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COMPLEMENTARITY, INSTABILITY, AND MULTIPLICITY

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Abstract

A multiplicity of equilibria, steady states, cycles, etc., are prominent features of models with complementarities. Multiplicity suggests the instability of an economic system and the sensitive dependence of the equilibrium behavior on the environment. In this lecture, I discuss some methodological questions concerning the economics of complementarity, with special emphasis on instability and multiplicity. A series of monopolistic competition models are developed to illustrate the argument.

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1. Introduction.

The economics of complementarity has been one of my major research interests in recent years. Broadly speaking, complementarities are said to exist when two or more phenomena reinforce each other. For example, if expansion of industry A leads to an expansion of industry B, which in turn leads to the further expansion of A, then the two industries are complementary to each other. The presence of such complementarity has profound implications on the stability of the system, by making the system highly sensitive to disturbances. If a change in a certain activity is initiated by some external shocks, this leads to a change in complementary activities and starts a process of chain reactions. In the presence of complementarities, even a small cause can be amplified to create a big effect.

The instability of an economic system and the sensitive dependence of economic behavior on external factors, induced by the presence of complementarity, are closely related to the problems of nonlinearity and multiplicity. Putting in some elements of complementarities generally makes a model nonlinear in a nontrivial way, thereby generating multiple equilibria (in both static and dynamic models), multiple steady states with different basins of attraction (in dynamic models), or cycles with multiple phases (in dynamic models). In a nonlinear model with multiple equilibria, even a small perturbation to the parameter value can lead to a discrete change in the number of equilibria, which makes the model useful for explaining the possibility that a drastic structural change may be caused by a minor disturbance to the environment. In a nonlinear dynamic model with multiple asymptotic behaviors, a small difference in the initial condition might fail to dissipate over time; instead, it could be magnified over time to generate sizable differences in the long run. Models with multiple equilibria or multiple steady states offer a most natural way of explaining the diversity observed across geographical regions and across historical periods. As an intellectual exercise, the inherent nonlinearity poses an exciting challenge for a theorist. The challenge is to make a carefully chosen set of assumptions to develop a nonlinear model, which is simple enough to be tractable and yet rich enough to capture the complex nature of the world with complementarities. Multiplicity also poses many intriguing conceptual and philosophical issues.

Multiplicity, instability, and sensitive dependence are all prominent features of the economics of complementarity. At least to me, they are precisely the features that make models with complementarities interesting.[1] Many economists, however, seem to think that these features make models with complementarities unworthy of serious attention. If there were any unstable equilibrium, the economy would never be able to stay close to it, and hence, as the argument goes, the possibility of such instability is of little empirical relevance. Models with the sensitive dependence on the parameter values are by nature the models that cannot yield any robust predictions. And models with multiple equilibria have little predictive contents, and yield few empirically testable implications. Hence, as a matter of principle, we should propose as candidate models of the economy only those with a unique equilibrium, which is robust to a slight perturbation to the parameters. For those who view multiplicity and sensitive dependence as defects of a model, it is thus natural to impose restrictions in their model, which rule out the possibility of complementarities. It is my purpose in this lecture to address some methodological issues, and offer my counterargument to such a familiar set of closely related criticisms and objections to the economics of complementarity.[2]

For this purpose, I am going to develop a series of models and use them to illustrate my argument. It should be emphasized that my goal here is to defend the idea of complementarity, instability and multiplicity, not necessarily the way in which the idea is sometimes implemented in the literature. The

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[1]It should quickly be pointed out that this view is my own, or at least does not seem to be shared by the majority of the economists working in this area. They are mainly interested in the economics of complementarity, because models with complementarities typically demonstrate the possibility of coordination failures, i.e., the existence of an inefficient equilibrium, Pareto-dominated by others. It is generally believed that this feature of the economics of complementarity justifies policy activism. Such a view, however, is unwarranted. Matsuyama (forthcoming) discusses in detail why the logic of coordination failures does not imply a call for more active government interventions.

[2]My view on these methodological issues is far from original. Yet, it has been shaped so gradually over the years through conversations as well as readings that I cannot attribute it to particular sources. In preparing this lecture, however, I found it useful re-reading some of the articles, particularly Romer (1994) and Woodford (1987). Matsuyama (1995b, forthcoming) also discuss some related issues in more detail.

literature on complementarity, just like any other literature, has more than its fair share of bad papers, and I have no intention of defending them.[3] The problem is that, if I defend the economics of complementarity in an abstract way, I would never know which particular study you had in the back of your mind. An explicit use of examples should minimize the risk of generating confusion and misunderstandings, which are often associated with methodological debates.

The models I have chosen for the purpose of illustration are all based on Dixit and Stiglitz's (1977) model of product differentiation and monopolistic competition. Dixit and Stiglitz were aware of the possibility of complementarities across differentiated products in their model, but they rejected it as "implausible," and imposed the restrictions that ensure uniqueness and stability of equilibrium. Many studies, which apply and extend their model, continue to impose similar restrictions. In the following sections, I will extend the Dixit and Stiglitz model by allowing the possibility of complementarities, hoping to convince the audience that there is no empirical support for ruling out the possibility of complementarity, and that one may throw away many predictions of interest by doing so.

The following sections could also be read as a dynamic version of my survey of the literature on monopolistic competition models (Matsuyama 1995a). I would like to emphasize, however, that the main objective here is on methodological issues, not on the models. (And it is not written as a survey, so that the reference is very scant.) I believe that my argument is general and goes well beyond this narrow class of models. I have chosen to use monopolistic competition, partly because this is the framework, in which

[3] I have started working on the economics of complementarity, partially because I found many existing studies in this area unsatisfactory for three reasons. First, they often abused the multiplicity of equilibria. The typical "crime" is to develop a model with multiple equilibria, in which almost anything can be an equilibrium--this naturally makes the model empirically unfalsifiable--, and then to pick one equilibrium, which happens to have the property they like, even though other equilibria do not share the property. (See footnote 10.) Second, they often draw wrong policy implications. The existence of Pareto-ranked equilibria is interpreted as a justification for the government intervention. (See footnote 1.) Third, in many models, the distance between the assumptions and the conclusions is very small. They often assume the presence of strong complementarities in technologies and discuss rather obvious implications, instead of modeling why complementarities may arise through equilibrium interactions. (See footnote 7.)

I happen to know how to build nonlinear models "simple enough to be tractable and yet rich enough to capture the complex nature of the world with complementarities," and partly because one could not often need to assume the presence of strong technological complementarities; complemetarities arise through the nature of equilibrium interactions, and for this reason, the presence of such complementarities is not immediately obvious from just looking at the assumptions on technologies. My points concerning complementary, instability and multiplicity are, I believe, model-independent: they can (and should) be illustrated in other frameworks, as well.

2. The Basic Model: Dixit-Stiglitz (1977).

Let us begin with a brief review of Dixit and Stiglitz (1977). At a formal level, the following exposition is nothing but a reproduction of their basic model. However, some variables are given different interpretations, as I intend to apply the model in the contexts different from what they had in mind.

The economy is endowed with L units of labor, the only primary factor. There is a single final good, Y, produced in a competitive sector. The technology satisfies constant returns to scale, and is expressed by a linear homogeneous production function,

$$Y = F(H, X) \quad ,$$

where the inputs used are labor, H, and a composite of differentiated intermediate inputs, X. The composite takes the form of a symmetric CES,

$$X = \left[\int_0^N [x(z)]^{1-\frac{1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} \quad ,$$

where z is an index of intermediates, N is the range of differentiated intermediate inputs available in the marketplace, x(z) is the amount of an intermediate, z, and $\sigma > 1$ the direct partial elasticity of substitution between each pair of intermediates. As it turns out, it is more convenient to express the technology in a dual form: that is, by its unit cost function,

$$\phi(w, P) \quad ,$$

where w is the wage rate, and P represents the price index for the intermediate input composite, X, defined by

$$P = \left[\int_0^N [p(z)]^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} \quad ,$$

where p(z) is the price of variety z.

The intermediate inputs are produced in the monopolistically competitive sector. Each variety is supplied solely by a single firm, hence N also represents the number of the firms in this sector. To produce x units of its product, the intermediate input firm needs to use ax(z) + F units of labor, where a is the marginal labor requirement and F the fixed cost. Aware of its monopoly power, each firm sets the price optimally, but as its market power is negligible relative to the aggregate economy, it does not take into account any strategic interaction with other firms. As is well-known, this specification leads to the following simple pricing rule for each monopolistically competitive firm, $p(z)[1-1/\sigma] = aw$. By choice of units, one can set $a = 1 - 1/\sigma$, which yields p(z) = p = w. Hence, the price index of the intermediate inputs becomes

$$P = wN^{\frac{1}{1-\sigma}} \quad .$$

Since all the inputs are priced equally and enter symmetrically into the production function, all the input producing firms operate at the same scale, x(z) = x.

The labor market clearing condition is hence written as

$$L = N(ax+F) + \phi_w(w,P)\,Y$$
$$= N(ax+F) + \phi_w(N^{\frac{1}{\sigma-1}},1)\,Y \ , \tag{1}$$

where the first term in the right hand ride represents the labor demand by the intermediate input sector, and the second terms the labor demand by the final goods sector. Market clearing for the intermediate input market is similarly written as

$$xN^{\frac{\sigma}{\sigma-1}} = \phi_P(w,P)\,Y = \phi_P(N^{\frac{1}{\sigma-1}},1)\,Y \ . \tag{2}$$

Solving (1) and (2) for x yields

$$x = \frac{\sigma\beta(N)}{\sigma-\beta(N)}\left[\frac{L}{N} - F\right] \ , \tag{3}$$

where

$$\beta(N) \equiv \frac{P\phi_P(w,P)}{\phi(w,P)} = \frac{\phi_P(w/P,1)}{\phi(w/P,1)} = \frac{\phi_P(N^{1/(\sigma-1)},1)}{\phi(N^{1/(\sigma-1)},1)}$$

is the cost share of intermediate inputs in the final goods production, expressed as a function of N.

The net profit of each intermediate input firm is $\pi = px - w(ax+F) = w(x/\sigma - F)$. There is an inducement to enter if net profit is positive and to exit if it is negative. In equilibrium, free entry and free exit ensure that net profit is zero, and that the firm operates at the break-even level:

$$x = \sigma F \ .$$

Inserting this condition into (3) yields

$$\frac{\beta(N)}{N} = \frac{\sigma F}{L} \ . \tag{4-1}$$

Eq.(4-1) can be solved for the equilibrium value of N, the variety of inputs and the number of input producing firms. Finally, inserting (4-1) into (1) or (2) yields the equilibrium aggregate output level of the economy:

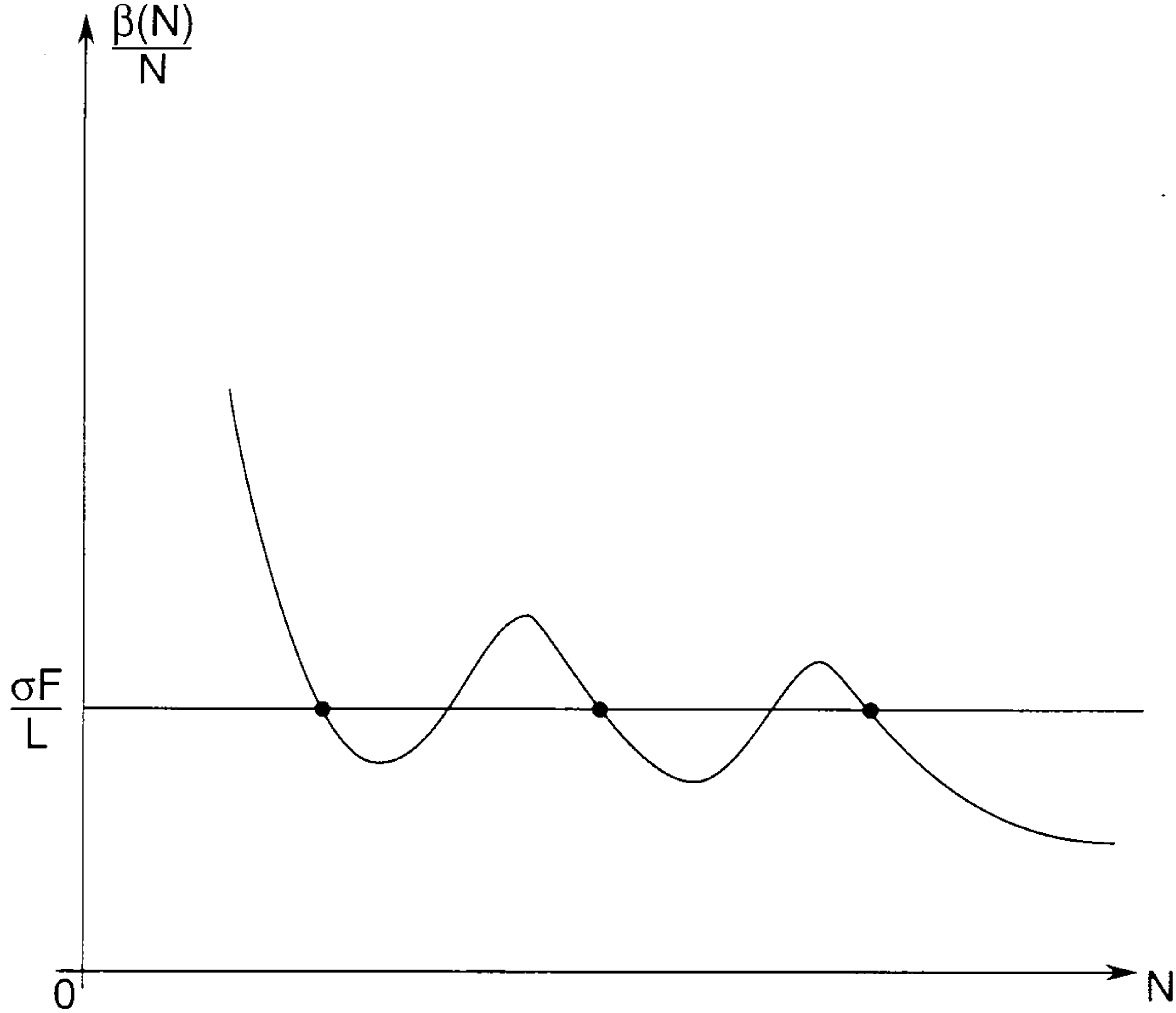

Fig. 1

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$$Y = \frac{L}{\phi(1, N^{1/(1-\sigma)})} . \tag{4-2}$$

As should be clear from eq.(4-1), the equilibrium of this model may not be unique; it depends on the shape of $\beta(N)/N$. In Matsuyama (1992), it is shown that, when the system of market demand for N differentiated inputs satisfies the Hicks-Allen notion of substitutability (complementarity), then $\beta(N)/N$ slopes downward (upward) at N. That is, an additional entry of firms into the sector decreases (increases) the market size of each firm, its net profit, and its incentive to stay in the sector. In other words, the entry decisions of firms are strategic substitutes (complements), whenever their products are Hicks-Allen substitutes (complements).

When we think of substitutes, we tend to think in terms of examples like "a hat and a cap," "a pen and a pencil," "coffee and tea," or "desk-top and lap-top computers." Likewise, as examples of complements, we think of "a left-shoe and a right-shoe," "nuts and bolts," "coffee and sugar," or "computers and printers." However, this is not a useful way of thinking about the notion of substitutes and complements discussed here. Generally, one cannot determine whether two goods are substitutes or complements solely by their physical characteristics; it must be determined as a part of general equilibrium interactions. A pair of goods can be substitutes in some circumstances and complements in others, depending on the set of goods available in the economy. In a simple set-up, where all the goods enter symmetrically, this means that they can be substitutes or complements, depending on the number of goods available. In other words, $\beta(N)/N$ may not be monotone and the multiplicity of equilibrium becomes a possibility, as illustrated in Figure 1.

Of course, not all of the equilibria depicted in the figure are equally plausible. In the neighborhood of the equilibria found in the region where goods are complements (i.e., the upward-sloping part of the curve), entry decisions of firms are strategic complements. If the number of firms is slightly below such an equilibrium, the firms make losses, which induces them to exit. Slightly above, they make profits, which induces other firms to enter. Under any reasonable entry-exit process, these equilibria are unstable, and they are unlikely to be observed under occasional perturbations. On the other hand, in the region where goods are substitutes (i.e., the downward-sloping part of the curve), equilibria are stable, because the profit per firm decreases with the number of firms. Therefore, any equilibrium observed should be in the downward-sloping part of the curve, where goods are substitutes.

Does this mean that we can forget about complementarities? The answer is "No." Even though the economy may never find itself at an unstable equilibrium, the presence of such an unstable equilibrium suggests the multiplicity of stable equilibrium. The hidden instability of the system due to the presence of complementarities suggests that a slight perturbation to the system may cause a drastic change in the equilibrium observed. And both the multiplicity and the potential sensitivity of equilibrium should have profound implications on the way we think about the economy.

Dixit and Stiglitz (1977) chose to consider only the case of substitutes, and did not discuss the implications of complementarities. I believe that this was a justifiable decision, given their purpose. First, in the spirit of Chamberlin, who proposed his theory of monopolistic competition as an alternative to the Marshallian theory of a competitive industry, they viewed their model as a contribution to industrial organization. From this perspective, the monopolistically competitive sector should be interpreted as only a small part of the entire economy, where general equilibrium feedback should not be important enough to make complementarities plausible. Second, the goal of their study was to examine the optimality of product diversity observed in an equilibrium. For this question, they did not need to be concerned with the structure of the set of stable equilibria. However, ruling out the possibility of complementarities is harder to justify, when the model is applied to macroeconomics, where the monopolistically competitive sector should be interpreted as capturing the broad industrial base of the entire economy. And, as the following examples illustrate, ruling out complementarities can be highly misleading, when the model is used to examine a variety of policy questions in the context of development, growth and business cycles.

3. Economic Development

One critical aspect of economic development is that productivity growth is generally associated with an ever greater roundaboutness in the production process and an ever increasing degree of specialization. In developed economies, consumer goods industries make superior use of highly specialized capital goods, particularly in machinery, and enjoy access to a wide range of producer services, such as equipment repair and maintenance, transportation and communication services, engineering and legal supports, accounting, advertising, and financial services. Many underdeveloped economies, on the other hand, are characterized by relatively simple production methods, and a small industrial base with the limited availability of specialized inputs. This aspect of development processes can be addressed naturally in the basic framework developed above. The variety of intermediate inputs available, N, can be interpreted as a broad measure of the industrial base; the cost share of intermediate inputs in the final goods production, β, as a measure of the indirectness (or the roundaboutness) of the production; and Y/L as the total factor productivity (and per capita income). The question is: how can we adopt the basic model to generate cross-country variations in N, β, and Y/L in such a way that they are positively correlated.

Suppose that you do not want to consider the possibility of complements between goods. This may be due to your belief that the case of substitutability is more empirically relevant. Or it may be due to your belief in the virtue of the uniqueness of equilibrium. Whatever the reason might be, this choice of modelling strategy would force you to introduce cross-country variations in some parameters of your model. With the uniqueness of equilibrium, any variations in endogenous variables needs to be explained by some variation in the exogenous variables.

For example, suppose that there are J different classes of countries,

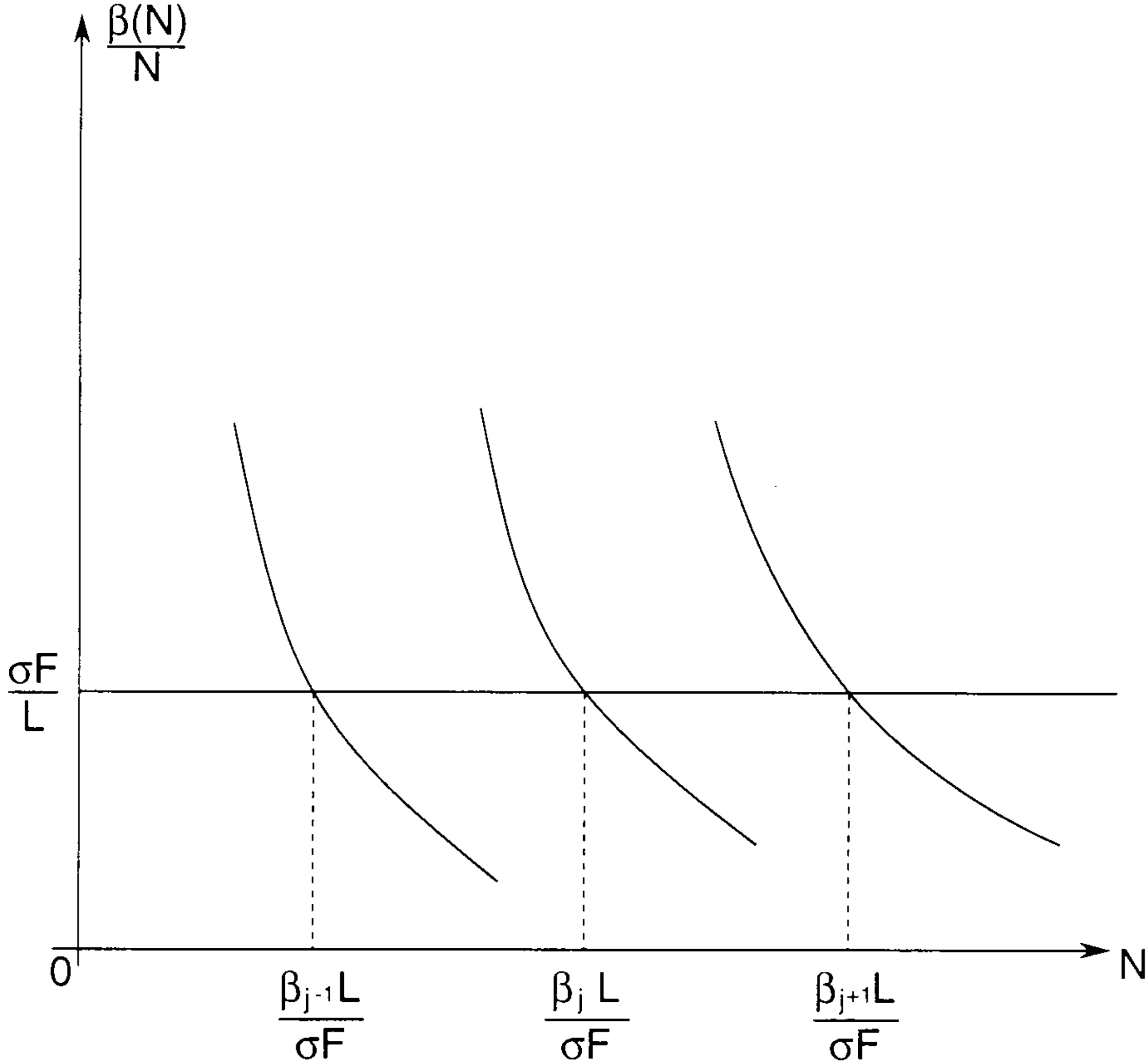

Fig. 2

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labelled by j = 1, 2,...,J. All the countries are identical, except that the final goods sector of the countries in j has the unit cost function, given by

$$\phi^j(w, P) = B_j w^{1-\beta_j} P^{\beta_j} ,$$

where β_j is increasing in j. Then, the cost share of the intermediate inputs sector in j is constant and equal to β_j. From (4-1), the equilibrium value of N is

$$N_j = \frac{\beta_j L}{\sigma F} , \tag{5-1}$$

as illustrated in Figure 2. Furthermore, if

$$\left[\frac{\beta_j L}{\sigma F}\right]^{\frac{1}{\sigma-1}} < \left[\frac{B_{j+1}}{B_j}\right]^{\frac{1}{\beta_{j+1}-\beta_j}} < \left[\frac{\beta_{j+1} L}{\sigma F}\right]^{\frac{1}{\sigma-1}}$$

for $1 \leq j \leq J - 1$, the total factor productivity (and the per capita income), derived from (4-2),

$$\frac{Y_j}{L} = \frac{1}{B_j}\left[\frac{\beta_j L}{\sigma F}\right]^{\frac{\beta_j}{\sigma-1}} \tag{5-2}$$

is increasing in j. Thus, introducing exogenous variations in technology help to explain cross-country correlations in N, β, and Y/L.

This is, however, not the only approach you could take. If you are willing to admit the possibility of complementarities, the possibility that β(N)/N may not be globally decreasing, then cross-country variations can be explained without assuming any inherent differences across countries.

Suppose that all countries are identical, including the technology of their final goods sector, whose unit cost function is now given by

$$\phi(w, P) = Min\{ B_j w^{1-\beta_j} P^{\beta_j} \}_{j=1}^{J} ,$$

where all the parameters satisfy the same restrictions as before. Instead of assigning different sets of technologies to different countries, as done in the previous approach, it is now assumed that all

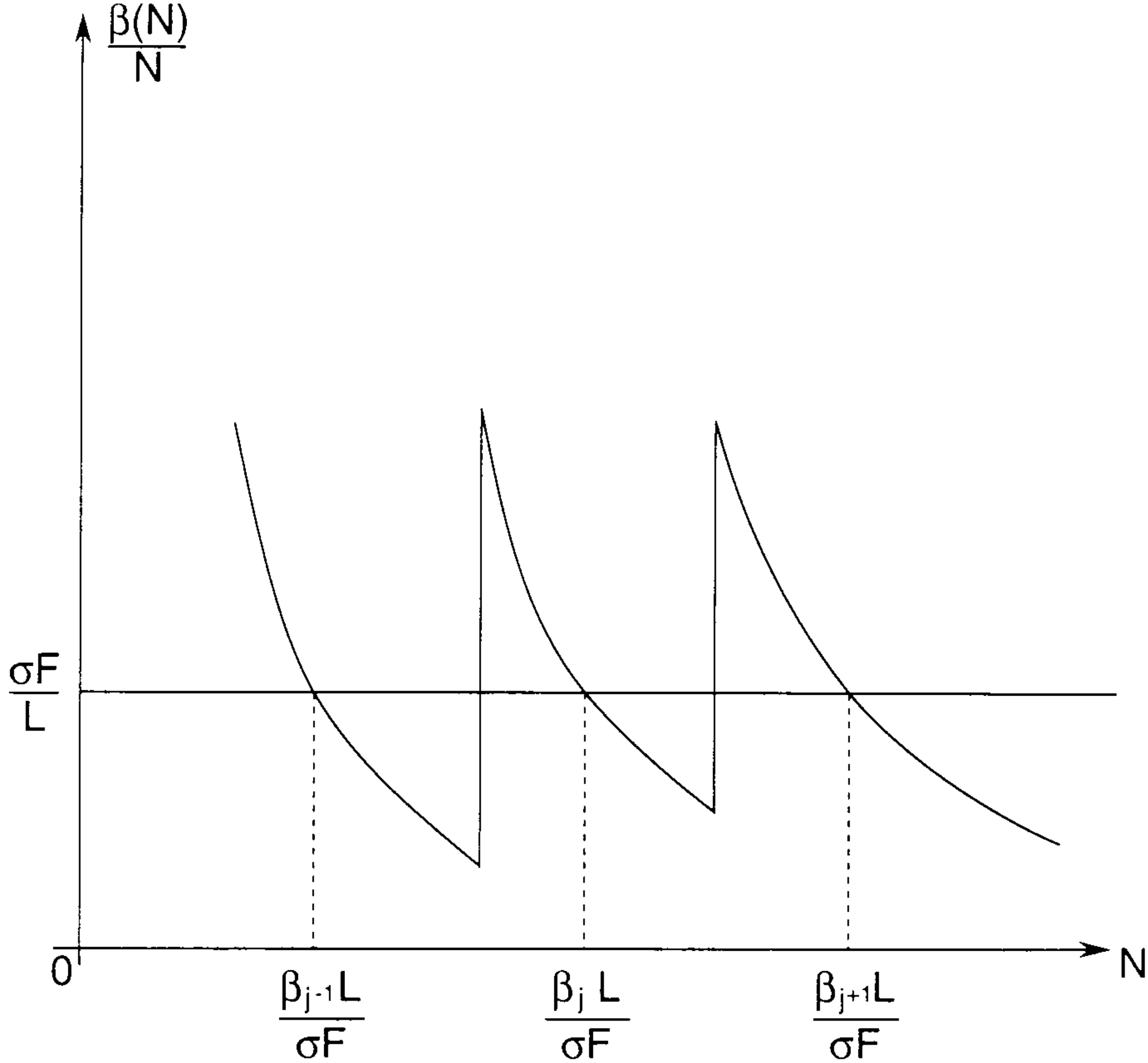
$\frac{\beta(N)}{N}$
$\frac{\sigma F}{L}$
0
N
$\frac{\beta_{j-1}L}{\sigma F}$
$\frac{\beta_j L}{\sigma F}$
$\frac{\beta_{j+1}L}{\sigma F}$

Fig. 3

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technologies are available in each country. (Note that the unit cost function above satisfies all the standard properties: it is linear homogeneous, and convex in (w,P), and it is increasing in both w and P.) Some algebra then verifies that $\beta(N)$ is a step function:

$$\beta(N) = \beta_j \qquad if \quad \left[\frac{B_j}{B_{j-1}}\right]^{\frac{\sigma-1}{\beta_j-\beta_{j-1}}} < N < \left[\frac{B_{j+1}}{B_j}\right]^{\frac{\sigma-1}{\beta_{j+1}-\beta_j}}$$

and that $\beta(N)/N$ intersects the horizontal line from above at

$$N_j = \frac{\beta_j L}{\sigma F} \tag{5-1}$$

for $1 \leq j \leq J$, as illustrated in Figure 3. If some countries find themselves at the j-th equilibrium, then their measures of the indirectness of production are equal to β_j and total factor productivity (and per capita income) is given by

$$\frac{Y_j}{L} = \frac{1}{B_j}\left[\frac{\beta_j L}{\sigma F}\right]^{\frac{\beta_j}{\sigma-1}} . \tag{5-2}$$

According to this story, the failure of some countries to adopt the more complex technologies used in advanced economies is due to their narrow industrial base. The lack of industrial support forces the use of relatively simple methods of production in downstream industries. This in turn implies a small market size for specialized inputs; the lack of demand prevents a network of support industries to emerge in the economy. Hence, the two factors, the lack of demand and the lack of support industries, reinforce each other, which creates underdevelopment traps.

Note that these two alternative stories yield the same cross-country implications; they are observationally equivalent. The only difference is that, in the first approach, the technologies are <u>assumed</u> to be different in different countries, while the second approach <u>explains</u> why different technologies may be chosen in different countries, in spite of the universal availability of technologies.

Because of their observational equivalence, empirical studies could not distinguish between the two stories. For example, demonstrating that the case of substitutes is empirically more relevant does not favor one over the other. Furthermore, the observational equivalence means that the two stories are equally falsifiable. In other words, the lack of falsifiability, a very common (and in my view, unwarranted) criticism against models with multiple equilibria, cannot be used to reject the second story.

In spite of their observational equivalence, the choice between the two approaches is not just a matter of taste. Although they may generate the same empirical implications, they project very different views of the world. According to the first, the lack of know-how, or of blue-prints is the source of underdevelopment, which would suggest a technology transfer as a solution. The second suggests that such a technology transfer cannot solve the problem, because all countries already have access to the same set of technological information. According to the first view of the world, "a take-off" or "an economic miracle" is a big puzzle. The second view is, by contrast, consistent with such a phenomenon. A sudden change in the performance of an economy may occur either because the economy somehow moves from one equilibrium to another, or because the equilibrium, where the economy has situated, disappeared due to a small change in the environment.

By demonstrating endogenous cross-country variation in a model with inherently identical countries, the second approach does not intend to argue that variations in the climate, the natural resources endowment, and in other factors external to the economic system, are not responsible for the variations in the economic performance. Instead, it attempts to show why variations in the external factors do not need to be big to generate the huge diversity observed in economic variables. In the presence of complementarity and its hidden instability of the system, even a small variation in the environment can be amplified to cause a big effect. This sensitivity also suggests the fundamental difficulty of evaluating the effects of policy interventions in economic development.[4]

[4]This statement, which may be interpreted as a call for a prudent approach to policy interventions, may surprise some readers. A model of this kind, with its demonstration of coordination failures, i.e., the co-existence of Pareto-rankable multiple equilibria, is typically interpreted as a call for policy activism. On

One possible criticism against the second approach might be that it explains only the possibility of endogenous diversity; it does not explain the inevitability: that is, why we should expect to see such cross-country variations. After all, it is possible that all countries find themselves in the same equilibrium in Figure 3. I think this is a legitimate objection, although it does not make the second approach less desirable than the first approach, which does not even try to explain the diversity. To respond to such a criticism, and to explain why it is inevitable that some countries become richer than others, we need to move out of a one-country setting, and develop a multi-country setting, where the interaction between the countries can be modelled explicitly. In a model of the multi-country world economy similar to the one presented here, I have shown that the symmetric equilibrium, in which inherently identical countries perform all equally well, is unstable; some countries must become richer than others in any stable equilibrium: Matsuyama (1996a). The recent literature on economic geography (Krugman 1991b) also emphasizes the process of "symmetry-breaking," as a cause of the regional disparities (Krugman 1995; Matsuyama 1995b).

4. A Dynamic Extension.

To further explore the implications of complementarity, instability and multiplicity, let us now extend the basic model into a dynamic one. Time is discrete. There are two primary factors of production, labor and the capital stock. As before, there is a single final good, which can be either consumed or invested. Let K_t denote the capital stock at the end of period t, i.e., the amount of the final good left unconsumed in period t, and carried over to period t+1. This implies that the amount of the capital stock available for use in period t is denoted by K_{t-1}. The labor supply is constant and equal to L.

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this issue, I simply refer the reader to Matsuyama (forthcoming), where I discuss at length the policy implications of the economics of coordination failures and of complementarity.

The competitive final goods sector may now use not only labor, and the intermediate inputs, but also capital services directly. Its unit cost is expressed as a linear homogeneous function

$$\phi^Y(w_t, r_t, P_t) \quad ,$$

where w_t, r_t, and P_t are the wage rate, the rental rate, and the price index for the intermediate inputs composite, at period t. The structure of the intermediate inputs sector is the same as before, except that it may use both labor and capital. The cost function of an intermediate input producing firm is given by

$$\phi^X(w_t, r_t)\ (F + a x_t) \quad ,$$

where ϕ^x satisfies the linear homogeneity and $a = 1 - 1/\sigma$.

The optimal pricing rule of the intermediate input producer is now $p(z) = \phi^x$, so that

$$P_t = N_t^{\frac{1}{1-\sigma}} \phi^X(w_t, r_t) \quad , \tag{6-1}$$

and the market clearing conditions for labor, capital and the intermediate inputs are given by

$$L = N_t \phi^X_w(w_t, r_t)(F + a x_t) + \phi^Y_w(w_t, r_t, P_t) Y_t \quad , \tag{6-2}$$

$$K_{t-1} = N_t \phi^X_r(w_t, r_t)(F + a x_t) + \phi^Y_r(w_t, r_t, P_t) Y_t \quad , \tag{6-3}$$

and

$$x_t N_t^{\frac{\sigma}{\sigma-1}} = \phi^Y_P(w_t, r_t, P_t) Y_t \quad , \tag{6-4}$$

respectively. The break-even level of the intermediate input sector is, as before,

$$x_t = \sigma F \quad . \tag{6-5}$$

To close the model, imagine that there are overlapping generations of equal size, and each generation lives for two periods. Every period, a new generation of workers enters the economy and inelastically supplies L units of labor. The workers who enter the economy in period t earn the wage income, w_tL, some of which is consumed in the first period, C_t^1, and the rest is saved to finance their second period consumption, C_{t+1}^2. Their preferences are given by $U^t = (1-s)\log(C_t^1) + s\log(C_{t+1}^2)$, which yields a simple saving function, $S_t = w_tL - C_t^1 = sw_tL$. Since the capital stock is the only store of value in this economy,

$$\phi^Y(w_t, r_t, P_t)\, K_t \;=\; s w_t L \,. \tag{6-6}$$

In addition, I choose the final goods to be a numeraire:

$$\phi^Y(w_t, r_t, P_t) \;=\; 1 \;. \tag{6-7}$$

Given K_{t-1}, the system of seven equations, (6-1) through (6-7), generically make the following seven endogenous variables, w_t, r_t, P_t, N_t, x_t, Y_t, and K_t, locally determinate. The equilibrium of the economy can thus be solved for by applying these equations repeatedly, once the initial condition, K_0, is given.

5. <u>Convergence</u>.

Let us now consider a special case of the dynamic model, where the intermediate inputs sector uses only labor:

$$\phi^X(w, r) \;=\; w \;.$$

Furthermore, the unit cost of the final goods sector is given by

$$\phi^Y(w_t, r_t, P_t) \;=\; \left[\frac{r_t}{\alpha}\right]^{\alpha}\left[\frac{\phi\,(w_t, P_t)}{1-\alpha}\right]^{1-\alpha} \;.$$

Note that this special case is indeed very similar to the static model developed in section 2. The only difference is that "the aggregate output" of that model is now combined with the capital stock by a Cobb-Douglas production function, where α is the share of the capital stock in GNP. It should thus be clear that eqs. (6-1) through (6-5) can be condensed to

$$\frac{\beta(N_t)}{N_t} = \frac{\sigma F}{L} \tag{7-1}$$

and

$$Y_t = \left[\frac{L}{\phi(1, N_t^{1/(1-\sigma)})}\right]^{1-\alpha} K_{t-1}^{\alpha} .$$

(Notice the analogy between these equations and (4-1) and (4-2).) Furthermore, the share of labor income earned by the younger generation in the GNP is equal to 1-α, of which 100s% is saved. Hence, the dynamics of the capital stock are

$$K_t = s(1-\alpha)\left[\frac{L}{\phi(1, N_t^{1/(1-\sigma)})}\right]^{1-\alpha} K_{t-1}^{\alpha} . \tag{7-2}$$

Let us now proceed with our analysis as in section 3. Imagine again that there are J different classes of countries, which are identical except that, for countries in j,

$$\phi^j(w, P) = B_j w^{1-\beta_j} P^{\beta_j} ,$$

with the same parameter restrictions as before. Then, for these countries, the equilibrium is described by

$$N_t = \frac{\beta_j L}{\sigma F} \tag{8-1}$$

and

$$K_t = (K_j^*)^{1-\alpha}(K_{t-1})^{\alpha} , \tag{8-2}$$

where K_j^* is defined by

$$K_j^* \equiv \{s(1-\alpha)\}^{\frac{1}{1-\alpha}}\left[\frac{\beta_j L}{\sigma F}\right]^{\frac{\beta_j}{\sigma-1}} \frac{L}{B_j} ,$$

which is increasing in j. The dynamics are depicted in Figure 4. Regardless of the initial capital stock, all the countries in class j converges to the steady state, K_j^*. Hence, the dynamics satisfies the property

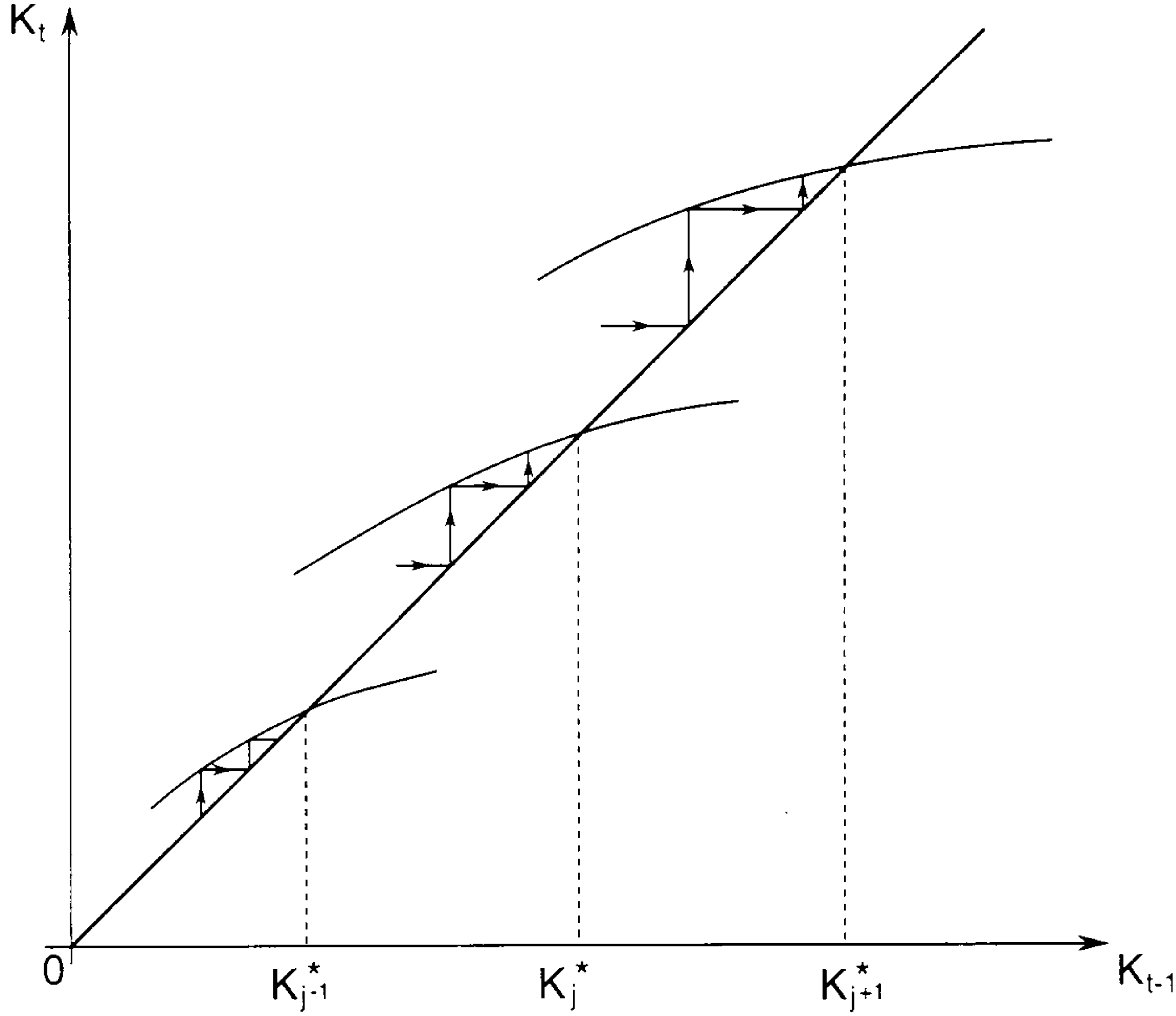

Fig. 4

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of conditional convergence, the term proposed by some economists to express the notion that cross-country variations in economic performance tend to converge over time, once all idiosyncratic factors are adjusted for.

The story told above is not, however, the only story consistent with conditional convergence. If all the countries are inherently identical, with

$$\phi(w, P) = Min\{ B_j w^{1-\beta_j} P^{\beta_j} \}_{j=1}^{J} ,$$

there are J stable equilibria, described by (8-1) and (8-2). In particular, eq. (8-2) can now be interpreted as the evolution of the capital stock, when a country finds itself at the j-th equilibrium in period t. If each country tends to stay in the same equilibrium over time, this specification of the model also exhibits the conditional convergence property.

All the remarks given at the end of section 3 are equally applicable, here. The two approaches are observationally equivalent. They are both consistent with the notion of conditional convergence. The second approach, due to its multiplicity, has the additional advantage of explaining the lack of unconditional convergence, which the first approach, due to the uniqueness of equilibrium, has to assume by introducing exogenous variations in the model. But more importantly, in spite of the observational equivalence, the two approaches project very different views of the world and policy implications.

6. Business Cycles.

With a slight reinterpretation, the dynamic model discussed above can be used to illustrate two competing theories of business cycles: Matsuyama (work in progress). Consider now an economy, whose technology satisfies

$$\phi^t(w , P) = B_t w^{1-\beta_t} P^{\beta_t} ,$$

where $\{ \beta_t, B_t \}$ are Markovian random variables with the transition probability,

$$Pr\{\,\beta_t = \beta_j\,;\,B_t = B_j \mid \beta_{t-1} = \beta_i\,;\,B_{t-1} = B_i\,\} \;=\; q_{ij} \;\geq\; 0 \quad ; \quad \sum_{j=1}^{J} q_{ij} \;=\; 1 \;.$$

Then, eq. (8-2) is now a stochastic difference equation, with K_j^* being interpreted as a realization of the Markovian processes,

$$Pr\{\,K_t = K_j^* \mid K_{t-1} = K_i^*\,\} \;=\; q_{ij} \;\geq\; 0 \quad ; \quad \sum_{j=1}^{J} q_{ij} \;=\; 1 \;.$$

This captures the real business cycle approach in its essentials.

Alternatively, one can imagine that there is no inherent shock to the economy and that the technologies available in the economy are always described by

$$\phi(w, P) \;=\; Min\{\,B_j w^{1-\beta_j} P^{\beta_j}\,\}_{j=1}^{J} \;.$$

Suppose further that there is a random variable, ξ_t, which may be called "expectations," "animal spirits," or "sunspots," which has no intrinsic effect on the economy. This random variable is publicly observable and follows the processes,

$$Pr\{\,\xi_t = \xi_j \mid \xi_{t-1} = \xi_i\,\} \;=\; q_{ij} \;\geq\; 0 \quad ; \quad \sum_{j=1}^{J} q_{ij} \;=\; 1 \;,$$

and, when $\xi_t = \xi_j$ are observed, the agents in the economy expect

$$N_t \;=\; \frac{\beta_j L}{\sigma F}$$

to occur in period t. Then, in equilibrium, their expectations indeed become self-fulfilling and the dynamics of the economy follows the stochastic difference equation, (8-2). This approach, which captures the theory of endogenous business cycles based on sunspots in its essentials, generates the same time-series implications as the real business cycle approach sketched above. (Kamihigashi 1996 also makes a similar point in a different class of models.)

By demonstrating the possibility that business fluctuations are driven by a random variable with no intrinsic effect on the economy, the theory of sunspots does not intend to argue that some variations in the climate, such as heat waves and snow storms, new scientific discoveries, fiscal and monetary policies, and other intrinsic shocks to the economy, are not responsible for business cycles. Instead, it attempts to show why these intrinsic shocks, even if their direct effects are small, can generate large effects on the economy by influencing expectations. Even if the central bank's open market operations may have a small direct effect on the financial markets, the public announcement of its action can have a large impact on the economy by affecting an investor's confidence. This sensitivity also suggests the fundamental difficulty of evaluating the effects of each policy intervention.

In spite of their observational equivalence, these two alternative explanations of business cycles have different macroeconomic policy implications.

According to the first view, there is little the government can do to stabilize the economy. According to the second view, it is difficult to evaluate the effect of each discretionary policy intervention, but the government can potentially change the frequency and magnitude of business cycles by developing and following systematic intervention strategies, which could help to stabilize the expectations of the agents (Woodford 1987).

One might criticize the theory based on sunspots in a manner similar to the criticism discussed in the last paragraph of section 3. Random realizations of sunspots cause business cycles only in a subset of equilibria of the model; cyclical behaviors are not inevitable. There are many other equilibria, including the one in which the sunspot variable is ignored by the agents: the belief that sunspots are irrelevant is also self-fulfilling. In such an equilibrium, there are no fluctuations. Again, I think this is a legitimate objection, although it does not make the real business cycles theory, which simply "blames" fluctuations on unobservable error terms, more desirable than the theory of sunspot fluctuations. One possible way of responding to this criticism is to develop a model of learning processes, under which the agents would "learn to believe in sunspots (Woodford 1990)." Another possibility is to develop a model with an intertemporal linkage in such a way that an increase in the level of activities in one period induces a decline in the level of activities in another. The presence of such an intertemporal linkage causes instability of no-fluctuation equilibria and leads to cyclical behavior. In section 8, I will discuss such a model of business cycles.

7. Instability and Multiple Asymptotic Behaviors.

Let us now consider the second special case of the dynamic model developed in section 4. The unit cost of the final goods sector is now equal to

$$\phi^Y(w_t, r_t, P_t) = \left[\frac{w_t}{1-\alpha}\right]^{1-\alpha}\left[\frac{\phi(r_t, P_t)}{\alpha}\right]^{\alpha} ,$$

while

$$\phi^X(w, r) = r .$$

This special case makes the factor intensity assumption, which is the polar opposite of the previous case, discussed in sections 5 and 6. That is, labor is used only in the final goods sector, and the intermediate input sector uses the capital stock only, while α is again the share of the capital income in GNP. Hence, the equilibrium conditions can now be written as

$$\frac{\beta(N_t)}{N_t} = \frac{\sigma F}{K_{t-1}} \tag{9-1}$$

and

$$K_t = s(1-\alpha)L^{1-\alpha}\left[\frac{K_{t-1}}{\phi(1, N_t^{1/(1-\sigma)})}\right]^{\alpha} . \tag{9-2}$$

This case displays the possibility of complementarity and instability, even when $\beta(N)/N$ is globally decreasing and hence the equilibrium dynamics are uniquely determined. To see this, let us assume that

$$\phi(w, P) = P \; .$$

Then, $\beta(N)/N = 1/N$ and

$$N_t = \frac{K_{t-1}}{\sigma F}$$

and hence

$$K_t = G(K_{t-1})^{\frac{\alpha\sigma}{\sigma-1}} \quad , \tag{10}$$

where

$$G \equiv s(1-\alpha)\left[\frac{1}{\sigma F}\right]^{\frac{\alpha}{\sigma-1}} L^{1-\alpha} \quad .$$

Figures 5a-5c illustrates (10). In Figures 5a and 5c, there is a unique steady state with a positive capital stock. In Figure 5a, where $\alpha < 1 - 1/\sigma$, the economy converges to the steady state, regardless of the initial condition. On the other hand, if $\alpha > 1 - 1/\sigma$, the output-capital ratio increases with the level of capital stock: there exists increasing returns at the aggregate level. This makes the dynamics of the model unstable, as shown in Figure 5c. When the economy starts with the capital stock even slightly below the steady state, it implodes to the zero level. When the economy starts with the capital stock even slightly above the steady state, it takes off to an unbounded growth. In this example, the complementarity due to increasing returns does not give rise to multiple equilibria. Nevertheless, the complementarity leads to an unstable steady state and makes the asymptotic behavior of the economy depending upon the initial condition. The uniqueness of equilibrium notwithstanding, this example thus captures basically the same idea as the other cases discussed before: the presence of complementarities and the implied instability of the system make the economy's performance highly sensitive to small perturbations to the environment.

Finally, consider the borderline case, where $\alpha = 1 - 1/\sigma$ holds. This case captures the essence of Romer's (1987) endogenous growth model. The dynamics become simply

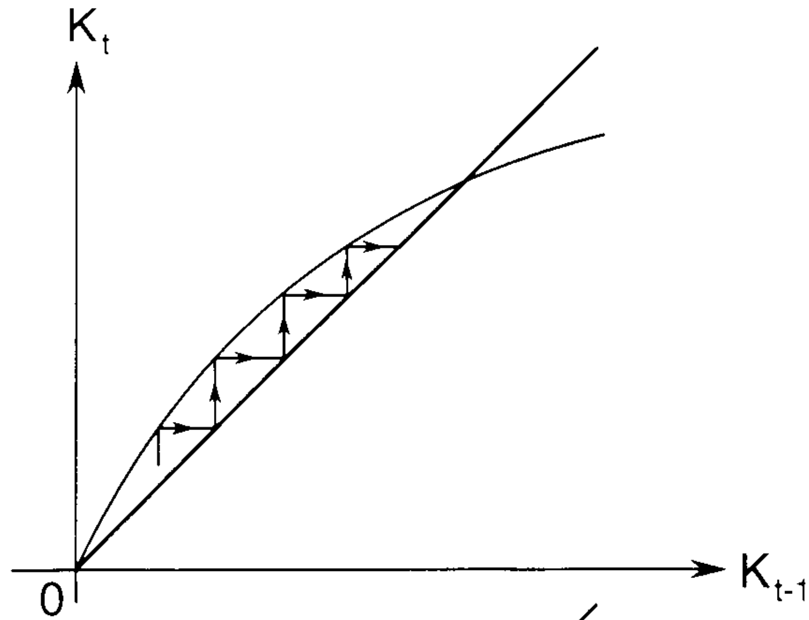

Figure 5a

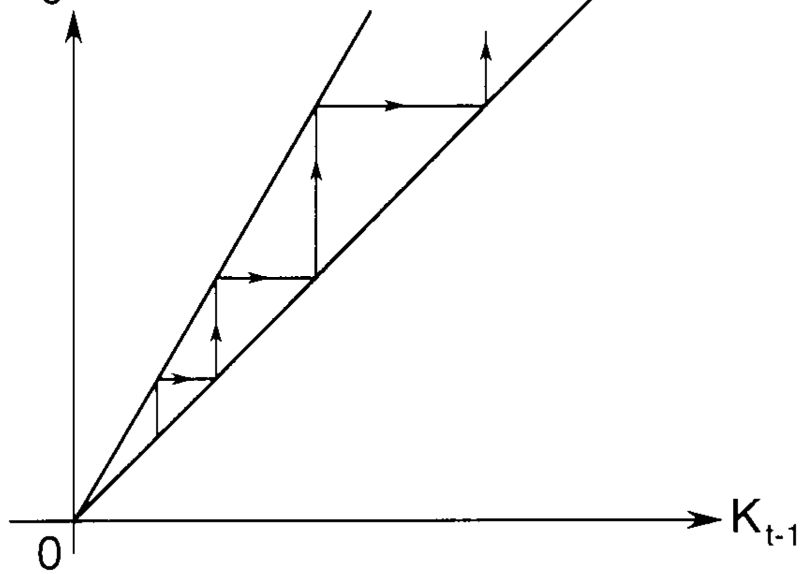

Figure 5b

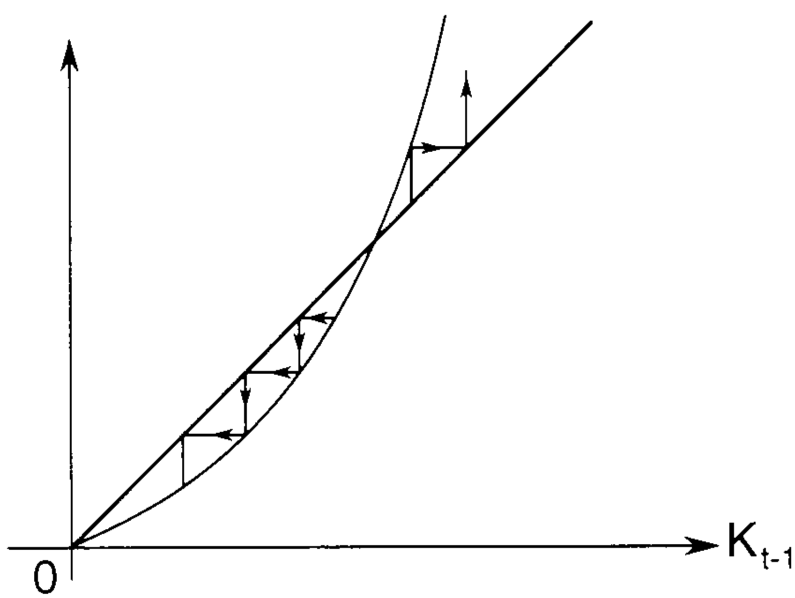

Figure 5c

$$K_t = K_0 G^t \quad ,$$

where

$$G = \frac{S}{\sigma}\left[\frac{L}{\sigma F}\right]^{\frac{1}{\sigma}} \quad . \tag{11}$$

If G > 1, the economy grows forever at a constant rate, as depicted in Figure 5b. That is, a small difference in the initial condition will magnify over time.

It is not difficult to demonstrate that, for other specifications of the unit cost function, ϕ, even if restrictions are imposed to ensure the uniqueness of the equilibrium path, the dynamics described by (9-1) and (9-2) could have an arbitrary number of stable steady states, each associated with a basin of attraction. And two adjacent basins of attraction are separated by an unstable steady state. Toward which stable steady state the economy converges is determined by the initial condition.

If we allow $\beta(N)/N$ to be nonmonotone, the model generates multiple equilibria, at least for some range of the initial capital stock. Then, which stable steady state the economy converges to is no longer solely determined by the initial condition. Basins of attraction for different steady states can overlap, and if the economy starts from the overlap region, expectational factors, such as "animal spirits," or cultural factors, such as "protestant ethics," can play an important role in determining the long run behavior of the economy (Boldrin 1992, Krugman 1991a, Matsuyama 1991).

8. Instability and Cycles.

The instability of a steady state is not always associated with multiple steady states, or multiple asymptotic behaviors, of the economy. Depending on the nature of complementarity, the loss of stability could give rise to cyclical behavior. To demonstrate this, let us modify the structure of the intermediate input producing sector in two respects. (This specification is borrowed from Deneckere and Judd 1992). First, the cost function for an intermediate input is equal to $\phi^x(w_t,r_t)(F+ax_t(z))$, only if z is produced for the first time in period t. On the other hand, it is equal to $\phi^x(w_t,r_t)ax_t(z)$, if z has been produced before. Hence, F should now be interpreted as the sunk cost incurred, when "new" inputs are introduced. Second, the monopoly power enjoyed by the firm that introduces a new input in period t and bears the sunk cost, lasts only one period. From period t+1 on, after the input becomes "old," other firms can also manufacture it by constant returns to scale technology, $\phi^x(w_t,r_t)ax_t(z)$ and hence it is supplied competitively.

With these modifications, N_{t-1} now represents the range of old, competitively priced, inputs available in period t, while the range of new, monopolistically priced inputs available in period t can be represented by $N_t - N_{t-1}$. The price of the old, and competitively priced inputs, is equal to its marginal cost: $p_t^c = \phi^x(w_t,r_t)a$. The price of the new, monopolistically priced inputs, is $p_t^m = \phi^x(w_t,r_t)$, so that the relative demand for the two types of inputs satisfies

$$\frac{x_t^c}{x_t^m} = \left[\frac{p_t^c}{p_t^m}\right]^{-\sigma} = \left[1-\frac{1}{\sigma}\right]^{-\sigma} .$$

The composite of the intermediate inputs, X_t, and its price index, P_t, are hence given by

$$\begin{aligned} X_t^{1-\frac{1}{\sigma}} &= N_{t-1}(x_t^c)^{1-\frac{1}{\sigma}} + (N_t - N_{t-1})(x_t^m)^{1-\frac{1}{\sigma}} \\ &= \{N_{t-1}\theta + N_t - N_{t-1}\}(x_t^m)^{1-\frac{1}{\sigma}} \end{aligned}$$

and

$$\begin{aligned} P_t^{1-\sigma} &= N_{t-1}\{\phi^x(w_t,r_t)\, a\}^{1-\sigma} + (N_t - N_{t-1})\{\phi^x(w_t,r_t)\}^{1-\sigma} \\ &= \{N_{t-1}\theta + N_t - N_{t-1}\}\{\phi^x(w_t,r_t)\}^{1-\sigma} , \end{aligned} \tag{12-1}$$

where

$$\theta \equiv \left[1 - \frac{1}{\sigma}\right]^{1-\sigma} > 1 .$$

The free entry condition is now modified to

$$N_t \geq N_{t-1} \quad ; \quad x_t^m \leq \sigma F \quad . \tag{12-2}$$

That is, new varieties are introduced in period t if and only if their output is expected to reach the break-even level.

The labor market clearing condition becomes

$$\begin{aligned} L &= N_{t-1}\phi_w^X(w_t, r_t)\, a x_t^c + \{N_t - N_{t-1}\}\phi_w^X(w_t, r_t)\left(F + a x_t^m\right) + \phi_w^Y(w_t, r_t, P_t)\, Y_t \\ &= \{N_{t-1}\theta + N_t - N_{t-1}\}\,\phi_w^X(w_t, r_t)\; x_t^m + \phi_w^Y(w_t, r_t, P_t)\, Y_t \quad . \end{aligned} \tag{12-3}$$

The market clearing condition for capital services is similarly written as

$$K_{t-1} = \{N_{t-1}\theta + N_t - N_{t-1}\}\,\phi_r^X(w_t, r_t)\; x_t^m + \phi_r^Y(w_t, r_t, P_t)\, Y_t \quad . \tag{12-4}$$

Finally, the intermediate input market clears when

$$x_t^m\{N_{t-1}\theta + N_t - N_{t-1}\}^{\frac{\sigma}{\sigma-1}} = \phi_P^Y(w_t, r_t, P_t)\, Y_t \tag{12-5}$$

Given K_{t-1} and N_{t-1}, the system of seven equations, (12-1) through (12-5) and eqs. (6-6) and (6-7), generically make the following seven endogenous variables, w_t, r_t, P_t, N_t, x_t^m, Y_t, and K_t, locally determinate. The equilibrium of the economy can thus be solved for by applying these equations repeatedly, once the initial condition, K_0 and N_0, is given.

In what follows, let us consider the case, where

$$\phi^Y(w_t, r_t, P_t) = \left[\frac{w_t}{\sigma^{-1}}\right]^{\frac{1}{\sigma}}\left[\frac{P_t}{1-\sigma^{-1}}\right]^{1-\frac{1}{\sigma}}$$

and

$$\phi^X(w, r) = r \quad .$$

Then, after some algebra, eqs. (12-1) through (12-5) as well as (6-1) and (6-2) can be condensed into the system of difference equations in K and N as follows:

$$K_t = G(K_{t-1})^{1-\frac{1}{\sigma}}\left[Max\{K_{t-1}\,,\ (\theta\sigma F)N_{t-1}\}\right]^{\frac{1}{\sigma}}$$

$$N_t = N_{t-1} + Max\left\{\frac{K_{t-1}}{\sigma F} - \theta N_{t-1}\,,\ 0\right\}\ ,$$

where G is given by eq. (11). By introducing a new variable, $k_t \equiv K_t/N_t(\theta\sigma F)$, this can be further simplified to the following one-dimensional map, Φ: $R_+ \to R_+$,

$$k_t = \Phi(k_{t-1}) \equiv \begin{cases} G\,(k_{t-1})^{1-\frac{1}{\sigma}} & if \quad k_{t-1} \le 1 \\ \dfrac{G\,k_{t-1}}{1+\theta(k_{t-1}-1)} & if \quad k_{t-1} \ge 1\ . \end{cases}$$

When $G > 1$, there exists a continuum of balanced growth paths in this economy; both K_t and N_t grow at the same rate, $K_t = K_0G^t$ and $N_t = N_0G^t$, if the initial condition satisfies $k_0 = K_0/N_0(\theta\sigma F) = 1+(G-1)/\theta$. The balanced growth paths are not necessarily stable. As discussed in Matsuyama (1996b), if

$$\theta - 1 > G > 1\ ,$$

they are unstable. Moreover, when this condition is met, there are a pair of period-2 cycles and, the economy fluctuates persistently for almost all initial conditions (see Figure 6).

The instability of the balanced growth path in this model is caused by complementarity in the timing of entry decisions. Timing matters in this model, because the firm could enjoy monopoly profits only temporarily. In order to recover the sunk cost, the firm must choose a period, in which the market for its product is sufficiently large, so that it can reach the break-even level of output. The size of the market depends on how the products with which it competes are priced. If the firm chooses to enter when other firms also enter, then some of the products are monopolistically priced. If the firm chooses to enter in the following period, then these products become competitively priced. This consideration gives an incentive for firms to enter when other firms also enter. When this complementarity is strong enough, it gives rise to a bunching of the entry activity, and hence the cyclicality.

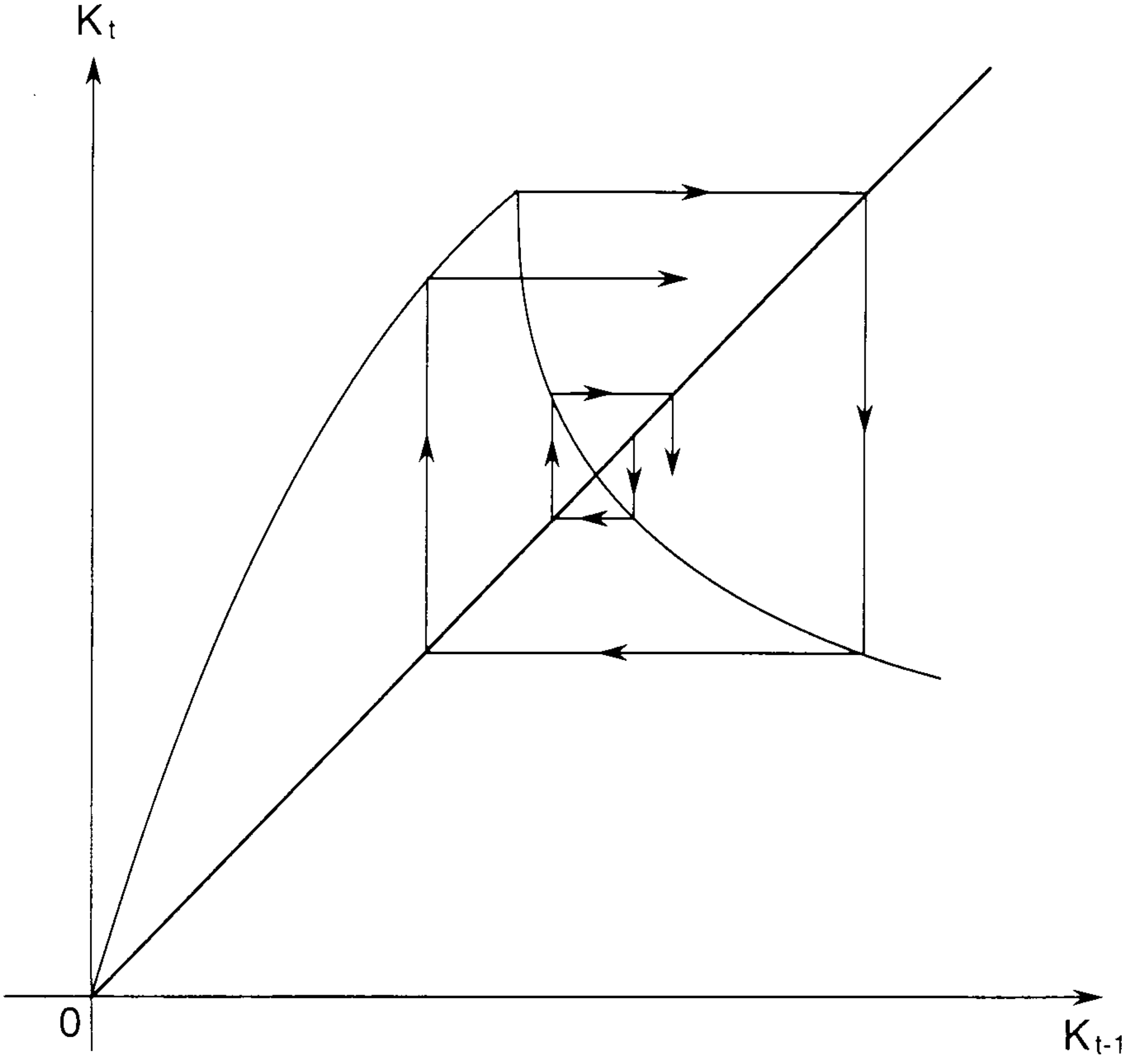

Fig 6

The equilibrium in this model is unique. Nevertheless, it captures essentially the same idea with all the models discussed above. Complementarity leads to the instability of a balanced growth path. The resulting cyclical behavior implies that the economy goes through multiple phases of cycles; it is difficult to predict in which phase of cycles the economy will find itself in a particular period, i.e., whether it is a boom or a bust, because of its sensitive dependence on the initial condition.[5]

9. Some Methodological Issues.

Multiplicity, instability, and the sensitive dependence of equilibrium on the parameter values are prominent features of the economics of complementarity. Many economists, however, regard these features as defeats of a model. From the very beginning of their modelling exercises, they tend to impose some restrictions that help to rule out the possibility of complementarities. This practice is common across many fields, one of which is the literature on monopolistic competition. In the previous sections, I have extended the Dixit and Stiglitz model to allow for the possibility of complementarities, and argued, by means of examples, that there is no empirical reason for ruling out complementarities and that, by ignoring such possibility, we unnecessarily narrow the scope of the models and throw out many predictions of great significance. Although monopolistic competition models are used for the purpose of illustration, I believe that the same argument can be made for other classes of models, as well. In this section, let me reiterate and elaborate on my defense of the economics of complementarity.

For those who believe that complementarity is an empirically irrelevant case, I should emphasize that this is not a good reason for ignoring the possibility of complementarity. The economics of complementarity is perfectly consistent with the observation that there is little complementarity in a neighborhood of a stable (hence potentially observable) state. What makes a complementarity important

[5]In this model, the output and investment growth rates are <u>lower</u> during the period of high entry activities. Complementarity in this model creates a synchronization of innovations, which leads to an asynchronization between investment and innovation.

is that its hidden presence in a system introduces an instability, which generates multiple stable states. There is no reason to expect that one can find the evidence for complementarities, once the economy has settled into one of many stable states.[6] Indeed, the unobservable nature of complementarities should be viewed as a virtue, not a defeat, of the models, because, arguably, entrepreneurs should be able to solve a coordination problem caused by the presence of complementarities that are obvious even to the outsider, such as the economist.[7]

For those who believe that models with sensitive dependence of equilibrium behaviors cannot yield robust predictions, my response is twofold. First, there is no reason to believe a priori that the real world is stable in small perturbations. Second, I do not know of any model, whose equilibrium is robust to all conceivable perturbations of the model. Of course, this is not to deny the value of demonstrating the robustness of an equilibrium against some perturbations of the model. Indeed, in a model with sensitive dependence, one may be able to make robust predictions about the class of perturbations against which the equilibrium behaviors are robust (or unrobust), which is exactly the goal of bifurcation analyses in nonlinear models.

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[6]An economist once wrote to me, "Your argument reminds me of the following proof for black holes: Black holes cannot be seen. Thus, as we do not see them they must exist." I should probably have responded by writing to him, "Your argument reminds me of the non-existence proof of black holes: Black holes cannot be seen. Thus, they don't exist." Actually, his comment reminded me of the following impossibility proof of a perfect crime: a crime is not perfect if it becomes known to be committed. The record shows that all the crimes have left some traces behind. Hence, a perfect crime is not possible.

[7]It should be emphasized that, in the models presented above, complementarities arise as an outcome of equilibrium interactions. In many cases, the direct partial elasticity of substitution between differentiated goods, σ, does not have to be low to generate complementarities. This is in sharp contrast with many studies in the literature, both in industrial organization and in macroeconomics, which analyze the performance of imperfect competitive markets in the presence of assumed technological complementarities. An extreme example would be the problems of two producers, each supplying a good to the customer that is a perfect complement to the other, as "nuts and bolts," or "left-shoe and right-shoe." The most successful application of this framework is the literature on the organization of firms, summarized by Milgrom and Roberts (1992). I find the macroeconomic applications of this framework unconvincing (and honestly speaking, uninteresting), since there would be a strong incentive to internalize this kind of complementary activities, as rightly pointed out by Milgrom and Roberts, and hence the aggregate implications would be, at best, dubious.

The most common criticism is concerned with multiplicity. Models with multiple equilibria by nature have little predictive contents. In particular, it is impossible to perform comparative static analyses. Models with multiple equilibria yield few testable implications, and hence are unfalsifiable.

A number of responses can be made to this objection.[8] First, the mere fact that one model has a unique equilibrium does not necessarily mean that it has more predictive content than another model with multiple equilibria. It depends on the distance between assumptions and conclusions. For example, suppose that we want to model the world economy, consisting of many countries with different standards-of-living. As demonstrated in sections 3 and 5, we can build a model with unique equilibrium, but in doing so, we may have to make the "realistic" assumption that different countries have different levels of total factor productivity. Alternatively, we can build a model with multiple equilibria, in which the technologies available are the same everywhere, and yet, different countries achieve different levels of total factor productivity in equilibrium. In this case, the second model clearly has more predictive content: it can explain everything the first model explains, plus what the first model assumes. Furthermore, the second model can also explain what kind of changes leads to a drastic improvement in the economic performance, "a take-off" or "an economic miracle,"--a possibility that the first model rules out by assumption.

Every time I hear someone complain that models with multiple equilibria have few predictive contents and testable implications, I wonder "Compared to what?" In order to have a fair contest between a model with uniqueness and a model with multiplicity, we must control for something. We cannot

[8]As shown in the previous sections, the presence of complementarities does not always imply multiple equilibria in a technical sense of the word. In a dynamic model, complementarities sometimes lead to a unique equilibrium with multiple steady states (section 7) or cycles with multiple phases (section 8). This is an important distinction to make when we try to understand the working of a particular model. As far as the methodological issues discussed here are concerned, these different types of multiplicity do not need to be distinguished. In what follows, I will use the terms "multiple equilibria" and "multiplicity" interchangeably. The reader may interpret an "equilibrium" loosely as a state of the economy that the model predicts is potentially observable.

simply say "other things being equal," because other things cannot be equal. The reason why the two models could have different numbers of equilibria is because the assumptions are different.

One way, and the only way I can think of, to have a fair contest between the two types of models is to compare two models, which are observationally equivalent, and hence equally empirically refutable. This is the reason why I have emphasized the observational equivalence of the two models throughout my formal presentations. Many examples I gave are meant to illustrate the following points. In comparing the two models that are observationally equivalent, the model with multiple equilibria has more predictive content than the model with a unique equilibrium. I believe that this is a fairly general point, although I don't know how to prove it formally. The point is this. Take any model with a unique equilibrium; you can always build a model with multiple equilibria, which is observationally equivalent, but makes less assumptions. Take any model with multiple equilibria; you can always build a model with unique equilibrium, which is observationally equivalent, but only at the cost of more stringent assumptions.

The observational equivalence of the two models implies that a common criticism against multiple equilibria, the unfalsifiablity, is unwarranted. When I present a model with multiple equilibria, people frequently ask me: "Can you test multiple equilibria?" I often find it difficult to answer this question, because the precise meaning of the question is not always clear. If they are asking "Does your model have any testable implication when multiple equilibria exist," then the answer would be "YES," because the multiple equilibrium model imposes the same restrictions on the data with the observationally equivalent unique equilibrium model. If they are asking, "is it possible to test whether the condition for the multiplicity holds," then the answer is "NO," because of the observational equivalence. But this does not mean that we can ignore the possibility of multiple equilibria and focus our attention on the case of a unique equilibrium, because one could also say that it is impossible to test the condition under which uniqueness holds.[9]

This is not to deny that, for any given model, a smaller set of equilibria means sharper predictions. It is thus desirable to "refine" the set of equilibria by throwing away some highly implausible or "unstable" equilibria. But this is different from saying that it is desirable to "select" the unique equilibrium.[10] "Few"-ness may be desirable, but there is nothing special about "unique"-ness. What determines the falsifiability of a theory is the size of the set of all feasible allocations that violate the equilibrium conditions, not the size of the set of all equilibrium allocations.

Since the falsifiability of a theory depends on the extent to which the theory forbids, not on the extent to which it permits, models with multiple equilibria can generate a large number of testable implications. At least three kinds of hypotheses can be tested. First, no matter how big the set of equilibria is, we can still test the model if all equilibria share some common properties. For instance, we can always test the equilibrium conditions directly. Second, one can test how a change in an exogenous variable affects the set of equilibrium values of an endogenous variable (Milgrom and Roberts 1994). For example, in Figure 3, a change in F shifts the set of equilibrium in the same direction across all stable equilibria. So, even though we may not know which equilibrium is actually chosen, we can still predict how the economy responds to a change in F <u>on average</u>, which can be tested by a simple regression. In

[9]Another possible interpretation of the question would be "Can we test whether the real world has multiple equilibria?" (A few have indeed asked me the question in these words.) If so, those who asked the question fail to understand that the uniqueness or the multiplicity of equilibrium is a property of the model, not a property of the world the model is trying to explain. After all, the two models explain exactly the same phenomena (remember: they are observationally equivalent); they differ only in perspective. Such a question would be just as absurd as "can we test whether the real world is in a partial equilibrium or in a general equilibrium?"

[10]Another common question when presenting a model with multiple equilibria is "How are we going to select an equilibrium?" My response is "Why do you want to select an equilibrium?" The main message of a multiple equilibrium model is the multiplicity: the equilibrium restrictions are not strong enough to eliminate all but a unique outcome. So, we must not select an equilibrium; we must deal with a set of equilibria as a whole; we have no justification to pick one of them. It is unfortunate that economic journals are littered with articles on multiple equilibrium models, in which the authors seem to feel free to select their favorite equilibrium.

this case, multiple equilibria even justify the presence of error terms, which a model with unique equilibrium would have to explain in a more ad hoc way, i.e., by introducing some "unobservable" variables (Jovanovic 1989). Third, the model may predict that a certain change in parameter values affects the number of equilibria, which could be tested in principle by comparing different policy and regulatory regimes across countries and across different periods.

More generally, the very fact that certain parameter values ensure uniqueness while others imply multiplicity of equilibria itself is a prediction of great significance. Prediction of this kind also provides us with a new way of conducting policy analysis. Depending on the objective, one may design policies, in the spirit of mechanism design, to generate a good equilibrium, to eliminate a bad equilibrium, or to ensure unique equilibrium. Analyses of this kind can be more useful than comparative analyses as a guide for policymaking. We may be throwing away many important policy implications if we look only at models with a unique equilibrium.

That a small parameter change can cause a discrete change in the set of equilibria also make models with multiple equilibria useful for explaining the possibility of a structural change, which may be initiated by a small change in the environment. In my view, this is the most important message of the economics of complementarity: through the process of mutual interaction of complementary activities, a small shock can be amplified to create a large effect.[11] Such sensitive dependence also has an important policy implication. It is generally viewed, both by its advocates and its critics, that the literature on coordination failures based on "strategic complementarity," justifies policy activism. However, as discussed in Matsuyama (forthcoming) the results in this literature can be interpreted as a case for a more prudent approach. The economics of complementarity may indeed demonstrate the possibility of

[11]I should point out that a model with multiple equilibria is a useful and simple way of demonstrating the sensitive dependence, but it is not the only way. A dynamic model can exhibit sensitive dependence, even when its equilibrium is unique, as shown in section 8. Even a static model with a unique equilibrium can capture the senstive dependence, if a model have a large number of state variables, as in Jovanovic (1987). See also Scheinkman and Woodford (1994).

coordination failures. It may explain why active government intervention seems to have been successful in certain cases. But, it also poses a serious question concerning the replicability of such experiences; each society, each region, or each industry is unique; without detailed knowledge of the environment, the effects of policy interventions are extremely difficult to predict. And the inefficiency of original allocation means that, if we make a mistake, the welfare loss due to such a policy error is of the first-order of magnitude.

10. Concluding Remarks.

Many debates in our profession remind me of the story of blind men palpating the elephant. Models with multiple equilibria differ from those with unique equilibrium mainly in perspectives, not necessarily in the set of the facts they are trying to explain. The critic of models with multiple equilibria, exhibiting instability and sensitive dependence, all prominent features of the economics of complementarity, seems to forget that they are designed to answer a different set of questions and to express different ideas from models with unique equilibrium. For example, models with multiple equilibria are the most natural framework for explaining diversity: why are there huge variations across regions and periods in productivity, standards-of-living, business practices, or the organization of industry? In a model with unique equilibrium, any attempt to explain the variations of per capita income forces us to introduce variations in other variables, such as saving rates and education, as is common in growth accounting exercises. Yet variations in these variables themselves are left unexplained, or need to be explained by introducing variations in another set of variables. Even worse, such an endless search for the cause may have the danger of encouraging the view that "we are so much richer than they are because we saved so much more and worked so much harder in the past, and because we are so much more prudent and industrious." In models with multiple equilibria, on the other hand, one can explain diversity across economies without assuming inherent differences, which tends to suggest a completely different view. There is nothing special about the world we actually observe; it is merely one of many different conceivable outcomes, that happened to be chosen as a consequence of a long series of small historical accidents; or "small initial advantages we had in natural conditions have started cumulative processes of more saving and more education, until we became much richer." Needless to say, there is nothing wrong with a unique equilibrium. There are many important questions that are better answered by models with unique equilibrium. But belief in the virtue of uniqueness has the danger of conditioning economists into certain prescribed patterns of thinking. It would force us to look only for problems that are easily predictable, which may lead us to think that the real world should be predictable and hence may be manipulated. It would lead us to believe that only a big change in the environment can cause a big effect, etc.

These responses may not have succeeded in convincing the critics. Behind their objections and skepticism, there may also be some psychological resistance. Accepting multiplicity means abandonment of the deterministic view of the world.[12] It means that there is nothing special about the world in which we live. It means that there are alternative forms of economic, political, and social organizations that are completely different from the way our society is actually organized. And it is highly unsettling to admit the possibility that a slight change in the environment may result in drastic changes. This may be one

[12]It should be noted that the term "deterministic" is used as a modifier to the word, "view," not to the word "model." The adjective, "deterministic," and its noun, "determinism," are the standard terminologies in the philosophy of science. Determinism is the doctrine that all events, including human choices and decisions, have sufficient causes. It is typically associated with the Newtonian-Laplacian mechanics, which projects the view that all future events can be uniquely determined once all the relevant initial conditions, that is, the initial positions and velocities of all particles in the universe, are known. This view is expressed in its mathematical form, that a completely specified model should have a unique equilibrium. The determinism doctrine does not require your model to be deterministic. To the contrary, the deterministic view calls for stochastic models, because all the forecast errors, according to this view, are attributed to our imperfect knowledge, which need to be modelled as stochastic error terms or "unobservables." On the other hand, those who believe in indeterminism, that is, those who believe that the laws of nature do not determine uniquely the world we live in, tend to use deterministic (meaning nonstochastic) models with multiple equilibria in order to express their idea, because they do not need stochastic shocks to generate fluctuations.

reason why many economists tend to impose at the beginning of their modelling exercises parameter restrictions that ensure uniqueness, even when there is no supporting evidence for such restrictions --a tendency that is so deeply entrenched that we are rarely aware of doing it.[13] And once these restrictions become standard, they become "natural" and "plausible." In many of the models presented above, multiple equilibria are generated simply by dropping "standard" parameter restrictions imposed in the literature. Some economists seem to be surprised how a small perturbation to standard assumptions can generate multiple equilibria in models of monopolistic competition.

[13]There may be another possible reason. Models of complementarities with multiple equilibria are inevitably highly nonlinear. For the sake of tractability, one often has to make many simplifying assumptions concerning the functional forms, keeping the number of variables at a minimal level. On the other hand, ruling out the possibility of complementarities and multiplicity allows one to write down a linear model or at least to solve a model by a linear approximation, which permits more general function forms with an arbitrary number of variables. This apparent generality of a model may give the researcher a false sense of achievement.

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