consensus government only if $\mid Y \mid$ is very large or q^0 is far from the Pareto set (in the sense that (24) is negative). Otherwise, period 1 formateurs form majoritarian governments.

A Majority Party in Period 2

As indicated in Section III, a formateur in period 1 could form a government with a policy x^* such that if it were also the formateur in period 2 it would choose the policy $\bar{z}$ and the other parties would form majoritarian governments. Then, in the period-two election the period 1 formateur would receive a majority of the votes, and as a majority would form a consensus government and implement $\bar{z}$. As demonstrated next, this is not an optimal strategy for the formateur in period 1.

Suppose that party 1 is the formateur in period 1 and chooses a policy x^* such that if it is the formateur in period 2 it will implement $\bar{z}$, whereas if selected as the formateur the other parties would form majoritarian governments; i.e., $q^1 = x^* \notin D^2 \cup D^3$ and $x^* \in D^1$. The two-period utility W^{1m} of party 1 if the status quo q^0 is such that it forms a government with party 2 with the policy x^* in period 1 then is

$$W^{1m} = u^1(x^*) + \delta v^{1*}(x^*) + u^2(x^*) + \delta v^{2*}(x^*) + Y - u^2(q^0) - \delta v^2(q^0),$$

where the continuation values $v^{i*}(x^*)$ corresponding to party 1 having a majority, being selected as formateur, and implementing $\bar{z}$ are

$$v^{1*}(x^*) = u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^2(x^*) - u^3(x^*)$$

$$v^{j*}(x^*) = u^j(x^*), \;\; j = 2, 3.$$

Party 1 chooses x^* to maximize W^{1m} subject to the constraints that in period 2 it would not form a majoritarian government with either party 2 or 3 and that parties 2 and 3 would form majoritarian governments. These conditions imply the following constraints:

$$u^j(x^*) \leq -\frac{1}{2}, \;\; j = 2, 3 \tag{25}$$

$$u^1(x^*) \geq -\frac{1}{2}. \tag{26}$$

For $q \in Q^V \cup Q^{VI}$ the relevant constraint in (25) is $u^2(x^*) \leq -\frac{1}{2}$, and party 1's optimal proposal is $x^* = (1 - \frac{1}{\sqrt{2}}, 0)$. Then,

$$W^{1m} = -2 + \sqrt{2} + \delta\Big(\frac{1}{2} - \frac{1}{\sqrt{2}}\Big) + (1+\delta)Y - u^2(q^0) - \delta v^2(q^0).$$

It is straightforward but tedious to show numerically that in period 1 party 1 prefers $\hat{x}^{12}(\delta)$ in (18) to x^* unless Y is very large; i.e., unless the officeholding gains are very large relative to the policy utility. To illustrate this, the expected utility $\hat{W}^1$ in (23) is

$$\begin{aligned}\hat{W}^1 =& 2\Big(1 + \delta\frac{m}{2}\Big)u^1(\hat{x}^{12}(\delta)) - \delta(1-m)u^3(\hat{x}^{12}(\delta)) + (1 + \delta(1-m))Y \\ & - \delta\Big(\frac{1-m}{2} + \frac{3}{4}\Big) - u^2(q^0) - \delta v^2(q^0).\end{aligned}$$

Then, evaluating $\hat{W}^1 - W^{1m}$ indicates that this difference is positive unless $\delta m Y$ is very large relative to the policy disutility. For example, if $m = 0$, then

$$\hat{W}^1 - W^{1m} = \frac{1}{4} + \frac{\sqrt{3}}{2} - \sqrt{2} + \frac{1}{\sqrt{2}} > 0.$$

As another example, if $\delta = 1$ and $m = 0.05$,

$$\hat{W}^1 - W^{1m} = 0.3156 - 0.05Y,$$

which is nonnegative unless $Y \geq 6.312$. For the purpose of comparison, the expected policy utility component of $\hat{W}^1$ is -0.4773. Thus, even though party 1 could position itself to win a majority in the period-2 election, it prefers not to do so unless Y is very large.

The intuition underlying this result centers both on what party 1 gives up in period 1 by choosing x^* rather than $\hat{x}^{12}(\delta)$ and on its gain as a majority government in period 2, given that its potential government partner in period one fully anticipates the period-two equilibrium. In period 1, party 2 anticipates a majority government in period 2 and thus has a lower continuation value (equal to $u^2(x^*) = -\frac{1}{2}$) than if a majoritarian government were formed. This raises the period-one cost of implementing x^* and eliminates the incentive for party 1 to position itself to win a majority in the final period. The only qualification is that a majority government provides officeholding benefits ($\delta m Y$) in period 2. This result is summarized as:

Proposition 7: Unless Y is very large relative to the policy utility, in period one the formateur forms a government with a policy such that a minority parliament is elected and the period-two governments are majoritarian with policies given in (6). Majority parties then do not result.

CONCLUSIONS

Parliamentary systems include four principal institutions three of which – elections, government formation, and legislation – determine representation, governments, and legislation – and the fourth – government termination procedures – determines when elections are held and whether governments change during the constitutional interelection period. The strategic theory presented here provides predictions about and explanations for election outcomes, governments formed, and their policies. The natural sequential structure of determining representation followed by governments followed by legislation allows equilibria to be characterized for multi-dimensional policy spaces.

The theory also provides insight to the dynamics of parliamentary systems under two conditions. If parties and voters are myopic, the equilibria characterized for the single-period model allow the examination of exogenous dynamics resulting from the policy choice in the previous period and random shocks. In the absence of random shocks, the governments that form are majoritarian, their policies maximize the utility of the government parties, and representation changes from period to period as a function of the policy of the previous government. With exogenous shocks the same outcomes result unless the shock is sufficiently great that one or all the parties would form a consensus government. A consensus government thus results only when the shock leaves the status quo sufficiently far from the center of preferences; e.g., when there is a crisis. If only one party would form a consensus government that party receives a majority in the election. If all parties would form a consensus government, each receives in expectation one-third of the seats.

When the parties and voters are fully rational and anticipate future actions, parliamentary systems have an internal, strategic dynamic even in the absence of shocks. In forming a government in one period, the formateur and its potential government partners anticipate the effect of the current policy on future elections, government formation, and legislation. In a two-period model the formateur in the first period could position the policy such that it receives a majority of the votes in the next period, but unless the officeholding benefits are very large, it prefers to choose a period-one policy such that minority parliaments result. The formateur, however, chooses that policy to advantage itself in the next election in the sense that it has as large a vote share as any other party. Moreover, among all the policies with that property it prefers one that places at least one of the other parties in a weak bargaining position in the period-two government formation. That policy is outside the Pareto set of the parties' preferences, so elections do not moderate policy outcomes. This result depends importantly on parties being unable to commit credibly to the policies they would implement if they were selected as formateur. The study of the endogenous generation of credible policy commitments thus is potentially important for the study of the dynamics of parliamentary systems.

Two features of the model considerably simplify the characterization of the equilibria and allow the dynamics of parliamentary systems to be characterized. One is that there are three parties. Then, any two of the three parties in a minority parliament constitute a majority, so their seat shares are irrelevant in government formation (once a formateur has been selected) and in legislation. If there were more than three parties so that the set of decisive coalitions is greater, seat shares can matter and the characterization becomes more complex. The second is that parties have policy and officeholding preferences that can be represented by a quasi-linear utility function. Then, with unrestricted transfers, the set of policies the possible governments will choose in any subgame equilibrium is finite and small. This contributes to the tractability of the model.

Appendix A

Robustness: Majoritarian Governments

Nonnegative Transfers

The government formation-legislative equilibria characterized in (6) and (4) have been established under the assumption that the transfers are unrestricted, which allows the formateur to form a government with a policy at the centroid of the preferences of the government coalition. The government formed is the one for which the formateur can strike the better bargain, and with symmetric locations of party ideal points that is the party that is the more disadvantaged by the status quo. This result also holds for the case in which transfers are restricted to be nonnegative.

To show this, suppose that party 1 is the formateur. Party 1 will either propose its ideal point z^1 if one of the other parties prefers it to the status quo or will propose a policy that it most prefers subject to the (binding) constraint that the other party joins the government. First, suppose that $u^\ell(z^1) > u^\ell(q), \ell = 2$ or $\ell = 3$. Then, since $y_\ell \geq 0, \ell = 2, 3$, party 1 will propose z^1 and it will be supported by the party that is the more disadvantaged by the status quo. Next, suppose that neither party 2 nor party 3 prefers z^1 to the status quo q. If party 1 forms a government with party ℓ, its policy $\tilde{x}^{1\ell}$ satisfies $u^\ell(\tilde{x}^{1\ell}) = u^\ell(q)$, and its utility is $\tilde{W}^{1\ell} = u^1(\tilde{x}^{1\ell}) + Y$. Party 1 will then form a government with party 2 if and only if

$$\tilde{W}^{12} - \tilde{W}^{13} = u^1(\tilde{x}^{12}) - u^1(\tilde{x}^{13}) \geq 0.$$

It is straightforward to see that if $u^2(q) \leq u^3(q)$, then $u^1(\tilde{x}^{12}) \geq u^1(\tilde{x}^{13})$. Party 1 thus prefers to form a government with the party that is the more disadvantaged by the status quo. Moreover, $y^i_\ell = 0, \ell = 2, 3$.

To illustrate that policies are more extreme when transfers are restricted, consider the government formation and policy choice problem in (3) with $y_j \geq 0$ and $q \in Q^{VI}$. For the case in which $z^1 = (0,0)$, $z^2 = (1,0)$, $z^3 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$, and $q = (\frac{1}{4}, \frac{\sqrt{3}}{4})$, the optimal proposal for $i = 1$ and $j = 2$ is $\tilde{x}^{12} = (1 - \frac{\sqrt{3}}{2}, 0)$. With unrestricted transfers party 1's proposal is $x^{12} = (\frac{1}{2}, 0)$, which is closer to the centroid of the preferences of the parties than is $\tilde{x}^{12}$.

Symmetry

Symmetry of the ideal points as in Figure 2 is important to the result in Proposition 1 that the formateur forms a government with the other party that is in the weaker bargaining position. Consider the case in which the ideal point z^3 of party 3 can be located anywhere. Party 1 prefers to form a majoritarian government with party 3 rather than party 2 if and only if

$$W^{13} - W^{12} = u^1(x^{13}) + u^3(x^{13}) - u^3(q) - u^1(x^{12}) - u^2(x^{12}) + u^2(q).$$

Since x^{13} and x^{12} are centroids of the preferences of the coalitions, $u^1(x^{13}) = u^3(x^{13})$ and $u^1(x^{12}) = u^2(x^{12})$. Consequently,

$$W^{13} - W^{12} = 2(u^1(x^{13}) - u^1(x^{12})) - (u^3(q) - u^2(q)).$$

Whether party 1 forms a government with party 2 or 3 thus depends on both how disadvantaged the parties are by the status quo and how close their ideal points are to that of party 1. A sufficient condition for party 1 to form a government with party 3 is that party 3 is the more disadvantaged by the status quo $u^3(q) \le u^2(q)$ and is also closer to party 1 than is party 2. A party with extreme preferences thus is not an attractive government partner unless it is quite disadvantaged by the status quo. Thus, with two parties equally distant from the status quo, the formateur forms a government with the party with the more closely-aligned policy preferences. That is, extreme parties are less likely to be in government.

As an example when the sufficient conditions are not satisfied, consider the case in which $z^1 = (0,0)$, $z^2 = (1,0)$, and $z^3 = (\frac{1}{2}, z_2^3)$, $z_2^3 \geq 0$. Party 3 is thus farther from party 1's ideal point than is party 2 if $z_2^3 > \frac{\sqrt{3}}{2}$, and party 1 forms a government with party 3 for all

$$z_2^3 \le \left(\frac{3}{4} - 2R\right)^{\frac{1}{2}}, \tag{A1}$$

where $R = u^3(q) - u^2(q)$ is party 3's relative disadvantage. If party 3 is more disadvantaged by the status quo than is party 2 (i.e., $R<0$), it will be in the government provided that its policy preferences are not too extreme. This is summarized as:

Corollary to Proposition 1: Let z^3 be equidistant from $z^1 = (0,0)$ and $z^2 = (1,0)$. Party 1 forms a government with party 3 if and only if $(A1)$ is satisfied; i.e., party 3's policy preferences are not too extreme.

Differing Preference Intensities

To examine the case of differing policy preference intensities, suppose the parties' policy preferences corresponding to $(1a)$ are given by

$$u^i(x) = -a_i(x_1 - z_1^i)^2 - (x_2 - z_2^i)^2, \tag{A2}$$

where $a_i, i = 1,2,3$, is a positive constant. Then, for the ideal points $z^1 = (0,0), z^2 = (1,0)$, and $z^3 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ the policies chosen by majoritarian governments are

$$x^{12} = \Big(\frac{a_2}{a_1 + a_2}, 0\Big), \qquad x^{13} = \Big(\frac{a_3}{2(a_1 + a_3)}, \frac{\sqrt{3}}{4}\Big), \qquad x^{23} = \Big(\frac{a_2 + \frac{1}{2}a_3}{a_2 + a_3}, \frac{\sqrt{3}}{4}\Big).$$

If party 1 is the formateur, it prefers to form a majoritarian government with party 2 rather than party 3 if

$$\Delta u^1 = a_1\left[\frac{-4a_2(a_1 + a_2)(a_1 + a_3)^2 + a_3(a_1 + a_3)(a_1 + a_2)^2}{4(a_1 + a_2)^2(a_1 + a_3)^2}\right] + \frac{3}{8} - u^2(q) + u^3(q) \geq 0, \tag{A3}$$

where $\Delta u^1 = u^1(x^{12}) - u^1(x^{13})$. To illustrate this condition, let parties 1 and 2 have equal policy intensities and let those intensities be $a_1 = a_3 = 1$. As the parameter a_2 increases, the policy x^{12} is such that x_1^{12} increases.

To characterize the set of status quos such that party 1 forms a government with party 2, evaluate $(A3)$ at $a_1 = a_3 = 1$ to obtain

$$\frac{-14a_2 + 2}{16(1 + a_2)} + \frac{3}{8} - u^2(q) + u^3(q) \geq 0. \tag{A4}$$

Solving the equality in $(A4)$ for $q_2(q_1)$ yields

$$q_2(q_1) = \frac{1}{\sqrt{3}}\Big(\frac{14a_2 - 2}{16(1 + a_2)} + \frac{3}{8} - a_2(q_1 - 1)^2 + \Big(q_1 - \frac{1}{2}\Big)^2\Big).$$

Evaluating this at $a_2 = 1$ yields the line in Figure 2 through z^1 and $\bar{z}$. Differentiating with respect to a_2 yields

$$\frac{dq_2}{da_2} = \frac{1}{\sqrt{3}}\Big(\frac{1}{(1 + a_2)^2} - (q_1 - 1)^2\Big).$$

When evaluated at $a_2 = 1$, this derivative is positive (zero) (negative) as $q_1 > (=) < \frac{1}{2}$. Thus, for a given q_1 the set of q_2 such that party 1 forms a government with party 2 is greater (smaller) if $q_1 > (<) \frac{1}{2}$. The line in Figure 2 thus rotates counterclockwise through the centroid $\bar{z} = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$.

Extended Bargaining

To examine the effect of multiple rounds of government formation, first consider two rounds in which if a government is not formed in the first round, a second round commences with parties again selected with probabilities equal to their seat shares. If no government is formed in either round, a caretaker government maintains the status quo. Let the continuation values if the first round fails to produce a government be denoted by $g^i(q), i = 1, 2, 3$. For a $q \in Q^{VI}$, for example, and such that each party would form a majoritarian government, the continuation values are

$$\begin{aligned} g^1(q) &= \rho_1[u^1(x^{12}) + u^2(x^{12}) + Y - u^2(q)] + (1 - \rho_1)u^1(x^{23}) \\ g^2(q) &= (1 - \rho_2)u^2(q) + \rho_2[u^2(x^{23}) + u^3(x^{23}) + Y - u^3(q)] \\ g^3(q) &= \rho_1 u^3(x^{12}) + \rho_2 u^3(q) + \rho_3[u^3(x^{23}) + u^2(x^{23}) + Y - u^2(q)]. \end{aligned} \tag{A5}$$

In round one suppose that party 1 is the formateur. If in round 1 party 1 forms a majoritarian government with policy x, party 2 will accept the proposal if

$$u^2(x) + y_2^1 \geq g^2(q),$$

so

$$y_2^1 = g^2(q) - u^2(x).$$

Party 1 then chooses the policy according to

$$\hat{x}^1 \in \arg\max_x W^{12} = u^1(x) + u^2(x) + Y - g^2(q),$$

so party 1 will choose the policy $\hat{x}^1 = x^{12}$. This verifies that the policies chosen by governments are independent of the number of rounds. Party 1 can also form a consensus government in which case the policy is $\bar{z}$ and the utility $\bar{W}^1$ is

$$\bar{W}^1 = u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - g^2(q) - g^3(q).$$

Party 1 will thus form a majoritarian government with party 2 rather than a consensus government if and only if $W^{12} \geq \bar{W}^1$ or

$$\frac{1}{2} \geq -g^3(q), \tag{A6}$$

which is directly analogous to the condition in (12). The condition in $(A6)$ defines a set $G^3(q) \equiv \{q \mid \frac{1}{2} \geq - g^3(q)\}$, which depends on the seat shares $\rho_i, i = 1,2,3$, as well as on Y. Thus, in general $G^3(q)$ is not a disk, as it is in the case with one round of government formation.

As an example, suppose that the seat shares are equal. Then, the condition in $(A6)$ is

$$\frac{1}{4} + Y \geq q_1 - \sqrt{3}\, q_2.$$

Consequently, the boundary of the set $G^3(q)$ is a linear function parallel to the line through z^1 and $\bar{z}$.[19]

Extended bargaining can also result in the transfer going from the formateur to the government parties. To illustrate this, suppose that $\rho_i = \frac{1}{3}, i = 1,2,3$ and q is near z^1 so that the continuation values in $(A5)$ hold. Then, majoritarian governments will form, and $g^2(q) = -\frac{1}{2} + \frac{1}{3}Y$ and $g^3(q) = -\frac{5}{4} + \frac{1}{3}Y$. If party 1 is the formateur, it will form a government with party 2 rather than a consensus government for $Y > -\frac{1}{4}$, using $(A6)$.[20] Then, $y_2^1 = -\frac{1}{4} + \frac{1}{3}Y$, which is nonnegative for $Y \geq \frac{3}{4}$. Consequently, if the gains Y from officeholding are sufficiently large, the possibility of multiple rounds of government formation attempts can lead the formateur to provide transfers to its government partner.

Next, consider the government formation process in which the formateur first forms a proto-coalition and then the coalition parties bargain sequentially over the division of the gains where the probabilities of being selected to make the next offer are proportional to the seat shares of the parties. Let the vote (seat) shares be ordered as $\rho_1 > \rho_2 \geq \rho_3 \geq m$, and let $r^{ij} \equiv \frac{\rho_i}{\rho_i + \rho_j}$. The ordering of vote shares is then:

$$r^{12} \geq r^{13}, \qquad r^{23} > r^{21}, \qquad r^{32} > r^{31}. \qquad (A7)$$

[19] If there were three rounds of government formation attempts before a caretaker government assumes office, the policies a government forms are the same as with one or two rounds. The condition analogous to $(A6)$ is

$$\frac{1}{2} \geq - h^3(q),$$

where $h^3(q)$ is the continuation value for the final two rounds of government formation.

[20] Note that with equal vote shares, party 1 will form a government with party 2 if it is the formateur in round 1 and if it is the formateur in round 2.

The aggregate gain G^{ij} for a coalition is

$$G^{ij} = u^i(x) + u^j(x) - u^i(q) - u^j(q),\ i \neq j. \tag{A8}$$

The coalition maximizes the aggregate gain which yields the policies in (6). The parties then bargain over the division of the maximal aggregate gain $\hat{G}^{ij}$, and the bargaining equilibrium is that party i obtains a share $r^{ij}\hat{G}^{ij}$ and j obtains a share $(1 - r^{ij})\hat{G}^{ij}$. If selected as formateur, party i will form a proto-coalition with party j rather than party k if and only if

$$r^{ij}\hat{G}^{ij} \geq r^{ik}\hat{G}^{ik},\ k \neq j.$$

Consider a status quo $q \in Q^{VI}$, which implies that

$$\hat{G}^{12} \geq \hat{G}^{13}, \qquad \hat{G}^{23} \geq \hat{G}^{21}, \qquad \hat{G}^{23} > \hat{G}^{13}. \tag{A9}$$

Using ($A7$) and ($A9$) in ($A8$) implies that as formateur party 1 will form a government with party 2, and parties 2 and 3 will form governments with each other. Since voters have preferences over policies and not over the division of the gain, the electoral model in Section IV implies that the vote shares are $\rho_1 = \frac{1}{2}$ and $\rho_2 = \rho_3 = \frac{1}{4}$. The seat shares satisfy the hypothesized seat shares and hence ($A7$). Other than the distribution of the gain, this is the same equilibrium as in the basic model. This result holds for all q such that majoritarian governments are formed in the basic model.

Appendix B
Proof of Proposition 4

(A) Suppose that q is such that only one party j would propose a consensus government if selected as formateur in a three-party parliament. By symmetry it is without loss of generality to suppose that $j = 3$, so $q \notin D^1 \cup D^2$. Then both parties 1 and 2 would propose x^{12} if selected. We now construct a class of equilibria where at least a majority of voters vote for 3 (these voters may be coalition pivotal) and at least M voters vote for each of the other two parties (these voters may be representation pivotal). In these equilibria when all voters play their equilibrium strategies the government policy is $\bar{z}$. First, consider voter i's incentive to vote for 3 given that both other parties are represented in the chamber. If a strict majority of other voters vote for 3, consider a set of voters who (collectively) are majority pivotal. It suffices to consider the case in which half the voters vote for party 3, so let voter i with ideal point $z(i)$ be majority pivotal. That is, if i votes for party 3, her expected utility (given that all other voters play their equilibrium strategies) is $u(\bar{z}; z(i))$. However, if i were to deviate, then a minority parliament would result and the outcome of the government formation game would be an even lottery over $\bar{z}$ and x^{12} giving i an expected utility of $EU_i = \frac{1}{2}u(\bar{z}; z(i)) + \frac{1}{2}u(x^{12}; z(i))$. Hence i will vote for 3 if and only

$$u(\bar{z}; z(i)) \;\geq\; u(x^{12}; z(i)). \tag{B1}$$

which holds if and only if

$$z_2(i) \;\geq\; \frac{1}{4\sqrt{3}}. \tag{B2}$$

Thus every voter above the horizontal line at $\frac{1}{6}z_2^3$ has a strict incentive to vote for 3, and every voter below the line has a strict incentive to vote for 1 or 2. The area above the horizontal line constitutes a strict majority of all voters. Consequently, i and any set of voters including i will not deviate from voting for party 3. Second, consider a voter i who is representation pivotal with respect to party k; i.e., i votes for party k and k has exactly M votes. (If i is not pivotal, a deviation does not affect outcomes.) If i were to deviate, then a two-party parliament would result with a majority of seats for 3 and outcome $\bar{z}$. This is the same outcome if i votes for party k, so there is no incentive to deviate. The suggested strategy combination is thus an equilibrium with outcome $\bar{z}$.

To demonstrate that that $\bar{z}$ is the unique strong Nash equilibrium outcome, it is sufficient to show that $\bar{z}$ is strictly preferred by a majority to every other equilibrium outcome. We have already seen that $\bar{z}$ is strictly preferred by a strict majority to any $x^{\ell k}$ $(\ell \neq k)$. What remains to be shown is that a majority prefers $\bar{z}$ to each party ideal point z^j, since the latter could be implemented by, for example, a voting strategy combination that would only have party j represented in the chamber. But since a strict majority of voters has

$$u(\bar{z}; z(i)) \;\geq\; u(z^j; z(i)),\; j + 1, 2, 3, \tag{B4}$$

a strict majority of the electorate strictly prefers $\bar{z}$ to any z^j.

(B) Consider a q such that no party would propose a consensus government if selected as formateur in a minority parliament. Consider the case in which $q \in Q^I$, so party 1 will propose x^{13} if selected, whereas parties 2 and 3 will propose x^{23}. Given symmetry and Proposition 1, this is without loss of generality. Given a proportional selection rule this implies a lottery over the two government policies with x^{13} occurring with probability ρ_1 and x^{23} occurring with probability $1 - \rho_1$. We now construct a class of equilibria with the following property: (a) exactly $\frac{N}{2}$ voters vote for party 1 with probability one, (b) at least M voters vote for party 3 with probability one, (c) at least M voters vote for party 2 with probability one, and (d) the remaining voters (if any) randomize between parties 2 and 3 with probability $r_i \in [0, 1]$. This implies an even lottery over x^{13} and x^{23}. So, in equilibrium all voters are selection pivotal, and depending on the r_i voters may also representation or coalition pivotal. First consider a voter i of type (a). Voter i has an incentive to vote for party 1 if and only if

$$u(x^{13}; z(i)) \geq u(x^{23}; z(i)). \tag{B5}$$

This defines a line that divides the set of voters into two equal halves. Every voter to the left $(z_1(i) < \frac{1}{2})$ has a strict incentive to vote for party 1, whereas every voter to the right $(z_1(i) > \frac{1}{2})$ votes either for party 2 or party 3. By symmetry each group comprises exactly half the electorate, and no voter has an incentive to deviate.

For a voter of type (b) suppose that (a), (c) and (d) hold and exactly $M - 1$ other voters cast their ballots for party 3. Voter i is thus representation pivotal with positive probability.[21] If i votes for party 1 or 2, then party 3 does not meet the threshold M, and party 1 will have a majority of seats. Depending on the location of q, party 1 either implements z^1 or x^{13}. In the latter case i will vote for party 3 if and only if

$$\frac{1}{2}u(x^{13}; z(i)) + \frac{1}{2}u(x^{23}; z(i)) \geq u(x^{13}; z(i)) \tag{B6}$$

or

$$u(x^{23}; i) \geq u(x^{13}; i) \tag{B7}$$

which, by the same argument as above, holds for half the electorate. Thus, the voters i voting for party 3 satisfy (B7). In the former case in which z^1 is implemented, i will vote

[21] Given the voting strategies and a finite number of voters, there is a positive probability that a voter in (b) will be representation pivotal.

for party 2 if and only if

$$\frac{1}{2}u(x^{13};i)+\frac{1}{2}u(x^{23};i)\geq u(z^1;i)$$

or

$$z_2(i)\geq\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}z_1(i) \tag{B8}$$

The right side of this inequality defines a line with half the slope of the line connecting z^2 and z^3 and which intersects the line connecting z^1 and z^2 at $(\frac{1}{2},0)$. Hence, more than half the electorate strictly prefers to vote for party 3. An analogous argument holds for (c). For (d) there is nothing left to prove. This shows that the strategy combination is a Nash equilibrium.

To see that it is also strong we need to show that no other implementable government policy would be strictly preferred to an even lottery over x^{13} and x^{23}. But given Propositions 2 and 3 the only other options are z^1, z^2, z^3,x^{13}, and x^{23}. We have shown the claim for z^1 in (B8). For z^2 and z^3 an analogous argument holds. The inequality in (B7) implies the claim for x^{13} and x^{23}.

(C) If q is such that all parties would propose $\bar{z}$ if selected as the formateur, then all voters are indifferent as to how they vote. Any allocation of votes is thus a strong equilibrium.

(D) Consider the case in which $q\in Q^I\cap Q^2\cap D^1\cap D^2$; i.e., the status quo is on the boundary of two of the regions in Figure 2. Then, as formateur party i will propose the policy $x^{i3}, i=1,2$, and party 3 will randomize between those two policies. A strong equilibrium is M voters voting for party 3, $\frac{1-m}{2}$ of the voters with $z_1(i)\leq\frac{1}{2}$ voting for party 1, and $\frac{1-m}{2}$ of the voters with $z_1(i)\geq\frac{1}{2}$ voting for party 2. The argument to show that these voting strategies constitute a strong Nash equilibrium is directly analogous to that given above for majoritarian governments. For example, a voter i who is to vote for party 3 is representation pivotal, and if i were to vote for party 1, that party would have a majority and choose policy x^{12} or z^1. A strict majority of voters prefers a lottery between x^{12} and x^{13} to either x^{12} or z^1. To show that there is no strong Nash equilibrium with more m voters voting for party 3, suppose that $\rho_3>m$. Each of those voters is selection pivotal, and a voter i with $z_1(i)\leq\frac{1}{2}$ prefers to vote for party 1, since that increases the probability that the policy will be x^{13}. Similarly, a voter with $z_1(i) > \frac{1}{2}$ prefers to vote for party 2. Consequently, $\rho_3 = m$. **Q.E.D.**

Appendix C

Proof that Party 1 prefers $\hat{x}^{12}(\delta^*(m)) = (\frac{1}{2}, -\frac{1}{2})$ for $\delta > \delta^*(m)$.

Suppose that party 1 forms a government in period 1 with an extreme policy such that all parties would form a consensus government in period 2 with policy $\bar{z}$. Then, the period-2 electoral equilibrium is $\rho_i = \frac{1}{3}, i = 1, 2, 3$, and the continuation values $v^i(q^1)$ for parties $i = 1, 2$, respectively, are

$$v^1(q^1) = \frac{1}{3}\left[u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^2(q^1) - u^3(q^1) + 2u^1(q^1)\right]$$

$$v^2(q^1) = \frac{1}{3}\left[u^1(\bar{z}) + u^2(\bar{z}) + u^3(\bar{z}) + Y - u^1(q^1) - u^3(q^1) + 2u^2(q^1)\right].$$

For $q^0 \in Q^V \cup Q^{VI}$ the optimal period-1 policy $\bar{x}^{12}$ for party 1 in forming a government with party 2 is

$$\bar{x}^{12} \in \arg\max_x u^1(x) + \delta v^1(x) + Y + u^2(x) + \delta v^2(x) - u^2(q^0) - \delta v^2(q^0).$$

This yields

$$\bar{x}^{12} = \left(\frac{1}{2}, -\frac{\delta}{2\sqrt{3}}\right).$$

This will be an equilibrium if given $q = \bar{x}^{12}$ all three parties prefer to form a consensus government in period 2. This requires that $\bar{x}^{12} \notin D^1 \cup D^2$ or $\frac{1}{2} \leq -\bar{x}_2^{12}$. This, however, is not satisfied for any $\delta \in [0, 1]$. The same result obtains for at least one party for each of the regions Q^ℓ. Consequently, in a two-period model there is no equilibrium in which all three parties form a consensus government in period 2.

Appendix D

Consensus and Majoritarian Governments

The conclusion drawn from (23) depends on the initial status quo q^0, and to explore the robustness of the conclusion, consider the case in which $q^0 \in Q^{VI}$, so if the period-one proposal is rejected, in period 2 party 1 forms a government with party 2 and parties 2 and 3 form governments with each other and the vote shares are $\rho_1 = \frac{1}{2}$ and $\rho_\ell = \frac{1}{4}, \ell = 1, 2$. The continuation values then are

$$v^1(q^0) = \frac{1}{2}[u^1(x^{12}) + Y + u^2(x^{12}) - u^2(q^0)] + \frac{1}{2}u^1(x^{23})$$

$$v^2(q^0) = \frac{3}{4}u^2(q^0) + \frac{1}{4}[u^2(x^{23}) + Y + u^3(x^{23}) - u^3(q^0)]$$

$$v^3(q^0) = \frac{1}{2}u^3(x^{12}) + \frac{1}{4}u^3(q^0) + \frac{1}{4}[u^3(x^{23}) + Y + u^2(x^{23}) - u^2(q^0)].$$

The expected utility $\bar{W}^1$ of party 1 when selected as the formateur in period 1 and proposes a consensus government is given in (22), and party 1's preferences are indicated by

$$\hat{W}^1 - \bar{W}^1 = -\frac{3(2+\delta m)}{4(2-\delta(1-2m))^2}(\delta^2(1-m)^2 + 2 + \delta m) + \frac{1}{2} - \frac{\delta}{2} + \delta Y(\frac{1}{4} - m) + \left(1 + \frac{\delta}{4}\right)u^3(q^0) - \frac{\delta}{4}u^2(q^0).$$

Unless $u^3(q^0)$ and/or Y is very negative, party 1 prefers to form a majoritarian government.

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