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Education Signalling with Preemptive Offers*

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Abstract

We analyze a version of Spence's job market signalling model in which firms can make job offers before workers complete their education. Workers cannot commit to turning down such offers. In contrast to previous work, we assume that firms can make these offers privately, so that workers are unable to use one firm's offer in an attempt to elicit better offers from other firms. In the unique sequential equilibrium outcome of the model, workers choose not to be educated. This suggests that wasteful education cannot serve as a signal of ability if firms are able to make such offers. We examine the robustness of our result to a variety of generalizations. When education is productive, the standard model predicts that more able individuals choose to become overeducated so as to separate themselves from less able workers. In contrast, in our model, less able workers become overeducated to (partially) pool with more able workers. The pooling mutes the incentives of high ability workers, who in consequence actually choose to become *undereducated*. Finally, we examine a model with grades. Here we find some scope for a Spence style equilibrium in which workers of high ability become overeducated in order to separate. The mechanism however is rather different than that of the standard model, and in particular is not in any way dependent on the inverse correlation of education costs and ability which is central to the Spence model.

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1. Introduction

In his path-breaking papers, Spence [7], [8] argued that people may rationally invest in education even when such education is wasteful in that it does not enhance productivity. In the simplest version of Spence's model, there are two types of worker, one having higher innate productivity than the other. Education is less costly for a worker with higher innate productivity. This negative correlation of the cost of education with worker productivity introduces the possibility of a *signalling* equilibrium: firms believe that an educated worker is more productive and so are forced by competition to pay a premium to such a worker. This premium is large enough to make education worthwhile for a high productivity worker, but not for a low one, justifying the belief of firms that an educated employee is more productive. Spence showed that there were many equilibria in his model, including signalling equilibria and equilibria in which no type of worker chooses to be educated.

Cho and Kreps [3] provide an explicitly game-theoretic analysis of the Spence model. They consider a model in which a worker, knowing his productivity, chooses a length of time to be educated. After education is complete, two firms that do not know the worker's productivity, but can observe the amount of education obtained, simultaneously bid for the worker's services. While there are many sequential equilibria of this game, only the minimal separating equilibrium the Riley equilibrium [6] satisfies the intuitive criterion. In this equilibrium the high productivity worker chooses just enough education so that it does not pay for the low productivity worker to obtain this level of education even if this convinces firms that he is of high productivity.[1] A low productivity worker chooses a zero level of education.

An interesting feature of this equilibrium was pointed out by Weiss [9] and Admati and Perry [1]: when a student arrives for the first day of class, the separation has already occurred. A student who begins education is of high type. Why, then, do corporate recruiters not intercept students as they arrive on campus? Since education is costly but does not increase the value of the worker to the employer, remaining in school at cost c and getting a wage w at the end is pareto dominated by any wage in the interval $(w-c, w)$ now. Of course, if merely arriving for the first day of class is enough to elicit the high wage offer, without actually having to incur the cost of education, then a low type will also find it

[1] With multiple types, each type chooses just enough education to separate himself from his next most able colleague.

worthwhile to "choose" education. So, separation with education is overturned by early wage offers, while separation followed by early wage offers is overturned by the incentive of low types to mimic high types. The Cho-Kreps model avoids this problem by implicitly assuming that individuals can commit to their chosen length of education and, in particular, to turning down offers while still in school.

In practice, it is difficult to see how such commitments can be made. It is true that many of the costs of education are incurred early in the education process, and are to some extent sunk – the cost of giving up one's current job, costs of moving, and some part of the tuition payment. These give some degree of commitment. However, the important costs for the functioning of the separating equilibrium are those which are negatively correlated with ability, and it is hard to imagine that costs such as moving are related in the necessary way to worker ability. Rather, the costs which allow differentiation are those involved in the actual acquisition of education – sitting in class, preparing assignments, taking exams – and these costs are necessarily incurred over time.

This casts doubt on the validity of the Spence model for actual educational experience. Without a degree of commitment which seems difficult to achieve in theory, and which certainly is not achieved by current institutional arrangements, the Riley outcome, and in fact any perfectly separating outcome, is not consistent with equilibrium. Of course, this does not tell us that education does not play any signalling role, only that perfectly separating equilibria do not survive in a model with unproductive education and no worker commitment. What then are the equilibria in a model without commitment?

One answer is provided by Nöldeke and van Damme [5] (henceforth NVD). In their model, there are again two types of worker and two firms who bid for the worker's services. A worker of type k, $k = 1, 2$, has productivity k and education cost per unit of time of $1/k$. The prior probability of a high productivity worker is μ_0. The maximum length of education is $E > 1$.[2] The innovation in the NVD model is splitting E into many small periods, each of length Δ. After each period, firms have the opportunity to make new offers. So, as Δ gets small, the model has less and less ability for the worker to commit to education.

A key feature of the NVD setup is that offers are public, so that a firm knows the precise value of every previously rejected offer by either firm. This allows considerable latitude for how beliefs depend on the history of rejected offers, creating a large set of sequential equilibria, including ones in which the firms earn positive profits.

[2] NVD also consider the case when $E < 1$. In this case full separation ceases to be incentive compatible because the difference between pre- and post-education wages is greater than the education costs of even the low types.

NVD view many of these equilibria as implausible and so adopt the never a weak best response requirement, hereafter referred to as NWBR (Kohlberg and Mertens [4]), to restrict beliefs.[3] For given Δ, there is a unique equilibrium outcome satisfying NWBR. Remarkably, as Δ goes to zero, this outcome converges to the Riley outcome.

For small Δ their equilibrium is roughly as follows. Both firms make an offer of 1 at the beginning of the game. The low type accepts this offer with sufficient probability to raise the productivity of the worker conditional on remaining to $2-\frac{\Delta}{2}$ (clearly as $\Delta \to 0$, this involves almost all of the low types accepting). There is education lasting $1-\frac{\Delta}{2}$. During the education period, both firms make wage offers each period of 1 and these offers are turned down by both types of worker. Finally, after the education period has ended, firms offer a wage of $2-\frac{\Delta}{2}$ and the worker accepts.

After the initial period, the average productivity of the worker is $2-\frac{\Delta}{2}$ and this does not subsequently change. So the Admati-Perry-Weiss question remains: an offer below $2-\frac{\Delta}{2}$ any time during the education period would earn positive profits if accepted by both types, and if sufficiently close to $2-\frac{\Delta}{2}$, will be preferable to both types to waiting until education is finished. Why don't firms make such offers?

It is here that the publicness of the offers and NWBR play their key role. During the education phase the worker receives offers which are unacceptable to either type, and so firms infer nothing from the rejection of the offers. Conversely, if a firm makes an offer near $2-\frac{\Delta}{2}$ this gives the high type worker a chance to signal his type by rejecting the offer. If such an offer is turned down, NWBR forces both firms to assume that the worker is of the high type, and thus competition forces both firms to offer 2 next period. Rejecting any offer strictly below $2-\frac{\Delta}{2}$ is thus preferable for the high type worker to accepting. Since the high type worker always rejects such an offer, the firm at best breaks even, and loses money if the low type accepts. Consequently, such offers are not made.

Suppose that offers could be made privately. That is, suppose that a firm can make an offer that is binding on itself but that the worker cannot credibly convey the existence of this offer to the other firm. One such example may be an oral offer. Another is the question familiar to many academics: "If we were to make you the following offer, would you accept?" With private offers, rejecting an offer from one firm does not change the other firm's information and therefore its actions. A worker cannot then confidently expect a higher future wage to result from turning down a current period offer, and the current offer becomes

[3] In games of this sort, NWBR can be roughly interpreted as requiring that any deviation is attributed to the type who has the most to gain from the deviation.

more attractive. So, it is no longer obvious that the NVD construction is an equilibrium.

In the NVD world, at any time after the beginning of the game, both the firm and the worker would find it in their interest to make such private offers possible. Both know that if such offers are possible, the remaining educational costs can be saved and divided between them. So, if NVD's extensive form is the right one, then the players in the game have an incentive to try to change it, and it seems to us likely that in this case they will succeed.[4] We thus think it important to understand the degree to which the rather surprising result of NVD depends on the assumption of public offers.

In this paper we analyze a model with private offers. We go to the opposite extreme of NVD and break the informational link between successive periods completely by assuming that different firms make offers at the beginning of each period. We discuss the role of this assumption further in Section 5.

In Section 2 we describe a model with unproductive education and a finite number of types. We show that, when the time interval between successive offers is sufficiently small, there is a unique sequential equilibrium outcome. In this equilibrium, the worker receives an offer equal to his expected productivity *at the beginning of the game* and accepts regardless of his type. The result differs from NVD in two important ways. First, even without equilibrium refinements, there is a unique equilibrium. Second, in this equilibrium, there is no wasteful education.

In Section 3, we turn to a model with productive education. For simplicity, we restrict ourselves to two types. In this setup, the standard result (if E is long enough for separation) is that the less able worker chooses an optimal level of education (i.e. one at which the marginal increase in productivity is just balanced by the marginal cost of education), while the more able worker overinvests in education so as to separate himself from the less able type. In our model, this result is turned on its head: the more able worker *never* overinvests in education. Rather, the more able worker becomes educated because this raises his true value, while less able worker becomes *overeducated* so as to partially pool with the more able worker. Finally, because the more able worker is unable to fully separate, his incentives to acquire additional education are muted, and he obtains less education than in a world with perfect information. An attractive feature of this equilibrium is that, unlike in a standard Spencian model, the set of equilibrium outcomes varies continuously as the probability of a low ability worker goes to 0.

[4] Of course, if the firm and the worker arrange things so that such an offer is possible, and the firm then makes an offer in the $(w - c, 2 - \frac{\Delta}{2})$ range, then the worker has a further incentive to try to circumvent the arrangement made with the firm and somehow make the offer public.

In Section 4, we consider education in which the student receives a grade at the end of his program. This is the setup which comes closest to validating the Spence result of wasteful education by the high ability worker. However, a major point distinguishing the results is that the qualitative features of the equilibrium here do not depend in any way on the high ability worker having lower educational costs. In a sense, the result is not one of signalling through education but rather one of signalling through a very expensive test, in which the time element plays a fairly minor role.

In Section 5, we discuss our assumption that different firms make offers at the beginning of each period. We then briefly consider a version of our model in which the same firms make offers in successive periods, but these offers are not known to the other firm. This is perhaps a more realistic model than either NVD or our model, but very hard to analyze. We satisfy ourselves with some preliminary comments.

2. Education Signaling with Unproductive Education

We begin our analysis in a world with unproductive education. The worker can have one of a finite set of types. Productivity is increasing in type, while education costs are decreasing. The only significant difference between the model of this section and that of NVD is in breaking the informational link between periods.

2.1. Description of the Model

The economy has a single worker. The feasible length of education is denoted by $E > 0$. This length is divided into T equal periods, each of length Δ, so that $T = E/\Delta$. Define $G(\Delta) = \{0, \Delta, 2\Delta, \cdots, E\}$. The worker is of one of K possible types $k \in \{1, \cdots, K\}$. The productivity of a worker of type k is $\pi(k)$, where $\pi(\cdot)$ is strictly increasing. We normalize productivity levels so that $\pi(1) = 1$ and $\pi(K) = 2$. The prior probability that the worker is of type k is $\mu_0(k)$.

There are a total of $2T$ firms. At each time $t \in G(\Delta)$, two of these firms simultaneously make wage offers to the worker without knowing the worker's type, or anything about previous offers except that they were rejected. Each firm makes an offer at only one time. At $t \in G(\Delta)$, $t < E$, the worker either accepts one of the offers or chooses additional education lasting Δ. At time E, the worker chooses which, if any, wage offer to accept. The worker cannot commit to education that lasts longer than Δ.

The utility of a worker of type k who accepts a wage of w at time t is given by

$$U(t, k, w) = w - [3 - \pi(k)]\, t$$

so that the cost of education of length t is $[3 - \pi(k)]\, t$, which is strictly decreasing in productivity.[5] The profit of a firm that employs a worker of type k at a wage w is given by $\pi(k) - w$.

Since the active firms at any given time know nothing about previous offers except that they were rejected, a (pure) strategy for each firm is simply a wage offer in the interval $[0, 2]$. A (pure) strategy for the worker is a choice of which offers, if any, to accept at each time t as a function of his type and history.[6] We allow for mixed strategies on the part of all agents. A sequential equilibrium is defined in the standard fashion.

2.2. Analysis

The main proposition of this section establishes that if Δ is sufficiently small, the worker does not choose any education in equilibrium regardless of his type. We begin by showing that in any sequential equilibrium, all workers follow reservation wage strategies. The proof does not depend on any restriction on beliefs, but does rely heavily on the assumption that firms making later offers cannot observe the wages the worker has previously rejected. In NVD's model, non-reservation wage strategies can be a part of a sequential equilibrium if one does not impose NWBR.

Lemma 2.1. *(Reservation Wage Strategies): In any sequential equilibrium, for each type k and for each time $t \in G(\Delta)$ there is a number $r_t(k)$ such that a worker of type k accepts one of the maximal offers at time t if the offer is strictly greater than $r_t(k)$ and rejects all offers strictly less than $r_t(k)$. Furthermore for all $t < E$, $r_t(k)$ is strictly increasing in k.*

Proof. Let $V_t(k)$ denote the continuation value for a worker of type k immediately prior to receiving offers at time t. This continuation value is independent of offers by previous firms since firms do not know the offers that were made in the past. The choices available to the worker and his payoffs from these choices are independent of the value of rejected offers. It follows that the worker follows a reservation wage strategy and the reservation wage of a worker of type k at time t is given by

$$r_t(k) = V_{t+\Delta}(k) - [3 - \pi(k)]\,\Delta$$

To see that reservation wages are strictly increasing in type note that a worker of type k can always choose the same continuation strategy as a worker of type

[5] Our specification of costs differs from NVD. The difference is for notational convenience and without significance for the results.

[6] As we will see shortly, it is strictly suboptimal for the worker to condition his behavior at any given time on any information other than his type.

$k-1$, but has lower education costs. Thus $V_{t+\Delta}(k) \geq V_{t+\Delta}(k-1)$ and $r_t(k) > r_{t-\Delta}(k)$. ■

Since reservation wages are increasing in type, a typical offer will attract all types up to some threshold type. So, given beliefs μ, let $\pi_k(\mu)$ denote the expected productivity conditional on the worker's type being between 1 and k. That is,

$$\pi_k(\mu) = \frac{\sum_{i=1}^{k} \mu(i)\pi(i)}{\sum_{i=1}^{k} \mu(i)}.$$

For a given equilibrium, let $\mu_t(k)$ denote the probability that the worker is of type k at the beginning of time t.

Then the next lemma follows immediately from Lemma 2.1.

Lemma 2.2. *(Stochastic Dominance): Let $t \in G(\Delta)$, $t < E$. If in equilibrium $t+\Delta$ is reached with positive probability, then $\mu_{t+\Delta}$ stochastically dominates μ_t.*

Proof. Let $p_t(k)$ denote the probability that a type k worker chooses further education at time t. Choose $j, k \in \{1, \cdots, K\}$, $j < k$. Since $r_t(k) > r_t(j)$,

$$p_t(k) \geq p_t(j). \tag{2.1}$$

By Bayes' rule,

$$\mu_{t+\Delta}(k) = \frac{p_t(k)\mu_t(k)}{\sum_{i=1}^{K} p_t(i)\mu_t(i)}.$$

Thus,

$$\frac{\mu_{t+\Delta}(k)}{\mu_{t+\Delta}(j)} = \frac{p_t(k)}{p_t(j)}\frac{\mu_t(k)}{\mu_t(j)}. \tag{2.2}$$

Equations (2.1) and (2.2) together imply stochastic dominance. ■

If a worker accepts an offer at any time, he clearly will accept the highest offer then available. Further, the utility of accepting this offer depends neither on the identity of the firm nor the value of the other offers. And, given the information structure, the continuation payoff to rejecting both offers is independent of the offers made. We thus make the following restriction on the worker's strategy.

Assumption. *(No Favoritism): (i) The probability that the worker accepts an offer at any time depends only on the worker's type and the highest offer then available. (ii) If at some t a worker of a given type receives the same offer from both firms, the probability with which he accepts each firm's offer is the same.*

None of the results which follow depend in an essential way on this restriction, but the proofs become more tedious without it.

Next, we show that any equilibrium wage offer by firms generates zero expected profits. The argument uses the Bertrand nature of competition between firms but is complicated by the possible dependence of average productivity on the wage offer.

Lemma 2.3. *(Zero Profits): In any sequential equilibrium, every wage offer w made by a firm earns zero expected profits.*

Proof. Fix $t \in G(\Delta)$, and let the time t firms be denoted by 1 and 2. Since firms can offer a wage of zero, their expected profits cannot be negative. Assume firm 2 earns expected profits $\rho > 0$. The optimality of firm 2's strategy implies that firm 2 earns ρ from each offer in the support of its offer distribution.

We begin by demonstrating that firm 1 also makes positive profits. Let $\overline{w}_2$ be the supremum of the support of firm 2's offers. Let $\overline{j}$ be the highest type for whom accepting an offer of $\overline{w}_2$ is a best response. So for $w \leq \overline{w}_2$, the expected productivity of a worker conditional on accepting an offer of w by firm 2 is at best $\pi_{\overline{j}}(\mu_t)$. This follows by the no favoritism assumption and Lemma 2.1. If w is an equilibrium offer for firm 2 it must be that

$$\pi_{\overline{j}}(\mu_t) - w \geq \rho.$$

Since equilibrium offers can be chosen arbitrarily close to $\overline{w}_2$, we have

$$\pi_{\overline{j}}(\mu_t) - \overline{w}_2 \geq \rho.$$

Consider the offer $\overline{w}_2 + \frac{\rho}{2}$ by firm 1. The offer is always maximal and thus any worker who accepts an offer accepts firm 1's offer. But since accepting $\overline{w}_2$ was a best response for $\overline{j}$, by Lemma 2.1 the unique best response for types $\{1, \cdots, \overline{j}\}$ is to accept $\overline{w}_2 + \frac{\rho}{2}$, and some types above $\overline{j}$ may also choose to accept. So, the average quality of the worker conditional on accepting this offer is at least $\pi_{\overline{j}}(\mu_t)$, and firm 1 earns at least $\frac{\rho}{2}$ from this offer.

Thus if one firm makes profits, the other must as well. Since profits conditional on acceptance are bounded, there exists a $\zeta > 0$ such that all equilibrium offers win with probability at least ζ. This implies that the firms' equilibrium offers have a common infimum $\underline{w}$ that both firms offer with positive probability and that winning with this offer is strictly profitable. But then an offer slightly higher than $\underline{w}$ is strictly better than $\underline{w}$, a contradiction. ■

It is important to realize why Lemma 2.3 does not hold in NVD's model. In their model there is an equilibrium in which firms make positive profits. This is supported by beliefs on the worker's part that above equilibrium wage offers now will be followed by even higher wage offers in the future. These future higher wage offers cause high types to reject the current offer and low types to accept with positive probability. This simultaneously renders current deviations unprofitable and justifies the higher wage offers in the future. In our model, future wage offers cannot depend on current offers and so this type of behavior cannot be part of an equilibrium.

Fix Δ, and an equilibrium of the game where the interval between offers is Δ. Let τ denote the last time which is reached with positive probability in the equilibrium.

Lemma 2.4. *Suppose $[\pi(K) - \pi(K-1)]\,\mu_0(K) > 2\Delta$, and assume $\tau > 0$. Then, $\mu_{\tau-\Delta}(\cdot) = \mu_\tau(\cdot)$.*

Proof. Suppose that $\tau > 0$. From stochastic dominance, $\mu_{\tau-\Delta}(K) > 0$. Assume $\mu_{\tau-\Delta}(K) = 1$. Then since $r_{\tau-\Delta}(K) \leq 2 - \Delta$ (as 2 is the most the worker could expect in the future), and since education is sometimes chosen at time $\tau - \Delta$, the time $\tau - \Delta$ firms must each be offering below $2 - \Delta$ with positive probability. But then an offer of $2 - \frac{\Delta}{2}$ is accepted with positive probability and makes positive profits, contradicting Lemma 2.3.

Suppose then that $\mu_{\tau-\Delta}(K) \in (0,1)$. Since firms make zero profits, both firms offer $\pi_K(\mu_\tau)$ with probability one in period τ. So, for all types k,

$$r_{\tau-\Delta}(k) = \pi_K(\mu_\tau) - [3 - \pi(k)]\,\Delta \geq \pi_K(\mu_\tau) - 2\Delta. \tag{2.3}$$

Now

$$\begin{aligned}
\pi_K(\mu_\tau) - \pi_{K-1}(\mu_\tau) &= \mu_\tau(K)\pi(K) + (1 - \mu_\tau(K))\pi_{K-1}(\mu_\tau) - \pi_{K-1}(\mu_\tau) \\
&= [\pi(K) - \pi_{K-1}(\mu_\tau)]\,\mu_\tau(K) \\
&\geq [\pi(K) - \pi(K-1)]\,\mu_0(K) > 2\Delta.
\end{aligned}$$

and so $\pi_K(\mu_\tau) > \pi_{K-1}(\mu_\tau) + 2\Delta$. But then from (2.3), we have

$$r_{\tau-\Delta}(k) \geq \pi_{K-1}(\mu_\tau)$$

for all types k. So to attract any worker type at time $\tau - \Delta$, firms must offer a wage exceeding $\pi_{K-1}(\mu_{\tau-\Delta})$. For firms to break even on such an offer, the average productivity of the worker who accepts the offer must exceed $\pi_{K-1}(\mu_{\tau-\Delta})$, and so, since reservation wages are increasing in type, the offer must attract the worker with positive probability when he is of type K (and with probability one when of types $1, \cdots, K-1$), and so must be at least $r_{\tau-\Delta}(K)$.

Suppose that in equilibrium a firm makes an offer of $r_{\tau-\Delta}(K)$, that this is maximal with positive probability, and that when maximal it is accepted with probability strictly less than one by a worker of type K. Then, since the offer breaks even,

$$\pi_K(\mu_{\tau-\Delta}) > r_{\tau-\Delta}(K).$$

But then, an offer by the firm of $w \in (r_{\tau-\Delta}(K), \pi_K(\mu_{\tau-\Delta}))$ is maximal with positive probability, and, when maximal, is accepted by all types and so earns positive profits, again contradicting Lemma 2.3.

So, in order for a worker who reaches time $\tau - \Delta$ to choose further education, it must be that the maximal offer made at time $\tau - \Delta$ is one which is turned down by both types. Thus, $\mu_{\tau-\Delta}(\cdot) = \mu_\tau(\cdot)$. ■

Theorem 2.5. *Suppose* $[\pi(K) - \pi(K-1)]\,\mu_0(K) > 2\Delta$. *Then in every sequential equilibrium firms offer a wage of* $\pi_K(\mu_0)$ *with probability one at time zero and all types of worker accept.*

Proof. We will argue that in equilibrium $\tau = 0$. The rest follows from Lemma 2.3.

Assume $\tau > 0$. Since by Lemma 2.4 $\pi_K(\mu_{\tau-\Delta}) = \pi_K(\mu_\tau)$, it follows that $r_{\tau-\Delta}(k) = \pi_K(\mu_{\tau-\Delta}) - \Delta$ for all types k. Since the worker chooses further education with positive probability at time $\tau - \Delta$, each firm must be offering a wage at or below $\pi_K(\mu_{\tau-\Delta}) - \Delta$ with positive probability. Suppose a firm offers $\pi_K(\mu_{\tau-\Delta}) - \frac{\Delta}{2}$. This offer attracts all types when it is maximal and thus makes positive profits, again contradicting Lemma 2.3. Thus the worker cannot be choosing education with positive probability at time $\tau - \Delta$, contradicting that $\tau > 0$. ■

This result is in sharp contrast to that of NVD. In their model, there is a multiplicity of sequential equilibrium outcomes, while in our model, there is a unique sequential equilibrium outcome even without equilibrium refinements. And, while in their model, the "reasonable" equilibrium converges to the minimal separating equilibrium, in our model, there is complete pooling at no education.

The contrast in these results illustrates the critical role of the differing information structure in the two models. In both models, education creates the possibility of mutually advantageous preemptive offers. However when offers are public, the worker can signal that he is of a high type by rejecting such an offer, thus getting an even better offer next round. Foreseeing this, firms do not make preemptive offers in the first place and the gains are not realized. In our model, the confidentiality of the offer prevents this kind of behavior by the worker, inducing firms to make preemptive offers in the face of wasteful education.

Our equilibrium path may also be puzzling to students of the refinements literature. Consider the following argument. In our model, the set of actions for the firms active in periods after the first which would induce the worker of type K to forego a wage of $\pi_K(\mu_0)$ at the beginning of the game and choose education is in a neighborhood of the set of actions that would induce the same choices by a worker of type k, for all $k < K$. Seemingly then, refinements like NWBR would imply that period $T-1$ firms should view education as signalling that the worker is of type K. They must then offer a wage of 2 which, for small Δ, would induce the worker to turn down $\pi_K(\mu_0)$ at the beginning of the game, upsetting what we have claimed to be the unique sequential equilibrium!

The flaw in this argument goes to the heart of the difference in our respective information structures. Since the time Δ firms do not observe the time zero offers, they are not compelled to believe that the rejected offer was $\pi_K(\mu_0)$. In particular, they can believe that what actually occurred was that the time zero firms both made an offer unacceptable to any type. And, since offer distributions which put positive mass on such an unacceptable offer can be chosen arbitrarily close to the distribution putting point mass on $\pi_K(\mu_0)$, it is in fact a best response for all types to choose education for strategies arbitrarily close to the equilibrium one, and so NWBR has no power. With public offers, unexpected rejections cannot be attributed to the firms, and so NWBR has bite.

3. Productive Education

We have seen that in a world with unproductive education, preemptive offers overturn the possibility of wasteful education. We turn now to the effect of preemptive offers when education enhances worker productivity.

3.1. Description of the Model

For simplicity, we analyze a two type model. Type L has productivity $\pi(L,t)$ after t periods of education, while type H has productivity $\pi(H,t)$, where $\pi(L,t)$ and $\pi(H,t)$ are increasing, strictly concave, and continuously differentiable functions of t. We assume $\pi(L,0) < \pi(H,0)$, and $\frac{\partial\pi(H,t)}{\partial t} > \frac{\partial\pi(L,t)}{\partial t}$ for all t. So, the high type is both initially more productive than the low type and benefits more from education. Education of length t costs the high type t, and the low type $2t$. Let μ_0 be the prior probability that the worker is of type H. Define

$$\pi(\mu, t) = \mu\pi(H, t) + (1 - \mu)\,\pi(L, t).$$

The rest of the model is as before. In particular, at intervals of length Δ, two firms simultaneously make offers to the worker with no knowledge of the offers of previous firms.

Recall that Cho and Kreps [3] show in the standard setting that the only equilibrium to survive equilibrium refinements is the Riley outcome: the low type worker obtains the efficient amount of education, while the high type worker obtains either the minimum amount of education required to separate from the low type or the efficient amount, whichever is greater. Thus the equilibrium involves overeducation by the high type worker for signalling purposes. Note that this remains the unique equilibrium (surviving equilibrium refinements) no matter how small the probability of a low type worker.

3.2. Analysis

It is immediate that the reservation wage lemma (Lemma 2.1) and the stochastic dominance lemma (Lemma 2.2) continue to hold. We also continue to require the no-favoritism assumption. It is then easily seen that the zero-profit lemma (Lemma 2.3) also obtains.

We will make the following assumption on Δ:

Assumption. $\pi(\mu_0, 0) - \pi(L, 0) > 2\Delta$.

This assumption plays the same role as the assumption in Theorem 2.5 that $[\pi(K) - \pi(K-1)]\,\mu_0(K) > 2\Delta$, but is simpler due to the restriction to two types.

Fix Δ satisfying the assumption, and an equilibrium of the resulting game. For this equilibrium let τ be the last time which is reached with positive probability by either type of worker. Denote the reservation wages of the high and low types at time t by $r_t(H)$ and $r_t(L)$, respectively, and let μ_t be the probability the worker is of the high type at the beginning of time t.

Lemma 3.1. *If $\tau > 0$, then $\mu_{\tau-\Delta} = \mu_\tau$.*

Proof. Since time τ is the last one reached in equilibrium, by the zero-profit lemma the time τ firms must offer $\pi(\mu_\tau, \tau)$ with probability one. Since $\frac{\partial\pi(H,t)}{\partial t} >$

$\frac{\partial \pi(L,t)}{\partial t}$, it follows that $\frac{\partial \pi(\mu,t)}{\partial t} \geq \frac{\partial \pi(L,t)}{\partial t}$ for all μ. But then, since $\pi(L,0) < \pi(\mu_0, 0) - 2\Delta$,

$$\pi(L, \tau - \Delta) < \pi(\mu_0, \tau - \Delta) - 2\Delta \leq \pi(\mu_\tau, \tau) - 2\Delta \tag{3.1}$$

where the second inequality arises from stochastic dominance and the fact that the $\pi(\cdot,\cdot)$ functions are increasing in their second argument. So, $r_{\tau-\Delta}(L) = \pi(\mu_\tau, \tau) - 2\Delta > \pi(L, \tau - \Delta)$ by (3.1). Since firms don't lose money in equilibrium, any equilibrium offer which is accepted by the low type must be accepted with the high type with positive probability. As before, profit maximization by the firms plus the zero-profit lemma implies that such an offer must attract both types. So, the only way a worker who reaches time $\tau - \Delta$ also reaches time τ is if the maximal offer made at $\tau - \Delta$ is one which is turned down by both types, giving $\mu_{\tau-\Delta} = \mu_\tau$. ■

The next lemma and corollary make precise the sense in which the high type never obtains more education than he would in a world with perfect information.

Lemma 3.2. $\pi(\mu_\tau, \tau) - \pi(\mu_\tau, \tau - \Delta) \geq \Delta$.

Proof. Assume not. Then, since by Lemma 3.1, $\mu_{\tau-\Delta} = \mu_\tau$, $\pi(\mu_{\tau-\Delta}, \tau - \Delta) > \pi(\mu_\tau, \tau) - \Delta = r_{\tau-\Delta}(H)$. By the definition of τ, the offers of both firms at time $\tau - \Delta$ must be below $r_{\tau-\Delta}(H)$ with positive probability. But then, since $r_{\tau-\Delta}(H) > r_{\tau-\Delta}(L)$, offers in the interval $(\pi(\mu_\tau, \tau) - \Delta, \pi(\mu_{\tau-\Delta}, \tau - \Delta))$ are maximal with positive probability, and are accepted by both types when maximal, violating the zero-profit lemma. ■

Remark 1. *Note that*

$$\pi(\mu_\tau, \tau) - \pi(\mu_{\tau-\Delta}, \tau - \Delta)$$
$$= \mu_\tau [\pi(H, \tau) - \pi(H, \tau - \Delta)] + (1 - \mu_\tau) [\pi(L, \tau) - \pi(L, \tau - \Delta)],$$

and so is the expected benefit of the last period of education, while Δ is the cost of education to the high type.

Corollary 3.3. *The high type worker never receives too much education. If the high and low type worker both reach time τ with positive probability, then the high type worker may receive too little education.*

Proof. Both assertions follow immediately from Lemma 3.2 and the fact that $\frac{\partial \pi(H,t)}{\partial t} \geq \frac{\partial \pi(\mu,t)}{\partial t}$ for all μ, with strict inequality if $\mu < 1$. ■

Remark 2. *The statement of the corollary is that the high type "may receive" too little education if the low type reaches time τ rather than "will receive". When $\mu_\tau < 1$,*

$$\pi(H, \tau) - \pi(H, \tau - \Delta) > \Delta. \tag{3.2}$$

However, to conclude that the high type is undereducated, we require

$$\pi(H, \tau + \Delta) - \pi(H, \tau) > \Delta. \tag{3.3}$$

Because $\Delta > 0$, (3.2) need not imply (3.3). However, the left hand sides of (3.2) and (3.3) converge in Δ. It is thus easily seen that there is a continuous $f(\Delta)$ with $f(0) = 1$ such that, if $\mu_\tau < f(\Delta)$, then the high type worker is indeed strictly undereducated.

Lemma 3.4. *The high type accepts offers only at τ.*

Proof. Choose $t \in \{0, \Delta, 2\Delta, \cdots, \tau - \Delta\}$. Arguing as before, at τ firms offer $\pi(\mu_\tau, \tau)$ with probability one, and so

$$r_t(H) \geq \pi(\mu_\tau, \tau) - (\tau - t).$$

By Lemma 3.2, $\pi(\mu_\tau, \tau - \Delta) \leq \pi(\mu_\tau, \tau) - \Delta$. But $\pi(\cdot, \cdot)$ is strictly concave in its second argument, and thus

$$\pi(\mu_\tau, t) < \pi(\mu_\tau, \tau) - (\tau - t) \leq r_t(H).$$

But, by stochastic dominance $\pi(\mu_t, t) \leq \pi(\mu_\tau, t)$. So, any offer acceptable to the high type before τ must lose money. ■

Define t_L as the most preferred level of education for the low type, given the grid $G(\Delta)$ and given he is paid his true productivity. That is,

$$t_L = \arg\max_{t \in G(\Delta)} \pi(L, t) - 2t.^{7}$$

Lemma 3.5. *The low type accepts offers only at t_L or at τ.*

Proof. Assume first that $t_L > \tau$. Choose $t \in \{0, \Delta, 2\Delta, \cdots, \tau - \Delta\}$. Since the worker can always wait until τ to accept,

$$r_t(L) \geq \pi(\mu_\tau, \tau) - 2(\tau - t) \geq \pi(L, \tau) - 2(\tau - t).$$

[7] Generically, t_L is singleton, but for specific Δ, t_L may be a set consisting of two successive elements of $G(\Delta)$. For simplicity, assume that Δ is chosen such that t_L is unique.

Since $\pi(L, \cdot)$ is strictly concave, and since $t_L > \tau$,

$$\pi(L, \tau) - 2(\tau - t) > \pi(L, t).$$

So, $r_t(L) > \pi(L, t)$. Since by Lemma 3.4, the high type turns down all equilibrium offers before τ, any offer that attracts the low type at t thus loses money.

Now consider the case $t_L < \tau$. Assume first that $r_{t_L}(L) < \pi(L, t_L)$. Then, by the zero-profit lemma, both firms offer at least $\pi(L, t_L)$ with probability one at time t_L. But then for each $t \in \{0, \Delta, 2\Delta, \cdots, t_L - \Delta\}$,

$$\begin{aligned} r_t(L) &\geq \max\{r_{t_L}(L), \pi(L, t_L)\} - 2(t_L - t) \\ &\geq \pi(L, t_L) - 2(t_L - t) \geq \pi(L, t) \end{aligned}$$

where the last inequality follows from the definition of t_L. As before, this plus Lemma 3.4 implies that in equilibrium no low type ever accepts an offer prior to t_L.

If the low type always accepts at t_L, then we are done. Assume the low type does not always accept at time t_L. Then at t_L, the worker must (weakly) prefer the continuation game to $\pi(L, t_L)$. But, in future periods other than τ, any equilibrium offer which is accepted with positive probability must be $\pi(L, t)$ since high types accept equilibrium offers only at τ. This is inferior to accepting $\pi(L, t_L)$ at t_L. So, the only future event with positive probability which could possibly make the low type better off is to accept the offer at τ. So, for the continuation to be preferred to accepting $\pi(L, t_L)$ at t_L it must be that the offer made at time τ is at least weakly preferred to $\pi(L, t_L)$. Since the offer is made with probability one, the low type who continues beyond t_L would thus never accept an offer giving him lower utility than $\pi(L, t_L)$ prior to τ. ■

So, the only possibilities for equilibria are equilibria in which complete separation occurs, equilibria in which the low type quits with positive probability at t_L and otherwise pools with the high type at τ, and equilibria in which both types pool at τ.

We next turn to the question of how little education can be received in equilibrium. Note first that it is possible to construct equilibria in which the worker receives no education whatever his type. Assume for example that $\pi(\mu_0, 0) \geq \pi(L, t) - t$ for all t. Then, it is an equilibrium for neither type of worker to get any education, supported by beliefs that a worker who chooses education at time zero is of the low type.

However, we argue that these beliefs are not reasonable: in particular, note that any combination of current offers by firms and beliefs of the worker about future firm actions for which makes it a best response for the low type worker to reject the current offer make rejection the unique best response for the high type. So, it seems unreasonable to update beliefs in the direction of the low type worker following such a rejection. (This is precisely the motivation behind the Banks and Sobel definition of Divinity [2]). In other words, the stochastic dominance which we have argued must hold along the equilibrium path should hold for beliefs off the equilibrium path as well.

Assumption. *(Belief restriction): μ_t is nondecreasing in t, whether on or off the equilibrium path.*

It can be shown that this (and no more) is precisely the implication of NWBR, so that Divinity and NWBR agree on this game. As discussed at the end of the last section, there are actions by the current firms (offering zero with small probability) which would induce both types to reject, and which are arbitrarily close to the equilibrium strategy. So, NWBR does not force $\mu_{\tau+\Delta} = 1$ after an unexpected deviation.

Lemma 3.6. *At the beginning of every period t, the high type has continuation utility at least $\pi(\mu_t, t)$. That is, the high type either has a reservation wage of $\pi(\mu_t, t)$, or receives an offer of $\pi(\mu_t, t)$ with probability one.*

Proof. If the high type's reservation wage is less than $\pi(\mu_t, t)$, then, if either firm is offering $\pi(\mu_t, t)$ with probability one, then an offer slightly below $\pi(\mu_t, t)$ by the other firm will be accepted with positive probability and earn positive profits, contradicting the zero-profit lemma. ■

Lemma 3.7. *Under the belief restriction,* $\pi(\mu_\tau, \tau + \Delta) - \pi(\mu_\tau, \tau) \leq \Delta$

Proof. Assume that $\pi(\mu_\tau, \tau + \Delta) - \pi(\mu_\tau, \tau) > \Delta$. Then by the belief restriction and Lemma 3.6, $r_\tau(H) \geq \pi(\mu_\tau, \tau + \Delta) - \Delta$. Since $\pi(\mu_\tau, \tau + \Delta) - \Delta > \pi(\mu_\tau, \tau)$, the worker will turn down an offer of $\pi(\mu_\tau, \tau)$ at time τ, contradicting the definition of τ. ■

Together, Lemmas 3.2 and 3.7 tell us that

$$\left.\frac{\partial \pi(\mu_\tau, t)}{\partial t}\right|_{t=\tau} \approx 1,$$

where the approximation becomes exact as Δ grows small. That is, education stops at approximately the point where the expected marginal benefit of education is equal to its cost to the high type worker.

This implies an interesting contrast between our and previous results. In the standard model, the low type worker receives the optimal level of education, while the high type worker gets excessive education so as to separate. In our setting, the high type worker, if anything, gets too little education, because his reward for extra education is tied to the *expected* value of extra education, rather than the value of education to them. The low types gets too much education in a desperate attempt not to be separated from the high type.

At some level, this seems more descriptive of real education than previous results. Most of us feel that our best students benefit greatly from our efforts, but doubt that our weakest students retain much more than their diploma after graduation.

By Lemmas 3.4 and 3.5, an equilibrium outcome can be completely specified by the last period reached with positive probability, and the probability of a high type worker conditional on reaching the last period. So, when we talk about an equilibrium outcome (μ, τ), for $\tau \in G(\Delta)$ and $\mu \geq \mu_0$, we mean the outcome in which at time τ both firms offer $\pi(\mu, \tau)$ and both types of worker accept, while at time t_L both firms offer $\pi(L, t_L)$ and the low type accepts with probability $\frac{\mu - \mu_0}{\mu(1-\mu_0)}$, so that the posterior probability of the high type is raised to μ.

We now provide a complete characterization of the set of sequential equilibrium outcomes. Using this characterization, we show that a sequential equilibrium exists for all parameter values.[8]

Let

$$t_\mu = \underset{t \in G(\Delta)}{\arg\max}\ \pi(\mu, t) - t.$$

That is, t_μ gives the high type's most preferred education level given that he is paid as if he is of the high type with probability μ. Note that $t_\mu \geq t_L$, and that t_μ is increasing as a correspondence.[9] So, t_μ is single-valued almost everywhere. Where t_μ is not a singleton, it consists of two consecutive elements of $G(\Delta)$. Let $\underline{U}_L = \pi(L, t_L) - 2t_L$.

Theorem 3.8. *Under the belief restriction,*

1. (μ_0, τ) *is a pooling equilibrium outcome if and only if*

   1. $\tau \in t_{\mu_0}$

   2. $\pi(\mu_0, \tau) - 2\tau \geq \underline{U}_L$

[8] Existence is at issue since the set of possible wage offers is infinite.

[9] That is, if $t \in t_\mu$, $t' \in t_{\mu'}$, and $\mu' > \mu$, then $t' \geq t$.

2. (μ, τ), $\mu \in (\mu_0, 1)$ *is a (partially) separating equilibrium outcome if and only if*

   1. $\tau \in t_\mu$
   2. $\pi(\mu, \tau) - 2\tau \geq \underline{U}_L$

3. $(1, \tau)$ *is a perfectly separating equilibrium outcome if and only if*

   1. $\tau \in t_1$
   2. $\pi(1, \tau) - 2\tau \geq \underline{U}_L$.

Proof. Let (μ, t) be one of the outcomes described above. (In case 1, $\mu = \mu_0$, and in case 3, $\mu = 1$.) To see that (μ, t) is consistent with an equilibrium that satisfies the belief restriction, consider the strategy profile in which both firms offer $\pi(L, t)$ for all $t < \tau$ and $\pi(\mu, t)$ for all $t \geq \tau$, supported by beliefs μ_0 for $t \leq t_L$, and μ for $t > t_L$. At periods other than t_L and τ, specify any behavior for the worker that is consistent with the reservation wage functions implied by the specified firm behavior. It is easy to verify that this is a sequential equilibrium. Note in particular that for all $t < \tau$, $r_t(H) \geq \pi(\mu_t, t)$ and so offering $\pi(L, t)$ is a best response. Lemmas 3.2, 3.4, 3.5, and 3.7, plus the obvious incentive constraints, imply that any equilibrium must be of this form. ■

Corollary 3.9. *There exists at least one sequential equilibrium.*

Proof. Take a selection θ_μ from the correspondence t_μ. Since t_μ is an increasing correspondence, θ_μ is an increasing function, and therefore continuous almost everywhere. Define the function $U(\cdot)$ by

$$U(\mu) = \pi(\mu, \theta_\mu) - 2\theta_\mu$$

for each $\mu \in [0, 1]$. If $U(\mu_0) \geq \underline{U}_L$, then (μ_0, θ_{μ_0}) is a pooling equilibrium outcome; if $U(1) \leq \underline{U}_L$, then $(1, \theta_1)$ is a separating equilibrium outcome.

Suppose that neither a pooling nor a separating equilibrium exists. Then, we know that $U(\mu_0) < \underline{U}_L$ and $U(1) > \underline{U}_L$. The function $U(\cdot)$ is continuous anywhere θ_μ is continuous. Since $\theta_\mu \in \arg\max_{t \in G(\Delta)} \pi(\mu, t) - t$, it follows that $\pi(\mu, \theta_\mu) - \theta_\mu$ is continuous in μ. Because θ_μ only jumps upward, it follows that any discontinuity in $U(\cdot)$ must be a downward jump. But, since $U(\mu_0) < \underline{U}_L$, $U(1) > \underline{U}_L$, and $U(\cdot)$ is continuous except for downward jumps, there exists $\mu \in (\mu_0, 1)$ such that $U(\mu) = \mu_0$, and so (μ, θ_μ) is a partially separating equilibrium outcome. ■

A troublesome property of the separating equilibria of the Spence game is that even a trivial probability of a low type worker causes overinvestment in education by the high type. In contrast, note that the utility received by the high type in our setting is at least $\pi(\mu_0, t_{\mu_0}) - t_{\mu_0}$, and that as $\mu_0 \to 1$ this converges to $\pi(H, t_H) - t_H$. The utility received by the low type similarly converges to $\max\{\pi(H, t_H) - 2t_H, \underline{U}_L\}$. It follows that when $\mu_0 \approx 1$, both payoffs and actions are close to those when $\mu_0 = 1$.

4. Education with Grades

The previous sections leave the implications of the Spence model in serious doubt. With preemptive offers and our information structure, the conclusions of the Spence model are essentially turned on their head. In this section, we turn to a model of education with grades. Here we find some support for the Spencian result of wasteful education by high ability workers.

As in the previous section, there are two types, H and L, with education of length t costing $2t$ to the low type and t to the high type. Here, however, education is completely unproductive. If the worker chooses education up to some specified time E he then takes an examination on which he gets an A or a B. For simplicity, we assume the grading standard is independent of the average ability of the student, so that an A is an absolute rather than a relative standard. Let α_H be the probability that a high type worker receives an A and let $\beta_H = 1 - \alpha_H$ be the probability that he receives a B, and similarly for α_L and β_L. We assume that $1 > \alpha_H > \alpha_L > 0$, so that a high quality worker is more likely to get an A than a low quality worker, but grades are not perfectly separating.

If the probability of a high type worker before grades are observed is μ, then the expected productivity conditional on an A is

$$\pi(A, \mu) \equiv P(H|A)\,\pi(H) + P(L|A)\,\pi(L), \tag{4.1}$$

where $P(H|A, \mu) = \frac{\mu\alpha_H}{\mu\alpha_H + (1-\mu)\alpha_L}$ is the probability of a high type conditional on an A, and analogously for $P(L|A)$, while productivity conditional on a B is

$$\pi(B, \mu) \equiv P(H|B)\,\pi(H) + P(L|B)\,\pi(L) \tag{4.2}$$

from which it is easily seen that $\pi(H) > \pi(A, \mu) > \pi(\mu) > \pi(B, \mu) > \pi(L)$ for $\mu \in (0, 1)$.

Using the zero-profit condition (which continues to hold in this setting), the expected wage to a high ability worker who takes the test is thus

$$w_H(\mu) = \alpha_H \pi(A, \mu) + \beta_H \pi(B, \mu).$$

while to the low ability worker it is

$$w_L(\mu) = \alpha_L \pi(A, \mu) + \beta_L \pi(B, \mu),$$

from which $\pi(H) > w_H(\mu) > \pi(\mu) > w_L(\mu) > \pi(L)$ for $\mu \in (0, 1)$.

Assume E is such that

$$w_H(\mu_0) - E > \pi(\mu_0) \text{ and } w_L(\mu_0) - 2E > \pi(L).$$

(Clearly for all $\mu_0 > 0$, there is a range of such E.) Then, it is a sequential equilibrium outcome for both types of the worker to choose to be educated to time E: because the high type is partially separated by the examination results, and E is relatively short, he is not willing to accept $\pi(\mu_0)$ at any time prior to E, and so there is no profitable opportunity to attract both types of the worker before E. On the other hand, because the low type is not completely separated by the examination, and E is relatively short, he is not willing to accept $\pi(L)$ at any time prior to E, and so there is no profitable opportunity to attract just the low type worker prior to E. Note further that this is the unique equilibrium satisfying the belief restriction. When E is longer, equilibria become substantially more complicated, involving intricate mixing. We will not subject the reader to a full characterization, but remark merely that for a wide variety of parameter values, it remains the case that there is wasteful education.

What are we to make of this? On the one hand, this clearly illustrates that there are settings in which the wasteful education result of Spence survives even with the possibility of preemptive offers. On the other hand, note that the equilibrium construction did not hinge in any significant way on the fact that education was more costly for the high type than for the low type. In fact, one could as well think about the test as being offered at time 0 with the price of admission to the exam not exceeding $\min\{w_H(\mu_0) - \pi(\mu_0), w_L(\mu_0) - \pi(L)\}$. So in a strong sense, this is a model not so much of wasteful education, but rather of a wasteful test where the particular form of waste is in the time spent in school before one is allowed to take the test.

Second, while a pure wage offer before E does not upset this equilibrium, this equilibrium remains susceptible to the same form of criticism as before: the spirit of the first section of the paper was that when there is wasteful education, the firm can create an offer which splits the gains from trade and so is profitable for both sides. So, assume that the firm can offer a test with the same discriminatory power as the exam offered by the school, and that the firm can prevent the worker from using his grade on this test with other firms. Then, wasteful education is once again upset.

Finally, this equilibrium does not seem particularly descriptive of real education. If, for example, the only reason business schools earn rents is that until education is completed, firms are unable to distinguish worker quality, then they would presumably not test as early or often as they do, and would work hard to keep early exam results confidential until late in the education process, rather than their current practice of making the revelation of such results as convenient as possible through placement offices, transcripts, etc.

5. The Informational Assumption

The analysis began with the Admati-Perry-Weiss observation that wasteful education seems to create an incentive for preemptive offers. However, when these offers are public, NVD show that firms do not make preemptive offers, because high ability workers turn down preemptive offers in the hope of signalling their ability to other firms.

We viewed this informational assumption as unrealistic, and wondered how sensitive the NVD result was to weakening this key assumption. As a first step, this paper shows that in a world where the informational link between periods is completely broken the results are opposite to the standard results. In the basic form of our model there is no wasteful education whatsoever, while in the analogous model with full information about previous offers, there is close to full separation.

However, while it seems quite plausible that a firm is able to keep its offers secret from other firms if it so wishes, it is less clear that a firm will never return to the worker if its initial offer is turned down.

In response to this, we note first that our assumption does not need to be as strong as we have made it. Think about a world in which the set of firms is constant over time, and firms can make offers in any period they wish. Then, even if most firms might make successive offers to the worker, all that is needed to preclude wasteful education is that in each period there is at least one firm which can commit to not making another offer if it is turned down today. The worker must then follow a reservation wage strategy with respect to that particular firm, which is enough to imply our results. Not making another offer might be credible if for example the firm has only one position to fill, and is also examining other candidates.

It is also easy to describe other modifications to our basic setting in which firms do not wish to return to a worker who has turned them down in the past. For example, assume that in addition to his productivity, the worker has a firm-specific match parameter known only to himself (for example, the worker knows if he likes Pittsburgh). If this parameter is unfavorable, then the worker will simultaneously be unhappy and less productive with that firm than with other firms. Then, turning down an offer can be interpreted as indicating an unfavorable match parameter rather than high ability.

Despite this defense, it would clearly be interesting to understand a model in which firms can return to the worker in several periods, and cannot commit not to do so. So, for example, consider a modification of our basic model (with unproductive education and no grades), in which the same two firms make offers at each t knowing their own past offers but not knowing the values of past offers made by the other firm. This turns out to be a very difficult model to analyze.

To see why, it is enough to consider the following reduced model: Firms 1 and 2 make offers at time 0. If the worker rejects both offers, he take education lasting Δ. At the end of Δ, the two firms again make offers, the worker decides on an employer, and the game ends. The first difficulty lies in the nature of interaction at time Δ: since a firm's inference about the quality of the worker at time Δ will in general depend on the firm's offer at time zero, the continuation game takes the form of a common value first price auction with asymmetric information. Consideration of the incentive constraints suggest that time zero offers will be drawn from a finite set of offers. The information structure in the second period thus involves atoms, a form of auction we know very little about. Further, to solve the whole game, one needs not only to solve this auction for all possible belief structures but also to solve an extremely ugly fixed point problem. To see this note that both the time 0 accept/reject rules for the worker and the optimal time zero offers for the firms depend on the solution to the continuation game, while the time Δ equilibrium is in turn a function of both the structure of time zero offers and the worker's accept/reject rules.

Several points of interest do arise from a partial analysis of the model in which the two same firms make offers, but do not know the value of the other firm's offers. First, it is a sequential equilibrium to pool at no education. This equilibrium even satisfies a limited form of forward induction: when a firm is turned down unexpectedly it can believe that the worker is of the high type with probability one. However, it can also believe that the other firm made a truly terrible offer – one that all types of worker would turn down – and thus that it is the only firm with this information. So, the firm can safely plan on bidding ε more than the population average productivity in succeeding periods. However, because these beliefs require disequilibrium play by both the worker and the other firm, the strong suspicion is that this equilibrium does not satisfy the full force of NWBR, and that under NWBR it would become common belief that the worker is of high ability when he turns down the time zero offer. So, the complete pooling at no education that we found does not survive equilibrium refinements in the modified setting.

On the other hand, an outcome of the type arising in NVD's framework is also not possible in this setting. In their equilibrium there is a sequence of periods during which both firms make offers unacceptable to either type, followed by a time at which both types are hired. Consider any period during which the worker is expected to turn down the offer he receives in equilibrium. Then, a firm which deviates and makes a better offer can say: "Look, if you turn me down you might convince me that you are of the high type. But the other firm was fully expecting you to turn me down, and so will learn nothing. Thus next time I'll be a monopolist on this information, and make you only a slightly better offer. Thus you might as well accept now."

So, neither pooling at zero nor essentially perfect separation seems to be an equilibrium. It would be interesting to know what the equilibria of this model are, and whether they approach one case or the other as Δ goes to 0.

6. Conclusion

Our results call into question the basic Spence result when firms are able to make confidential preemptive offers. On the whole, the addition of these offers seems to be efficiency enhancing, although there are cases where it is not.

The analysis of this paper serves as yet another example of the sensitivity of game theoretic predictions to the specification of the extensive form. The combination of this sensitivity with uncertainty on the modeler's part about the *right* extensive form is troubling. One response to this problem—and one we have followed to a limited extent here—is to examine the sensitivity of our results to changes in the game which agents in the game might be able to effect, such as the addition of the preemptive offers we use. A purist might argue that changing the game is meaningless in a fully specified extensive form game. We argue that models, by their very nature, must and should be abstractions from a very complicated universe. A reasonable test of whether one has made the right abstraction is to ask if the players in the game we write down will be content to play the game as written. Formalization and exploration of this idea seems both challenging and rewarding.

Our central proposition holds as long as the time interval between successive offers Δ is small relative to the mass of the highest types. Thus, the proof of this result relies quite heavily on the discrete nature of the formulation. This introduces the possibility that the no education result depends on this perhaps artificial feature of our model.[10] Preliminary exploration of a model with a continuous type space suggests that if the density function over types is either uniformly bounded away from zero or concave near the top of its support, then the probability that the worker chooses education goes to zero as Δ goes to zero. So the no-education results do not seem to be merely an artifact of the discrete type space.

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[10] Note that the argument that at some fundamental level everything is discrete, even if correct, is no defense of the discrete type model. Knowing that ability levels lie on a discrete grid is only helpful if one knows that the grid on which offers can be made is sufficiently fine relative to the grid on types.