Discussion Paper No. 1181

Incentives for Procrastinators

by

Ted O'Donoghue
Northwestern University
and
Matthew Rabin
University of California, Berkeley

February 25, 1997

Incentives for Procrastinators

Ted O'Donoghue
The Center for Mathematical Studies in Economics and Management Sciences
Northwestern University

and

Matthew Rabin
Department of Economics
University of California, Berkeley

February 25, 1997

Abstract

People procrastinate. Psychological research and common intuition suggest that this propensity reflects a time inconsistency: People often put off unpleasant tasks because they pursue immediate gratification in a way that their long-run selves do not appreciate. This paper examines the implications of procrastination for the design of temporal incentive schemes, where an agent is rewarded according to when he completes some task. A risk-neutral agent is hired by a risk-neutral principal to complete some task. Delay in completion of the task is costly to the principal, but the agent faces a stochastic cost of completing the task, so that it is efficient to wait when the task cost is high. We assume the principal designs temporal incentive schemes to avoid inefficient delay. We mainly consider an environment where the incremental cost of delay is constant over time. When the principal knows the distribution of task costs, she can always design a stationary incentive scheme that achieves first-best efficiency. However, while for time-consistent agents this scheme will reflect the true delay cost, for procrastinators this scheme must punish delay more severely to counteract procrastination. When the agent is privately informed about the task-cost distribution, again a stationary scheme can induce efficiency for time-consistent agents. But for procrastinators, different task-cost distributions imply different propensities for procrastination, so the first-best may be no longer achievable. Furthermore, second-best optimal incentive schemes will typically not be a stationary scheme, but rather a sort of generalized deadline contract: Delay will be punished moderately at first, but after some time it will be punished severely. Such schemes encourage those with little propensity to procrastinate to wait until the cost of completing the task is low, while deadlines assure that those with a severe propensity to procrastinate don't delay too long.

Keywords: Deadlines, Incentive Schemes, Procrastination

JEL Classifications: A12, B49, C70, D60, D82, NC-17

Acknowledgments: We thank Steven Blatt and seminar participants at Berkeley's IO/Strategy Seminar for helpful comments, and Erik Eyster and especially Steven Blatt for valuable research assistance. O'Donoghue thanks the Alfred P. Sloan Foundation and Rabin thanks the Alfred P. Sloan and Russell Sage Foundations for financial support.

Mail: Ted O'Donoghue / The Center for Mathematical Studies in Economics and Management Sciences / Northwestern University / 2001 Sheridan Road, Room 371 / Evanston, IL 60208-2014. Matthew Rabin / Department of Economics / 549 Evans Hall #3880 / University of California, Berkeley / Berkeley, CA 94720-3880.

E-mail: eodonogh@skew2.kellogg.nwu.edu and rabin@econ.berkeley.edu.

1. Introduction

While the standard economics model assumes that the desire to delay an unpleasant task must be time-consistent, people tend to *procrastinate* – connoting a time-*inconsistent* preference for delay. Today we feel we should write a referee report tomorrow, but tomorrow we tend to delay again. A small set of economists and psychologists have over the years proposed formal models of time-inconsistent preferences and self-control problems, where people have a tendency to pursue their immediate well-being in a way that their "long-run selves" do not appreciate. O'Donoghue and Rabin (1996) build from this prior research and show that, when a person is unaware of the time inconsistency, such preferences lead a person to procrastinate in completing an unpleasant task.[1]

In this paper, we examine the implications of time-inconsistent procrastination for the design of *temporal incentive schemes*, which reward an agent based on when he completes some task. Temporal incentive schemes are a central aspect to organizational design and many types of contracts. People face punishments for delay, sometimes explicitly in the form of decreased compensation, and more often implicitly in the form of admonitions from supervisors and decay in reputation. Such incentives are needed when an agent finds a task unpleasant and does not intrinsically value its timely completion.

We first show that if principals designing contracts wish to induce agents to behave efficiently, they will make incentives "steeper" than if the agent did not procrastinate: Whereas for time-consistent agents optimal incentives would exactly reflect the principal's true cost of delay, to counteract procrastination the principal must punish procrastinators more severely than the true cost of delay. But our main conclusion concerns a prevalent feature of temporal incentive schemes: They often involve *deadlines*, which we define broadly as dates after which the incremental punishment for delay in completing a task becomes more severe than it was before that date. We show that deadlines can be (second-best) optimal given time inconsistency, even in stationary environments where they would never be optimal with time consistency. While we suspect there are other significant explanations for the prevalence of deadlines, we feel that the logic behind our results suggests that deadlines arise in part to battle procrastination.

In Section 2, we formalize a simplified version of time-inconsistent preferences: A person always values her well-being *now* more than in any future moment, and values her well-being at all future moments equally. In Section 3, we introduce our model of temporal incentive schemes. We suppose a risk-neutral principal hires a risk-neutral agent to complete some task. Because the principal faces a *delay cost*, she prefers that the task be done sooner rather than later. But because the agent faces a stochastic *task cost*, it may be better for him to delay when the task-cost realization is particularly high. Efficient behavior then will minimize the sum of expected delay costs and expected task costs. Because the principal cannot observe task-cost realizations, she must compensate the agent based solely on observed delay, yet she does not know whether observed delay is an efficient response to a high task-cost realization, or inefficient procrastination. In addition, the principal is limited to the temporal incentive schemes discussed above. We explore whether temporal incentive schemes can induce efficient behavior, and, if not, what temporal incentive schemes are

[1] This finding replicates and extends a similar example of procrastination by Akerlof (1991), who motivates a mathematically similar model of choice behavior by emphasizing how the costs of doing a task are more salient when they are immediate than when they are delayed.

second-best optimal.

Time-*consistent* agents do not procrastinate, and therefore the optimal incentive scheme is straightforward: If the incremental punishment for delay exactly mirrors the principal's delay costs, then the agent will internalize those delay costs, balance them against his task costs, and behave efficiently. Importantly, the optimal incentive scheme is independent of the probability distribution of the agent's task costs. Time-*inconsistent* agents procrastinate, so things are more complicated. Incentive schemes must deter inefficient procrastination, yet still encourage efficient delay when the task-cost realization is high. Nonetheless, we show that as long as the principal knows the distribution of task costs, she can exactly counteract the agent's tendency to procrastinate with incentives that punish delay by more than the actual delay cost. Despite the agent's propensity to procrastinate, therefore, a fully efficient incentive scheme can be implemented. However, this (first-best) optimal incentive scheme for time-inconsistent agents very much depends on the distribution of task costs. Specifically, higher task costs (on average) make the agent more prone to procrastinate, in which case the principal must impose a more severe punishment for delay to counteract procrastination.

In Sections 4 and 5, we assume the agent has private information about the distribution of task costs, and investigate the nature of incentive schemes when the principal's incremental cost of delay is stationary. For time-consistent agents, a stationary delay cost implies that a stationary incentive scheme can induce first-best efficiency even with uncertainty over the distribution of task costs. Furthermore, if the principal could observe the distribution of task costs, then stationary incentive schemes could induce first-best efficiency for time-inconsistent agents as well. But when the principal is uncertain about a time-inconsistent agent's task-cost distribution, she is uncertain about the agent's propensity to procrastinate, creating a problem: Punishment for delay that is harsh enough to prevent excessive procrastination by severe procrastinators may be so harsh that moderate procrastinators complete the task when it would be more efficient to wait. As a result, first-best optimality typically will not be feasible.

In Section 4, we consider the case where task-cost distributions differ only in their means, with the same probability distribution around this mean. We show that second-best optimal incentive schemes typically will not be stationary, but rather will be a deadline scheme: Incremental delay will be punished moderately early on, but after some "deadline" it will be punished more severely. Such a scheme early on encourages those with little propensity to procrastinate to wait until it is efficient to do the task, while the deadline assures that severe procrastinators do not delay too long. Put differently, the longer an agent delays completing a task, the more likely the delay is because of procrastination rather than efficient waiting; it is therefore optimal for the principal to punish incremental delay more severely as time goes by. In Section 5, we relax the assumption that the distribution of task costs around the mean is the same for all task-cost distributions. We show that for the more natural case – where agents with lower average task costs also have lower variance – the "deadline result" from Section 4 holds.

An important issue in modeling time-inconsistent preferences is downplayed in all the discussion above: How aware are people that they might behave in the future against their current preferences? In our context: Do people predict their tendency to procrastinate? In Section 2, we discuss two extreme assumptions that have appeared in the literature – that people are fully aware of their future self-control problems, and that they are completely *un*aware of their future self-control problems. O'Donoghue and Rabin (1996) compare the two assumptions. We show that while self-control problems lead unambiguously to procrastination when an agent is naive about his tendency to procrastinate, an agent who is sophisticated about his tendency to procrastinate may do a task *earlier* than a more patient, time-consistent agent would. Because we feel that naivete is often the more realistic assumption – and because it is far more tractable than the sophistication assumption – our formal model assumes naivete.[2]

The assumption that people are naive about their self-control problem gives rise to an important additional issue: Since naive people overestimate their payoff from an incentive scheme (because they don't realize that they'll get low wages due to procrastination), principals aware of this procrastination might hire people merely to bilk them of money rather than to efficiently complete a task. We discuss in Section 3 how reputational pressures and other factors might lead a principal to care only about efficiency, as assumed in the discussion above, and most of our analysis in the paper assumes that the principal prefers to induce efficient behavior. Even so, in Section 3 we examine the alternative assumption that the principal wishes to bilk the agent. We show that she can always do so with an incentive scheme that is sufficiently lenient in punishing delay so that the agent is lulled into severe and costly procrastination.

We conclude in Section 6 with a discussion of some caveats and possible extensions to the model of this paper, and a discussion of some other implications of procrastination for organizational design.

2. Presently Preferences

O'Donoghue and Rabin (1996) coined the term "presently preferences" for the class of time-inconsistent preferences where a person puts greater and greater weight on his well-being at an earlier moment rather than a later moment as the earlier moment gets closer. Variants of such preferences have been studied by many researchers.[3] Consider the choice between doing 7 hours of an unpleasant task on April 1 versus spending 8 hours to complete the same task on April 15. The task could be completing your taxes - on April 15 you would have to take the extra time to go to the post office to mail your returns, whereas on April 1 you could simply mail it without hassle on your way to work the next day.

If asked to commit on February 1 to one or the other, most people would prefer to do less work in April, and therefore choose 7 hours on April 1. If they must choose on April 1, however, most people are inclined to put off the task two weeks rather than doing it right away. When April 1 arrives, people have a preference for immediate gratification –

---

[2] Economists examining self-control issues seem to be inclined to assume sophistication, well beyond what we feel behavioral evidence supports. In part this inclination derives from a desire to depart from familiar economic assumptions one step at a time -- naivete is two steps away by simultaneously assuming time-inconsistent preferences and "irrational expectations" about those preferences. Indeed, the analysis in this paper to some extent reinforces this worry: Many of the issues to which naivete gives rise are unfamiliar and problematic for economic analysis. Yet, in this and related research, we have discovered a pattern: In many models, naivete is *far* more tractable than sophistication in terms of the practical logistics of formal models. We fear, therefore, that a conservative weddedness to the sophistication assumption may not only be tenuous behaviorally, but hamper the incorporation of self-control and time-inconsistency issues into economics.

[3] Casual observation, introspection, and psychological research all indicate such time inconsistency. See Chung and Herrnstein (1967), Ainslie (1974, 1975, 1987, 1992), Ainslie and Herrnstein (1981), Thaler (1991), Funder and Block (1989), Hoch and Loewenstein (1991), Ainslie and Haslam (1992a, 1992b), Loewenstein and Prelec (1992), and Kirby and Herrnstein (1995). For early economics papers focusing on time-inconsistent discounting, see Strotz (1955), Koopmans (1960), Phelps and Pollak (1968), Pollak (1968), and Goldman (1979,1980).

not doing the unpleasant task today – with which their long-run selves disagree.

Phelps and Pollak (1968) put forward an elegant model (later employed by Laibson (1994)) which can be used to capture this time-inconsistent taste for immediate gratification. They proposed a simple two-parameter model that slightly modifies exponential discounting. Let u_t be the instantaneous utility a person gets in period t. Then her intertemporal preferences at time t, U^t, can be represented by the following utility function:

$$\text{For all } t,\ U^t(u_t, u_{t+1}, \ldots, u_T) \equiv \delta^t u_t + \beta \sum_{\tau=t+1}^{T} \delta^\tau u_\tau.$$

The parameter δ represents "time-consistent" impatience, so for $\beta = 1$ these preferences are simply (the discrete version of) exponential discounting. But for $\beta < 1$, these preferences parsimoniously capture the time-inconsistent preference for immediate gratification. Since we shall focus in this paper on relatively short horizons, we assume $\delta = 1$ – so there is no time-consistent discounting. Hence, the intertemporal utility function is

$$\text{For all } t,\ U^t(u_t, u_{t+1}, \ldots, u_T) \ = \ u_t + \beta \sum_{\tau=t+1}^{T} u_\tau.$$

Consider again the above example. Assume that your instantaneous disutility from doing work is simply the number of hours of work, so that $u_t(7) = -7$ and $u_t(8) = -8$ for all t. Suppose also that $\beta = .8$: You are willing to forego a given loss in utility *in the future* for a gain in utility *now* that is only 80% as large. Consider your decision on February 1. Because on February 1 you discount both dates by β, you will choose to work 7 hours on April 1 rather than 8 hours on April 15. Contrast this with what your decision would be on April 1. You can experience a utility of -7 by working today, or experience a discounted utility of $.8(-8) = -6.4$ by delaying the work until 2 weeks from now. You will, therefore, delay work. Hence, for the exact same problem, your choice on April 1 is different than your choice on February 1. Irrespective of its specific prediction, exponential discounting would predict that your choice would be the same whether you made that choice on February 1 or April 1.

To examine dynamic choice given time-inconsistent preferences, researchers have converged on a simple modeling strategy: For each point in time, a person is modeled as a separate "agent" who chooses her current behavior to maximize her current preferences, predicting how her future selves will behave. In such a framework, an important issue arises: What are a person's beliefs about how her future selves will behave? Of course the answer to this question depends on a vast array of beliefs about future selves' preferences and beliefs about future selves' beliefs. Two extreme assumptions have appeared in the literature to deal with the issue of beliefs about future behavior. *Sophisticated* people are fully aware of their future self-control problems and therefore know exactly how their future selves will behave. *Naive* people are fully *un*aware of their future self-control problems and therefore believe their future selves will behave exactly as they currently would like them to behave.[4]

[4] Strotz (1955) and Pollak (1968) carefully lay out these two assumptions (and develop the labels), but do not much consider the implications of assuming one versus the other. More recent papers have either assumed one or the other, without attempting to justify the choice with behavioral evidence. For instance, Akerlof (1991) assumes naive beliefs, while Laibson (1994,1995) assumes sophisticated beliefs. O'Donoghue and Rabin (1996) consider both, and explicitly contrast the two, but likewise do not provide behavioral evidence for either.

There seems to be elements of both sophistication and naivete in people. Some degree of sophistication is implied by the fact that people often pay to commit themselves to smaller choice sets (e.g., joining fat farms or Christmas clubs, or buying small rather than large packages of enticing goods). A naive person would never worry that her tomorrow self might choose an option that she doesn't like today, and therefore would find committing herself unattractive. On the other hand, people do seem to overestimate the degree to which they will abide by their current preferences for future self-control. For example, people will repeatedly not have the "will power" to forego tempting foods or quit smoking while predicting that tomorrow they will have this will power.

O'Donoghue and Rabin (1996) examine the implications of assuming sophistication versus naivete. One of their conclusions is that sophistication often leads to complicated behavior. For the context of this paper, small changes in incentive schemes can lead to dramatic changes in behavior, and incentive schemes that yield stationary behavior for both time-consistent agents and naive time-inconsistent agents can yield highly nonstationary behavior for sophisticated agents. This makes the search for optimal incentive schemes a much more difficult exercise. In part to avoid getting lost in such difficulties, we shall focus mostly in this paper on the case of naive beliefs. While extreme, we do not believe our focus on naive beliefs is without behavioral foundation. We believe that much day-to-day procrastination is characterized by a large degree of naivete: We procrastinate today believing we will complete some task tomorrow, but tomorrow we decide to delay again. Importantly, even when we are aware of a general tendency to procrastinate we seem capable of underestimating this tendency on a case-by-case basis. We return in Section 6 to comment briefly on how sophistication might affect our results.

3. A Model of Temporal Incentive Schemes

Suppose a principal hires an agent to complete some task. They sign a contract specifying how the principal will compensate the agent, where wages can be contingent only on information available to both parties. In contrast to the standard principal-agent paradigm, we assume that there is no uncertainty about whether an action has been taken, nor about the level of effort by the agent, so that there is no moral hazard of the traditional sort. Rather, we focus on the problem of *when* the agent completes the task if there is day-to-day uncertainty over the cost to the agent of completing the task. For example, on any given day the agent may be sick or may have more pressing projects to complete. Efficiency may require that the agent wait on days with a high task cost, and do it when the task cost is low. Moral hazard can arise if the principal cannot observe the task-cost realizations. We explore the role of *temporal incentive schemes* – contracts where wages are contingent on when the agent completes the task – in such an environment.[5]

The trade-off typically studied in principal-agent models is between incentives and insurance. In our context, creating incentives not to delay can impose risk on the agent, since he will get low wages if he has unusually bad luck in completing the task early on. Because we wish to focus solely on the procrastination issue, we will assume the agent is risk-neutral, so insurance is not an issue.

[5] Although incentive schemes have been studied extensively in organizational and mechanism-design literatures, to our knowledge this literature has not examined temporal incentive schemes.

Suppose a task can be completed in any period $t \in \{1, 2, \ldots, T\}$, where T can be finite or infinite. The principal prefers to have the task done sooner because she faces a cost of delay. The principal's exogenously-determined gross payoff schedule is $\mathbf{X} = (X_1, X_2, \ldots, X_T)$, where she gets payoff X_t if the agent completes the task in period t. The marginal delay cost is captured by $x_t^{\Delta} \equiv X_t - X_{t+1} > 0$ for all t. In the case of $T = \infty$, we assume the principal's payoff if the agent never completes the task is less than X_t for all t.[6] We often assume a *stationary delay cost* x^{Δ}, where $T = \infty$ and $x_t^{\Delta} = x^{\Delta}$ for all t.

Although the principal prefers to have the task done sooner, there can be benefits to delay if it yields a lower task cost to the agent; lowering the agent's expected task cost allows the principal to pay a smaller expected wage (i.e., it will be easier to satisfy the agent's participation constraint). In period t, the *task cost* to the agent, c_t, is drawn from a stationary distribution C with support $[\underline{c}, \overline{c}]$, $\underline{c} \geq 0$, and cumulative density function $F(c_t)$. In this section, to simplify the arguments we assume $F(\underline{c}) = 0$, but the results all hold if we relax this assumption.[7] The task cost is meant to capture any disutility to the agent for completing the task, including opportunity cost. Importantly, we assume the task costs are "salient" to time-inconsistent agents, causing the agents to give the costs exaggerated weight in their decisions, and hence causing procrastination. In other words, our model examines tasks which involve long-term gains and short-term costs.

The agent's behavior can be described by a strategy that is a vector of cutoff costs $\mathbf{s} \equiv (s_1, s_2, \ldots, s_T)$, where the agent completes the task in period t if and only if $c_t \leq s_t \in [\underline{c}, \overline{c}]$.[8] Before considering incentive schemes, we characterize first-best efficient behavior for a given $\mathbf{X}$ and C. Efficient behavior $\gamma^* \equiv (\gamma_1^*, \gamma_2^*, \ldots, \gamma_T^*)$ will minimize the sum of expected task costs and expected delay costs.[9] Throughout we will denote a generic strategy by s and specific strategies (i.e., the efficient strategy and "equilibrium" strategies) by γ's. To solve for γ^*, we need some notation. First, we define a hazard function $h(\tau \mid t, \mathbf{s})$, which represents the probability that the agent has not completed the task before period τ conditional on not completing the task before period $t \leq \tau$, given the strategy s. Then

$$h(\tau \mid t, \mathbf{s}) = \begin{cases} 1 & \text{if } \tau = t \\ \displaystyle\prod_{i=t}^{\tau-1} (1 - F(s_i)) & \text{if } \tau > t. \end{cases}$$

[6] We can think of this as the principal receiving X_1 in period 1, and paying x_τ^{Δ} in each period τ that the task is delayed. This interpretation implies that the principal cannot avoid losses relative to X_1 by inducing the agent never to complete the task.

[7] This assumption implies that "completing the task when $c_t \leq \underline{c}$" is equivalent to "waiting" – not doing the task for sure. Without this assumption, we have to define an action to represent waiting (as we do in Sections 4 and 5).

[8] By defining the strategy this way, we are restricting the set of possible strategies that the agent could employ. More generally, each action s_t could be a function of the history of task costs $(c_1, c_2, \ldots, c_{t-1})$. This simplification is unrestrictive, however. Since we assume throughout that the agent knows the expected distribution of task costs C, continuation payoffs are independent of past task costs.

[9] I.e., by efficiency we mean minimizing the sum of all ex ante costs incurred by either party. Therefore, given the structure of ex ante negotiations and the "reputation constraint" (both to be discussed shortly), the efficient strategy will be equivalent to the strategy that would maximize the principal's payoff if she could observe task-cost realizations. We assume throughout the paper that γ^* exists and is (as we define it below) "unique" in the sense that all efficient strategies will yield equivalent behavior. These conditions hold in all cases we've considered, and we suspect that this holds without loss of generality, but have not proven any such results.

Let $\chi^t(\mathbf{s})$ be the *expected delay cost* from waiting in period t given the strategy $\mathbf{s}$ (so the expected gross payoff to the principal if the agent waits in period t is $X_t - \chi^t(\mathbf{s})$). Then

$$\chi^t(\mathbf{s}) \;=\; \sum_{\tau=t+1}^{T} h(\tau|t+1,\mathbf{s})\, x^{\Delta}_{\tau-1}.$$

Let $\zeta^t(\mathbf{s})$ be the *expected task cost* from waiting in period t given the strategy $\mathbf{s}$. Then,

$$\zeta^t(\mathbf{s}) \;=\; \sum_{\tau=t+1}^{T} h(\tau|t+1,\mathbf{s})\, F(s_\tau)\, E(c|c \le s_\tau),$$

where $E(c|c \le s) \equiv \frac{1}{F(s)} \int_{\underline{c}}^{s} c\, dF(c)$.

Efficient behavior minimizes the sum of the expected delay cost and the expected task cost. In other words, in each period t the agent should complete the task if the task cost c_t is less than the total expected costs for delay, so γ^* satisfies for each $t < T$

$$\gamma^*_t \;=\; \begin{cases} \chi^t(\gamma^*) + \zeta^t(\gamma^*) & \text{if } \; \underline{c} \le \chi^t(\gamma^*) + \zeta^t(\gamma^*) \le \overline{c} \\ \underline{c} & \text{if } \; \chi^t(\gamma^*) + \zeta^t(\gamma^*) < \underline{c} \\ \overline{c} & \text{if } \; \chi^t(\gamma^*) + \zeta^t(\gamma^*) > \overline{c}. \end{cases}$$

We next describe temporal incentive schemes. We shall always assume that the agent can observe c_t in period t before choosing whether or not to perform the task. The principal never observes the task costs. As discussed above, this is the source of moral hazard: The principal would like to require the agent to complete the task in period t if and only if $c_t \le \gamma^*_t$. But since the principal cannot observe c_t, the contract cannot specify a wage contingent on the task cost, but only on when the agent completes the task. We denote a *temporal incentive scheme* by $\mathbf{W} = (W_1, W_2, \ldots, W_T)$, where the agent receives wage W_t if he completes the task in period t. The *incremental wage* w^{Δ}_t is defined by $w^{\Delta}_t \equiv W_t - W_{t-1}$. The principal is assumed to be risk-neutral, so her overall payoff from a contract under which the agent follows strategy $\mathbf{s}$ will be $\sum_{\tau=1}^{T} h(\tau|1,\mathbf{s})\, F(s_\tau)\, [X_\tau - W_\tau]$.

The agent has presently preferences, as described in Section 2, with $\delta = 1$. We consider two types of agents. *TCs* have $\beta = 1$, so they have standard time-consistent preferences. (*TC* stands for time consistency.) *Naifs* have $\beta < 1$, but they are naive so they believe they will behave like TCs beginning next period. We examine the behavior of TCs mostly as a benchmark against which to compare behavior for people with presently preferences and because TC behavior represents naifs' perceived future behavior. Naifs are time-inconsistent, and are unaware of this time inconsistency. They are also more impatient than TCs. It will become clear that the main results are driven by the combination of time-inconsistency and naivete, and *not* the relative impatience.

To analyze an agent's reaction to an incentive scheme, we must convert wages and task costs into instantaneous utilities. Because we assume the task cost is incurred immediately, while the wage is received at some future date, the agent's utilities for completing the task in period $\hat{t}$ are given by $u_{\hat{t}} = -c_{\hat{t}}$, $u_{\hat{t}+1} = W_{\hat{t}}$, and $u_t = 0$ for all $t \notin \{\hat{t}, \hat{t}+1\}$.[10] The assumption that the task cost is incurred immediately while the wage is delayed is crucial because it implies that

[10] We assume for simplicity that the wage is received in period $\hat{t}+1$. This means that the wage is not salient – the agent does not have an extra incentive to do the task to get the wage *now* as opposed to sometime in the future. Because we have assumed no time-consistent discounting, the agent does not care when in the future he receives the wage.

naifs have a tendency to procrastinate.[11]

Before considering ex ante negotiations, we first discuss how the agent will behave given an incentive scheme $\mathbf{W}$. We use the concept of *perception-perfect strategies* introduced by O'Donoghue and Rabin (1996). Rather than give the general definition, we describe the implications for the two types of agents in the model here. For TCs a perception-perfect strategy is the standard, simple decision-theoretic prediction: At all times TCs maximize their expected utility given their current information, so they complete the task now if the utility from doing so is higher than the expected utility from waiting. Naifs similarly compare their utility from completing the task now to their *perceived* expected utility from waiting; but because naifs think they will behave like TCs in the future, their perceived utility is systematically wrong (and, in particular, overoptimistic). That is, naifs misperceive their future behavior and consequently their future utility from waiting.

Let $\hat{\gamma} \equiv (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_T)$ be a perception-perfect strategy for TCs. The period-t expected utility from waiting for TCs will depend on the expected task cost from waiting as well as the expected lost wages from waiting. The expected task cost from waiting for TCs is $\zeta^t(\hat{\gamma})$. Let $p^t(\mathbf{s})$ be the *expected wage cost* from waiting in period t when the agent perceives he will follow strategy $\mathbf{s}$ in the future (so the expected wage if the agent waits in period t is $W_t - p^t(\mathbf{s})$). Then

$$p^t(\mathbf{s}) \;=\; \sum_{\tau=t+1}^{T} h(\tau \mid t-1, \mathbf{s})\, w^{\Delta}_{\tau-1}.$$

In period t, TCs compare the utility from completing the task now $W_t - c_t$ to the expected utility from waiting $W_t - p^t(\hat{\gamma}) - \zeta^t(\hat{\gamma})$, and therefore complete the task if and only if $c_t \leq p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma})$. Then $\hat{\gamma} \equiv (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_T)$ satisfies for each $t < T$

$$\hat{\gamma}_t \;=\; \begin{cases} p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma}) & \text{if } \;\underline{c} \leq p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma}) \leq \overline{c} \\ \underline{c} & \text{if } \; p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma}) < \underline{c} \\ \overline{c} & \text{if } \; p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma}) > \overline{c}. \end{cases}$$

Let $\gamma \equiv (\gamma_1, \gamma_2, ..., \gamma_T)$ be a perception-perfect strategy for naifs. Naifs believe they will behave like TCs in the future, so their *perceived* expected task cost from waiting is $\zeta^t(\hat{\gamma})$, and their *perceived* expected wage cost from waiting is $p^t(\hat{\gamma})$. Then $\gamma \equiv (\gamma_1, \gamma_2, ..., \gamma_T)$ satisfies for each $t < T$.[12]

In the case of $T = \infty$, we assume (as we did for the principal) that never completing the task carries consequences for the agent, and in particular if the agent never completes the task his payoff is below W_t for all t. An interpretation is that from time to time the agent must "settle up" incurred wage costs.

11 Even if the agent gets his wages immediately, they are not salient if he cannot enjoy the benefits of those wages immediately; wages will therefore be salient only if the agent can both consume the wages immediately *and* does not have liquid wealth that is already available to consume. We discuss the use of salient rewards such as breaks or parties in the concluding section.

12 $\hat{\gamma}$ and γ as defined in the text are unique, but there could be other strategies that would yield identical observed behavior (and would be perception-perfect strategies under a more general definition). Throughout, we refer to $\hat{\gamma}$ and γ as *the* perception-perfect strategies. For $T = \infty$, under some $\mathbf{W}$ $\hat{\gamma}$ and/or γ may not exist; but for all $\mathbf{W}$ we consider they do exist.

$$\gamma_t = \begin{cases} \beta\left[p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma})\right] & \text{if } \underline{c} \leq \beta\left[p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma})\right] \leq \bar{c} \\ \underline{c} & \text{if } \beta\left[p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma})\right] < \underline{c} \\ \bar{c} & \text{if } \beta\left[p^t(\hat{\gamma}) + \zeta^t(\hat{\gamma})\right] > \bar{c} \end{cases}$$

The final component of the model is the ex ante negotiation and participation constraint. We assume the principal proposes a contract which the agent can accept or reject. If the agent accepts, the agent will be compensated according to the incentive scheme. If the agent rejects, then there is no contract. We assume the agent gets utility $\bar{U} = 0$ if he rejects the contract. What contracts will the agent accept?

For TCs, the answer is clear. Given an incentive scheme $\mathbf{W}$, TCs correctly predict their future behavior (i.e., $\hat{\gamma}$), so the utility to TCs from accepting the contract is the ex ante expected wage minus the ex ante expected task cost. Given our definition of ζ^t, ζ^0 represents the ex ante expected task cost. We can denote the ex ante expected wage given strategy s by $W^0(\mathbf{s}) \equiv W_1 - (1 - F(s_1))p^1(\mathbf{s})$. Hence, TCs accept the contract if and only if $W^0(\hat{\gamma}) - \zeta^0(\hat{\gamma}) \geq \bar{U} = 0$, so the participation constraint for TCs is $W^0(\hat{\gamma}) \geq \zeta^0(\hat{\gamma})$.

For naifs, several issues arise. Since naifs have time-inconsistent preferences, it matters whether the contract is signed in period 1 or prior to period 1. We assume ex ante negotiations occur prior to period 1, so the agent's "long-run utility" is relevant when signing the contract. In our model, we can interpret this as ex ante negotiations occurring in period 0, where the agent's preferences are described by U^0.

More importantly, we must determine the right perspective from which to consider payoffs. Naifs incorrectly perceive future behavior – they believe they will behave like TCs. Consequently, they will generally be overoptimistic about their utility from signing a contract. Should the participation constraint for naifs be based on the utility naifs perceive at the time they sign, or on the expected utility they actually get from the contract? In other words, should we use utility from an *ex ante* view ("perceived utility") or an *ex post* view ("experienced utility")?

Throughout our analysis, we assume that the principal knows about the agent's procrastination. We primarily take the point of view that the participation constraint for naifs should be based on experienced utility of the agent. But we first consider the perceived-utility perspective. When using perceived utility, naifs could be exploited as a "money pump": You could hire a naif to do some task and get the naif to *pay you* a large sum of money (i.e., a large negative wage) to do the task. Consider the following example:

Example 1

Suppose there is some task that is "meaningless" to the principal (i.e., $X_t = 0$ for all t). Suppose further that the distribution of task costs for the agent has support $[\underline{c}, \bar{c}]$, with mean E_C, and satisfies $\frac{1}{\beta}\underline{c} > \bar{c}$.

Consider an incentive scheme $\mathbf{W} = (E_C, E_C - w, \ldots, E_C - (T-1)w)$ with $\bar{c} - E_C \leq w < \frac{1}{\beta}\underline{c} - E_C$. Since $\bar{c} \leq E_C + w$, TCs will complete the task for sure in period 1. Hence, naifs' ex ante utility from the contract is $W_1 - E_C = 0 = \overline{U}$, so naifs would be willing to sign the contract. However, since $\underline{c} > \beta(E_C + w)$, in fact naifs procrastinate until period T. Hence, the principal could use this "meaningless" task to bilk an arbitrarily large amount of money from the agent, since $\lim_{T\to\infty} W_T = -\infty$.

In Example 1, it is possible for the principal to offer a contract that the agent will accept under the premise that he will complete the task immediately, when in fact he will for sure procrastinate until the very end. By proposing a contract with T arbitrarily large, the principal can guarantee an arbitrarily large negative wage. This example assumes the support of the task-cost distribution is small (relative to β), and for general task-cost distributions creating such a stark contrast between beliefs and behavior will not be possible. Nonetheless, the following lemma establishes that the principal can bilk arbitrarily large sums of money from naifs as long as the task cost is bounded away from zero.

Lemma 1 *Suppose there exists a task such that $\underline{c} > 0$. Then for any $\overline{W} < 0$ and any $\beta < 1$ there exists a contract* **W** *that naifs perceive to yield expected utility $U^p > 0$, but the actual realized wage is less than $\overline{W}$.*

All proofs are in the Appendix. The intuition for Lemma 1 is similar to the reasoning in Example 1. For any cost distribution, the principal can create an incentive scheme such that naifs believe they will complete the activity with positive probability in all periods (not necessarily with probability one), but in fact procrastinate until period T. For T sufficiently large, naifs believe they will complete the task long before period T, so they will accept a contract with a large negative wage in period T.

Lemma 1, taken literally, says that principals can become arbitrarily rich by hiring naive agents not to produce any useful function but rather to exploit their overoptimism. While it *is* plausible that firms take some advantage of such bilking opportunities, we think studying efficiency-oriented temporal incentive schemes is probably more important than studying bilking-oriented schemes. There are several reasons for this perspective. For one thing, because successful bilking requires delay in the agent completing the task, a principal who wishes *both* to bilk the agent *and* to get a task efficiently completed would need to separately offer "bilking contracts" and "efficiency contracts."

Furthermore, there are likely to be reputational pressures that induce firms to offer incentive contracts that are *ex post* acceptable to agents, which will also imply that firms wish to induce efficient behavior. A firm (i.e., the principal) will likely have many tasks that it needs completed. To maintain a pool of willing agents, the firm might need to develop a reputation of making agents on average ex post pleased with the outcomes, which means the expected *experienced* utility for the agent must be at least $\bar{U}$. Such a "reputation constraint" for principals would imply that, effectively, the participation constraint for naifs is based on experienced utility. If working at a firm involves completing, say, 100 tasks over the course of a year, and the most important "participation constraint" by employees is not whether they wish to take a job but whether they wish to change jobs, then the experienced-utility perspective is clearly more appropriate.

Another reason to downplay the principal's bilking opportunities is somewhat more subtle, and somewhat slippery to model formally, but seems psychologically realistic. A pattern for many psychological biases is the coexistence of day-to-day errors with a "meta-awareness" of these errors. In terms of procrastination, people seem to be "meta-sophisticated" about their tendency to procrastinate, and yet exhibit day-to-day naivete. With this conceptualization, considering ex post efficiency may not be a bad approximation: Agents may sign contracts aware of their tendency to procrastinate, but they are not sophisticated enough to overcome procrastination on a day-to-day basis. This conceptualization also accords well with our assumption that the principal is aware of the procrastination problem, because it allows that the principal is no more aware than the agent himself, only that they are both meta-aware.

Finally, the often invoked (if rarely formalized) presumption that efficient institutions and production schemes tend to survive over time, even if people are not aware of why they work, may be used to suggest that we should focus on efficiency contracts rather than bilking contracts. Principals may use deadline contracts and employees may tend to accept such contracts even if neither party knows why they work. In any event, for the rest of this paper, we assume the participation constraint for naifs is based on experienced utility, so the participation constraint uses actual behavior γ and is therefore $\beta\left[W^0(\gamma) - \zeta^0(\gamma)\right] \geq \beta\,\bar{U} = 0$, or $W^0(\gamma) \geq \zeta^0(\gamma)$.

We conclude this section by asking whether temporal incentive schemes can induce efficient behavior when the principal has complete information about the agent. In other words, the principal knows the agent's *inherent* propensity to procrastinate β, as well as the agent's distribution of task costs.

Since TCs are time-consistent, TC behavior minimizes the sum of the expected task cost and the expected wage cost. Efficient behavior minimizes the sum of the expected task cost and the expected delay cost. Hence, TCs will behave efficiently if the expected wage cost is identical to the expected delay cost – i.e., if the incentive scheme internalizes the principal's payoff schedule. With a stationary delay cost, this means a stationary incentive scheme that reflects the true delay cost. We formalize this intuition in Proposition 1:

Proposition 1 *TCs behave efficiently under any incentive scheme* $\mathbf{W}$ *satisfying* $w_t^{\Delta} = x_t^{\Delta}$ *for all* t*; and if* $\mathbf{X}$ *has a stationary delay cost* x^{Δ}*, TCs behave efficiently under any stationary incentive scheme with* $w^{\Delta} = x^{\Delta}$.

Proposition 1 implies TCs behave efficiently if the incentive scheme internalizes the principal's preferences. For naifs, however, if wages merely reflected true delay costs, in each period the cutoff cost would be lower than the efficient cutoff γ_t^* because naifs overweigh current costs. When the principal has complete information about the agent, however, she can in fact induce efficient behavior for naifs with an incentive scheme that exactly counteracts the tendency to procrastinate. With a stationary delay cost, this means a stationary incentive scheme reflecting a delay cost larger than the true delay cost. We formalize this intuition in Proposition 2:

Proposition 2 *For every* $\mathbf{X}$, C *and* $\beta < 1$,

(i) There exists an incentive scheme $\mathbf{W}$ *such that naifs behave efficiently; and*

(ii) If $\gamma_t^* < \bar{c}$ *for all* t*, then any such* $\mathbf{W}$ *satisfies* $w_t^{\Delta} > x_t^{\Delta}$ *for all* $t \leq T$*; and if* $\mathbf{X}$ *has a stationary delay cost* x^{Δ}*, there exists a stationary incentive scheme* $\mathbf{W}$ *with* $w^{\Delta} > x^{\Delta}$ *such that naifs behave efficiently.*

The following example demonstrates the intuition for Propositions 1 and 2, and also illustrates that a "steeper" incentive scheme for naifs relative to TCs implies that the initial wage for naifs must be larger in order to satisfy the participation constraint. Since both types are induced to behave efficiently, they face the same expected task cost.

Example 2

Suppose $T = \infty$, $x^{\Delta} = \frac{1}{32}$, and C is distributed uniformly on $[\underline{c}, \underline{c}+1]$, so $F(c) = c - \underline{c}$ for $c \in [\underline{c}, \underline{c}+1]$.

Efficiency: Clearly, the efficient cutoff cost will be stationary. Let $\gamma^* \equiv (\gamma^*, \gamma^*, \ldots)$ denote the efficient strategy. For each t, $X^t(\gamma^*) = \frac{1}{F(\gamma^*)}x^{\Delta} = \frac{1}{\gamma^*-\underline{c}}\frac{1}{32}$ and $\zeta^t(\gamma^*) = E(c \mid c \leq \gamma^*) = \frac{\underline{c}+\gamma^*}{2}$. Hence, $\gamma^* = \frac{1}{\gamma^*-\underline{c}}\frac{1}{32} + \frac{\underline{c}+\gamma^*}{2}$, implying $\gamma^* = \underline{c} + \frac{1}{4}$.

TCs: Proposition 1 establishes that a stationary incentive scheme with incremental wage $w^{\Delta} = x^{\Delta} = \frac{1}{32}$ will induce efficient behavior. The principal will offer an incentive contract such that the agent behaves efficiently and the expected wage equals the expected task cost. Hence, the incentive scheme must satisfy $\zeta^0(\gamma^*) = W^0(\gamma^*) = W_1 + (1 - F(\gamma^*))\, p^1(\gamma^*)$. We have $p^1(\gamma^*) = \frac{1}{\gamma^* - \underline{c}} w^\Delta = 4w^\Delta$ and $\zeta^0(\gamma^*) = \frac{\underline{c} + \gamma^*}{2} = \underline{c} + \frac{1}{8}$, so $W_1 = 3w^\Delta + \underline{c} + \frac{1}{8} = \underline{c} + \frac{7}{32}$. Hence, the first-best contract for TCs is $\mathbf{W} = (\underline{c} + \frac{7}{32}, \underline{c} + \frac{6}{32}, \underline{c} + \frac{5}{32}, \underline{c} + \frac{4}{32}, \ldots)$.

Naifs: Suppose $\beta = \frac{1}{2}$. Proposition 2 establishes that a stationary incentive scheme can induce efficient behavior for naifs. We search for the incremental wage w^Δ that will induce efficient behavior. Given w^Δ, $\hat{\gamma}$ will clearly be stationary (i.e., cutoff $\hat{\gamma}$ in all periods) and satisfies $\hat{\gamma} = \frac{1}{\hat{\gamma} - \underline{c}} w^\Delta + \frac{\underline{c} + \hat{\gamma}}{2}$, or $\hat{\gamma} = \underline{c} + \sqrt{2w^\Delta}$ (as long as $w^\Delta \leq \frac{1}{2}$ so $\hat{\gamma} \leq \underline{c} + 1$). In general, γ satisfies $\gamma_t = \beta\hat{\gamma}_t$ for all t, so $\gamma_t = \gamma^*$ for all t if $\gamma^* = \beta\left(\underline{c} + \sqrt{2w^\Delta}\right)$ or $w^\Delta = \frac{1}{2}\left(\underline{c} + \frac{1}{2}\right)^2$ (and $w^\Delta \leq \frac{1}{2}$ as long as $\underline{c} \leq \frac{1}{2}$). As for TCs, the incentive scheme must satisfy $W_1 = (1 - F(\gamma^*))\, p^1(\gamma^*) + \zeta^0(\gamma^*) = 3w^\Delta + \underline{c} + \frac{1}{8}$. So for any $\underline{c} \leq \frac{1}{2}$, the first-best contract for naifs is described by $W_1 = \underline{c} + \frac{1}{8} + 3w^\Delta$ and $w^\Delta = \frac{1}{2}\left(\underline{c} + \frac{1}{2}\right)^2$.

The contracts used to induce efficiency in Example 2 vary according to $\underline{c}$, which determines how high the average task cost is. Figure 1 illustrates the incentive schemes that will be chosen for both naifs and TCs, for two different values of $\underline{c}$, a low cost of $\underline{c} = 0$ and a high cost of $\underline{c} = \frac{1}{2}$.[13] For each task-cost distribution, the optimal incentive scheme is steeper for naifs than for TCs; therefore naifs must have a larger intercept to satisfy the participation constraint. Another feature of Figure 1 is crucial for the intuition of the next section: For TCs changing the task-cost distribution changes the intercept (i.e., the participation constraint) but *not* the slope. In contrast, for naifs changing the task-cost distribution changes both the intercept *and* the slope. Intuitively, higher average task costs imply a greater propensity to procrastinate, so steeper incentives are required to overcome procrastination. This difference, that changing the average task cost affects the optimal incentives for naifs but not for TCs, implies a qualitative difference in how principals deal with uncertainty over average task costs for the two types of agents.

We conclude this section with another example illustrating that in nonstationary environments where the principal faces an absolute deadline, an agent's propensity to procrastinate can induce the principal to impose incremental punishments for delay even before the deadline.

Example 3

Suppose the principal faces a pure deadline: $X_t = K > 0$ for all $t \leq D$, $X_t = 0$ for all $t > D$. For TCs, the optimal incentive scheme will clearly punish the agent by K if and only if he delays past period D (i.e., $w_t^\Delta = 0$ for all $t < D$ and $w_t^\Delta = K$ for $t = D$). For naifs, however, the optimal incentive scheme may punish delay even before the deadline (i.e., $w_t^\Delta > 0$ for $t < D$). That is, the principal must "falsely" punish the agent for delay if she wants to induce efficient behavior.

The point is that to induce efficient behavior a principal may need to punish delay even if she does not care at all directly about delay. Even if a professor feels that it is only important for a student to understand the material by exam time, she may still want to grade problem sets throughout the semester. Although such a policy may punish a few students who *would* successfully learn the material at the last moment, it benefits the many students who would put off learning the material until it becomes so late that they cannot adequately do so.

[13] For $\underline{c} = 0$, the first-best contract is $\mathbf{W} = (\frac{7}{32}, \frac{6}{32}, \frac{5}{32}, \ldots)$ for TCs and $\mathbf{W} = (\frac{4}{8}, \frac{3}{8}, \frac{2}{8}, \ldots)$ for naifs; for $\underline{c} = \frac{1}{2}$, the first-best contract is $\mathbf{W} = (\frac{23}{32}, \frac{22}{32}, \frac{21}{32}, \ldots)$ for TCs and $\mathbf{W} = (\frac{17}{8}, \frac{9}{8}, \frac{1}{8}, \ldots)$ for naifs.

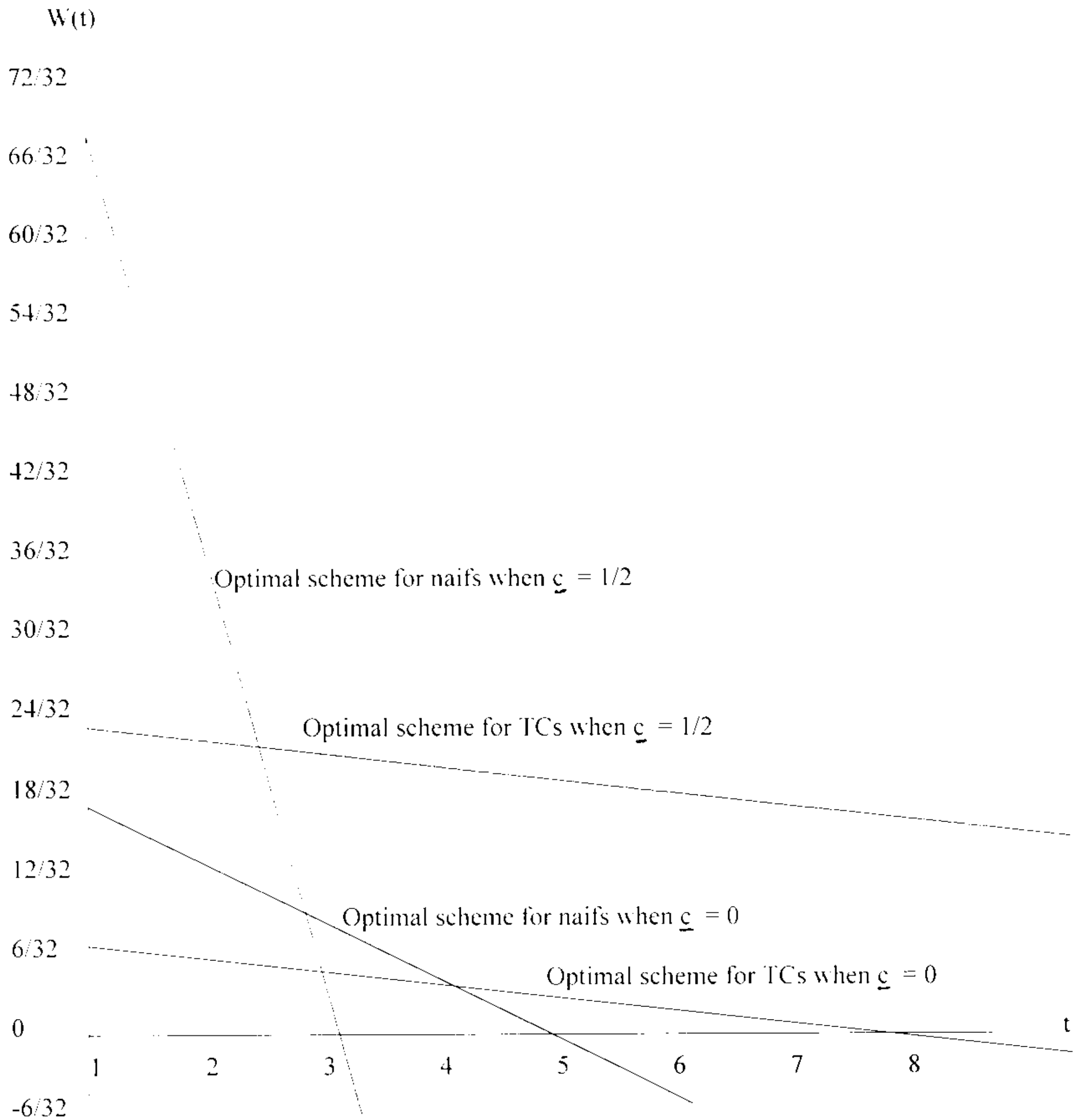

Figure 1: Optimal Incentive Schemes for Example 2

4. Heterogeneous Propensities to Procrastinate

In this section we relax the assumption that the principal has complete information about the agent, so that the principal is uncertain about the agent's propensity to procrastinate. Specifically, we suppose there is uncertainty regarding the task-cost distribution C. We assume C is unknown to both parties in period 0 when the contract is signed, but is revealed to the agent but not to the principal sometime before period 1. The specific C realized determines an agent's "type." As before, all types of the agent face a stochastic task cost, but now we assume that some types may average a larger task cost than other types. This difference is important because high-cost types are more prone to procrastinate.[14] We assume in this section that there are two types of agents who differ only in their mean task costs, and have the exact same distribution of task costs around the mean. Perhaps the best interpretation of this situation is that the principal (and, ex ante, the agent) is unsure how hard the agent will find the task, but that she has a rough sense of the day-to-day variance in the opportunity cost for the agent to do the task. In the next section we consider what happens in the case where not only the mean of the task costs may differ, but also the distribution around the mean.

In the previous section, we found that we can induce efficiency for both TCs and naifs if there is complete information about task costs. Private information about task costs does not cause a problem for TCs: TCs behave efficiently no matter the distribution of task costs as long as the incentive scheme internalizes the principal's preferences. As illustrated by Example 2 and Figure 1, the slope of the optimal incentive scheme for TCs does not depend on the task-cost distribution C. For naifs, on the other hand, a problem arises. Unlike TCs, the optimal incentive scheme for naifs depends on the distribution of task costs. As illustrated by Example 2 and Figure 1, agents with higher average task costs (who are more prone to procrastinate) require a "steeper" incentive scheme. Hence, when the propensity to procrastinate is unknown, incentives must be steep to prevent high-cost types from procrastinating too much, while still shallow enough to induce low-cost types to wait when waiting is efficient.[15]

In this section and the next section, we consider the case in which the principal's incremental cost of delay is stationary: $X_t - X_{t-1} = x^\Delta$ for all t, with $T = \infty$. Our focus on stationary environments allows us to highlight an interesting non-stationarity result. Proposition 1 establishes that the optimal incentive scheme for TCs will be

---

[14] Our assumption of uncertainty about the agent's task-cost distribution creates uncertainty about the agent's *induced* propensity to procrastinate. Our model would be essentially the same if we assumed that all agents had identical task-cost distributions but differed in their *inherent* propensity to procrastinate β. We prefer the former for a couple reasons. First, we consider it a result of direct interest that a disparity between efficient waiting and inefficient procrastination can arise purely as a function of the environment, rather than solely as a function of an agent's inherent procrastinatory tendencies. Second, we suspect that for long-employed agents, uncertainty over the environment can persist in the long run while uncertainty over the inherent propensity to procrastinate may not. Consider a single employee who is given many tasks over time to complete. Eventually his supervisor may figure out his inherent propensity to procrastinate β. But if an agent will have to perform a long series of idiosyncratic tasks of uncertain difficulty, then uncertainty over the propensity to procrastinate can remain in the long run.

[15] We do not consider direct mechanisms – contracts where the agent reveals his type to the principal and the incentive scheme is then type-dependent. In the model of this section, the two types can never strictly prefer to reveal themselves because for any incentive scheme they perceive the same behavior. Moreover, direct mechanisms seem unreasonable in this environment. Our analysis may be most applicable to situations where an agent is hired to complete many tasks over time, or when many agents in an organization are given the same incentive schemes. In such environment, renegotiation of each incentive scheme to take account of case-specific information seems unrealistic.

stationary, and Proposition 2 establishes that the optimal "full-information" incentive scheme for naifs will also be stationary. We show, however, that if the agent has private information about the distribution of task costs, then the second-best optimal incentive scheme for naifs will generally *not* be stationary. Rather, it will often be a "deadline scheme": An agent is initially punished only moderately for delay, but there is a date after which punishment for delay becomes more severe. By deadline schemes, therefore, we do not mean imposing an absolute deadline, in the sense of an infinite punishment for not meeting the deadline. In practical terms we doubt there are many absolute deadlines. Modeling deadlines as jumps in how severely one is punished for delay seems a more natural interpretation. However, a more general model than the simple one we present would not generate deadline schemes even by this interpretation: This section's prediction of a unique date when there is a shift in the severity of punishment is an artifact of our simple model. Our qualitative result, therefore, is that second-best optimal incentive schemes will involve increasingly severe punishments over time, with no special role for simple deadline schemes of the sort commonly observed. We discuss the relationship between our model and such simple deadlines in the concluding section.

For tractability reasons, we consider a highly simplified model where there are two types of agent and each type faces two possible costs. Let $i \in \{L, H\}$ denote an agent's type, and let π denote the ex ante probability that the agent is type L (so $1 - \pi$ is the ex ante probability that the agent is type H). The cost distribution C_i for each $i \in \{L, H\}$ is

$$c = \begin{cases} \underline{c}_i \equiv c_i - k & \text{with probability } \frac{1}{2} \\ \overline{c}_i \equiv c_i + k & \text{with probability } \frac{1}{2}, \end{cases}$$

where $c_H > c_L$ and $k > 0$. Hence, the H agent has a higher average task cost than the L agent, and therefore the H agent is more prone to procrastinate. In this 2×2 model, the agent could be a high-cost type or a low-cost type, and each type can have a high-cost realization or a low-cost realization. To clarify our discussion, we use the following terminology to describe the three possible plans that the agent might employ in any given period.[16]

Terminology	**Action**
do it for sure (d):	complete task if $c = \underline{c}_i$ or $c = \overline{c}_i$
be selective (s):	complete task only if $c = \underline{c}_i$
wait for sure (w):	do not complete task

We redefine strategies in terms of these three plans. A strategy is therefore $\mathbf{s} = (s_1, s_2, ...)$ such that $s_t \in \{d, s, w\}$ for all t. Since $\mathbf{X}$ and $\mathbf{W}$ are the same for both types, we can define $h(\tau \mid t, \mathbf{s})$, $\chi^t(\mathbf{s})$, and $p^t(\mathbf{s})$ exactly as in Section 3, where $F(d) = 1$, $F(s) = \frac{1}{2}$, and $F(w) = 0$. Since the two types face different task-cost distributions, however, the expected task cost is type-dependent. Let $\zeta_i^t(\mathbf{s})$ be identical to $\zeta^t(\mathbf{s})$ except that $E_i(c \mid c \leq s_t)$ replaces $E(c \mid c \leq s_t)$, where we define $E_i(c \mid c \leq s_t)$ as

$$E_i(c \mid c \leq s_t) \equiv \begin{cases} c_i & \text{if } s_t = d \\ c_i - k & \text{if } s_t = s \\ 0 & \text{if } s_t = w. \end{cases}$$

Given $\mathbf{X}$, let $\gamma^{*i} \equiv (\gamma_1^{*i}, \gamma_2^{*i}, ...)$ be the efficient strategy for type $i \in \{L, H\}$. Similarly, given $\mathbf{W}$ let $\hat{\gamma}^i \equiv (\hat{\gamma}_1^i, \hat{\gamma}_2^i, ...)$ and $\gamma^i \equiv (\gamma_1^i, \gamma_2^i, ...)$ be, respectively, the perception-perfect strategies for TCs and naifs of type $i \in \{L, H\}$. Then, for all t, γ^{*i}, $\hat{\gamma}^i$, and γ^i satisfy:[17]

[16] The fourth possible plan, complete the task only if $c = \overline{c}_i$, would obviously be neither optimal nor chosen.

$$\gamma_t^{*i} = \begin{cases} d & \text{if} \quad c_i + k \leq x^t(\gamma^{*i}) + \zeta_i^t(\gamma^{*i}) \\ s & \text{if} \quad c_i - k \geq x^t(\gamma^{*i}) + \zeta_i^t(\gamma^{*i}) \end{cases}$$

$$\hat{\gamma}_t^i = \begin{cases} d & \text{if} \quad c_i + k \leq p^t(\hat{\gamma}^i) + \zeta_i^t(\hat{\gamma}^i) \\ s & \text{if} \quad c_i - k \geq p^t(\hat{\gamma}^i) + \zeta_i^t(\hat{\gamma}^i) \end{cases}$$

$$\gamma_t^i = \begin{cases} d & \text{if} \quad c_i + k \leq \beta\left[p^t(\hat{\gamma}^i) + \zeta_i^t(\hat{\gamma}^i)\right] \\ s & \text{if} \quad c_i - k \leq \beta\left[p^t(\hat{\gamma}^i) + \zeta_i^t(\hat{\gamma}^i)\right] < c_i + k \\ w & \text{if} \quad c_i - k \geq \beta\left[p^t(\hat{\gamma}^i) + \zeta_i^t(\hat{\gamma}^i)\right] \end{cases}$$

The assumption that task-cost distributions differ in only the means is important for two reasons. First, efficient behavior depends on the distribution of task costs but *not* the mean, so efficient behavior will be the same for both types (i.e., $\gamma^{*H} = \gamma^{*L} \equiv \gamma^*$). Second, TC behavior also depends on the distribution of task costs but *not* the mean, so TC behavior will be the same for both types (i.e., $\hat{\gamma}^H = \hat{\gamma}^L \equiv \hat{\gamma}$). This second result has an important implication: Naifs perceive that they will behave like TCs in the future, so L's and H's perceive the same continuation strategies. Hence, for any incentive scheme, we have for all t, $p^t(\hat{\gamma}) + \zeta_H^t(\hat{\gamma}) - c_H = p^t(\hat{\gamma}) + \zeta_L^t(\hat{\gamma}) - c_L$. That is, both types of the agent have the exact same perceptions of how delaying will affect their net gain or loss in expected cost.

Efficient behavior weighs the cost of delay against the value of perhaps getting a lower task cost in the future. Since there is a stationary delay cost x^Δ, efficient behavior will call for *either* doing it for sure in all periods *or* being selective in all periods (i.e., $\gamma^* = (d, d, ...)$ or $\gamma^* = (s, s, ...)$). Analysis of the first case is trivial, since efficiency can be achieved simply with a very steep incentive scheme in the first period. We focus instead on the case where efficiency calls for being selective in all periods, which holds as long as $k > x^\Delta$.

We now ask what incentive schemes can induce efficiency for a naif of given type. We begin by considering stationary incentive schemes, and Lemma 2 establishes that naive behavior is straightforward: For small incremental wages, naifs wait for sure in all periods; for moderate incremental wages, naifs are selective in all periods; and for large incremental wages, naifs do it for sure in all periods.

Lemma 2 *Suppose there is a stationary incentive scheme with incremental wage w^Δ. Then for agent $i \in \{L, H\}$:*

$\gamma^i = (w, w, ...)$ if and only if $w^\Delta < \underline{w}_i$;
$\gamma^i = (s, s, ...)$ if and only if $\underline{w}_i \leq w^\Delta \leq \overline{w}_i$;
$\gamma^i = (d, d, ...)$ if and only if $w^\Delta > \overline{w}_i$;

where $\underline{w}_i$ and $\overline{w}_i$ are given by

$$\overline{w}_i = \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k \text{ and } \underline{w}_i = \begin{cases} \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k & \text{if } c_i \geq \frac{1+\beta}{1-\beta}k \\ \frac{1-\beta}{2\beta}c_i - \frac{1-\beta}{2\beta}k & \text{if } c_i \leq \frac{1+\beta}{1-\beta}k. \end{cases}$$

[17] Notice that these definitions do not specify what the agent should do if she is indifferent between two actions. Hence, unlike the previous section there could be multiple perception-perfect strategies. Here, we follow the incentive-design literature by assuming that when indifferent the agent behaves as the principal would like her to behave.

Lemma 2 establishes that for type i a range of stationary incentive schemes can induce efficient behavior $\gamma^i = (s, s, ...)$. Given the discrete nature of the model, it is not surprising that multiple incentive schemes can induce efficient behavior. The equation for $\underline{w}_i$ depends critically on the relationship between c_i and $\frac{1-\beta}{1-\beta}k$ because this determines whether $w^\Delta = \underline{w}_i$ implies perceptions $\hat{\gamma}^i = (s, s, ...)$ or perceptions $\hat{\gamma}^i = (d, d, ...)$. For $c_i < \frac{1-\beta}{1-\beta}k$ we have $\underline{w}_i < k$, so $w^\Delta = \underline{w}_i$ implies perceptions $\hat{\gamma}^i = (s, s, ...)$; and for $c_i > \frac{1-\beta}{1-\beta}k$ we have $\underline{w}_i > k$, so $w^\Delta = \underline{w}_i$ must imply $\hat{\gamma}^i = (d, d, ...)$. In contrast, for any $\beta < 1$ we must have $\overline{w}_i > k$, so $w^\Delta = \overline{w}_i$ implies $\hat{\gamma}^i = (d, d, ...)$. Intuitively, $\overline{w}_i$ represents the cutoff between the agent being selective always and the agent doing it for sure always, and naifs do it for sure only if TCs do it for sure.

The values $\underline{w}_i$ and $\overline{w}_i$ represent the minimum and maximum *stationary* incremental wages that induce the agent to be selective in all periods. With *non-stationary* incentive schemes, however, there may exist t such that $\gamma_t^i = s$ even though $w_t^\Delta \notin [\underline{w}_i, \overline{w}_i]$. For instance, with a stationary incremental wage $\overline{w}_i$, $w_t^\Delta = \overline{w}_i$ implies $\gamma_t^i = s$ (rather than $\gamma_t^i = d$) because the agent perceives he will complete the task for sure in period $t+1$ (i.e., the future looks bad). But with a non-stationary scheme under which the agent perceives he will be selective in period $t+1$, the future looks better, the agent is less willing to complete the task in period t, and therefore we can have $\gamma_t^i = s$ even if $w_t^\Delta > \overline{w}_i$. Similarly, with a stationary incremental wage $\underline{w}_i$, $w_t^\Delta = \underline{w}_i$ implies $\gamma_t^i = s$ (rather than $\gamma_t^i = w$) when the agent perceives he will be selective in all future periods (i.e., the future looks good). But with a non-stationary scheme under which the agent perceives he will do it for sure in period $t+1$, the future looks worse, the agent is more willing to complete the task in period t, and therefore we can have $\gamma_t^i = s$ even if $w_t^\Delta < \underline{w}_i$.

While Lemma 2 implies that a stationary incentive scheme that induces efficiency for type i must have incremental wage w^Δ in the range $[\underline{w}_i, \overline{w}_i]$, the previous paragraph suggests that a non-stationary incentive scheme can induce efficiency for type i even if there are periods where $w_t^\Delta < \underline{w}_i$ or $w_t^\Delta > \overline{w}_i$. However, the following lemma establishes that for any incentive scheme that induces efficiency for type i, the "average" incremental wage must be in the range $[\underline{w}_i, \overline{w}_i]$. Intuitively, an agent will be selective rather than do it for sure when $w_t^\Delta > \overline{w}_i$ only if there are future periods where the incremental wage is much smaller than $\overline{w}_i$ (i.e., we must make the future look better); similarly, the agent will be selective rather than wait when $w_t^\Delta < \underline{w}_i$ only if there are future periods where the incremental wage is much larger than $\underline{w}_i$ (i.e., we must make the future look worse). Noting that a constant incremental wage of w^Δ implies that the ex ante expected wage cost of a selective agent is $2w^\Delta$, we have the following lemma formalizing the claim that optimal incentive schemes must "on average" reflect the incentives of stationary incentive schemes:

Lemma 3 *If an incentive scheme* $\mathbf{W}$ *induces* $\gamma_t^i = s$ *for all* t, *then* $2\underline{w}_i \leq \sum_{t=1}^{\infty} \left(\frac{1}{2}\right)^{t-1} w_t^\Delta \leq 2\overline{w}_i$.

Lemma 2 and Lemma 3 characterize the types of incentive schemes that can induce efficiency for a given type. We now ask when can we induce efficiency for both types. In fact, it follows directly from Lemma 2 and Lemma 3 that either a stationary incentive scheme can induce efficiency for both types (when $\underline{w}_H \leq \overline{w}_L$), or no incentive scheme can induce efficiency for both types (when $\underline{w}_H > \overline{w}_L$). Which case holds depends on how big is the difference in the mean task costs for the two agents:

Proposition 3 *$\underline{w}_H \leq \overline{w}_L$ if and only if $c_H - c_L \leq \frac{2k}{1-\beta}$. Hence, if $c_H - c_L \leq \frac{2k}{1-\beta}$, there exists a stationary incentive scheme under which both types behave efficiently (by being selective every period); and if $c_H - c_L > \frac{2k}{1-\beta}$, then no incentive scheme can induce both types to behave efficiently (by being selective every period).*

As discussed above, each type will behave efficiently for a range of stationary incentive schemes. If the types are not sufficiently differentiated, these ranges will overlap, so a stationary incentive scheme can induce efficient behavior for both types.[18] And Lemma 3 establishes that any non-stationary incentive scheme that induces efficiency for a given type must have an "average" incremental wage in the same range as the stationary incremental wages that induce efficiency for that type. Hence, if the ranges do not overlap for stationary incentive schemes, then they do not overlap for non-stationary incentive schemes either.

Given we cannot induce first-best efficiency for both types when $c_H - c_L > \frac{2k}{1-\beta}$, we search for second-best optimal incentive schemes. We are particularly interested in the use of "deadline schemes":

Definition 1 *A **deadline scheme** with **deadline** $D \geq 1$ is an incentive scheme such that there exists w^* such that $w_t^\Delta < w^*$ for all $t < D$ and $w_t^\Delta > w^*$ for all $t \geq D$.*

Definition 2 *A (w_A^Δ, w_B^Δ)**–deadline scheme** is a deadline scheme in which $w_t^\Delta = w_A^\Delta$ for all $t < D$ and $w_t^\Delta = w_B^\Delta$ for all $t \geq D$ for some $w_A^\Delta < w_B^\Delta$.*

A deadline scheme is an incentive scheme where the incremental wage before some period D is everywhere smaller than the incremental wage after period D. A (w_A^Δ, w_B^Δ) deadline scheme is a two-part linear incentive scheme where w_A^Δ and w_B^Δ are the respective slopes. We interpret these two-part incentive schemes as deadline incentive schemes because agents are punished relatively lightly for delay up to some date D (the "deadline"), after which further delay leads to more severe punishment.

To give some intuition as to the advantages of a deadline scheme, we first consider what happens under stationary incentive schemes when $c_H - c_L > \frac{2k}{1-\beta}$. Since efficiency calls for both L's and H's to be selective in all periods, clearly the best stationary incentive scheme should induce either L's or H's to be selective each period. But if L's are selective every period then H's never complete the task (which occurs when $\underline{w}_L \leq w^\Delta \leq \overline{w}_L < \underline{w}_H$); and if H's are selective every period, the L's complete the task immediately (which occurs when $\overline{w}_L < \underline{w}_H \leq w^\Delta \leq \overline{w}_H$). Hence, a stationary incentive scheme has the limitation that *either* H's procrastinate forever *or* L's get no efficiency value of waiting. Deadline schemes can prevent either of these from happening. The initial small incremental wage gives L's some efficiency value of waiting, while the eventual large incremental wage prevents H's from procrastinating forever.

Might other non-deadline, non-stationary incentive schemes be second-best optimal? For instance, one might imagine an incentive scheme that alternates between a large incremental wage and a small incremental wage such that L's are selective always (i.e., behave efficiently) and H's are selective whenever the incremental wage is large. Lemma 4 rules out such possibilities:

[18] Clearly, this result is an artifact of having a discrete cost distribution. For full-support cost distributions and "interior" efficient behavior, we cannot achieve efficiency for both types for any differentiation (assuming $\beta < 1$); the analogue to Proposition 3 in a more continuous model would be a convergence result that we get closer to efficiency as the distribution of types becomes less dispersed.

Lemma 4 *Suppose $c_H - c_L > \frac{2k}{1-\beta}$. Then for any t, $\gamma_t^H \neq w$ implies $\gamma_t^L = d$.*

Lemma 4 follows from the intuition discussed earlier that for any incentive scheme we have $\hat{\gamma}^H = \hat{\gamma}^L = \hat{\gamma}$ and therefore for all t, $p^t(\hat{\gamma}) - \zeta_H^t(\hat{\gamma}) - c_H = p^t(\hat{\gamma}) - \zeta_L^t(\hat{\gamma}) - c_L$. This property guarantees that if we can induce both types to be selective in *any* period, then we can do so in *every* period by adjusting the incremental wages appropriately. Lemma 4 implies the second-best optimality of deadline incentive schemes, because it implies that the choice for a principal in any period is either to have L's be selective and have H's for sure not do it, or to get L's to do it for sure and have H's be selective. The first option is more attractive if and only if it is likely that the agent is an L. This yields the second-best optimality of deadline schemes: At the beginning, when it is likely that the agent is an L, the principal wishes to induce selectivity by L's, tolerating the fact that H's are behaving inefficiently by waiting. As the likelihood that the agent is an L becomes smaller, however, it is eventually more efficient to make L's (inefficiently) do it for sure, and start getting H's to be selective. That point is the deadline. To summarize:

Proposition 4 *Suppose $c_H - c_L > \frac{2k}{1-\beta}$, so no incentive scheme can induce efficiency for both types. Then:*

(i) There exists $D^* \geq 1$ such that the $(\overline{w}_L, \overline{w}_H)$-deadline scheme with deadline D^* is second-best optimal. This incentive scheme will induce $\gamma_t^L = s$ and $\gamma_t^H = w$ for all $t < D^*$ and $\gamma_t^L = d$ and $\gamma_t^H = s$ for all $t \geq D^*$. And D^* is one of the two integers satisfying $\alpha - 1 \leq D^* \leq \alpha - 1$ where

$$\alpha \equiv 1 + \frac{ln\left(\frac{\pi}{1-\pi}\right) + ln\left(\frac{k-x^{\Delta}}{x^{\Delta}}\right) + ln\,(ln\,2)}{ln\,2}$$

(ii) If $c_L \geq \frac{1+\beta}{1-\beta}k$, then all second-best optimal incentive schemes are deadline schemes.

Part (i) of Proposition 4 establishes that a $(\overline{w}_L, \overline{w}_H)$–deadline scheme is always second-best optimal. While there can be other incentive schemes that are second-best optimal, they all induce exactly the same behavior except for knife-edge parameter values. While part (ii) of Proposition 4 establishes that sometimes all second-best optimal incentive schemes are deadline schemes, when $c_L < \frac{1+\beta}{1-\beta}k$ second-best behavior can be induced with some non-deadline schemes.[19] Intuitively, before the "deadline" there can be a period with a large incremental wage if it is followed by at least one period with a small incremental wage (note that this implies the incremental wage in period $D^* - 1$ must always be smaller than the "post-deadline" incremental wages). In general, the principal can mimic the outcome of the simple deadline scheme in Proposition 4 part (i) with bizarre schemes, but the principal would never have an incentive to depart from this scheme.

The optimal incentive scheme in Proposition 4 implies the following observed pattern of behavior: A number of people complete the task immediately (half the L's), and another large group completes the task in period D, just before the more severe punishment kicks in (the remaining L's and half the H's). In between, we would observe smaller (and decreasing) numbers of agents doing it. In other words, people less prone to procrastinate complete the task at the first convenient time, or just before the deadline if no convenient time arises. People more prone to procrastinate wait until the deadline (and often beyond) before completing the task.

Also implied by Proposition 4 are some comparative statics for the parameter values that reflect the intuition of why these schemes are attractive. Note that the optimal deadline is independent of c_L, c_H and β except insofar as

[19] Even so, a result analogous to Lemma 3 could be formalized – the "average" incremental wage before the deadline must be less than the "average" incremental wage after the deadline.

they determine whether we can achieve efficiency. That is, if $c_H - c_L < \frac{2k}{1-\beta}$, then a stationary incentive scheme is optimal; otherwise, a deadline scheme is optimal. But once those cases are determined, the optimal D^* does not depend on c_L, c_H and β. Rather, the optimal deadline depends on the relative likelihood of L's vs. H's, and on the relative benefits of discouraging delay vs. encouraging selective performance. As π approaches 1, D^* approaches ∞, and as π approaches 0, D^* approaches 1. If the population is predominantly L's, then it is optimal to give them more opportunities to get a low-cost realization. Alternatively, if the population is predominantly H's, it is optimal to have no delay before they are selective. Finally, consider comparative statics on the term $\left(\frac{k-x^{\Delta}}{x^{\Delta}}\right)$, maintaining $c_H - c_L \geq \frac{2k}{1-\beta}$. As k becomes large relative to the delay cost x^{Δ}, D^* approaches ∞. In other words, as task-cost considerations swamp delay-cost considerations, we first give L's a very long time to find a small cost, and we can still get H's to eventually find a small cost.

This section has shown in a simple model how deadline schemes can be second-best optimal for time-*in*consistent agents in a way they would not be for time-consistent agents. We have worked out some generalizations of our model as well. For instance, we have considered full-support task-cost distributions rather than discrete task-cost distributions, and we have considered what happens if there are more than two types. In both cases, the optimal incentive schemes for naifs need not be the simple two-part linear schemes described above; they will, however, be concave, so they can be seen as a "generalized deadline scheme" where the punishment for delay gets increasingly harsh. Beyond indicating the specialness of our two-part linear scheme, these generalizations yield no qualitatively different results or insights, so we have omitted them from the paper. In the next section, however, we turn to a qualitatively-different generalization.

5. Different Task-Cost Distributions

In this section we generalize the model of Section 4 to allow different types of agents to face different variances to their task-cost distributions. Consider a model identical to that in Section 4 except that the task-cost distribution is

$$c = \begin{cases} c_i - k_i & \text{with probability } \frac{1}{2} \\ c_i + k_i & \text{with probability } \frac{1}{2}, \end{cases}$$

for type $i \in \{L, H\}$, where $c_L < c_H$. We assume $k_L < c_L$ and $k_H < c_H$, guaranteeing that the task cost is always positive. While we consider both $k_L > k_H$ and $k_H \geq k_L$, we feel that $k_H \geq k_L$ is probably the more natural case (so a higher mean task cost is associated with increased day-to-day variance).[20] For the majority of this section, we also assume $k_L > x^{\Delta}$ and $k_H > x^{\Delta}$ so that, as in Section 4, efficiency calls for both types to be selective in all periods. We briefly comment at the end of this section on the cases $k_H > x^{\Delta} \geq k_L$ and $k_L > x^{\Delta} \geq k_H$.

The goal of this section is to examine the robustness of our results in Section 4 when we relax the special assumption that task-cost distributions differ only in their means. As such, this section has two main points. First, we will show that for the case $k_L > k_H$ our deadline result no longer holds: In Example 4 below, we show that non-deadline schemes can

[20] For example, suppose we take our task-difficulty interpretation literally (i.e., that the task is more difficult for H's than for L's), and assume that the stochastic opportunity costs for each hour of the day are identically and independently distributed with a mean of one. If we then interpret c_H and c_L as the number of hours required to complete the task, then H's face a higher variance.

be superior to all deadline schemes. Second, we will show that for the more natural case of $k_H \geq k_L$, our main result does hold: Propositions 6 and 7 below establish that whenever a stationary incentive scheme cannot induce efficiency for both types, deadline schemes are second-best optimal. Since much of the analysis here is exactly analogous to that in Section 4, we do not provide as much detail here.

As a preliminary step, we first establish the conditions under which we can induce efficiency for both types. Recall that Lemma 2 and Lemma 3 characterize the types of incentive schemes that can induce type i to be selective in all periods. These lemmas remain intact for this section, except that in the equations for $\underline{w}_i$ and $\overline{w}_i$ we must replace k with k_i. In other words, for type i there exists $\underline{w}_i$ and $\overline{w}_i$ such that i's are selective in all periods for any stationary incremental wage $w^\Delta \in [\underline{w}_i, \overline{w}_i]$, and *any* incentive scheme that induces i's to be selective in all periods must have an "average" incremental wage in the same range. Hence, we have a result in Proposition 5 below that is analogous to that in Proposition 3: Either a stationary incentive scheme can induce efficiency for both types (when $\underline{w}_H \leq \overline{w}_L$) or no incentive scheme can induce efficiency for both types (when $\underline{w}_H > \overline{w}_L$).[21]

Proposition 5 *First-best efficiency can be induced if and only if it can be induced with a stationary incentive scheme.*

Next, we suppose $\underline{w}_H > \overline{w}_L$ so no incentive scheme can induce efficiency, and ask whether deadline schemes will be second-best optimal. As foreshadowed above, the answer may be no when $k_L > k_H$. Given $k_L > k_H$, we can have periods where H's perceive they will do it next period (i.e., $\hat{\gamma}^H_{t-1} = d$) while L's perceive they will be selective next period (i.e., $\hat{\gamma}^L_{t-1} = s$). As a result, L's may be more optimistic about the benefits of waiting, and therefore L's may be selective even if the incremental wage w^Δ_t is large enough to induce H's not to wait. Hence, we can have (non-deadline) incentive schemes under which L's are selective always and yet H's do *not* procrastinate forever. Example 4 illustrates two such schemes, and in this example deadline schemes are never second-best optimal.

Example 4

Suppose $\beta = \frac{1}{2}$, $c_L = 8$, $k_L = 6$, $c_H = 22$, $k_H = \frac{1}{2}$, $x^\Delta \in (0, \frac{1}{2})$, and $\pi = \frac{1}{2}$.

Both $k_L > x^\Delta$ and $k_H > x^\Delta$, so efficiency calls for both to be selective in all periods. We have $\underline{w}_L = 1$, $\overline{w}_L = 20$, $\underline{w}_H = 21$, and $\overline{w}_H = 23$. Hence, $\underline{w}_L < \overline{w}_L < \underline{w}_H < \overline{w}_H$, so Proposition 5 implies we cannot induce efficiency for both types. Consider the following two non-stationary incentive schemes, neither of which is a deadline scheme:

- Scheme A: $w^\Delta_t = 22$ for $t \in \{1, 3, 5, ...\}$ and $w^\Delta_t = 1$ for $t \in \{2, 4, 6, ...\}$. Then $\gamma^L = (s, s, ...)$ and $\gamma^H = (s, w, s, w, ...)$, so L's behave efficiently and H's are selective every other period.
- Scheme B: $w^\Delta_1 = 24$ and $w^\Delta_t = 1$ for $t \in \{2, 3, 4, ...\}$. Then $\gamma^L = (s, s, ...)$ and $\gamma^H = (d, w, w, w, ...)$, so L's behave efficiently and H's (inefficiently) complete the task immediately.

It is straightforward to show that for any $x^\Delta \in (0, \frac{1}{2})$, scheme A is better than any deadline scheme. And in fact scheme B is better than scheme A for $x^\Delta \in (\frac{3}{8}, \frac{1}{2})$.

Example 4 shows that our deadline result does not hold when $k_L > k_H$. We now consider the more natural case of $k_H \geq k_L$. As discussed above, the crucial intuition for Example 4 is that there can be periods where $\hat{\gamma}^H_{t-1} = d$ while

[21] In this model, however, there is no simple condition in terms underlying parameters for when we can induce efficiency.

$\hat{\gamma}^L_{t-1} = s$ so that L's can be more optimistic than H's about the benefits of waiting. But for $k_H \geq k_L$ we can have $\hat{\gamma}^H_{t-1} = d$ only if $\hat{\gamma}^L_{t-1} = d$. Hence, schemes like those in Example 4 will not be useful, and deadline schemes will again be second-best optimal. However, we must separate out two cases, $c_H \geq \frac{1-\beta}{1-\beta}k_H$ and $c_H < \frac{1-\beta}{1-\beta}k_H$. Lemma 5 below establishes that when $c_H \geq \frac{1+\beta}{1-\beta}k_H$, a result identical to Lemma 4 holds, and therefore a deadline result identical to Proposition 4 holds (except k_L replaces k in the equation for α because only L's can incur cost $c_i + k_i$ under a deadline scheme).

Lemma 5 *Suppose $k_H > k_L$ but $\underline{w}_H > \overline{w}_L$, so no incentive scheme can induce efficiency for both types.*
If $c_H \geq \frac{1-\beta}{1-\beta}k_H$, then for any t, $\gamma^H_t \neq w$ implies $\gamma^L_t = d$.

Proposition 6 *Suppose $k_H \geq k_L$ but $\underline{w}_H > \overline{w}_L$, so no incentive scheme can induce efficiency for both types. If $c_H \geq \frac{1-\beta}{1-\beta}k_H$, then:*

(i) There exists $D^* \geq 1$ such that the $(\overline{w}_L, \overline{w}_H)$–deadline scheme with deadline D^* is second-best optimal. This incentive scheme will induce $\gamma^L_t = s$ and $\gamma^H_t = w$ for all $t < D^*$ and $\gamma^L_t = d$ and $\gamma^H_t = s$ for all $t \geq D^*$. And D^* is one of the two integers satisfying $\alpha - 1 \leq D^* \leq \alpha + 1$ where

$$\alpha \equiv 1 - \frac{ln\left(\frac{\pi}{1-\pi}\right) - ln\left(\frac{k_L - r\Delta}{r\Delta}\right) - ln\,(ln\,2)}{ln\,2}$$

(ii) If $c_L \geq \frac{1-\beta}{1-\beta}k_L$, then all second-best optimal incentive schemes are deadline schemes.

For $c_H < \frac{1+\beta}{1-\beta}k_H$, we do not get the strong result that *for any individual period* t, if L's are selective (or wait) in period t then H's must wait in period t. However, we do have a slightly weaker result, Lemma 6: If L's are selective (or wait) *in all periods* then H's must wait *in all periods*. And even though Lemma 6 is weaker than Lemmas 4 and 5, it is still sufficient to establish the second-best optimality of deadlines: Proposition 7 shows that there must be some period D (which is the "deadline") in which L's complete the task for sure because otherwise H's wait forever (which implies infinite delay costs).

Lemma 6 *Suppose $k_H \geq k_L$ but $\underline{w}_H > \overline{w}_L$, so no incentive scheme can induce efficiency for both types.*
If $c_H < \frac{1-\beta}{1-\beta}k_H$, then if $\gamma^L_t \neq d$ for all t, then $\gamma^H_t = w$ for all t.

Proposition 7 *Suppose $k_H \geq k_L$ but $\underline{w}_H > \overline{w}_L$, so no incentive scheme can induce efficiency for both types. If $c_H < \frac{1+\beta}{1-\beta}k_H$, then:*

(i) There exists $D^{**} \geq 1$ such that the $(\overline{w}_L, \overline{w}_H)$–deadline scheme with deadline D^{**} is second-best optimal. This incentive scheme will induce $\gamma^L_t = s$ for $t < D^{**}$, $\gamma^L_t = d$ for $t \geq D^{**}$, $\gamma^H_t = w$ for $t < D^{**} - \hat{d}$, and $\gamma^H_t = s$ for $t \geq D^{**} - \hat{d}$, where

$$\hat{d} = \min\left\{n \in \{0, 1, 2, \ldots\} \;\middle|\; \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k_H > k_H - \sum_{j=0}^{n}\left(\frac{1}{2}\right)^j(\overline{w}_L - k_H)\right\},$$

and $D^{**} = \max\left\{D^*, \hat{d} - 1\right\}$, where D^* is defined in Proposition 6.

(ii) If $c_L \geq \frac{1-\beta}{1-\beta}k_L$, then all second-best optimal incentive schemes are deadline schemes.

Hence, we have established that for the case $k_H \geq k_L$ our main result holds: If $\underline{w}_H > \overline{w}_L$ so no incentive scheme can induce efficient behavior for both types, then there is always a second-best optimal $(\overline{w}_L, \overline{w}_H)$–deadline scheme. We note, however, that behavior under second-best optimal deadline schemes can be slightly different (and "better") than in the basic model. Given a deadline D, L's behave the same as they do in the basic model – they are selective before the deadline and do it at the deadline. In contrast, H's can behave differently: If k_H is large enough, then the impending deadline can induce H's to sometimes be selective, in the $\hat{d}$ periods just prior to the deadline where $\hat{d}$ is defined in Proposition 7.[22]

Throughout this section, we have assumed that efficiency requires both types to be selective in all periods, as in Section 4. In fact, when $k_L \neq k_H$ there are two other interesting cases for efficient behavior: $k_H > x^{\triangle} \geq k_L$, so L's should complete the task and H's should be selective in all periods; and $k_L > x^{\triangle} \geq k_H$, so L's should be selective and H's should complete the task in all periods. We conclude this section by briefly discussing these cases.

It turns out that the case $k_H > x^{\triangle} \geq k_L$ is not very interesting because we can always induce first-best efficiency: $k_H > k_L$ implies that $\overline{w}_H > \overline{w}_L$ (and we always have $\underline{w}_L < \overline{w}_H$); efficiency will be induced with any stationary incremental wage $w^{\triangle} \in [\max\{\underline{w}_L, \underline{w}_H\}, \min\{\overline{w}_L, \overline{w}_H\}]$. The intuition is straightforward: The problem is that getting H's to be selective makes L's complete the task for sure, so the solution is easy if that behavior is efficient.

The case $k_L > x^{\triangle} > k_H$ is more interesting. (Of course, like Example 4 this case violates the more natural assumption that $k_H \geq k_L$.) First, since $k_L > k_H$ it is possible to have $\overline{w}_L \geq \overline{w}_H$, in which case efficiency can be achieved with any stationary incremental wage $w^{\triangle} \in [\overline{w}_H, \overline{w}_L]$. If $\overline{w}_L < \overline{w}_H$, a stationary incentive scheme cannot induce efficiency. However, it may be possible to induce efficiency with a non-stationary incentive scheme: In Example 4 above, scheme B induces efficiency when $x^{\triangle} \in (\frac{1}{2}, 6)$.[23] Even so, for some parameter values where no incentive scheme can induce efficiency, stationary incentive schemes can sometimes be second-best optimal (under which both types are selective always) and deadline schemes can sometimes be second-best optimal (under which H's complete the task for sure at the deadline).

6. Discussion and Conclusion

We conclude by discussing various aspects of incentives for procrastinators that are not incorporated into our specific model. Our analysis is all based on agents that are naive about their time inconsistency. How might sophistication affect the results presented in this paper?[24] Just as with naifs, when the principal has complete information about a sophisticated agent, she can always induce efficient behavior with an incentive scheme that exactly counteracts the tendency to procrastinate. And also as with naifs, if the principal faces uncertainty over the distribution of task costs and therefore uncertainty over the propensity to procrastinate, first-best optimality typically cannot be achieved.

[22] In Proposition 7, the optimal deadline $D^{**} = \max\{D^*, \hat{d}+1\}$ because $\hat{d}$ does not affect the marginal cost/benefits of changing the deadline unless the deadline is less than $\hat{d}-1$. Hence, the optimal deadline D^{**} will be equal to D^* as long as $D^* > \hat{d}$. For $D^* \leq \hat{d}$, the optimal deadline is $D^{**} = \hat{d}+1$: A deadline of $\hat{d}+1$ implies that H's are selective always and L's are selective for the first $\hat{d}$ periods, so any deadline less than $\hat{d}-1$ is clearly inferior because H's are unaffected but L's are selective for fewer periods

[23] That non-stationary schemes can induce efficiency when stationary schemes cannot is not too surprising since this is case where *want* players to behave differently.

[24] Sophisticates, like naifs, tend to procrastinate unpleasant tasks, although this tendency is *not* universal, and is always less pronounced for sophisticates than for naifs – because sophisticates correctly predict the costs of procrastination.

Sophistication, however, complicates the nature of second-best-optimal incentive schemes.[25] In particular, the strong deadline result no longer holds: Although deadline schemes are sometimes second-best optimal, there are examples where deadline schemes are not optimal. For example, consider a stationary incentive scheme that induces efficient behavior for L's (i.e., L's are selective in all periods). For naifs, such an incentive scheme would induce H's to procrastinate forever. Sophisticates, on the other hand, know how they will behave in the future and will never procrastinate forever if incremental wages are positive. Instead, such an incentive scheme will induce a cyclical perception-perfect strategy for sophisticates of the sort "be selective in periods $1, 1+m, 1+2m, \ldots$, wait for sure in all other periods." If the length of the cycle m is short enough, this stationary incentive scheme will be better than the best deadline scheme.

For a variety of reasons, we feel that the potentially different predictions for sophisticated agents is not too damning for the relevance of our model. Mostly, we think that, of the two extremes, naivete is the more realistic assumption. Moreover, recall our discussion following Lemma 1: Our model is best interpreted as assuming that people are "meta-sophisticated" – aware of their general propensity to procrastinate. It is "moment-by-moment sophistication" – where the full costs of delay resonate with the agent at each moment – that might negate our basic results. Note that, in any event, if an agent were *extremely* sophisticated, then procrastination would not be a severe problem.[26]

Even maintaining the assumption of naivete, several important issues are ignored in our analysis. One is the delivery date of rewards. We assume that the reward the principal offers the agent for completing the task is not salient to the agent, which seems realistic in most contexts – especially if the reward is money. But the principal might offer in addition some sort of immediate non-monetary reward, such as breaks or parties once a task is completed. Such "salient" rewards may be a cheap way for the principal to overcome procrastination: The same preference for immediate gratification that tempts the agent to put off incurring the task cost will tempt him to grab this reward. Of course, such salient rewards will be inefficient in the sense that the agent does not value them from a long-run perspective as much as non-salient rewards, such as income. We are therefore skeptical that such rewards will be used merely as a way for principals to extract surplus from agents, for the same reason that we make the *ex post* break-even assumption throughout the paper: In the long run, agents will find a job unsatisfying if they are getting mostly short-run rewards they do not value highly in the long run.

But salient rewards can potentially be efficiency-enhancing as well, because they can be used to align incentives for heterogeneous agents. Suppose – unlike the model in Section 4 – that agents differ in their *innate* preference for immediate gratification, as measured by the parameter β. As in our model, punishments for delay harsh enough to induce efficient behavior for those with large self-control problems (i.e., small β) may be so harsh that those with small self-control problems (i.e., large β) complete the task when waiting is efficient. If there is some salient reward,

[25] This discussion refers to a model analogous to that in Section 4, where there is a stationary delay cost x^{Δ} and $T = \infty$. There is a slight complication because an infinite horizon can imply multiple perception-perfect strategies for sophisticates which yield different observed behavior. This discussion assumes that in such cases the agent will follow the principal's desired perception-perfect strategy.

[26] O'Donoghue and Rabin (1996) formalize an argument that naive agents might procrastinate even for very mild self-control problems while sophisticated agents must have large self-control problems before they will significantly procrastinate.

however, then exactly those agents with large self-control problems will react most to this incentive. Hence, the use of salient rewards may provide a second-best mechanism for aligning incentives for heterogeneous agents, and could in some cases be more efficient than the deadline schemes analyzed in our model.

This focus on the nature of the rewards brings us to important issue: We have spoken throughout this paper as if the reward schemes used are monetary incentives, yet agents in organizations are rarely given explicit monetary incentives to complete specific tasks early. More often, an employee's basic incentive scheme is that he is either fired or not fired, or his promotion in a firm is dependant vaguely on his performance. Whatever the "unit of account" by which we keep track of whether an employee has been successful or not, we feel that our model has something to tell us: The essential prediction is that organizations will wish to induce a sense of more and more severe marginal incentives to complete a task as completion is delayed further and further.[27]

Clearly, there are many reasons for deadlines other than combatting procrastination. A major one, intuitively, is coordination among agents – it is useful for others in an organization to know a date at which a project will (almost) surely be done. A second potential reason for deadlines is their simplicity: It may be easier to monitor whether somebody met or missed a deadline, rather than to monitor exactly when a project was completed. If an organization needs to have simple rules of operation, then deadlines may be natural. Our analysis does *not* plausibly predict simple deadline schemes – that simple deadline schemes were sufficient in Sections 4 and 5 was clearly an artifact of our stylized model – so we suspect simplicity considerations are important in explaining the use of deadlines. But we think that these reasons for deadlines complement rather than contradict the message of this paper. Imposing "lumpy" deadlines in environments where the actual marginal cost of delay is relatively constant over time might be a necessary evil because of organizational and transactions-costs explanations. But even if the ideal incentive contract according to our model is to have a smoothly concave incentive scheme, the concavity implies that, among simple schemes, simple deadlines may be *better* than simple linear schemes.

Finally, there is a subtext to this paper which we suspect might generate some of the paper's interest to many readers: Not only might the paper shed light on how "principals" cope with the procrastination of "agents", but it might also help address how individuals cope with their own procrastination. In other words, we can interpret the "principal" as our current self and the "agent" as our future self.[28] Many people who procrastinate only moderately do so not because of intrinsic self-control, but because they have developed schemes to overcome procrastination. Some such schemes may use external commitment devices: People commit to giving a seminar in the hopes that this will force them to finish a paper. Other such schemes are internal: People try to fool themselves into believing in false deadlines, exaggerating to themselves ahead of time how crucial it is that they adhere to some timetable. It is somewhat subtle to conceptualize self-incentives, but we hope the analysis of this paper might be useful in this regards.

---

[27] Of course, monetary incentives are not often used in many of the contexts considered by formal principal-agent models. Insofar as we are not invoking risk aversion to drive our results, perhaps our model suffers less from inapplicability to non-monetary incentives than standard principal-agent problems.

[28] Implicit in this interpretation is an issue we have discussed previously: A person may be "meta-sophisticated" and aware of her general propensity to procrastinate, but naive about day-to-day procrastination. We can think of the "meta-sophisticated" person as setting self-incentives to overcome future day-to-day naivete.

Appendix: Proofs

Proof of Lemma 1: If $\frac{1}{\beta}\underline{c} > \bar{c}$, then the result follows from Example 1. Suppose $\frac{1}{\beta}\underline{c} \le \bar{c}$, and consider **W** with $w_T^{\Delta} = \infty$ (so the agent must complete the task in or before period T), $w_{T-1}^{\Delta} = \frac{1}{\beta}\underline{c} - Ec$, and $w_t^{\Delta} = F(\frac{1}{\beta}\underline{c})\left[\frac{1}{\beta}\underline{c} - E(c|c < \frac{1}{\beta}\underline{c})\right]$ for all $t \in \{1, 2, ..., T-2\}$. How would TCs behave? Clearly, $\hat{\gamma}_T = \bar{c}$. Then

$$\hat{\gamma}_{T-1} = p^{T-1}(\hat{\gamma}) - \varsigma^{T-1}(\hat{\gamma}) = w_{T-1}^{\Delta} + Ec = \tfrac{1}{\beta}\underline{c}.$$
$$\hat{\gamma}_{T-2} = p^{T-2}(\hat{\gamma}) - \varsigma^{T-2}(\hat{\gamma}) = w_{T-2}^{\Delta} + F(\tfrac{1}{\beta}\underline{c})E(c|c < \tfrac{1}{\beta}\underline{c}) + \left(1 - F(\tfrac{1}{\beta}\underline{c})\right)\left[p^{T-1}(\hat{\gamma}) - \varsigma^{T-1}(\hat{\gamma})\right] = \tfrac{1}{\beta}\underline{c}$$
$$\hat{\gamma}_{T-3} = p^{T-3}(\hat{\gamma}) - \varsigma^{T-3}(\hat{\gamma}) = w_{T-3}^{\Delta} + F(\tfrac{1}{\beta}\underline{c})E(c|c < \tfrac{1}{\beta}\underline{c}) + \left(1 - F(\tfrac{1}{\beta}\underline{c})\right)\left[p^{T-2}(\hat{\gamma}) - \varsigma^{T-2}(\hat{\gamma})\right] = \tfrac{1}{\beta}\underline{c}$$

Continuing this logic, we have $\hat{\gamma} = (\frac{1}{\beta}\underline{c}, \frac{1}{\beta}\underline{c}, ..., \frac{1}{\beta}\underline{c}, \bar{c})$. For naifs, we must have $\gamma_T = \bar{c}$ and $\gamma_t = \beta\hat{\gamma}_t$ for all $t < T$, so $\gamma = (\underline{c}, \underline{c}, ..., \underline{c}, \bar{c})$. Hence, under **W** naifs complete the task in period T, while TCs complete the task with positive probability in all periods.

When naifs sign the contract, they believe they will behave like TCs. Hence, naifs perceive $U^P \ge 0$ if and only if $W_1 \ge (1 - F(\hat{\gamma}_1))p^1(\hat{\gamma}) - \varsigma^0(\hat{\gamma})$. Consider the contract where this holds with equality. Since $\varsigma^0(\hat{\gamma}) = F(\hat{\gamma}_1)E(c|c < \frac{1}{\beta}\underline{c}) + (1 - F(\hat{\gamma}_1))\varsigma^1(\hat{\gamma})$, we have

$W_1 = F(\frac{1}{\beta}\underline{c})E(c|c < \frac{1}{\beta}\underline{c}) + \left(1 - F(\frac{1}{\beta}\underline{c})\right)\left[p^1(\hat{\gamma}) - \varsigma^1(\hat{\gamma})\right] = F(\frac{1}{\beta}\underline{c})E(c|c < \frac{1}{\beta}\underline{c}) + \left(1 - F(\frac{1}{\beta}\underline{c})\right)\frac{1}{\beta}\underline{c}$, and

$W_T = W_1 - (T-2)F(\frac{1}{\beta}\underline{c})\left[\frac{1}{\beta}\underline{c} - E(c|c < \frac{1}{\beta}\underline{c})\right] - \left[\frac{1}{\beta}\underline{c} - Ec\right] = -(T-1)F(\frac{1}{\beta}\underline{c})\left[\frac{1}{\beta}\underline{c} - E(c|c < \frac{1}{\beta}\underline{c})\right] + Ec.$

Naifs do the task in period T for wage W_T, and the result follows from $\lim_{T\to\infty} W_T = -\infty$. □

Proof of Proposition 1: Efficient behavior satisfies for each t, $\gamma_t^* = x^t(\gamma^*) - \varsigma^t(\gamma^*)$. TC behavior satisfies for each t, $\hat{\gamma}_t = p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma})$. If $w_t^{\Delta} = x_t^{\Delta}$ for each t, then $p^t(\mathbf{s}) = x^t(\mathbf{s})$ for each t and $\mathbf{s}$. The result follows. □

Proof of Proposition 2: (i) Consider $T < \infty$. The proof is straightforward: Use backwards-induction logic, noting that γ_τ is independent of w_t^{Δ} for all $t < \tau$. Set w_T^{Δ} very large so $\gamma_T = \bar{c}$. Then set w_{T-1}^{Δ} appropriately so $\gamma_{T-1} = \gamma_{T-1}^*$, then set w_{T-2}^{Δ} appropriately so $\gamma_{T-2} = \gamma_{T-2}^*$, and so on.

Consider $T = \infty$. Given the definition of γ, $\gamma = \gamma^*$ if and only if $p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma}) = \frac{1}{\beta}\gamma_t^*$ for all t. Given the definition of $\hat{\gamma}$, $p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma}) = \frac{1}{\beta}\gamma_t^*$ for all t implies $\hat{\gamma}_t = \min\left\{\frac{1}{\beta}\gamma_t^*, \bar{c}\right\}$ for all t.

Suppose $\frac{1}{\beta}\gamma_{t+1}^* \ge \bar{c}$ so $\hat{\gamma}_{t+1} = \bar{c}$ and therefore $p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma}) = w_t^{\Delta} + Ec$. Then $p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma}) = \frac{1}{\beta}\gamma_t^*$ if and only if $w_t^{\Delta} = \frac{1}{\beta}\left[\gamma_t^* - \beta Ec\right]$.

Suppose $\frac{1}{\beta}\gamma_{t+1}^* < \bar{c}$ so $\hat{\gamma}_{t+1} = \frac{1}{\beta}\gamma_{t+1}^*$ and therefore

$$p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma}) = w_t^{\Delta} + \int_{\underline{c}}^{\hat{\gamma}_{t+1}} c\,dF(c) + (1 - F(\hat{\gamma}_{t+1}))\left[p^{t+1}(\hat{\gamma}) - \varsigma^{t+1}(\hat{\gamma})\right]$$
$$= w_t^{\Delta} + \int_{\underline{c}}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) + \left(1 - F(\frac{1}{\beta}\gamma_{t+1}^*)\right)\frac{1}{\beta}\gamma_{t+1}^*$$

Then $p^t(\hat{\gamma}) - \varsigma^t(\hat{\gamma}) = \frac{1}{\beta}\gamma_t^*$ if and only if $w_t^{\Delta} = \frac{1}{\beta}\left[\gamma_t^* - \beta\int_{\underline{c}}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) - \left(1 - F(\frac{1}{\beta}\gamma_{t+1}^*)\right)\gamma_{t+1}^*\right]$.

Hence, $\gamma = \gamma^*$ if and only if $\mathbf{W}$ satisfies for all t

$$w_t^{\Delta} = \begin{cases} \frac{1}{\beta}\left[\gamma_t^* - \beta Ec\right] & \text{if } \frac{1}{\beta}\gamma_{t+1}^* \geq \bar{c} \\ \frac{1}{\beta}\left[\gamma_t^* - \beta\int_{\underline{c}}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) - \left(1 - F(\frac{1}{\beta}\gamma_{t+1}^*)\right)\gamma_{t+1}^*\right] & \text{if } \frac{1}{\beta}\gamma_{t+1}^* \leq \bar{c} \end{cases} \tag{A-1}$$

(ii) If $\gamma_t^* < \bar{c}$ for all t, then $\gamma_t^* = \chi^t(\gamma^*) - \zeta^t(\gamma^*)$ for all t. Given this, we show that w_t^{Δ} from equation (A-1) is strictly greater than x_t^{Δ} for all $t < T$, where $\gamma_T^* = \bar{c}$ for the $T < \infty$ case. For all $t < T$, we have

$$\gamma_t^* = \chi^t(\gamma^*) - \zeta^t(\gamma^*) = x_t^{\Delta} + \int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c) + (1 - F(\gamma_{t+1}^*))\,\gamma_{t+1}^*$$

$$\text{or} \quad x_t^{\Delta} = \gamma_t^* - \int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c) - (1 - F(\gamma_{t+1}^*))\,\gamma_{t+1}^*$$

Suppose $\frac{1}{\beta}\gamma_{t+1}^* \leq \bar{c}$. Then $\int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c) + (1 - F(\gamma_{t+1}^*))\,\gamma_{t+1}^* \geq \beta\int_{\underline{c}}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) + \left(1 - F(\frac{1}{\beta}\gamma_{t+1}^*)\right)\gamma_{t+1}^*$ implies $w_t^{\Delta} > x_t^{\Delta}$. We can rewrite this inequality as

$\left[F(\frac{1}{\beta}\gamma_{t+1}^*) - F(\gamma_{t+1}^*)\right]\gamma_{t+1}^* + (1-\beta)\int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c) - \beta\left[\int_{\underline{c}}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) - \int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c)\right] \geq 0.$

We have $\beta\left[\int_{\underline{c}}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) - \int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c)\right] = \beta\int_{\gamma_{t+1}^*}^{\frac{1}{\beta}\gamma_{t+1}^*} c\,dF(c) < \left[F(\frac{1}{\beta}\gamma_{t+1}^*) - F(\gamma_{t+1}^*)\right]\gamma_{t+1}^*$, so the above inequality holds.

Suppose $\frac{1}{\beta}\gamma_{t+1}^* > \bar{c}$. Then $\int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c) + (1 - F(\gamma_{t+1}^*))\,\gamma_{t+1}^* \geq \frac{\gamma_{t+1}^*}{\bar{c}}Ec > \beta Ec$ implies $w_t^{\Delta} > x_t^{\Delta}$. We can rewrite this inequality as $(1 - F(\gamma_{t+1}^*))\,\gamma_{t+1}^* + \frac{\bar{c}-\gamma_{t+1}^*}{\bar{c}}\int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c) - \frac{\gamma_{t+1}^*}{\bar{c}}\left[\int_{\underline{c}}^{\bar{c}} c\,dF(c) - \int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c)\right] \geq 0.$

We have $\frac{\gamma_{t+1}^*}{\bar{c}}\left[\int_{\underline{c}}^{\bar{c}} c\,dF(c) - \int_{\underline{c}}^{\gamma_{t+1}^*} c\,dF(c)\right] = \frac{\gamma_{t+1}^*}{\bar{c}}\int_{\gamma_{t+1}^*}^{\bar{c}} c\,dF(c) < (1 - F(\gamma_{t+1}^*))\,\gamma_{t+1}^*$, so the above inequality holds.

Finally, we must show $w_{t+1}^{\Delta} = w_t^{\Delta}$ for all t when $\mathbf{X}$ has a stationary delay cost x^{Δ}. A stationary delay cost implies $\gamma_t^* = \gamma_{t+1}^*$ for all t. Using the equation for w_t^{Δ} above, the result follows. □

Proof of Lemma 2: First, suppose $w^{\Delta} < k$. $w^{\Delta} < k$ implies $\hat{\gamma}^i = (s, s, \ldots)$, so for all t we must have $\zeta_i^t(\hat{\gamma}^i) = c_i - k$ and $p^t(\hat{\gamma}^i) = 2w^{\Delta}$. Then for any t, $\gamma_t^i = d$ if and only if $c_i - k < \beta(c_i - k + 2w^{\Delta})$; but $w^{\Delta} < k$ implies $c_i - k > \beta(c_i - k + 2w^{\Delta})$, so $w^{\Delta} < k$ implies $\gamma_t^i \neq d$. For any t, $\gamma_t^i = w$ if and only if $c_i - k > \beta(c_i - k + 2w^{\Delta})$ or $w^{\Delta} < \frac{1-\beta}{2\beta}c_i - \frac{1-\beta}{2\beta}k$, and $\frac{1-\beta}{2\beta}c_i - \frac{1-\beta}{2\beta}k < k$ if and only if $c_i < \frac{1+\beta}{1-\beta}k$. Hence, if $c_i \geq \frac{1+\beta}{1-\beta}k$ then $\gamma^i = (w, w, \ldots)$ for any $w^{\Delta} < k$, and if $c_i < \frac{1+\beta}{1-\beta}k$ then $\gamma^i = (s, s, \ldots)$ for any $\frac{1-\beta}{2\beta}c_i - \frac{1-\beta}{2\beta}k \leq w^{\Delta} < k$ and $\gamma^i = (w, w, \ldots)$ for any $w^{\Delta} < \frac{1-\beta}{2\beta}c_i - \frac{1-\beta}{2\beta}k$.

Second, suppose $w^{\Delta} \geq k$. $w^{\Delta} \geq k$ implies $\hat{\gamma}^i = (d, d, \ldots)$, so for all t we must have $\zeta_i^t(\hat{\gamma}^i) = c_i$ and $p^t(\hat{\gamma}^i) = w^{\Delta}$. Then for any t, $\gamma_t^i = d$ if and only if $c_i - k < \beta(c_i + w^{\Delta})$ or $w^{\Delta} > \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k$. For any t, $\gamma_t^i = w$ if and only if $c_i - k > \beta(c_i + w^{\Delta})$ or $w^{\Delta} < \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k$, and $\frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k > k$ if and only if $c_i > \frac{1+\beta}{1-\beta}k$. Hence, if $c_i > \frac{1+\beta}{1-\beta}k$ then $\gamma^i = (w, w, \ldots)$ for any $k \leq w^{\Delta} < \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k$, $\gamma^i = (s, s, \ldots)$ for any $\frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k \leq w^{\Delta} \leq \frac{1-\beta}{\beta}c_i + \frac{1}{\beta}k$, and $\gamma^i = (d, d, \ldots)$ for any $w^{\Delta} > \frac{1-\beta}{\beta}c_i + \frac{1}{\beta}k$. If $c_i \leq \frac{1+\beta}{1-\beta}k$ then $\gamma^i = (s, s, \ldots)$ for any $k \leq w^{\Delta} \leq \frac{1-\beta}{\beta}c_i + \frac{1}{\beta}k$, and $\gamma^i = (d, d, \ldots)$ for any $w^{\Delta} > \frac{1-\beta}{\beta}c_i + \frac{1}{\beta}k$.

Combining the cases $w^{\Delta} < k$ and $w^{\Delta} \geq k$, the result follows. □

Proof of Lemma 3: We first show $\gamma_t^i = s$ for all t implies $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \leq 2\overline{w}_i$. By Lemma 2, if $w_t^{\Delta} = \overline{w}_i$ for all t then $\gamma_t^i = s$ for all t, in which case $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} = \sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} \overline{w}_i = 2\overline{w}_i$. We now prove that if for any τ $w_\tau^{\Delta} > \overline{w}_i$ then there exists $\mu \in \{\tau+1, \tau+2, \ldots\}$ such that $\sum_{t=\tau}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} w_t^{\Delta} < \sum_{t=\tau}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} \overline{w}_i$, from which the result follows. We prove this by construction. Let $\mu = \min\{t \geq \tau : \hat{\gamma}_{t+1}^i = d\}$ if this exists, otherwise let $\mu = \infty$. We must have $\mu > \tau$, because otherwise $\gamma_\tau^i = s$ implies $c_i - k_i \geq \beta(c_i + w_\tau^{\Delta})$ or $w_\tau^{\Delta} \leq \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k_i = \overline{w}_i$. So given $\mu > \tau$, $\gamma_\tau^i = s$ implies $c_i - k_i \geq \beta\left[c_i + \sum_{t=\tau}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} w_t^{\Delta} - \sum_{t=\tau+1}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} k_i\right]$ or $\sum_{t=\tau}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} w_t^{\Delta} \leq \overline{w}_i + \sum_{t=\tau+1}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} k_i < \sum_{t=\tau}^{\mu}\left(\frac{1}{2}\right)^{t-\tau} \overline{w}_i$, where the last inequality follows from $k_i < \overline{w}_i$.

Next, we show $\gamma_t^i = s$ for all t implies $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq 2\underline{w}_i$. By Lemma 2, if $w_t^{\Delta} = \underline{w}_i$ for all t then $\gamma_t^i = s$ for all t, in which case $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} = \sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} \underline{w}_i = 2\underline{w}_i$. Furthermore, if $c_i \geq \frac{1+\beta}{1-\beta}k_i$ then for any t $\gamma_t^i = s$ only if $w_t^{\Delta} \geq \underline{w}_i$, and the result follows. So suppose $c_i < \frac{1+\beta}{1-\beta}k_i$, in which case $k_i > \underline{w}_i = \frac{1-\beta}{2\beta}(c_i - k_i)$. We prove that $\gamma_1^i = s$ implies $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq \sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} \underline{w}_i = 2\underline{w}_i$. If $\hat{\gamma}_t^i = s$ for all $t > 1$, then $\gamma_1^i = s$ only if $c_i - k_i < \beta\left[c_i - k_i + \sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta}\right]$ or $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq \frac{1-\beta}{\beta}(c_i - k_i) = 2\underline{w}_i$. If $\mu = \min\{t > 1 : \hat{\gamma}_{t+1}^i = d\}$ exists, then $\gamma_1^i = s$ only if $c_i - k_i \leq \beta\left[c_i - \sum_{t=2}^{\mu-1}\left(\frac{1}{2}\right)^{t-1} k_i + \sum_{t=1}^{\mu-1}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta}\right]$ or $\sum_{t=1}^{\mu-1}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k_i + \sum_{t=2}^{\mu-1}\left(\frac{1}{2}\right)^{t-1} k_i$. We also have $\hat{\gamma}_\mu^i = d$ only if $c_i - k_i \leq p^{\mu}(\hat{\gamma}^i) - \varsigma^{\mu}(\hat{\gamma}^i) \leq c_i - k_i + \sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-\tau} w_t^{\Delta}$, where the second inequality follows because TCs must do at least as well as they would from being selective in all periods. Hence, $\hat{\gamma}_\mu^i = d$ only if $\sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-\tau} w_t^{\Delta} \geq 2k_i = \sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-\tau} k_i$, which we can rewrite as $\sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq \sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-1} k_i$. Using $\sum_{t=1}^{\mu-1}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} > \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k_i + \sum_{t=2}^{\mu-1}\left(\frac{1}{2}\right)^{t-1} k_i$ and $\sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq \sum_{t=\mu}^{\infty}\left(\frac{1}{2}\right)^{t-1} k_i$, we have $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k_i + \sum_{t=2}^{\infty}\left(\frac{1}{2}\right)^{t-1} k_i = \frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k_i + k_i = \frac{1-\beta}{\beta}(c_i - k_i) = 2\underline{w}_i$. □

Proof of Proposition 3: We first prove we can induce efficiency with a stationary scheme if and only if $\underline{w}_H \leq \overline{w}_L$. We can induce efficiency with a stationary incentive scheme if and only if $\max\{\underline{w}_L, \underline{w}_H\} \leq \min\{\overline{w}_L, \overline{w}_H\}$, in which case any stationary incentive scheme with incremental wage $w^{\Delta} \in [\max\{\underline{w}_L, \underline{w}_H\}, \min\{\overline{w}_L, \overline{w}_H\}]$ will induce efficiency. We always have $\overline{w}_H > \underline{w}_L$ since we have $c_H > c_L$, $\overline{w}_H = \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k$, and $\underline{w}_L < \frac{1-\beta}{\beta}c_L$. Given $\overline{w}_H > \underline{w}_L$, $\underline{w}_H \leq \overline{w}_L$ implies $\max\{\underline{w}_L, \underline{w}_H\} \leq \min\{\overline{w}_L, \overline{w}_H\}$, and the result follows.

Next, we prove $\underline{w}_H > \overline{w}_L$ implies no incentive scheme can induce efficiency. Using Lemma 3, $\gamma_t^L = s$ for all t only if $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} < 2\overline{w}_L$, and $\gamma_t^H = s$ for all t only if $\sum_{t=1}^{\infty}\left(\frac{1}{2}\right)^{t-1} w_t^{\Delta} \geq 2\underline{w}_H$. Given $\underline{w}_H > \overline{w}_L$, the result follows.

Finally, we prove that $\underline{w}_H \leq \overline{w}_L$ if and only if $c_H - c_L \leq \frac{2k}{1-\beta}$. Suppose $c_H - c_L \leq \frac{2k}{1-\beta}$. $c_H - c_L \leq \frac{2k}{1-\beta}$ implies $\frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k \leq \frac{1-\beta}{\beta}c_L + \frac{1}{\beta}k = \overline{w}_L$, so if $c_H \geq \frac{1+\beta}{1-\beta}k$ then $\underline{w}_H \leq \overline{w}_L$. If $c_H < \frac{1+\beta}{1-\beta}k$, then $\underline{w}_H < k < \overline{w}_L$. Hence, if $c_H - c_L \leq \frac{2k}{1-\beta}$ then $\underline{w}_H \leq \overline{w}_L$. Suppose $c_H - c_L > \frac{2k}{1-\beta}$ so $\frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k > \frac{1-\beta}{\beta}c_L + \frac{1}{\beta}k = \overline{w}_L$. If $c_H \geq \frac{1+\beta}{1-\beta}k$ then $\underline{w}_H = \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k > \overline{w}_L$. And if $c_H < \frac{1+\beta}{1-\beta}k$ then $\underline{w}_H = \frac{1-\beta}{2\beta}c_H - \frac{1-\beta}{2\beta}k > \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k$. Hence, if $c_H - c_L \geq \frac{2k}{1-\beta}$ then $\underline{w}_H > \overline{w}_L$. □

Proof of Lemma 4: As discussed in the text, under any incentive scheme we must have $\hat{\gamma}^L = \hat{\gamma}^H = \hat{\gamma}$, and therefore for all t, $p^t(\hat{\gamma}) - \zeta^t_H(\hat{\gamma}) - c_H = p^t(\hat{\gamma}) - \zeta^t_L(\hat{\gamma}) - c_L$. H's are selective in period t *only if* $c_H - k \leq \beta\left(p^t(\hat{\gamma}^H) - \zeta^t_H(\hat{\gamma}^H)\right)$ or $\frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k \leq p^t(\hat{\gamma}) - \zeta^t_H(\hat{\gamma}) - c_H$. L's complete the task for sure in period t *if* $c_L + k \leq \beta\left(p^t(\hat{\gamma}^L) - \zeta^t_L(\hat{\gamma}^L)\right)$ or $\frac{1-\beta}{\beta}c_L + \frac{1}{\beta}k \leq p^t(\hat{\gamma}) - \zeta^t_L(\hat{\gamma}) - c_L$. $c_H - c_L \geq \frac{2k}{1-\beta}$ implies $\frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k \geq \frac{1-\beta}{\beta}c_L + \frac{1}{\beta}k$, and the result follows. □

Proof of Proposition 4: (i) First, we argue that Lemma 4 implies that we cannot do better than the following outcome: There is some period $D \geq 1$ such that L's are selective and H's wait for sure in periods $t < D$, and L's complete the task for sure and H's are selective in periods $t \geq D$. Lemma 4 says that if H's are selective in period t then L's must complete the task for sure in period t. If D is the first period in which H's are selective, clearly we want H's to be selective (i.e., to behave efficiently) in all periods $t \geq D$ because the probability of L's reaching period $t > D$ is zero. By the definition of D, H's wait for sure in any period $t < D$. Clearly, we would like L's to be selective in any period $t < D$.

Second, we show that for any D this outcome can be achieved with the $(\overline{w}_L, \overline{w}_H)$–deadline scheme with deadline D. Given $\overline{w}_H > \overline{w}_L > k$, clearly $\hat{\gamma}^L_t = \hat{\gamma}^H_t = d$ for all t. Hence, for any t $\gamma^i_t = s$ if and only if $c_i - k \leq \beta(w^\Delta_t - c_i) \leq c_i + k$ or $\frac{1-\beta}{\beta}c_i - \frac{1}{\beta}k \leq w^\Delta_t \leq \frac{1-\beta}{\beta}c_i + \frac{1}{\beta}k = \overline{w}_i$. And $c_H - c_L > \frac{2k}{1-\beta}$ implies $\underline{w}_H = \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k > \overline{w}_L$. Therefore, $w^\Delta_t = \overline{w}_L$ for $t < D$ implies $\gamma^L_t = s$ and $\gamma^H_t = w$ for all $t < D$, and $w^\Delta_t = \overline{w}_H$ for all $t \geq D$ implies $\gamma^L_t = d$ and $\gamma^H_t = s$ for all $t > D$.

Finally, we solve for the optimal deadline. Given a deadline D, the expected costs are:

Task cost for L's $= \frac{1}{2}(c_L - k) + \frac{1}{4}(c_L - k) + \cdots + \left(\frac{1}{2}\right)^D(c_L - k) + \left(\frac{1}{2}\right)^D(c_L + k) = c_L - k + \left(\frac{1}{2}\right)^{D-1}k$

Delay cost for L's $= \frac{1}{2}(0x^\Delta) + \frac{1}{4}(1x^\Delta) + \ldots + \left(\frac{1}{2}\right)^{D-1}((D-2)x^\Delta) + \left[1 - \sum_{n=1}^{D-1}\left(\frac{1}{2}\right)^n\right](D-1)x^\Delta = \left[1 - \left(\frac{1}{2}\right)^{D-1}\right]x^\Delta$

Task cost for H's $= c_H - k$ (because H's are selective in all periods $t \geq D$)

Delay cost for H's $= \frac{1}{2}(D-1)x^\Delta + \frac{1}{4}(D)x^\Delta + \frac{1}{8}(D+1)x^\Delta + \ldots = Dx^\Delta$

Hence, the expected total costs are

$$\pi\left(c_L - k + \left(\frac{1}{2}\right)^{D-1}k + \left[1 - \left(\frac{1}{2}\right)^{D-1}\right]x^\Delta\right) + (1-\pi)\left(c_H - k + Dx^\Delta\right) \equiv Z(D)$$

The only component of the incentive scheme that affects Z is the deadline D, so all second-best incentive schemes will have the same deadline. Z is continuous, twice-differentiable, and $\frac{d^2Z}{dD^2} > 0$. We have

$$argmin_D \quad Z(D) \quad = \quad \alpha \quad = \quad 1 + \frac{ln\left(\frac{\pi}{1-\pi}\right) + ln\left(\frac{k-x^\Delta}{x^\Delta}\right) + ln\,(ln\,2)}{ln\,2}$$

However, the optimal deadline D^* must be an integer. Since Z is continuous and $\frac{d^2Z}{dD^2} > 0$, the optimal deadline is either the largest integer less than α or the smallest integer greater than α.

(ii) If $c_L \geq \frac{1+\beta}{1-\beta}k$, then $\gamma^L_t = s$ only if $w^\Delta_t \geq \underline{w}_L \geq k$. Hence, any incentive scheme that induces the second-best optimal outcome (i.e., $\gamma^L_t = s$ and $\gamma^H_t = w$ for all $t < D^*$ and $\gamma^L_t = d$ and $\gamma^H_t = s$ for all $t \geq D^*$) will imply $\hat{\gamma}^L_t = \hat{\gamma}^H_t = d$. Then for any $t \geq D^*$ $\gamma^H_t = s$ only if $w^\Delta_t \geq \underline{w}_H$ and for any $t < D^*$ $\gamma^L_t = s$ only if $w^\Delta_t \leq \overline{w}_L$. By Proposition 3, $c_H - c_L > \frac{2k}{1-\beta}$ implies $\underline{w}_H > \overline{w}_L$, and the result follows. □

Proof of Proposition 5: Essentially identical to proof of Proposition 3, so omitted.

Proof of Lemma 5: Define $X_t^i(\hat{\gamma}^i)$ as any perceived reductions in expected future task costs below c_i net of additional lost incremental wages. In other words, the period-t continuation payoff from waiting for type i is $\beta\left[w_t^\Delta - c_i - X_t^i(\hat{\gamma}^i)\right]$. Clearly $X_t^i(\hat{\gamma}^i) \geq 0$, and if $\mu^i = \min\{\tau > t : \hat{\gamma}_\tau^i = d\}$ then

$$X_t^i(\hat{\gamma}^i) = \begin{cases} 0 & \text{for } \mu^i = 1 \\ \sum_{n=1}^{\mu^i-1}\left(\frac{1}{2}\right)^n k_i - \sum_{n=1}^{\mu^i-1}\left(\frac{1}{2}\right)^n w_{t+n}^\Delta & \text{for } \mu^i \in \{2,3,\ldots\} \end{cases}$$

Using a revealed preference argument, we can prove that $k_H \geq k_L$ implies $X_t^H(\hat{\gamma}^H) \geq X_t^L(\hat{\gamma}^L)$: $k_H \geq k_L$ clearly implies $X_t^H(\hat{\gamma}^L) \geq X_t^L(\hat{\gamma}^L)$, and $\hat{\gamma}^H$ represents how TCs would behave and therefore maximizes X_t^H. Hence, we have $X_t^H(\hat{\gamma}^H) \geq X_t^H(\hat{\gamma}^L) > X_t^L(\hat{\gamma}^L)$.

For any t, $\gamma_t^L \neq d$ only if $c_L - k_L > \beta\left[w_t^\Delta + c_L - X_t^L(\hat{\gamma}^L)\right]$ or $w_t^\Delta < \frac{1-\beta}{\beta}c_L - \frac{1}{\beta}k_L + X_t^L(\hat{\gamma}^L) = \overline{w}_L + X_t^L(\hat{\gamma}^L)$. Similarly, for any t $\gamma_t^H = w$ if $c_H - k_H > \beta\left[w_t^\Delta + c_H - X_t^H(\hat{\gamma}^H)\right]$ or $w_t^\Delta < \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k_H + X_t^H(\hat{\gamma}^H)$. And $c_H \geq \frac{1+\beta}{1-\beta}k_H$ implies $\underline{w}_H = \frac{1-\beta}{\beta}c_H - \frac{1}{\beta}k_H$, so the inequality becomes $w_t^\Delta < \underline{w}_H + X_t^H(\hat{\gamma}^H)$. Using $\underline{w}_H > \overline{w}_L$ and $X_t^H(\hat{\gamma}^H) \geq X_t^L(\hat{\gamma}^L)$, the result follows. □

Proof of Proposition 6: Essentially identical to the proof of Proposition 4, so omitted. (Proposition 6 follows from Lemma 5 in exactly the same way that Proposition 4 follows from Lemma 4.)

Proof of Lemma 6: We first prove that for any t, $\gamma_t^L \neq d$ implies $\hat{\gamma}_t^H = s$. Define $X_t^i(\hat{\gamma}^i)$ as in the proof of Lemma 5, and again $k_H \geq k_L$ implies $X_t^H(\hat{\gamma}^H) > X_t^L(\hat{\gamma}^L)$ for all t. For any t, $\gamma_t^L \neq d$ only if $c_L - k_L > \beta\left[w_t^\Delta + c_L - X_t^L(\hat{\gamma}^L)\right]$ or $w_t^\Delta < \overline{w}_L + X_t^L(\hat{\gamma}^L)$. For any t, $\hat{\gamma}_t^H = s$ if $c_H - k_H > w_t^\Delta + c_H - X_t^H(\hat{\gamma}^H)$ or $w_t^\Delta < k_H - X_t^H(\hat{\gamma}^H)$. $c_H < \frac{1-\beta}{\beta}k_H$ implies $k_H > \underline{w}_H > \overline{w}_L$, which along with $X_t^H(\hat{\gamma}^H) \geq X_t^L(\hat{\gamma}^L)$ establishes that $\gamma_t^L \neq d$ implies $\hat{\gamma}_t^H = s$.

Now suppose $\gamma_t^L = s$ for all t, so $\hat{\gamma}_t^H = s$ for all t. Then for any t, $\gamma_t^H \neq w$ only if $c_H - k_H \leq \beta\left[\sum_{\tau=t}^{\infty}\left(\frac{1}{2}\right)^{\tau-t} w_\tau^\Delta + c_H - k_H\right]$ or $\sum_{\tau=t}^{\infty}\left(\frac{1}{2}\right)^{\tau-t} w_\tau^\Delta \geq \frac{1-\beta}{\beta}(c_H - k_H) = 2\underline{w}_H$ (since $c_H < \frac{1+\beta}{1-\beta}k_H$). But the logic of Lemma 3 implies $\gamma_t^L \neq d$ for all t only if $\sum_{\tau=t}^{\infty}\left(\frac{1}{2}\right)^{\tau-t} w_\tau^\Delta < 2\overline{w}_L$ for all τ. Given $\underline{w}_H > \overline{w}_L$, the result follows. □

Proof of Proposition 7: (i) Lemma 6 implies there must be some period in which L's complete the task for sure because otherwise H's wait forever. Let $D \equiv \min\{t : \gamma_t^L = d\}$. We now ask for any given D what incentive schemes make H's most likely not to wait in period $\tau < D$.

For each τ, $\gamma_\tau^H \neq w$ if and only if $c_H - k_H < \beta\left(p^\tau(\hat{\gamma}^H) - \zeta_H^\tau(\hat{\gamma}^H)\right)$, so we are most likely to have $\gamma_\tau^H \neq w$ under incentive schemes that maximize $p^\tau(\hat{\gamma}^H) - \zeta_H^\tau(\hat{\gamma}^H)$. As argued in the proof of Lemma 6, for any t $\gamma_t^L \neq d$ implies $\hat{\gamma}_t^H = s$. Hence, for any $\tau < D$ we have

$$p^\tau(\hat{\gamma}^H) - \zeta_H^\tau(\hat{\gamma}^H) = \sum_{t=\tau}^{D-1}\left(\frac{1}{2}\right)^{t-\tau} w_t^\Delta + \left(\frac{1}{2}\right)^{D-\tau} p^D(\hat{\gamma}^H) - \sum_{t=\tau+1}^{D-1}\left(\frac{1}{2}\right)^{t-\tau}(c_H - k_H) - \left(\frac{1}{2}\right)^{D-\tau}\zeta_H^D(\hat{\gamma}^H)$$

First note that $\gamma_D^L = d$ implies $\hat{\gamma}_D^L = d$ (since naifs complete task only if TCs complete task). Given $\hat{\gamma}_D^L = d$, w_t^Δ for any $t \geq D$ does not affect the set of incremental wages for which $\gamma_t^L \neq d$ for all $t < D$. Hence, for any fixed D it is optimal to choose w_t^Δ for each $t \geq D$ to maximize $p^D(\hat{\gamma}^H) + \zeta_H^D(\hat{\gamma}^H)$, so $w_t^\Delta = \overline{w}_H$ for each $t \geq D$ is optimal.

Having maximized $p^D(\hat{\gamma}^H) + \zeta_H^D(\hat{\gamma}^H)$, we now maximize $p^\tau(\hat{\gamma}^H) + \zeta_H^\tau(\hat{\gamma}^H)$ by choosing the w_t^Δ for $t < D$ to maximize $\sum_{t=\tau}^{D-1} \left(\frac{1}{2}\right)^{t-\tau} w_t^\Delta$ given $\gamma_D^L = d$. Using the same logic as in the proof of Lemma 3 part (i), it is straightforward to show that if $\gamma_D^L = d$ and $\gamma_t^L \neq d$ for all $t < D$, then for any $\tau < D$ we must have $\sum_{t=\tau}^{D-1} \left(\frac{1}{2}\right)^{t-\tau} w_t^\Delta \leq \sum_{t=\tau}^{D-1} \left(\frac{1}{2}\right)^{t-\tau} \overline{w}_L$. In other words, $w_t^\Delta = \overline{w}_L$ for each $t < D$ maximizes the likelihood that $\gamma_\tau^H \neq w$.

Hence, for each $\tau < D$ H's are most likely not to wait in period τ if $w_t^\Delta = \overline{w}_L$ for each $t < D$ and $w_t^\Delta = \overline{w}_H$ for each $t \geq D$. This implies that for any $\tau < \tau' < D$, having $w_t^\Delta = \overline{w}_L$ for each $t < D$ and $w_t^\Delta = \overline{w}_H$ for each $t \geq D$ maximizes $p^t(\hat{\gamma}^H) + \zeta_H^t(\hat{\gamma}^H)$ for both $t = \tau$ and $t = \tau'$. It follows that there always exists a $(\overline{w}_L, \overline{w}_H)$–deadline scheme that is second-best optimal.

Now consider behavior under the $(\overline{w}_L, \overline{w}_H)$ deadline scheme with deadline D. It is obvious that $\gamma_t^L = s$ for $t < D$ and $\gamma_t^L = d$ for $t \geq D$. It is also obvious that $\gamma_t^H = s$ for all $t \geq D$. Consider γ_t^H for $t < D$. In period $D - n$, $n \in \{0, 1, 2, \ldots\}$, we have $\gamma_t^H = s$ if and only if $c_H - k_H \leq \beta \left[\sum_{j=0}^{n} \left(\frac{1}{2}\right)^j \overline{w}_L + c_H - k_H - \sum_{j=0}^{n} \left(\frac{1}{2}\right)^j k_H\right]$ or $\frac{1-\beta}{\beta} c_H - \frac{1}{\beta} k_H \leq k_H + \sum_{j=0}^{n} \left(\frac{1}{2}\right)^j (\overline{w}_L - k_H)$. Hence, given the definition of $\dot{d}$, $\gamma_t^H = d$ for $t < D - \dot{d}$, and $\gamma_t^H = s$ for $t \geq D - \dot{d}$.

Finally, consider the optimal deadline. Using the same method as in the proof of Proposition 4, we get the following equation for expected total costs, which we denote by a function $\dot{Z}$:

$$\dot{Z}(D, \dot{d}) = \begin{cases} \pi\left(c_L + k_L + \left(\frac{1}{2}\right)^{D-1} k_L + \left[1 - \left(\frac{1}{2}\right)^{D-1}\right] x^\Delta\right) + (1 - \pi)\left(c_H - k_H + (D - \dot{d}) x^\Delta\right) & \text{for } D > \dot{d} + 1 \\ \pi\left(c_L + k_L + \left(\frac{1}{2}\right)^{D-1} k_L + \left[1 - \left(\frac{1}{2}\right)^{D-1}\right] x^\Delta\right) + (1 - \pi)\left(c_H - k_H + x^\Delta\right) & \text{for } D < \dot{d} + 1 \end{cases}$$

For $D \leq \dot{d} - 1$, $\dot{Z}$ is increasing in D, so we must have the optimal deadline $D^{**} \geq \dot{d} + 1$. For $D > \dot{d} + 1$, it is straightforward to show that the optimal deadline is D^* defined in Proposition 6 provided $D^* > \dot{d} + 1$. Hence, the optimal deadline $D^{**} = \min\{D^*, \dot{d} + 1\}$.

(ii) Essentially identical to the proof of Proposition 4 part (ii), so omitted. ☐

References

Ainslie, G. (1974). "Impulse control in pigeons." *Journal of the Experimental Analysis of Behavior* 21(3): 485-489.

Ainslie, G. (1975). "Specious reward: A behavioral theory of impulsiveness and impulse control." *Psychological Bulletin* 82(4): 463-496.

Ainslie, G. (1987). "Self-reported tactics of impulse control." *International Journal of the Addictions* 22(2): 167-179.

Ainslie, G. (1991). "Derivation of 'Rational' Economic Behavior from Hyperbolic Discount Curves." *American Economic Review* 81(2): 334-340.

Ainslie, G. (1992). *Picoeconomics: The strategic interaction of successive motivational states within the person.* Cambridge University Press; New York, NY, US.

Ainslie, G. and N. Haslam (1992a). "Self-control." in *Choice over Time*, G. Loewenstein and J. Elster, eds., Russell Sage Foundation, New York, NY, US: 177-209.

Ainslie, G. and N. Haslam (1992b). "Hyperbolic discounting." in *Choice over Time*, G. Loewenstein and J. Elster, eds., Russell Sage Foundation, New York, NY, US: 57-92.

Ainslie, G. and R. J. Herrnstein (1981). "Preference reversal and delayed reinforcement." *Animal Learning & Behavior* 9(4): 476-482.

Akerlof, G. A. (1991). "Procrastination and Obedience." *American Economic Review* 81(2): 1-19.

Chung, S-H., and Herrnstein (1967). "Choice and Delay of Reinforcement." *Journal of the Experimental Analysis of Behavior* 10: 67-74.

Funder, D. C. and J. Block (1989). "The role of ego-control, ego-resiliency, and IQ in delay of gratification in adolescence." *Journal of Personality & Social Psychology* 57(6): 1041-1050.

Goldman, S. M. (1979). "Intertemporally Inconsistent Preferences and the Rate of Consumption." *Econometrica* 47(3): 621-626.

Goldman, S. M. (1980). "Consistent Plans." *Review of Economic Studies* 47: 533-537.

Hoch, S. and G. Loewenstein (1991). "Time-inconsistent Preferences and Consumer Self-Control." *Journal of Consumer Research* 17: 492-507.

Kirby, K. and R. Herrnstein (1995). "Preference reversals due to myopic discounting of delayed reward." *Psychological Science* 6(2): 83-89.

Koopmans, T. (1960). "Stationary Ordinal Utility and Impatience." *Econometrica* 28: 287-309.

Laibson, D. (1994). "Essays in Hyperbolic Discounting." Economics. MIT.

Laibson, D. (1995). "Hyperbolic Discount Functions, Undersaving, and Savings Policy," Harvard University.

Loewenstein, G. and D. Prelec (1992). "Anomalies in Intertemporal Choice: Evidence and an Interpretation." *Quarterly Journal of Economics* 107(2): 573-597.

O'Donoghue, Ted and Matthew Rabin (1996). "Doing It Now or Later," Northwestern University and University of California at Berkeley.

Phelps, E. S. and R. A. Pollak (1968). "On Second-best National Saving and Game-equilibrium Growth." *Review of Economic Studies* 35: 185-199.

Pollak, R. A. (1968). "Consistent Planning." *Review of Economic Studies* 35: 201-208.

Strotz, R. H. (1955). "Myopia and Inconsistency in Dynamic Utility Maximization." *Review of Economic Studies* 23(3): 165-180.

Thaler, R. H. (1991). "Some Empirical Evidence on Dynamic Inconsistency." *Quasi Rational Economics*, Russell Sage Foundation: 127-133.